Non-perturbative study of the viscosity in $SU(2)$ lattice gluodynamics.

V. V. Braguta, A. Yu. Kotov

Семинар "Теория адронного вещества при экстремальных условиях"

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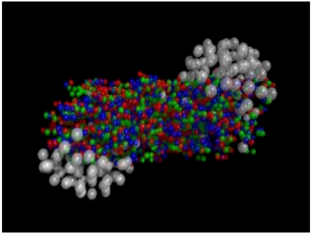
Outline

- Introduction
- Transport coefficients in lattice calculations
- Improving statistical accuracy of the results
- Analytical continuation problem
- Numerical setup
- Results and discussion

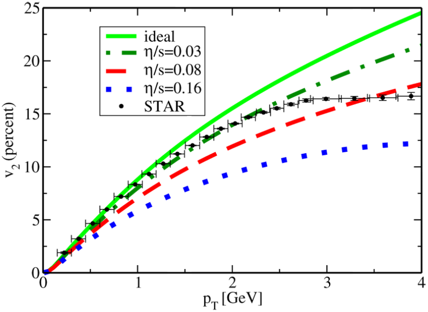
Introduction. Hydrodynamical description.



Hydrodinamical description of the distribution of final particles

- One heavy ion collision produces a huge number of final particles
- Large number of particles $\Rightarrow$ hydrodynamical description can be used
- In hydrodinamics transport coefficients control flow of energy, momentum, electrical charge and other quantities

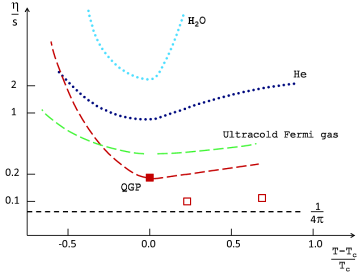
Shear viscosity. Value and bounds.



Teaney D., *Viscous Hydrodynamics and the Quark Gluon Plasma*, arXiv:0905.2433

- Experimentally preferred value: $\frac{\eta}{s} \sim (1 \leftrightarrow 3)\frac{1}{4\pi}$
- Experimental bound: $\frac{\eta}{s} < 5\frac{1}{4\pi}$
- KSS-bound: $\frac{\eta}{s} \geq \frac{1}{4\pi}$

Shear viscosity. Value and bounds.



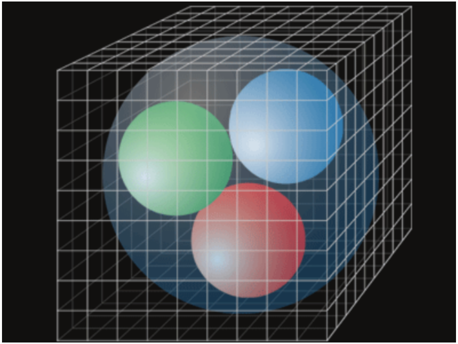
Cremonini S., Gursoy U. and Szepietowski P., *On the Temperature Dependence of the Shear Viscosity and Holography*, arXiv:1206.3581

Comparison of different liquids

QGP the most superfluid liquid

The aim: first principle calculation of transport coefficients

Lattice simulations of QCD.



- Allows to study strongly interacting systems
- Based on the first principles of quantum field theory
- Acknowledged approach to study QCD
- Very powerful due to the development of computer systems

Previous lattice calculations ($SU(3)$ gluodynamics).

- F. Karsch, H. W. Wyld. Phys. Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev.Lett. 100 (2008) 162001

Viscosity in lattice calculations.

Green-Kubo relation:

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho_{12,12}(\omega, \mathbf{q}=0)}{\omega}$$

Green function measured on the lattice(Eucledian):

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int d^3\mathbf{x} e^{i\mathbf{p}x} \langle T_{12}(0) T_{12}(x_0, \mathbf{x})\rangle$$

Spectral function and correlator of stress-energy tensor:

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega(\frac{1}{2}L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega$$

Stress-energy tensor for gluodynamics:

$$T_{\mu\nu} = 2\,\mathrm{tr}(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

Asymptotic behaviour - perturbation theory: $\rho(\omega) = \frac{1}{10}\frac{3}{(4\pi)^2}\omega^4, \omega \to \infty$

Main difficulties.

- Large statistical errors in measuring correlator $C_{12,12}(x_0, 0)$
  - Improved action
  - Multilevel algorithm
- Extracting spectral function $\rho_{12,12}$ from

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega(\frac{1}{2} L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega$$

  - Fit by model function
  - Maximum entropy method
  - Linear method
  - ...

Statistical error of the correlator $C_{12,12}$. Improved action.

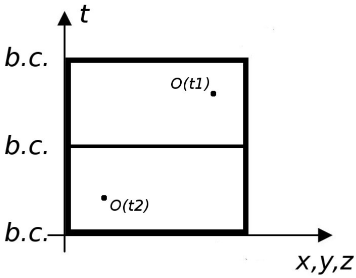
$$S_{unimpr} = \beta \sum_{pl} S_{pl}$$

$$S_{impr} = \beta_{impr} \sum_{pl} S_{pl} - \frac{\beta_{impr}}{20 u_0^2} \sum_{rt} S_{rt}$$

$$S_{pl,rt} = \frac{1}{2} \mathrm{tr}(1 - U_{pl,rt})$$

Increases accuracy but is not enough.

Statistical error of the correlator $C_{12,12}$. Multilevel algorithm.



