Transversal and longitudinal gluon spectral functions across the phase transition from twisted mass lattice QCD with $N_f = 2 + 1 + 1$ flavors

E.-M. Ilgenfritz[1], J. M. Pawlowski[2],
A. Rothkopf[2] and A. M. Trunin[1]

[1]Joint Institute for Nuclear Research, Laboratory of Theoretical Physics, Dubna, Russia
[2]Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, Germany

Bogolubov Laboratory for Theoretical Physics
Seminar "Theory of Hadronic Matter under Extreme Conditions"
JINR Dubna, 18 October 2017

Outline

Outline

Outline

Outline

Outline

Outline

Outline

Outline

Outline

Outline

Physical picture of QCD phases above and below the crossover

Below T_c : Confinement and chiral symmetry breaking
Modelled by Hadron Resonance Gas (Remarkably: even with masses taken from $T = 0$! Apparently no other degrees of freedom ?)
Above T_c : (gradual) Deconfinement and chiral symmetry restoration
Modelled by colored degrees of freedom with strong interaction.
However, there are - in addition - remnants of mesonic objects, not-yet melted charmonia, glueballs ?
Kinetic description : gluon- and quark-like quasi particles (one needs their spectral functions !)
Lattice theory of extremal hadron matter in recent years went far beyond sketching the phase structure.
Dynamical and transport properties of hadron and quark-gluon matter in the respective phases and near the borderline are now requested !

More about the quasiparticle picture of QGP

- early attempts: an ideal gas of "dressed" massive gluons
  A. Peshier et al. Phys. Rev. D 54 (1996)
- current quasiparticle models: PHSD (parton-hadron-string dynamics)
  W. Cassing and E. Bratkovskaya, Phys. Rev. C 78 (2008) 034919
  with quark or gluon spectral functions

$$\rho_{q/g}(\omega, T) \sim \frac{4\,\omega\,\Gamma_{q/g}}{\left(\omega^2 - p^2 - M^2_{q/g}(T)\right)^2 + 4\,\omega^2\Gamma^2_{q/g}(T)}$$

- transport coefficients in terms of spectral functions : Why ?
  Direct lattice calculation of viscosity η/s ? Hardly possible.
  Barely possible from quenched simulations (most recently by V. Braguta, A. Kotov et al., ITEP) via Kubo-type correlators of the EM tensor and analytical continuation to $\rho_{TT}(\mathrm{limit}\ \omega \to 0)$.
  For full QCD, this program is near to science fiction (hopeless ?)

More about the quasiparticle picture of QGP

For non-lattice calculation of transport coefficients the knowledge of the spectral function of quasi particles is necessary.

This is much more than just the in-medium dispersion relations :

- T-dependent mass
- T-dependent width
- also important : strength, sign of the spectral function $\rho(\omega, \vec{q})$ for all 3-momenta $\vec{q}$

Transport coefficients in terms of spectral functions

Recent achievement of the Functional Renormalization Group (FRG) approach : (Heidelberg and Giessen Universities)

They derived a closed (2-loop) expression in terms of the non-perturbative gluon spectral function, to be extended to full QCD (including then the non-perturbative quark spectral function as well).

"Transport Coefficients in Yang-Mills Theory and QCD",
N. Christiansen, M. Haas, J. M. Pawlowski, and N. Strodthoff,
Phys. Rev. Lett. 111 (2015) 112002

Diagrammatic prescription and numerical result for η/s as function of T

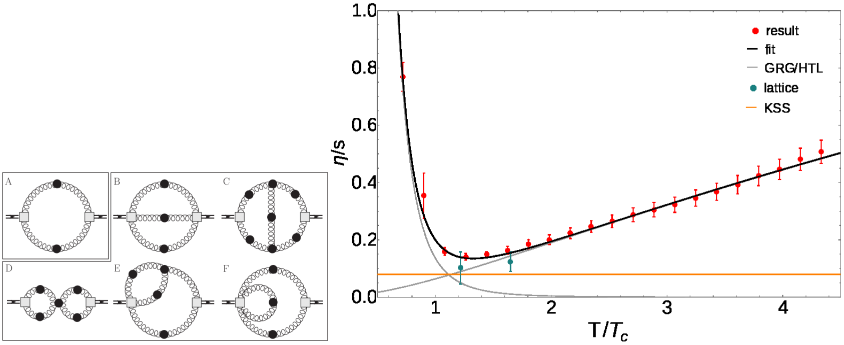


Figure: Left: Types of diagrams contributing to the correlation function of the energy momentum tensor up to two-loop order; squares denote vertices derived from the EMT; all propagators and vertices are fully dressed. Right: Full Yang-Mills result (red) for η/s in comparison to lattice results (H. Meyer 2007 and 2009) (blue) and the AdS/CFT bound (orange).

A step towards the Euclidean lattice calculation of spectral functions of QCD constituents

The aim is to evaluate spectral functions of gluons and quarks ...

- ... for different sets of dynamical quarks
- ... in all regions of the phase diagram (across the phase transition)

"Finite temperature gluon spectral functions from $N_f = 2 + 1 + 1$ lattice QCD"
by E.-M. Ilgenfritz, J. M. Pawlowski, A. Rothkopf, A. Trunin
(Dubna-Heidelberg collaboration in the Heisenberg-Landau Program)
e-Print: arXiv:1701.08610 [hep-lat]

- Work is in progress for other lattice ensembles (other M_π) of the "twisted mass at finite temperature" (tmfT) collaboration.
- Temperature is introduced in a fixed-scale approach : when all masses and the gauge coupling β are fixed, temperature is chosen by N_τ.

