
Screening mass of gluons in presence of external Abelian chromomagnetic field

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Magnetic Mass

Magnetic (electric) mass shows how fast magnetic (electric) field decreases with distance in plasma.

$$F_{\mu\nu} = C(r) e^{-m^k r^k} \quad (1)$$

$$m = \frac{1}{\lambda}, \quad \lambda - \text{screening length} \quad (2)$$

Example: QED

$$m_{\text{el}}^2 = \frac{1}{3} e^2 T^2 \quad - \text{electric (Debye) mass} \quad (3)$$

$$m_{\text{magn}}^2 = 0 \quad - \text{magnetic mass} \quad (4)$$



Screened



Long range

Magnetic Mass in SU(N) Gauge Theory

$$F_{\mu\nu} = \sum_{a=1}^3 F_{\mu\nu}^{(a)} t_a, \quad t_a - \text{generators of SU(N) group} \quad (5)$$

// D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53**, 43 (1981)

1-loop calculations:

$$m_{\text{el}}^2 = \frac{1}{3} g^2 T^2 \left(N + \frac{N_f}{2} \right) \quad (6)$$

$$m_{\text{magn}}^2 = 0 \quad (7)$$

Higher orders, nonperturbative calculations:

$$m_{\text{magn}}^2 \sim g^4 T^2 \quad (8)$$

$m_{\text{magn}} = 0$ is not excluded

Hypothesis: not all color components of chromomagnetic field contribute to the magnetic mass

Magnetic Mass in the Presence of External Field

External chromomagnetic field: $H_\mu^a = H \delta_{\mu 3} \delta^{a 3}$

$SU(2)$

High temperature: $gH/T^2 \ll 1$

Neutral gluon field:

$$m_{\text{el}}^2 \sim g^2 T^2 \left(1 - C \sqrt{gH}/T\right), \quad m_{\text{magn}}^2 = 0 \quad (9)$$

// M. Bordag and V. Skalozub, Phys. Rev. D **75**, 125003 (2007) [hep-th/0611256]

// S. Antropov, M. Bordag, V. Demchik and V. Skalozub, Int. J. Mod. Phys. A **26**, 4831 (2011) [arXiv:1011.3147 [hep-ph]]

Color-charged gluon fields:

$$m_{\text{el}}^2 \sim g^2 T^2 \left(1 - C \sqrt{gH}/T\right), \quad m_{\text{magn}}^2 \sim g^2 T \sqrt{gH} \quad (10)$$

// M. Bordag and V. Skalozub, Phys. Rev. D **77**, 105013 (2008) [arXiv:0801.2306 [hep-th]]

// M. Bordag and V. Skalozub, Phys. Rev. D **85**, 065018 (2012) [arXiv:1201.1978 [hep-th]]

$$\sqrt{gH} \sim g^2 T \quad \Rightarrow \quad m_{\text{magn}}^2 \sim g^4 T^2$$



Spontaneous field generation

- 1 T. A. DeGrand and D. Toussaint, “The Behavior of Nonabelian Magnetic Fields at High Temperature,” Phys. Rev. D **25**, 526 (1982)
 - Screening of the chromomagnetic field of the monopole-antimonopole string was shown
 - Color structure could not be clarified by this method
- 2 S. Antropov, M. Bordag, V. Demchik and V. Skalozub, “Long range chromomagnetic fields at high temperature,” Int. J. Mod. Phys. A **26**, 4831 (2011) [arXiv:1011.3147 [hep-ph]]
 - Zero magnetic mass of the Abelian chromomagnetic field was shown
 - Non-Abelian components of the chromomagnetic field were not investigated

Magnetic Mass: Analytical Calculations vs Lattice

$m \neq 0$
 $m^2 \sim g^4 T^2$
Perturb. On the lattice

$m_{\text{neut}} = 0$
Perturb. On the lattice

$m_{\text{ch}} \neq 0$
 $m_{\text{ch}}^2 \sim g^2 T \sqrt{gH}$
Perturb.

