Study of the influence of external effects on the properties of QCD by means of lattice simulations

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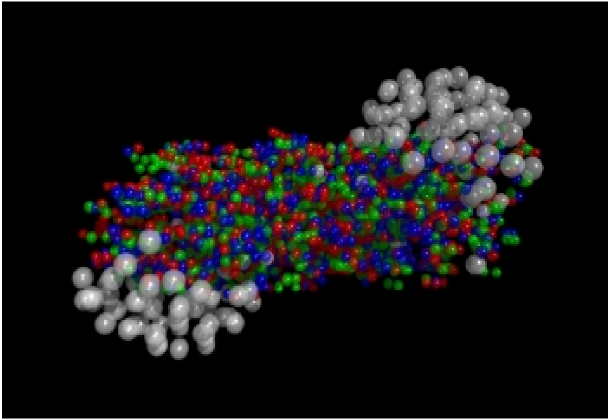
(based on the PhD thesis)

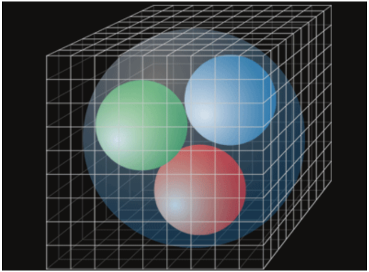
JINR

20 April 2016

Motivation



- Temperature
- Baryonic density
- Chiral density
- Magnetic field



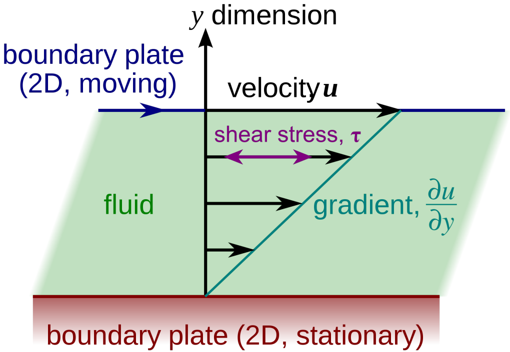
Lattice simulations in QCD

- Allow to study strongly coupled systems
- Based on the first principles of QFT
- Acknowledged approach to QCD
- Very powerful method due to the development of computer systems

Discussed problems

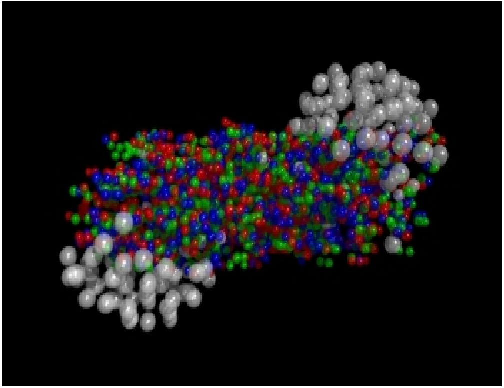
- Viscosity of Quark-Gluon Plasma
- Two-Color QCD with nonzero baryon density
- QCD with nonzero chiral density
- Superconductivity of QCD vacuum in superstrong magnetic fields

Viscosity of Quark-Gluon Plasma



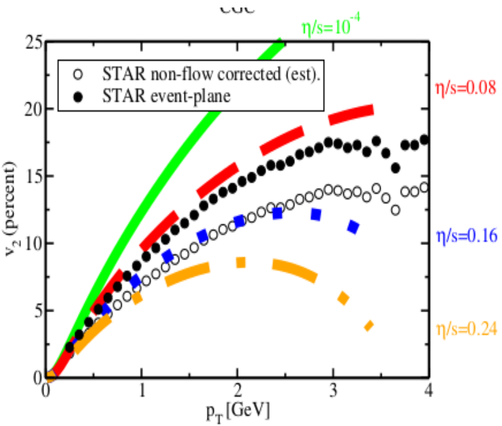
Viscosity

- $F_x = -\eta \cdot \frac{du}{dy} \cdot S$, η–viscosity
- Viscosity is connected with T_{xy}



Hydrodynamical description

- One heavy ion collision produces a huge number of final particles
- Large number of particles $\Rightarrow$ hydrodynamical description can be used
- In hydrodynamics transport coefficients control flow of energy, momentum, electrical charge and other quantities

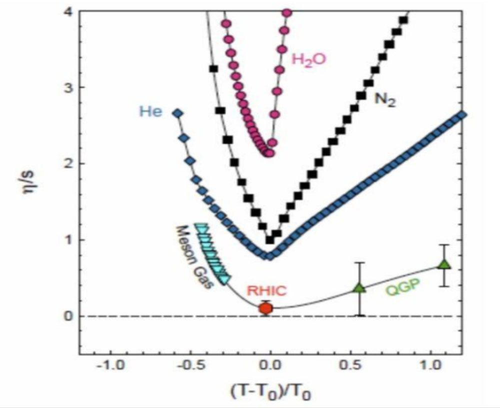


Elliptic flow at STAR (Nucl. Phys. A 757, 102 (2005))