Spectral function of gluons (and quarks)

This work tries to extract the in-medium gluon spectral function from Euclidean gluon correlation data.
Below T_c, non-positivity of the gluon spectral function demonstrates, that the gluon is not a "particle as usual".
Violation of spectral positivity is an important feature (over and over observed in studies of the gluon propagator) of confinement.
Non-positivity seen also in the Laplace transform ($p_4 \to$ Euclidean time) of the $T = 0$ gluon propagator, $G(\tau, \vec{p})$ (non positive).

Non-positivity thoroughly discussed in :

- R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281 [hep-ph/0007355]
- J. M. Cornwall, Mod. Phys. Lett. A **28** (2013) 1330035 [arXiv:1310.7897 (hep-ph)]

In general, spectral function can be obtained by analytic continuation of Euclidean correlation functions.

The ill-posedness

Notoriously known as an ill-posed problem :

- finite number (actually, only a very small number) of data points $N_{q_4} = N_\tau$ at finite T, unless one uses highly anisotropic lattices
- wanted: a continuous spectral function $\rho(\omega)$
- usually the data is very noisy !
- What is helpful in other cases ? For physical (bound state) particles (light, heavy-light mesons, charmonia) the spectral function is positive semidefinite (giving a number of distinct gauge-invariant states per mass interval).
- Gluons, in contrast, are unphysical particles : violate spectral positivity, *i.e.* $\rho(\omega)$ may irregularly (in ω) assume positive and negative values.

This complicates our task.
Superconvergent sum rule $\int_0^\infty \rho_T(m^2)dm^2 = 0$ (Reinhard Oehme)

Outline

Gauge potential and propagator from lattice links

only briefly :

$$A_\mu(x+\hat{\mu}/2) = \frac{1}{2iag_0}(U_{x\mu} - U^\dagger_{x\mu})\,|_{traceless}$$

Fourier transform of the gauge potential on lattice

$$\tilde{A}^a_\mu(q)$$

Fourier transformed gluon propagator : correlator of two Fourier transformed gauge potentials

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(q)\widetilde{A}^b_\nu(-q) \right\rangle .$$

Discrete lattice momenta k and physical momenta q

$$k_\mu a = \frac{\pi n_\mu}{N_\mu}, \quad n_\mu \in (-N_\mu/2, N_\mu/2]\,, \quad q_\mu(n_\mu) = \frac{2}{a}\sin\left(\frac{\pi n_\mu}{N_\mu}\right) .$$

Transversal and longitudinal projectors

At non-zero temperature : no (approx.) $O(4)$ rotational symmetry anymore ! QGP has its own rest frame.

Define transversal and longitudinal polarization tensors

$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right)$$

$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P^T_{\mu\nu} \; .$$

This tensor structure defines two propagators :
D_L (longitudinal, electric) and D_T (transversal, magnetic)

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left(P^T_{\mu\nu} D_T(q_4^2, \vec{q}^{\,2}) + P^L_{\mu\nu} D_L(q_4^2, \vec{q}^{\,2})\right) \; .$$

Transversal and longitudinal propagators

The explicit expressions for the propagators $D_{T,L}$ read (if $q_4 \neq 0$)

$$D_T(q) = \frac{1}{2N_g} \left\langle \sum_{i=1}^{3} \widetilde{A}_i^a(q)\widetilde{A}_i^a(-q) - \frac{q_4^2}{\vec{q}^{\,2}} \widetilde{A}_4^a(q)\widetilde{A}_4^a(-q) \right\rangle$$

and

$$D_L(q) = \frac{1}{N_g} \left(1 + \frac{q_4^2}{\vec{q}^{\,2}} \right) \left\langle \widetilde{A}_4^a(q)\widetilde{A}_4^a(-q) \right\rangle$$

In the past, propagators were mostly studied as function of spatial $|\vec{q}|$ (and, moreover, restricted to $q_4 = 0$).
However, zero Matsubara frequency data is not sufficient for the task of extracting the spectral function from full $D(q_4, |\vec{q}|)$.

$$D(q_4, |\vec{q}|) \neq D\left(0, \sqrt{\vec{q}^2 + q_4^2}\right) \text{ (as is often assumed!)} \qquad (1)$$

Källan-Lehmann representation

We may relate the gluon correlators for imaginary frequencies q_4 to their spectral function via the Källen-Lehmann representation (at any temperature, $T = 0$ and $T \neq 0$, for each momentum $\mathbf{q}$)

$$
\begin{aligned}
D_{T,L}(q_4, \mathbf{q}) &= \int_{-\infty}^{\infty} \frac{1}{iq_4 + \omega} \, \rho_{T,L}(\omega, \mathbf{q}) \, d\omega \\
&= \int_{0}^{\infty} \frac{2\omega}{q_4^2 + \omega^2} \, \rho_{T,L}(\omega, \mathbf{q}) \, d\omega \; ,
\end{aligned}
$$

with the spectral function being antisymmetric around the origin of real-time frequencies, $\rho(-\omega) = -\rho(\omega)$.

Inverting this relation using the simulated correlator data represents the spectral function depending on the temperature.

Obviously, knowledge of the q_4 dependence becomes now crucial !

Reconstruction method for the spectral function

Our method is a Bayesian Reconstruction method.

