Dispersive approach to non-Abelian axial anomaly and η, η' production in heavy ion collisions

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- η and η' mesons are known to be deeply related to Abelian and non-Abelian axial anomalies.
- We generalize the exact anomaly sum rules to the case of non-Abelian axial anomaly and
  apply the results to the processes of η and η' radiative decays and their production in heavy ion collisions.

Outline

Axial anomaly

In QCD, for a given flavor q, the divergence of the axial current $J_{\mu 5}^{(q)} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and strong anomalous terms:

$$\partial_\mu J_{\mu 5}^{(q)} = m_q \bar{q}\gamma_5 q + \frac{e^2}{8\pi^2} e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi} G\tilde{G}, \quad (1)$$

An octet of axial currents

$$J_{\mu 5}^{(a)} = \sum_q \bar{q}\gamma_5\gamma_\mu \frac{\lambda^a}{\sqrt{2}} q$$

Singlet axial current $J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$:

$$\partial^\mu J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(0)} N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi} G\widetilde{G}, \quad (2)$$

The diagonal components of the octet of axial currents
$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$,
$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$
acquire an electromagnetic anomalous term only:

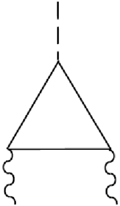
$$\partial^\mu J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(m_u\overline{u}\gamma_5 u - m_d\overline{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi}C^{(3)}N_c F\tilde{F}, \tag{3}$$

$$\partial^\mu J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d - 2m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(8)}N_c F\tilde{F}. \tag{4}$$

The electromagnetic charge factors $C^{(a)}$ are

$$\begin{aligned} C^{(3)} &= \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}, \\ C^{(8)} &= \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}, \\ C^{(0)} &= \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \end{aligned} \tag{5}$$

Anomaly sum rule for the singlet axial current



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$e^2 T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0 | T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\} | 0 \rangle; \qquad (6)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

Anomalous axial-vector Ward identity for the singlet component of axial current:

$$p_\alpha T^{\alpha\mu\nu} = 2mG\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma + \frac{C_0 N_c}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma + N(p^2, q^2, k^2)\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma, \tag{7}$$

where $2mG\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma = \langle 0| \sum_{q=u,d,s} m_q \bar{q}\gamma_5 q|\gamma\gamma\rangle$,

$$\langle 0|\frac{\sqrt{3}\alpha_s}{4\pi}G\tilde{G}|\gamma(k)\gamma(q)\rangle = e^2 N(p^2, k^2, q^2)\epsilon^{\mu\nu\rho\sigma}k_\mu q_\nu \epsilon^{(k)}_\rho \epsilon^{(q)}_\sigma, \tag{8}$$

$$\langle 0|F\tilde{F}|\gamma(k)\gamma(q)\rangle = 2\epsilon^{\mu\nu\rho\sigma}k_\mu q_\nu \epsilon^{(k)}_\rho \epsilon^{(q)}_\sigma. \tag{9}$$

The VVA triangle graph amplitude presented as a tensor decomposition:

$$
\begin{aligned}
T_{\alpha\mu\nu}(k,q) &= F_1\ \varepsilon_{\alpha\mu\nu\rho}k^{\rho} + F_2\ \varepsilon_{\alpha\mu\nu\rho}q^{\rho} \\
&+\ F_3\ k_{\nu}\varepsilon_{\alpha\mu\rho\sigma}k^{\rho}q^{\sigma} + F_4\ q_{\nu}\varepsilon_{\alpha\mu\rho\sigma}k^{\rho}q^{\sigma} \\
&+\ F_5\ k_{\mu}\varepsilon_{\alpha\nu\rho\sigma}k^{\rho}q^{\sigma} + F_6\ q_{\mu}\varepsilon_{\alpha\nu\rho\sigma}k^{\rho}q^{\sigma},
\end{aligned}
\tag{10}
$$

$F_j = F_j(p^2, k^2, q^2; m^2)$, $p = k + q$.
In the kinematical configuration with one real photon ($k^2 = 0$) the anomalous Ward identity can be rewritten in terms of form factors F_j as follows ($N(p^2, q^2) \equiv N(p^2, q^2, k^2 = 0)$):

$$
(q^2 - p^2)F_3 - q^2 F_4 = 2mG + \frac{C_0 N_c}{2\pi^2} + N(p^2, q^2). \tag{11}
$$

– G, F_3, F_4 can be rewritten as dispersive integrals without subtractions. [Horejsi, Teryaev '94]
– N : rewrite it in the form with one subtraction,

$$
N(p^2, q^2) = N(0, q^2) + p^2 R(p^2, q^2), \tag{12}
$$

where the new form factor R can be written as an unsubtracted dispersive integral.

