Beam energy scan using a viscous hydro+cascade model

Iurii KARPENKO

Frankfurt Institute for Advanced Studies/
Bogolyubov Institute for Theoretical Physics

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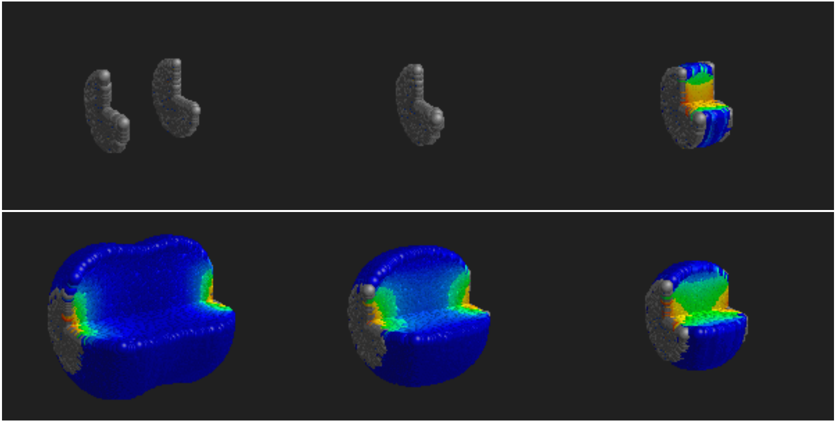
In collaboration with M. Bleicher, P. Huovinen, H. Petersen
`arXiv:1310.0702` `arXiv:1311.0133` `arXiv:1312.4160 [nucl-th]`

Introduction: heavy ion collision in pictures[1]



Typical size
10 fm $\propto 10^{-14}$m

Typical lifetime
10 fm/c $\propto 10^{-23}$s

10^{-8}sec after the collision: hadrons are detected

[1] https://www.jyu.fi/fysiikka/tutkimus/suurenergia/urhic/anim1.gif/image_view_fullscreen

“Stages of Heavy Ion Collision”

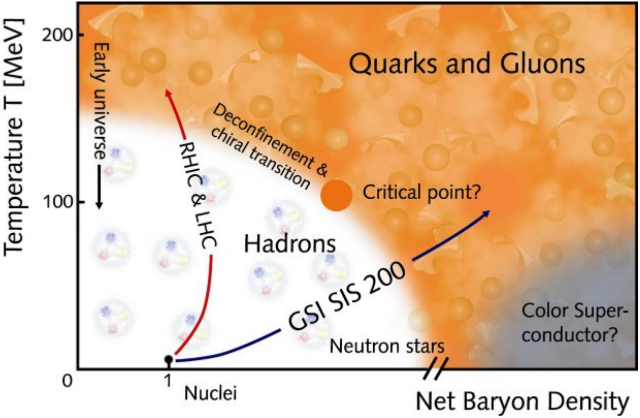
1. Initial(pre-thermal) stage
   - Thermalization
2. Hydrodynamic expansion
   - Quark-gluon plasma phase
   - Phase transition
   - Hadron Gas phase
   - Chemical freeze-out
   - End of hydrodynamic regime
3. Kinetic stage
   Kinetic freeze-out
   $\Downarrow$
   Free streaming, then hadrons are detected

$\Leftrightarrow$

1. **Ingredients of hydro+cascade model**:

1. Initial stage model
   Enforced thermalization
2. Hydrodynamic solution
   - Equation of state for hydrodynamics
   - transport coefficients
3. Particlization and switching to a cascade

Where do we want to apply our model



- small net baryon density: hydro(+cascade) model is well established
  arXiv: “hydrodynamic” + “RHIC” = 44 manuscripts. Existing codes (by author):
  Kolb, Song, Hirano, Nonaka, Chaudhuri, Mota, Luzum, Holopainen, Schenke, Bozek, Molnar, Del Zanna, IuK,...

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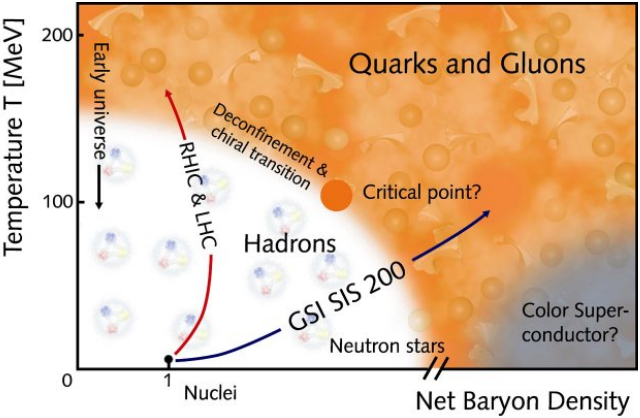

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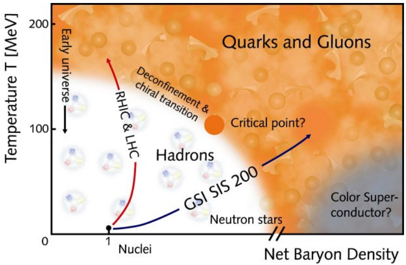
- large net baryon density:
  arXiv: “hydrodynamic” + “SPS” = 8 manuscripts
  arXiv: “hydrodynamic” + “FAIR” = 3 manuscripts



Ingredients essential for beam energy scan studies are marked red.

EoS reference: J. Steinheimer, S. Schramm and H. Stocker, J. Phys. G 38, 035001 (2011).

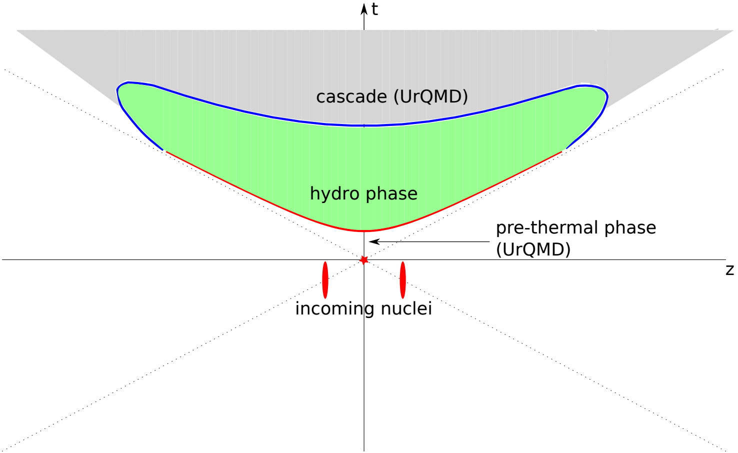
1. **Ingredients of the model**:

1. Initial stage:
   UrQMD
2. Hydrodynamic solution
   - Equation of state for hydrodynamics:
     Chiral model coupled to Polyakov loop to include the deconfinement phase transition
     - ★ good agreement with lattice QCD data at $\mu_B = 0$
     - ★ Applicable also at finite baryon densities
   - transport coefficients
3. Particlization and switching back to cascade (UrQMD)

Initial conditions for hydrodynamic evolution

Time to switch from UrQMD to fluid description: $\tau = \frac{2R}{\gamma v_z} = \frac{2R}{\sqrt{(\sqrt{s}/2m_N)^2 - 1}}$

where $\tau = \sqrt{t^2 - z^2}$. Switching surface is the red curve



$\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid = averaged $\{T^{0\mu}, N_b^0, N_q^0\}$ of particles

