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Based on works:

[1] Phys. Rev. Lett., 129(15):151601, 2022.

[2] arXiv: 2211.03865

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Gravitational chiral anomaly and Kinematical Vortical Effect

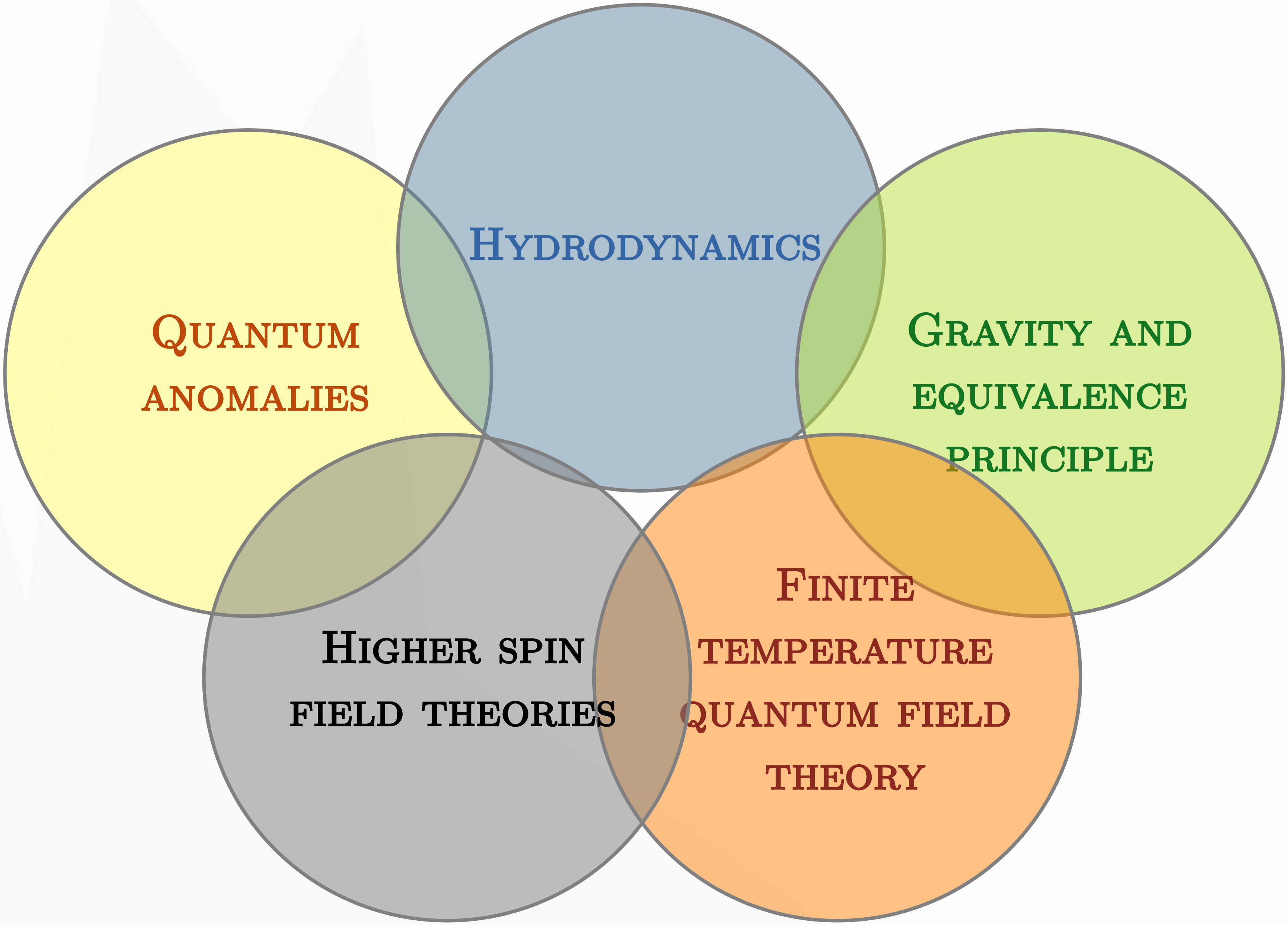
Seminar "Theory of Hadronic Matter under Extreme Conditions", BLTP, JINR, Dubna, 16 November, 2022

Contents

Part 1

Introduction

What will be discussed?



Gravity in flat spacetime: Cheshire cat grin

“Well! I've often seen a cat without a grin,' thought Alice 'but a grin without a cat! It's the most curious thing i ever saw in my life!”

— Lewis Carroll, Alice in Wonderland

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GRADIENT EXPANSION IN HYDRODYNAMICS

Hydrodynamics is constructed as a gradient expansion. For an ideal fluid, the stress-energy tensor does not contain gradients (on this slide signature (-1,1,1,1)):

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + p\eta^{\mu\nu}$$

energy density

pressure

4-velocity of the fluid

However, for a viscous fluid, linear gradients arise. Can be obtained from the second law of thermodynamics: **[L. D. Landau, E. M. Lifshitz, Fluid Mechanics, Vol. 6, 1987]**

$$\partial_\mu s^\mu \geqslant 0$$

entropy current

Shear viscosity

Bulk viscosity

$$\tau_{\mu\nu} \;=\; -\eta P^\alpha_\mu P^\beta_\nu(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - (\zeta - \frac{2}{3}\eta)P_{\mu\nu}\partial_\alpha u^\alpha$$

Projector

$$P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$

From string theory:

$$\eta/s \geq \hbar/(4\pi k_B)$$

Quark-gluon plasma – to be near this limit.

[P. Kovtun, D. T. Son, A. O. Starinets, Phys.Rev.Lett. 94 (2005), 111601]

CVE and CME – new anomalous transport

However there are gradient terms that are not related to dissipation.

[V. I. Zakharov, Lect. Notes Phys.871,295(2013), 1210.2186]

Chiral magnetic effect (CME)

CME: $j^\mu = C\mu_5 B^\mu$

Current flows along the magnetic field

Chiral vortical effect (CVE)

CVE: $j_A^\mu = C\mu^2\omega^\mu$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$

Current flows along the vorticity

Consistency with quantum anomaly modifies hydrodynamic equations
[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]:

- Generalization of [L. D. Landau, E. M. Lifshitz, Fluid Mechanics, Vol. 6, 1987]

Conservation equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

+

Quantum chiral anomaly

$$\langle \partial_\mu \hat{j}_A^\mu \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

+

Second law of thermodynamics

$$\partial_\mu s^\mu \geq 0$$

Modern development and the problem

A whole set of similar effects has been found at the intersection of quantum field theory and hydrodynamics [M. N. Chernodub et al. 2110.05471].

Experimental search is in progress:

- CME not yet found in ion collisions at RHIC [D.E. Kharzeev et al. 2205.00120].
- Condensed matter copies of the effects are found in semimetals [Qiang Li et al. Nature Phys. 12 (2016)].

What about the **gravitational chiral anomaly**?

- The gravitational chiral anomaly grows **rapidly** with **spin**:

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

[S. M. Christensen, M. J. Duff, Nucl. Phys. B 154, 301–342 (1979)]

How does the **gravitational** chiral anomaly manifest itself in **hydrodynamics**?

Is it possible to see the **factor** $S - 2S^3$ in hydrodynamics?

