Pion dissociation and Levinson's theorem in hot PNJL quark matter

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in collaboration with
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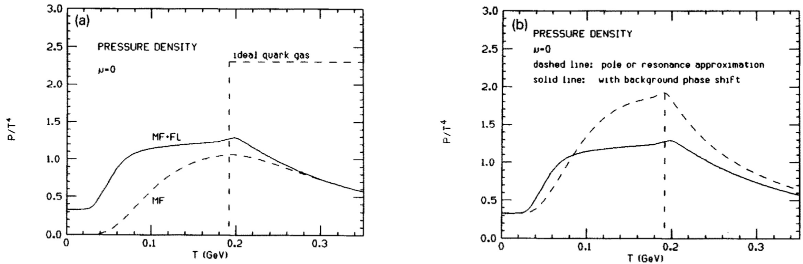
JINR Dubna, Feb. 6, 2013

The state of the art in January 1994

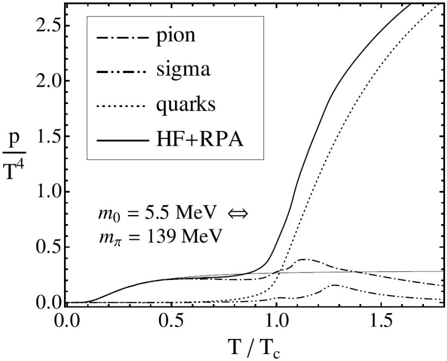
1) The NJL model:



(P. Zhuang, J. Hufner and S. P. Klevansky, Nucl. Phys. A **576** (1994) 525)

The state of the art in January 2008

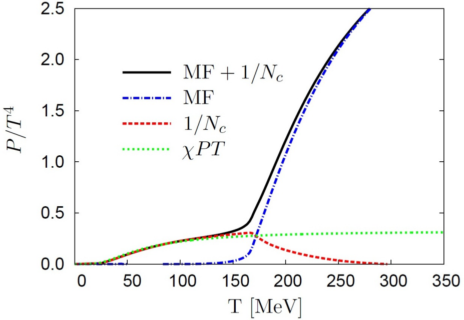
2) The PNJL model:



(S. Roessner, T. Hell, C. Ratti and W. Weise,
Nucl. Phys. A **814** (2008) 118 [arXiv:0712.3152 [hep-ph]])

The state of the art in January 2011

3) The nonlocal PNJL model:



(A. E. Radzhabov, D. Blaschke, M. Buballa and M. K. Volkov, Phys. Rev. D **83** (2011) 116004 [arXiv:1012.0664 [hep-ph]])

The PNJL model

Everything begins with a Lagrangian:

$$\mathcal{L}_{PNJL} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m_0 - \gamma^0\mu\right)q + \sum_{M=\sigma',\vec{\pi}'} G_M\left(\bar{q}\Gamma_M q\right)^2 - U(\Phi[A];T),$$

where

$$D_{\mu} = \partial_{\mu} - iA_{\mu},$$

$$U(\Phi;T) = T^4\left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{3}\Phi^3 + \frac{b_4}{4}\Phi^4\right],$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3,$$

a_0	a_1	a_2	b_3	b_4	T_0 [MeV]
6.75	-1.95	-7.44	0.75	7.5	208

(C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019,

B.-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D **76** (2007) 074023.)

The PNJL model

The partition function in the PNJL model:

$$\mathcal{Z}_{PNJL}[T, V, \mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{ \int_0^\beta d\tau \int_V d^3x \left[\bar{q}\left(i\gamma^\mu(\partial_\mu - iA_\mu) - m_0 - \gamma^0\mu\right) q + \right.\right.$$

$$\left.\left. + G_S\left(\bar{q}\Gamma_{\sigma'} q\right)^2 + G_S\left(\bar{q}\vec{\Gamma}_{\pi'} q\right)^2 - U(\Phi[A]; T)\right]\right\}$$