Hydrodynamic phase

The hydrodynamic equations in arbitrary coordinate system:

$$\partial_{;\nu} T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu_{\nu\lambda} T^{\nu\lambda} + \Gamma^\nu_{\nu\lambda} T^{\mu\lambda} = 0, \quad \partial_{;\nu} N^\nu = 0 \tag{1}$$

where (we choose Landau definition of velocity)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p+\Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu} \tag{2}$$

and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Evolutionary equations for shear/bulk, coming from **Israel-Stewart** formalism:

$$< u^\gamma \partial_{;\gamma} \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\text{NS}}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma}u^\gamma \tag{3a}$$

where

$$< A^{\mu\nu} >= (\frac{1}{2}\Delta^\mu_\alpha \Delta^\nu_\beta + \frac{1}{2}\Delta^\nu_\alpha \Delta^\mu_\beta - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})A^{\alpha\beta}$$

* Bulk viscosity $\zeta = 0$, charge diffusion=0

Coordinate transformations (hydro phase)

Milne coordinates

The coordinate system is defined as follows:

0) $\tau = \sqrt{t^2 - z^2}$

1) $x = x$

2) $y = y$

3) $\eta = \frac{1}{2} ln \frac{t+z}{t-z}$

$$g^{\mu\nu} = diag(1, -1, -1, -1/\tau^2)$$

Nonzero Christoffel symbols are:

$$\Gamma^{\eta}_{\tau\eta} = \Gamma^{\eta}_{\eta\tau} = 1/\tau, \quad \Gamma^{\tau}_{\eta\eta} = \tau$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p \cdot g^{\mu\nu}, \text{ where}$$

$$u^\mu = \{\cosh(\eta_f - \eta)\cosh\eta_T, \sinh\eta_T\cos\phi, \sinh\eta_T\sin\phi, \frac{1}{\tau}\sinh(\eta_f - \eta)\cosh\eta_T\}$$

(cf. $u^i_{\text{Cart}} = \{\cosh(\eta_f)\cosh\eta_T, \sinh\eta_T\cos\phi, \sinh\eta_T\sin\phi, \sinh(\eta_f)\cosh\eta_T\}$)

EM conservation equations are

$$\partial_{;\nu} T^{\mu\nu} = 0$$

or

$$\mu = 0: \quad \partial_\nu T^{\tau\nu} + \tau T^{\eta\eta} + \frac{1}{\tau} T^{\tau\tau} = 0$$

$$\mu = 1: \quad \partial_\nu T^{x\nu} + \frac{1}{\tau} T^{x\tau} = 0$$

$$\mu = 2: \quad \partial_\nu T^{y\nu} + \frac{1}{\tau} T^{y\tau} = 0$$

$$\mu = 3: \quad \partial_\nu T^{\eta\nu} + \frac{3}{\tau} T^{\eta\tau} = 0$$

Additional transformations:

$T^{\mu\eta} \to T^{\mu\eta}/\tau,\ \mu \neq \eta,$
$T^{\eta\eta} \to T^{\eta\eta}/\tau^2$

$\Downarrow\Downarrow$

$$\partial_\nu(\tau T^{\tau\nu}) + \frac{1}{\tau}(\tau T^{\eta\eta}) = 0$$

$$\partial_\nu(\tau T^{x\nu}) = 0$$

$$\partial_\nu(\tau T^{y\nu}) = 0$$

$$\partial_\nu(\tau T^{\eta\nu}) + \frac{1}{\tau}\tau T^{\eta\tau} = 0$$

Conservative variables are
$Q^\mu = \tau \cdot T^{\tau\mu}$

Closer to numerics:

$$\partial_\mu(T^{\mu\nu}_{\rm id}+\delta T^{\mu\nu})=S^\nu, \qquad \text{S=geometrical source terms}$$

$$\partial_\tau\underbrace{(T^{\tau i}_{\rm id}+\delta T^{\tau i})}_{Q_i}+\partial_j\underbrace{(T^{ji})}_{\text{id.flux}}+\partial_j\underbrace{(\delta T^{ji})}_{\text{visc.flux}}=\underbrace{S^\nu_{\rm id}+\delta S^\nu}_{\text{source terms}}$$

Finite-volume realization:

$$\frac{1}{\Delta\tau}(Q^{n+1}_{\rm id}+\delta Q^{n+1}-Q^n_{\rm id}-\delta Q^n)+\frac{1}{\Delta x}(\Delta F^{n+1/2}_{\rm id}+\Delta\delta F^{n+1/2})=S^{n+1/2}_{\rm id}+\delta S^{n+1/2}$$

now, a small trick:

$$\frac{1}{\Delta\tau}(Q^{n+1}_{\rm id}+\delta Q^{n+1}\underbrace{-Q^{*n+1}_{\rm id}+Q^{*n+1}_{\rm id}}_{=0}-Q^n_{\rm id}-\delta Q^n)+\frac{1}{\Delta x}(\Delta F_{\rm id}+\Delta\delta F)=S_{\rm id}+\delta S$$

Then,split the equation into two parts[2]:

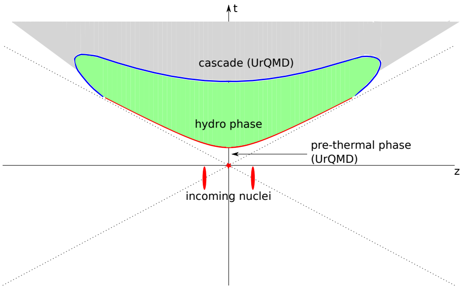
$$\frac{1}{\Delta t}(Q^{*n+1}_{\rm id}-Q^n_{\rm id})+\frac{1}{\Delta x}\Delta F_{\rm id}=S_{\rm id} \quad \text{(using finite volume, HLLE approx)} \tag{4}$$

$$\frac{1}{\Delta t}(Q^{n+1}_{\rm id}+\delta Q^{n+1}-Q^{*n+1}_{\rm id}-\delta Q^n)+\frac{1}{\Delta x}\Delta\delta F=\delta S \quad \text{(upwind/Lax-Wendroff)} \tag{5}$$

[2] Makoto Takamoto, Shu-ichiro Inutsuka, J.Comput.Phys. 230 (2011), 7002

Fluid→particle transition

$\varepsilon = \varepsilon_{sw} = 0.5$ GeV/fm^3 (blue curve):
$\{T^{0\mu}, N_b^0, N_q^0\}$ of hadron-resonance gas = $\{T^{0\mu}, N_b^0, N_q^0\}$ of fluid



▷ Space and momentum distribution from Cooper-Frye prescription:

$$p^0 \frac{d^3 n_i}{d^3 p} = \int \left(f_{\rm l.eq.}(x,p) + \delta f(x,p) \right) p^\mu d\sigma_\mu$$

▷ Cornelius subroutine* is used to compute $\Delta\sigma_i$ on transition hypersurface.
▷ UrQMD cascade is employed after particlization surface.