Gravitational anomaly in thermal CVE

The answer was obtained in different approaches:

- From the holography. **[K. Landsteiner, E. Megias, F. Pena-Benitez, "Gravitational Anomaly and Transport," Phys. Rev. Lett. 107, 021601 (2011)]**

- In **[S.P. Robinson, F. Wilczek. Phys. Rev. Lett., 95:011303, 2005]** Hawking radiation is associated with a gravitational anomaly: it is necessary to integrate the anomaly from the horizon to infinity + the condition for the consellation of the currents on the horizon.

  In **[M. Stone, J. Kim. Phys. Rev., D98(2):025012, 2018]** the derivation was generalized to 3+1 dimensional gravitational chiral anomaly and (analogue) of rotating black hole.

- From the condition of the translational invariance of the Euclidean vacuum.

  [K. Jensen, R. Loganayagam, A. Yarom, JHEP 02, 088 (2013)]

$$j_A^\nu = (\sigma_T T^2 + \sigma_\mu \mu^2)\omega^\nu$$

$$\epsilon^{\mu\nu\alpha\beta} = \frac{1}{\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}$$

$$\sigma_T = 64\pi^2 \mathcal{N} \quad \Longleftrightarrow \quad \nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

Gravitational anomaly in thermal CVE

Verified for the Dirac field

$$j_A^\mu = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \right) \omega^\mu$$

[L. Alvarez-Gaume, E. Witten,
Nucl. Phys. B234 (1984) 269]

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

$$\sigma_T = 64\pi^2 N$$

A **problem** arose for fields with spin 3/2 (within the framework of the Rarita-Schwinger-Adler theory) [S. L. Adler, Phys. Rev. D 97 (4) (2018) 045014]

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov,
Phys. Rev. D 105 (4) (2022) L041701]

$$j_A^\nu = \left(\frac{5}{6} T^2 + \frac{5}{2\pi^2} \mu^2 \right) \omega^\nu$$

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov,
Phys. Rev. D 106 (2) (2022) 025022]

$$\nabla_\mu j_A^\mu = -\frac{19}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

$$\sigma_T \neq 64\pi^2 N$$

In hydrodynamics, the **cubic** dependence on the spin from the gravitational anomaly **is not visible**?

Part 2

Gravitational chiral anomaly and cubic gradients

Hydrodynamics in curved space-time

Consider an uncharged liquid of particles with an arbitrary spin in a **gravitational field**:

fluid	
4-velocity of the fluid	$u_\mu(x)$
Proper temperature	$T(x)$
Inverse temperature vector	$\beta_\mu = u_\mu / T$
Thermal vorticity tensor (analogous to the acceleration tensor)	$\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu \beta_\nu - \nabla_\nu \beta_\mu)$

space-time

Curved space-time metric

$g_{\mu\nu}(x)$

Riemann tensor

$R_{\mu\nu\kappa\lambda}$

We consider a medium in a state of
(global) thermodinamic equilibrium

[F. Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75, 191 (2015)]

[F. Becattini, Acta Phys. Polon. B 47, 1819 (2016)]

Killing equation

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

For example, we can find the second derivative:

$$\nabla_\mu \nabla_\nu \beta_\alpha = R^\rho{}_{\mu\nu\alpha} \beta_\rho$$

Decomposition of the tensors: hydrodynamics

Components of the thermal vorticity tensor

6 components

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP 10 (2017) 091]

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

Similar to the expansion for the electromagnetic field

Inverse formulas

"thermal" acceleration

3 components

$$\alpha_\mu = \varpi_{\mu\nu} u^\nu$$

Usual "kinematic" vorticity

"thermal" vorticity pseudovector

3 components

$$w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} u^\nu \varpi^{\alpha\beta} \quad \Longleftrightarrow \quad w_\mu = \frac{\omega_\mu}{T}$$

In a state of global equilibrium

$$\alpha_\mu = \frac{a_\mu}{T}$$

Usual "kinematic" acceleration

Decomposition of the tensors: gravity

We also decompose the Riemann tensor into components

$$\begin{aligned} R_{\mu\nu\alpha\beta} =\ & u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\ & +\epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\ & +\epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\ & +\epsilon_{\mu\nu\lambda\rho}\epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta} \end{aligned}$$

20 components

Generalization of 3d tensors from

Coincide with 3d tensors in the fluid rest frame.

[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]

Inverse formulas

$$A_{\mu\nu} = u^\alpha u^\beta R_{\alpha\mu\beta\nu}$$

Symmetric tensor

6 components

$$B_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\eta\rho} u^\alpha u^\beta R_{\beta\nu}{}^{\eta\rho}$$

Nonsymmetric traceless pseudotensor dual to the Riemann tensor

8 components

$$C_{\mu\nu} = \frac{1}{4}\epsilon_{\alpha\mu\eta\rho}\epsilon_{\beta\nu\lambda\gamma} u^\alpha u^\beta R^{\eta\rho\lambda\gamma}$$

Double dual symmetric Riemann tensor

6 components

Decomposition of the tensors: gravity

Properties

$$A_{\mu\nu} = A_{\nu\mu}\,, \quad C_{\mu\nu} = C_{\nu\mu}\,, \quad B^{\mu}{}_{\mu} = 0\,,$$
$$A_{\mu\nu}u^{\nu} = C_{\mu\nu}u^{\nu} = B_{\mu\nu}u^{\nu} = B_{\nu\mu}u^{\nu} = 0\,.$$

The gravitational field is external. For simplicity, we impose an **additional condition**:

(for example, the field around a black hole) $$R_{\mu\nu} = 0$$

Additional properties appear, similar to

[L. D. Landau and E. M. Lifschits,
The Classical Theory of Fields, Vol. 2, 1975]

$$A_{\mu\nu} = -C_{\mu\nu}\,, \quad A^{\mu}_{\mu} = 0\,, \quad B_{\mu\nu} = B_{\nu\mu}$$

There are 10 independent components left

Gradient expansion in the curved spacetime

The gravitational chiral anomaly has the **4th order** in gradients – it is to be related to the **3rd order** terms in gradient expansion of the axial current.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:

Survive in flat spacetime

$$j_\mu^{A(3)} = \xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu$$
$$+ \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu$$

arbitrary coefficients

"gravitational" currents

See also gradient expansion for the fluid in the gravitational field, e.g.:

[P. Romatschke, Class. Quant. Grav. 27, 025006 (2010)]

[S. M. Diles, L. A. H. Mamani, A. S. Miranda, V. T. Zanchin, JHEP 2020, 1-40 (2020)]

Anomaly matching: principle

Following [D.T. Son, P. Surowka, PRL, 103 (2009) 191601]
- it is necessary to construct the **entropy current**.