Other methods to find the spectral function are being used:

- Maximal entropy method (prevented by non-positivity)
- Tikhonov regularization (used by the Coimbra-Leuven group: P. Silva, O. Oliveira, D. Dudal)

A method directly relating data given in the Euclidean time domain to the corresponding spectral function is the

- Gilbert-Backus method, being used
  - by the Mainz group (H. Meyer et al.)
  - by the ITEP group (V. Braguta, A. Kotov, N. Astrakhantsev) and by M. Ulybyshev, Regensburg

Reconstruction from data in the Euclidean time domain

Our method is in contrast to correlators primarily obtained in Euclidean time domain, *e.g.* for the calculation of viscosity (or electric conductivity),

$$C_{TT}(x_0) = T^{-5} \int d^3\mathbf{x} \, \langle T_{12}(0) T_{12}(x_0, \mathbf{x}) \rangle \ ,$$

where the correlation function can be written in terms of the corresponding spectral function $\rho_{TT}(\omega)$ as follows :

$$C_{TT}(x_0) = T^{-5} \int_0^{\infty} \rho_{TT}(\omega) \, \frac{\cosh \omega(\frac{1}{2T} - x_0)}{\sinh \frac{\omega}{2T}} \, d\omega \ . \qquad (2)$$

In our case, the kernel has no explicit temperature dependence !

Outline

Bayesian spectral reconstruction

In quasi-continuum version: we have to reproduce a finite and noisy set of data points by D_i^ρ, an ω-integral over ρ (as Riemann integral).
Frequency bins : $N_\omega = O(1000)$ bins.
Lattice data points : only $N_{q_4} \in [4 \dots 20]$ values (the number depending on temperature for a fixed scale, *i.e.* in a fixed-β setting).

Hence, our task is inverting a bin-discretized Källen-Lehmann relation

$$D_i^\rho = \sum_{l=1}^{N_\omega} K_{il}\, \rho_l\, \Delta\omega_l, \quad i \in [0, N_{q_4}], \quad N_\omega \gg N_{q_4}$$

Using a naive χ^2-fit for the (binwise constant-valued) ρ_l values would yield an infinite number of degenerate solutions.

Bayesian spectral reconstruction

It starts by writing the probability of a test spectral function ρ to be the correct spectral function, given the measured data (D_i) and given further, so called prior information (I). This probability is proportional to the product of two terms

$$P[\rho|D, I] \propto P[D|\rho, I] \; P[\rho|I].$$

This expression follows from the multiplication theorem of conditional probabilities and formally allows the prior information I (in other words, a default model) to influence both factors on the right hand side.

The functional to maximize

The first factor of it,

$$P[D|\rho, I] = \exp[-L]$$

refers to the likelihood probability, where the likelihood L measures the χ^2-distance between the correlator points D_i^ρ (as obtained from the test function ρ) and the actually simulated data D_i (for both D_T or D_L)

$$L = \frac{1}{2} \sum_{i,j=1}^{N_{q_4}} (D_i - D_i^\rho) C_{ij}^{-1} (D_j - D_j^\rho),$$

where C_{ij} is the usual covariance matrix of the simulated D_i's.
Prior information (I) enters here only implicitely.
The L functional (as functional of ρ_i) possesses $N_\omega - N_{q_4}$ flat directions. In any Bayesian approach this must be regularised by a prior probability, which is specified in terms of an "entropy" functional.

The standard Bayesian Reconstruction method

The *a priori* probability of ρ is $P[\rho|I] = \exp[-\alpha(S(\omega)]$ specified by some "entropy functional".

- Maximal entropy method (MEM)
  Here, the Shannon-Jaynes relative entropy plays this role. It is applicable in case of replacing the default model $m(\omega)$ by some freely chosen $\rho(\omega)$

$$S_{\mathrm{SJ}} = \int d\omega \left(\rho(\omega) - m(\omega) - \rho(\omega) \log\left[\frac{\rho(\omega)}{m(\omega)}\right]\right)$$

The prior knowledge enters through the parametrization given by the default spectral density $m(\omega)$ (given in binned form).
The coefficient α (multiplying the relative entropy) expresses the importance given to the prior information.
For $\alpha \to \infty$, the most probable $\rho(\omega)$ turns out to be $\rho(\omega) = m(\omega)$, independently of any data.

The improved regulator

Non-positivity is a problem ! The Shannon-Jaynes entropy above might be used even if there are regions of positive and negative $\rho(\omega)$, but in our case these regions are not known *a priori* !

- Standard Bayesian method (BR)
  Here, the Shannon-Jaynes relative entropy is replaced by a regulating functional for which - in the absence of simulation data - the most probable $\rho(\omega)$ would be again $\rho(\omega) = m(\omega)$.

$$S_{\rm BR} = \int d\omega \big(1 - \frac{\rho(\omega)}{m(\omega)} + \log\big[\frac{\rho(\omega)}{m(\omega)}\big]\big)$$

  Only the ratio $\rho(\omega)/m(\omega)$ matters here !
  "Bayesian Approach to Spectral Function Reconstruction for Euclidean Quantum Field Theories",
  Yannis Burnier and Alexander Rothkopf,
  Phys. Rev. Lett. 111 (2013) 18200331, arXiv:1307.6106

The improved regulator

- The Novel Bayesian method, which accounts for the non-positivity of $\rho(\omega)$; here the generalized entropy functional

$$S^g_{\mathrm{BR}} = \int d\omega \Big(- \frac{|\rho(\omega) - m(\omega)|}{h(\omega)} + \log\Big[\frac{|\rho(\omega) - m(\omega)|}{h(\omega)} + 1\Big]\Big).$$

  takes over the role of regulator and relative entropy.
  $r_l = \frac{|\rho_l - m_l|}{h_l}$ is the difference of ρ from the default model m, to be taken relative to h, which encodes the confidence in the default model.
  "Bayesian inference of nonpositive spectral functions in quantum field theory",
  Alexander Rothkopf,
  Phys. Rev. D95 (2017) 056016, arXiv:1611.00482

