Study of properties of two color QCD at nonzero baryon density by means of lattice simulation

A.Yu. Kotov

in collaboration with V.G. Bornyakov, V.V. Braguta, E.-M. Ilgenfritz, I.E. Kudrov, A.V. Molochkov, A.A. Nikolaev, R.N. Rogalyov, A. Rothkopf

arXiv:1605.04090, Phys.Rev. D94 (2016) no.11, 114510
arXiv:1711.01869, sent to JHEP

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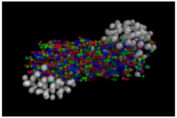
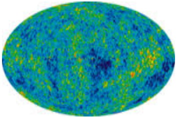
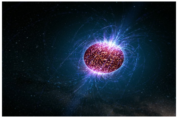
What happens if we compress matter as much as possible?

Introduction

- Baryon density $n_B - n_{\bar{B}}$
- Excess of baryons over antibaryons
- Excess of quarks over antiquarks
- Applications
  - Heavy ion collisions
  - Neutron stars
  - Early Universe

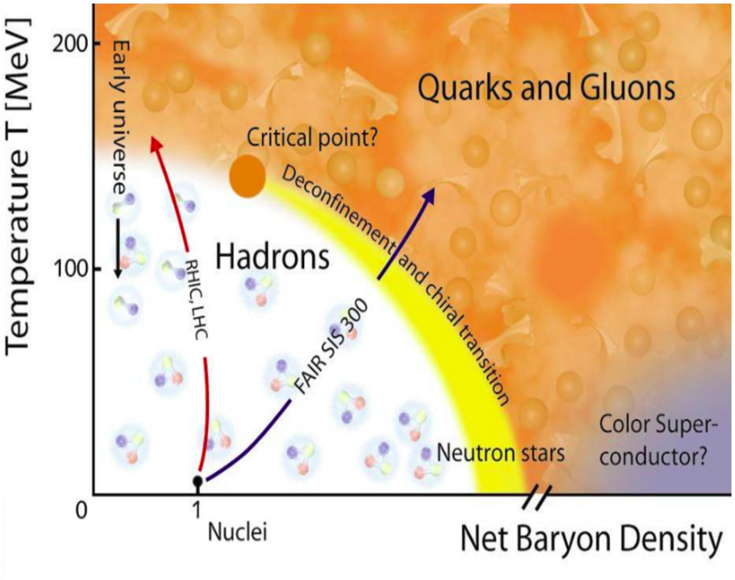




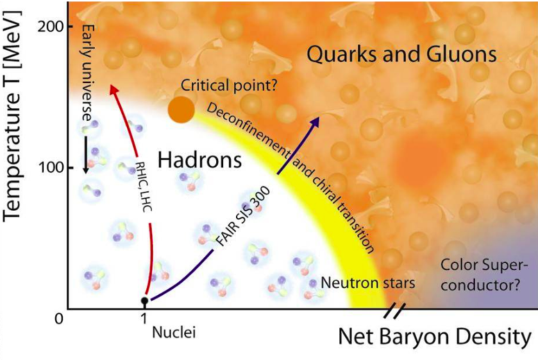


- Baryon chemical potential $\mu_B = N_c \mu_q$

QCD phase diagram



Phenomena(nonperturbative) at large baryon density



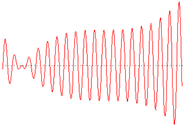
Predicted by phenomenological models, e.g.