The imaginary part of AWI (11) w.r.t. p^2 (s in the complex plane) reads

$$(q^2 - s)ImF_3 - q^2 ImF_4 = 2mImG + sImR. \tag{13}$$

– Divide every term of Eq. (13) by $(s - p^2)$ and integrate:

$$\frac{1}{\pi}\int_0^\infty \frac{(q^2 - s)ImF_3}{s - p^2}ds - \frac{q^2}{\pi}\int_0^\infty \frac{ImF_4}{s - p^2}ds = \frac{1}{\pi}\int_0^\infty \frac{2mImG}{s - p^2}ds + \frac{1}{\pi}\int_0^\infty \frac{sImR}{s - p^2}ds \tag{14}$$

– After transformation and making use of the dispersive relations for the form factors F_3, F_4, G, R:

$$(q^2 - p^2)F_3 - \frac{1}{\pi}\int_0^\infty ImF_3 ds - q^2 F_4 = 2mG + p^2 R + \frac{1}{\pi}\int_0^\infty ImRds. \tag{15}$$

Comparing (15) with (11) we arrive at the anomaly sum rule for the singlet current:

$$\frac{1}{\pi}\int_0^\infty ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0, q^2) - \frac{1}{\pi}\int_0^\infty ImR(s, q^2)ds, \tag{16}$$

ASR and meson contributions

Saturating the l.h.s. of (16) with resonances according to global quark-hadron duality, we write out the first resonances' contributions explicitly, while the higher states are absorbed by the integral with a lower limit s_0,

$$\Sigma f_M^0 F_{M\gamma}(q^2) + \frac{1}{\pi}\int_{s_0}^{\infty} ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0,q^2) - \frac{1}{\pi}\int_0^{\infty} ImR(s,q^2)ds, \tag{17}$$

where

$$\int d^4x e^{ikx}\langle M(p)|T\{J_\mu(x)J_\nu(0)\}|0\rangle = e^2\epsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma F_{M\gamma}(q^2)\ , \tag{18}$$

$$\langle 0|J_{\alpha 5}^{(a)}(0)|M(p)\rangle = ip_\alpha f_M^a\ . \tag{19}$$

- "Continuum threshold" $s_0(q^2)$ [KOT'11],[Oganesian,Pimikov,Stefanis,Teryaev'15]. $s_0 \gtrsim 1$ GeV^2.
- If one saturates with resonances the last term in the ASR: the glueball-like states.

Low-energy theorem

The matrix element $\langle 0|G\tilde{G}(p)|\gamma(k)\gamma(q)\rangle$?

- No rigorous calculation from the QCD.
- Possible to estimate it in the limit $p^\mu = 0$. [Shifman'88].

We consider the case of two real photons ($q^2 = k^2 = 0$). Supposing that there are no massless particles in the singlet channel in the chiral limit (i.e. *no admixture of the* η):

$$\lim_{p\to 0} p^\mu \langle 0|J_{\mu 5}(p)|\gamma\gamma\rangle = 0,$$

$$\langle 0|\partial^\mu J_{\mu 5}|\gamma\gamma\rangle = 0.$$

Using the explicit expression for the divergence of axial current in the chiral limit (put $m_q = 0$), one can relate the matrix elements of $\langle 0|G\tilde{G}|\gamma\gamma\rangle$ and $\langle 0|F\tilde{F}|\gamma\gamma\rangle$ in the considered limits.

- Mixing: η *spoils the theorem!*

Low-energy theorem for mixing states

Take into account mixing.

$$J_{\mu 5}^{(x)} = aJ_{\mu 5}^{(0)} + bJ_{\mu 5}^{(8)}, \quad \langle 0|J_{\mu 5}^{(x)}|\eta\rangle = 0. \tag{20}$$

$$J_{\mu 5}^{(x)} = b(J_{\mu 5}^{(8)} - \frac{f_\eta^8}{f_\eta^0} J_{\mu 5}^{(0)}), \tag{21}$$

$$\langle 0|J_{\mu 5}^{(i)}(0)|M(p)\rangle = ip_\mu f_M^i. \tag{22}$$

The current (21) gives no massless poles in the matrix element $\langle 0|J_{\mu 5}^{(x)}|\gamma\gamma\rangle$ even in the chiral limit, and therefore

$$\lim_{p\to 0}\langle 0|\partial_\mu J_{\mu 5}^{(x)}(p)|\gamma\gamma\rangle = 0. \tag{23}$$