Getting rid of the factor α

This analytic form of $S^g_{\rm BR}$ allows one to integrate out α in a straight forward fashion, allowing full ignorance about the values of α (putting the corresponding distribution $W[\alpha] = const$):

$$P[\rho|D, I, m] \propto P[D|\rho, I] \int_0^\infty d\alpha \; P[\rho|m, \alpha] \; W[\alpha]$$

Variational problem

Once $m(\omega)$ and $h(\omega)$ are specified, say as $m(\omega) = 0$ and $h(\omega) = 1$, we have to carry out a numerical search for the most probable Bayesian spectrum according to

$$\left.\frac{\delta P[\rho|D, I]}{\delta \rho}\right|_{\rho=\rho^{\mathrm{Bayes}}} = 0,$$

- $m(\omega) = \pm 0$ is chosen as unbiased prior for the non-positive spectral function (with zero sum rule).
- Alternative choices $m(\omega) = \pm 1$ and changing the confidence $h(\omega) = 1 \to h(\omega) = 2$ allow to check the influence of the prior (provides a reliability estimate for the spectral function)

Shape of the prior probability distribution

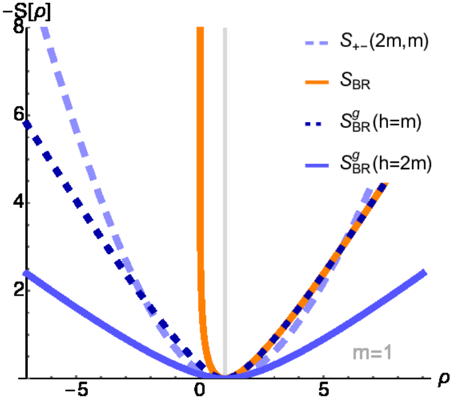


Figure: Comparison of different priors

Outline

Configurations taken from the tmfT collaboration

Aim of the tmfT collaboration : improve the lattice thermodynamics of Wilson fermions by employing the twisted-mass improvement.

Main results obtained for $N_f = 2$

- Localization and characterization of the crossover for various light quark masses (or "pion" mass values)
- Equation of State (EoS) for two light flavors
- Unquenching effect on the gluon propagator

Main results obtained for $N_f = 2+1+1$

- Localization and characterization of the crossover for various light quark masses (or "pion" mass values) in presence of *s* and *c* quarks (with realistic masses of strange and charmed hadrons)
- Equation of State (EoS) including two light flavors and additional *s* and *c* quarks with realistic mass (not yet finished)
- T dependence of the topological susceptibility with four flavors

Topological susceptibilty

"Topological susceptibility from Nf=2+1+1 lattice QCD at nonzero temperature",
Anton Trunin, Florian Burger, Ernst-Michael Ilgenfritz, Maria Paola Lombardo, Michael Müller-Preussker,
J.Phys.Conf.Ser. 668 (2016) no.1, 012123, arXiv:1510.02265
(Strangeness in QuarkMatter 2015)

"Topology (and axion's properties) from lattice QCD with a dynamical charm",
Florian Burger, Ernst-Michael Ilgenfritz, Maria Paola Lombardo, Michael Müller-Preussker, Anton Trunin,
arXiv:1705.01847 (Quark Matter 2017)

The fermionic action is improved compared with unimproved Wilson fermions coming in four flavours

tmfT exists parallel to ETMC (European twisted mass collaboration, for $T = 0$)

Fermions are grouped in one (for $N_f = 2$) or two twisted doublets.

- The light doublet action (degenerate u and d quarks) with mass tuned by the twisted mass parameter μ_l

$\kappa_l = \kappa_c(\beta)$ (*i.e.* "maximal twist")

$$S_f^l[U, \chi_l, \overline{\chi}_l] = \sum_{x,y} \overline{\chi}_l(x)[\delta_{x,y} - \kappa_l D_{\mathrm{W}}(x,y)[U] + 2i\kappa_l a\mu_l \gamma_5 \delta_{x,y} \tau_3]\chi_l(y)$$

The fermionic action is improved compared with unimproved Wilson fermions coming in four flavours

- The heavy doublet action (non-degenerate s and c quarks) with masses tuned by two twisted mass parameters μ_σ and μ_δ (fixed by strange and charmed hadron masses at $T = 0$)

  whereas again $\kappa_h = \kappa_c(\beta)$ (*i.e.* "maximal twist")

$$S_f^h[U, \chi_h, \overline{\chi}_h] = \sum_{x,y} \overline{\chi}_h(x)[\delta_{x,y} - \kappa_h D_W(x,y)[U] + 2i\kappa_h a\mu_\sigma \gamma_5 \delta_{x,y}\tau_1 + 2\kappa_h a\mu_\delta \delta_{x,y}\tau_3]\chi_h(y)$$

In both actions, τ_i are Pauli matrices in the respective doublet (*i.e.* flavor) space.

The fermionic action is a Wilson-type action

The term $D_W[U]$ denotes the standard gradient term for Wilson fermions

$$D_W[U] = \frac{1}{2a}[\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \nabla^*_\mu \nabla_\mu]$$

Simulation algorithm : PHMC

"Numerical simulation of QCD with u, d, s and c quarks in the twisted-mass Wilson formulation",
T. Chiarappa, F. Farchioni, K. Jansen, I. Montvay, E. E. Scholz, L. Scorzato, T. Sudmann, and C. Urbach,
Eur. Phys. J. C50 (2007) 373, arXiv:hep-lat/0606011

Lattice setting

Configurations taken from simulations of the "twisted mass at finite temperature" (tmfT) collaboration (M. Müller-Preussker et al.).
For this first spectral paper, only the ensembles for $M_\pi \approx 370$ MeV have been analysed for all three lattice spacings (will be extended ...).

