Turbulent nonabelian matter in high energy nuclear collisions

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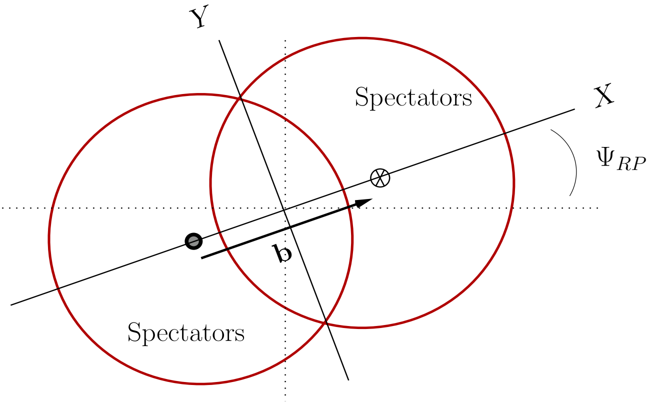
- Elliptic flow in heavy ion collisions
- Emergence of Kolmogorov spectrum in glasma
- Emergence of Kolmogorov spectrum in the toy CGC model
- Emergence of Kolmogorov spectrum in QGP
- Turbulent instability in QED plasma

Elliptic flow



Definition of the reaction plane

Elliptic flow

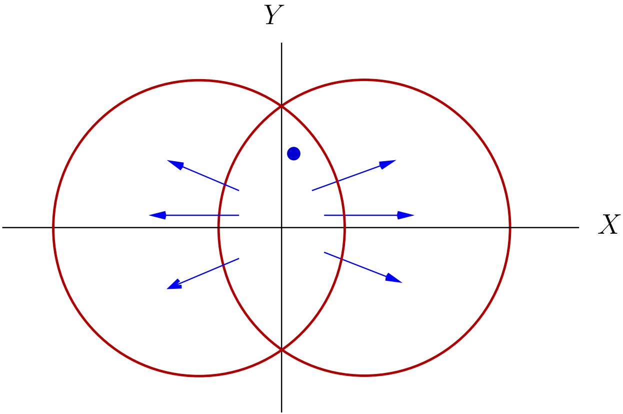
Spatial asymmetry of the reaction zone

- $$\epsilon_{\mathrm{s,part}} = \frac{\left\langle y^2 - x^2 \right\rangle}{\left\langle y^2 + x^2 \right\rangle}$$
- $$\left\langle y^2 - x^2 \right\rangle = \frac{1}{N_p} \int dxdy \, (y^2 - x^2) \frac{dN_p}{dx\, dy}$$

Momentum asymmetry: elliptic flow

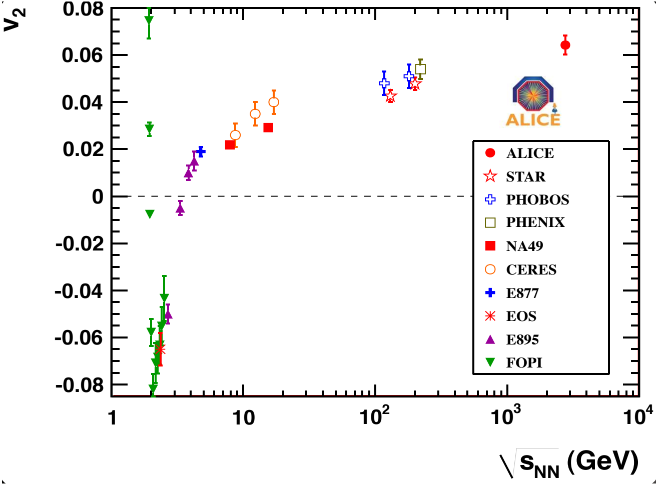
- $$v_2 \equiv \left\langle \frac{p_X^2 - p_Y^2}{p_X^2 + p_Y^2} \right\rangle$$
- $$\frac{1}{p_T} \frac{dN}{dydp_T d\phi} = \frac{1}{2\pi p_T} \frac{dN}{dydp_T} (1 + 2v_2(p_T) \cos 2(\phi - \Psi_{RP}) + \ldots)$$

Elliptic flow



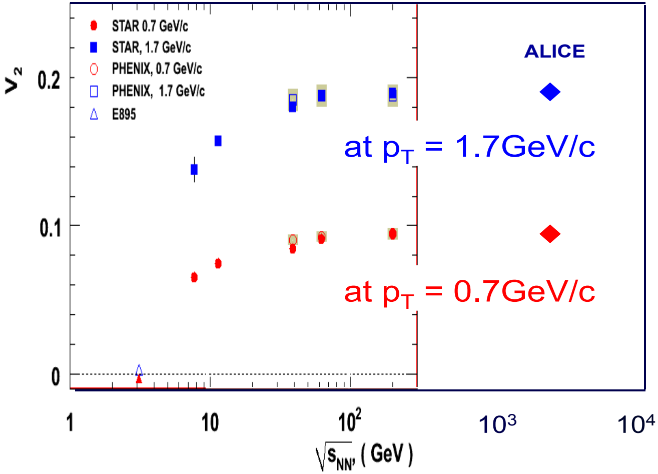
Hydrodynamic origin of the elliptic flow: anisotropic pressure converts spatial anisotropy is into momentum one

