Analysis of the $e^+e^- \to \pi^0\gamma$ process using anomaly sum rules approach

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Outline

Introduction

Modern situation with $\gamma^*\gamma \to \pi^0$ and $\gamma^* \to \pi^0\gamma$ TFF:

- **SPACE-LIKE**: CELLO $0.7 - 2.2$ GeV^2, CLEO $1.6 - 8.0$ GeV^2; BaBar and Belle up to 40 GeV^2. At the $1.0 \leq Q^2 \leq 9$ all the experiments show same behaviour and agree with theoretical expectations [Lepage& Brodsky,1980]. But at large Q^2 there is contradiction between BaBar and Belle! The more precise data expected in the future from BES-III and KLOE-2.
- **TIME-LIKE**: SND2016 and CMD2 $0.36 - 1.9$ GeV^2 of q^2. Theoretical desrcpition of data were done in the NJL-model [Arbuzov et al.,2011] and by using the methods of the dispersive theory [Hoferichter et al.,2014]. More precise data expected in the future from CLAS.

ASR approach

The VVA-amplitude,

$$T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0| T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\}|0\rangle, \qquad (1)$$

contains an axial current $J^3_{\alpha 5}(0) = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u - \bar{d}\gamma_\alpha\gamma_5 d)$ and two EM currents $J_\mu = \sum_{i=u,d} \bar{q}_i\gamma_\mu q_i$ (in units $|e| = 1$). k and q are the $\gamma's$ momenta, and one of the photons is real $k^2 = 0$. $T_{\alpha\mu\nu}(k,q)$ can be presented as a tensor decomposition:

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\, \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\, \varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\, k_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\, q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \qquad (2) \\ &+ F_5\, k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\, q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned}$$

where coef. $F_j = F_j(k^2, p^2, q^2; m^2), j = 1, ..., 6$ are Loretz invariant amplitudes(formfactors).

ASR approach

As it was shown in [Hořejší&Teryaev'95] the imaginary part $A_3 \equiv \frac{1}{2} Im(F_3 - F_6)$ of the of the invariant amplitude at the tensor structure $k_\nu \varepsilon_{\alpha\mu\rho\sigma} k^\rho q^\sigma$ in the variable $(k+q)^2 = s > 0$ satisfy the relation:

$$\int_0^\infty A_3(s, q^2; m_i^2) ds = \frac{1}{2\pi} \frac{1}{\sqrt{2}}. \quad (3)$$

- The relation (3) is exact!
- Holds for any q^2 and any m^2.
- Does not have α_s corrections (Adler-Bardeen theorem).
- Does not have non-perturbative corrections (t'Hooft consistency principle).

ASR approach

Supposing that A_3 decreases fast enough at $|q^2| \to \infty$ and is analytical everywhere except the cut $q^2 \in (0, +\infty)$, it can be expressed as the dispersive integral without subtractions

$$A_3(s, q^2) = \frac{1}{2\pi} \int_0^\infty dy \frac{\rho(s, y)}{y - q^2 + i\epsilon}, \tag{4}$$

where $\rho = 2Im_{q^2} A_3$. Then, ASR (3) for time-like is given by the double dispersive integral:

$$\int_0^\infty ds \int_0^\infty dy \frac{\rho(s, y)}{y - q^2 + i\epsilon} = \frac{1}{\sqrt{2}} \tag{5}$$

The real and imaginary parts are:

$$p.v. \int_0^\infty ds \int_0^\infty dy \frac{\rho(s, y)}{y - q^2} = \frac{1}{\sqrt{2}}, \tag{6}$$

$$\int_0^\infty ds \rho(s, q^2) = 0, \; . \tag{7}$$

Saturating the lhs of the three-point correlation function (1) with the resonances in the axial channel, singling out the first (pion) contribution and replacing the higher resonance's contributions with the integral of the spectral density, the ASR leads to

$$\pi f_\pi Re F_{\pi\gamma}(q^2) + \int_{s_3}^{\infty} A_3(s,q^2) ds = \frac{1}{2\pi}\frac{1}{\sqrt{2}}, \tag{8}$$

where s_3 is duality region of the pion in the isovector channel, the meson decay constant f_π is

$$\langle 0|J_{\alpha 5}(0)|\pi^0(p)\rangle = i p_\alpha f_\pi, \tag{9}$$

and $F_{\pi\gamma}$

$$\int d^4x e^{ikx} \langle \pi^0(p)|T\{J_\mu(x)J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F(q^2)\ , \tag{10}$$

ASR approach

As the integral of A_3 in eq.(8) is over the region $s > s_3$, we expect that nonperturbative corrections to A_3 in this region are small enough and we can use the one-loop expression for it:

$$A_3 = \frac{e_i^2 N_c}{2\pi} \frac{\Theta(s - 4m_i^2)}{(s - q^2)^2} (-q^2 R^{(i)} + 2m_i^2 ln \frac{1 + R^{(i)}}{1 - R^{(i)}}), \qquad (11)$$

where $R^{(i)} = \sqrt{1 - \frac{4m_i^2}{s}}$. The corresponding double spectral density in the case of massless quarks is

$$\rho^{(i)}(s, y) = e_i^2 N_c \delta'(s - y), \qquad (12)$$

where $\delta'(x) = \frac{d\delta(x)}{dx}$.

