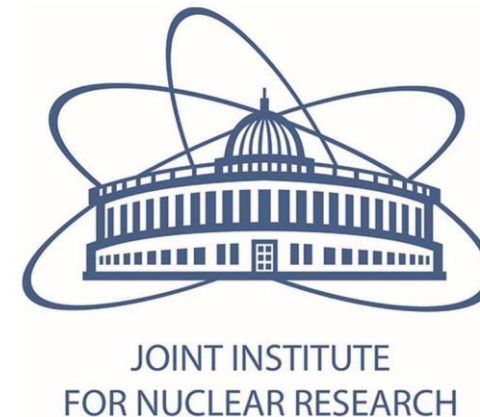


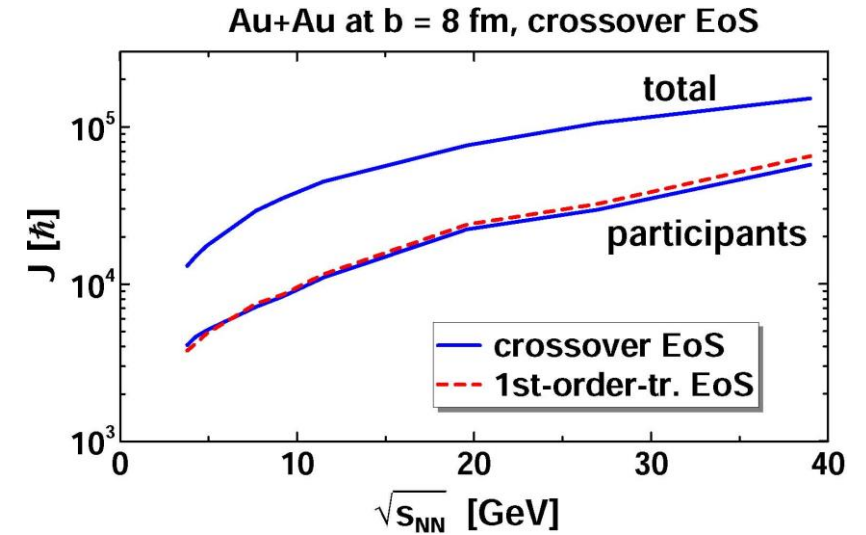
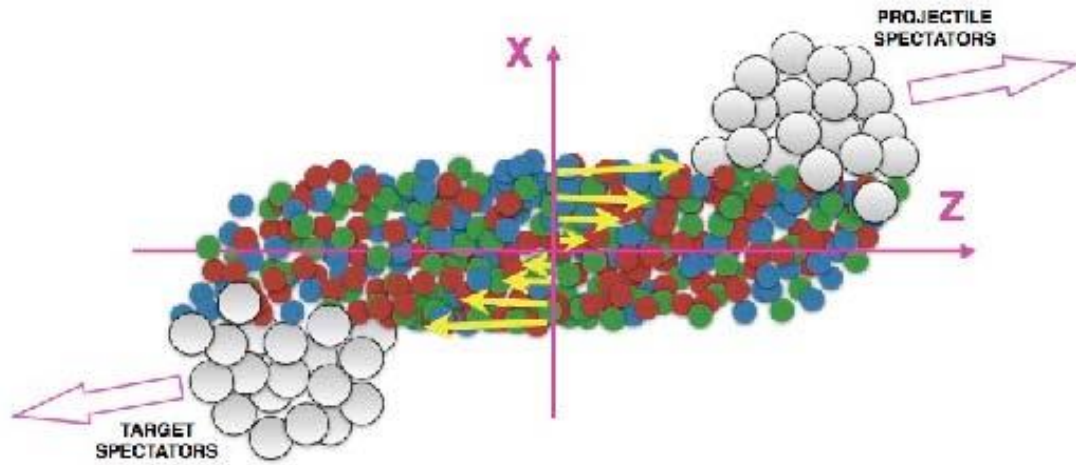
Particle polarization in heavy-ion collisions at moderately relativistic energies

Yuri B. Ivanov



BLTP seminar "Theory of Hadronic Matter under Extreme Conditions", March 31, 2021

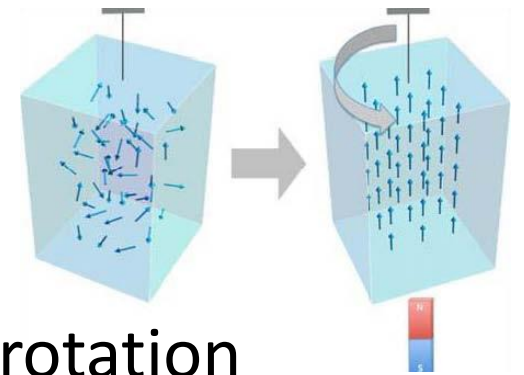
Vortical motion of nuclear matter



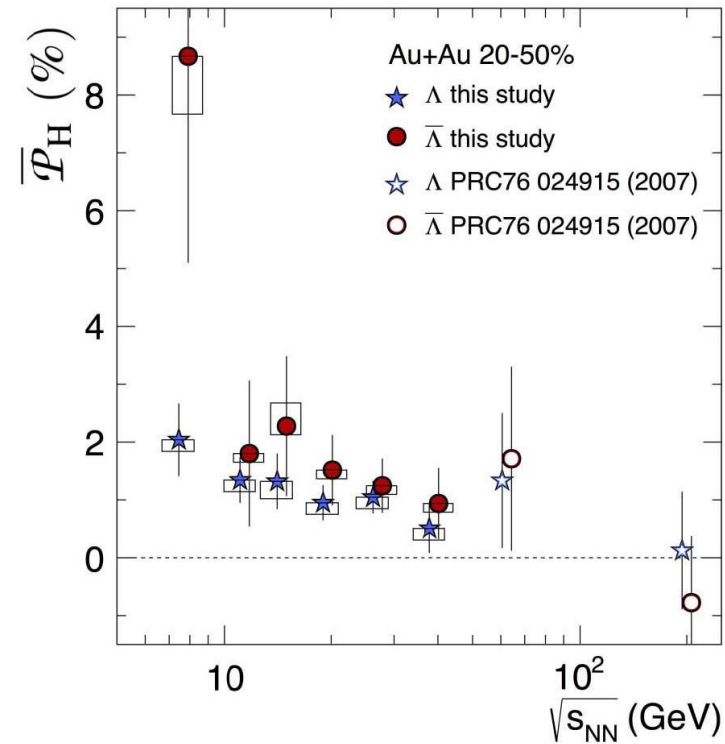
Vortical motion: $\boldsymbol{\omega} = (1/2) \nabla \times \mathbf{v} = \mathbf{Vorticity}$

Relativistic Vorticity = $\omega_{\mu\nu} = \frac{1}{2} (\partial_\nu u_\mu - \partial_\mu u_\nu)$

- Angular momentum \rightarrow spin polarization
- Similarly to Barnett effect (1915): magnetization by rotation



Polarization Measurements



STAR

- ✓ Global Λ and anti- Λ polarization [[Nature 548, 62 \(2017\)](#)]
- ✓ Local polarization of hyperons along the beam direction [[PRL 123, 132301 \(2019\)](#)]
- ✓ Measurement of global spin alignment of vector Mesons [[NPA 1005 \(2021\) 121733](#)]
- ✓ Global polarization of Ξ and Ω hyperons at 200 GeV [[2012.13601](#)]

