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QUARK-ANTIQUARK SYSTEM in ULTRA-INTENSE MAGNETIC FIELD (MF)

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Motivations and Purposes:

- MF ~ 10^{18}–10^{20} G generated in heavy-ion collisions
- Nontrivial behavior of Positronium (Shabad-Usov) and Hydrogen (Vysotsky) in MF
- Vacuum instability for charged vector particles in MF (Ambjorn-Olesen, Schramm-Mueller, Chernodub)
- Chiral dynamics in MF
- Collapse?

Motivations And Purposes

MF Hierarchy (in G)

Medical MPI scan	10^4
ATLAS at LHC	$4 \cdot 10^4$
Lab. (without destruction)	10^6
Lab. (explosion)	$28 \cdot 10^6$
Atomic: $l_B = (eB)^{-1/2} = a_{Bohr}$	$2.35 \cdot 10^9$
Schwinger: $eB = m_e^2$	$4.4 \cdot 10^{13}$
Surface of magnetars	$10^{14} - 10^{15}$
Squeezing ρ: $(eB)^{-1/2} \sim a_\rho \sim 0.6\,fm$	$2 \cdot 10^{19}$
Formal ρ-condensation: $m_\rho^2 + eB(1-g_\rho) = 0$	10^{20}
RHIC and LHC	$10^{18} - 10^{20}$
Early Universe	10^{24}

- A side remark: what is the real MF duration at RHIC and LHC?

$\tau \sim 0.2$ fm – max overlap of colliding nuclei

(Kharzeev-McLerran-Warringa
Skokov-Illarionov-Toneev)

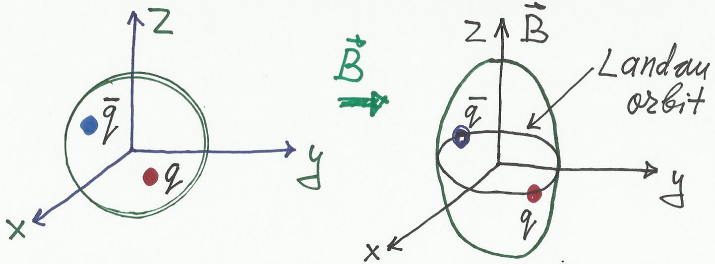
But Magnetic response of the produced matter

$\tau' \simeq \sigma L$, let $\sigma/T \simeq 0.5$, $T \simeq 200$ MeV, $L \simeq 2$ fm

$\tau' \simeq 2$ fm

(Baym, Tuchin, Andreichikov-B.K.)

- Strong MF reveals quark structure of mesons



A quest for: $r_\perp(B)$, $r_\parallel(B)$, $M(B)$

? Magnetic collapse: $M(u\uparrow, \bar{u}\downarrow) \to 0$?

$(u\bar{u})$ as a mock ρ^0-meson

Starting from scratch: Pseudomomentum and w.f. factorization

in QM: Lamb, Gor'kov–Dzyaloshinskii, Simon

$$\hat{H}_{QM} = \frac{1}{2m}\left(\hat{\vec{p}}_1 - e\vec{A}(\vec{x}_1)\right)^2 + \frac{1}{2m}\left(\hat{\vec{p}}_2 + e\vec{A}(\vec{x}_2)\right)^2 + V(\vec{\eta} = \vec{x}_1 - \vec{x}_2), \quad Q = 0$$

$$\vec{R} = \frac{\vec{x}_1 + \vec{x}_2}{2}, \quad \vec{\eta} = \vec{x}_1 - \vec{x}_2, \quad \underline{P} = \vec{p}_1 + \vec{p}_2, \quad \hat{\vec{\pi}} = -i\frac{\partial}{\partial \vec{\eta}}, \quad \hat{\underline{\vec{P}}} = -i\frac{\partial}{\partial \vec{R}}$$

$$\vec{A} = \frac{1}{2}(\vec{B} \times \vec{\Sigma}) \Rightarrow \hat{H} = \frac{1}{4m}\left(\underline{\vec{P}} - \frac{e}{2}(\vec{B} \times \vec{\eta})\right)^2 + \frac{1}{m}\left(-i\frac{\partial}{\partial \vec{\eta}} - \frac{e}{2}(\vec{B} \times \vec{R})\right)^2 + V(\vec{\eta})$$

Pseudomomentum $\hat{\vec{F}} = \underline{\vec{P}} + \frac{e}{2}(\vec{B} \times \vec{\eta}), \quad [\hat{F}, \hat{H}] = 0$