The aim of this investigation: to show that m_{magn} is produced by the charged components of the gluon field on the lattice

Quantum Gluodynamics on the Lattice

| | | |
|---------------------------------|---|--|
| Continuous Minkovsky space-time | → | Euclidean 4D discrete lattice |
| Continuous operators | → | Discrete operators on the lattice |
| Gluon fields | → | SU(N) matrices at the links of the lattice |

Expectation value of a measured quantity \mathcal{O} :

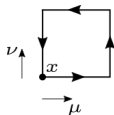
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{O}[U] e^{-S[U]} \quad \longrightarrow \quad \langle \mathcal{O} \rangle \approx \frac{1}{K} \sum_{U_k} \mathcal{O}[U_k] \quad (11)$$

$$Z = \int \mathcal{D}U e^{-S[U]}, \quad \int \mathcal{D}U = \prod_{x, \mu} \int dU_\mu(x), \quad \text{configurations } U_k \text{ are distributed with probability } \propto e^{-S[U_k]}.$$

$$\text{Lattice Wilson action: } S_W = \beta \sum_{\mu > \nu} \sum_x \left[1 - \frac{1}{N} \text{Re Tr } U_{\mu\nu} \right] \quad (12)$$

$$S_W \xrightarrow{a \rightarrow 0} \frac{1}{4g^2} \int d^4x F_{\mu\nu}^{(c)}(x) F_{\mu\nu}^{(c)}(x) \quad (13)$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + \hat{\mu}) U_\mu^\dagger(x + \hat{\nu}) U_\nu^\dagger(x) - \text{plaquette.} \quad (14)$$



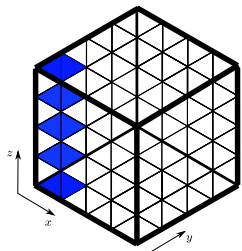
Idea of the investigation:

Two external chromomagnetic fields are introduced on the lattice:

- field of monopole-antimonopole string;
- Abelian field flux.

Screening of combination of that fields is investigated.

Monopole-Antimonopole String on the Lattice



// T. A. DeGrand and D. Toussaint

// M. Srednicki and L. Susskind, Nucl. Phys. **B179**, 239 (1981)

$$S = \beta \sum_n \sum_{\mu > \nu} \left[1 - \frac{1}{N} \text{Re Tr } U_{\mu\nu}(n) \Xi_{\mu\nu}(n) \right],$$

$$\Xi_{\mu\nu}(n) \in Z(N)$$

Center of the SU(N) group:

$$Z(N) = \{ \sqrt[N]{1} \cdot I \}$$

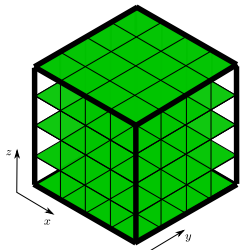
$$\text{SU(2) case: } Z(2) = \{ 1 \cdot I, -1 \cdot I \}$$

$$\text{SU(3) case: } Z(3) = \{ e^{-\frac{2}{3}\pi i} \cdot I, 1 \cdot I, e^{\frac{2}{3}\pi i} \cdot I \}$$

$$\Xi_{\mu\nu}(n) \neq I \quad \text{if string} \cap U_{\mu\nu}(n)$$

$$\Xi_{\mu\nu}(n) = -I \quad \text{if } x = 0, y = 0, \forall z, t$$

Abelian Field Flux on the Lattice



Plaquette:

// S. Antropov et al.

$$\left. \begin{aligned} U_{\mu\nu}(x) &= U_\mu(x)U_\nu(x + \hat{\mu})U_\mu^\dagger(x + \hat{\nu})U_\nu^\dagger(x) \\ U_\mu(n) &= e^{iaA_\mu(n)} \end{aligned} \right\} \Rightarrow U_{\mu\nu}(n) = e^{ia^2 F_{\mu\nu}(n)}$$

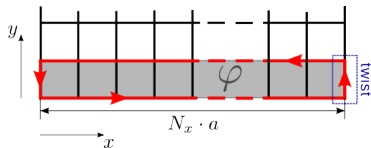
$$U'_{xy} = e^{ia^2(H_z + H_z^{\text{ext}})} = U_{xy} e^{ia^2 H_z^{\text{ext}}}$$

$$U'_y(0, n_y, n_z, n_t) = U_y(0, n_y, n_z, n_t) e^{i\varphi}$$

$$\varphi = a^2 N_x H_z^{\text{ext}}$$

Twisted boundary conditions:

$$\left\{ \begin{aligned} U_y(N_x, n_y, n_z, n_t) &= U_y(0, n_y, n_z, n_t) e^{i\varphi}, \\ U_\mu(N_x, n_y, n_z, n_t) &= U_\mu(0, n_y, n_z, n_t), \quad \mu \neq y, \\ U_\mu(n_x, N_y, n_z, n_t) &= U_\mu(n_x, 0, n_z, n_t), \\ U_\mu(n_x, n_y, N_z, n_t) &= U_\mu(n_x, n_y, 0, n_t), \\ U_\mu(n_x, n_y, n_z, N_t) &= U_\mu(n_x, n_x, n_z, 0). \end{aligned} \right.$$



$$e^{i\varphi} = e^{i\varphi_3 \sigma_3 / 2} = \begin{pmatrix} e^{i\varphi_3/2} & 0 \\ 0 & e^{-i\varphi_3/2} \end{pmatrix}$$

Abelian Field Flux on the Lattice

$SU(3)$

$$e^{i\varphi} = e^{i(\varphi_3\lambda_3 + \varphi_8\lambda_8)/2} = \begin{pmatrix} e^{i(\varphi_3 + \varphi_8/\sqrt{3})/2} & 0 & 0 \\ 0 & e^{i(-\varphi_3 + \varphi_8/\sqrt{3})/2} & 0 \\ 0 & 0 & e^{-i\varphi_8/\sqrt{3}} \end{pmatrix} \quad (15)$$

Ext. Field through the Flux vs Ext. Field through the Strength

TBC

$$U_{\mu\nu}(n) = e^{ia^2 F_{\mu\nu}(n)}$$

$$a^2 F_{xy}(n) \rightarrow a^2 [F_{xy}(n) + H_z^{\text{ext}}]$$

Cosmai & Cea

// P. Cea and L. Cosmai, Phys. Rev. D **60**,
094506 (1999) [hep-lat/9903005]

$$U_\mu(n) = e^{iaA_\mu(n)}$$

$$A_\mu(n) \rightarrow A_\mu(n) + H_z^{\text{ext}} x \delta_{\mu 2} t_3 / 2$$

$$U_\mu(x + N_x, y, z, t) = U_\mu(x, y, z, t)$$

⇓

$$\frac{H_z^{\text{ext}} N_x a}{2} = 2\pi k, \quad k \in \mathbb{Z}$$

Outline of the Investigation

Lattices: $N_t \times N^3$, $N_t = \text{const}$

Measured quantity: $\langle U \rangle = \langle \text{Re Tr } U_{\mu\nu} \rangle$

Investigated quantity: $f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$

$$U_{\mu\nu}(n) = e^{ia^2 F_{\mu\nu}(n)} \approx 1 + ia^2 F_{\mu\nu}(n), \quad a \rightarrow 0 \quad (16)$$

$$\Delta U_{\mu\nu}(n) \approx ia^2 \Delta F_{\mu\nu}(n) \quad (17)$$

$$F(\varphi) = -T \ln \frac{\mathcal{Z}(\varphi)}{\mathcal{Z}(0)}, \quad - \text{ free energy}, \quad \mathcal{Z}(\varphi) = \int \mathcal{D}U e^{-S(\varphi)} \quad (18)$$

$$S(\varphi) = S(0) + \sum_n \frac{1}{n!} \left. \frac{\partial^n S(\varphi)}{\partial \varphi^n} \right|_{\varphi=0} \varphi^n = S(0) + S_\varphi(\varphi) \quad (19)$$

$$\mathcal{Z}(\varphi) = \int \mathcal{D}U e^{-S(0)} e^{-S_\varphi(\varphi)} \quad \frac{\mathcal{Z}(\varphi)}{\mathcal{Z}(0)} = e^{-\bar{S}_\varphi(\varphi)} \quad (20)$$

$$F(\varphi) = T\bar{S}_\varphi(\varphi) = T(\bar{S}(\varphi) - \bar{S}(0)) \quad f(n) \sim \frac{\partial F(n)}{\partial \beta} \quad (21)$$

Outline of the Investigation

Lattices: $N_t \times N^3$, $N_t = \text{const}$

Measured quantity: $\langle U \rangle = \langle \text{Re Tr } U_{\mu\nu} \rangle$

Investigated quantity: $f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$

Possibilities for f :

- $f \sim 1/N^2$ – flux tubes, the flux is conserved;
- $f \sim 1/N^4$ – Coulombic behavior, flux spreads out over the available area;
- $f \sim e^{-kN^2}$ – screening of the field; $k = m_{\text{magn}}^2$;
- $f \sim 1/N$ – spontaneous field generation, flux increases with distance.