For t_1 and t_2 in different areas

$$\langle O(t_1)O(t_2)\rangle = \frac{1}{N_{bc}}\sum_{bc}\langle O(t_1)\rangle_{b.c}\langle O(t_2)\rangle_{b.c}$$

Analytical continuation problem.

- Fit by model function
- Maximum entropy method
- Linear method
- ...

Analytical continuation problem. Fit by model function.

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega(\frac{1}{2} L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega$$

Proposed in the first work on transport coefficients:
F. Karsch, H. W. Wyld. Phys. Rev. D35 (1987)

$$\rho(\omega)/\omega = A \left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2} \right)$$

A, m, γ - parameters
Clearly ignores asymptotic behaviour

Analytical continuation problem. Maximum entropy method.

$$C_{12,12}(x_0, \mathbf{p}) = \beta^5 \int_0^\infty \rho_{12,12}(\omega, \mathbf{p}) \frac{\cosh \omega(\frac{1}{2}L_0 - x_0)}{\sinh \frac{\omega L_0}{2}} d\omega$$

We discretize ω, $N_\omega \sim O(10^3)$

$$\rho(\omega) \xrightarrow{K(\omega, x_i)} G(x_i) = G_i$$

$$\chi^2 = \sum_{i,j} (G_i - G_i^{(0)})(S^{-1})_{ij}(G_j - G_j^{(0)})$$

$$\min \chi^2 \quad \to \quad \sim O(10) \text{ equations}$$

Analytical continuation problem. Maximum entropy method.

Instead of χ^2 we minimize

$$\chi^2 + \alpha S,$$

Entropy S determines how our function is close to some model function $\mu(\omega)$ (which summarizes our prior knowledge about the spectral function).

$$S = \sum_{m=1}^{N_w} \left(\rho_m - \mu_m - \rho_m \log \frac{\rho_m}{\mu_m} \right)$$

Doesn't work for small lattice sizes.

Analytical continuation problem.

Linear method:

$$\rho(\omega) = m(\omega)(1 + a(\omega)) = m(\omega)(1 + \sum_l a_l u_l(\omega)),$$

where $m(\omega)$ is an initial approximation :

$$m(\omega) = \frac{A\omega^4}{\tanh^2 \frac{\omega}{4T} \tanh \frac{\omega}{2T}},$$

and $u_l(\omega)$ are eigenmodes of $H(\omega, \omega') = \sum_i K(t_i, \omega) K(t_i, \omega')$ with
$K(t, \omega) = m(\omega) \frac{\cosh(\omega(\frac{1}{2T} - t))}{\sinh \frac{\omega}{2T}}$
a_l are selected to minimize χ^2.

Analytical continuation problem. Resolution function.

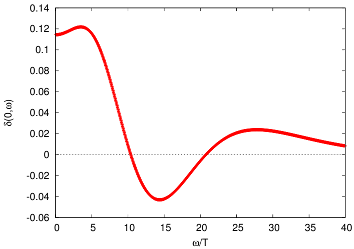
$$\rho(\omega) = m(\omega)(1 + a(\omega))$$

Let $\hat{\rho}(\omega)$ be a true spectral function.

$$\hat{\rho}(\omega) = m(\omega)(1 + \hat{a}(\omega)) \xrightarrow{K(t_i,w)} G_i \xrightarrow{linear} \rho(\omega) = m(\omega)(1 + a(\omega))$$

Resolution function:

$$a(\omega) = \int d\omega \hat{a}(\omega)\delta(\omega, \omega')$$



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  - ~~Fit by model function~~
  - ~~Maximum entropy method~~
  - Linear method
  - ...

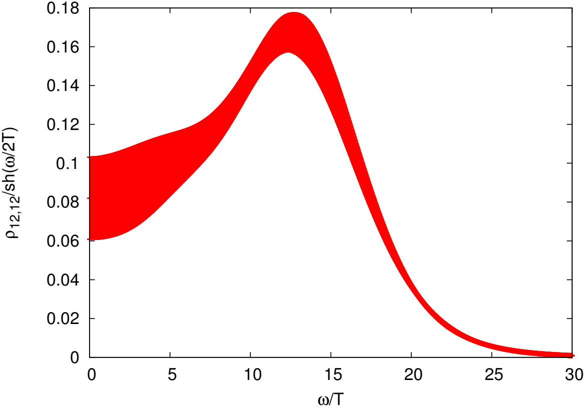
Numerical setup.

- $SU(2)$-gluodynamics with Wilson action:

$$S = \frac{\beta}{2} \sum_{pl} \text{tr}(1 - U_{pl})$$

- Lattice 8×32^3
- $\beta = 2.643$
- $T/T_c \approx 1.2$
- Clover-shaped discretization for $F_{\mu\nu}$
- Two-level algorithm for measuring stress-energy tensor correlator.

Spectral function $\rho_{12,12}$.



Numerical results.

$$\frac{\eta}{s} = 0.111 \pm 0.032$$

KSS-bound:

$$\eta/s \geq \frac{1}{4\pi} \approx 0.08$$

Perturbative result:

$$\eta/s \sim 2$$

Experimental bound and preferred value:

$$\eta/s < 5\frac{1}{4\pi} \approx 0.4$$

$$\eta/s \sim (1 \leftrightarrow 3)\frac{1}{4\pi}$$

Unsatisfactory attempts to increase lattice size