Elliptic flow



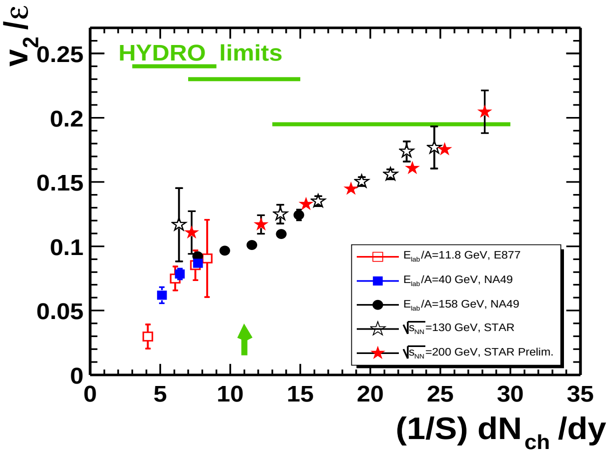
Average elliptic flow as a function of $\sqrt{s}$

Elliptic flow



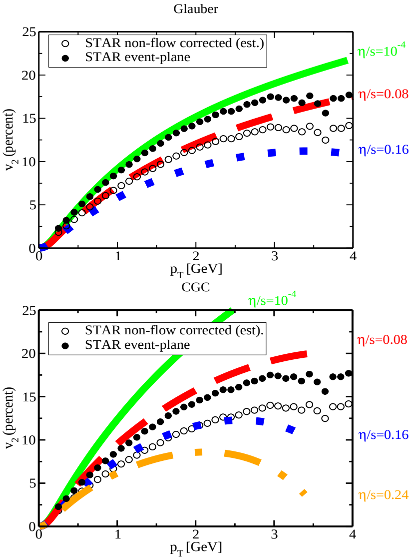
Differential elliptic flow as a function of $\sqrt{s}$

Elliptic flow

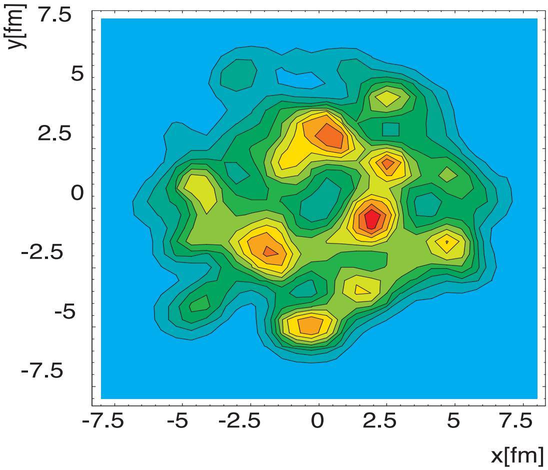


Hydro limit for ideal liquid for v_2 reached at RHIC

Elliptic flow

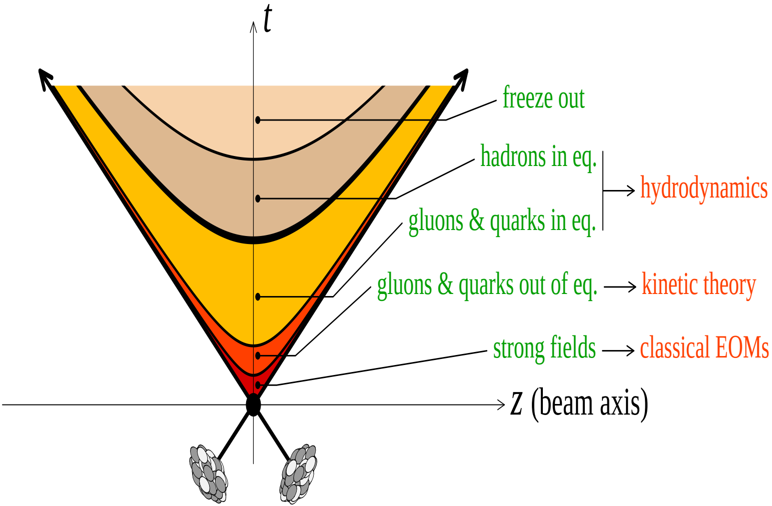


Initial conditions

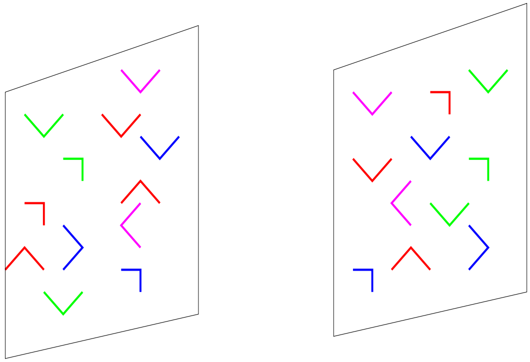


Initial transverse energy density for AuAu collisions at $\sqrt{s} = 200\ \mathrm{GeV}$

Little Bang: collision stages

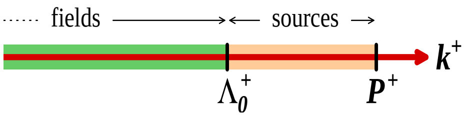


Little Bang: before the collision



Initial state at $t < 0$

Nuclei before collision: degrees of freedom



- Characteristic evolution time for parton modes

$$\Delta x^+ \sim \frac{1}{k^-} \sim \frac{2k^+}{\mathbf{k}_\perp^2} = \frac{2P^+}{\mathbf{k}_\perp^2} x$$

- Static modes (sources):

$$x \sim 1$$

- Fluctuational modes (fields):

$$x \ll 1$$

QCD physics at high energies is that of fields with $x \ll 1$

Nuclei before collision: fields

The fields A^a_μ and the source J^a_μ are related by the equation

$$[D_\mu, F^{\mu\nu}] = J^\nu \;\Leftrightarrow\; J^\mu = \delta^{\mu +}\rho_1(\mathbf{x}_\perp, x^-)$$