Simulations are performed

- in absence of external Abelian field flux φ ;
- in presence of external Abelian field flux φ :
 φ is directed parallel to the monopole-antimonopole string.

Simulations Setup; SU(2)

Lattices used: $4 \times N^3$, $N = 6, 8, \dots, 72$

External Abelian field flux $\varphi = 0.08$ ($\sim 10^4$ MeV²)

$\beta = 2.835$ ($T \sim 1.2$ GeV)

$\beta = 3.020$ ($T \sim 1.9$ GeV)

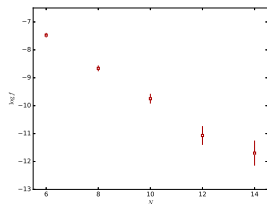
$\beta = 3.091$ ($T \sim 2.3$ GeV)

Simulations are performed with the QCDGPU program

(<https://github.com/vadimdi/QCDGPU>, V. Demchik, N. K., Comp. Sc. and Appl., **1**, 1 (2014) [arXiv:hep-lat/1310.7087])



χ^2 -analysis of the Data



$$f_i = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|_i$$

The data are fitted through minimization of χ^2 function:

$$\chi^2(a) = \sum_{i=1}^K \frac{[y_i - \log f(N_i; a)]^2}{\sigma_i^2}, \quad (22)$$

$$y_i = \log f_i, \quad f(N_i; a) = \frac{A}{N^b} e^{-kN^q}.$$

$$\chi_{min}^2 = \chi^2(\hat{a}) \sim \chi_{\nu}^2; \quad \nu = K - L; \quad L = \text{Length } a$$

Hypothesis testing:

- H_0 : $f(N_i; a)$ describes the data;
- H_1 : $f(N_i; a)$ does not describe the data.

$$\Leftrightarrow \chi_{min}^2 \leq \chi_{\nu; 0.05}^2$$

$$\Leftrightarrow \chi_{min}^2 > \chi_{\nu; 0.05}^2$$

Functions
describing
the data

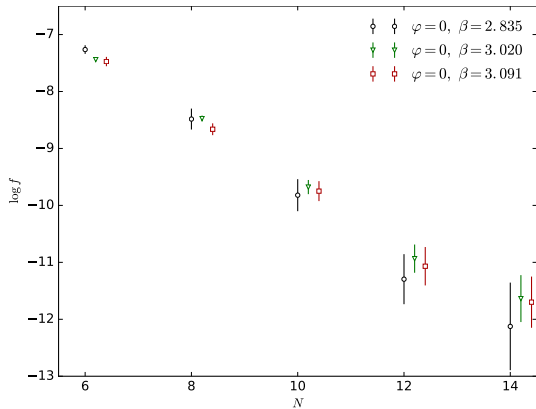
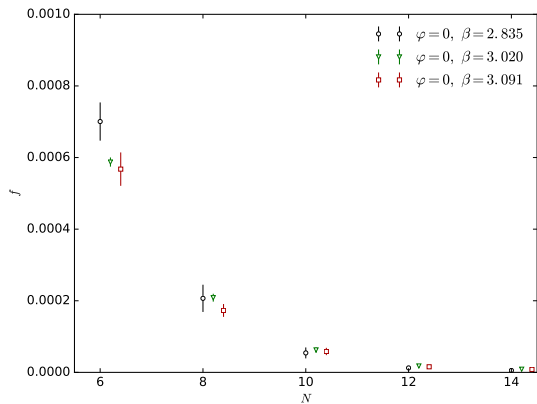
$$\Delta\chi^2 = \chi^2(a) - \chi_{min}^2 \sim \chi_L^2$$

$$\chi^2(a) \leq \chi_{min}^2 + \chi_{L; 0.05}^2 \Rightarrow 95\% \text{ CIs for } a$$