$\frac{dN}{d\phi} \sim (1 + 2v_1 cos(\phi) + 2v_2 cos^2(\phi))$, ϕ-scattering angle

QGP is close to ideal liquid $(\frac{\eta}{s} = (1-3)\frac{1}{4\pi})$

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)



Comparison of different liquids, arXiv:nucl-ex/0609025

QGP is the most superfluid liquid

The aim: first principle calculation of transport coefficients

Previous lattice calculations (SU(3) gluodynamics)

- Karsch, F. et al. Phys.Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev. D76 (2007) 101701

H. B. Meyer, Phys.Rev. D76 (2007) 101701

$$\frac{\eta}{s} = 0.134 \pm 0.033 \; (T/T_c = 1.65)$$

$$\frac{\eta}{s} = 0.102 \pm 0.056 \; (T/T_c = 1.24)$$

Green-Kubo formula

$$\langle T_{12} T_{12} \rangle_E(\tau) = \int_0^\infty \rho(\omega) \frac{\cosh \omega(\frac{1}{2T} - \tau)}{\sinh \frac{\omega}{2T}} d\omega$$

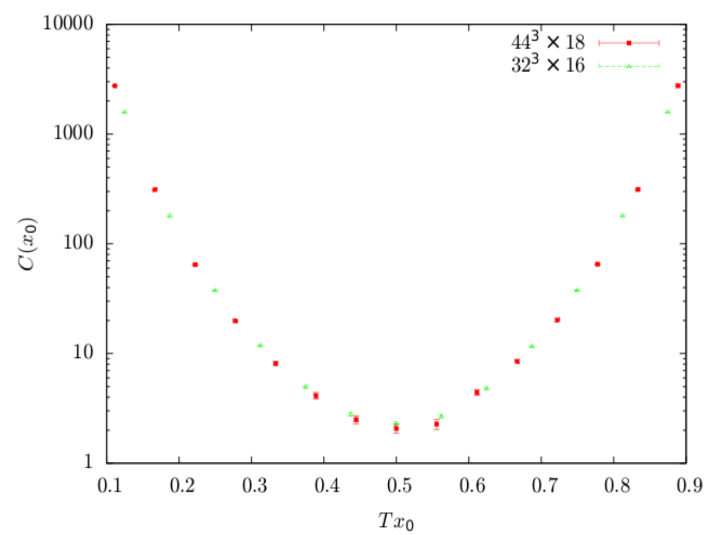
$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Lattice calculation of transport coefficients

- Lattice measurement of the correlator $C(t) = \langle T_{12}(t) T_{12}(0) \rangle$
- Calculation of the spectral function $\rho(\omega)$ from
  $C(t) = T^5 \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$
  Hydrodynamical approximation $\rho(\omega)|_{\omega \to 0} \sim \frac{\eta}{\pi} \omega$
- Viscosity $\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$

SU(2) gluodynamics, $T/T_c \simeq 1.2$

Correlation function



Calculation of the spectral function

$$C(t) = T^5 \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

Properties:

- $\rho(\omega) \geq 0, \; \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|^{NLO}_{\omega\to\infty} = \frac{1}{10}\frac{d_A}{(4\pi)^2}\omega^4\left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  7/8 of the whole correlator at $t = \frac{1}{2T}$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{\eta}{\pi}\omega$

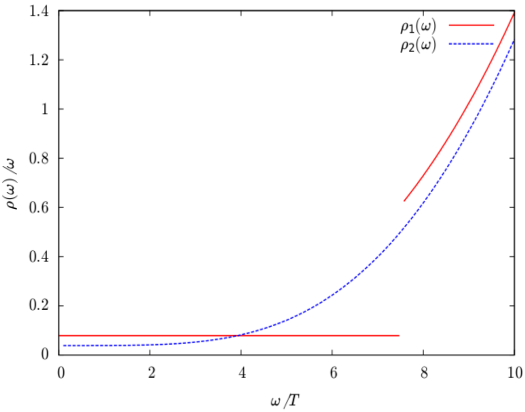
Ansatz for the spectal function

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + \theta(\omega - \omega_0)A\rho_{asym}(\omega)$$

Ansatz for the spectal function

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + \theta(\omega - \omega_0)A\rho_{asym}(\omega)$$