In [M. Buzzegoli, Lect. Notes Phys. 987, 53–93 (2021)]
[Shi-Zheng Yang, Jian-Hua Gao, and Zuo-Tang Liang, Symmetry 14, 948 (2022)]

it is shown that it is possible to use the global equilibrium condition

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

And it is enough to consider ***only*** the equation for the current

Simplifies analysis and allows viewing effects in **curved space**

We use only:

$$\nabla_\mu j^\mu_A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

We substitute the gradient expansion:

$$\nabla^\mu \Big(\xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu + \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu \Big) = 32 \mathcal{N} A_{\mu\nu} B^{\mu\nu}$$

Derivatives

Using the condition of global equilibrium and relations for the gravitational field (the Bianchi identity, etc.), we obtain for the derivatives:

Luttinger relation

[J. M. Luttinger, Phys. Rev. 135, A1505–A1514 (1964)]

$$
\begin{aligned}
\nabla_\mu T &= T^2 \alpha_\mu \, , \\
\nabla_\mu u_\nu &= T(\epsilon_{\mu\nu\alpha\beta} u^\alpha w^\beta + u_\mu \alpha_\nu) \, , \\
\nabla_\mu w_\nu &= T(-g_{\mu\nu}(w\alpha) + \alpha_\mu w_\nu) - T^{-1} B_{\nu\mu} \, , \\
\nabla_\mu \alpha_\nu &= T\big(w^2 (g_{\mu\nu} - u_\mu u_\nu) - \alpha^2 u_\mu u_\nu - w_\mu w_\nu \\
&\quad - u_\mu \eta_\nu - u_\nu \eta_\mu\big) + T^{-1} A_{\mu\nu} \, , \\
\nabla^\mu (A_{\mu\nu} w^\nu) &= -3T B_{\mu\nu} w^\mu w^\nu - T^{-1} A_{\mu\nu} B^{\mu\nu} \, , \\
\nabla^\mu (B_{\mu\nu} \alpha^\nu) &= 3T A_{\mu\nu} w^\mu \alpha^\nu + T^{-1} A_{\mu\nu} B^{\mu\nu} \\
&\quad - T B_{\mu\nu} w^\mu w^\nu - T B_{\mu\nu} \alpha^\mu \alpha^\nu
\end{aligned}
$$

Anomaly matching: system of equations

The divergence of the axial current transforms into the sum of **independent** terms:

$$\begin{aligned}
\nabla_\mu j^\mu_{A(3)} &= (\alpha w)w^2\left(-3T\xi_1 + T^2\xi_1' + 2T\xi_3\right)\\
&\quad +(\alpha w)\alpha^2\left(-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3'\right)\\
&\quad +A_{\mu\nu}\alpha^\mu w^\nu\left(T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3\right)\\
&\quad +B_{\mu\nu}w^\mu w^\nu\left(-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5\right)\\
&\quad +B_{\mu\nu}\alpha^\mu\alpha^\nu\left(T^2\xi_5' - T\xi_5 - T^{-1}\xi_3\right)\\
&\quad +A_{\mu\nu}B^{\mu\nu}\left(-T^{-1}\xi_4 + T^{-1}\xi_5\right)\\
&= 32\mathcal{N}A_{\mu\nu}B^{\mu\nu}\,.
\end{aligned}$$

Principle:

In this case, the divergence is equal to the anomaly: additional terms - *macroscopic* - cannot violate the equation from the fundemental *microscopic* theory.

The coefficient in front of each pseudocalar must be equal to zero - a **system of equations** for the unknown coefficients $\xi_n(T)$.

Anomaly matching: system of equations

This **system of linear differential equations** has the form:

$$
\begin{aligned}
-3T\xi_1 + T^2\xi_1' + 2T\xi_3 &= 0 \\
-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3' &= 0 \\
T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 &= 0 \\
-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 &= 0 \\
T^2\xi_5' - T\xi_5 - T^{-1}\xi_3 &= 0 \\
-T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathscr{N} &= 0
\end{aligned}
$$

Includes the factor from the **gravitational chiral anomaly**

Anomaly matching: solution

Since the theory does not include dimensional parameters other than temperature:

$$\xi_1 = T^3\lambda_1 \quad \xi_2 = T^3\lambda_2 \quad \xi_3 = T^3\lambda_3 \quad \xi_4 = T\lambda_4 \quad \xi_5 = T\lambda_5$$

The solution looks like

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N} \qquad \lambda_4 = -8\mathcal{N} - \frac{\lambda_1}{2}$$

$$\lambda_3 = 0 \qquad \lambda_5 = 24\mathcal{N} - \frac{\lambda_1}{2}$$

was also shown in

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, JHEP 02 (2019)]

The current:

$$j_\mu^{A(3)} = \lambda_1\omega^2\omega_\mu + \lambda_2 a^2\omega_\mu + \lambda_3(a\omega)\omega_\mu + \lambda_4 A_{\mu\nu}\omega^\nu + \lambda_5 B_{\mu\nu}a^\nu$$

Flat space limit:
Kinematical Vortical Effect (KVE)

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains a contribution** to the axial current induced by the gravitational chiral anomaly:

$$
\begin{cases}
j^A_\mu = \lambda_1(\omega_\nu \omega^\nu)\omega_\mu + \lambda_2(a_\nu a^\nu)w_\mu & \Leftarrow \quad R_{\mu\nu\alpha\beta} = 0 \\
\dfrac{\lambda_1 - \lambda_2}{32} = \mathcal{N} & \Leftarrow \quad \nabla_\mu j^\mu_A = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}
\end{cases}
$$

- A new type of anomalous transport – the **Kinematical Vortical Effect (KVE)**.
- Does not explicitly depend on temperature and density → determined only by the **kinematics** of the flow.

Discussion

It also turns out that in the current

$$j_\mu^{A(3)} = \lambda_1 \omega^2 \omega_\mu + \lambda_2 a^2 \omega_\mu + \lambda_3 (a\omega) \omega_\mu + \lambda_4 A_{\mu\nu} \omega^\nu + \lambda_5 B_{\mu\nu} a^\nu$$

- **Difference** of **flat-space terms** is to be equal to **difference** of **curved-space terms:**

$$\lambda_1 - \lambda_2 = \lambda_5 - \lambda_4$$

- At the finite mass, λ begin to depend on mass and temperature:

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, JHEP 02 (2019)]

Discussion

- Arbitrary fields with arbitrary spin were considered:

General exact result

- Only gradient decomposition and anomaly were used.

- Although the effect is associated with an anomaly - it exists in flat space-time (the *Cheshire cat grin*).

- In contrast to CVE and the gauge anomaly case, the factor from the gravitational anomaly is split into **two conductivities:**

$$\lambda_1 - \lambda_2 = 32\mathscr{N}$$

KVE and Unruh effect

- It is possible to distinguish **conserved** and **anomalous** parts of the current:

$$j^A_\mu = j^A_{\mu(\text{conserv})} + j^A_{\mu(\text{anom})}$$

Thermal vorticity tensor squared $\omega^2 - a^2 = -\frac{1}{2}\varpi_{\mu\nu}\varpi^{\mu\nu}$

$$j^A_{\mu(\text{anom})} = 16\mathcal{N}\left\{(\omega^2 - a^2)\omega_\mu - A_{\mu\nu}\omega^\nu + B_{\mu\nu}a^\nu\right\} \Rightarrow \nabla^\mu j^A_{\mu(\text{anom})} = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$j^A_{\mu(\text{conserv})} = \frac{\lambda_1+\lambda_2}{2}\left\{(\omega^2 + a^2)\omega_\mu - \frac{1}{2}A_{\mu\nu}\omega^\nu - \frac{1}{2}B_{\mu\nu}a^\nu\right\} \Rightarrow \nabla^\mu j^A_{\mu(\text{conserv})} = 0$$

- Consider the term with acceleration from the anomalous part of the current:

$$j^A_{\mu(\text{anom})} = -16\mathcal{N}a^2\omega_\mu$$

- **Unruh effect** [W.G. Unruh, 1976] – in an accelerated frame there is a thermal bath of particles with the **Unruh temperature**:

$$T_U = |a|/(2\pi)$$

Substitute $|a| \to 2\pi T_U$:

$$j^A_{\mu(\text{anom})} = 64\pi^2\mathcal{N}T_U^2\omega_\mu$$

$j^A_{\mu(\text{anom})} = \frac{T_U^2}{6}\omega_\mu$ for spin ½ → standard CVE

thermal CVE current is **proportional** to the **anomaly**!