CIs for screening parameters

SU(2) Results: Data at $\varphi = 0$

$$f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$$



SU(2) Results: Fitting at $\varphi = 0$

| Function | $\beta = 2.835$ | | | | $\beta = 3.020$ | | | | $\beta = 3.091$ | | | |
|---------------------|-----------------|---------------------|-------|--|-----------------|---------------------|-------|--|-----------------|---------------------|-------|--|
| | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ |
| A/N | 137 | 9.49 | ✗ | – | 509 | 9.49 | ✗ | – | 190 | 9.49 | ✗ | – |
| A/N^2 | 80.4 | 9.49 | ✗ | – | 247 | 9.49 | ✗ | – | 102 | 9.49 | ✗ | – |
| A/N^4 | 14.0 | 9.49 | ✗ | – | 19.5 | 9.49 | ✗ | – | 9.53 | 9.49 | ✗ | – |
| $A e^{-kN}$ | 0.40 | 7.81 | ✓ | 63.9 ± 10.9 | 2.19 | 7.81 | ✓ | 54.4 ± 4.5 | 0.98 | 7.81 | ✓ | 56.8 ± 8.0 |
| $A e^{-kN^2}$ | 3.18 | 7.81 | ✓ | 3.68 ± 0.63 | 11.8 | 7.81 | ✗ | 3.33 ± 0.28 | 10.3 | 7.81 | ✗ | 3.18 ± 0.45 |
| $(A/N) e^{-kN}$ | 0.60 | 7.81 | ✓ | 51.7 ± 10.9 | 4.14 | 7.81 | ✓ | 41.5 ± 4.5 | 0.64 | 7.81 | ✓ | 44.8 ± 8.0 |
| $(A/N) e^{-kN^2}$ | 1.49 | 7.81 | ✓ | 2.99 ± 0.63 | 4.09 | 7.81 | ✓ | 2.55 ± 0.28 | 5.10 | 7.81 | ✓ | 2.52 ± 0.45 |
| $(A/N^2) e^{-kN}$ | 0.99 | 7.81 | ✓ | 39.5 ± 10.9 | 7.29 | 7.81 | ✓ | 28.6 ± 4.5 | 0.77 | 7.81 | ✓ | 32.7 ± 8.0 |
| $(A/N^2) e^{-kN^2}$ | 0.63 | 7.81 | ✓ | 2.30 ± 0.63 | 2.16 | 7.81 | ✓ | 1.78 ± 0.28 | 1.89 | 7.81 | ✓ | 1.85 ± 0.45 |
| $(A/N^4) e^{-kN}$ | 2.32 | 7.81 | ✓ | 15.1 ± 10.9 | 17.2 | 7.81 | ✗ | 2.80 ± 4.52 | 2.45 | 7.81 | ✓ | 8.65 ± 7.96 |
| $(A/N^4) e^{-kN^2}$ | 1.36 | 7.81 | ✓ | 0.91 ± 0.63 | 15.5 | 7.81 | ✗ | 0.23 ± 0.28 | 1.58 | 7.81 | ✓ | 0.52 ± 0.45 |