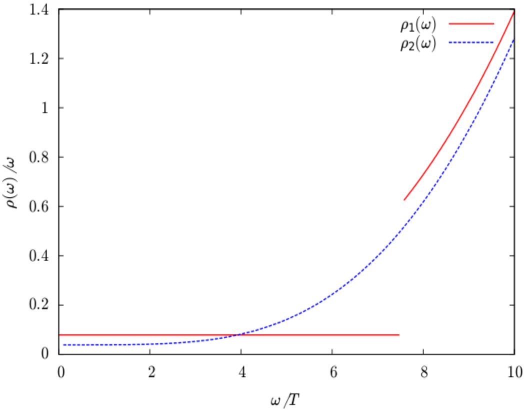
- $\chi^2/dof \sim 1, \; A = 0.723 \pm 0.004, \; \omega_0 = 2.7\text{GeV}$
- $\frac{\eta}{s} = 0.18 \pm 0.04$



Other variants of the spectral function

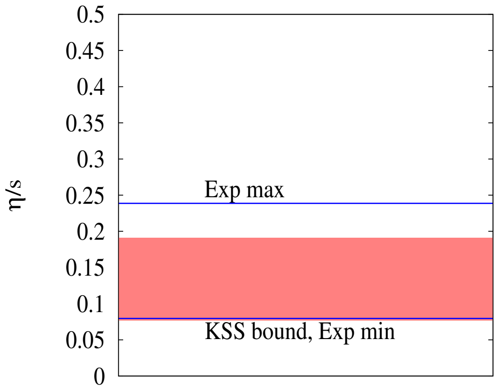
$$\rho(\omega) = \frac{\eta}{\pi}\omega + \text{th}^2 \frac{\omega}{\omega_0} A\rho_{asym}(\omega)$$

- $\chi^2/dof \sim 1,\ A = 0.723 \pm 0.003,\ \omega_0 = 2.0\text{GeV}$
- $\frac{\eta}{s} = 0.09 \pm 0.03$



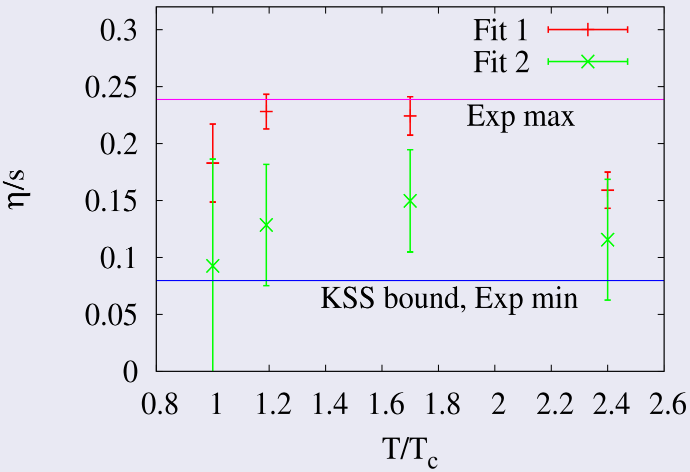
Other variants of the spectral function

- $\frac{\eta}{s} < 0.18 \pm 0.04$
- $\frac{\eta}{s} \in (0.09, 0.18)$
- $\frac{\eta}{s} = 0.134 \pm 0.057$



SU(3) gluodynamics, 16×32^3

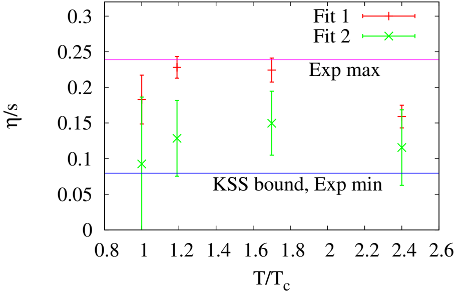
η/s vs T



Results ($T/T_c = 1.2$)

- $\frac{\eta}{s} = 0.134 \pm 0.057$ SU(2)
- $\frac{\eta}{s} = 0.178 \pm 0.06$ SU(3)

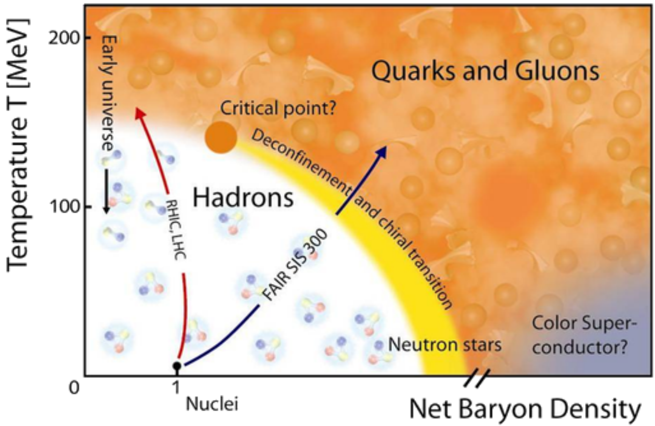
- $\frac{\eta}{s} = \frac{1}{4\pi} \simeq 0.08$ N=4 SYM $\lambda = \infty$ (Phys. Rev. Lett. 87 (2001) 081601)
- $\frac{\eta}{s} = (1-3)\frac{1}{4\pi} \simeq 0.08 - 0.24$ Experiment (Phys. Rev. C 78, 034915 (2008))
- $\frac{\eta}{s} \sim 2$ Perturbative result (JHEP 11 (2000) 001)
- $\frac{\eta}{s} = 0.102 \pm 0.056$ (SU(3), Phys.Rev. D76 (2007) 101701)