Solution of classical equations:

$$\begin{aligned} A^+ &= 0, \quad A^- = 0 \\ A^i &= \frac{i}{g} U(\mathbf{x}_\perp, x^-)\partial^i U^\dagger(\mathbf{x}_\perp, x^-) \end{aligned}$$

where

$$\begin{aligned} U(\mathbf{x}_\perp, x^-) &= P \exp\left\{ ig \int_{-\infty}^{x^-} dy^- \alpha(\mathbf{x}_\perp, x^-) \right\} \\ \alpha(\mathbf{x}_\perp, x^-) &= -\rho(\mathbf{x}_\perp, x^-)/\nabla^2_\perp \end{aligned}$$

Nuclei before collision: observable quantities

- Charge density $\rho(\mathbf{x}_\perp, x^-)$ is random. Event-by-event averaging with respect to $\rho(\mathbf{x}_\perp, x^-)$ is described by some functional $W_{\Lambda^+}[\rho]$
- For the simplest Gaussian ensemble

$$\langle \rho^a(\mathbf{x}_\perp, x^-)\rho^b(\mathbf{y}_\perp, y^-)\rangle = g^2\mu_A^2\delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)\delta(x^- - y^-))$$

- Structure function:

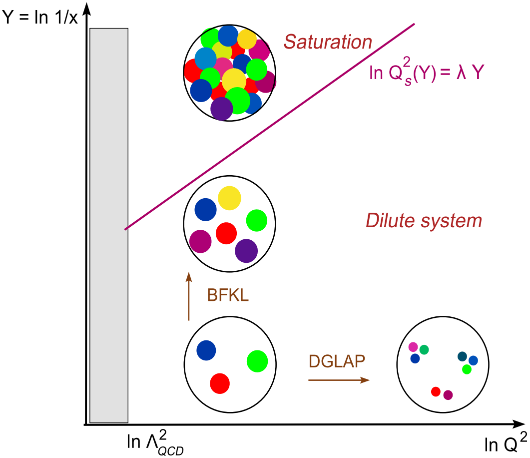
$$\frac{dN}{d^3k} = \frac{2k^+}{(2\pi)^3}\langle A_a^i(k, x^+))A_a^i(-k, x^+)))\rangle_{W_{\Lambda^+}}$$

$$\langle A_a^i(0))A_a^i(x))\rangle \sim \frac{1}{\mathbf{x}_\perp^2}\left(1 - \exp\left[-\mathbf{x}_\perp^2 Q_S^2 \ln(\mathbf{x}_\perp^2\mu^2)\right]\right)$$

- Q_S^2 - saturation scale,

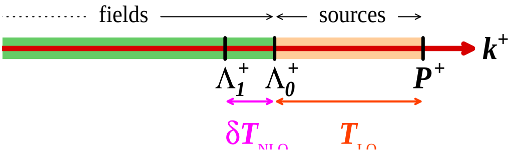
$$Q_S^2(Y) \simeq Q_0^2 \mathrm{e}^{\lambda_s Y}, \quad Q_0^2 \sim A^{1/3}$$

Nuclei before collision: quantum evolution



$$x = \frac{k^+}{P^+} \qquad \delta S_\perp \sim \frac{1}{Q^2}$$

Nuclei before collision: quantum evolution



- Structure function, classical approximation

$$< AA >= \int [d\rho]\, W_{\Lambda^+}[\rho]\, A_{\mathrm{cl.}}(\rho)\, A_{\mathrm{cl.}}(\rho)$$

- Arbitrary observable, classical approximation

$$\langle \mathcal{O} \rangle_Y = \int [d\alpha] \mathcal{O}[\alpha] W_Y[\alpha]$$

- Quantum evolution: JIMWLK equation:

$$\frac{\partial \langle \mathcal{O}[\alpha] \rangle_Y}{\partial Y} = \langle \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \alpha_Y^a(x_\perp)} \chi^{ab}_{x_\perp, y_\perp}[\alpha] \frac{\delta}{\delta \alpha_Y^b(y_\perp)} \mathcal{O}[\alpha] \rangle_Y$$

Nuclei before collision: quantum evolution

- The JIMWLK equation is Hamiltonian:

$$\frac{\partial \langle \mathcal{O}[\alpha] \rangle_Y}{\partial Y} = \langle \mathcal{H}_{\mathrm{JIMWLK}} \mathcal{O}[\alpha] \rangle_Y$$

- Kernels of JIMWLK equation:

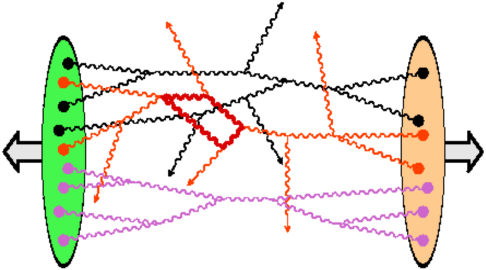
$$\chi^{ab}_{\mathbf{x}_\perp \mathbf{y}_\perp}[\alpha] = \int \frac{d^2\mathbf{z}_\perp}{4\pi^3} \frac{(\mathbf{x}_\perp - \mathbf{z}_\perp)(\mathbf{y}_\perp - \mathbf{z}_\perp)}{(\mathbf{x}_\perp - \mathbf{z}_\perp)^2 (\mathbf{y}_\perp - \mathbf{z}_\perp)^2} \left[\left(1 - U^\dagger_{\mathbf{x}_\perp} U_{\mathbf{z}_\perp}\right) \left(1 - U^\dagger_{\mathbf{z}_\perp} U_{\mathbf{y}_\perp}\right) \right]$$