At moderately relativistic energies

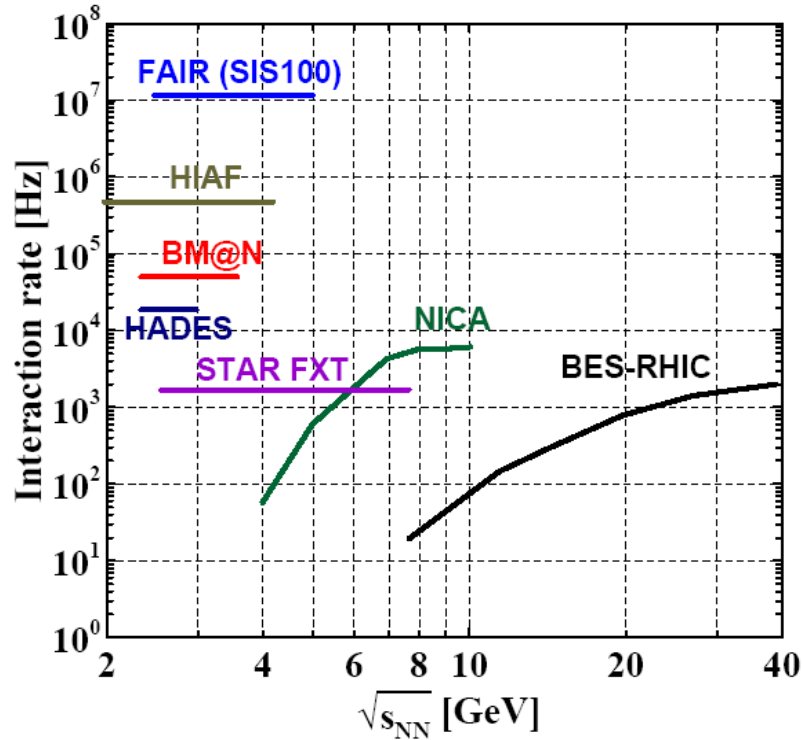
- HADES: Λ Polarization at 2.4 GeV [[Springer Proc.Phys. 250 \(2020\) 435](#)]
- STAR-FXT: in progress
- NICA: planned in approx. 2025

Motivations

Study of

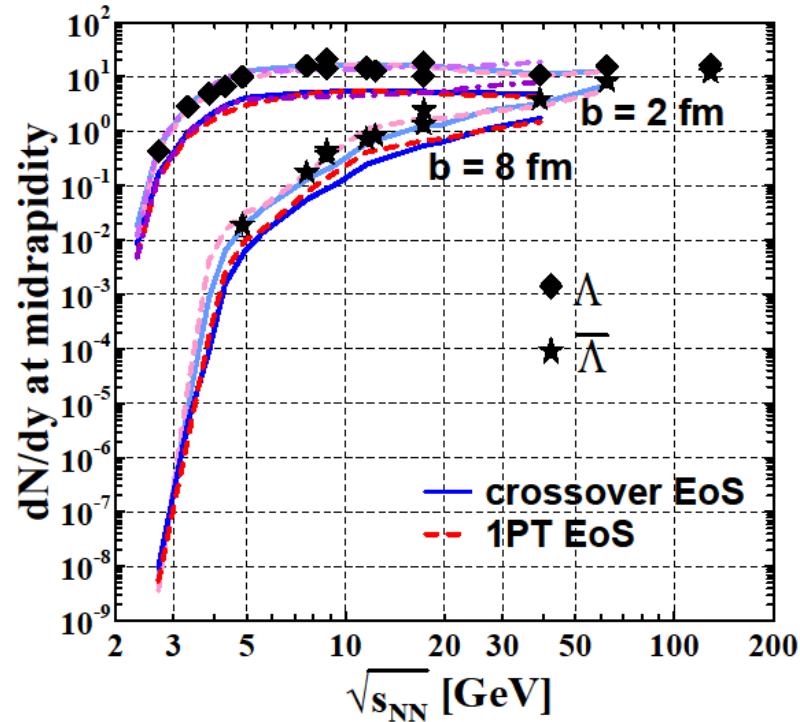
- ✓ **vortical motion in heavy-ion collisions**
- ✓ **mechanism of angular-momentum transfer from orbital one to spin**
 - Thermodynamic approach
 - Chiral Vortical Effect
 - ...

Feasibility of polarization measurements



CBM, *Eur.Phys.J.A* 53 (2017) 3, 60

Threshold collision energies, above which measurements are feasible.



STAR and HADES experience

global polarization:

$$(dN/dy)(\text{interaction rate}) \geq 1 \text{ s}$$

local polarization:

$$(dN/dy)(\text{interaction rate}) \geq 10^4 \text{ s}$$

3FD simulations

Facility	BM@N	HIAF	FAIR	NICA
$\sqrt{s_{NN}}$ [GeV]	2.3 – 3.5	2.3 – 4	2.7 – 4.9	4 – 11
global Λ , $\sqrt{s_{NN}} \gtrsim$	2.3 GeV	2.3 GeV	2.7 GeV	4 GeV
global $\bar{\Lambda}$, $\sqrt{s_{NN}} \gtrsim$	no	3.5 GeV	3 GeV	5 GeV
local Λ , $\sqrt{s_{NN}} \gtrsim$	2.7 GeV	2.5 GeV	2.7 GeV	6 GeV
local $\bar{\Lambda}$, $\sqrt{s_{NN}} \gtrsim$	no	no	4 GeV	no

3-Fluid Dynamics (3FD)

Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$
 Leading particles carry bar. charge exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0$, $\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0$, $\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$
 Baryon-free fluid Source term Exchange
 The **source term** is delayed due to a formation time τ

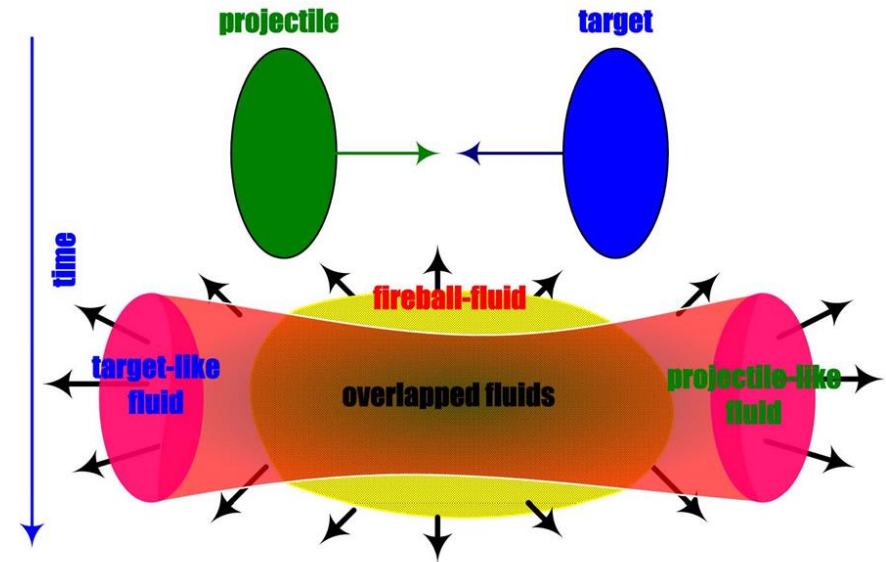
Total energy-momentum conservation:

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

YI, Russkikh, Toneev, PRC 73, 044904 (2006)

Physical Input

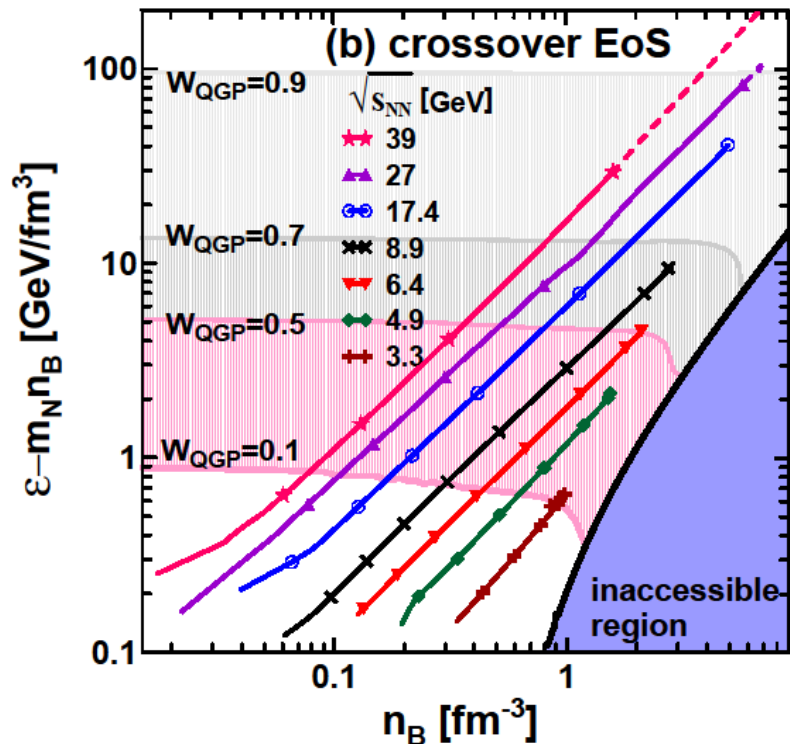
- ✓ Equation of State
- ✓ Friction
- ✓ Freeze-out energy density $\mathcal{E}_{\text{frz}} = 0.4 \text{ GeV/fm}^3$