- Color-Flavour Locking
- Nonuniform phases
- Chiral symmetry restoration
- Deconfinement

μ and Lattice QCD

SU(3) QCD

- $Z = \int DUD\bar{\psi}D\psi \; \exp\left(-S_G - \int d^4x\bar{\psi}(\hat{D}+m)\psi\right) =$
  $= \int DU \exp\left(-S_G\right) \times \det\left(\hat{D}+m\right)$
- Eigenvalues go in pairs $\hat{D}$: $\pm i\lambda \Rightarrow$
  $\det\left(\hat{D}+m\right) = \prod_\lambda(\lambda^2+m^2) > 0$
  i.e. one can use lattice simulation



- Introduce chemical potential: $\det\left(\hat{D}+m\right) \to \det\left(\hat{D}-\mu\gamma_4+m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

SU(2) QCD

- $(\gamma_5 C\tau_2)\cdot D^* = D\cdot(\gamma_5 C\tau_2)$
- Eigenvalues go in pairs $\hat{D}-\mu\gamma_4$: λ, λ^*
- For even N_f $\det\left(\hat{D}-\mu\gamma_4+m\right) > 0$ $\Rightarrow$ **free from sign problem**

Differences between SU(3) and SU(2) QCD

- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons $(N_f = 2)$ $\pi^+, \pi^-, \pi^0, d, \bar{d}$

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However, in dense medium:

- **Chiral symmetry is restored**
  symmetry breaking pattern is not important
- **Relevant degrees of freedom are quarks and gluons**
  rather than goldstone bosons

$SU(2)$ & $SU(3)$ QCD have many common properties

- There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- A lot of observables are very close:

Topological susceptibility (Nucl.Phys.B715(2005)461):
$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40)\ (SU(2)),\quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35)\ (SU(3))$

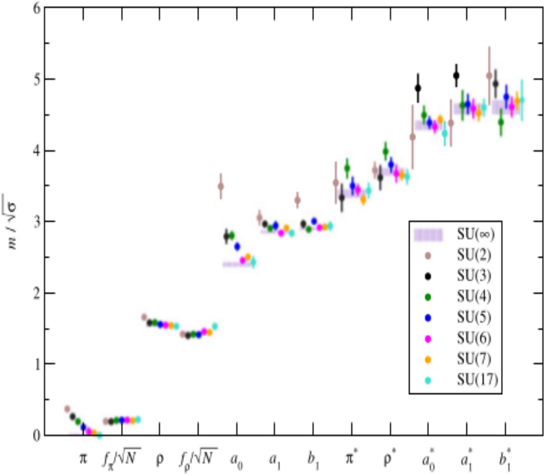
Critical temperature (Phys.Lett.B712(2012)279):
$T_c/\sqrt{\sigma} = 0.7092(36)\ (SU(2)),\quad T_c/\sqrt{\sigma} = 0.6462(30)\ (SU(3))$

Shear viscosity :
$\eta/s = 0.134(57)\ (SU(2)),\quad \eta/s = 0.102(56)\ (SU(3))$

JHEP 1509(2015)082 Phys.Rev. D76(2007)101701

$SU(2)$ & $SU(3)$ QCD have many common properties

- Spectroscopy (Phys.Rep.529(2013)93)

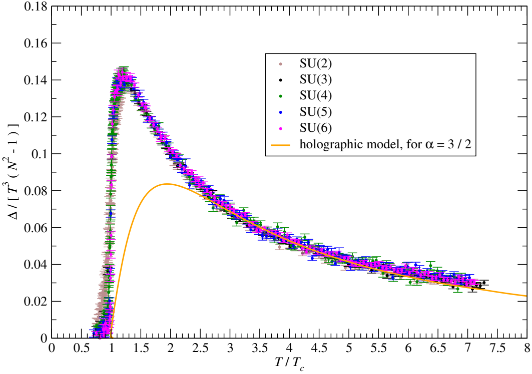


$SU(2)$ & $SU(3)$ QCD have many common properties

- Thermodynamic properties (JHEP 1205(2012)135)
- Some properties of dense medium (Phys.Rev.D59(1999)094019):

$$\Delta \sim \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

Trace of the energy-momentum tensor



$SU(2)$ & $SU(3)$ QCD

- Dense SU(2) QCD can be used to study dense SU(3) QCD
  - Calculation of different observables
  - Study of different physical phenomena
- Lattice study of SU(2) QCD contains full dynamics of real system (contrary to phenomenological models)