- Match with [K. Landsteiner, et al. PRL, 2011] and [M. Stone, J. Kim. PRD, 2018], where thermal CVE $\mathbf{j}_A \sim T^2\mathbf{\Omega}$ is associated with the gravitational chiral anomaly!

Part 3

Verification: spin 1/2

Transport coefficients and anomaly: spin 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

KVE

$$j_\mu^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_\mu$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

$$\Rightarrow \quad \left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

Correspondence between gravity and hydrodynamics is shown!

TRANSPORT COEFFICIENTS AND ANOMALY: SPIN 1/2

Using another pair of equations
$$\lambda_4 = -8\mathcal{N} - \frac{\lambda_1}{2}$$
$$\lambda_5 = 24\mathcal{N} - \frac{\lambda_1}{2}$$

we obtain a current for the Dirac field that includes "gravitational" terms (taking into account also linear terms):

$$j_\mu^{A,S=1/2} = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega^\mu + \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega_\mu + \frac{1}{24\pi^2}\epsilon_{\alpha\mu\eta\rho}R^{\beta\nu\eta\rho}u^\alpha u_\beta a_\nu$$

- Exact result - shown outside of perturbation theory

(But does not work at low temperatures $T \sim |a|, |\omega|$ due to instability.

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D100, 125009 (2019)]

- In most of the cases - polynomiality in $\omega, a, \mu, T \dots$

- Describes the **current** at sufficiently high temperatures, in the **Ricci-flat space-time** $R_{\mu\nu} = 0$ e. g. in the space around the black hole?

Part 4 Verification: spin 3/2

Rarita-Schwinger-Adler model of spin 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

Generalized Hamiltonian dynamics: **Dirac bracket** instead of **Poisson bracket**

$$[F(\vec{x}), G(\vec{y})]_D = [F(\vec{x}), G(\vec{y})] - \int d^3w d^3z [F(\vec{x}), \chi^\dagger(\vec{w})] M^{-1}(\vec{w}, \vec{z}) [\chi(\vec{z}), G(\vec{y})]$$

$$M(\vec{x}, \vec{y}) = [\chi(\vec{x}), \chi^\dagger(\vec{y})]$$

There is **singularity** in a Dirac bracket in weak gauge field limit for RS-theory!

Doesn't allow to construct perturbation theory!

Solved in **[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]** by introducing of interaction with additional spin ½ field:

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i\bar{\lambda}\gamma^\mu \partial_\mu \lambda - im\bar{\lambda}\gamma^\mu \psi_\mu + im\bar{\psi}_\mu \gamma^\mu \lambda \right)$$

Rarita-Schwinger-Adler model of spin 3/2

- The interaction **shifts the pole** in the Dirac bracket!

$$[\Psi_i(\vec{x}), \Psi_j^\dagger(\vec{y})]_D = -\,i\left[(\delta_{ij} - \frac{1}{2}\sigma_i\sigma_j)\delta^3(\vec{x}-\vec{y}) - \overrightarrow{D}_{\vec{x}\,i}\frac{\delta^3(\vec{x}-\vec{y})}{m^2 + g\vec{\sigma}\cdot\vec{B}(\vec{x})}\overleftarrow{D}_{\vec{y}\,j}\right]$$

Contribution of interaction with an additional field

- The **stress-energy tensor** can be obtained by varying with respect to the metric

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}$$

$$T^{\mu\nu} = \frac{1}{2}\varepsilon^{\lambda\nu\beta\rho}\bar{\psi}_\lambda\gamma_5\gamma^\mu\partial_\beta\psi_\rho + \frac{1}{8}\partial_\eta\Big(\varepsilon^{\lambda\alpha\nu\rho}\bar{\psi}_\lambda\gamma_5\gamma_\alpha[\gamma^\eta,\gamma^\mu]\psi_\rho\Big) + \frac{i}{4}\Big(\bar{\lambda}\gamma^\nu\partial^\mu\lambda - \partial^\mu\bar{\lambda}\gamma^\nu\lambda\Big)$$
$$+\frac{i}{2}m\Big(\bar{\psi}^\mu\gamma^\nu\lambda - \bar{\lambda}\gamma^\nu\psi^\mu\Big) + (\mu\leftrightarrow\nu)\,.$$

Traceless unlike the usual Rarita-Schwinger field $\quad T^\mu_\mu = 0$

- The currents can be obtained from Noether's theorem. The **axial current** can be constructed for the $U(1)_A$ transformation:

$$j^\mu_A = -i\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_\nu\psi_\rho + \bar{\lambda}\gamma_\mu\gamma_5\lambda$$

Chiral anomaly in RSA theory: gauge part

- Since the problem with the Dirac bracket is solved – **perturbation theory** can be constructed

- The **chiral (gauge) quantum anomaly** was obtained by the shift method:

[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]

see also

[S. L. Adler, P. Pais, Phys. Rev. D 99, 095037 (2019)]

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Also by the method of conformal three-point functions:

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

- The factor "**5**" differs from what is expected according to the prediction "**3**" based on supergravity [M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

However the correspondence is restored if we take into account that there are two additional degrees of freedom with spin ½: then ***5=3+2***

Gravitational chiral anomaly in RSA theory

For a conformally symmetric theory, if

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu j_V^\mu = 0\,, \quad \partial_\mu j_A^\mu = 0\,,$$
$$T^\mu_\mu = 0\,, \quad T_{\mu\nu} = T_{\nu\mu}\,.$$

It is proven in [J. Erdmenger.Nucl. Phys. B, 562:315–329, 1999], that the three-point function $\langle T\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\rho}(y)\hat{j}^A_\omega(z)\rangle_c$ has the **universal form**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^\omega_A(z)\rangle_c = \frac{1}{(x-z)^8(y-z)^8}$$
$$\times \mathscr{I}_T^{\mu\nu,\mu'\nu'}(x-z)\mathscr{I}_T^{\sigma\rho,\sigma'\rho'}(y-z)t^{TTA}_{\mu'\nu'\sigma'\rho'}{}^{\omega}(Z)$$

where the notations are introduced:

$$\mathscr{I}^T_{\mu\nu,\sigma\rho}(x) = \mathscr{E}^T_{\mu\nu,\alpha\beta} I^\alpha_\sigma(x) I^\beta_\rho(x)\,,$$
$$\mathscr{E}^T_{\mu\nu,\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta}\,,$$
$$t^{TTA}_{\mu\nu\sigma\rho\omega}(Z) = \frac{\mathscr{A}}{Z^6}(\mathscr{E}^T_{\mu\nu,\eta}{}^{\varepsilon}\mathscr{E}^T_{\sigma\rho,\kappa\varepsilon}\varepsilon_\omega{}^{\eta\kappa\lambda}Z_\lambda$$
$$-6\,\mathscr{E}^T_{\mu\nu,\eta\gamma}\mathscr{E}^T_{\sigma\rho,\kappa\delta}\varepsilon_\omega{}^{\eta\kappa\lambda}Z^\gamma Z^\delta Z_\lambda Z^{-2})$$