In the chiral limit, at $p^\mu = 0$:

$$\langle 0|\frac{\sqrt{3}\alpha_s}{4\pi} G\tilde{G}|\gamma\gamma\rangle = \frac{N_c}{f_\eta^8}(f_\eta^0 C^{(8)} - f_\eta^8 C^{(0)})\langle 0|\frac{\alpha_e}{2\pi} F\tilde{F}|\gamma\gamma\rangle. \tag{24}$$

$$N(0,0,0) = \frac{N_c}{2\pi^2 f_\eta^8}(f_\eta^0 C^{(8)} - f_\eta^8 C^{(0)}). \tag{25}$$

Hadron contributions and analysis of the ASR

$$\Sigma f_M^0 F_{M\gamma}(q^2) + \frac{1}{\pi}\int_{s_0}^{\infty} ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0, q^2) - \frac{1}{\pi}\int_0^{\infty} ImR(s, q^2) ds$$

The first hadron contributions to the ASR: η and η'. For real photons, the transition form factors determine the 2-photon decay amplitudes A_M $(M = \eta, \eta')$:

$$A_M \equiv F_{M\gamma}(0) = \sqrt{\frac{64\pi\Gamma_{M\to2\gamma}}{e^4 m_M^3}}. \tag{26}$$

The ASR for the octet channel [KOT'12] for real photons:

$$f_\eta^8 A_\eta + f_{\eta'}^8 A_{\eta'} = \frac{1}{2\pi^2} N_c C^{(8)}. \tag{27}$$

The ASR in the singlet channel:

$$f^0_\eta A_\eta + f^0_{\eta'} A_{\eta'} = \frac{1}{2\pi^2} N_c C_0 + B_0 + B_1, \qquad (28)$$

where

$$B_0 \equiv N(0,0,0),\ B_1 \equiv -\frac{1}{\pi}\int_0^\infty ImR(s)ds - \frac{1}{\pi}\int_{s_0}^\infty ImF_3 ds. \qquad (29)$$

- The B_0 term stands for a subtraction constant in the dispersion representation of gluon anomaly;
- The B_1 term consists of two parts: spectral representation of gluon anomaly and the integral covering higher resonances. The latter is proportional to α_s^2: F_3 is described by a triangle graph (no α_s corrections) plus diagrams with additional boxes ($\propto \alpha_s^2$ for the first box term). The α_s^2 suppression of the box graph contribution is due to $s > s_0 \gtrsim 1$ GeV^2.
- In the case of both real photons in the chiral limit the triangle amplitude is zero ($\propto q^2$). So, B_1 is represented by the integral with the lower limit $s_0 \sim 1$ GeV^2 and is suppressed at least as α_s^2 on the scale of 1 GeV^2.

Combining ASRs for the octet and singlet channels, we obtain the 2-photon decay amplitudes:

$$A_{\eta} = \frac{1}{\Delta}\left(\frac{N_c}{2\pi^2}(C^{(8)}f^0_{\eta'} - C^{(0)}f^8_{\eta'}) - (B_0 + B_1)f^8_{\eta'}\right), \tag{30}$$

$$A_{\eta'} = \frac{1}{\Delta}\left(\frac{N_c}{2\pi^2}(C^{(0)}f^8_{\eta} - C^{(8)}f^0_{\eta}) + (B_0 + B_1)f^8_{\eta}\right), \tag{31}$$

where $\Delta = f^8_{\eta} f^0_{\eta'} - f^8_{\eta'} f^0_{\eta}$.
Making use of the result of the LET for B_0:

$$A_{\eta} = \frac{N_c C^{(8)}}{2\pi^2 f^8_{\eta}} - \frac{B_1 f^8_{\eta'}}{\Delta}, \tag{32}$$

$$A_{\eta'} = \frac{B_1 f^8_{\eta}}{\Delta}. \tag{33}$$

Note, that low energy theorem leads to the cancellation of the photon anomaly term with subtraction part of gluon anomaly B_0 in (31), so the amplitude $\eta' \to \gamma\gamma$ (in the chiral limit) is entirely determined by B_1, i.e., predominantly by the spectral part of the gluon anomaly.