Conclusion

- Model-independent calculation of viscosity in $SU(2)$ and $SU(3)$ gluodynamics (no free parameters)
- Results are in agreement with experimental estimations
- QGP is strongly coupled system close to SYM and far from weakly interacting plasma

Phase diagram of QCD

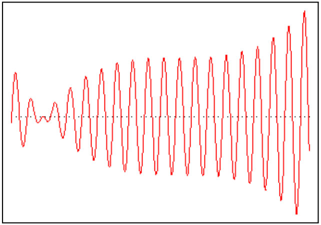


Lattice QCD

$$Z = \int D\psi D\bar{\psi} DA_\mu e^{-S_g - \bar{\psi} D \psi} = \int DA_\mu e^{-S_g} \det D$$

Lattice QCD + $\mu_B \longrightarrow$ Sign problem!

$\det D \notin \mathbf{R}$



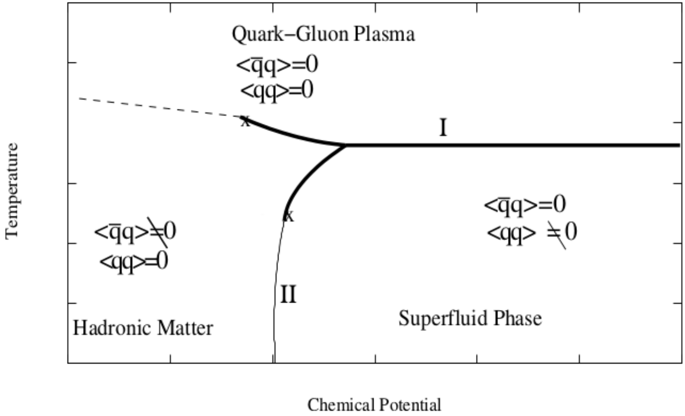
Lattice Monte-Carlo Calculations

Gauge group: $SU(2) \longrightarrow$ No sign problem!

$$\det D^\dagger(\mu_B^*) = \det\left[(\tau_2 C\gamma_5) D(\mu_B)(\tau_2 C\gamma_5)\right]$$

c $C = \gamma_2\gamma_4$ - charge conjugation matrix
$\det D(\mu_B) \in \mathbf{R}, > 0.$