ETMC ens. ($T=0$)	A60.24	B55.32	D45.32
tmfT ens. ($T \neq 0$)	A370	B370	D370
β	1.90	1.95	2.10
a [fm]	0.0936	0.0823	0.0646
m_π [MeV]	364(15)	372(17)	369(15)
$T_{\rm deconf}$ [MeV]	202(3)	201(6)	193(13)
$N_\tau = N_{q_4}$ in range	4-14	10-14	4-20

Table: Properties of the three sets of finite-temperature ensembles used in our study, among them the deconfinement crossover temperature $T_{\rm deconf}$ (defined by the peak of Polyakov loop susceptibility).

Grid sizes for $D370$, *i.e.* $\beta = 2.10$ and $M_\pi \approx 370$ MeV

For the first evaluation of the spectral function one set of lattice ensembles has been selected :

$D370$										
N_τ	4	6	8	10	11	12	14	16	18	20
T [MeV]	762	508	381	305	277	254	218	191	170	152
N_s	32	32	32	32	32	32	32	32	40	48
N_{meas}	310	400	120	410	420	380	790	610	590	280

Table: Grid sizes and temperatures of the $D370$ ensembles used for the computation of the correlation functions in this work. N_{meas} refers to the number of available correlator measurements (uncorrelated configurations).

Gauge condition : Landau gauge

One essential detail :

Propagators require gauge fixing: we specify the Landau gauge.
This corresponds to the following discretized local condition

$$\nabla_\mu A_\mu = \sum_{\mu=1}^{4} (A_\mu(x + \hat{\mu}/2) - A_\mu(x - \hat{\mu}/2)) = 0 \, ,$$

to be imposed on the gauge fields defined in terms of link variables.
This can be achieved by using the freedom of performing suitable gauge transformations acting on the links.

Iterative gauge fixing

This condition may be fulfilled by iteratively applying local gauge transformations g_x

$$U_{x\mu} \overset{g}{\mapsto} U^g_{x\mu} = g^\dagger_x U_{x\mu} g_{x+\mu} \,, \qquad g_x \in SU(3)\,,$$

in order to maximize the gauge functional

$$F_U[g] = \frac{1}{3} \sum_{x,\mu} \mathrm{Tr} \left(g^\dagger_x U_{x\mu} g_{x+\mu} \right) .$$

We apply the convergence criterium

$$\max_x \mathrm{Tr} \left[\nabla_\mu A_{x\mu} \nabla_\nu A^\dagger_{x\nu} \right] < 10^{-13} \,.$$

This procedure has been carried out by means of the cuLGT (CUDA) library (Schröck 2012), adapted by A. Trunin for the use for our lattice configurations.

Outline

Longitudinal gluon correlation functions for $\beta = 2.1$ at zero Matsubara frequency

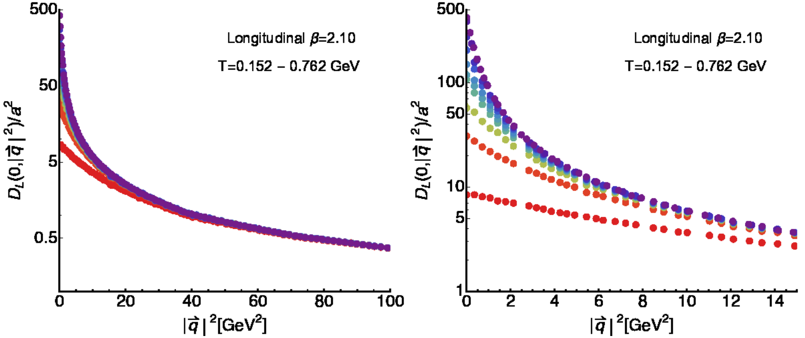


Figure: The longitudinal gluon correlators at $\beta = 2.10$ evaluated for different temperatures $T = 152 \ldots 762$ MeV at **vanishing** imaginary frequency $q_4 = 0$ for finite spatial momenta $|\vec{q}|^2$. The right panel is zoomed in towards the origin.

Longitudinal gluon correlation functions for $\beta = 2.1$ including nonzero Matsubara frequencies

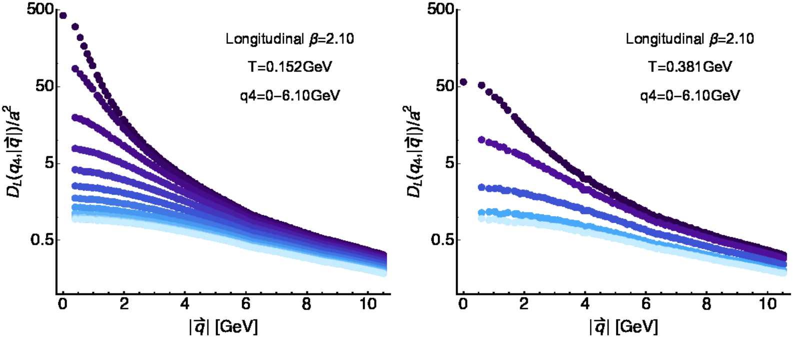


Figure: The longitudinal gluon propagators at $\beta = 2.10$ evaluated for two temperatures (left $T = 152$ MeV, right $T = 381$ MeV) showing the $|\vec{q}|$ dependence at various fixed q_4 values. Darkest colors are assigned to the lowest value of the corresponding parameter q_4, *i.e.* Matsubara $q_4 = 0$.