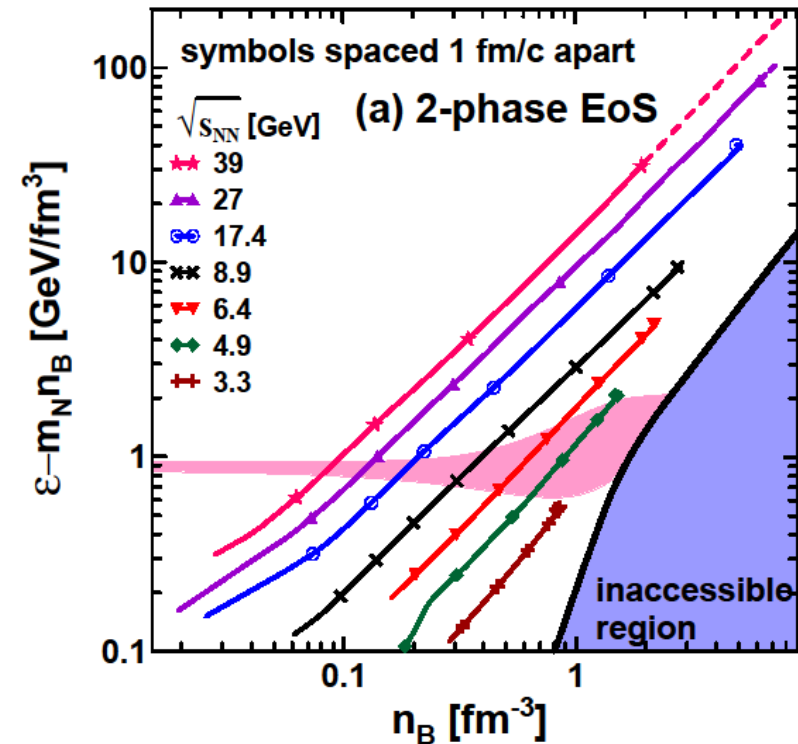
QGP Transition in central region [Y.I., Phys.Rev.C 87 (2013) 6, 064904]

$|x| \leq 2 \text{ fm}$, $|y| \leq 2 \text{ fm}$ and $|z| \leq \gamma_{\text{cm}} 2 \text{ fm}$, γ_{cm} = Lorentz factor of initial motion in cm frame

EoS's: Khvorostukhin, Skokov, Redlich, Toneev, EPJ C48, 531 (2006)

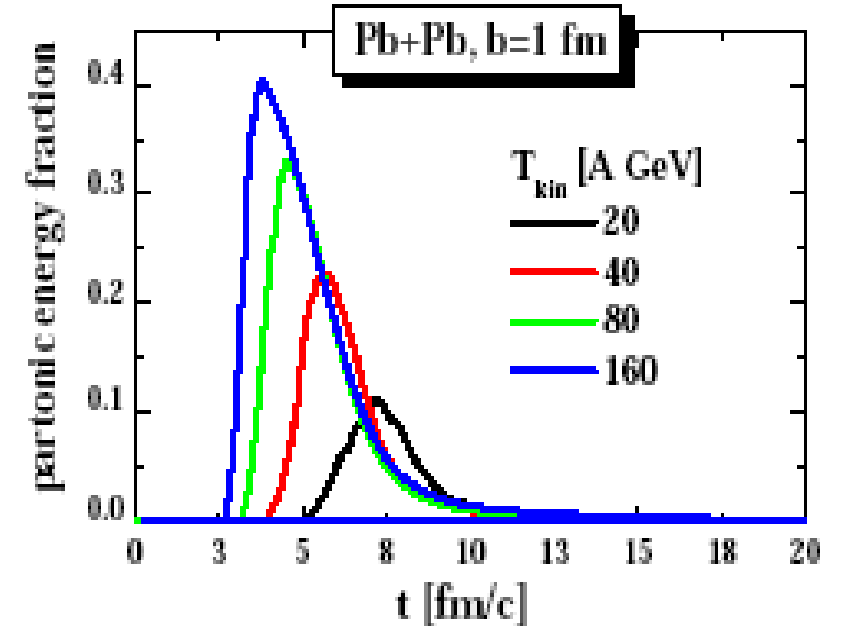
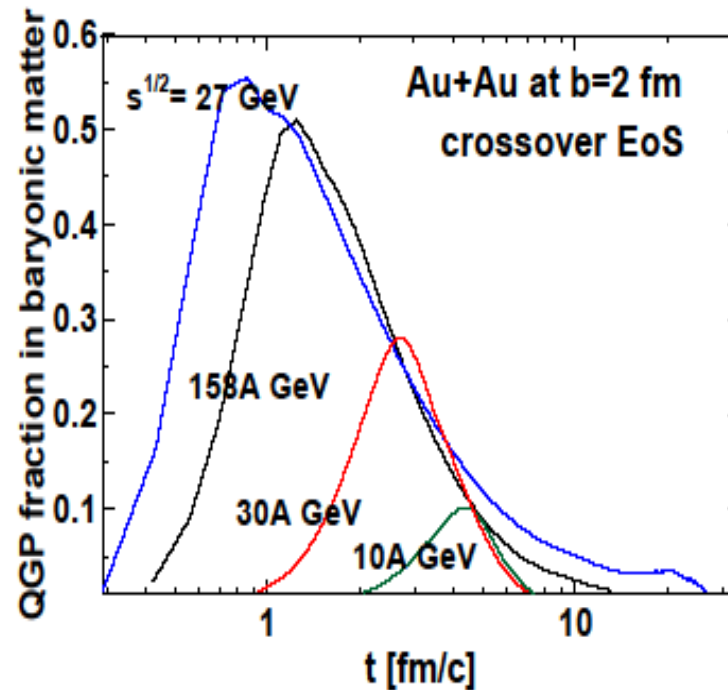
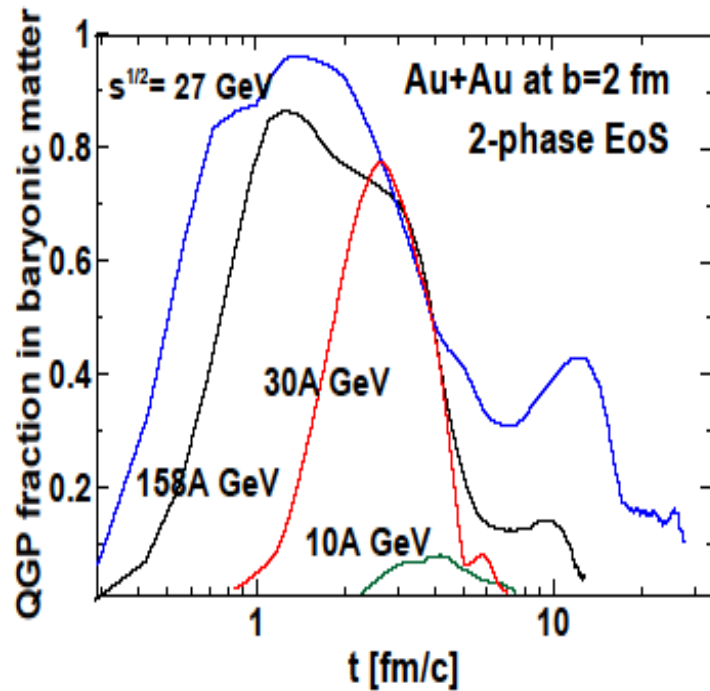


Slow crossover EoS
lattice QCD: fast crossover



deconfinement transition starts at top AGS energies in both cases.

QGP Transition in bulk



Deconfinement transition starts at top AGS energies.

PHSD: Cassing&Bratkovskaya,
NPA 831, 215 (2009).

Alternative viewpoint: Seck, Galatyuk, et al., arXiv:2010.04614 [nucl-th]
Dilepton Signature of a First-Order Phase Transition already at 1-2A GeV.