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$$\mathcal{Z}_{PNJL}[T,V,\mu] = \int \mathcal{D}\sigma'\mathcal{D}\vec{\pi}' \exp\left\{ -\left[\int_0^\beta d\tau \int_V d^3x \left(\frac{\sigma'^2 + \vec{\pi}'^2}{4G_S} + U(\Phi[A];T) \right) \right] + \right.$$

$$\left. + \mathrm{Tr}\ln\left[\beta S^{-1}[\sigma',\vec{\pi}']\right] \right\}$$

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$$\Omega_{FL}^{(2)}[T,V,\mu] = \frac{T}{V}\ln\left[\det\left(\frac{1}{2G_S} - \Pi_\sigma\left(q_0,\vec{q}\right)\right)\right]^{-\frac{1}{2}} + \frac{T}{V}\ln\left[\det\left(\frac{1}{2G_S} - \Pi_{\vec{\pi}}\left(q_0,\vec{q}\right)\right)\right]^{-\frac{3}{2}}$$

The Generalized Beth-Uhlenbeck approach

$$P_M^{(2)} = -\Omega_M^{(2)}(T,\mu) = \frac{N_M}{2}\int \frac{d^3q}{(2\pi)^3}\left(\int_0^{+\infty}\frac{d\omega}{\pi}\left[\omega + 2T\ln\left(1-e^{-\beta\omega}\right)\right]\frac{d\Phi_M(\omega,\vec{q})}{d\omega}\right)$$

Mott-effect: bound state (delta function)→ resonance (spectral broadening)

$$\frac{d\Phi_M(\omega,\vec{q})}{d\omega} \quad \rightarrow \quad \begin{cases} \pi\ \delta(\omega - E_M), & T < T_{\mathrm{Mott}} \\ \frac{d\phi_R(\omega,\vec{q})}{d\omega}, & T > T_{\mathrm{Mott}} \end{cases}$$

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The Breit-Wigner ansatz for the phase shift derivative:

$$\frac{d\phi_R(s,T)}{ds} = A_R(s,T) = \frac{\pi}{\frac{\pi}{2} + \arctan\left(\frac{\vec{q}^2 + M_M^2}{M_M\Gamma_M}\right)}\ \frac{M_M\Gamma_M}{\left(s - M_M^2\right)^2 + (M_M\Gamma_M)^2}$$

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and the corresponding meson pressure ($\omega = \sqrt{\vec{q}^2 + s}$)

$$P_M(T) = \frac{N_M}{2}\int\frac{d^3q}{(2\pi)^3}\int_{4m^2}^{+\infty}\frac{ds}{\pi}\left(\omega + 2T\ln\left(1-e^{-\beta\omega}\right)\right)A_R(s,T)$$

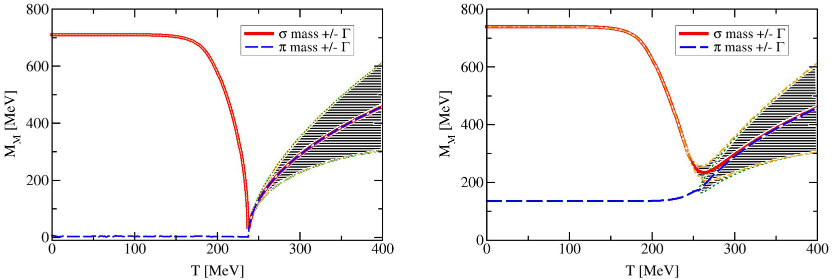
Meson masses with spectral broadening

Separating the real and imaginary part of $\Pi_M\left(q_0,\vec{0}\right) = 4N_cN_f\ I_1 - 2N_cN_fP_M\ I_2$ results in coupled Bethe-Salpeter equations:

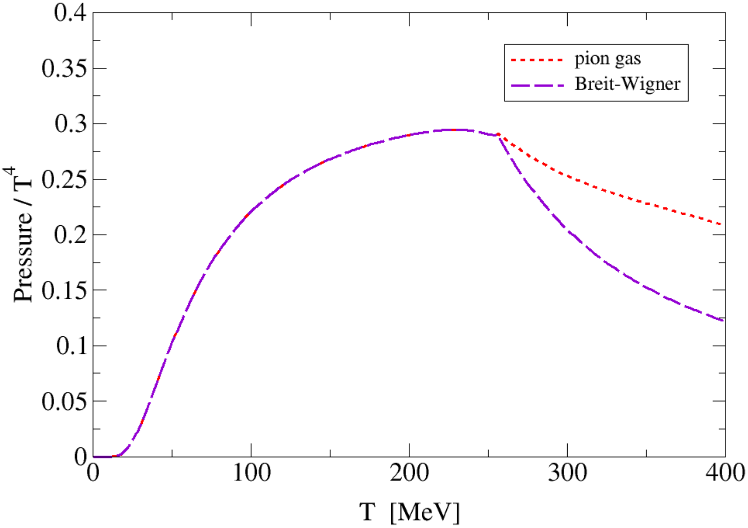
$$M_M^2 - \frac{1}{4}\Gamma_M^2 - \begin{pmatrix} 4m^2 \\ 0 \end{pmatrix} = \frac{\frac{1}{4N_cN_fG_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \mathrm{Re}\ I_2(q_0),$$

$$M_M\Gamma_M = \frac{\frac{1}{4N_cN_fG_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \mathrm{Im}\ I_2(q_0).$$

See, e.g., D. Blaschke, M. Jaminon, Yu.L. Kalinovsky, *et al.*, NPA **592** (1995) 561



Pion pressure: massive pion gas and Breit-Wigner ansatz



Levinson's theorem

Breit-Wigner ansatz $\rightarrow$ ϕ_R is

$$\phi_R(s) = \frac{\pi}{\frac{\pi}{2} - \arctan\left(\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right)} \left(\arctan\left[\frac{s - M_M^2}{M_M \Gamma_M}\right] - \arctan\left[\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right] \right)$$

[it fulfills $\phi_R(s \to 4m^2) = 0$ and $\phi_R(s \to \infty) = \pi$]

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violates Levinson's theorem which would require

$$\phi(s_{\text{threshold}} = 4m^2) - \phi(\infty) = n\pi = 0 \ ,$$

since the number of bound states below threshold vanishes (n=0) for $T > T_{\text{Mott}}$
$\rightarrow$ Solution: phase shift coresponding to scattering states missing!

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Two contributions to the scattering phase shift: $\Phi_M = \phi_R + \phi_{sc}$

$$\Phi_M = -\arctan\left(\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2} \cdot \frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}\right) .$$

(P. Zhuang, J. Hufner, S. P. Klevansky, Nucl. Phys. A576, 525-552 (1994).)

Zhuang formula - derivation

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Using

$$\Pi_M \left(q_0, \vec{0} \right) = 4N_c N_f \ I_1 - 2N_c N_f P_M \ I_2 = \widetilde{I}_1 - P_M \widetilde{I}_2,$$

and

$$\frac{i}{2} \ln \left(\frac{1 - ix}{1 + ix} \right) = \arctan x$$

we show that

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \mathrm{Im} \widetilde{I}_2}{1 - 2G_S \widetilde{I}_1 + 2G_S P_M \mathrm{Re}\, \widetilde{I}_2} \right].$$

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(several steps more)

$$\Phi_M = -\arctan\left[\frac{\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2} - \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2}{P_M+\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}}{1+\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2^2}{P_M\mathrm{Re}\,\widetilde{I}_2+\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2^2}}\right].$$

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Using

$$-(\arctan\alpha \pm \arctan\beta) = -\arctan\left[\frac{\alpha\pm\beta}{1\mp\alpha\beta}\right]$$

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Using

$$-(\arctan\alpha \pm \arctan\beta) = -\arctan\left[\frac{\alpha\pm\beta}{1\mp\alpha\beta}\right]$$

we get

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Our approach

Now then

$$\Phi_M = \phi_{sc} + \phi_R = -\arctan\left(\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2}\right) + \arctan\left(\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}\right)$$