Factorization: $\Psi(\vec{R}, \vec{\eta}) = \varphi(\vec{\eta}) \exp\left\{ i\vec{P}\vec{R} - i\frac{e}{2}(\vec{B} \times \vec{\eta})\vec{R} \right\}$

$$\hat{H}\Psi = E\Psi \rightarrow \frac{1}{4m}\left(\vec{P} - e\vec{B} \times \vec{\eta}\right)^2 \varphi - \frac{1}{m}\frac{\partial^2 \varphi}{\partial \vec{\eta}^2} + V(\vec{\eta})\varphi = E\varphi.$$

Relativistic Problem

How to solve? • Bethe-Salpeter – perplexing, maybe worth to try

Fock-Schwinger-Feynman (FSR, better FFSR)

Fock 1937 – proper time, Schwinger 1951 – proper time Dirac propagator
Feynman 1950-1951 – path integral

in QCD: Yu.A. Simonov and co-authors – $q\bar{q}$ with confinement, meson and baryon spectrum, decays, magnetic moments

FFSR is more tractable, though has its own problems

Strategy: Path Integral FFSR for $q\bar{q}$ Green's function $\rightarrow$ Minkowski space Hamiltonian $\rightarrow$ w.f. and spectrum, strong MF limit.

Key points:

1) The lowest order gluon correlator
2) Minimal area Wilson loop

$\Rightarrow V(\vec{r}) = \sigma r$, $\sigma = 0.2\ GeV^2$

3) Einbein fields $\omega_i(t) \rightarrow$ Hamiltonian without square roots
4) Stationary point for $\int D\omega_i(t)$
5) $\hat{H}\psi = M(\omega_i)\psi$, $\dfrac{\partial M(\omega_i)}{\partial \omega_i} = 0 \Rightarrow M(B)$.

FFSR – a Reminder

a toy example: $\mathcal{L}_\varphi = \frac{1}{2}\left|(\partial_\mu - igA_\mu)\varphi\right|^2 + \frac{1}{2}m^2\varphi^2$, $A_\mu = A_\mu^a(x)t^a$

non-abelian

$$G(x,y) = (m^2 - D_\mu^2)^{-1}_{xy} = \langle x | \underline{P} \int_0^\infty ds\, e^{-s(m^2 - D_\mu^2)} | y \rangle$$

second step

$$G(x,y) = \int_0^\infty ds\, (\mathcal{D}z)_{xy}\, e^{-K}\, P\,\exp\left(ig\int_y^x A_\mu(z)\, dz_\mu\right)$$

$$K = m^2 s + \frac{1}{4}\int_0^s d\tau \left(\frac{dz_\mu}{d\tau}\right)^2$$

$(\mathcal{D}z)_{xy}$: $\langle x | \int_0^\infty ds\, e^{sD_\mu^2} | y \rangle = \langle x | e^{\varepsilon D_\mu^2(N)} | x_{n-1} \rangle \times \ldots$

$\ldots \langle x_1 | e^{\varepsilon D_\mu^2(1)} | y \rangle.$

FFSR for quark in QCD+QED

For Dirac particle $S = (m+\hat{D})^{-1} = (m-\hat{D})(m^2-D^2)^{-1} =$

$$= (m-\hat{D}) \int_0^\infty ds\, e^{-s(m^2-D^2)}$$

QCD+QED: $S(x,y) = (m+\hat{\partial} - ig\hat{A} - ie\hat{A}^{(e)})^{-1}_{xy} \equiv (m+\hat{D})^{-1}_{xy} =$

$$= (m-\hat{D}) \int_0^\infty ds\, (\mathcal{D}z)_{xy}\, e^{-K}\, \Phi(x,y) \equiv (m-\hat{D})\, G(x,y)$$

$$\Phi(x,y) = P_A P_F \exp\left(ig \int_y^x A_\mu dz_\mu + ie \int_y^x A_\mu^{(e)} dz_\mu\right) \times$$

$$\times \exp\left(\int_0^s d\tau\, \sigma_{\mu\nu} (g F_{\mu\nu} + e B_{\mu\nu})\right), \quad \sigma_{\mu\nu} F_{\mu\nu} = \begin{pmatrix} \vec{\sigma}\vec{H} & \vec{\sigma}\vec{E} \\ \vec{\sigma}\vec{E} & \vec{\sigma}\vec{H} \end{pmatrix}$$

QCD QED

$$\sigma_{\mu\nu} B_{\mu\nu} = \begin{pmatrix} \vec{\sigma}\vec{B} & 0 \\ 0 & \vec{\sigma}\vec{B} \end{pmatrix}$$

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$$\sigma_{\mu\nu}B_{\mu\nu} = \begin{pmatrix} \vec{\sigma}\vec{B} & 0 \\ 0 & \vec{\sigma}\vec{B} \end{pmatrix}$$