Longitudinal gluon correlation functions for $\beta = 2.1$ including nonzero Matsubara frequencies

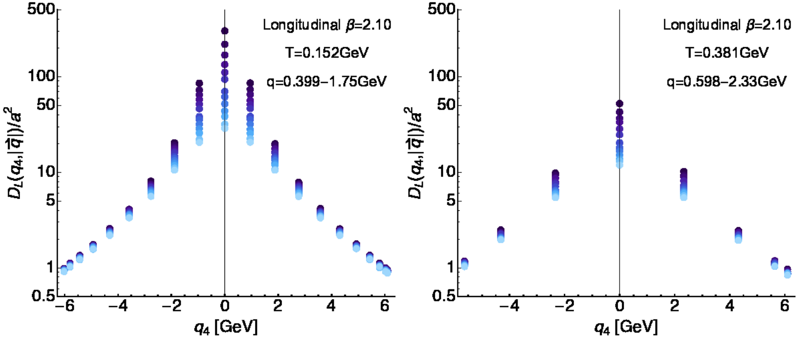


Figure: The longitudinal gluon propagators at $\beta = 2.10$ evaluated for two temperatures (left $T = 152$ MeV, right $T = 381$ MeV) showing the q_4 dependence for fourteen lowest $|\vec{q}|$ momentum values. Darkest colors are assigned to the lowest value of the corresponding 3-momentum $|\vec{q}|$.

Outline

Transversal gluon correlation functions for $\beta = 2.1$ at zero Matsubara frequency

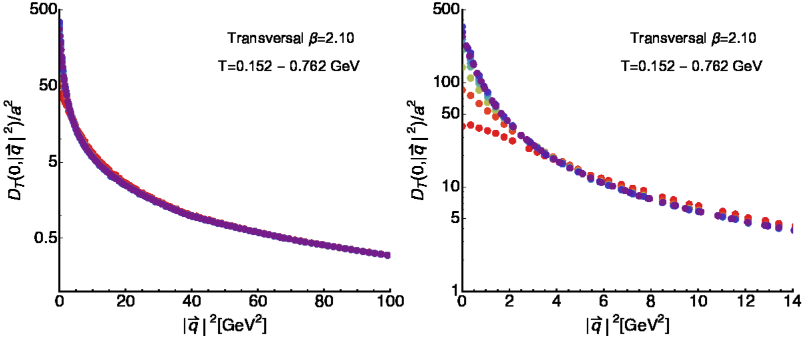


Figure: The transversal gluon correlators at $\beta = 2.10$ evaluated for different temperatures $T = 152 \ldots 762$ MeV at **vanishing** imaginary frequency $q_4 = 0$ for finite spatial momenta $|\vec{q}|^2$. The right panel is zoomed in towards the origin.

Transversal gluon correlation functions for $\beta = 2.1$ including nonzero Matsubara frequencies

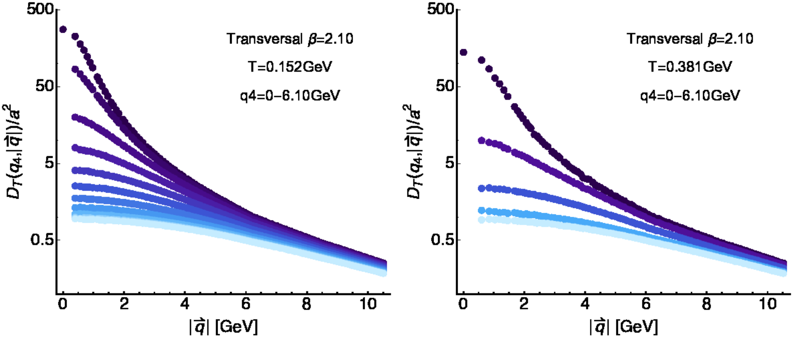


Figure: The transversal gluon propagators at $\beta = 2.10$ evaluated for two temperatures (left $T = 152$ MeV, right $T = 381$ MeV) showing the $|\vec{q}|$ dependence at various fixed q_4 values. Darkest colors are assigned to the lowest value of the corresponding parameter q_4, *i.e.* Matsubara $q_4 = 0$.

Transversal gluon correlation functions for $\beta = 2.1$ including nonzero Matsubara frequencies

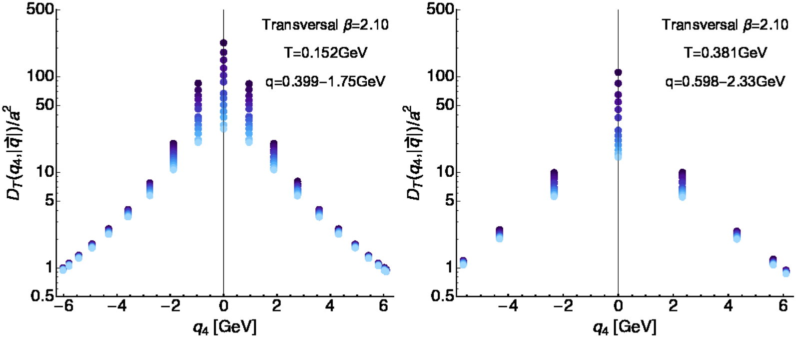


Figure: The transversal gluon propagators at $\beta = 2.10$ evaluated for two temperatures (left $T = 152$ MeV, right $T = 381$ MeV)