“6” – consequence of $T^\mu_\mu = 0$

Gravitational chiral anomaly in RSA theory

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

Summing 9 correlates (contributions of different), we will obtain:

$$\langle T\,\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = -19\Big(4\pi^6(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

⟺

Matches the form we want!

$$\langle T\,\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle = \mathscr{A}\Big(4(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

(points on the same axis)

We can determine the factor in the anomaly:

$$\mathscr{A}_{RSA} = -19\mathscr{A}_{s=1/2} = -\frac{19}{\pi^6}$$

$$\langle\nabla_{\mu}\hat{j}^{\mu}_{A}\rangle_{RSA} = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

–19 times different from the anomaly for spin ½

Gravitational chiral anomaly in RSA theory

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_{RS} = \frac{-21}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

"ghostless" contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

Contribution of spin 1/2

$$-19 = -21 + 2$$

Zubarev density operator

- The density operator $\hat{\rho}$ plays a central role in the statistical description of a medium.
- Integration in the imaginary time plane: Quantum field theory in imaginary time.
- General covariant form of the density operator in local equilibrium.

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\,\widehat{j}^\mu(x) \right) \right] \quad \Rightarrow \quad \text{maximal} \quad S = -\mathrm{tr}(\widehat{\rho} \log \widehat{\rho})$$

[D.N. Zubarev, A.V. Prozorkevich and S.A. Smolyanskii, Theor. Math. Phys. 40 (1979) 821.]

[M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017).]

⇩ On condition

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

⇨

$$\beta_\mu(x) = b_\mu + \varpi_{\mu\nu} x^\nu \qquad \zeta = \mathrm{const}$$

$$b_\mu = \mathrm{const} \qquad \varpi_{\mu\nu} = \mathrm{const}$$

⇩

$\widehat{\rho}$ does not depend on the choice of hypersurface $\mathrm{d}\Sigma_\mu$

⇩

global thermodynamic equilibrium

Zubarev density operator

Global Equilibrium Conditions

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

Form of the density operator for a medium with rotation and acceleration

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

4-momentum operator

Thermal vorticity tensor

charge operator

Lorentz Transform Generators

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right)$$

$$\varpi_{\mu\nu} \hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (related to acceleration)

$\hat{J}^\mu$ – angular momentum (related to vorticity)

Zubarev density operator

Quantum statistical mean value

$$\langle \widehat{O}(x)\rangle = \frac{1}{Z}\mathrm{tr}(\widehat{\rho}\,\widehat{O}(x))_{\mathrm{ren}}$$

statistical sum:
cancellation of disconnected correlators

Perturbation theory in the third order

$$\langle \widehat{O}(x)\rangle = \langle \widehat{O}(x)\rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2|\beta|}\int_0^{|\beta|} d\tau \langle T_\tau J^{\mu\nu}_{-i\tau u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}}{8|\beta|^2}\int_0^{|\beta|} d\tau_x d\tau_y \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u}\widehat{O}(0)\rangle_{\beta(x),c}$$

$$+\frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3}\int_0^{|\beta|} d\tau_x d\tau_y d\tau_z \langle T_\tau J^{\mu\nu}_{-i\tau_x u} J^{\rho\sigma}_{-i\tau_y u} J^{\alpha\beta}_{-i\tau_z u}\widehat{O}(0)\rangle_{\beta(x),c} + \ldots$$

Imaginary time $\tau = i\,t$ *ordering*

Connected correlators

$$\langle \widehat{J}\widehat{O}\rangle_c = \langle \widehat{J}\widehat{O}\rangle - \langle \widehat{J}\rangle\langle \widehat{O}\rangle$$

KVE IN RSA THEORY: CALCULATION

- Our *goal* is to calculate the conductivities λ_1 and λ_2 in the KVE current:

$$j^{\mu}_{A,KVE} = \lambda_1(\omega_\nu \omega^\nu)\omega^\mu + \lambda_2(a_\nu a^\nu)\omega^\mu$$

- Using the described perturbation theory, we obtain:

$$\lambda_1 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\langle T_\tau \hat{J}^3_{-i\tau_x}\hat{J}^3_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}$$

$$\lambda_2 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\Big\{\langle T_\tau(\hat{K}^1_{-i\tau_x}\hat{J}^3_{-i\tau_y} + \hat{J}^3_{-i\tau_x}\hat{K}^1_{-i\tau_y})\hat{K}^1_{-i\tau_z u}\hat{j}^3_A(0)\rangle_{T,c} + \langle T_\tau \hat{K}^1_{-i\tau_x}\hat{K}^1_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}\Big\}$$

- Representing $\hat{J}_\sigma$, $\hat{K}^\mu$ through the stress-energy tensor, we obtain

$$\lambda_1 = -\frac{1}{6T^3}\Big(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} + C^{01|01|02|3|221} - C^{01|01|01|3|222} - C^{01|02|02|3|211} - C^{02|01|02|3|121} - C^{02|02|01|3|112}\Big),$$

$$\lambda_2 = -\frac{1}{6T^3}\Big(C^{02|00|00|3|111} + C^{00|02|00|3|111} + C^{00|00|02|3|111} - C^{01|00|00|3|211} - C^{00|01|00|3|121} - C^{00|00|01|3|112}\Big).$$

Typical correlator to be found: 4-point one-loop function

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = T^3\int[d\tau]d^3x\, d^3y\, d^3z\, x^i y^j z^k\langle T_\tau \hat{T}^{\alpha_1\alpha_2}(-i\tau_x,\mathbf{x})\hat{T}^{\alpha_3\alpha_4}(-i\tau_y,\mathbf{y})\hat{T}^{\alpha_5\alpha_6}(-i\tau_z,\mathbf{z})\hat{j}^\lambda_5(0)\rangle_{T,c}$$

When expanding the density operator, a shift occurs along the imaginary axis - field theory at finite temperatures.

KVE in RSA theory: calculation

- Apply **point splitting** to all operators (no additional contributions arise - operators satisfy free field equations):

$$\hat{T}^{\mu\nu}(X) = \lim_{X_1,X_2\to X} \mathcal{D}^{\mu\nu}_{ab(IJ)}(\tilde{\partial}_{X_1},\tilde{\partial}_{X_2})\bar{\Psi}_{aI}(X_1)\Psi_{bJ}(X_2)\,,$$
$$\hat{j}^{\mu}_{A}(X) = \lim_{X_1,X_2\to X} \mathcal{J}^{\mu}_{ab(IJ)}\bar{\Psi}_{aI}(X_1)\Psi_{bJ}(X_2)\,,$$

where $X_\mu = (\tau_x, -\mathbf{x})$

Fields are combined into one vector $\Psi_I = \{\tilde{\psi}_\mu, \lambda\}$ $(I = 0...4)$

The matrix element has the form of a product of **vertices** and **propagators**.

