



3FD model

JINR,  
20.02.13

MFD  
Models

2-Fluid Models

3-Fluid Models

3FD

Friction

Freeze-out

Phys. Input

Results

Baryon Stopping

Part. Production

Hadron Ratios

pt spectra

Flow

Summary

# Heavy-Ion Collisions within Multi-Fluid Simulations: Scenarios with and without Deconfinement Transition

**Yu.B. Ivanov**

**Kurchatov Inst.**



# Outline

## 3FD model

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## MFD Models

2-Fluid Models  
3-Fluid Models  
3FD  
Friction  
Freeze-out

## Phys. Input

## Results

Baryon Stopping  
Part. Production  
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pt spectra  
Flow

## Summary

- 1 Multi-Fluid Models:**  
minimal extension of hydrodynamics required by heavy-ion dynamics
  - 2-Fluid Models
  - 3-Fluid Models
  - 3-Fluid Dynamics (3FD), present version
  - Friction
  - Freeze-out
- 2 Physical Input of the Model**
- 3 Results**
  - Baryon Stopping
  - Part. Production
  - Hadron Ratios
  - pt spectra
  - Flow
- 4 Summary**



# Exploring Nuclear Phase Diagram

## 3FD model

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## MFD Models

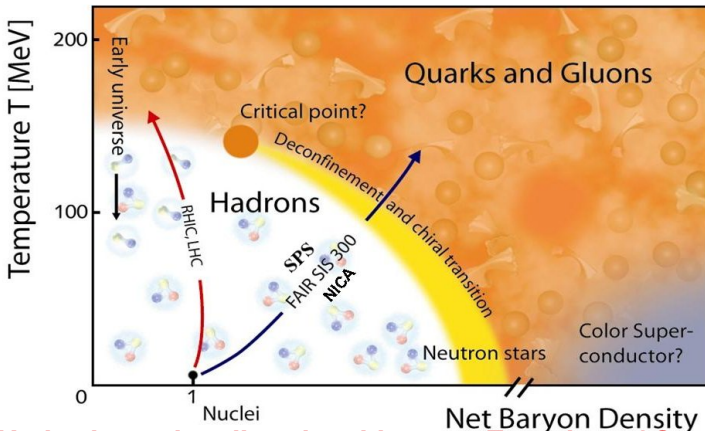
- 2-Fluid Models
- 3-Fluid Models
- 3FD
- Friction
- Freeze-out

## Phys. Input

## Results

- Baryon Stopping
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- Flow

## Summary



**Hydrodynamics directly addresses Equation of State**

However, nonequilibrium prevents direct application of Hydrodynamics



# Hydrodynamics versus Kinetics

## 3FD model

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## Summary

## Why we are not satisfied with kinetics?

- In practice, kinetics  $\Rightarrow$  only binary collisions  
mean free path  $\lambda \approx 1/(n_B \sigma)$   
if  $\sigma \approx 4 \text{ fm}^2$  and  $n_B \approx 5n_0 \Rightarrow \lambda \approx 0.3 \text{ fm} \sim$  nucleon core  
( $n_0 = 0.15 \text{ fm}^{-3} =$  normal nuclear density)

Approximation of binary collisions is bad!

- Phase transition into QGP is inaccessible in kinetics as a rule

Two exceptions based on simple combinatorics of quarks:

A Multi-Phase Transport (AMPT) model [Lin, Ko and Pal, PRL 89, 152301 (2002)]

Parton-Hadron-String Dynamics [Cassing, Bratkovskaya, arXiv:0907.5331 (2009)]

## Hydrodynamics

- takes into account any multi-particle interactions
- directly addresses **Equation of State (EoS)**!
- Phase transition in QGP is accessible through EoS
- However, there are certain **problems**