FFSR for $q\bar{q}$

$$G(x_1,x_2,y_1,y_2)=\int ds_1\int ds_2\, \mathcal{D}z_1\mathcal{D}z_2\, e^{-K_1-K_2}\langle W\rangle_A \Phi_1\Phi_2$$

$$\Phi_j=\exp\left\{ie_j\int_{y_j}^{x_j}A_\mu^{(e)}dz_\mu^{(i)}+e_j\int_0^{s_j}d\tau_j(\vec{\sigma}\vec{B})\right\},\quad j=1,2$$

$$\langle W\rangle_A=\exp\left(-\int_0^{T_E}dt_E\left[\sigma\left|\vec{z}^{(1)}-\vec{z}^{(2)}\right|-\frac{4}{3}\frac{\alpha_s}{\left|\vec{z}^{(1)}-\vec{z}^{(2)}\right|}\right]\right)$$

The next task – Hamiltonian. To get it in a workable form one introduces Einbein auxilary fields (effective masses), Brink, Di Vecchia, Howe 1977 Dirac

one-legged

The uses of Einbein–Examples

Classical relativistic particle, proper time Lagrangian

1. $L = -m\sqrt{\dot{x}^2} \to -\frac{m^2}{2\omega} - \frac{\omega\dot{x}^2}{2}$

$\omega_0 : f'(\omega) = 0, \ \frac{m^2}{2\omega^2} - \frac{\dot{x}^2}{2} = 0, \ \omega_0 = \frac{m}{\sqrt{\dot{x}^2}} \Rightarrow L(\omega_0) = -m\sqrt{\dot{x}^2}$

2. $\int_0^\infty \frac{d\omega}{\sqrt{\omega}} e^{-\frac{\Delta t}{2}\left(\omega + \frac{\vec{p}^2 + m^2}{\omega}\right)} =$ stationary point $=$

$= \sqrt{\frac{2\pi}{\Delta t}}\, e^{-\Delta t\sqrt{\vec{p}^2 + m^2}}$

Einbein for $q\bar{q}$

(derivation of $\hat{H}$)

Two suppositions:

1) $\int \mathcal{D}\omega$ – stationary point
2) $\omega > 0$ – no backtracking – no pair creation

(1) is harmless, proven by numerous calculations

(2) prevents to come close to lower continuum

Einbein for $V(r) = \sigma r \to \sigma\left(\frac{r^2}{2\gamma} + \frac{\gamma}{2}\right)$

Permits analytical solution

Pseudomomentum with ω_1, ω_2 Einbein fields

$$\vec{R} = \frac{\omega_1 \vec{z}_1 + \omega_2 \vec{z}_2}{\omega_1 + \omega_2}, \quad \vec{\eta} = \vec{z}_1 - \vec{z}_2$$

$$\bar{\omega} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}, \quad s = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}$$

$$\hat{\vec{p}}_1 = \hat{\vec{\pi}} + \frac{\bar{\omega}}{\omega_2} \hat{\vec{P}}, \quad \hat{\vec{p}}_2 = -\hat{\vec{\pi}} + \frac{\bar{\omega}}{\omega_1} \hat{\vec{P}}, \quad \vec{B} \equiv e\vec{B}$$

$$\hat{H} = \frac{1}{2\bar{\omega}} \left(\hat{\vec{\pi}} - \frac{1}{2} \vec{B} \times \vec{R} + \frac{s}{2} \vec{B} \times \vec{\eta} \right)^2 + \frac{1}{2(\omega_1 + \omega_2)} \left(\hat{\vec{P}} - \frac{1}{2} \vec{B} \times \vec{\eta} \right)^2 + \frac{\sigma}{2} \left(\frac{\eta^2}{\gamma} + \gamma \right)$$

the Hamiltonian

$$\hat{H} = H_0 + V_{conf} + V_{Coul} + H_\sigma + V_{\sigma\sigma} + W$$

$$H_0 = \frac{1}{2\omega}\left(-\frac{\partial^2}{\partial \vec{\eta}^{\,2}} + \frac{e^2}{4}(\vec{B}\times\vec{\eta})^2\right)$$

$$H_{conf} = \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma} + \gamma\right), \quad H_\sigma = -\frac{e\vec{\sigma}\vec{B}}{2\omega_1} + \frac{e\vec{\sigma}\vec{B}}{2\omega_2},$$

$$V_{\sigma\sigma} = \frac{8\pi}{9}\,\frac{\alpha_{hf}|\psi(0)|^2}{\omega_1\omega_2}(\vec{\sigma}_1\vec{\sigma}_2),$$

$$W = \frac{m_1^2+\omega_1^2}{2\omega_1} + \frac{m_2^2+\omega_2^2}{2\omega_2} - \frac{2\sigma}{\pi\omega_1} - \frac{2\sigma}{\pi\omega_2}$$