Equilibration at low energies

- Thermodynamic approach
 - Chiral Vortical Effect
- Require equilibrium

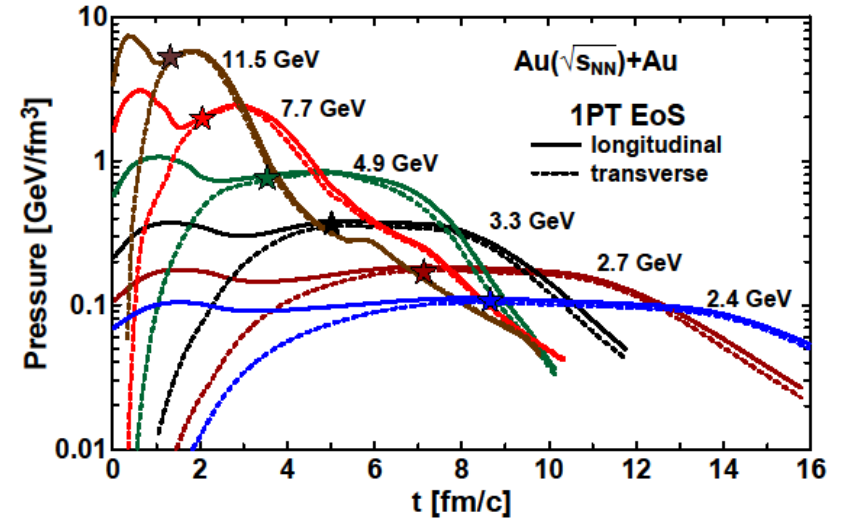
Longitudinal and transverse pressure in the center

Mechanical equilibration time (★) is comparatively long

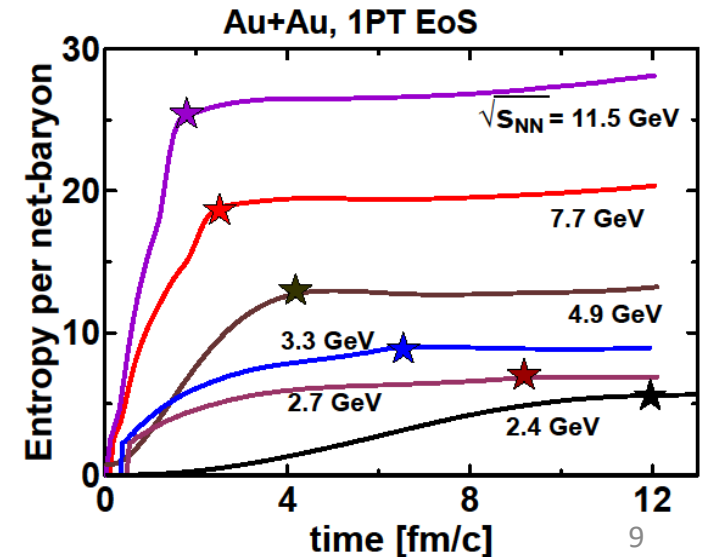
Freeze-out is mechanically equilibrium.
This of prime importance for the models.

Chemical equilibration (★)
(and hence thermalization) takes longer

YI, Soldatov, PRC C 101, 024915 (2020)



YI, Soldatov, EPJA 52 (2016) 12, 367

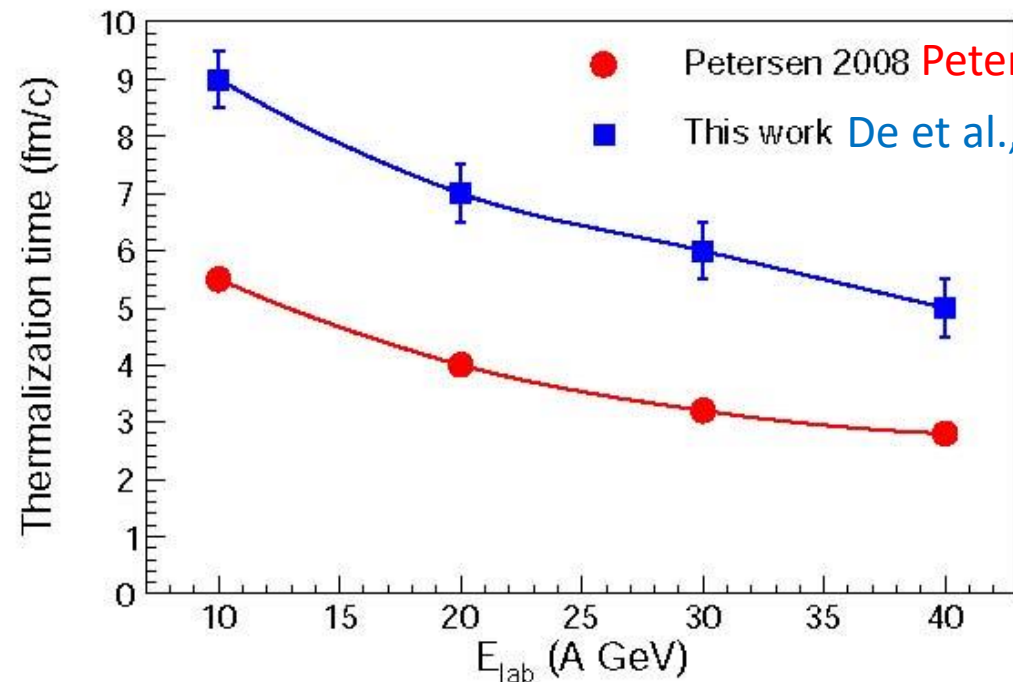


Thermalization at NICA energies

Other models result in similar thermalization times

Bravina et al., PRC 78, 014907 (2008); De et al., PRC 94, 054901 (2016);

Khvorostukhin, Toneev, Phys.Part.Nucl.Lett. 14 (2017), 9; Teslyk et al., PRC 101, 014904 (2020)



● Petersen 2008 Petersen et al., PRC 78, 044901 (2008) [twice overlap time]

■ This work De et al., PRC 94, 054901 (2016) [UrQMD]

For comparison:

Mechanical Equilibration at 10 A GeV

≈ 3.5 fm/c

The system is thermalized at the freeze-out stage, although it can be reached right before the freeze-out

Chiral vortical effect (CVE)

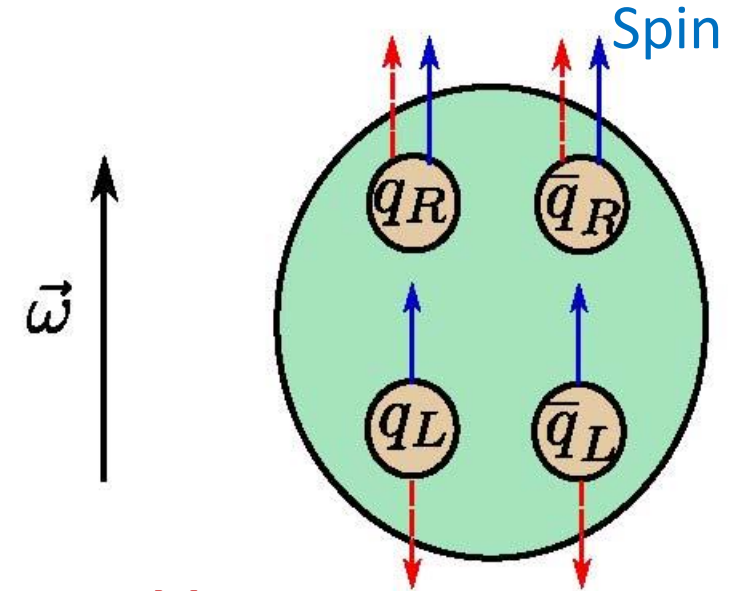
Axial current

$$J_5^\nu(x) = -N_c \left(\frac{\mu^2}{2\pi^2} + \kappa \frac{T^2}{6} \right) \epsilon^{\nu\alpha\beta\gamma} u_\alpha \omega_{\beta\gamma}$$

induced by vorticity $\omega_{\mu\nu} = \frac{1}{2} (\partial_\nu u_\mu - \partial_\mu u_\nu)$

Vilenkin, PRD 20, 1807 (1979); 21, 2260 (1980).