# 2-Fluid Models

## 3FD model

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## MFD Models

2-Fluid Models

3-Fluid Models

3FD

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Freeze-out

## Phys. Input

## Results

Baryon Stopping

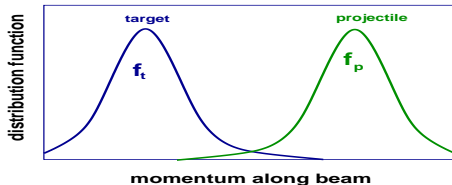
Part. Production

Hadron Ratios

pt spectra

Flow

## Summary



- Distributions are separated in momentum space  
⇒ **different fluids**
- Leading particles carry baryon charge  
⇒ **2 baryon-rich fluids: projectile-like and target-like**
- Los Alamos 1978–1986: Amsden, Harlow, Nix, Clare, Strottman  
**2-fluid hydrodynamics**
- Kurchatov Inst. 1988–1991: Mishustin, Russkikh, and Satarov  
**2-fluid hydro with free-streaming radiation of pions**
- GSI 1991–1997: Iv., Russkikh, Nörenberg  
**2-fluid mean-field hydro with hadrochemistry for SIS energies**



# From 2 Fluids to 3 Fluids

## 3FD model

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## MFD Models

2-Fluid Models

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3FD

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## Phys. Input

## Results

Baryon Stopping

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Hadron Ratios

pt spectra

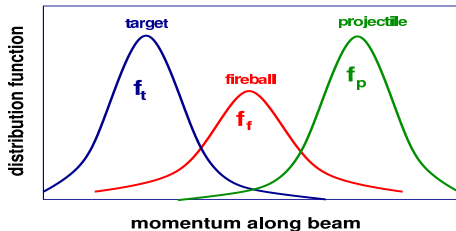
Flow

## Summary

- Distributions are separated in momentum space  
⇒ different fluids
- Leading particles carry baryon charge  
⇒ 2 baryon-rich fluids: **projectile-like** and **target-like**

At high incident energies ( $E_{lab} \gtrsim 10A$  GeV)

- **Produced particles populate mid-rapidity**  
⇒ **fireball fluid**





# 3-Fluid Models

## 3FD model

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## MFD Models

2-Fluid Models

3-Fluid Models

3FD

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## Phys. Input

## Results

Baryon Stopping

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pt spectra

Flow

## Summary

- Kurchatov Inst. 1988–1991:  
**2-fluid hydro with free-streaming radiation of pions**  
Mishustin, Russkikh, and Satarov
- Frankfurt University 1993–2000:  
**3-fluid hydrodynamics with instant formation of fireball**  
Brachmann, Katscher, Dumitru, Rischke, Maruhn, Stöcker, Greiner,  
Mishustin, Satarov, *et al.*
- GSI 2003–now:  
**3-fluid hydrodynamics with delayed formation of fireball**  
Iv., Russkikh, Toneev

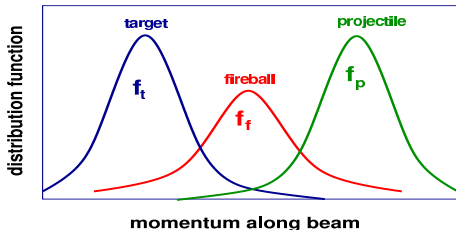


# 3-Fluid Dynamics, present version

## 3FD model

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Produced particles  
populate mid-rapidity  
⇒ **fireball** fluid



**Target-like fluid:**

$$\partial_\mu J_t^\mu = 0$$

Leading particles carry bar. charge

$$\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$$

exchange/emission

**Projectile-like fluid:**

$$\partial_\mu J_p^\mu = 0,$$

$$\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$$

**Fireball fluid:**

$$J_f^\mu = 0,$$

Baryon-free fluid

$$\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$$

Source term      Exchange

The **source term** is delayed due to a formation time  $\tau$

**Total energy-momentum conservation:**

$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

## MFD Models

2-Fluid Models  
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3FD

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# Hydrodynamic densities

## 3FD model

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MFD  
Models

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Summary

**Baryon current:**

$$J_{\alpha}^{\mu} = n_{\alpha} u_{\alpha}^{\mu}$$

$n_{\alpha}$  = baryon density of  $\alpha$ -fluid

$u_{\alpha}^{\mu}$  = 4-velocity of  $\alpha$ -fluid

**Energy-momentum tensor:**

$$T_{\alpha}^{\mu\nu} = (\varepsilon_{\alpha} + P_{\alpha}) u_{\alpha}^{\mu} u_{\alpha}^{\nu} - g_{\mu\nu} P_{\alpha}$$