Outline

Longitudinal spectral function in confinement and deconfinement

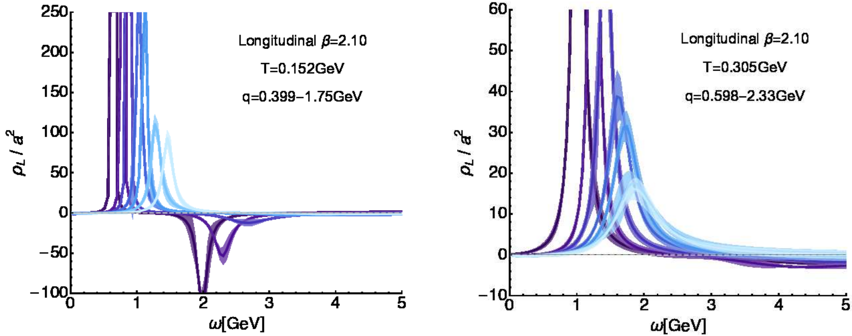


Figure: Reconstructed longitudinal gluon spectral function from the $\beta = 2.10$ ensembles for (left) $T = 152$ MeV and (right) $T = 305$ MeV. The different curves refer to seven lowest spatial momenta. The y-axis is shifted to allow to see strong negative "trough" contributions in confinement, significantly reduced in deconfinement. Error bands arose from varying the default.

Transversal spectral function in confinement and deconfinement

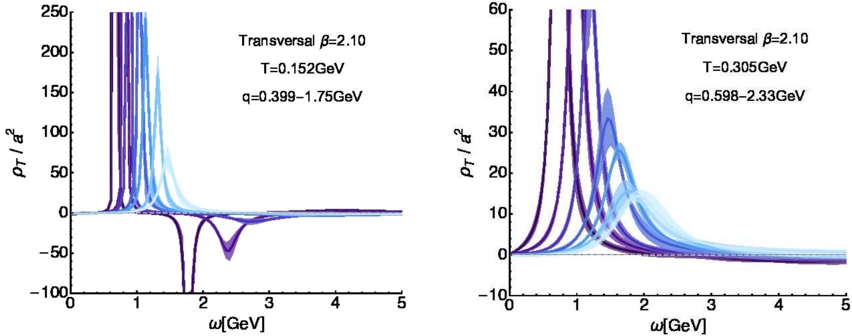


Figure: Reconstructed transversal gluon spectral function from the $\beta = 2.10$ ensembles for (left) $T = 152$ MeV and (right) $T = 305$ MeV. The different curves refer to seven lowest spatial momenta. The y-axis is shifted to allow to see strong negative “trough” contributions in confinement, significantly reduced in deconfinement. Error bands arose from varying the default.

Robustness of reconstruction against dilution of data

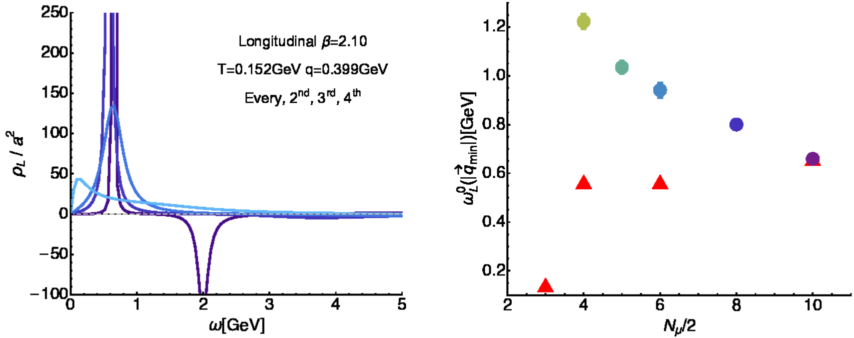


Figure: Reconstructed longitudinal gluon spectral function (left) from only sparse data: All data are kept, every second Matsubara frequency, every third Matsubara frequency, every fourth Matsubara frequency data is kept. Right: the corresponding peak position (red triangles) remains stable unless eventually only every fourth data point is kept.

Robustness of reconstruction against dilution of data

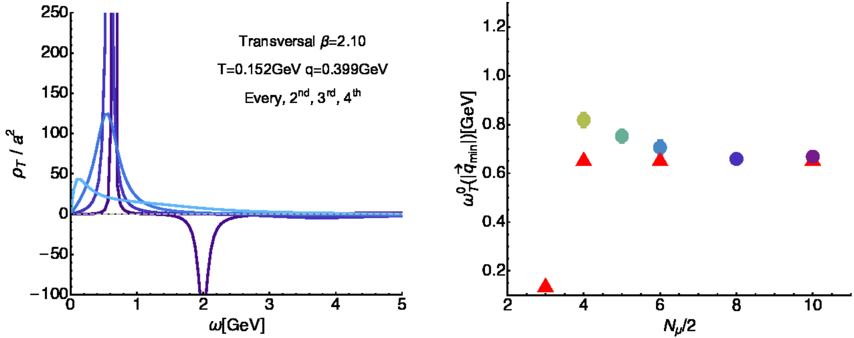


Figure: Reconstructed transversal gluon spectral function (left) from only sparse data: All data are kept, every second Matsubara frequency, every third Matsubara frequency, every fourth Matsubara frequency data is kept. Right: the corresponding peak position (red triangles) remains stable unless eventually only every fourth data point is kept.

Comparison of transversal vs. longitudinal spectral functions

- We find a clear structure with peak and trough in both (electric and magnetic) sectors at low temperature (confinement).
- The negative ("trough") contribution appears slightly stronger in the transversal (magnetic) sector at these low temperatures.
- The negative "trough" is significantly reduced at $T > T_c$ (in deconfinement).
- We can use the peak position (at lower momentum) to define the dispersion relation of longitudinal and transversal gluons.
- The $|\vec{q}|$ dependence is the same for both sectors at large spatial momenta. A remarkable splitting appears at low momentum.