- Nonlinear dependence on sources

$$U^\dagger(\mathbf{x}_\perp, x^-) = P \exp \left\{ ig \int_{-\infty}^{x^-} dy^- \alpha^a(\mathbf{x}_\perp, x^-) T^a \right\}$$

- In the limit of small α JIMWLK turns into BFKL

Nuclear collision: classical solution



$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\mu = \delta^{\mu +}\rho_1(\mathbf{x}_\perp, x^-) + \delta^{\mu +}\rho_2(\mathbf{x}_\perp, x^-)$$

Look for a solution in all orders in $\rho_{1,2}$

Boost-invariant classical solution

- Coordinates τ, η

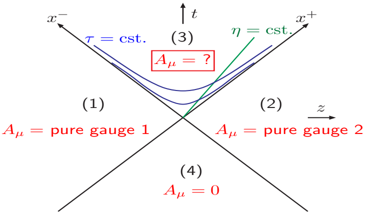
$$x^0 + x^3 = \tau e^{\eta}, \qquad x^0 - x^3 = \tau e^{-\eta}$$

- For a single source one uses gauges $A^{\pm} = 0$
- For the two-source problem it is convenient to use the mixed gauge $A^{\tau} = 0$

$$A_{\tau} = A^{\tau} \equiv \frac{1}{\tau}(x^{+}A^{-} + x^{-}A^{+})$$

- Boost-invariant solution does not depend on η

Boost invariant classical solution



- Look for the η - independent solution of the form:

$$A^i = \theta(-x^+)\theta(x^-)A^i_{(1)} + \theta(x^+)\theta(-x^-)A^i_{(2)} + \theta(x^+)\theta(x^-)A^i_{(3)}$$

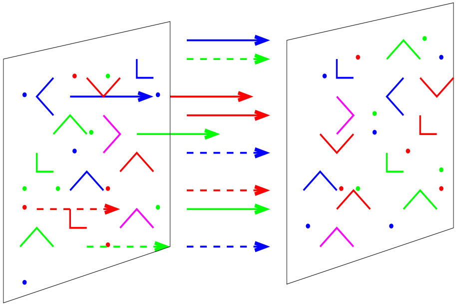
$$A^\eta = \theta(x^+)\theta(x^-)A^\eta_{(3)}$$

- Matching conditions at $\tau = 0$:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}\left[A^i_{(1)}, A^i_{(2)}\right]$$

Immediately after collision there form longitudinal chromoelectric and chromomagnetic fields - glasma :



$$E^z = ig\left[A^i_{(1)}, A^i_{(2)}\right]$$
$$B^z = ig\epsilon^{ij}\left[A^i_{(1)}, A^j_{(2)}\right]$$

Initial conditions: hydrodynamics?

- Equations of motion

$$\partial_\mu T^{\mu\nu} = 0$$

- Equation of state

$$p = f(\epsilon)$$

- Initial conditions set at some $\tau = \tau_0$

$$T^{\mu\nu}(\tau = \tau_0, \eta, \mathbf{x}_\perp)$$

- Generic structure of $T^{\mu\nu}$:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & \frac{\epsilon}{3} & & \\ & & \frac{\epsilon}{3} & \\ & & & \frac{\epsilon}{3} \end{pmatrix}$$

Initial conditions, Color Glass Condensate

For a configuration $\mathbf{E}^a_\mu = \lambda \mathbf{B}^a_\mu$

$$\langle T^{\mu\nu}(\tau = 0^+, \eta, \mathbf{x}_\perp) \rangle = \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & -\epsilon \end{pmatrix}$$

Does not look as hydro at all but is very similar to QCD string models (negative p_z !)

Glasma flux tubes	strings
Negative p_z	string tension
Glasma instabilities	string breaking

Isotropisation mechanism?

Boost-invariant solution: numerics. Lattice formulation

K. Fukushima, F. Gelis, Nucl. Phys. A874 (2012), 108

- Link variable: $U_\mu(x) \equiv \mathrm{e}^{-\mathrm{i}gaA_\mu(x)}$
- Plaquette variable:

$$U_{\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x) \approx \exp\left[-\mathrm{i}ga^2 F_{\mu\nu}(x)\right]$$

- Canonical momenta

$$\partial_\tau U_i(x) = \frac{-\mathrm{i}g}{\tau} E^i(x)U_i(x) \,, \qquad \partial_\tau U_\eta(x) = -\mathrm{i}ga_\eta \tau \, E^\eta(x)U_\eta(x) \,.$$

- Equations of motion ($\tau' = \tau + \Delta\tau$):

$$\begin{aligned} E^i(\tau') &= E^i(\tau - \Delta\tau) + 2\Delta\tau \frac{\mathrm{i}}{2ga_\eta^2\tau}\Big[U_{\eta i}(x) + U_{-\eta i}(x) - (\mathrm{h.c.})\Big]_\tau \\ &+ 2\Delta\tau \frac{\mathrm{i}\tau}{2g} \sum_{j\neq i}\Big[U_{ji}(x) + U_{-ji}(x) - (\mathrm{h.c.})\Big]_\tau \\ E^\eta(\tau') &= E^\eta(\tau - \Delta\tau) + 2\Delta\tau \frac{\mathrm{i}}{2ga_\eta\tau} \sum_{j=x,y}\Big[U_{j\eta}(x) + U_{-j\eta}(x) - (\mathrm{h.c.})\Big]_\tau \end{aligned}$$