*Huovinen P and Petersen H 2012, *Eur.Phys.J.* A **48** 171

Particle sampling

For each surface element:

$$\Delta N_i = \Delta\sigma_\mu u^\mu n_{\mathrm{i,th}} = \Delta\sigma_0^* n_{\mathrm{i,th}}$$

Momentum distribution in a fluid rest frame[3]:

$$\frac{d^3N_i}{dp^* d(cos\theta) d\phi} = \underbrace{\frac{\Delta\sigma_\mu^* p^{*\mu}}{p^{*0}}}_{W_{\mathrm{residual}}} \underbrace{p^{*2} f_{\mathrm{eq}}(p^{*0}; T, \mu_i)}_{\mathrm{isotropic}} \underbrace{\left[1 + (1 \mp f_{\mathrm{eq}}) \frac{p_\mu^* p_\nu^* \pi^{*\mu\nu}}{2T^2(\varepsilon + p)}\right]}_{W_{\mathrm{visc}}} \qquad (6)$$

Ideal case: $max\left(\frac{\Delta\sigma_\mu^* p^{*\mu}}{p^{*0}}\right) = max(\Delta\sigma_0^* + |\Delta\vec{\sigma}_i^*|)$

Momentum generation procedure:

- momentum sampling according to isotropic part of DF
- correction according to W_{residual} or $W_{\mathrm{residual}} \cdot W_{\mathrm{visc}}$
- Lorentz boost to global frame

[3] N.S. Amelin et al, Phys.Rev.C74:064901, 2006 (FASTMC event generator)

Finally, generated particle are fed to UrQMD cascade.

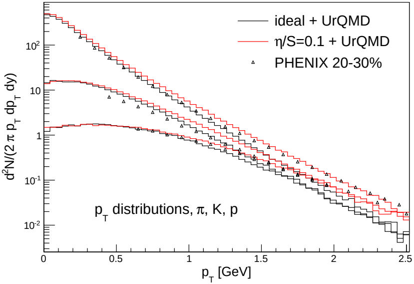
Model validation at $\sqrt{s} = 200$ GeV RHIC energy

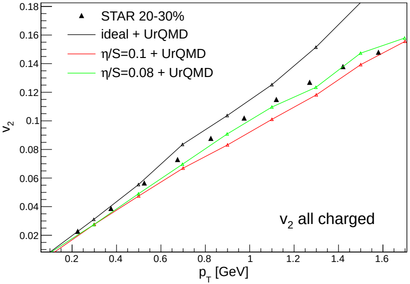
Setup: smooth 3D initial conditions:
2D Monte Carlo Glauber + parametrized rapidity dependence
no UrQMD for initial state

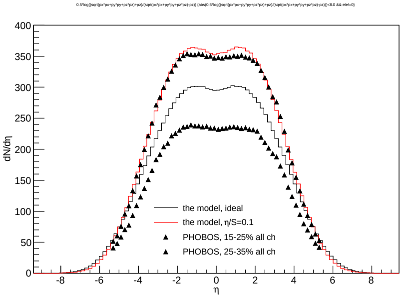
$$\varepsilon(\tau_0, \vec{r}_T, \eta) = \varepsilon_{\rm MCG}(\vec{r}_T) \cdot \theta(Y_b - |\eta|) \exp\left[-\theta(|\eta| - \Delta\eta) \frac{(|\eta| - \Delta\eta)^2}{\sigma_\eta^2} \right]$$

Y_b is beam rapidity, parameters: $\Delta\eta = 1.3$, $\sigma_\eta = 2.1$
(chosen from the fit to PHOBOS $dN_{\rm ch}/d\eta$)

Model validation at $\sqrt{s} = 200$ GeV RHIC energy







⇑

experimental data (points), 15-25% and 25-35% central

ideal hydro+cascade (black curve), 20-30% central
viscous hydro+cascade (red curve), 20-30% central

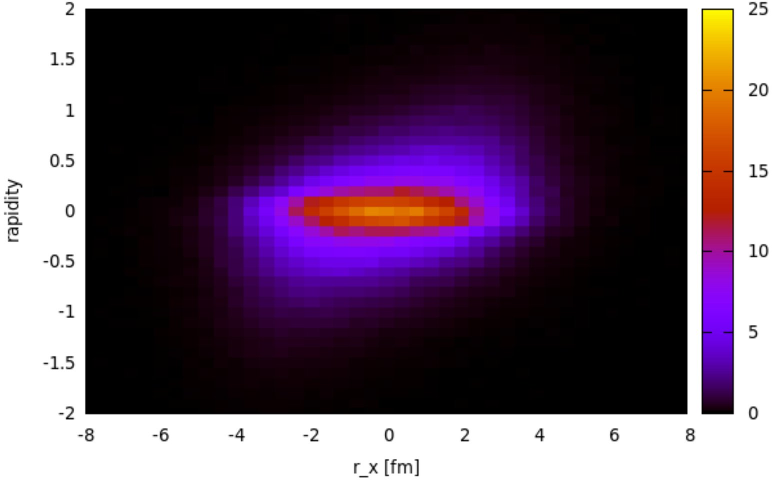
Beam energy scan

First round of simulations:

- single-shot hydro (1 hydro simulation for a given energy and centrality)
- smooth initial conditions taken as an average from many UrQMD initializations

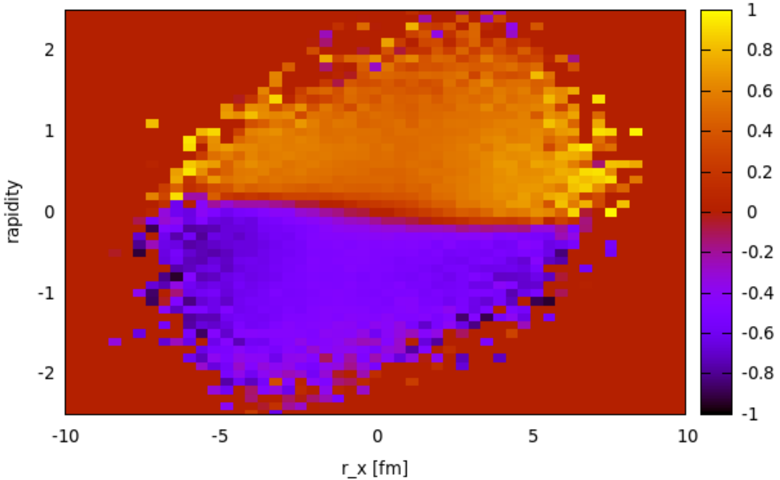
Typical smooth (event-averaged) initial condition for $E_{\mathrm{lab}} = 168$ A GeV midcentral SPS collisions.

energy density [GeV/fm^3] distribution:



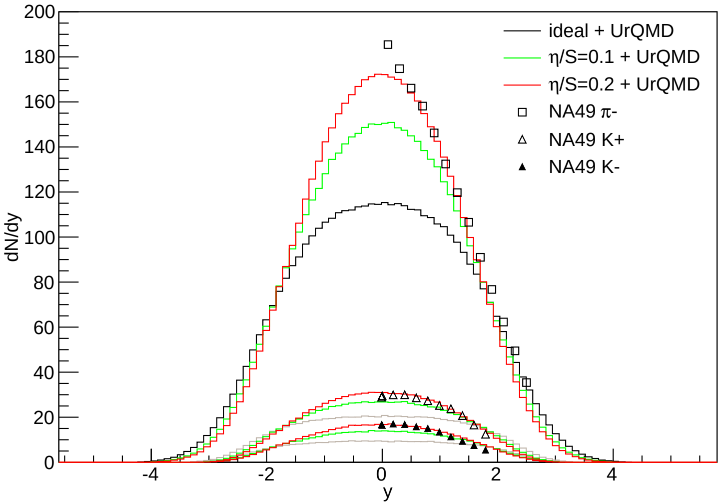
Typical smooth (event-averaged) initial condition for $E_{\rm lab} = 168$ A GeV midcentral SPS collisions.

v_η distribution (notice nonzero angular momentum!):



Results: $E_{\text{lab}} = 158$ A GeV Pb-Pb (SPS)