$\varepsilon_{\alpha}$  = energy density

$P_{\alpha}$  = pressure

**+ Equation of state:**

$$P = P(n, \varepsilon)$$

**Final Aim: To find a proper EoS**, which reproduces all data



# Friction

## 3FD model

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$$F_{\alpha}^{\nu} = \rho_p^{\xi} \rho_t^{\xi} [(u_{\alpha}^{\nu} - u_{\bar{\alpha}}^{\nu}) D_P + (u_p^{\nu} + u_t^{\nu}) D_E],$$

$s_{pt} = m_N^2 (u_p^{\nu} + u_t^{\nu})^2$ ,  $\alpha = p$  or  $t$ ,  $\bar{p} = t$  and  $\bar{t} = p$ , and  $\rho_{\alpha}^{\xi}$  is

$$\rho_{\alpha}^{\xi}(s_{pt}) = \left( \rho_{\alpha}^{bar.} + \frac{2}{3} \rho_{\alpha}^{mes.} \right) \xi_h(s_{pt}) + \frac{1}{3} (\rho_{\alpha}^q + \rho_{\alpha}^g) \xi_q(s_{pt}),$$

## MFD

### Models

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## Phys. Input

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## Summary

$\rho_{\alpha}^{bar.}$ ,  $\rho_{\alpha}^{mes.}$ ,  $\rho_{\alpha}^q$  and  $\rho_{\alpha}^g$  are scalar densities of all baryons, all mesons, quarks and gluons, respectively,

$$\rho_{\alpha}^a(x) = m_a \int \frac{d^3p}{\rho_0} f_{\alpha}^a(x, p)$$

in terms of where equilibrium distribution function  $f_{\alpha}^a$ .

Factors like 2/3 and 1/3 are from naive valence-quark count.

$D_P$  and  $D_E$  were estimated based on proton-proton cross sections [Satarov, 1990]

In view of uncertainties of the estimated friction, tuning factors are introduced for hadronic and quark-gluon phases:  $\xi_h$  and  $\xi_q$ , respectively.



# Freeze-out

## 3FD model

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20.02.13

## MFD Models

2-Fluid Models  
3-Fluid Models  
3FD  
Friction  
Freeze-out

## Phys. Input

## Results

Baryon Stopping  
Part. Production  
Hadron Ratios  
pt spectra  
Flow

## Summary

When system becomes dilute, we have to stop hydro

## Freeze-out

Spectra of observed particles

$$E \frac{dN}{d^3p} = \sum_{\alpha} \int dV (u_{\alpha}^{\mu} p^{\mu}) \frac{\text{degeneracy factor}}{\exp[(u_{\alpha}^{\mu} p^{\mu} - \mu_{\alpha}) T_{\alpha}] \pm 1}$$

where  $u_{\alpha}^{\mu}, \mu_{\alpha}, T_{\alpha}$  are taken at freeze-out instant