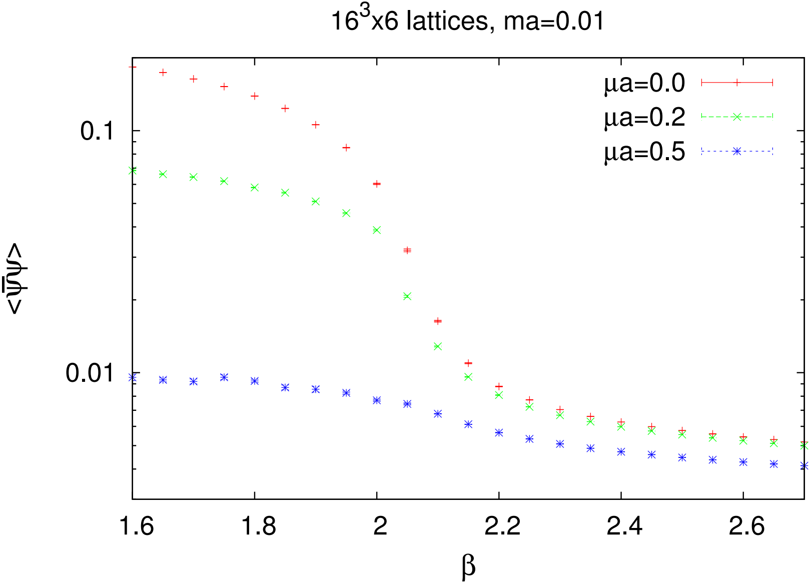
Phase diagram of *SU*(2) QCD



J. B. Kogut, D. Toublan, D.K. Sinclair, Nucl.Phys. B642 (2002) 181-209

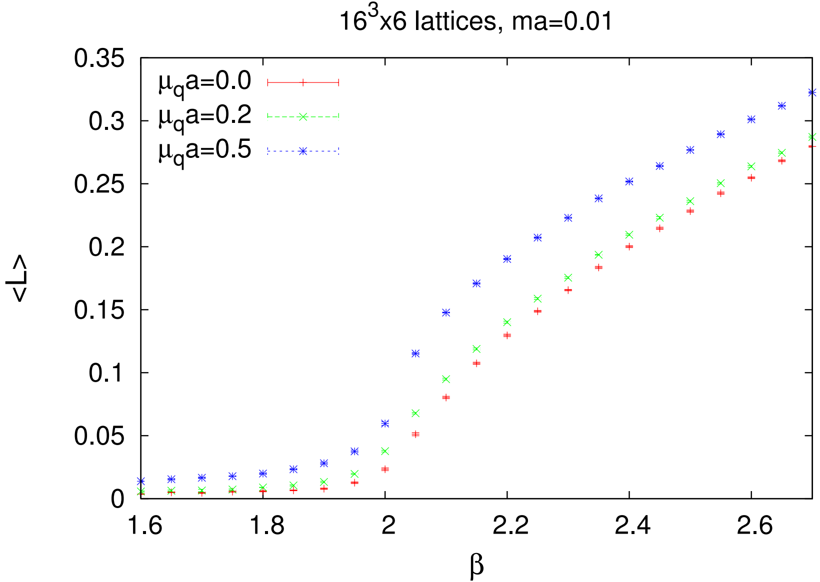
μ_B, confinement and chiral symmetry breaking

Chiral condensate:



μ_B, confinement and chiral symmetry breaking

Polyakov loop:

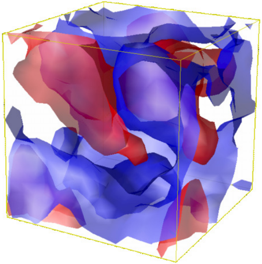


Conclusion

- First results about SU(2) QCD phase diagram
- μ_B disfavours chiral symmetry breaking
- Critical temperature decreases

QCD with nonzero chiral density

Topological fluctuations in QCD



(a) arXiv:1111.6733, P.V. Buividovich, T. Kalaydzhyan, M.I. Polikarpov

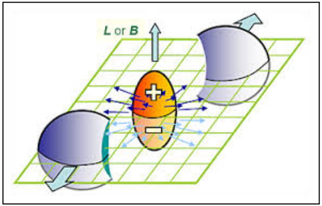
Anomaly:

$$\partial_\mu j^{(5)}_\mu = C F^{(a)}_{\mu\nu} \tilde{F}^{\mu\nu}_{(a)} \longrightarrow \text{Nonzero chiral density } \rho_5$$

CME

Possible manifestation: Chiral Magnetic Effect (CME)
$\rho_5 \;\&\; \vec{B} \quad \rightarrow \quad \vec{\jmath} \;\parallel\; \vec{B}$

$\vec{\jmath} = \frac{N_c}{2\pi^2}\mu_5 \vec{B}$



K. Fukushima, D. Kharzeev, H. J. Warringa,

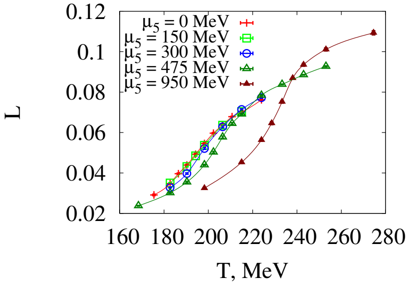
PRD 78, arXiv: 0808.3382 (hep-ph)
Phase is important!

Phase diagram of QCD with nonzero μ_5

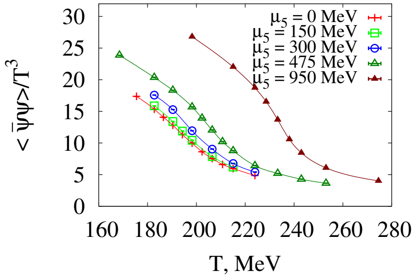
- Effective models (NJL, PNJL, PLSM$_q$ etc)
  arXiv: 1102.0188, 1110.4904, 1305.1100, 1310.4434
- Dyson-Schwinger equations
  arXiv:1505.00316
- Large N_c Universality
  arXiv:1111.3391
- Lattice QCD (no sign problem)