SU(2) & *SU*(3) QCD

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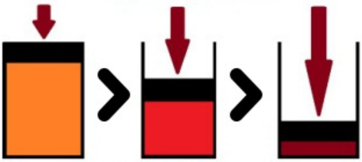
The aim: **numerical study of dense SU(2) QCD within lattice simulation**



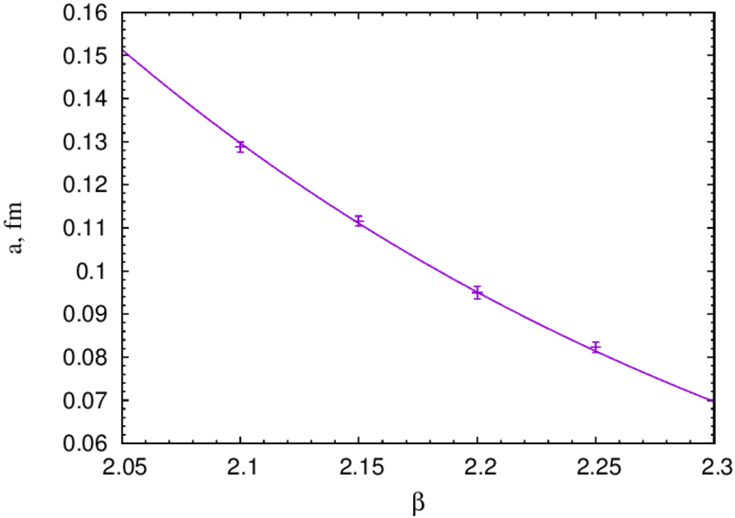
Parameters. Part I.

Details of the simulation:

- Staggered fermions with rooting: $N_f = 2$
- Wilson gauge action
- Lattice $16^3 \times 32$, $a = 0.11$ fm, $m_\pi = 362(4)$ MeV,
- Diquark source in the action $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$

- The symmetry breaking is different
  - Continuum: $SU(2N_f) \to Sp(2N_f)$
  - Staggered fermions: $SU(2N_f) \to O(2N_f)$
- Correct symmetry is restored in continuum limit
  - Naive limit $a \to 0$: two copies of $N_f = 2$ fundamental fermions
  - Correct β function for $a < 0.17$ fm
- See arXiv:1701.04664 by L. Holicki, J. Wilhelm, D. Smith, B. Wellegehausen, L. von Smekal for detailed discussion

β-function



Small chemical potential
$\mu < 350$ MeVV

Chiral Perturbation Theory in $SU(2)$

$$\mathcal{L} = \bar{\psi}\gamma_\nu D_\nu \psi = \mathrm{i}\begin{pmatrix}\psi_L^* \\ \psi_R^*\end{pmatrix}^T \begin{pmatrix}\sigma_\nu D_\nu & 0 \\ 0 & -\sigma_\nu^\dagger D_\nu\end{pmatrix}\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}$$

$$\mathcal{L} = \mathrm{i}\begin{pmatrix}\psi_L^* \\ \tilde{\psi}_R^*\end{pmatrix}^T \begin{pmatrix}\sigma_\nu D_\nu & 0 \\ 0 & \sigma_\nu D_\nu\end{pmatrix}\begin{pmatrix}\psi_L \\ \tilde{\psi}_R\end{pmatrix} = \mathrm{i}\Psi^\dagger \sigma_\nu D_\nu \Psi,$$

$$\Psi \equiv \begin{pmatrix}\psi_L \\ \sigma_2\tau_2\psi_R^*\end{pmatrix} \equiv \begin{pmatrix}\psi_L \\ \tilde{\psi}_R\end{pmatrix}$$