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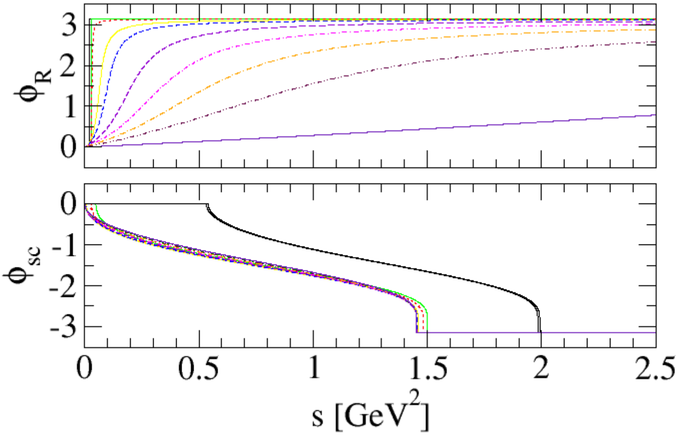
Our analysis is a combined approach:

$$D_M(s) = \frac{1}{\pi}\frac{d\phi_M(s)}{ds} = \begin{cases} \delta(s-M_M^2) + \frac{1}{\pi}\frac{d}{ds}\phi_{sc}(s)\ , & T < T_{\mathrm{Mott}}\ , \\ \frac{a_R}{\pi}\frac{\Gamma_M M_M}{(s-M_M^2)^2+\Gamma_M^2 M_M^2} + \frac{1}{\pi}\frac{d}{ds}\phi_{sc}(s)\ , & T > T_{\mathrm{Mott}}\ . \end{cases}$$

Phase shifts



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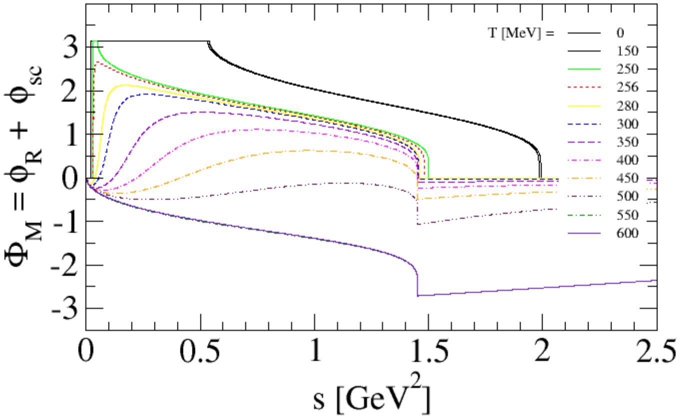
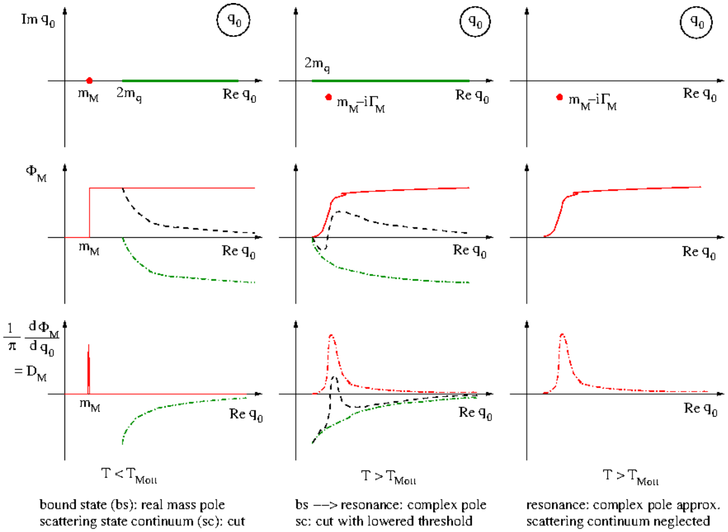
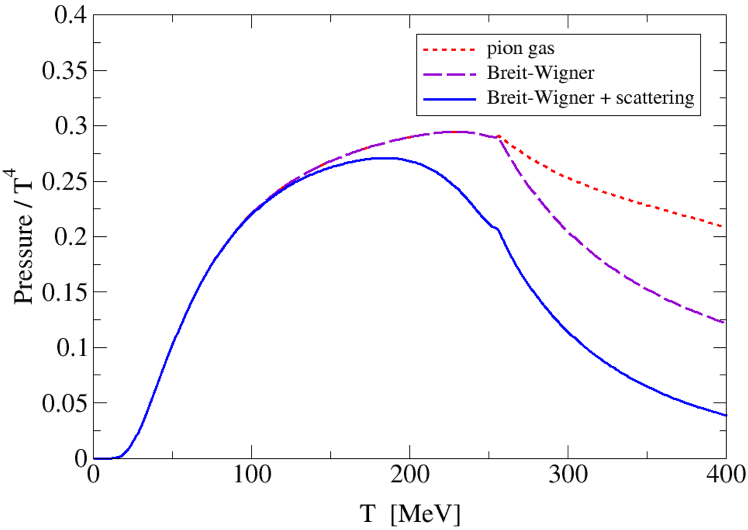