Numerical analysis

Gluon anomaly term contributions for different sets of meson decay constants

	$\begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} \frac{1}{f_\pi}$	$B_0 \times 10^2$	$B_1 \times 10^2$	$(B_0 + B_1) \times 10^2$
[KOT'12], free analysis	$\begin{pmatrix} 1.11 & -0.42 \\ 0.16 & 1.04 \end{pmatrix}$	-5.55	4.91	-0.64
[KOT'12], OS mix. sch.	$\begin{pmatrix} 0.85 & -0.22 \\ 0.20 & 0.81 \end{pmatrix}$	-5.36	3.84	-1.53
[KOT'12], QF mix. sch.	$\begin{pmatrix} 1.38 & -0.63 \\ 0.18 & 1.35 \end{pmatrix}$	-5.58	6.39	0.81
[Escribano,Frere'05], free analysis	$\begin{pmatrix} 1.39 & -0.59 \\ 0.054 & 1.29 \end{pmatrix}$	-5.77	5.86	0.095
[Feldmann,Kroll'98], QF mix. sch.	$\begin{pmatrix} 1.17 & -0.46 \\ 0.19 & 1.15 \end{pmatrix}$	-5.51	5.47	-0.047

- The contribution of gluon anomaly and higher order resonances (expressed by $B_0 + B_1$ term) to the 2-photon decay amplitudes appears to be rather small numerically in comparison with the contribution of electromagnetic anomaly $(1/2\pi^2)N_c C^{(0)} \simeq 0.058$.
- B_0 and B_1 enter the ASR with different signs and almost cancel each other, giving only a small total contribution to the two-photon decay widths of the η and η'.

η/η' ratio in heavy ion collisions

$\langle 0 \mid G\widetilde{G} \mid \eta(\eta')\rangle$ enter J/Ψ decays:

$$R_{J/\Psi} = \frac{\Gamma(J/\Psi \to \eta'\gamma)}{\Gamma(J/\Psi \to \eta\gamma)} = \left| \frac{\langle 0 \mid G\widetilde{G} \mid \eta' \rangle}{\langle 0 \mid G\widetilde{G} \mid \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3, \tag{34}$$

$p_{\eta(\eta')} = M_{J/\Psi}(1 - m^2_{\eta(\eta')}/M^2_{J/\Psi})/2$. [Novikov et al. '80]
Can be evaluated in terms of the decay constants:

$$R_{J/\Psi} = \left(\frac{f^8_{\eta'} + \sqrt{2} f^0_{\eta'}}{f^8_\eta + \sqrt{2} f^0_\eta} \right)^2 \left(\frac{m_{\eta'}}{m_\eta} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \tag{35}$$

$$R_{J/\Psi} = 4.67 \pm 0.15, \quad \left(\frac{p_{\eta'}}{p_\eta} \right)^3 \sim 0.81$$

(used as an additional constraint in [KOT'12])
Similarly, ratio of production of η/η' from gluons (CGC) in HIC: no kinematical factor.

η/η' ratio in heavy ion collisions

Possible sources of $G\tilde{G}$:
– rotating gluon-dominated plasma [Torrieri'18, *"η' Production in Nucleus-Nucleus collisions as a probe of chiral dynamics"*, suggested η'/π^0 as a probe – we use η/η'],
– self-dual fields [Nedelko et al.]
– inclusive process $(G\tilde{G})^2 \sim G^4$.

Multihadron production in HIC – universal thermal pattern with $T \sim 160 - 170$ MeV for hadron abundances and transverse momentum spectra $\rightarrow$ Less η' than η.
Direct gluonic production should dominate at larger transverse momentum. We expect growth of the ratio η'/η at larger transverse momentum. Detailed calculations are still required.

Conclusions

- Employing the dispersive approach to axial anomaly in the singlet current, we obtained the sum rule with photon and gluon anomaly contributions.
- The contributions of gluon and electromagnetic parts of axial anomaly in the $\eta(\eta') \to \gamma\gamma$ decays have been evaluated using the ASR for the singlet axial current.
- LET was generalized for the mixing states and the estimation for the subtraction constant of the gluon anomaly contribution in the dispersive form of axial anomaly was obtained.
- In HIC, the abundance ratio η'/η is expected to grow at larger transverse momentum.

Thank you for your attention!