Boost-invariant solution: numerics. Initial conditions

K. Fukushima, F. Gelis, Nucl. Phys. A874 (2012), 108

- Basic building blocks - single source solutions ($m = 1, 2$):

$$
\begin{aligned}
U_i^{(m)}(\mathbf{x}_\perp) &= V^{(m)}(\mathbf{x}_\perp) V^{(m)\dagger}(\mathbf{x}_\perp + \Delta x_i) \\
V^{(m)\dagger}(\mathbf{x}_\perp) &= \exp\left[\mathrm{i} g \Lambda^{(m)}(\mathbf{x}_\perp)\right]; \qquad \partial_\perp^2 \Lambda^{(m)}(\mathbf{x}_\perp) = -\rho^{(m)}(\mathbf{x}_\perp)
\end{aligned}
$$

- Averaging over initial configurations

$$
\langle \rho^{(n)}(\mathbf{x}_\perp) \rho^{(m)}(\mathbf{x}_\perp{}') \rangle = \delta^{nm} g^2 \mu^2 \delta(\mathbf{x}_\perp - \mathbf{x}_\perp{}')
$$

- Initial conditions:

$$
\begin{aligned}
U_i &= \left(U_i^{(1)} + U_i^{(2)}\right)\left(U_i^{(1)\dagger} + U_i^{(2)\dagger}\right)^{-1} \\
E^\eta &= \frac{-\mathrm{i}}{4g} \sum_{i=x,y} \Big[\left(U_i - 1\right)\left(U_i^{(2)\dagger} - U_i^{(1)\dagger}\right) \\
&+ \left(U_i^\dagger(x - \Delta x_i) - 1\right)\left(U_i^{(2)\dagger}(x - \Delta x_i) - U_i^{(1)\dagger}(x - \Delta x_i)\right) - (\mathrm{h.c.}) \Big]
\end{aligned}
$$

Glasma. Observables

- Gauge invariant observables (continuum)

$$\begin{aligned} \varepsilon &= \langle T^{\tau\tau} \rangle = \langle \mathrm{tr}\left[E_L^2 + B_L^2 + E_T^2 + B_T^2 \right] \rangle \\ P_T &= \frac{1}{2} \langle T^{xx} + T^{yy} \rangle = \langle \mathrm{tr}\left[E_L^2 + B_L^2 \right] \rangle \\ P_L &= \langle \tau^2 T^{\eta\eta} \rangle = \langle \mathrm{tr}\left[E_T^2 + B_T^2 - E_L^2 - B_L^2 \right] \rangle \end{aligned}$$

- Here

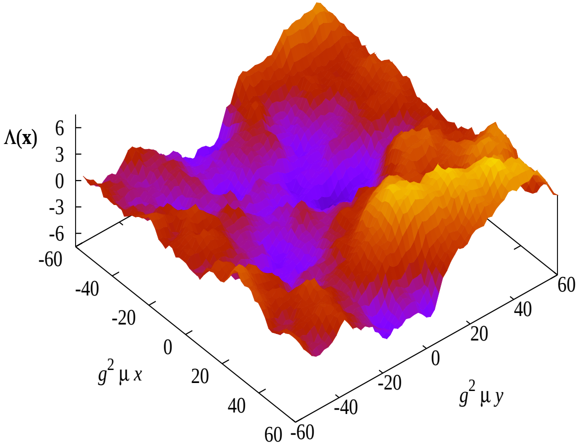
$$E_L^2 \equiv E^{\eta a} E^{\eta a} \, , \qquad E_T^2 \equiv \frac{1}{\tau^2} \left(E^{xa} E^{xa} + E^{ya} E^{ya} \right)$$

- and

$$B_L^2 = \frac{2}{g^2} \mathrm{tr}\left(1 - U_{xy} \right) \, , \qquad B_T^2 = \frac{2}{(g a_\eta \tau)^2} \sum_{i=x,y} \mathrm{tr}\left(1 - U_{\eta i} \right)$$

Boost-invariant solution: numerics. Initial conditions

K. Fukushima, F. Gelis, Nucl. Phys. A874 (2012), 108



Boost-invariant solution: numerics. Initial conditions

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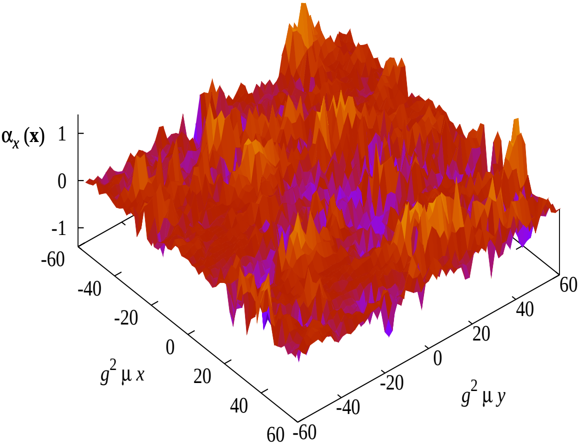
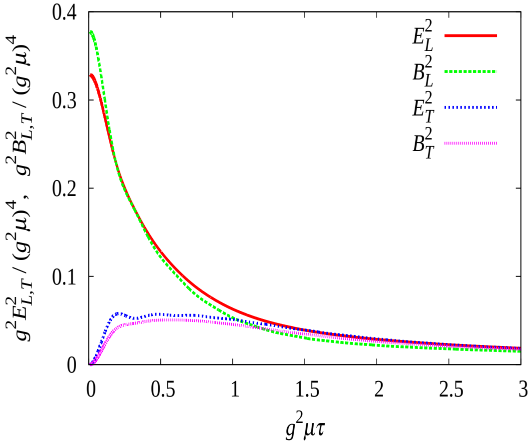