# In-plane evolution of energy density

## 3FD model

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## MFD Models

- 2-Fluid Models
- 3-Fluid Models
- 3FD

- Friction
- Freeze-out

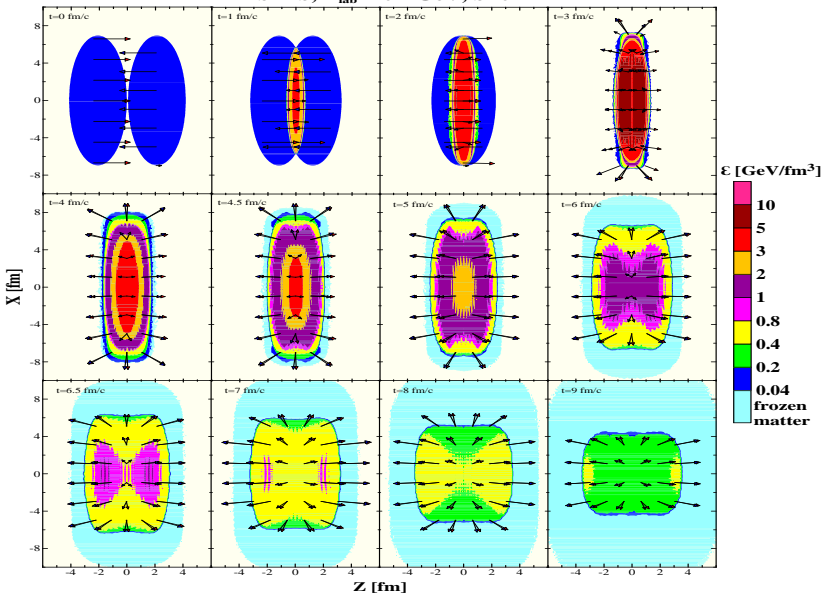
## Phys. Input

## Results

- Baryon Stopping
- Part. Production
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- pt spectra
- Flow

## Summary

**Pb+Pb,  $E_{\text{lab}}=20A$  GeV,  $b=0$**





# Physical Input I

## 3FD model

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## MFD Models

2-Fluid Models  
3-Fluid Models  
3FD

Friction  
Freeze-out

## Phys. Input

## Results

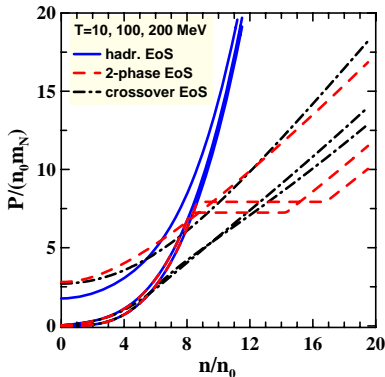
Baryon Stopping  
Part. Production  
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pt spectra  
Flow

## Summary

## I. Equation of State

- **Hadronic EoS**  
Galitsky&Mishustin (1979)
- 1st-order transition to QGP  
**(2-phase EoS\*)**
- **crossover EoS\***

\*[Khvorostukhin, Skokov,  
Redlich, Toneev, (2006)]



**Phase transition  $\implies$  EoS softening**



# Physical Input II

## 3FD model

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## MFD Models

2-Fluid Models  
3-Fluid Models  
3FD

Friction  
Freeze-out

## Phys. Input

## Results

Baryon Stopping  
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pt spectra  
Flow

## Summary

## II. Friction was fitted to reproduce the baryon stopping

### ● Hadronic EoS

$$\xi_h^2(s) = 1 + \left[ \ln \left( \frac{s^{1/2}}{2m_N} \right) \right]^{1/4}, \quad \tau = 2 \text{ fm/c}, \quad \varepsilon_{\text{frz}} = 0.4 \text{ GeV/fm}^3.$$

Friction is enhanced to reproduce the baryon stopping at  $\sqrt{s_{NN}} > 5 \text{ GeV}$ .

**Though the enhancement looks too high:**  $\xi_h^2 = 2.2$  at  $\sqrt{s_{NN}} = 17.3 \text{ GeV}$ .

### ● 2-phase EoS

$$\xi_h^2(s) = 1, \quad \xi_q^2(s) = 60 \frac{4m_N^2}{s}, \quad \tau = 0.17 \text{ fm/c}, \quad \varepsilon_{\text{frz}} = 0.4 \text{ GeV/fm}^3.$$

### ● crossover EoS

$$\xi_h^2(s) = 1, \quad \xi_q^2(s) = 200 \frac{4m_N^2}{s}, \quad \tau = 0.17 \text{ fm/c}, \quad \varepsilon_{\text{frz}} = 0.4 \text{ GeV/fm}^3.$$