Thus the ASR leads to the pion TFF:

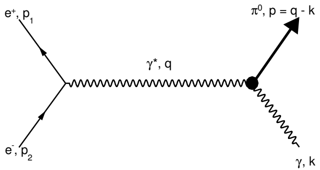
$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_3}{s_3 - q^2}. \quad (13)$$

This result is valid in the region far from the pole.
The numerical value of $s_3 = 4\pi^2 f_\pi^2 \approx 0.67\ GeV^2 \pm 10\%$ was obtained in the limit $-q^2 \to \infty$ of the space-like ASR [Klopot et al.,2011].
Matching of LCSR and ASR in space-like shows that $s_3 \approx 0.61\ GeV^2$ is more preferable at low q^2 [Oganesian et al,2016], but within the $\pm 10\%$ error of the s_3 calculation the value $s_3 = 0.67\ GeV^2$ also agrees with the experiment.

Advantages of the ASR approach

- based on the exact relation, implied by the axial anomaly;
- π^0 TFF can be described in the whole region of q^2(both space-like and time-like);
- model-independent method;
- not relying on the QCD factorization.

Calculation of total cross section



Ampliude of the process is $M_{\pi\gamma}$:

$$M_{\pi\gamma} = ie^3 \bar{u}\gamma^\alpha u \frac{g_\alpha^\mu}{q^2} \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F(q^2)(e^\nu)^* \tag{14}$$

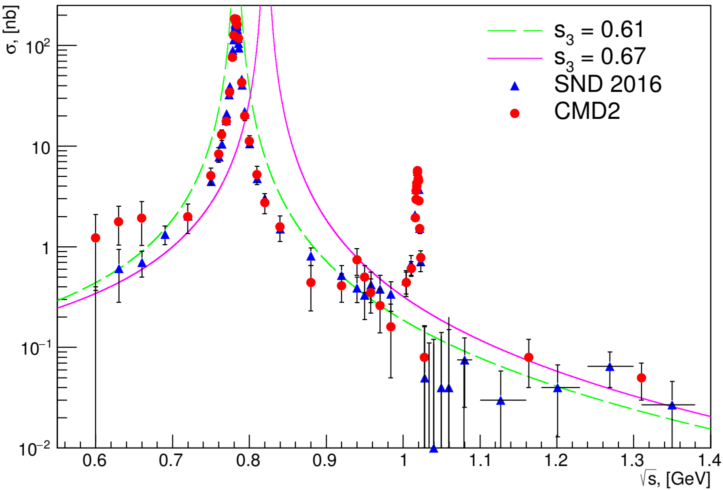
Total cross section:

$$\sigma_{theor} = \frac{2}{3}\pi^2 \alpha_{QED}^3 |F^2(q^2)| = \frac{\alpha_{QED}^3}{12\pi^2 f_\pi^2} \frac{s_3^2}{(s_3 - q^2)^2}. \tag{15}$$

And the angular distribution:

$$\frac{d\sigma}{d\cos\theta}\Big|_\pi^0 = \frac{\alpha_{QED}^3}{32\pi^2 f_\pi^2} \frac{s_3^2}{(s_3 - q^2)^2}(1 + \cos^2\theta). \tag{16}$$

Comparisson with data



Is $s_3(m_v^2) = m_v^2$?

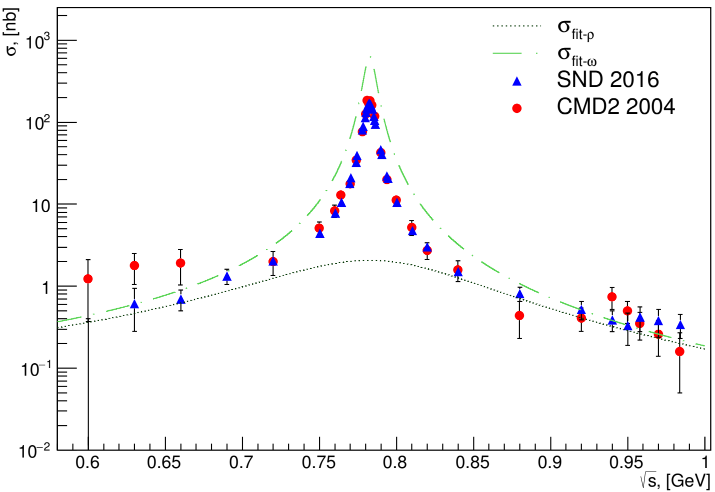
Fit with 1 resonance

Consider first peak. The equation of the pion TFF (13) was obtained using zero width of ρ-meson, so the $ImF(q^2) \sim \delta(s_3 - q^2)$.
We modify (13) by adding to denominator the term of the form $im_v\Gamma_v$:

$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_3}{s_3 - q^2 + im_v\Gamma_v}. \qquad (17)$$

Masses and widths of the ρ , ω mesons are taken from PDG: $m_\rho = 0.77526\ GeV, \Gamma_\rho = 0.149\ GeV, m_\omega = 0.78265\ GeV, \Gamma_\omega = 0.00849\ GeV$.). We obtain two fits for total cross sections:

$$\sigma_{fit-\rho,\omega} = \frac{\alpha_{QED}^3}{12\pi^2 f_\pi^2} \frac{s_{3\rho,\omega}^2}{(s_{3\rho,\omega} - q^2)^2 + m_{\rho,\omega}^2 \Gamma_{\rho,\omega}^2}. \qquad (18)$$



σ, [nb]
10^3
10^2
10
1
10^{-1}
10^{-2}
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
$\sqrt{s}$, [GeV]
$\sigma_{fit-\rho}$
$\sigma_{fit-\omega}$
SND 2016
CMD2 2004

Fit with 2 resonances

Let's take linear combination of terms type (17):

$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\left(\alpha\frac{s_{3\rho}}{s_{3\rho} - q^2 + i m_\rho \Gamma_\rho} + \beta\frac{s_{3\omega}}{s_{3\omega} - q^2 + i m_\omega \Gamma_\omega}\right). \quad (19)$$

The total cross section takes form:

$$\sigma_{fit-2resonances} = \frac{\alpha_{QED}^3}{12\pi^2 f_\pi^2}\Big(\frac{\alpha^2 s_{3\rho}^2}{(s_{3\rho} - q^2)^2 + m_\rho^2\Gamma_\rho^2} + \frac{\beta^2 s_{3\omega}^2}{(s_{3\omega} - q^2)^2 + m_\omega^2\Gamma_\omega^2} + \\ + \frac{2\alpha\beta s_{3\rho}s_{3\omega}((s_{3\rho} - q^2)(s_{3\omega} - q^2) + m_\rho\Gamma_\rho m_\omega\Gamma_\omega)}{((s_{3\rho} - q^2)^2 + m_\rho^2\Gamma_\rho^2)((s_{3\omega} - q^2)^2 + m_\omega^2\Gamma_\omega^2)}\Big). (20)$$

We perform one more fit, supposing that $m_\rho = m_\omega$ and $s_{3\rho} = s_{3\omega} = m_\rho^2 = m_\omega^2 \approx 0.61\ GeV^2$, but with PDG widths: $\Gamma_\rho = 0.149\ GeV, \Gamma_\omega = 0.00849\ GeV$.

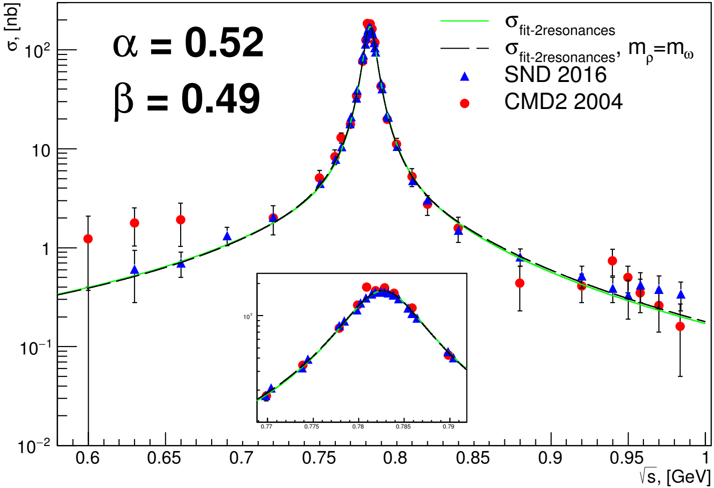


Table: χ^2 for 2 res.

	CMD2	SND2016	CMD2+SND2016
$\chi^2/d.o.f.$	2.29	1.46	1.71

Table: χ^2 for 2 res. with $m_\rho = m_\omega$.