Results

$SU(2)$, $N_f = 4$ fermionic flavours
Lattice size 6×20^3, $m_q = 12$ MeV

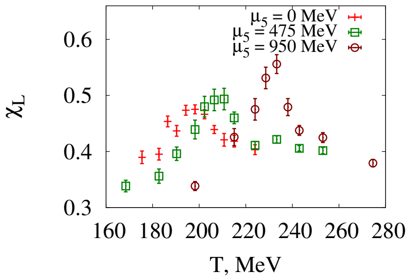


Polyakov loop

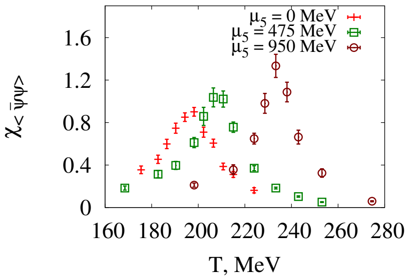


chiral condensate

Results. Susceptibilities

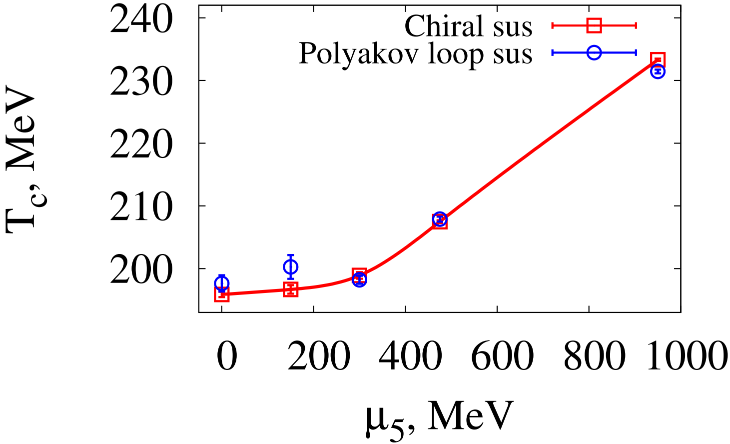


Polyakov loop susceptibility

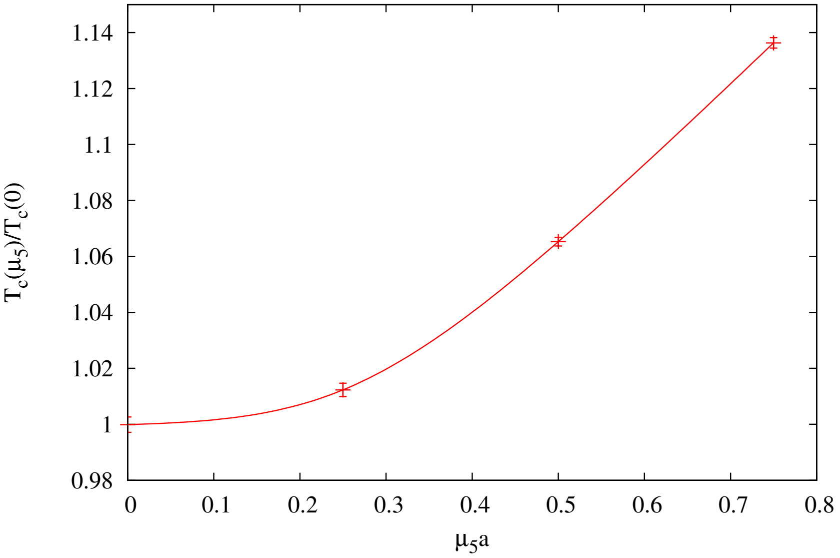


chiral susceptibility

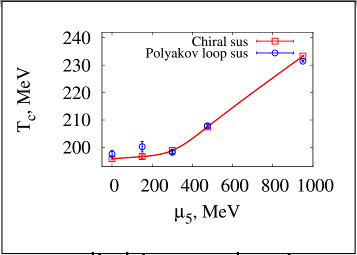
Critical temperature vs μ_5



Results for $SU(3)$ gauge group and $N_f = 2$ Wilson fermions



Conclusions



- Phase diagram with nonzero μ_5 was studied in two theories: $SU(2)$, $N_f = 4$ и $SU(3)$, $N_f = 2$
- $T_c \uparrow$ when $\mu_5 \uparrow$
- The same behaviour in the chiral limit
- The transition seems to become sharper
- No splitting of χS and confinement transitions

Superconductivity of QCD vacuum in strong magnetic fields

In the background of strong magnetic field
QCD vacuum turns into a superconductor
(due to condensation of charged ρ-mesons)

M. Chernodub, arXiv:1008.1055, arXiv:1101.0117

Superconductivity of QCD vacuum in strong magnetic fields

1. Emerges spontaneously at magnetic fields larger then critical

$$B_c \approx 10^{16}\text{T}$$
$$eB_c \approx m_\rho^2 \approx 0.6\ \text{GeV}^2$$

2. No Meissner effect (though vortices are formed)
3. Zero resistance along magnetic field
4. Isolator in other (perpendicular) directions

Approaches to problem

1. General ideas
2. Effective models (M. Chernodub, arXiv:1008.1055, arXiv:1101.0117);
3. Gauge-gravity duality (N. Callebaut, D. Dudal, H. Verschelde, arXiv:1105.2217; M. Ammon, J. Erdmenger, P. Kerner, M. Strydom , arXiv:1106.4551; ...)
4. Numerical calculations