**$\xi_h^2(s) = 1$  is advantage of deconfinement scenarios**



# Phase Evolution

## 3FD model

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## MFD Models

2-Fluid Models  
3-Fluid Models  
3FD

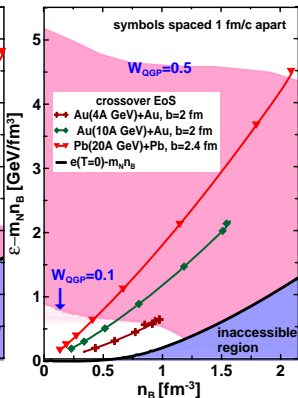
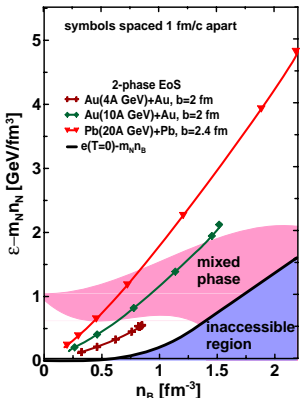
Friction  
Freeze-out

## Phys. Input

## Results

Baryon Stopping  
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## Summary



Dynamical trajectories  
of matter in the central  
box of colliding nuclei  
(4fm × 4fm ×  $\gamma_{cm}$  4fm)

## Crossover transition by Toneev et al. is too smooth

Lattice QCD predicts a fast crossover.

Therefore, a true EoS is somewhere in between the "Toneev"-crossover and "Toneev"-2-phase EoS's.

**Onset of deconfinement happens at top-AGS–low-SPS energies.**



# Net-Proton Rapidity distributions

## 3FD model

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20.02.13

## MFD Models

2-Fluid Models

3-Fluid Models

3FD

Friction

Freeze-out

## Phys. Input

## Results

Baryon Stopping

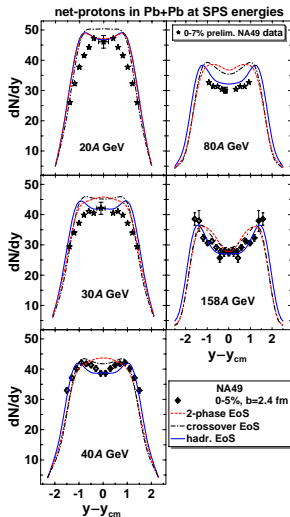
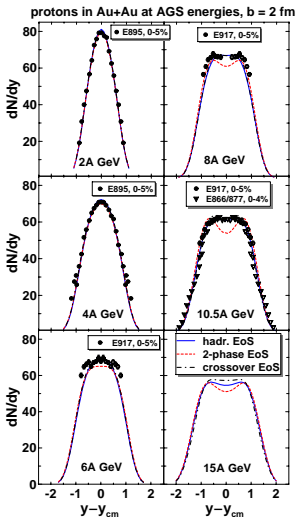
Part. Production

Hadron Ratios

pt spectra

Flow

## Summary



- Predictions of different scenarios differ to the largest extent in the energy region  $8A \text{ GeV} \leq E_{lab} \leq 40A \text{ GeV}$ .

**“peak-dip-peak-dip” irregularity at midrapidity**





# Reduced Curvature at Midrapidity

## 3FD model

To quantify the “peak-dip-peak-dip” irregularity, net-proton rapidity distributions are fitted by ( $a$ ,  $y_s$  and  $w_s$  are parameters of the fit)

$$\frac{dN}{dy} = a \left( \exp \left\{ -\left(1/w_s\right) \cosh(y - y_{cm} - y_s) \right\} + \exp \left\{ -\left(1/w_s\right) \cosh(y - y_{cm} + y_s) \right\} \right)$$