Son and Zhitnitsky, PRD 70, 074018 (2004)



Momentum

Gao, et al., PRL 109 (2012) 232301

$\frac{\mu^2}{2\pi^2}$ = axial anomaly term is topologically protected

$\kappa \frac{T^2}{6}$ = holographic gravitational anomaly

Landsteiner, Megias, Melgar, Pena-Benitez, JHEP 1109, 121 (2011) [Gauge-gravity correspondence]

Lattice QCD results in $\kappa = 0$ in confined phase and $\kappa \leq 0.1$ in deconfined phase

[Braguta, et al., PRD 88, 071501 (2013); 89, 074510 (2014)]

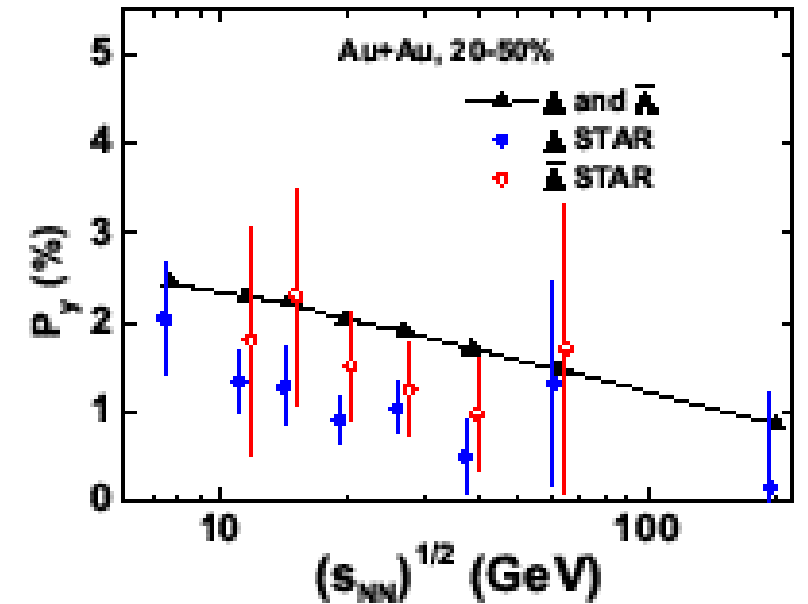
Chiral vortical effect (CVE): Coalescence

Coalescence-like hadronization:

quarks coalesce into hadrons,
keeping their polarization.

Λ -- $\bar{\Lambda}$ polarization splitting is not explained

Only BES-RHIC energies were studied



Sun and Ko, PRC 96, 024906 (2017)

Axial-vortical-effect (AVE):

Axial-charge conservation at hadronization

$$P_{\Lambda} = \int d^3x (J_{5s}^0 / u_y) / (N_{\Lambda} + N_{anti-K^*})$$
$$P_{anti-\Lambda} = \int d^3x (J_{5s}^0 / u_y) / (N_{anti-\Lambda} + N_{K^*})$$

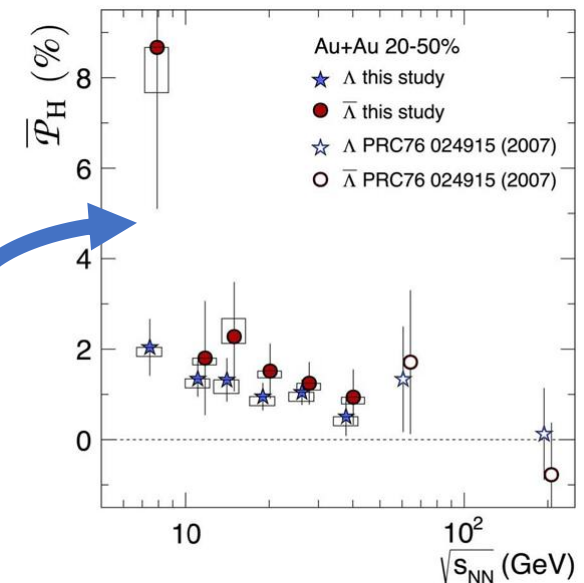
u_y results from boost to the local rest frame of the matter

Sorin and Teryaev, PRC 95, 011902 (2017)

P_{Λ} and $P_{anti-\Lambda}$ are quite different

Therefore,

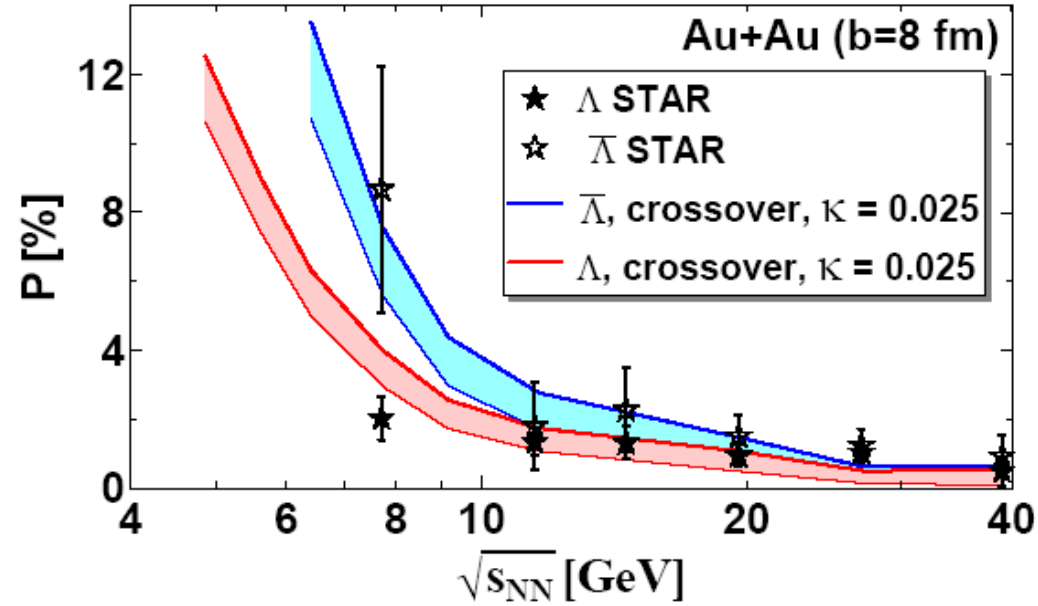
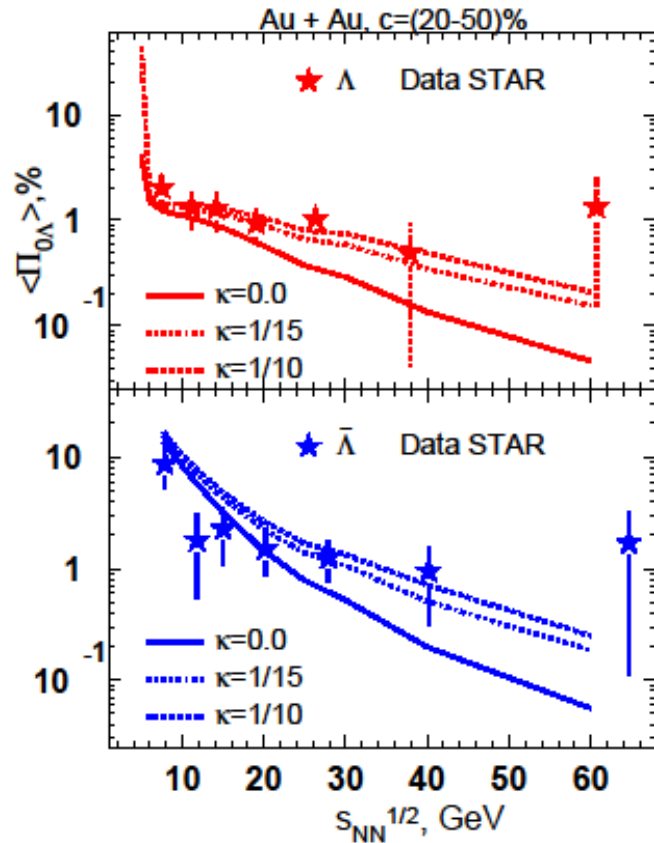
Λ -- $\bar{\Lambda}$ splitting can be addressed



Axial-vortical-effect (AVE) polarization

Baznat, Gudima, Sorin, Teryaev,
PRC 97, 041902 (2018)

YI, PRC 102 (2020) 4, 044904



Λ -- $\bar{\Lambda}$ splitting
is explained

- **CVE and AVE are hardly applicable below NICA range**
- because the chiral symmetry is spontaneously broken.