Boost-invariant solution: numerics. Time evolution

K. Fukushima, F. Gelis, Nucl. Phys. A874 (2012), 108

- Time evolution of chromoelectric and chromomagnetic fields ($g^2\mu = 2$ GeV):



Boost-invariant solution: numerics. Time evolution

- For thermally equilibrated system one expects $P_L = \epsilon/3$, $\varepsilon \propto \tau^{-4/3}$ and $P_T = P_L$
- The energy density is actually decreasing as $1/\tau$ (free streaming)
- The quantities E_L^2, B_L^2, E_T^2, and B_T^2 are all approaching a common value for $g^2\mu\tau > 1$, so that the system remains anisotropic (isotropy requires $E_T^2 = 2E_L^2$ and $B_T^2 = 2B_L^2$)!

Boost-invariant solution: numerics. Time evolution

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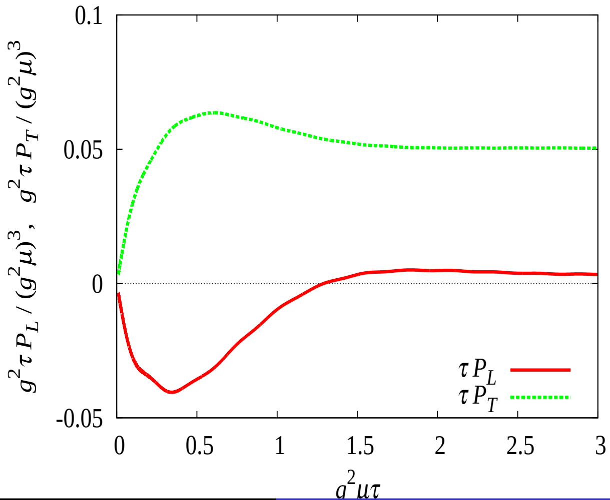
- Pressure



Boost-invariant solution. Spectral decomposition

- Transverse spectral decomposition of energy

$$\varepsilon_E = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\, \varepsilon_E(k_\perp), \qquad \varepsilon_B = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\, \varepsilon_B(k_\perp)$$

- Energy density in the mode $k_\perp$ is given by

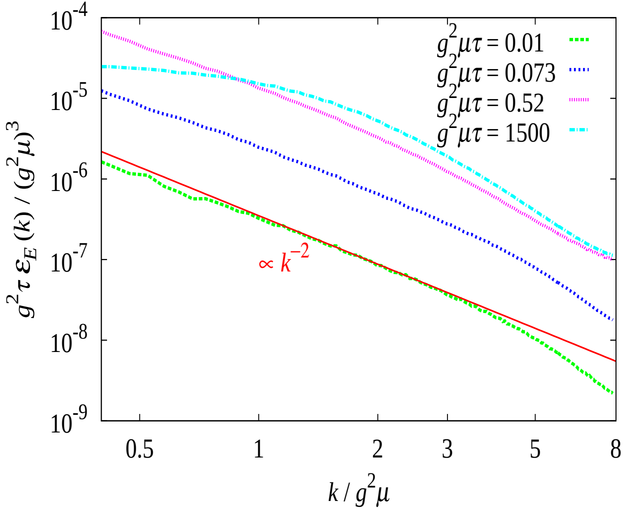
$$\begin{aligned} \varepsilon_E(k_\perp) &\equiv \left\langle \mathrm{tr}\left[E^{\eta a}(-\mathbf{k}_\perp)E^{\eta a}(\mathbf{k}_\perp) + \tau^{-2}\left(E^{ia}(-\mathbf{k}_\perp)E^{ia}(\mathbf{k}_\perp)\right)\right]\right\rangle, \\ \varepsilon_B(k_\perp) &\equiv \left\langle \mathrm{tr}\left[B^{\eta a}(-\mathbf{k}_\perp)B^{\eta a}(\mathbf{k}_\perp) + \tau^{-2}\left(B^{ia}(-\mathbf{k}_\perp)B^{ia}(\mathbf{k}_\perp)\right)\right]\right\rangle \end{aligned}$$

- Initial energy density

$$\varepsilon = \frac{3(g^2\mu)^4}{2\pi g^2} \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2} \frac{1}{k_\perp\sqrt{k_\perp^2 + 4m^2}} \ln\left[\frac{\sqrt{k_\perp^2 + 4m^2} + k_\perp}{\sqrt{k_\perp^2 + 4m^2} - k_\perp}\right]$$

Boost-invariant solution. Spectral decomposition

- Spectral energy density



Glasma instability

- Rapidity dependent fluctuations

$$\begin{aligned}\delta E^i(x) &= a_\eta^{-1}\left[f(\eta - a_\eta) - f(\eta)\right]\xi^i(\mathbf{x}_\perp) \\ \delta E^\eta(x) &= -f(\eta)\sum_{i=x,y}\left[U_i^\dagger(x-\hat{i})\xi^i(\mathbf{x}_\perp - \hat{i})U_i(x-\hat{i}) - \xi^i(\mathbf{x}_\perp)\right]\end{aligned}$$

- The functions $\xi^i(\mathbf{x}_\perp)$ are random:

$$\langle \xi^i(\mathbf{x}_\perp)\xi^j(\mathbf{x}_\perp{}')\rangle = \delta^{ij}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{x}_\perp{}')$$