$\sqrt{s_{NN}} = 17.3$ GeV, 0-5% central collisions ($b = 0 \ldots 3.4$ fm)



→ strong viscous entropy production

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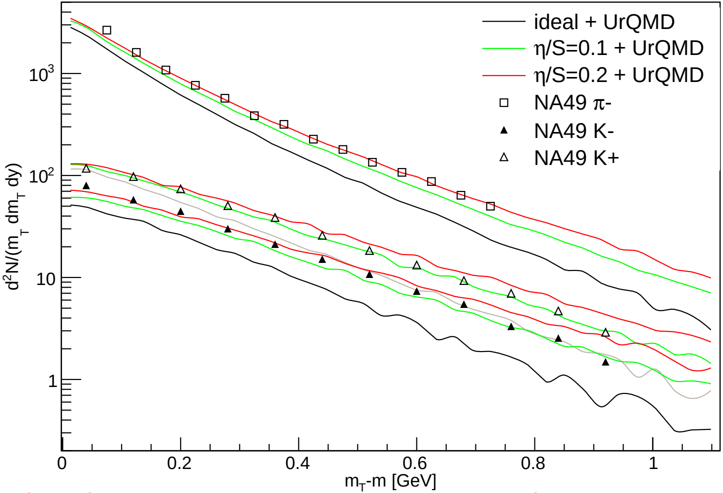
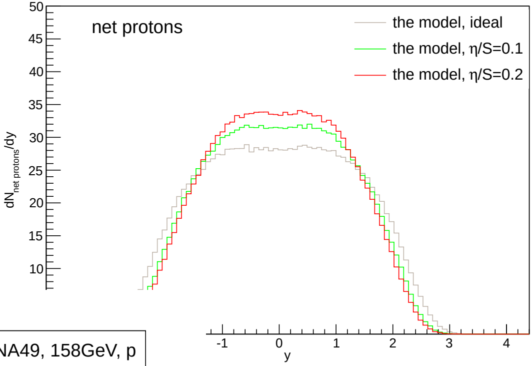


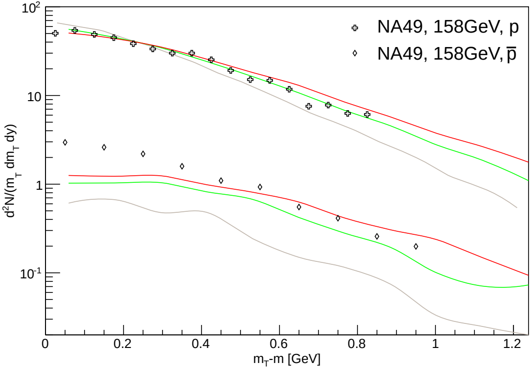
→ viscosity causes stronger transverse expansion

Results: 158 GeV SPS

protons & antiprotons

most central events ($b = 0..3.4$ fm)

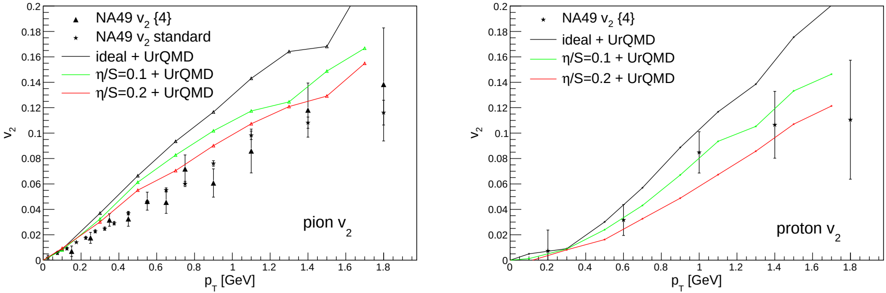




Hydrodynamic
$\tau_{start} = 1.42$ fm/c

Results: $E_{lab} = 158$ A GeV Pb-Pb (SPS)

Mid-central events as defined by NA49 ($c = 12.5 - 33.5\%$)

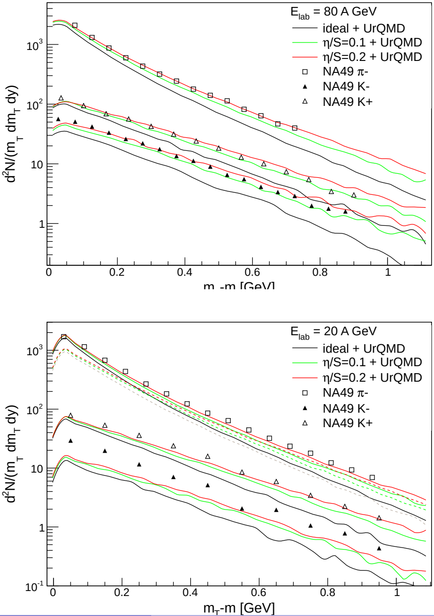


Results: $E_{\text{lab}} = 80, 40, 20$ A GeV Pb-Pb (SPS)

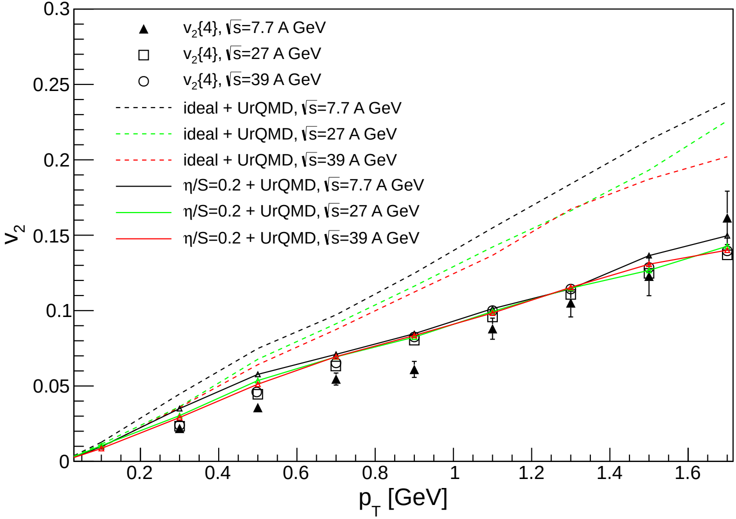
Corresp. $\sqrt{s_{NN}} = 12.3,\ 8.8,\ 6.3$ GeV

Pion & kaon pt-distributions for most central events ($c = 0-5\%$, $b = 0\ldots 3.4$ fm)

Overall good description with $\eta/S = 0.2$ except for K^- for lowest energies



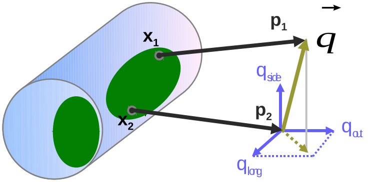
v_2 for BES at RHIC ($\sqrt{s_{NN}} = 7.7, 27, 39$ GeV Au-Au)



(20-30% central) $\eta/S \geq 0.2$ is required in hydro phase for all BES energies.