Spectrum and w.f.

$$\hat{H}\Psi = M(\omega_i)\Psi, \quad \frac{\partial M(\omega_i)}{\partial \omega_i} = 0, \quad \frac{\partial M(\gamma)}{\partial \gamma} = 0.$$

Example: single q in MF

$$M(\omega) = \frac{p_z^2 + m^2 + |eB|(2n+1) - eB\sigma_z}{2\omega} + \frac{\omega}{2}$$

$$\frac{dM(\omega)}{d\omega} = 0 \Rightarrow M_n = \left(p_z^2 + m^2 + |eB|(2n+1) - eB\sigma_z\right)^{1/2}$$

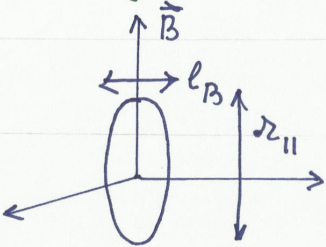
O.K.

Spectrum and w.f.

$$(H_0 + V_{conf})\,\psi(\vec{\eta}) = M(\omega_1 \omega_2 \gamma)\,\psi(\vec{\eta})$$

$$\psi(\vec{\eta}) = \frac{1}{\sqrt{\pi^{3/2} r_\perp^2 r_\parallel}} \exp\left(-\frac{\eta_\perp^2}{2 r_\perp^2} - \frac{\eta_z^2}{2 r_\parallel^2}\right)$$

strong MF: $r_\perp \to (eB)^{-1/2} = l_B$, $r_\parallel \to \frac{1}{\sqrt{\sigma}}$



elongated "ρ"-meson

Color Coulomb collapse

$$V_{Coul} = -\frac{4}{3}\frac{\alpha_s(r)}{r}, \quad \alpha_s(q) = \frac{4\pi}{\beta \ln\left(\frac{q^2+M^2}{\Lambda^2_{QCD}}\right)}$$

$M \propto \sqrt{\sigma}$, $M \sim 1\,\mathrm{GeV}$

ΔM_{Coul} – upper bound:

$$\Delta M_{Coul} = \int \frac{d\vec{q}}{(2\pi)^3} V_{Coul}(q)\,\varphi^2(q) \Rightarrow -\sqrt{\sigma}\ln\ln\frac{eB}{\sigma}$$

1. $M \to 0$ slowly, $(eB)_{crit} \simeq 2.5 \cdot 10^{23}\,G$
2. Stabilization due to loops (à la Hydrogen, Vysotsky et al)

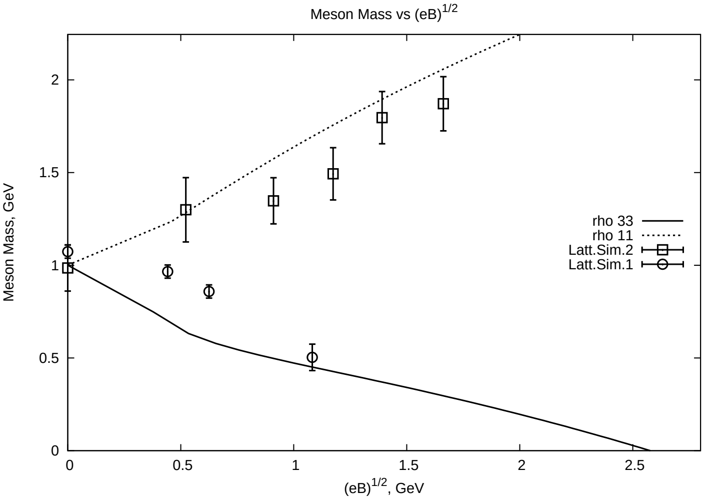
Hyperfine collapse

$$V_{\delta\sigma} \propto |\psi(0)|^2 \propto eB \,!$$

Very preliminary: loops do not save

Results:

- Two spin configurations considered $|u\uparrow \bar{u}\downarrow\rangle$ and $|u\downarrow \bar{u}\uparrow\rangle$
- Massless quarks
- See the figure $\rightarrow$



Meson Mass vs (eB)^1/2
Meson Mass, GeV
(eB)^1/2, GeV
rho 33
rho 11
Latt.Sim.2
Latt.Sim.1
0
0.5
1
1.5
2
2.5

Conclusions

- In strong MF mesons should be treated as composite objects
- In MF mesons change their masses and shape – elongated ellipsoid
- Spin and isospin get violated, $\rho-\pi$ mixing
- $u\bar{u}$ and $d\bar{d}$ are splitted
- Collapse for the lowest state calls for further analysis