## MFD Models

2-Fluid Models

3-Fluid Models

3FD

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## Phys. Input

## Results

Baryon Stopping

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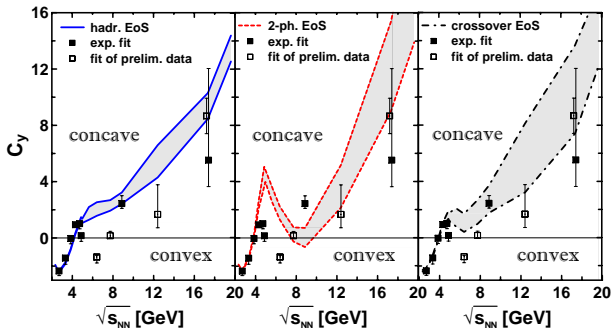
pt spectra

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## Summary

## A reduced curvature of the spectrum at midrapidity

$$C_y = \left( y_{cm}^3 \frac{d^3 N}{dy^3} \right)_{y=y_{cm}} / \left( y_{cm} \frac{dN}{dy} \right)_{y=y_{cm}} = (y_{cm}/w_s)^2 \left( \sinh^2 y_s - w_s \cosh y_s \right).$$



Updated experimental results at energies 20A and 30A GeV are badly needed



# Physical Origin of the Irregularity

## 3FD model

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## MFD Models

2-Fluid Models  
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## Phys. Input

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## Summary

### The irregularity is a combined effect of

- **softest point** of a EoS  
(a minimum of the sound speed)

spherical fireball

⇒ essentially 3D expansion

⇒ **a peak at midrapidity**

strongly deformed fireball

⇒ approximately 1D expansion

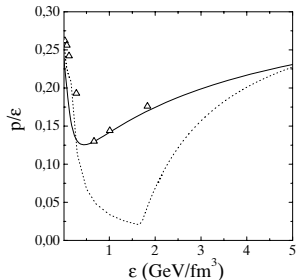
⇒ **a dip at midrapidity**

the more softer matter

⇒ the more deformed fireball

⇒ **a dip at midrapidity**

- **a change in the nonequilibrium regime** from hadronic to partonic one  
a change in cross sections ⇒ an irregularity in baryon stopping



from E. G. Nikonov, A. A. Shanenko and V. D. Toneev, Heavy Ion Phys. 8, 89 (1998)

**This irregularity is a signal from hot and dense stage of nuclear collision**



# Particle Production

## 3FD model

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20.02.13

## MFD Models

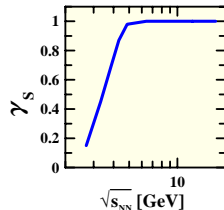
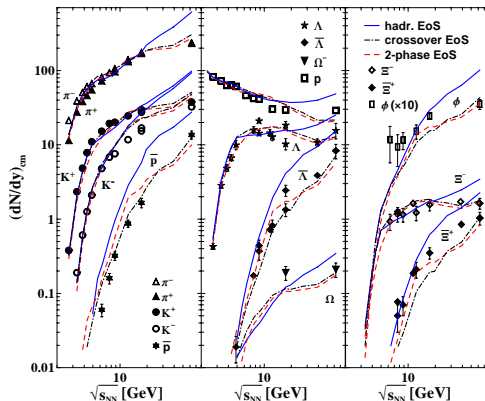
2-Fluid Models  
3-Fluid Models  
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## Phys. Input

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## Summary



$\gamma_s$  = strangeness suppression  
factor

- Hadr. scenario considerably overestimates experimental antibaryons already at lower SPS energies, i.e.  $\approx 20A$  GeV.
- At  $\sqrt{s_{NN}} > 17.4$  GeV, hadronic scenario overestimates available RHIC data on all species.
- Problem with  $\phi$ -meson yields at lower SPS energies

**hadronic scenario fails at high incident energies**

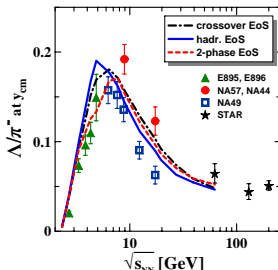
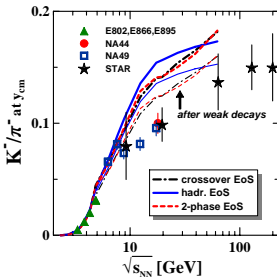
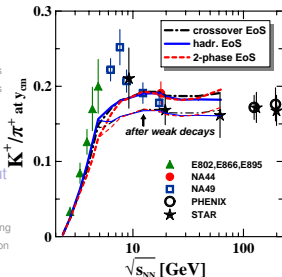


# Hadron Ratios

## 3FD model

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## Ratios of midrapidity yield of various hadrons



- “Horn” anomaly in the  $K^+/\pi^+$  ratio is not reproduced in any scenario.