Vertices $\mathcal{J}^{\mu}_{(ij)} = i^{1-\delta_{0\mu}}\varepsilon^{ij\mu\nu}\tilde{\gamma}_\nu$

Euclidean Dirac matrices $\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2\delta_{\mu\nu}$

$$\mathcal{D}^{\mu\nu}_{(ij)} = -\frac{1}{2}(-i)^{\delta_{0\mu}+\delta_{0\nu}}\varepsilon^{ij\nu\beta}\left(\gamma_5\tilde{\gamma}_\mu\tilde{\partial}^{X_2}_\beta - \frac{1}{4}\gamma_5\tilde{\gamma}_\beta[\tilde{\gamma}_\vartheta,\tilde{\gamma}_\mu]\left(\tilde{\partial}^{X_1}_\vartheta + \tilde{\partial}^{X_2}_\vartheta\right)\right) + (\mu\leftrightarrow\nu)$$
$0 \le (i,j) < 4$

Propagators

$$\langle T_\tau\tilde{\psi}_{a\mu}(X_1)\bar{\tilde{\psi}}_{b\nu}(X_2)\rangle_T = \sum\!\!\!\!\!\!\!\int_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{i}{2P^2}\left(\tilde{\gamma}_\nu \not{P}\tilde{\gamma}_\mu + 2\left[\frac{1}{m^2}-\frac{2}{P^2}\right]P_\mu P_\nu \not{P}\right)_{ab}$$

$$\langle T_\tau\tilde{\psi}_{a\mu}(X_1)\bar{\lambda}_b(X_2)\rangle_T = \sum\!\!\!\!\!\!\!\int_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{-P_\mu \not{P}_{ab}}{mP^2}$$

Mixed terms are non-zero

here $P_\mu = (p_n, -\mathbf{p})\,,\ p_n = (2n+1)\pi T$

$$\langle T_\tau\lambda_a(X_1)\bar{\lambda}_b(X_2)\rangle_T = 0$$

Field λ is **non-propagating**

KVE IN RSA THEORY: CALCULATION

Substituting the **split** form of the operators into the typical correlator C, we obtain:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = -T^3 \int [d\tau] d^3x d^3y d^3z \, x^i y^j z^k \lim_{X,Y,Z,F} \mathcal{D}^{\alpha_1\alpha_2}_{a_1a_2(I_1I_2)}(\tilde{\partial}_{X_1}, \tilde{\partial}_{X_2}) \mathcal{D}^{\alpha_3\alpha_4}_{a_3a_4(I_3I_4)}(\tilde{\partial}_{Y_1}, \tilde{\partial}_{Y_2}) \mathcal{D}^{\alpha_5\alpha_6}_{a_5a_6(I_5I_6)}(\tilde{\partial}_{Z_1}, \tilde{\partial}_{Z_2})$$
$$\times \mathcal{J}^{\lambda}_{a_7a_8(I_7I_8)} \langle T_\tau \overline{\Psi}_{a_1I_1}(X_1)\Psi_{a_2I_2}(X_2)\overline{\Psi}_{a_3I_3}(Y_1)\Psi_{a_4I_4}(Y_2)\overline{\Psi}_{a_5I_5}(Z_1)\Psi_{a_6I_6}(Z_2)\overline{\Psi}_{a_7I_7}(F_1)\Psi_{a_8I_8}(F_2)\rangle_{T,c}$$

We transform the average of 8 fields using the **Wick theorem**

$$\langle \overline{\Psi}(X)\Psi(X)\overline{\Psi}(Y)\Psi(Y)\overline{\Psi}(Z)\Psi(Z)\overline{\Psi}(F)\Psi(F)\rangle_c = -\langle \Psi(Y)\overline{\Psi}(X)\rangle\langle \Psi(X)\overline{\Psi}(F)\rangle\langle \Psi(Z)\overline{\Psi}(Y)\rangle\langle \Psi(F)\overline{\Psi}(Z)\rangle + (5 \text{ terms})$$

- Initially there are 6×5^8 terms, but we should take into account, that:

1) some terms are zero because they include propagator $\langle \lambda \bar{\lambda} \rangle = 0$
2) some vertices are zero, e.g. $\mathcal{D}^{\mu\nu}_{(ij)} = 0$ if $i = j \neq 4$
3) some terms are zero in the limit $m \to \infty$ if the total number of propagators $\langle \lambda \bar{\psi} \rangle$ and $\langle \psi \bar{\lambda} \rangle$ is greater than the total number of vertices $\mathcal{D}_{(4i)}$ and $\mathcal{D}_{(i4)}$

Then there are:

94752 terms for each C correlator in λ_1

31152 terms for each C correlator in λ_2

KVE IN RSA THEORY: CALCULATION

Propagator at finite temperature

$$\langle T_\tau \Psi_{aI_1}(X)\overline{\Psi}_{bI_2}(Y)\rangle = \sum\!\!\!\!\!\!\int_P e^{iP(X-Y)} G_{ab(I_1 I_2)}(P)$$

contains either **2nd or 4th order poles:**

$$G(P) = G_1(P) + G_2(P)\,,\; G_1 \sim 1/P^2\,,\; G_2 \sim 1/P^4$$

Formulas for summation over Matsubara frequencies

[M. Buzzegoli, thesis 2020. arXiv: 2004.08186]

$$\sum_{\omega_n=(2n+1)\pi T} \frac{f(\omega_n)e^{i\omega_n\tau}}{\omega_n^2+E^2} = \frac{1}{2ET}\sum_{s=\pm 1} f(-isE)e^{\tau sE}\Big[\theta(-s\tau) - n_F(E)\Big]$$

$$\sum_{\omega_n=(2n+1)\pi T} \frac{f(\omega_n)e^{i\omega_n\tau}}{(\omega_n^2+E^2)^2} = \frac{1}{T}\sum_{s=\pm 1} e^{\tau sE}\left\{\frac{f(-isE)}{4E^2}n_F'(E) + \frac{(1-s\tau E)f(-isE)+isEf'(-isE)}{4E^3}\Big[\theta(-s\tau) - n_F(E)\Big]\right\}$$

Fermi-Dirac distribution

KVE IN RSA THEORY: CALCULATION

Let us consider (for illustration) the contribution of the first of the term from the Wick's theorem and only from $G_1 \sim 1/P^2$. After **summation over the Matsubara** frequencies, we obtain:

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk}_{\text{Wick1,G1}} = -T^3 \int [d\tau] d^3x d^3y d^3z\, x^i y^j z^k \times \int \frac{d^3p_1 d^3p_2 d^3p_3 d^3p_4}{16 E_1 E_2 E_3 E_4} e^{-i\mathbf{p}_1(\mathbf{y}-\mathbf{x}) - i\mathbf{p}_2\mathbf{x} - i\mathbf{p}_3(\mathbf{z}-\mathbf{y}) + i\mathbf{p}_4\mathbf{z}}$$
$$\times \sum_{s_n=\pm 1} e^{s_1E_1(\tau_y-\tau_x)+s_2E_2\tau_x+s_3E_3(\tau_z-\tau_y)-s_4E_4\tau_z} \Big\{\theta(-s_1[\tau_y-\tau_x]) - n_F(E_1)\Big\}$$
$$\times \Big\{\theta(-s_2\tau_x) - n_F(E_2)\Big\}\Big\{\theta(-s_3[\tau_z-\tau_y]) - n_F(E_3)\Big\}\Big\{\theta(-s_4\tau_z) - n_F(E_4)\Big\}$$
$$\times \operatorname*{tr}_{I,a}\Big\{\mathcal{D}^{\alpha_1\alpha_2}(-i\widetilde{P}_1, i\widetilde{P}_2) G_1(\widetilde{P}_2) \mathcal{J}^\lambda G_1(\widetilde{P}_4) \mathcal{D}^{\alpha_5\alpha_6}(-i\widetilde{P}_4, i\widetilde{P}_3) G_1(\widetilde{P}_3) \mathcal{D}^{\alpha_3\alpha_4}(-i\widetilde{P}_3, i\widetilde{P}_1) G_1(\widetilde{P}_1)\Big\}$$

where

$$\widetilde{P}^n_\mu = (-is_nE_1, -\mathbf{p}_1),\ E_n = |\mathbf{p}_n|$$

Contains an **explicit dependence on coordinates**.