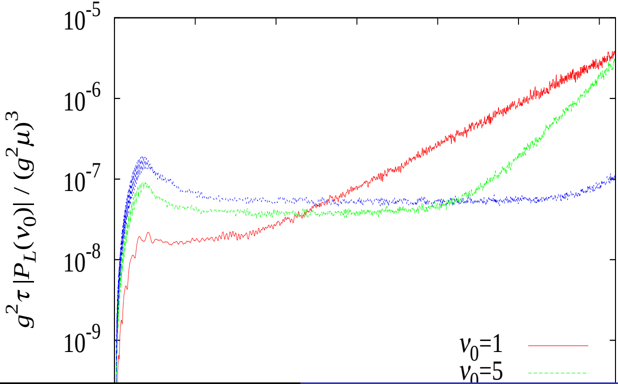
- Perturbation with a mode with fixed longitudinal wave number:

$$f(\eta) = \Delta\cos\left(\frac{2\pi\nu_0}{L_\eta}\eta\right)$$

K. Fukushima, F. Gelis, Nucl. Phys. A874 (2012), 108

- Let us study the evolution of $|g^2\tau P_L(\nu=\nu_0)/(g^2\mu)^3|$, where

$$P_L(\nu) \equiv \frac{1}{L^2}\int d^2\mathbf{x}_\perp \frac{1}{L_\eta}\int_0^{L_\eta} d\eta\, P_L(\eta,\mathbf{x}_\perp)\, e^{i(2\pi\nu/L_\eta)\eta}$$



Glasma instability: Kolmogorov spectrum

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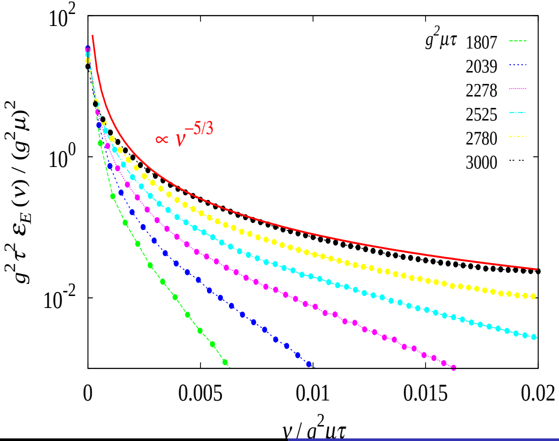
- At late stages of evolution glasma develops Kolmogorov mode spectrum



Scalar field model: tree level

T. Epelbaum et al., Nucl. Phys. **A872** (2011), 210

- Evolution of scalar field generated by strong time-dependent source:
- Lagrangian:

$$\mathcal{L} \equiv \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi \qquad J \sim \theta(-x^0)\frac{Q^3}{g}$$

- Tree-level energy-momentum tensor

$$\begin{aligned} T^{\mu\nu}_{\mathrm{LO}}(x) &= \partial^\mu \varphi \partial^\nu \varphi - g^{\mu\nu}\left[\frac{1}{2}(\partial_\alpha \varphi)^2 - \frac{g^2}{4!}\varphi^4\right], \\ \square \varphi + \frac{g^2}{3!}\varphi^3 &= J, \qquad \lim_{x^0 \to -\infty} \varphi(x^0, \mathbf{x}) = 0 \end{aligned}$$

Scalar field model: resummation of secular divergences

- One-loop quantum corrections bring in exponentially growing instabilities due to the parametric resonance.
- Resummation of these secular divergencies:

$$T^{\mu\nu}_{\rm resum}(x) \equiv \exp\left[\int d^3\mathbf{u}\, \beta\cdot\mathbb{T}_\mathbf{u} + \frac{1}{2}\int d^3\mathbf{u}d^3\mathbf{v}\int\frac{d^3\mathbf{k}}{(2\pi)^3 2k}[a_{+\mathbf{k}}\cdot\mathbb{T}_\mathbf{u}][a_{-\mathbf{k}}\cdot\mathbb{T}_\mathbf{v}]\right] T^{\mu\nu}_{\rm LO}(x)$$

- $\mathbb{T}_\mathbf{u}$ is the generator of shifts of the classical field on the $x^0 = 0$ hypersurface:

$$a\cdot\mathbb{T}_\mathbf{u} \equiv a(0,\mathbf{u})\frac{\delta}{\delta\varphi_0(\mathbf{u})} + \dot{a}(0,\mathbf{u})\frac{\delta}{\delta\dot{\varphi}_0(\mathbf{u})} \quad\Rightarrow\quad a(x) = \int d^3\mathbf{u}\,[a\cdot\mathbb{T}_\mathbf{u}]\,\varphi(x)$$

Scalar field model: resummation of secular divergences

- The fields $a_{\pm\mathbf{k}}$ are small perturbations propagating on top of the classical field φ, that are plane waves at $x^0 \to -\infty$, and β is the 1-loop correction to φ,

$$\Big[\square + V''(\varphi)\Big] a_{\pm\mathbf{k}} = 0 \; , \qquad \lim_{x^0 \to -\infty} a_{\pm\mathbf{k}}(x) = e^{\pm ik\cdot x} \; ,$$

$$\Big[\square + V''(\varphi)\Big] \beta = -\frac{1}{2} V'''(\varphi) \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \; a_{-\mathbf{k}} a_{+\mathbf{k}},$$

$$\lim_{x^0 \to -\infty} \beta(x) = 0$$

Scalar field model: resummation of secular divergences

- Resummation of these secular instabilities is equivalent to averaging over an ensemble of initial field configurations