Thermodynamic approach to polarization

Spin is in thermal equilibrium with other degrees of freedom

[F. Becattini, et al., Ann. Phys. 338, 32 (2013)]

Chemical potential for angular momentum $\varpi_{\mu\nu} = \frac{1}{2}(\partial_\nu\beta_\mu - \partial_\mu\beta_\nu)$ = Thermal Vorticity

$$\beta_\mu = u_\mu / T = \text{4-velocity/Temperature}$$

Mean spin vector of a spin of Λ particle in a relativistic fluid

$$S^\mu = \frac{1}{8m_\Lambda} \frac{\int d\Sigma_\lambda p^\lambda n_\Lambda p_\sigma \varepsilon^{\mu\nu\rho\sigma} \varpi_{\rho\nu}}{\int d\Sigma_\lambda p^\lambda n_\Lambda}$$

n_Λ = Fermi-Dirac distribution function, integration over freeze-out hypersurface

Formulation in terms of frozen-out hadronic matter!

Thermodynamic global polarization in 3FD

$$\mathbf{P}_\Lambda^\mu = \langle \mathbf{S}_\Lambda^\mu \rangle / S_\Lambda \quad \text{Polarization of } \Lambda \text{ particle, } S_\Lambda = 1/2$$

Polarization is measured **in the rest frame (*)** of Λ particle $\mathbf{S}_\Lambda^* = \mathbf{S}_\Lambda - \frac{\mathbf{p}_\Lambda \cdot \mathbf{S}_\Lambda}{E_\Lambda (E_\Lambda + m_\Lambda)} \mathbf{p}_\Lambda$

$$\mathbf{P}_\Lambda = \frac{1}{2m_\Lambda} \left\langle \left(E_\Lambda - \frac{1}{3} \frac{\mathbf{p}_\Lambda^2}{E_\Lambda + m_\Lambda} \right) \varpi_{zx} \right\rangle \quad \text{Global polarization is directed along the } y \text{ axis}$$

Approximation 1: Averaging of (...) and ϖ_{zx} are decoupled $\mathbf{P}_\Lambda \simeq \frac{\langle \varpi_{zx} \rangle}{2} \left(\mathbf{1} + \frac{2}{3} \frac{\langle \mathbf{E}_\Lambda \rangle - m_\Lambda}{m_\Lambda} \right)$

Approximation 2: Averaging over central region $[z_{\text{left}}, z_{\text{right}}]$ confined by $|y| < \Delta y_h / 2$

Hydrodynamical rapidity: $y_h(z, t) = \frac{1}{2} \ln \frac{\langle u^0 + u^3 \rangle}{\langle u^0 - u^3 \rangle} \quad \Delta y_h(t) = y_h(z_{\text{right}}, t) - y_h(z_{\text{left}}, t)$

Freeze-out for polarization calculation

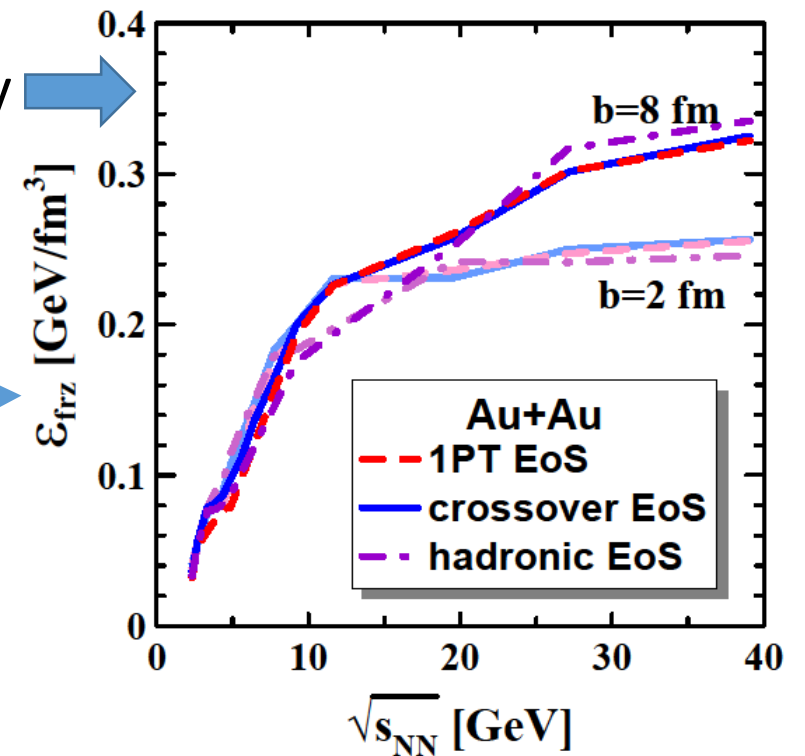
Usually it is a **local freeze-out**, i.e. cell-by-cell.

The freeze-out procedure starts when the **local energy density** $< 0.4 \text{ GeV}/\text{fm}^3$:

- (1) This criterion should be met **in the cell and in eight surrounding cells**.
- (2) At least one of the surrounding cells is empty (**border with vacuum**).

Therefore, the actual mean freeze-out energy density

**For the polarization calculation
global freeze-out at ϵ_{frz}
in the central region**



Calculations of polarization below BES-RHIC energies

Only few calculations at $\sqrt{s_{NN}} < 7.7$ GeV

✓ Within thermodynamic approach [*Becattini et al., Ann. Phys. 338, 32 (2013)*]

YI, et al., PRC 100, 014908 (2019); PRC 102, 024916 (2020); YI, PRC 103, L031903 (2021) [3FD model]

Deng, Huang, Ma, Zhang, PRC 101, 064908 (2020) [UrQMD, mean vorticity]

below NICA energies

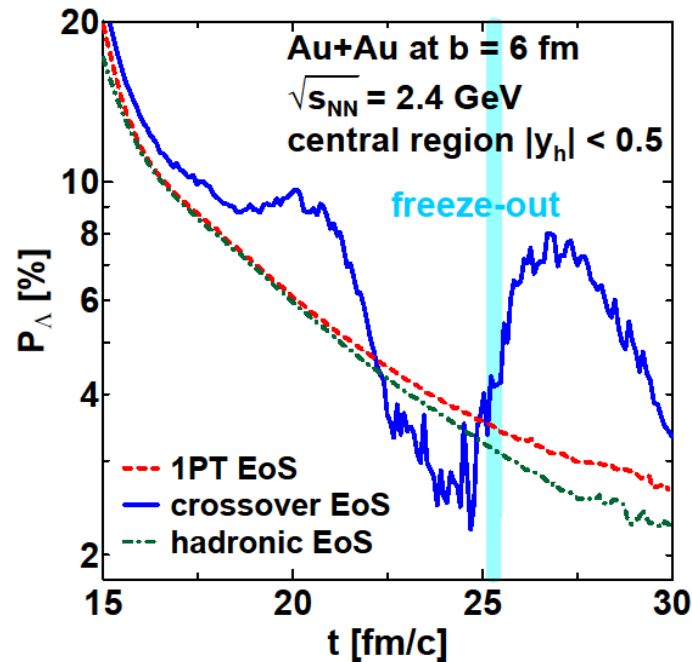
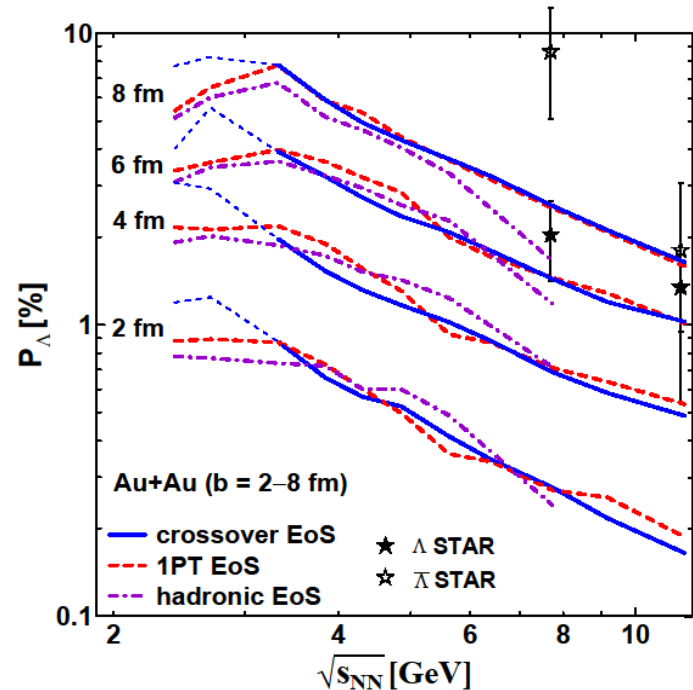
✓ Within axial-vortical-effect approach [*Sorin&Teryaev, PRC 95, 011902 (2017)*]