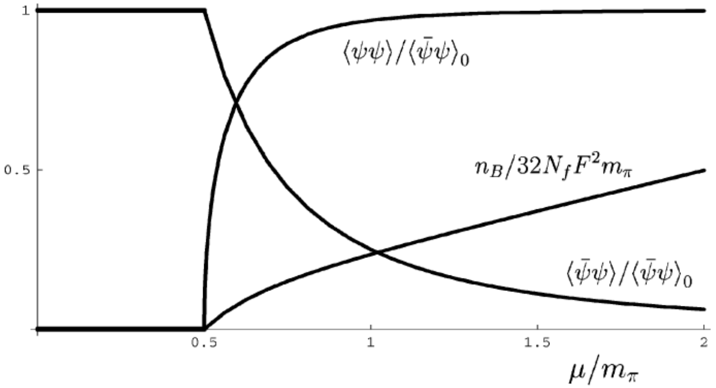
Symmetry of the Lagrangian: $SU(2N_f)$
contrary to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
Goldstone bosons ($N_f = 2$) $\pi^+, \pi^-, \pi^0, d, \bar{d}$

Chiral Perturbation Theory in $SU(2)$

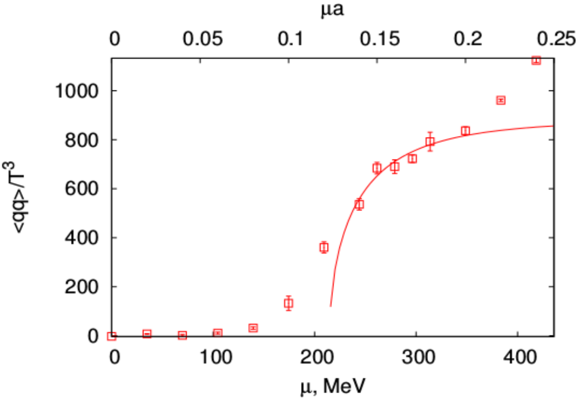
Low-energy theory for (pseudo-)Goldstone modes
(constructed from symmetry breaking pattern):

$$L_{eff} = \frac{F^2}{2}\,\mathrm{tr}\,\partial_\nu\Sigma\partial_\nu\Sigma^\dagger + 2\mu F^2\,\mathrm{tr}\,B\Sigma^\dagger\partial_0\Sigma -$$
$$-F^2\mu^2\,\mathrm{tr}(\Sigma B^T\Sigma^\dagger B + BB) - F^2 m_\pi^2\,\mathrm{re}\,\mathrm{tr}\,\hat{M}\Sigma$$
$$B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Predictions of ChPT