HBT(interferometry) measurements

The only tool for space-time measurements at the scales of 10^{-15}m, 10^{-23}s



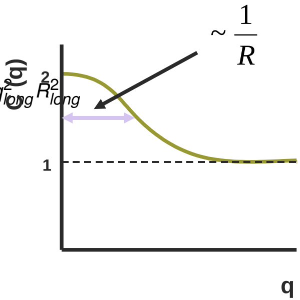
$$\vec{q} = \vec{p}_2 - \vec{p}_1$$

$$\vec{k} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2)$$

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}}$$

Gaussian approximation of CFs ($q \to 0$):

$$C(\vec{k}, \vec{q}) = 1 + \lambda(k) e^{-q_{out}^2 R_{out}^2 - q_{side}^2 R_{side}^2 - q_{long}^2 R_{long}^2}$$



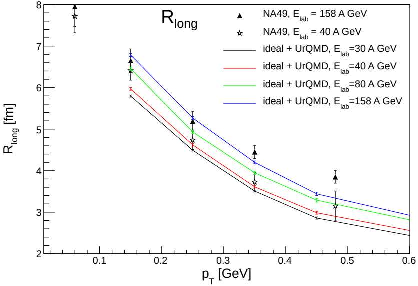
$R_{out}, R_{side}, R_{long}$ (HBT radii) correspond to *homogeneity lengths*, which reflect the space-time scales of emission process

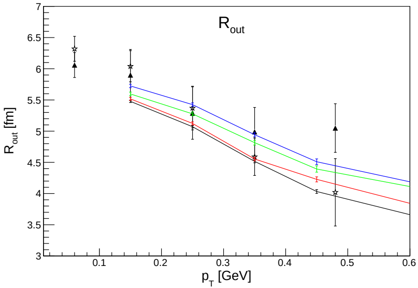
In an event generator, BE/FD two-particle amplitude (anti)symmetrization must be introduced

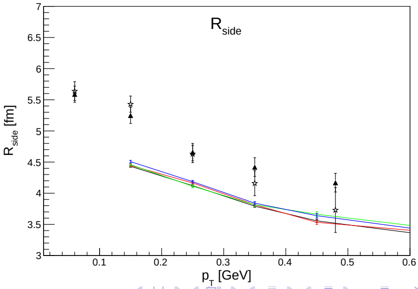
Femtoscopy at SPS energies

Corresponding $\sqrt{s_{NN}} = 12.3, 8.8, 6.3$ GeV, NA49, most central collisions ($c = 0 - 5\%$)

Femtoscopic radii for $\pi^-\pi^-$ pairs: $R_{\text{long}}, R_{\text{out}}$ consistent with NA49 data, R_{side} underestimated.



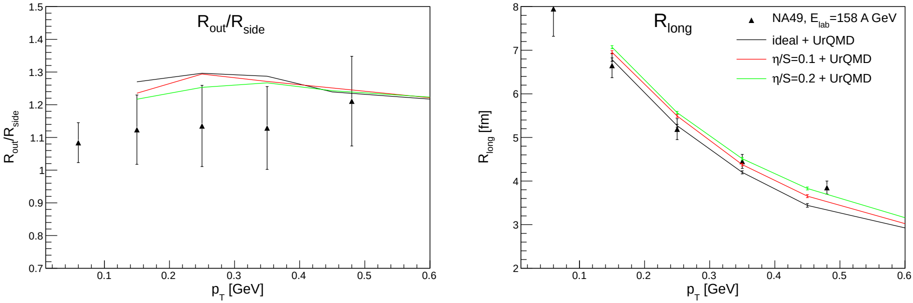




Femtoscopy at top SPS energy

$E_{\mathrm{lab}} = 158$ A GeV SPS ($\sqrt{s_{NN}} = 17.3$ GeV)

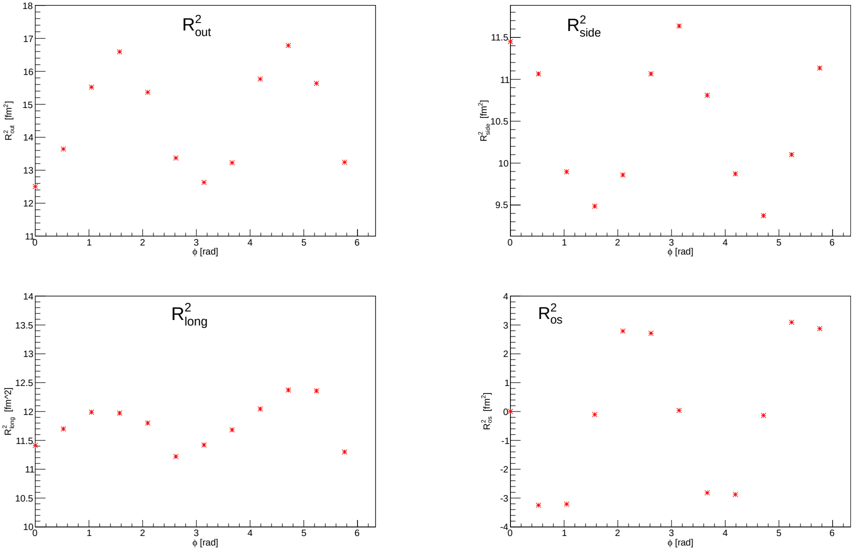
Dependence on η/S



R_{long} is increased and $R_{\mathrm{out}}/R_{\mathrm{side}}$ is slightly improved by viscosity

Azimuthally-sensitive femtoscopy

$\sqrt{s_{NN}} = 7.7$ GeV, 10-30% central AuAu; $p_T = 0.15 \ldots 0.6$ GeV; $\phi = \psi_{\rm pair} - \Psi_{\rm RP}$



Azimuthally-sensitive femtoscopy

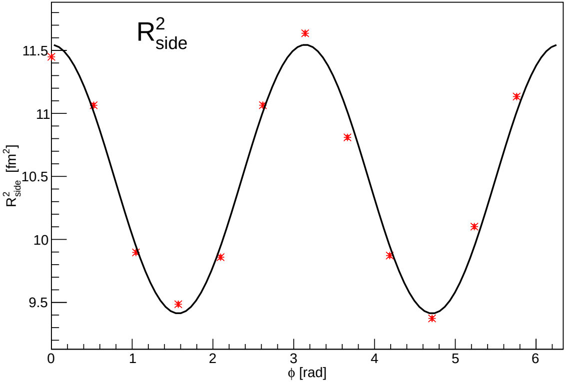
$$R_i^2(\phi) = R_{i,0}^2 + 2\sum_{n=2,4,6...} R_{i,n}^2 \cos(n\phi), \quad i = \text{out,side,long}$$

$$R_i^2(\phi) = 2\sum_{n=2,4,6...} R_{i,n}^2 \sin(n\phi), \quad i = \text{os}$$

solid curve:

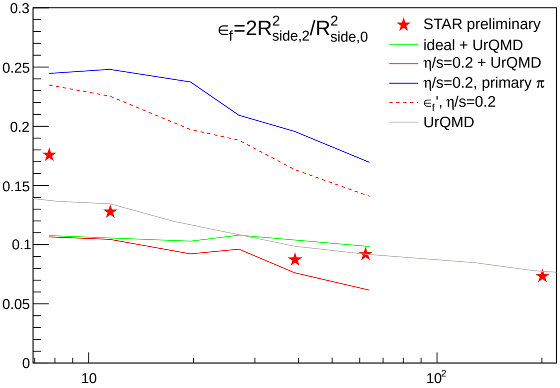
$R_{s,0}^2 + 2R_{s,2}^2 \cos(2\phi) \Rightarrow$