?????????

## MFD Models

2-Fluid Models  
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Freeze-out

## Phys. Input

## Results

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Flow

## Summary



# Pt Spectra

## 3FD model

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## MFD Models

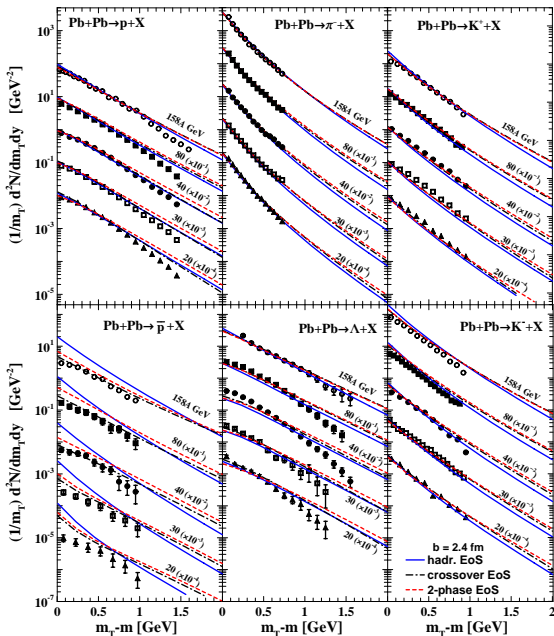
- 2-Fluid Models
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## Phys. Input

## Results

- Baryon Stopping
- Part. Production
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- pt spectra
- Flow

## Summary



hadronic scenario  
again fails for  
antiprotons



# Inverse Slopes and Mean Transverse Masses

## 3FD model

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$$\frac{d^2 N}{m_T dm_T dy} \propto \exp\left(-\frac{m_T}{T}\right),$$

$$\langle m_T \rangle = \frac{\int d^2 p_T m_T \left(\frac{d^2 N}{m_T dm_T dy}\right)}{\int d^2 p_T \left(\frac{d^2 N}{m_T dm_T dy}\right)}$$

## MFD

### Models

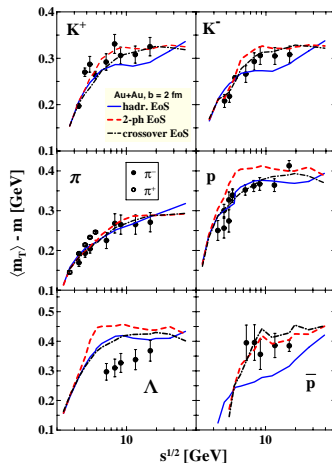
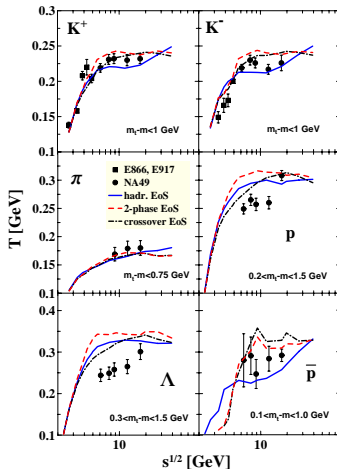
- 2-Fluid Models
- 3-Fluid Models
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### Phys. Input

### Results

- Baryon Stopping
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### Summary