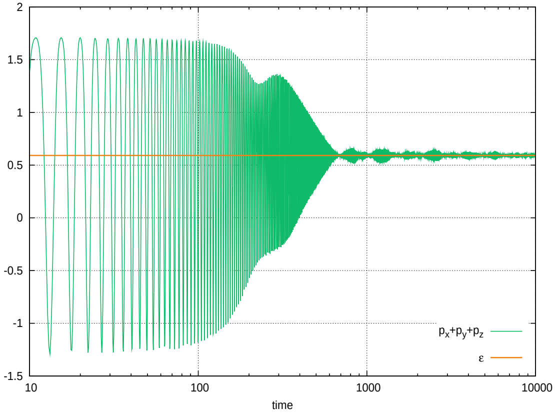
$$T^{\mu\nu}_{\rm resum} = \int [D\alpha(\mathbf{x}) D\dot{\alpha}(\mathbf{x})] \, F[\alpha, \dot{\alpha}] \; T^{\mu\nu}_{\rm LO}[\varphi_0 + \beta + \alpha]$$

- The distribution $F[\alpha, \dot{\alpha}]$ is Gaussian in $\alpha(\mathbf{x})$ and $\dot{\alpha}(\mathbf{x})$:

$$\begin{aligned} \langle \alpha(\mathbf{x}) \alpha(\mathbf{y}) \rangle &= \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \, a_{+\mathbf{k}}(0, \mathbf{x}) a_{-\mathbf{k}}(0, \mathbf{y}) \; , \\ \langle \dot{\alpha}(\mathbf{x}) \dot{\alpha}(\mathbf{y}) \rangle &= \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \, \dot{a}_{+\mathbf{k}}(0, \mathbf{x}) \dot{a}_{-\mathbf{k}}(0, \mathbf{y}) \end{aligned}$$

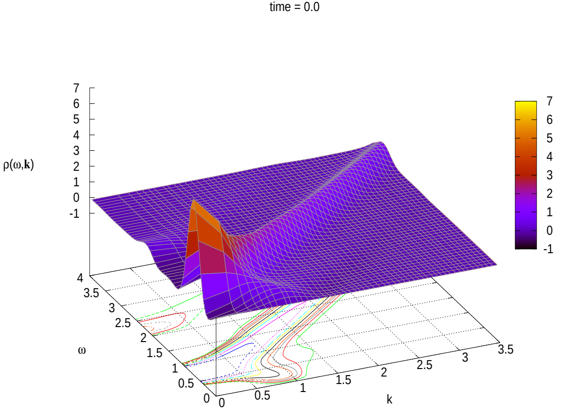
Scalar field model: relaxation of pressure

- Resummed pressure leads to one-to-one relation between pressure and energy (EOS):



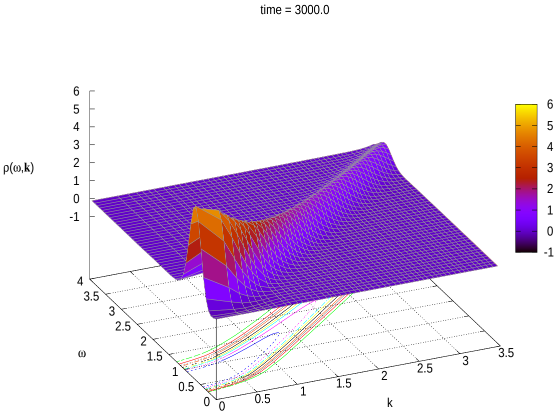
Scalar field model: spectral function

- Initial spectral density - not reducible to quasiparticles



Scalar field model: spectral function

- Emergence of quasiparticles



Scalar field model: Kolmogorov spectrum

- Occupation numbers:

$$f_{\mathbf{k}} = -\frac{1}{2} + \frac{1}{2\omega_{\mathbf{k}} V} \int \left[D\alpha D\dot{\alpha} \right] F[\alpha, \dot{\alpha}] \left| \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \left(\dot{\varphi}(x^0, \mathbf{x}) + i\omega_{\mathbf{k}} \varphi(x^0, \mathbf{x}) \right) \right|^2_{\varphi_0 + \alpha}$$

- Bose-Einstein

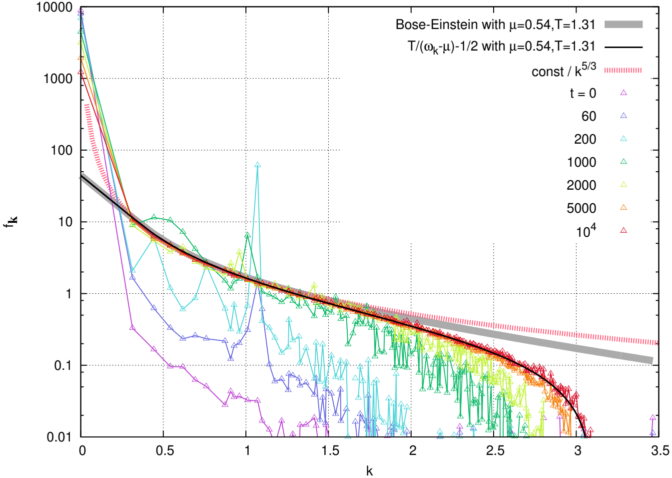
$$f_{\mathrm{BE}}(k) = \frac{1}{e^{\beta(\omega_{\mathbf{k}} - \mu)} - 1}$$

- Classical distribution

$$f_{\mathrm{class}}(k) = \frac{T}{\omega_{\mathbf{k}} - \mu} - \frac{1}{2}.$$

Scalar field model: Kolmogorov spectrum

- Emergence of Kolmogorov spectrum



Scalar field model: combined picture