$$\varepsilon_f = 2\frac{R_{\text{side},2}^2}{R_{\text{side},0}^2}$$



F. Retiere and M. Lisa, Phys.Rev. **C**70:044907, 2004

Azimuthally-sensitive femtoscopy



STAR: C. Anson,
J.Phys. G**38**:124148,2011

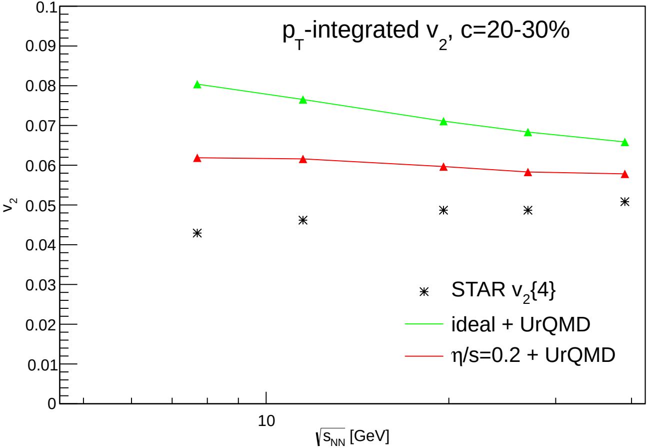
10-30% central AuAu,
$p_T = 0.15\ldots0.6$ GeV

$$\varepsilon' = \frac{\int(y^2-x^2)u^\mu d\sigma_\mu}{\int(y^2+x^2)u^\mu d\sigma_\mu}$$ [1]

[1] C. Shen, U. Heinz, Phys.Rev. C 85, 054902 (2012)
[2] UrQMD: M.A. Lisa, et al., New J.Phys.13:065006,2011

Rescatterings and resonance decays decrease the eccentricity

...and p_T-integrated elliptic flow



Large v_2 for $\eta/s = 0$ in hydro phase: feature of ICs used?

Second round of simulations:

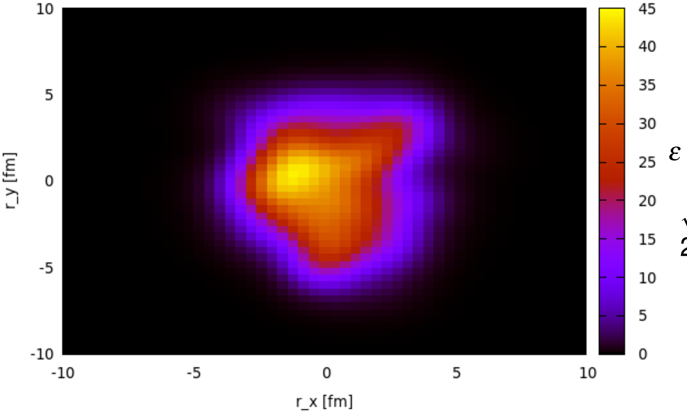
- event-by-event hydrodynamic evolution
- fluctuating initial conditions taken from single UrQMD initialization each

Fluctuating initial state

Fluctuating, but smoothed initial state:

$$E \propto \exp\left(-\frac{(x-x_{part})^2+(y-y_{part})^2+\gamma_z^2(z-z_{part})^2}{2R^2}\right), \text{ where } R=1 \text{ fm}$$

see e.g. H. Petersen et al., Phys.Rev. C78 (2008) 044901



$\sqrt{s_{NN}}=39$ GeV, 20-30% central

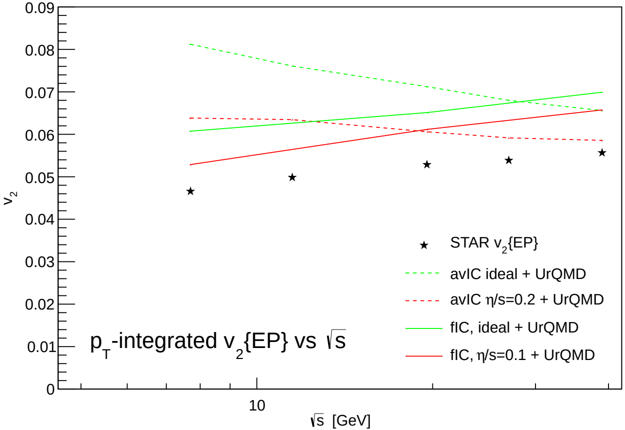
23% larger $dS/dy(y=0)$ than averaged IC.

Fluctuating vs. averaged initial state

Fluctuating, but smoothed initial state:

$$E \propto \exp\left(-\frac{(x-x_{part})^2+(y-y_{part})^2+\gamma_z^2(z-z_{part})^2}{2R^2}\right), \text{ where } R=1 \text{ fm}$$

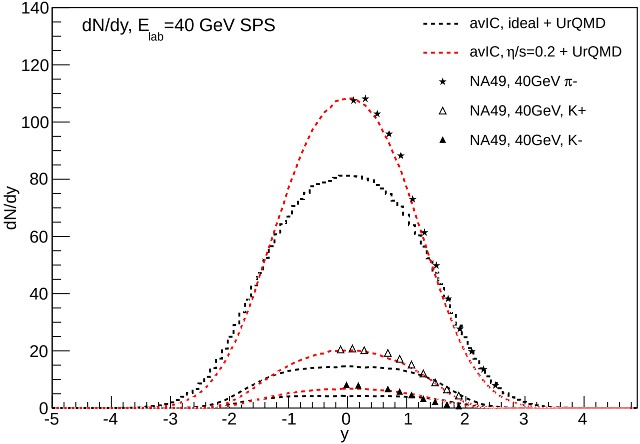
see e.g. H. Petersen et al., Phys.Rev. C78 (2008) 044901



$v_2(\sqrt{s})$ depends on how the initial state is constructed.