Baznat, Gudima, Sorin, Teryaev, PRC 97, 041902 (2018) [QGSM model]

YI, PRC 102 (2020) 4, 044904 [3FD model]

Thermodynamic polarization at moderate energies

YI, PRC **103**, L031903 (2021)

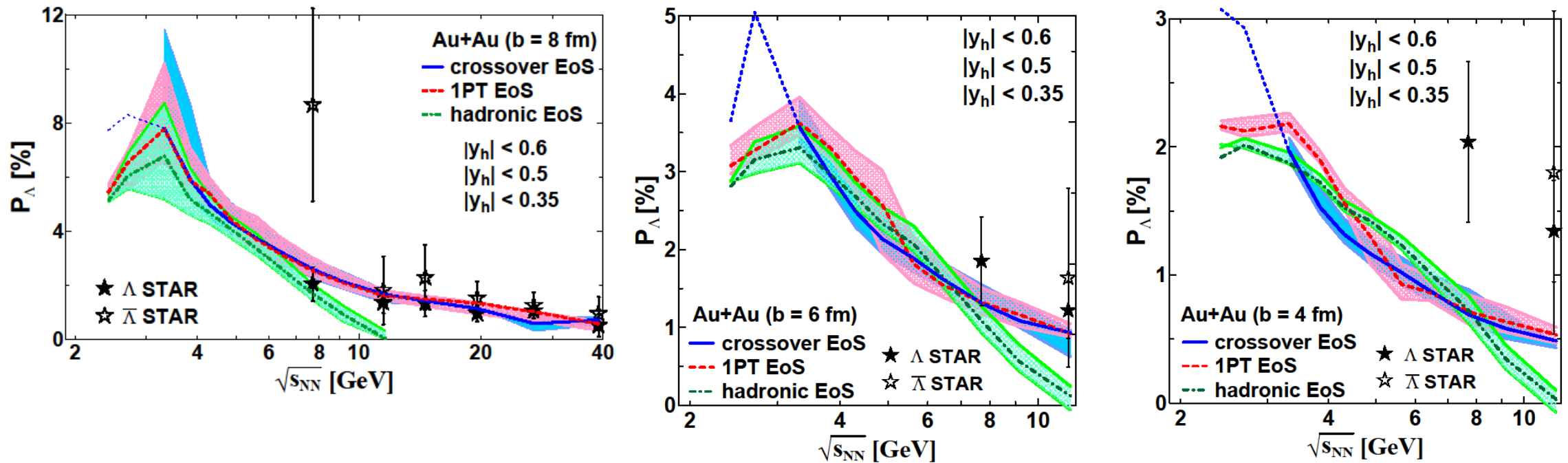


Unstable numerics for
crossover EoS

**Polarization reaches a maximum or a plateau
(depending on EoS and centrality) at $\sqrt{s_{NN}} \approx 3$ GeV.**

Rapidity window dependence

$|y_h| < 0.6$ upper border, $|y_h| < 0.5$ center line, $|y_h| < 3.5$ lower border



Global polarization increases with increasing width of rapidity window around the midrapidity

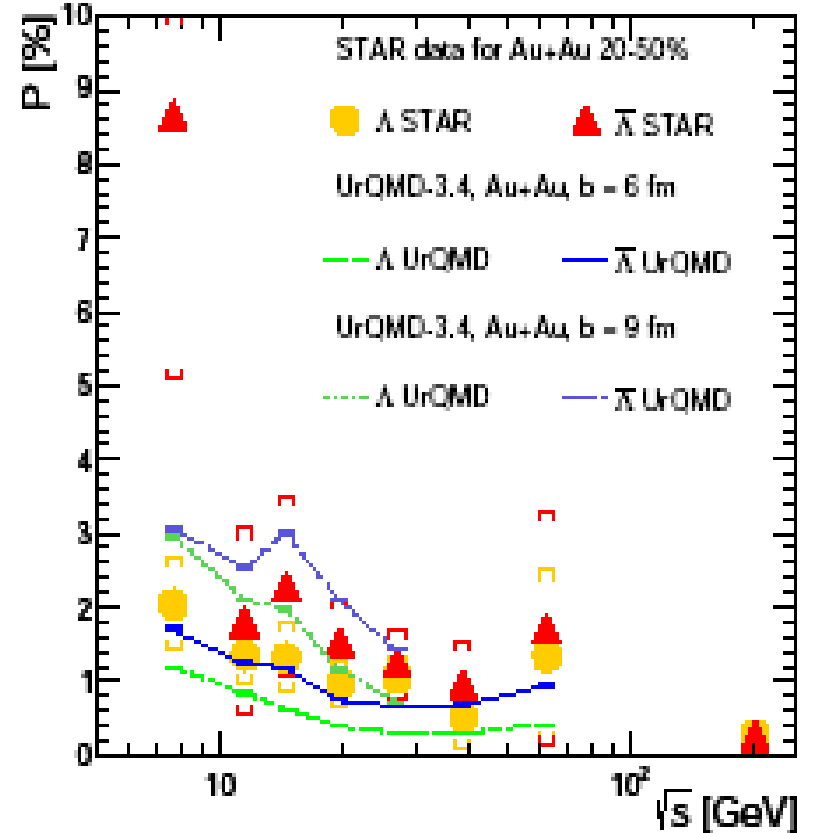
Problem: Λ -- $\bar{\Lambda}$ polarization splitting

In the standard thermodynamic approach this splitting is either very small

or simply small, if different freeze-out for Λ and $\bar{\Lambda}$ is taken into account,

Vitiuk, Bravina and Zabrodin, *Phys. Lett. B* 803, 135298 (2020)

while exp. difference is large at 7.7 GeV, although error bars for $\bar{\Lambda}$ are also large.