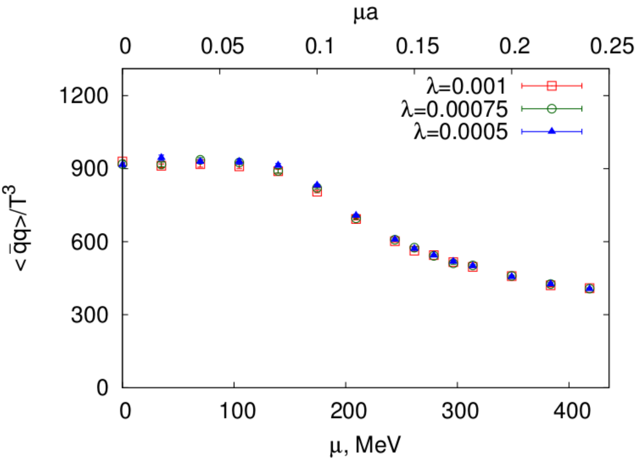


Diquark condensate



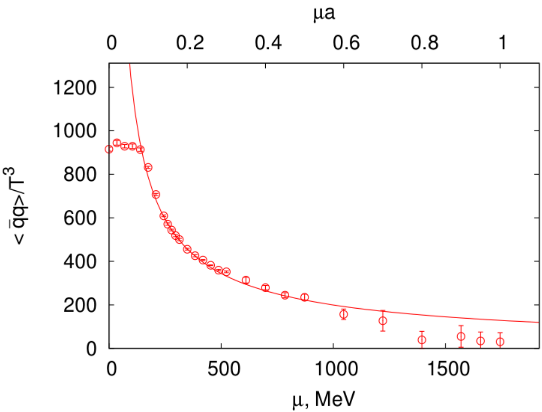
- Good agreement with ChPT $\langle\psi\psi\rangle/\langle\bar{\psi}\psi\rangle_0 = \sqrt{1 - \frac{\mu_c^4}{\mu^4}}$
- Phase transition at $\mu_c \sim m_\pi/2$
- Bose Einstein condensate (BEC) phase $\mu \in (200, 350)$ MeV

Chiral condensate



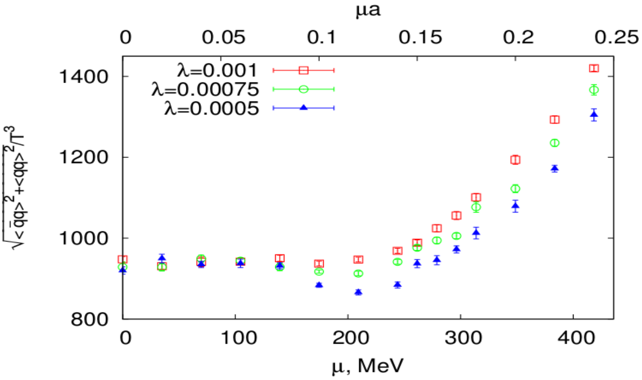
Good agreement with ChPT

Chiral condensate



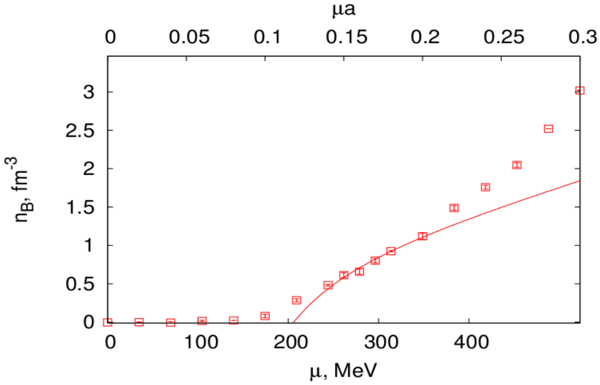
- ChPT prediction $\langle\bar{\psi}\psi\rangle \sim \frac{m_\pi^2}{\mu^2}$
- We observe $\langle\bar{\psi}\psi\rangle \sim \frac{1}{\mu^\alpha}, \alpha \sim 0.6 - 1.0$

Circle relation



Circle relation: $\langle\bar{\psi}\psi\rangle^2 + \langle\psi\psi\rangle^2 = \mathit{const}$

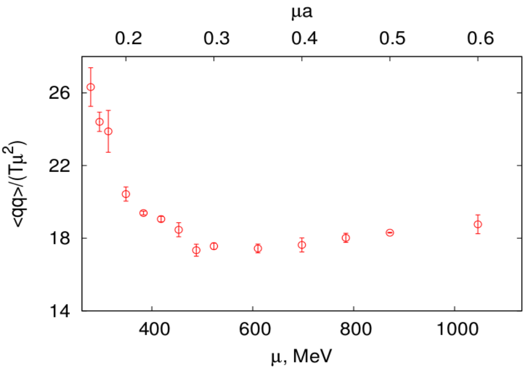
Baryon density



- Good agreement with ChPT $n \sim \mu - \frac{\mu_c^4}{\mu^3}$
- Phase transition at $\mu_c \sim m_\pi/2$
- Deviations from ChPT prediction start from $n \sim 1\ \mathrm{fm}^{-3}$