Outline

Longitudinal quasi-particle peak position as function of momentum

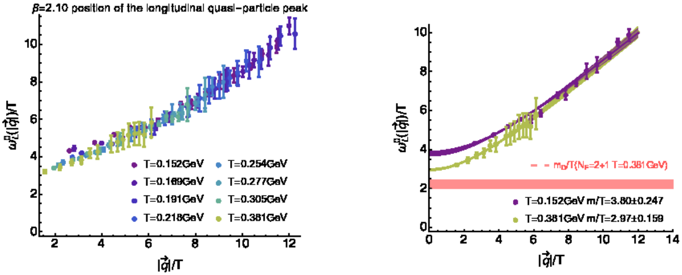
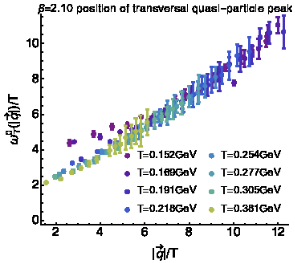


Figure: Left: Momentum dependence of the longitudinal quasi-particle peak position at $\beta = 2.10$ with a non-zero intercept. At the lowest temperatures within the hadronic phase one finds always a larger intercept than in deconfinement. Right: Fit of the lowest and highest temperature curves with the free-field ansatz $\omega^0_L(|\vec{q}|) = A\sqrt{B^2 + |\vec{q}|^2}$. (Quasiparticle mass defined as $m = AB$.) Debye mass from $N_f = 2 + 1$ lattice QCD given for comparison.

Transversal quasi-particle peak position as function of momentum



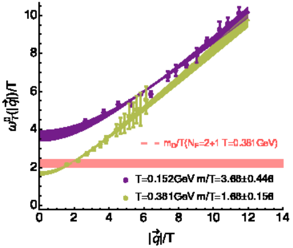


Figure: Left: Momentum dependence of the transversal quasi-particle peak position at $\beta = 2.10$ with a non-zero intercept. At the lowest temperatures within the hadronic phase one finds always a larger intercept than in deconfinement. Right: Fit of the lowest and highest temperature curves with the free-field ansatz $\omega_L^0(|\vec{q}|) = A\sqrt{B^2 + |\vec{q}|^2}$. (Quasiparticle mass defined as $m = AB$.) Debye mass from $N_f = 2 + 1$ lattice QCD given for comparison.

Masses : some Observations

- At the lowest temperatures (within the hadronic phase) one finds a larger intercept (mass) than in deconfinement.
- These (confinement) masses are larger for the longitudinal gluons than for the transversal gluons.
- We have fitted the curves for the lowest and highest temperature with the free-field ansatz $\omega^0_{L/T}(|\vec{q}|) = A\sqrt{B^2 + |\vec{q}|^2}$. (Quasiparticle mass is defined as $m = AB$.)
- The present statistics is not sufficient to study the width of the peak as function of temperature more in detail.
- The Debye mass from the heavy-quark potential measured in $N_f = 2 + 1$ lattice QCD is given for comparison.
- Measuring the $Q\bar{Q}$ potential for $N_f = 2 + 1 + 1$ lattice QCD (Debye screening) is under preparation (last months resuming the tmfT project).

Longitudinal and Transversal Mass

$$\begin{aligned} m_L/T|_{T=0.152\mathrm{GeV}} &= 3.80 \pm 0.25 \\ m_L/T|_{T=0.381\mathrm{GeV}} &= 2.97 \pm 0.16 \end{aligned} \tag{3}$$

$$m_{\mathrm{el}} \sim gT$$

$$\begin{aligned} m_T/T|_{T=0.152\mathrm{GeV}} &= 3.68 \pm 0.45 \\ m_T/T|_{T=0.381\mathrm{GeV}} &= 1.68 \pm 0.16 \end{aligned}$$

$$m_{\mathrm{mag}} \sim g^2 T$$

... in agreement with weak-coupling calculations

Outline

Summary

- Investigating gluon properties gives complementary insight into the QGP.
- Lattice QCD simulations with gauge fixing are a suitable non-perturbative tool.
- Extracting spectral properties from lattice data is an ill-posed inverse problem, where positivity violation precludes application of standard approaches (MEM, ...).
- Novel Bayesian approaches (BR) are available for positively definite and non-definite spectra.
- We did not tacitly rely on $O(4)$ rotational invariance of Euclidean correlators !!!
- The study has provided a clear observation of quasi-particle structure at small frequencies, which (in the confinement phase) is followed by a negative “trough”.
- Masses (from dispersion relations) are in qualitative agreement with weak coupling predictions.

Outlook

This pioneer study ...

- shall be sytematically extended to lower light quark masses : repeat the investigation for the $m_\pi \approx 210$ MeV ensembles next.
- shall be critically questioned : influence of quality of gauge fixing.

There should be a ...

- methodical study to compare with other tools of analytical continuation (*e.g.* Tichonov regularization). A collaborative effort together with the Coimbra group is planned.
- Hopefully, in future we will be able to extend the study to the quark spectral function from lattice data, following the recent study using the quark propagator from the Dyson-Schwinger equation (proof of principle, due to lack of lattice data) "Bayesian analysis of quark spectral properties from the Dyson-Schwinger equation", Christian S. Fischer, Jan M. Pawlowski, Alexander Rothkopf, and Christian A. Welzbacher, arXiv:1705.03207

Outlook

- How to extend this to $\mu \neq 0$? One needs high statistics of gluon propagator measurements, configuration by configuration !
- Last but not least : calculate applications of $\rho_{L/T}(\omega, \vec{q})$ for transport coefficients (Heidelberg group, under way)

Thank you for your attention !