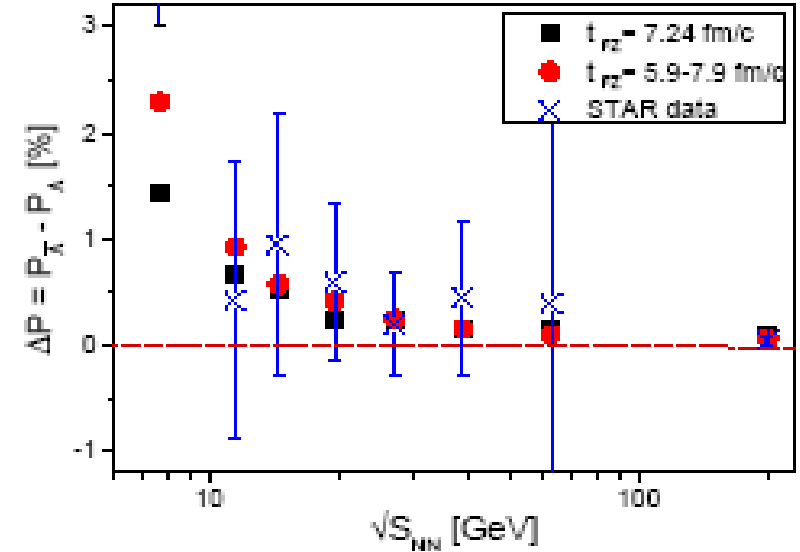


$\Lambda - \bar{\Lambda}$ polarization splitting: possible solutions

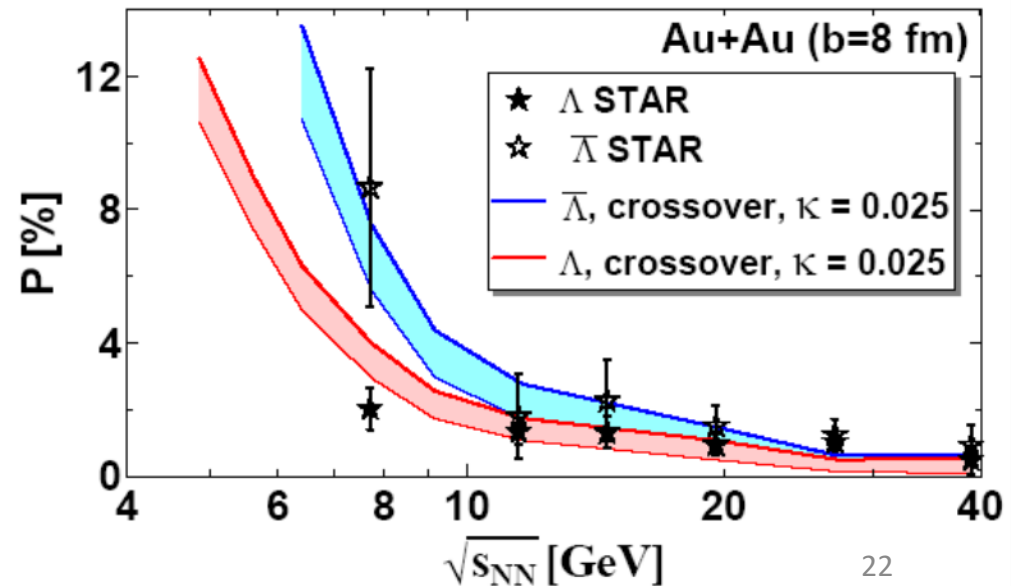
Interaction mediated by massive vector and scalar bosons (Walecka-like model)

Csernai, Kapusta, Welle, PRC 99, 021901 (2019)

Xie, Chen, Csernai, EPJC 81, 12 (2021) 



AVE naturally explains the $\Lambda - \bar{\Lambda}$ splitting



Fixed-target experiments

BM@N at JINR, CBM at FAIR, STAR FXT, HADES

Rapidity dependence of polarization is still under debates

[Becattini and Lisa, arXiv:2003.03640]

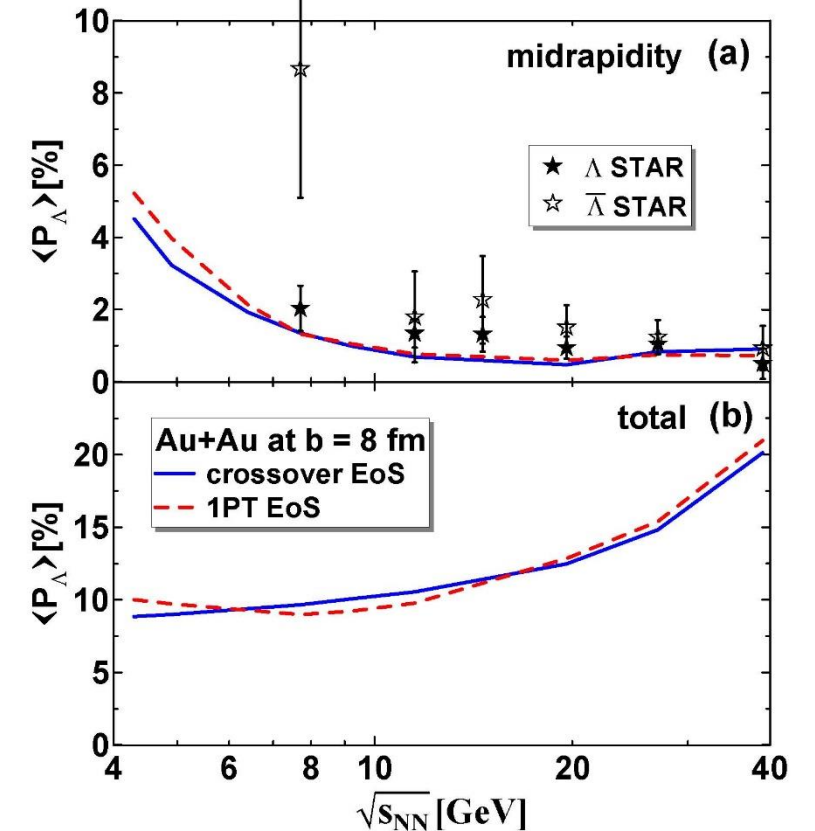
3FD: **total Λ polarization (i.e. averaged over all rapidities) increases with collision energy rise, in contrast to midrapidity polarization.**

In means

- ✓ Λ polarization in target fragmentation region is higher than the midrapidity one
- ✓ It increases with collision energy rise

It would be interesting to check these predictions

YI, et al., PRC 100, 014908 (2019)



Summary

- ✓ Prerequisite for polarization models:
The system is thermalized at the freeze-out stage
- ✓ Prediction: Λ polarization rises with collision energy decrease at $\sqrt{s_{NN}} \leq 7.7$ GeV
- ✓ Prediction: Λ polarization reaches a maximum or a plateau at $\sqrt{s_{NN}} \leq 3$ GeV
- ✓ Prediction: Λ polarization increases from midrapidity to forward/backward rapidities
- ✓ Measurements at moderate energies can clarify the nature of the $\Lambda - \bar{\Lambda}$ splitting

Other problems related to Polarization

- Thermodynamic approach predicts **the wrong sign of the local longitudinal polarization** as compared with that measured by STAR
It seems to be resolved [Becattini et al., 2103.10917, 2103.14621] two days ago.
Additional shear term resolves the problem and also contributes to global polarization
- **Spin alignment of vector mesons (ϕ and K^*):** it is large as compared to the tiny alignment predicted by the thermodynamic model.
- Conceptual problems

Additional shear term

[F. Becattini, M. Buzzegoli, A. Palermo. 2103.10917;

F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, 2103.14621]

Standard vortical term

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda \beta_\nu(x) (y-x)^\lambda$$

and replacing into the (5), with $\zeta = 0$ which is a good approximation for the purpose of this work:

$$\hat{\rho}_{\text{LE}} \simeq \frac{1}{Z_{\text{LE}}} \exp \left[-\beta_\nu(x) \hat{P}^\nu + \right. \\ \left. - \partial_\lambda \beta_\nu(x) \int_\Sigma d\Sigma_\mu(y) (y-x)^\lambda \hat{T}^{\mu\nu}(y) \right], \quad (7)$$

additional shear term

$$S_{\omega}^{\mu}(p) = -\frac{1}{8m} e^{\mu\nu\sigma\tau} p_\tau \frac{\int_{\Sigma} d\Sigma \cdot p n_F (1 - n_F) \varpi_{\rho\sigma}}{\int_{\Sigma} d\Sigma \cdot p n_F}, \quad (1)$$

where thermal vorticity is defined as the anti-symmetric derivative of the four-temperature field:

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu). \quad (2)$$

$$S_{\xi}^{\mu}(p) = -\frac{1}{4m} e^{\mu\nu\sigma\tau} \frac{p_\tau p^\lambda}{\varepsilon} \frac{\int_{\Sigma} d\Sigma \cdot p n_F (1 - n_F) \hat{t}_\rho \xi_{\sigma\lambda}}{\int_{\Sigma} d\Sigma \cdot p n_F} \quad (3)$$

where $\varepsilon = \sqrt{m^2 + p^2}$, \hat{t} is the time direction in the QGP or center-of-mass frame, and ξ is the symmetric derivative of the four-temperature, defined as *thermal shear tensor*:

$$\xi_{\mu\nu} = \frac{1}{2} (\partial_\mu \beta_\nu + \partial_\nu \beta_\mu). \quad (4)$$

Plans

- **Effect of the additional shear term on the global polarization**
- **Nature of the Λ -- $\bar{\Lambda}$ splitting** (interaction mediated by massive vector and scalar bosons)
- **Rapidity dependence of the global polarization**

Backup

Nuclotron-based Ion Collider Facility (NICA)

Dubna 2020



MultiPurpose Detector (MPD)

Au+Au

$$\sqrt{s_{NN}} = 4 - 11 \text{ GeV}$$

Bi(A=209) beam 2022

Au beam is planned later

Data taking at MPD 2023

**Polarization measurements
are planned (approx. 2025)**