- Can be absorbed into **derivatives** by integration by parts

$$\int d^3p_1 d^3p_2 d^3p_3 d^3p_4 d^3x d^3y d^3z\, x^i y^j z^k e^{-i\mathbf{p}_1(\mathbf{y}-\mathbf{x}) - i\mathbf{p}_2\mathbf{x} - i\mathbf{p}_3(\mathbf{z}-\mathbf{y}) + i\mathbf{p}_4\mathbf{z}} f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) =$$
$$= i(2\pi)^9 \int d^3p \left(\frac{\partial^3}{\partial p_4^k \partial p_2^i \partial p_3^j} + \frac{\partial^3}{\partial p_4^k \partial p_2^i \partial p_4^j}\right) f(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)\Big|_{\substack{\mathbf{p}_4=\mathbf{p}_1\\ \mathbf{p}_3=\mathbf{p}_1\\ \mathbf{p}_2=\mathbf{p}_1}} .$$

There remains only **one** integral over the momentum.

KVE IN RSA THEORY: CALCULATION

- The remaining actions are done explicitly: integration over imaginary time τ_x, τ_y and τ_z, over the angles in $d^3p = \sin(\vartheta)p^2 dp\, d\vartheta\, d\phi$, differentiation over the momentum variables. Finally, we obtain, in particular, for $C^{02|02|02|3|111}$

$$C^{02|02|02|3|111} = \frac{T}{480\pi^2}\int \frac{dp\, p\, e^{p/T}}{(1+e^{p/T})^5}\Bigg\{126 - 291\frac{p}{T} - 472\frac{p^2}{T^2} + \left[126 + 873\frac{p}{T} + 5192\frac{p^2}{T^2}\right]e^{p/T}$$
$$+\left[-126 + 873\frac{p}{T} - 5192\frac{p^2}{T^2}\right]e^{2p/T} + \left[-126 - 291\frac{p}{T} + 472\frac{p^2}{T^2}\right]e^{3p/T}\Bigg\} = \frac{177T^3}{80\pi^2}$$

Calculating other diagrams we obtain

$$\lambda_1 = -\frac{1}{6}\left(2\cdot\frac{177}{80\pi^2} + 6\cdot\frac{353}{240\pi^2}\right) = -\frac{53}{24\pi^2}\,,$$
$$\lambda_2 = -\frac{1}{6}\left(\frac{33}{40\pi^2} + \frac{53}{80\pi^2} + \frac{1}{2\pi^2} + \frac{3}{4\pi^2} + \frac{47}{80\pi^2} + \frac{17}{40\pi^2}\right) = -\frac{5}{8\pi^2}$$

Thus, the KVE in the RSA theory has the form:

$$j^{\mu}_{A,KVE} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^{\mu}$$

KVE vs Gravitational Anomaly

The obtained formula for **cubic gradients** (KVE):

$$j_\mu^{A(3)} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

Gravitational chiral **anomaly**:

$$\nabla_\mu j_A^\mu = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathscr{N}$$

Direct **verification**:

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)\Big/ 32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- For the RSA theory, the relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown: the factor **-19** from the anomaly is reproduced.
- Verification of the obtained formula in a very **nontrivial** case with higher spins and interaction.

KVE vs Gravitational Anomaly

- Considering also the linear terms, we obtain

$$j^{\mu}_{A,RSA} = \left(\frac{5}{6}T^2 + \frac{5}{2\pi^2}\mu^2 - \frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^{\mu}$$

A similar formula in the case of Dirac fields led to the expression in "all orders":

$$j^{\mu}_{A,\mathrm{Dirac}} = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega^{\mu}$$

(at low temperatures $T \sim |a|, |\omega|$ instability was found)

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D100, 125009 (2019)]

Also gives the "all-orders" formula?

- Using the remaining pair for the transport coefficient, we obtain:

$$j^{\mu}_{A,RSA} = \left(\frac{5T^2}{6} + \frac{5\mu^2}{2\pi^2}\right)\omega_{\mu} + \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_{\mu} + \frac{3}{2\pi^2}A_{\mu\nu}\omega^{\nu} - \frac{1}{12\pi^2}B_{\mu\nu}a^{\nu}$$

where

$$A_{\mu\nu} = u^{\alpha}u^{\beta}R_{\alpha\mu\beta\nu}$$

$$B_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\eta\rho}u^{\alpha}u^{\beta}R_{\beta\nu}{}^{\eta\rho}$$

Current (at high temperatures) in gravitational field for approximately empty space?

Part 5

KVE: ARBITRARY SPIN

KVE: ARBITRARY SPIN

Is it possible to obtain a general formula for KVE for an arbitrary spin?

General formulas

$$\nabla_\mu j^\mu_{A,S} = \frac{(S - 2S^3)}{96\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

$$\lambda_1 - \lambda_2 = 32\mathscr{N}$$

Special cases

$$j^\mu_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega^\mu$$

$$j^\mu_{A,RSA} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega^\mu$$

But it should be taken into account that the RSA theory includes **two degrees** of freedom **with spin ½**, then:

$$j^\mu_{A,S=3/2} = \left(-\frac{51}{24\pi^2}\omega^2 - \frac{3}{8\pi^2}a^2\right)\omega^\mu$$

Hypothesis:

$$\lambda_2 \sim S$$

$$\lambda_2^S = -\frac{S}{4\pi^2}$$

$$\lambda_1^S = \frac{S - 8S^3}{12\pi^2}$$

Differs from a cubic dependence on spin, which could be *naively* expected from the term