Naive approach

Energy of ρ-meson in magnetic fiels

$$E^2 = m_\rho^2 + eB(2n + 1 - 2S_z) + p_z^2$$

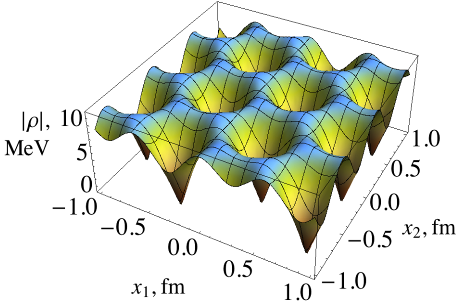
$$\left.\begin{array}{lr} \text{Zero Landau level:} & n = 0 \\ & p_z = 0 \\ \text{Spin along the field:} & S_z = 1 \end{array}\right\} \quad E^2 = m_\rho^2 - eB$$

If $eB > eB_c = m_{\rho^2} \approx 0.6 GeV^2 \Rightarrow E^2 < 0 \Rightarrow$ Condensation

Structure of the condensate

Effective bosonic model

$B = 1.01 B_c$



(b) Charged ρ-mesons

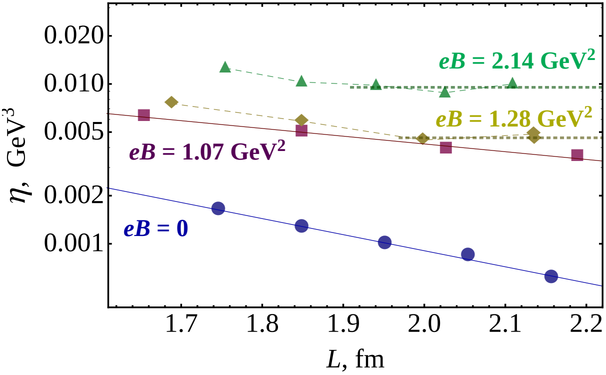
M. Chernodub, J. Doorsselaere, H. Verschelde, arXiv:1111.4401.
Similar results in holography Y.-Y. Bu, J. Erdmenger, J. P. Shock, M. Strydom, arXiv:1210.6669

Numerical calculations in quenched QCD

- Two quark flavours: u, d
- ρ-meson operator: $\rho_\mu = \bar{u}\gamma_\mu d$
- Spin ± 1 along magnetic field: $\rho_\pm = \frac{1}{2}(\rho_1 \pm i\rho_2)$
- Correlator: $G_\pm(z) = \langle \rho_\pm^\dagger(0)\rho_\pm(z)\rangle$
- Condensate: $\lim_{|z|\to\infty} G_+(z) = |\langle\rho\rangle|^2$

Superconducting condensate

Quenched QCD



Mass (arXiv:hep-lat/9803003)

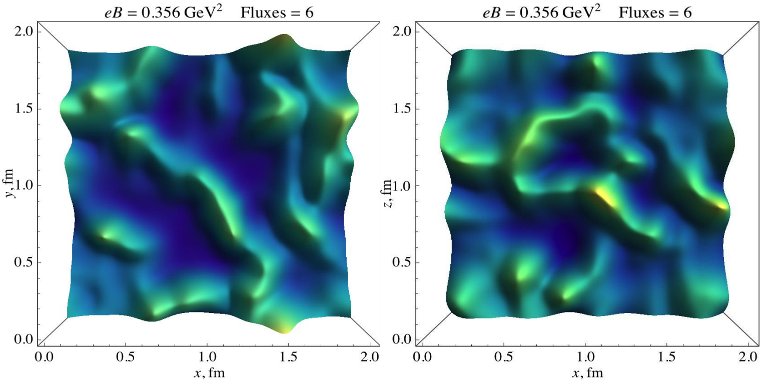
$$m_\rho \sim 1.1 GeV$$

Superconducting vortices

Observables:

- Field: $\rho_\mu = \bar{u}\gamma_\mu d$
- Correlator: $\rho_+(x) \to \phi(x) = \langle \rho_+^\dagger(0)\rho_+(x)\rangle_f$
- Energy density: $E(x) = \frac{|D_\mu \phi(x)|^2}{|\phi(x)|^2}$, $D_\mu = \partial_\mu - ieA_\mu$
- Electric current: $j_\mu(x) = \frac{\phi^* D_\mu \phi - \phi (D_\mu \phi)^*}{2i|\phi(x)|^2}$
- Vortex density: $v(x) = \operatorname{sing} \arg \phi(x) = \frac{\epsilon^{ab}}{2\pi} \frac{\partial}{\partial x_a} \frac{\partial}{\partial x_b} \arg \phi(x)$