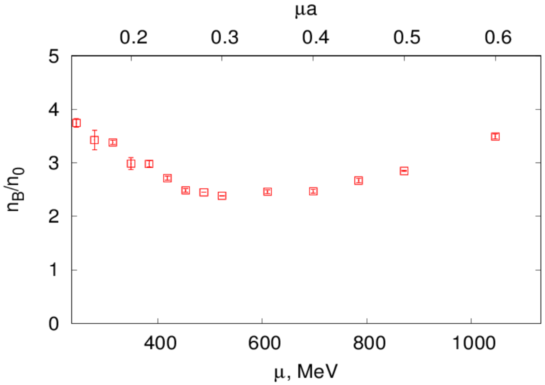
Large chemical potential
$\mu > 350$ MeV

Diquark condensate



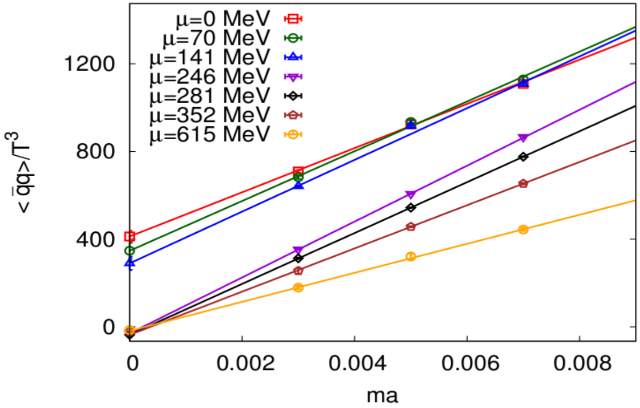
- Bardeen Cooper Schrieffer (BCS) phase $\mu > 500$ MeV, $\langle \psi\psi \rangle \sim \mu^2$

Baryon density



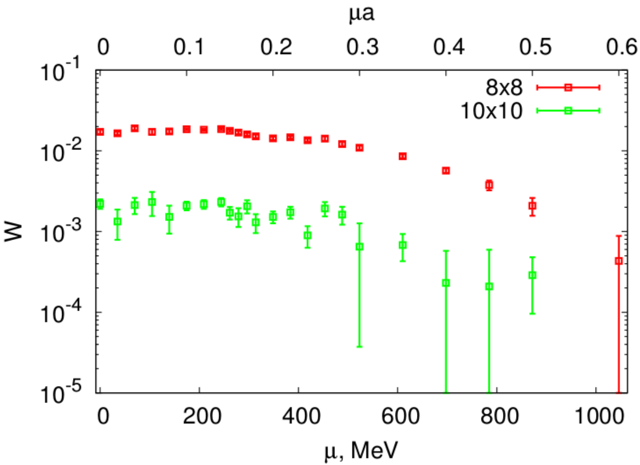
- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3} \theta(|p| - \mu) = \frac{4}{3\pi^2}\mu^3$
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$

Chiral condensate (chiral limit $m \to 0$)



Chiral symmetry is restored

Wilson loop



Polyakov loop is zero. The system seems to be in the confinement phase

Conclusion. Part I

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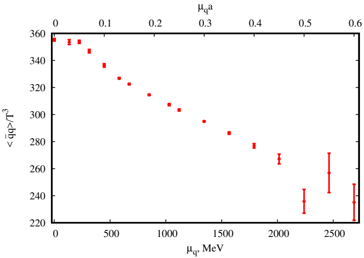
- We observe $\mu < m_\pi/2$ hadronic phase
- Transition to superfluid phase $\mu \simeq m_\pi/2$ (BEC)
- $\mu > m_\pi/2, \mu < m_\pi/2 + 150$ MeV dilute baryon gas
- Hadronic phase and BEC phase are well described by ChPT
- Deviation from ChPT from $\mu > 350$ MeV (dense matter)
- BCS phase $\mu \sim 500$ MeV, transition BEC$\rightarrow$BCS is smooth
- Always confinement ????