**“Step” is not a signal of deconfinement**



# Directed Flow

## 3FD model

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## MFD Models

2-Fluid Models

3-Fluid Models

3FD

Friction

Freeze-out

## Phys. Input

## Results

Baryon Stopping

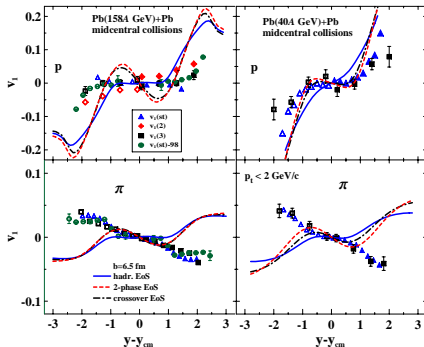
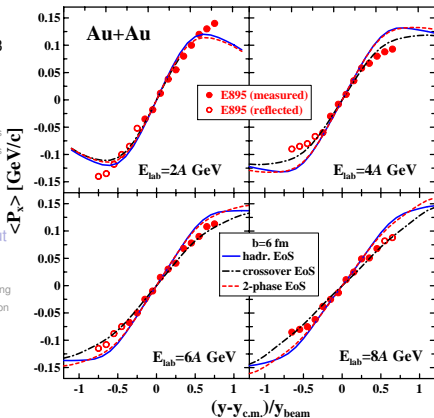
Part. Production

Hadron Ratios

pt spectra

Flow

## Summary



Deconfinement scenarios look preferable everywhere **except for protons at 158A GeV**

**No comments as yet**



# Elliptic Flow

## 3FD model

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20.02.13

## MFD Models

- 2-Fluid Models
- 3-Fluid Models
- 3FD

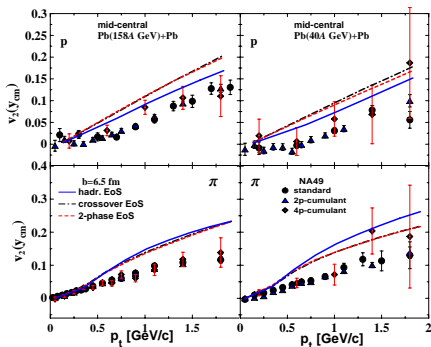
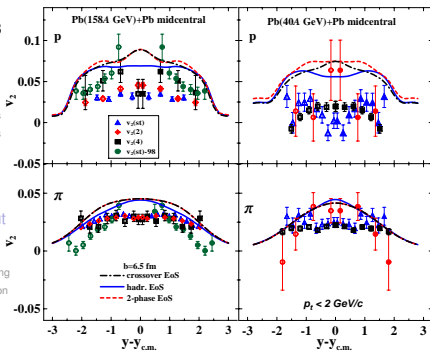
- Friction
- Freeze-out

## Phys. Input

## Results

- Baryon Stopping
- Part. Production
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## Summary



Afterburner (Hirano, et al., nucl-th/0701075) or viscosity reduces  $v_2$  achieved at hydro stage.

Therefore,  $v_2$  at 158A GeV is reasonable.

However,  $v_2$  at 40A GeV is too high.

**Critical point?**

E.Shuryak, arXiv:hep-ph/0504048





# Summary

## 3FD model

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## MFD

### Models

2-Fluid Models

3-Fluid Models

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Freeze-out

### Phys. Input

### Results

Baryon Stopping

Part. Production

Hadron Ratios

pt spectra

Flow

## Summary

## Deconfinement scenarios look preferable at $\sqrt{s_{NN}} > 5$ GeV

- net-proton rapidity distributions:  
**Irregularity signaling deconfinement onset** (not yet seen)

- particle production: hadronic scenario overestimates

exp. antibaryons yields already at  $\sqrt{s_{NN}} > 5$  GeV,  
and at  $\sqrt{s_{NN}} > 17.4$  GeV – yields of all species

- $m_T$ -spectra: “Step” is not a signal of deconfinement

- Directed and elliptic flow:  
reasonably well reproduced so far, analysis is still in progress

- (i) Disappearance of  $v_2$  at 40A GeV (semicentral collisions) and  
(ii) maximum in  $K^+/\pi^+$  ratio at 20A – 30A GeV (central collisions)  
cannot be reproduced by any scenario

- Analysis is still in progress