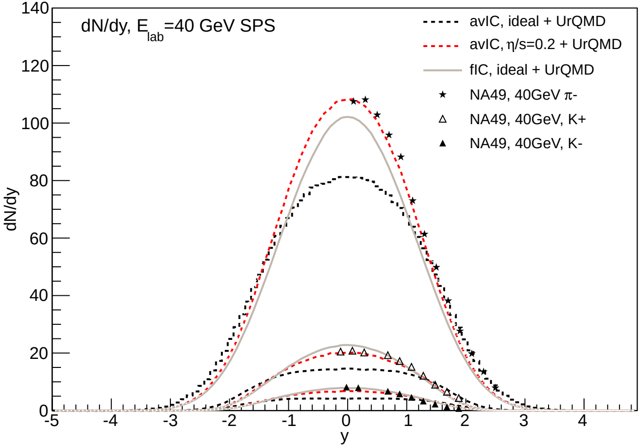
Influence on rapidity distribution

Results from single-shot hydro runs



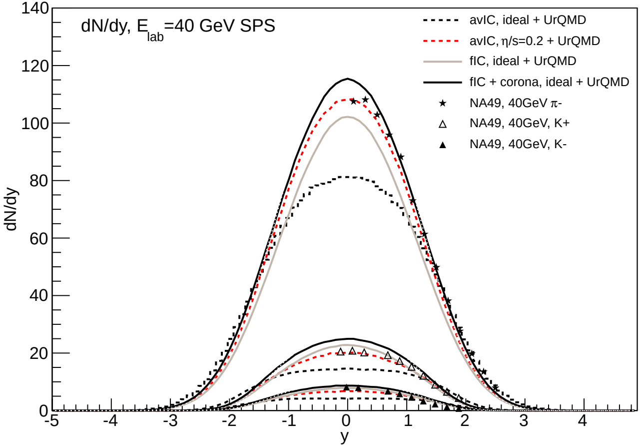
Influence on rapidity distribution

Single-shot hydro vs EbE ideal hydro



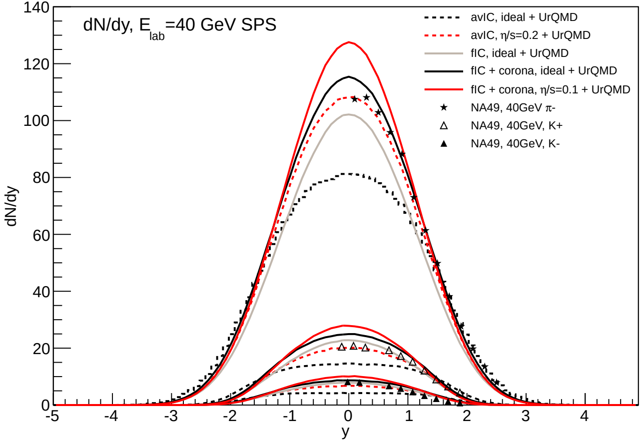
Influence on rapidity distribution

Single-shot hydro vs EbE id. hydro vs EbE id. hydro + corona



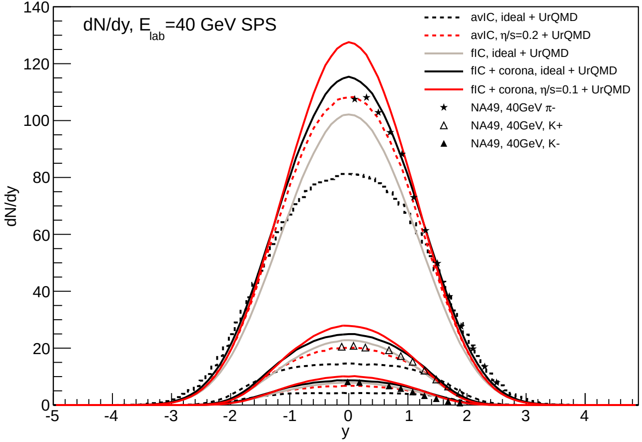
Influence on rapidity distribution

All that vs EbE viscous hydro + corona



Influence on rapidity distribution

All that vs EbE viscous hydro + corona



Too large initial entropy to accommodate viscous hydro phase

⇓

This can be regulated by decreasing Gaussian radius R.

Summary

UrQMD + 3D viscous hydro + UrQMD model including EoS at finite μ_B

Conclusions:

- model validated at top RHIC energy, and applied for Beam Energy Scan.
- averaged IC: and single-shot hydro
  shear viscosity in hydro phase improves description of
  - p_T-spectra
  - dN/dy
  - elliptic flow
  - femtoscopic radii
- v_2 from RHIC BES suggests $\eta/S \geq 0.2$

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  - femtoscopic radii
- v_2 from RHIC BES suggests $\eta/S \geq 0.2$
- EbE hydro with fluctuating IC corrects $v_2(\sqrt{s})$
- in EbE case the a simultaneous fit to dN/dy and v_2 should allow to fix both initial state granularity and shear viscosity in hydro phase (provided that the initial state is from UrQMD).

Outlook:

- Effects of EoS with 1^{st} order PT?

Thank you for your attention!

Extra slides

Viscous hydrodynamic equations

The hydrodynamic equations in arbitrary coordinate system:

$$\partial_{;\nu} T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu_{\nu\lambda} T^{\nu\lambda} + \Gamma^\nu_{\nu\lambda} T^{\mu\lambda} = 0 \tag{7}$$

where (we choose Landau definition of velocity)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p+\Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu} \tag{8}$$

and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Evolutionary equations for shear/bulk, coming from Israel-Stewart formalism:

$$< u^\gamma \partial_{;\gamma} \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\rm NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_{;\gamma}u^\gamma \tag{9a}$$

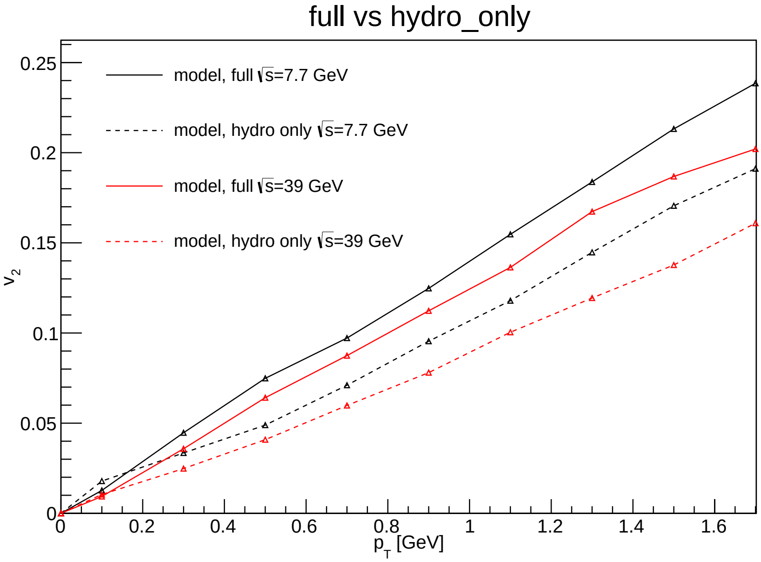
$$u^\gamma \partial_{;\gamma} \Pi = -\frac{\Pi - \Pi_{\rm NS}}{\tau_\Pi} - \frac{4}{3}\Pi\partial_{;\gamma}u^\gamma \tag{9b}$$

where

$$< A^{\mu\nu} >= (\frac{1}{2}\Delta^\mu_\alpha \Delta^\nu_\beta + \frac{1}{2}\Delta^\nu_\alpha \Delta^\mu_\beta - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})A^{\alpha\beta}$$

v_2 before and after the cascade

$\eta/S = 0$

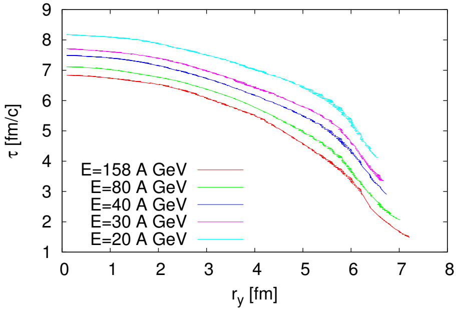


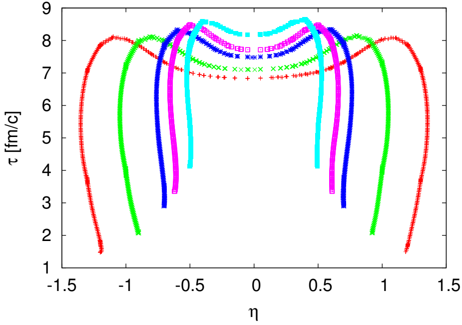
Transition surfaces

hydro→cascade transition

Most central collisions,
$E_{\mathrm{lab}} = 20$ GeV (cyan)...158 GeV (red)
$\sqrt{s_{NN}} = 6.27 \ldots 17.3$ GeV

Transition criterion: $\varepsilon = \varepsilon_{\mathrm{crit}} = 0.5$ GeV/fm^3, same for all energies



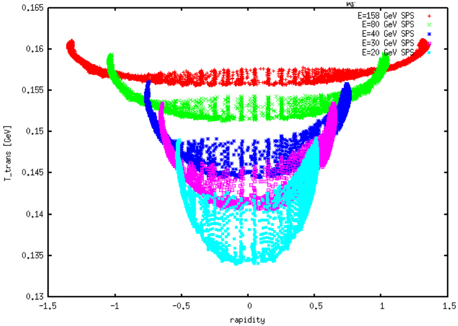


System squeezes in rapidity with decreasing collision energy,
hydro phase still lasts about 4.5 fm/c at lowest SPS energy.