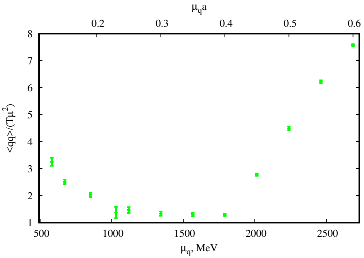
Parameters. Part II.

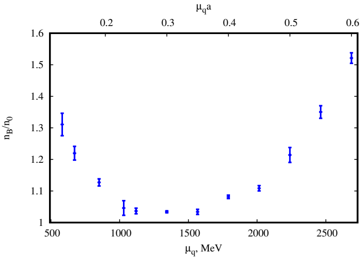
Details of the simulation:

- Staggered fermions
- Tree-level improved gauge action
- $a = 0.044$ fm
  $\Rightarrow$ **close to continuum limit**
  one can reach larger density without lattice artifacts
  $\mu > 2000$ **MeV**
- $m_\pi = 740(40)$ MeV
- Lattice: $32^3 \times 32$

It looks the same...

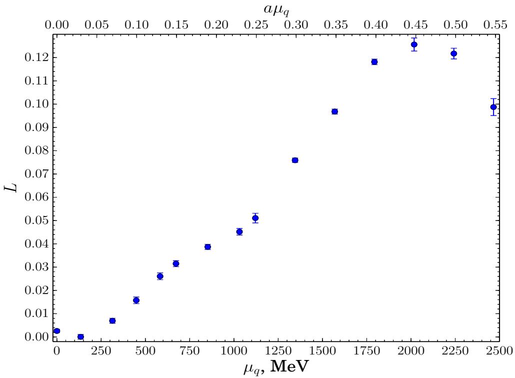


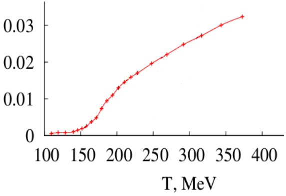




BUT

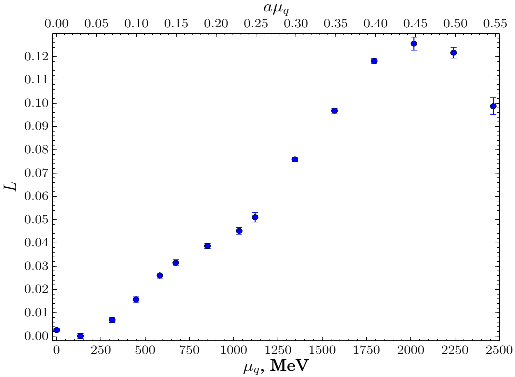
Polyakov loop





Compare with L at $\mu = 0$ vs T

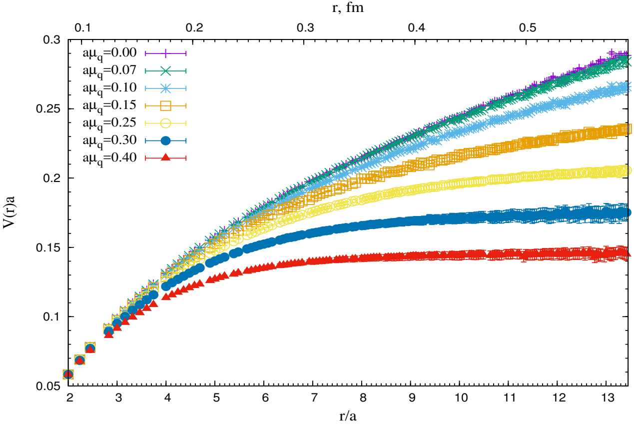
Polyakov loop



Rich physics?