	CMD2	SND2016	CMD2+SND2016
$\chi^2/d.o.f.$	2.12	1.07	1.42

Fit with 3 resonances

Let's take into account second peak, so, in addition to the terms corresponds to ρ and ω mesons, add a term corresponding to the ϕ meson (the value of $m_\phi = 1.0194\ GeV$ and the value of $\Gamma_\phi = 0.00426\ GeV$ were taken from PDG). Suppose, that $s_{3\phi}$ will be close to the m_ϕ^2 and $m_\rho = m_\omega, s_{3\rho} = s_{3\omega} = m_\rho^2 = m_\omega^2 = s_3 \approx 0.61\ GeV^2$. Thus, we got:

$$F(q^2) = \frac{1}{2\sqrt{2}\pi^2 f_{\pi^0}} \times \\ \times \left(\frac{\alpha s_3}{s_3 - q^2 + i m_\rho \Gamma_\rho} + \frac{\beta s_3}{s_3 - q^2 + i m_\omega \Gamma_\omega} + \frac{\gamma s_{3\phi}}{s_{3\phi} - q^2 + i m_\phi \Gamma_\phi} \right). \quad (21)$$

Note, due to the far from the poles one should obtain (10) :$\alpha + \beta + \gamma \approx 1$

α = 0.556, β = 0.49, γ = -0.036

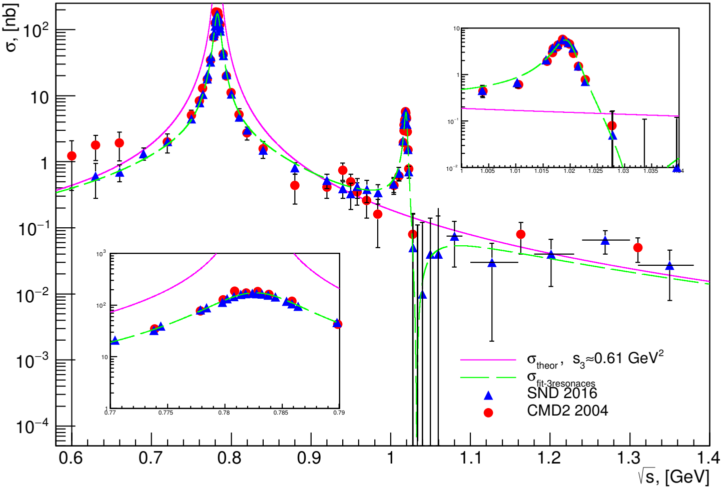


Table: χ^2 for fit with 3 resonances.

	CMD2	SND2016	CMD2+SND2016
$\chi^2/d.o.f.$	2.53	1.52	1.87

Note, that the contribution of the third term of (21) is sufficient only at the region of ϕ meson. So, the littleness of γ is compensates by the littleness of Γ_ϕ.
The third term in (21) can appear due to effects of $\pi - \eta - \eta'$ mixing. So, the coefficient γ corresponds to the mixing angle $\theta_{\pi-\eta}$. Theoretical prediction was done in [IOFFE,Oganesian 2007]:

$$\theta_{\pi-\eta} = \frac{1}{\sqrt{3}} \frac{m_u - m_d}{m_u + m_d} \cdot \frac{m_\pi^2}{m_\eta^2} = -0.0150 \pm 0.020. \qquad (22)$$

So, the values of γ and $\theta_{\pi-\eta}$ are consistent by the order of magnitude.
Also note, that with increasing of q^2 difference between (15) and the total cross section with (21) is decreasing.

Asymptotic at $q^2 \to 0$

In the limit of real photon $q^2 \to 0$ and neglecting $\Gamma_\rho, \Gamma_\omega, \Gamma_\phi$ the pion TFF takes form:

$$F(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}(\alpha + \beta + \gamma).$$

And correspondingly one can find

$$\Gamma(\pi^0 \to 2\gamma) \approx 7.9\ eV,$$

which is in the perfect agreement with experimental data [JAGER 2008].

Summary

- π^0 time-like TFF obtained by using ASR can be used to describe the data in the regions far from the pole, and the place of the pole coincides with the experimental peak.
- $s_3 \approx 0.61\ GeV^2$ has better agreement with the data than $s_3 \approx 0.67\ GeV^2$. This one coincides with the result of matching ASR and LCSR, that s_3 should vary between $0.67\ GeV^2$ at large q^2 and $0.61\ GeV^2$ at low q^2.
- There was proposed method to describe data in the region of the poles. Equation of pion TFF should have 3 terms. To obtain 3 terms in the π^0 TFF within the ASR approach one should include the effects of the $\pi - \eta - \eta'$ mixing.
- The values of the coef. γ and $\theta_{\pi-\eta}$ are consistent by the order of magnitude, so it confirms our assumption.
- Modified π^0 TFF has correct limit in the case of real photons ($q^2 \to 0$).

Thank you for the attention!