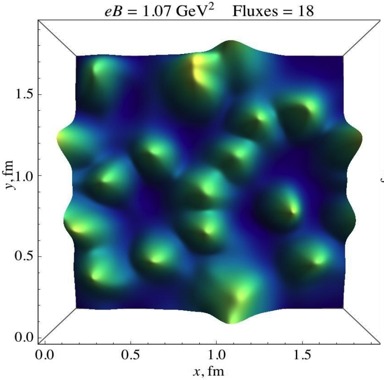
Superconducting vortices. Energy density. $eB = 0.36 GeV^2$



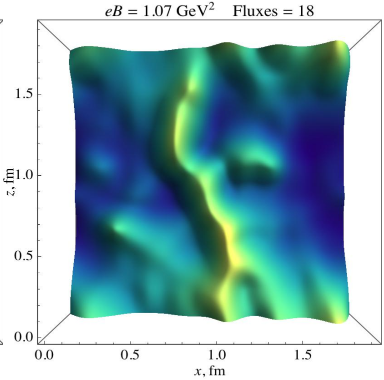
(c) $\perp B$ (d) $|| B$

arXiv:1301.6590

Superconducting vortices. Energy density. $eB = 1.07 GeV^2$

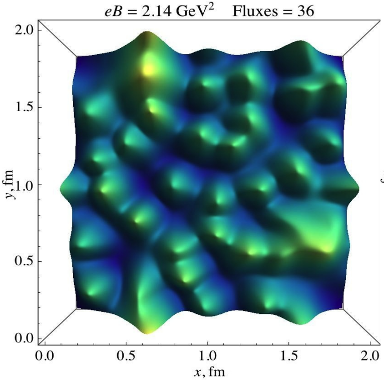


(e) $\perp B$

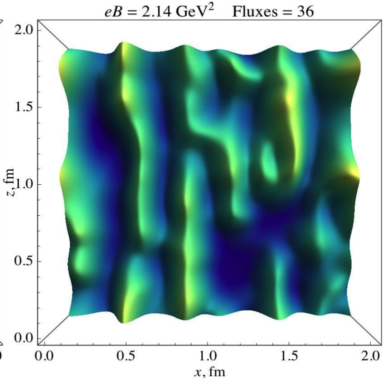


(f) $\parallel B$

Superconducting vortices. Energy density. $eB = 2.14 GeV^2$

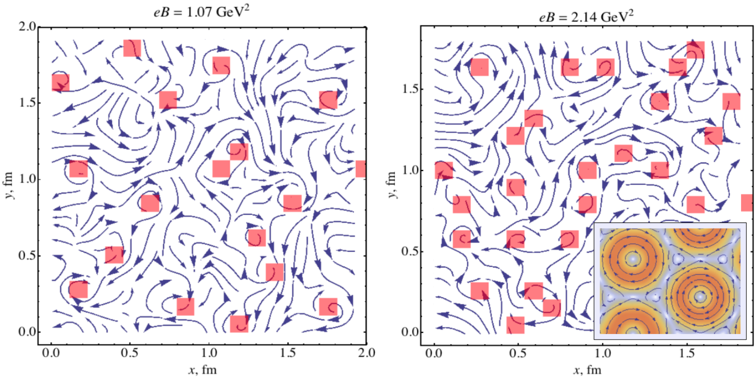


(g) $\perp B$

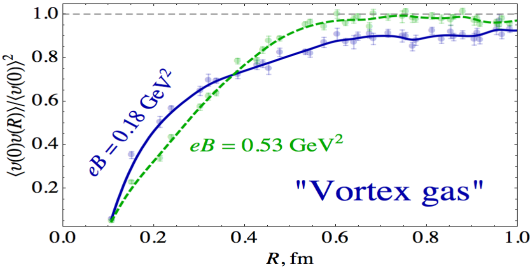


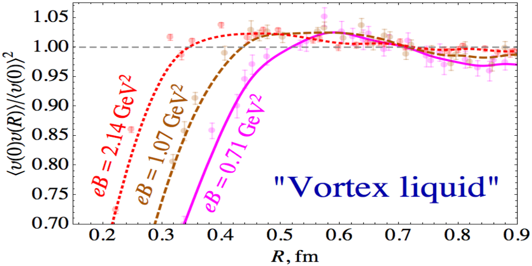
(h) $|| B$

Electric current around the vortices



Correlations between positions of vortices





Conclusions

- In a sufficiently strong magnetic field ρ-meson condensate is formed simultaneously
- New type of topological defects, "ρ-vortices emerge
- Liquid of ρ-vortices is observed in quenched lattice calculations(cf. theory: trigonal lattice)

Total conclusion

- Viscosity in SU(2) and SU(3) gluodynamics was measured (in particular, with respect to the temperature)
- Influence of the baryonic chemical potential on the temperature of the confinement-deconfinement transition was studied
- The phase diagram of QCD with nonzero chiral density was investigated
- The hypothesis about superconductivity of QCD vacuum in strong magnetic fields was studied