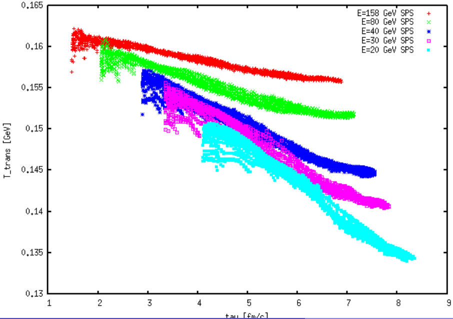
Thermodynamics on transition surface

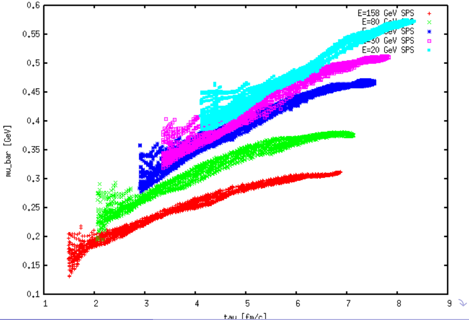
Procedure (for each surface element):

$$\{\varepsilon = \varepsilon_{\text{crit}}, n_B, n_Q\} \xrightarrow{EoS} \{T, \mu_B, \mu_Q, \mu_S\}$$



Most central collisions,
E_{lab} =20 GeV (cyan)...158 GeV (red)
T(rapidity) (top), $T(\tau)$ (bottom left), $\mu_B(\tau)$ (bottom right)

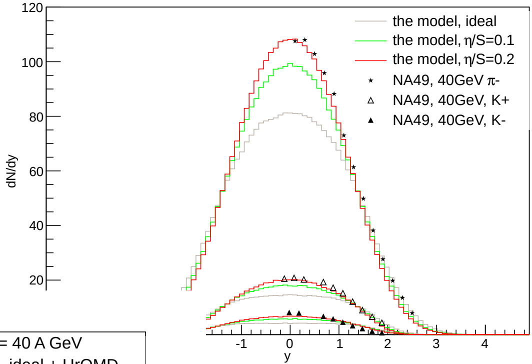


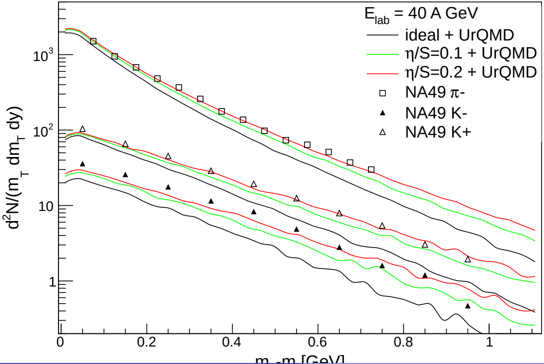


Results: 40 GeV SPS

pions & kaons

most central events
($b = 0..3.4$ fm)



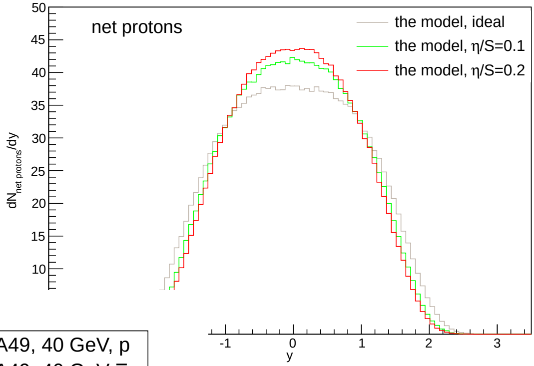


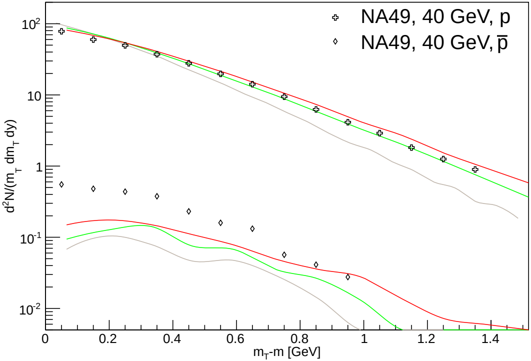
Hydrodynamic
$\tau_{start} = 2.83$ fm/c

Results: 40 GeV SPS

protons & antiprotons

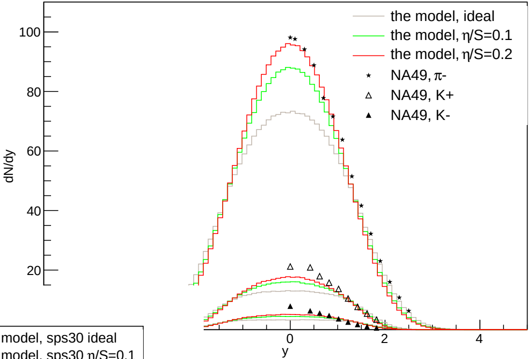
most central events
($b = 0..3.4$ fm)





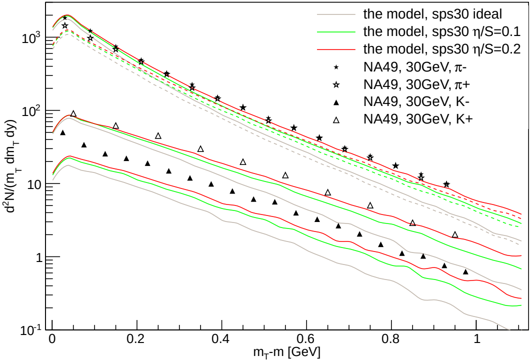
Hydrodynamic
$\tau_{start} = 2.83$ fm/c

Results: 30 GeV SPS



pions & kaons

most central events
($b = 0..3.4$ fm)

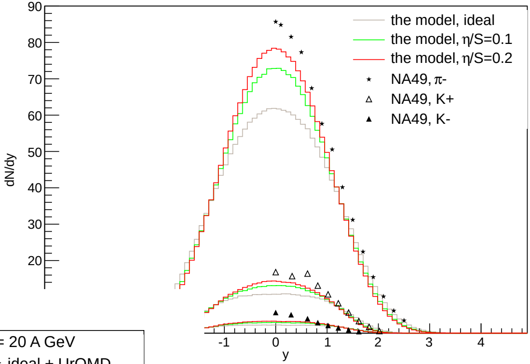


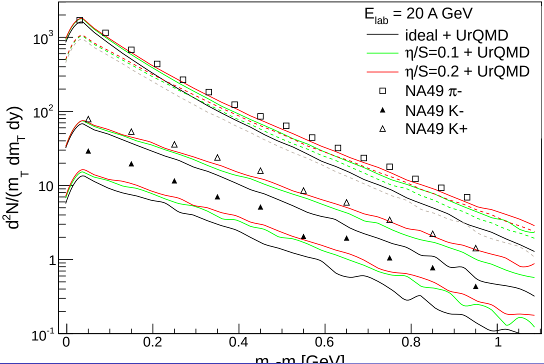
Hydrodynamic
$\tau_{start} = 3.28$ fm/c

Results: 20 GeV SPS

pions & kaons

most central events
($b = 0..3.4$ fm)





Hydrodynamic
$\tau_{\rm start} = 4.05$ fm/c