SU(2) Results: Fitting at $\varphi = 0$

| Function | $\beta = 2.835$ | | | | $\beta = 3.020$ | | | | $\beta = 3.091$ | | | |
|---------------------|-----------------|---------------------|-------|--|-----------------|---------------------|-------|--|-----------------|---------------------|-------|--|
| | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | $\hat{k} \pm 2\sigma$ CI $\times 10^{-2}$ |
| A/N | 137 | 9.49 | ✗ | — | 509 | 9.49 | ✗ | — | — | — | — | — |
| A/N^2 | 80.4 | 9.49 | ✗ | — | 247 | 9.49 | ✗ | — | — | — | — | — |
| A/N^4 | 14.0 | 9.49 | ✗ | — | 19.5 | 9.49 | ✗ | — | 9.53 | — | — | — |
| $A e^{-kN}$ | 0.40 | 7.81 | ✓ | 63.9 ± 10.9 | 2.19 | 7.81 | ✓ | 54.4 ± 4.5 | 0.98 | — | — | — |
| $A e^{-kN^2}$ | 3.18 | 7.81 | ✓ | 3.68 ± 0.63 | 11.8 | 7.81 | ✗ | 3.33 ± 0.28 | 10.3 | — | — | — |
| $(A/N) e^{-kN}$ | 0.60 | 7.81 | ✓ | 51.7 ± 10.9 | 4.14 | 7.81 | ✓ | 41.5 ± 4.5 | 0.64 | — | — | — |
| $(A/N) e^{-kN^2}$ | 1.49 | 7.81 | ✓ | 2.99 ± 0.63 | 4.09 | 7.81 | ✓ | 2.55 ± 0.28 | 5.10 | — | — | — |
| $(A/N^2) e^{-kN}$ | 0.99 | 7.81 | ✓ | 39.5 ± 10.9 | 7.29 | 7.81 | ✓ | 28.6 ± 4.5 | 0.77 | — | — | — |
| $(A/N^2) e^{-kN^2}$ | 0.63 | 7.81 | ✓ | 2.30 ± 0.63 | 2.16 | 7.81 | ✓ | 1.78 ± 0.28 | — | — | — | — |
| $(A/N^4) e^{-kN}$ | 2.32 | 7.81 | ✓ | 15.1 ± 10.9 | 17.2 | 7.81 | ✗ | 2.80 ± 0.28 | — | — | — | — |
| $(A/N^4) e^{-kN^2}$ | 1.36 | 7.81 | ✓ | 0.91 ± 0.63 | 15.5 | 7.81 | ✗ | 0.25 ± 0.28 | — | — | — | — |

PHYSICAL REVIEW D
 VOLUME 25, NUMBER 2
 15 JANUARY 1982
 Behavior of non-Abelian magnetic fields at high temperature
 T. A. DeGrand* and D. Toussaint

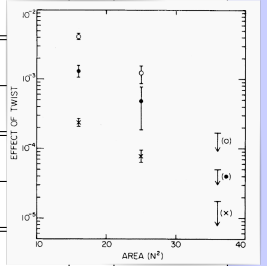
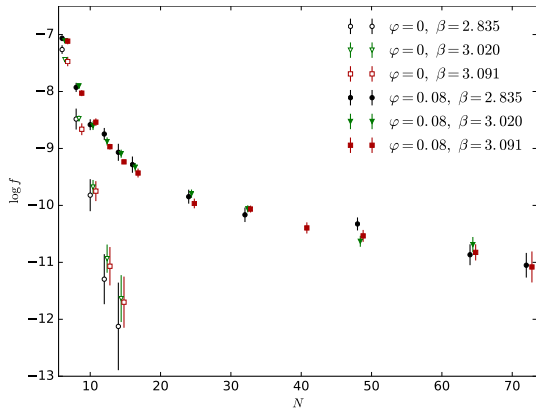
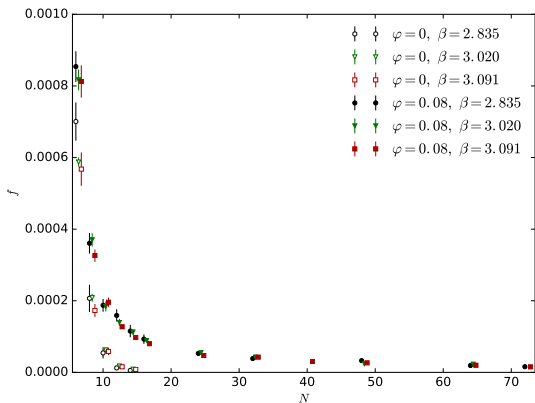


TABLE II. Fits to quantities to a/N^2 , b/N^4 , or Ce^{-kN^2} , with χ^2 .

| Quantity | a | χ^2 | b | χ^2 | c | k | χ^2 |
|--|---------------------|----------|-----------------|----------|--------|-------------------|----------|
| $\langle U_{xy} - \frac{1}{2}(U_{xx} + U_{yy}) \rangle_{tw}$ | 0.0216 ± 0.0027 | 26.5 | 0.53 ± 0.06 | 7.5 | 0.021 | 0.136 ± 0.021 | 1.1 |
| $\langle U \rangle_{tw} - \langle U \rangle_{no}$ | 0.0112 ± 0.0028 | 12.6 | 0.28 ± 0.06 | 5.1 | 0.0187 | 0.165 ± 0.054 | 1.3 |
| $\langle U_{xy} \rangle_{tw} - \langle U_{xy} \rangle_{no}$ | 0.0367 ± 0.045 | 38.6 | 0.89 ± 0.09 | 12.3 | 0.0507 | 0.157 ± 0.031 | 1.5 |