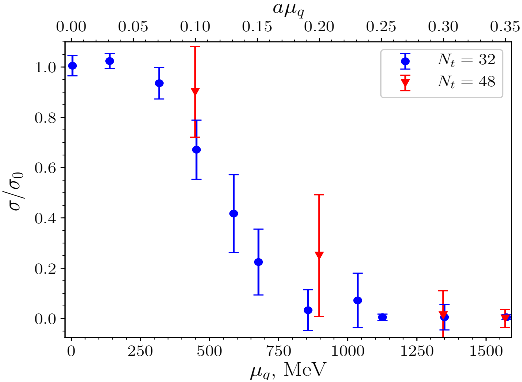
- Critical chemical potential $\mu \simeq$1100-1300 MeV ($a\mu \sim 0.25 - 0.3$)

Potential between static quark-antiquark pair



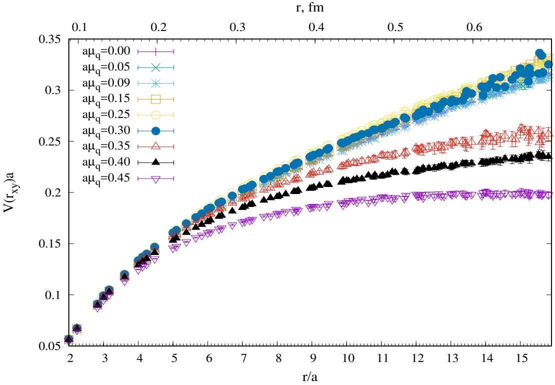
We observe deconfinement in dense medium!

String tension

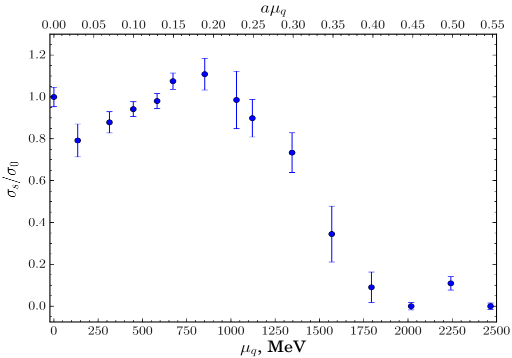


- Good fit by the Cornell potential: $V(r) = A + \frac{B}{r} + \sigma r$
  $\mu \leq 1100$ MeV
- Good fit by the Debye potential: $V(r) = A + \frac{B}{r} e^{-m_D r}$
  $\mu \geq 1300$ MeV

Spatial potential $V(r)$

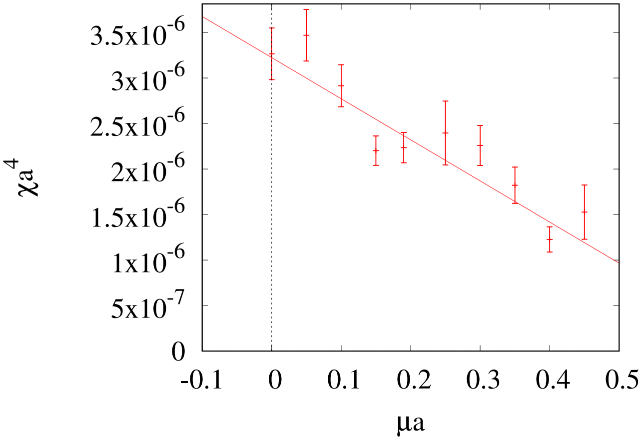


Spatial string tension



- Deconfinement at $\mu > 1100 - 1300$ MeV ($a\mu > 0.25 - 0.3$)?
- Spatial string tension disappears at $\mu \geq 2000$ MeV ($a\mu > 0.45$)
- Behaviour differs from finite-T transition

Topological susceptibility (preliminary!)



- Signatures of $U_A(1)$ restoration
- Further investigation is required (meson masses)

Conclusion. Part II

- **Deconfinement in dense medium**
- Difficult to determine critical chemical potential $\mu \sim 1100 - 1300$ MeV
- Spatial string tension disappears $\mu \geq 2000$ MeV
- Deconfinement at large density is different from the finite temperature deconfinement
- Quark-gluon plasma at large density is perturbative (gas of quarks and gluons)