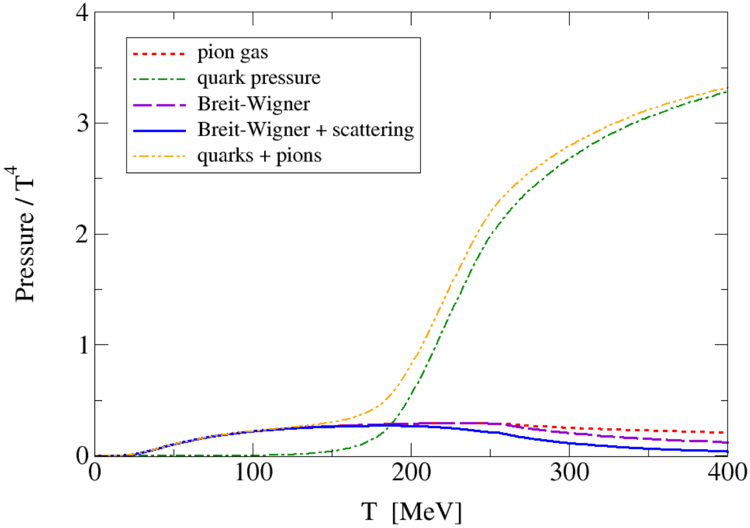
Summary: Levinson's Theorem & analytical properties



Pion pressure



Quark + pion pressure



Conclusions

- PNJL model: suitable for describing χSB and restoration at finite temperature, it describes pions as $q\bar{q}$ bound states and pseudo-Goldstone bosons $\quad \rightarrow \quad m(T),\, M_M(T),\, \Gamma_M(T)$

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- an analytic formula for the continuum states' contribution to the scattering phase shift together with the Breit-Wigner ansatz for the resonance
- resulting phase shift obeys the Levinson theorem
  $\rightarrow$ pressure reduction (ideally to zero) for large enough $T > T_{Mott}$

Conclusions

- PNJL model: suitable for describing χSB and restoration at finite temperature, it describes pions as $q\bar{q}$ bound states and pseudo-Goldstone bosons $\rightarrow$ $m(T)$, $M_M(T)$, $\Gamma_M(T)$
- pressure $P(T)$ for quark mean-field: suppresion of quarks for $T < T_c$, correct SB limit
- Gaussian fluctuations in σ, $\vec{\pi}$: Generalized Beth-Uhlenbeck
- resonance approximation for pionic mode above T_{Mott} **violates** Levinson's theorem!
- an analytic formula for the continuum states' contribution to the scattering phase shift together with the Breit-Wigner ansatz for the resonance
- resulting phase shift obeys the Levinson theorem
  $\rightarrow$ pressure reduction (ideally to zero) for large enough $T > T_{Mott}$
- **outlook**: semi-microscopic approach to implement Mott effect for hadrons (here: only pions) consistent with Levinson's theorem into hadron resonance gas (HRG) models

The end