SU(2) Results: Data at $\varphi = 0.08$

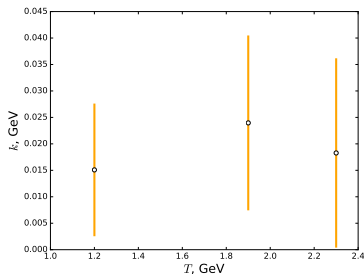
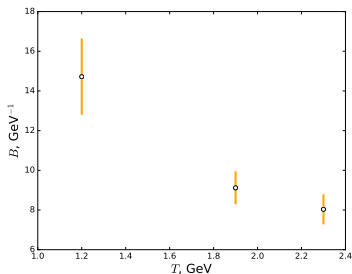
$$f(N) = |\langle U \rangle_{\text{field}} - \langle U \rangle_0|$$



SU(2) Results: Fitting at $\varphi = 0.08$

| Function | $\beta = 2.835$ | | | $\beta = 3.020$ | | | $\beta = 3.091$ | | |
|--------------------|-----------------|---------------------|-------|-----------------|---------------------|-------|-----------------|---------------------|-------|
| | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r |
| A/N^b | 91.2 | 16.9 | ✗ | 170 | 15.5 | ✗ | 223 | 18.3 | ✗ |
| $(A/N^b)e^{-kN}$ | 30.0 | 15.5 | ✗ | 44.9 | 14.1 | ✗ | 69.0 | 16.9 | ✗ |
| $(A/N^b)e^{-kN^2}$ | 47.2 | 15.5 | ✗ | 73.7 | 14.1 | ✗ | 118 | 16.9 | ✗ |
| $Ae^{B/N}e^{-kN}$ | 5.33 | 15.5 | ✓ | 7.14 | 14.1 | ✓ | 7.00 | 16.9 | ✓ |

| | | |
|--------------------------------------|--------------------------------------|--------------------------------------|
| $B = 20.3 \pm 2.64$ | $B = 20.1 \pm 1.84$ | $B = 21.3 \pm 2.01$ |
| $k = (1.09 \pm 0.91) \times 10^{-2}$ | $k = (1.08 \pm 0.75) \times 10^{-2}$ | $k = (6.90 \pm 6.76) \times 10^{-3}$ |

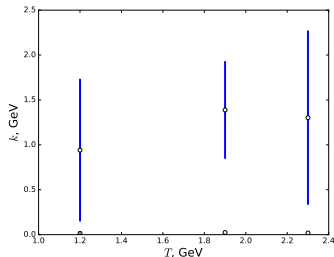
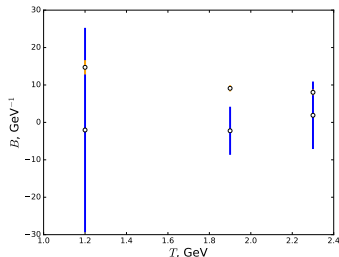


Comparison of the results

$$f(N) = A e^{B/N} e^{-kN}$$

$\varphi = 0$

| $\beta = 2.835$ | | | $\beta = 3.020$ | | | $\beta = 3.091$ | | |
|--------------------------------------|---------------------|-------|--------------------------------------|---------------------|-------|--------------------------------------|---------------------|-------|
| χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r | χ^2_{min} | $\chi^2_{\nu;0.05}$ | a/r |
| 0.36 | 5.99 | ✓ | 1.34 | 5.99 | ✓ | 0.64 | 5.99 | ✓ |
| $B = -2.81 \pm 37.6$ | | | $B = -4.95 \pm 14.2$ | | | $B = 5.04 \pm 23.9$ | | |
| $k = (6.83 \pm 5.76) \times 10^{-1}$ | | | $k = (6.29 \pm 2.46) \times 10^{-1}$ | | | $k = (4.92 \pm 3.66) \times 10^{-1}$ | | |
| $\varphi = 0.08$ | | | $\varphi = 0.08$ | | | $\varphi = 0.08$ | | |
| 5.33 | 15.5 | ✓ | 7.14 | 14.1 | ✓ | 7.00 | 16.9 | ✓ |
| $B = 20.3 \pm 2.64$ | | | $B = 20.1 \pm 1.84$ | | | $B = 21.3 \pm 2.01$ | | |
| $k = (1.09 \pm 0.91) \times 10^{-2}$ | | | $k = (1.08 \pm 0.75) \times 10^{-2}$ | | | $k = (6.90 \pm 6.76) \times 10^{-3}$ | | |