$$\Delta H = -\mathbf{\Omega} \cdot \mathbf{S}$$

Part 6

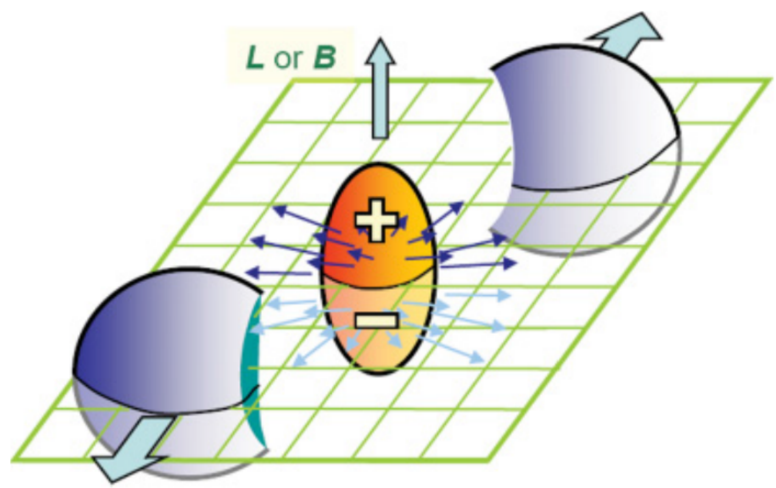
Experiment: few words

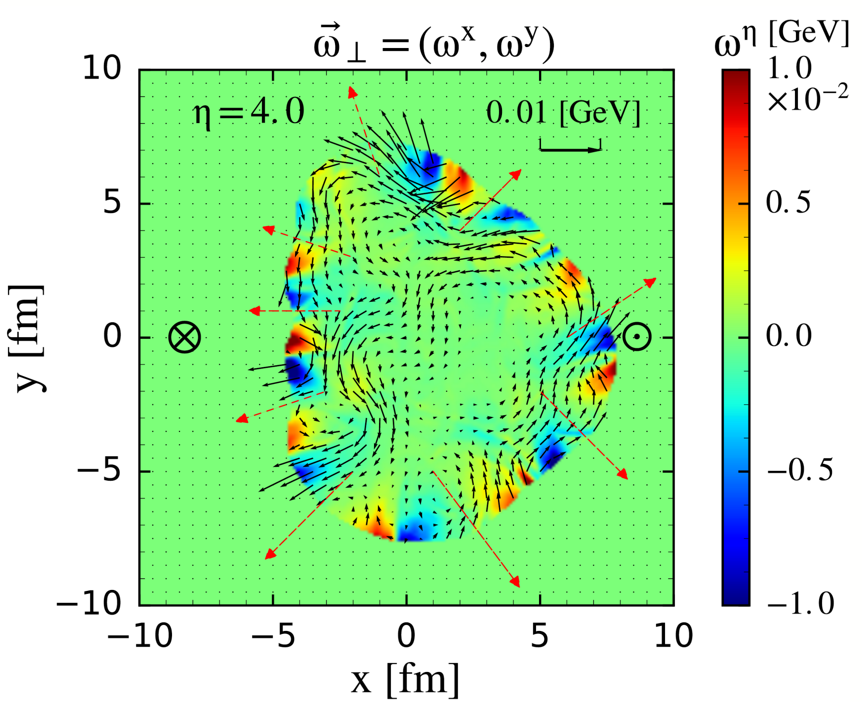
Experiment: few words

In non-central collisions of heavy ions, huge magnetic fields and huge angular momentum arise. Differential rotation - different at different points: **vorticity** and **vortices**.

– Rotation 25 orders of magnitude faster, than the rotation of the earth:

– The vorticity has order 10^{22} sec^{-1}

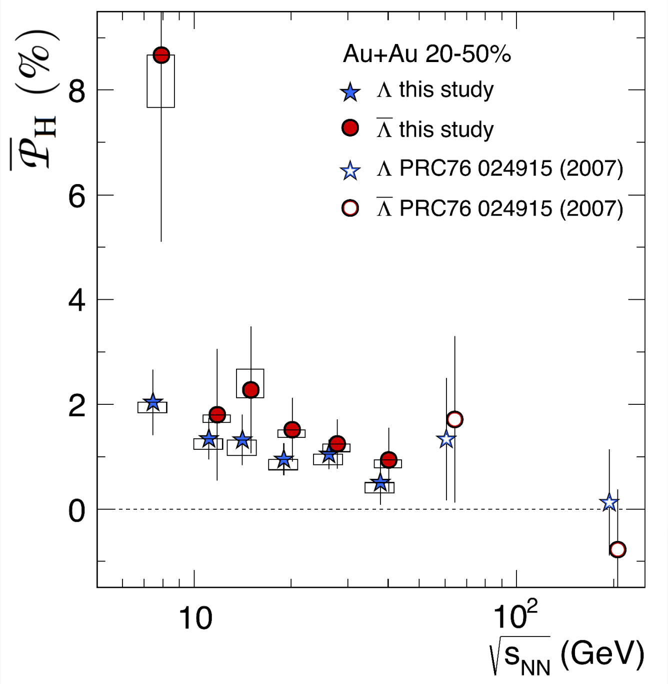




[Phys. Rev. Lett. 117, 192301 (2016)]

Experiment: few words

Vorticity transforms into polarization



[Nature 548 (2017) 62-65 arXiv:1701.06657 [nucl-ex]]

- Generation of **hyperon polarization**.
- Both **vorticity** and **acceleration** are essential for polarization.
- Also described based on **Chiral Vortical Effect** (**CVE**) **[Rogachevsky, Sorin, Teryaev, Phys.Rev.C82(2010) 054910], [Baznat, Gudima, Sorin, Teryaev, Phys.Rev.C93, no.3,031902 (2016)]**

CVE: $$\langle j^5_\mu \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega_\mu$$

– **Qualitative and quantitative correspondence!**

– Polarization from quantum anomaly ~ *spin crisis* and *gluon anomaly*: **[Efremov, Soffer, Teryaev, Nucl.Phys.B 346 (1990) 97-114]**

proton spin → hyperon polarization,
gluon field → chemical potential*4-velocity

Also described on the basis of a thermodynamic approach (Wigner function):

[I. Karpenko, F. Becattini, Nucl.Phys.A 982 (2019) 519-522]

Experiment: few words

- Is it possible to observe KVE in experiment?
- Is it possible to observe a **gravitational chiral anomaly** in the hydrodynamics of the matter, produced in heavy ion collisions?
- To observe cubic terms, it is necessary that the gradients (acceleration, vorticity, magnetic field...) give a contribution at least of the order of temperature

$$\omega, a \sim (0.1 - 2)T$$

[A. Zinchenko, A. Sorin, O. Teryaev, M. Baznat, J.Phys.Conf.Ser. 1435 (2020)]

[F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, Phys.Rev.Lett. 127 (2021) 27, 272302]

However, the cubic terms are suppressed by the numerical factor

KVE: $$j^{\mu}_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2\right)\omega^{\mu}$$

The **good** news: for spin 3/2 it is enhanced by cubic growth with spin:

The **bad** news: should be suppressed by mass $e^{-m/T}$ (omega baryon is heavy).

Idea: consider massless **quasiparticles** with spin 3/2 in semymetals?

[I. Boettcher, Phys. Rev. Lett. 124, 127602 (2020)]

Part 7

Conclusion

Conclusion

- The relationship between the hydrodynamic current in the third order of gradient expansion $(\omega_\nu \omega^\nu)\omega_\mu$ and $(a_\nu a^\nu)w_\mu$, the *Kinematical Vortical Effect* **(KVE)**, and the **gravitational chiral anomaly** has been proven:
  - The axial current in a flat space-time in a vortical and accelerated fluid turns out to be associated with a quantum violation of current conservation in a curved space-time.
- The obtained formula has been **verified** directly for **spin 1/2**.
- The obtained formula has been **verified** for **spin 3/2** using the **RSA** theory:
  - Cubic transport coefficients were derived using the statistical density operator expansion $-53/(24\pi^2)\omega^3$ and $-5/(8\pi^2)a^2\omega$.
  - Correspondence between the KVE and the gravitational chiral anomaly is directly shown.