$$m_0 = 1.26 \pm 0.41 \text{ GeV}$$

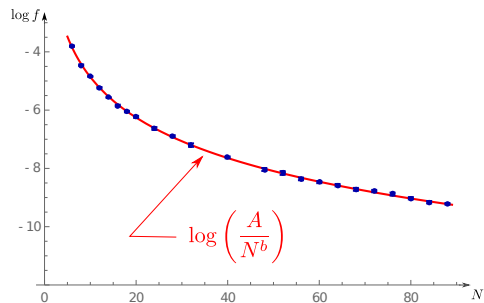
$$m_{0.08} = (1.83 \pm 0.87) \times 10^{-2} \text{ GeV}$$

at 95% CL

SU(3) Results: Data at $\varphi = 0$

$4 \times N^3$ lattices, $N = 6, \dots, 88$

$\beta = 6.4$ ($T \sim 580$ MeV)



$$\chi^2 = 20.2 \quad \chi_{\nu;0.05}^2 = 32.7$$

$$b = 2.00 \pm 0.02$$

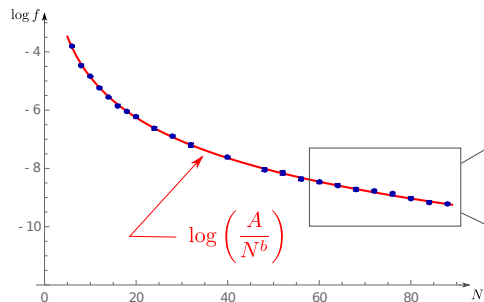
flux tubes

$\chi^2 > \chi_{\nu;0.05}^2 \Rightarrow$ rejection at 95% CL

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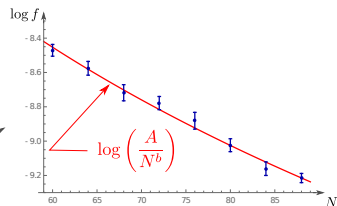


$$\chi^2 = 20.2 \quad \chi_{\nu;0.05}^2 = 32.7$$

$$b = 2.00 \pm 0.02$$

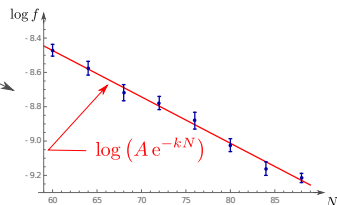
flux tubes

$\chi^2 > \chi_{\nu;0.05}^2 \Rightarrow$ rejection at 95% CL



$$\chi^2 = 3.05 \quad \chi_{\nu;0.05}^2 = 12.6$$

$$b = 1.99 \pm 0.23$$



$$\chi^2 = 2.24 \quad \chi_{\nu;0.05}^2 = 12.6$$

$$b = (2.70 \pm 0.32) \times 10^{-2}$$

- B. Grossman, S. Gupta, U. M. Heller and F. Karsch, “Glueball - like screening masses in pure $SU(3)$ at finite temperatures,” Nucl. Phys. B **417**, 289 (1994) [hep-lat/9309007].
 - External field sources are not introduced
 - m_{el} and m_{magn} are measured through Polyakov loop correlators
 - $2m_{magn} = 5.8(4)T$ @ $T = 1.5T_c$

Conclusions

- Both monopole-antimonopole string and external Abelian field flux are introduced on the lattice.
- Results of the previous investigations for $SU(2)$ gauge group are reproduced.
- In $SU(2)$ it is shown that adding of the Abelian field flux weakens the screening of the string field. This confirms that
 - for the Abelian field $m_{\text{magn}} = 0$;
 - m_{magn} of the monopole-antimonopole string field is produced by its non-Abelian components.
- In $SU(3)$ formation of flux tubes is obtained.

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Thank you for your attention!