Lattice study of gluon and ghost propagators in Landau gauge QCD

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Outline of the talk

1. Introduction, motivation:
   the infrared QCD debate and the change of a paradigm

2. How to compute Landau gauge gluon and ghost propagators on the lattice

3. Results for gluon, ghost propagators and the running coupling in lattice quenched and full QCD at $T = 0$ (2005 − 2015)

4. Systematic effects: Gribov copies, finite-volume effects, multiplicative renormalization, continuum limit

5. Results for gluon, ghost propagators and the running coupling in lattice quenched and full QCD at $T > 0$ (2010 − ?)

6. Conclusion and outlook

1. Introduction, motivation: the infrared debate

Why do we consider Landau gauge gluon, ghost, quark propagators and vertex functions?

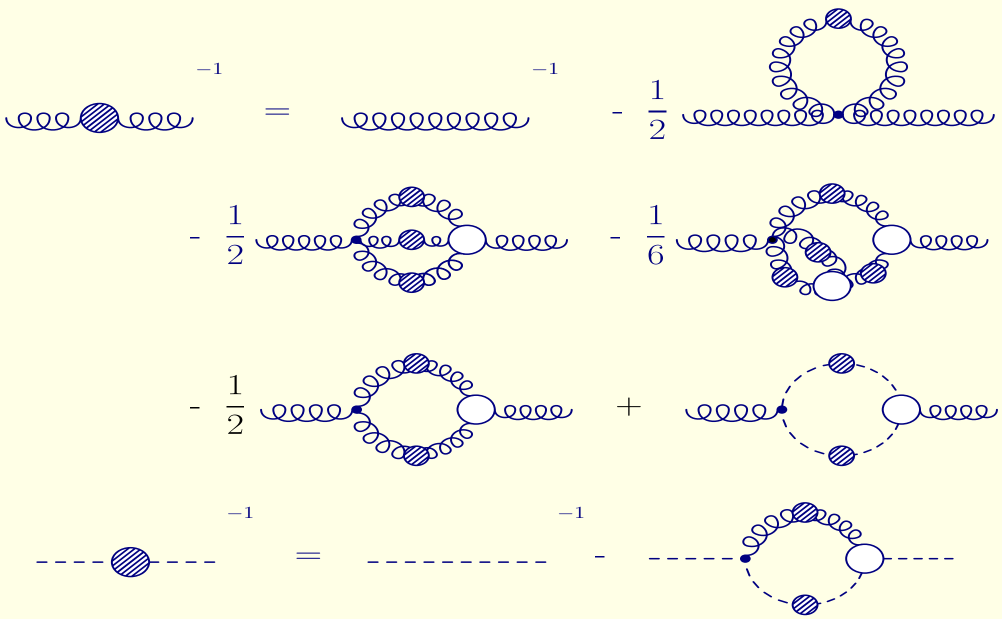
- $\Rightarrow$ Fixing of basic QCD parameters by comparison with continuum pert. theory: Λ_{QCD}, $\langle\overline{\psi}\psi\rangle$, quark masses, $\langle A^2\rangle$ (?), etc.
- $\Rightarrow$ Using them as input for hadron phenomenology: bound state calculations through Bethe-Salpeter and Faddeev eqs. for mesons and baryons, also $T > 0$, cf. review Alkofer, Eichmann, Krassnigg, Nicmorus, Chin. Phys. C34 (2010), arXiv:0912.3105 [hep-ph].
- $\Rightarrow$ Their infrared behaviour has been related to confinement criteria/scenarios: Gribov-Zwanziger, Kugo-Ojima, violation of positivity,....
- $\Rightarrow$ Propagators at $T > 0$ allow for determining screening lengthes etc.

$\Longrightarrow$ Intensive non-perturbative investigations in the continuum and on the lattice over many years.

$\Longrightarrow$ Infrared (IR) limit of special interest, here the particular impact of our work.

Landau gauge Green's functions in the continuum can be determined from (truncated) Dyson-Schwinger (DS) and funct. renorm. group (FRG) eqs. taking into account Slavnov-Taylor identities (STI)

[Alkofer, Aguilar, Boucaud, Dudal, Fischer, Pawlowski, von Smekal, Zwanziger,.. ('97 - '09)]



$$\Rightarrow \quad D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}$$

$$\Rightarrow \quad G^{ab} = \delta^{ab}\,\frac{J(q^2)}{q^2}$$

Running coupling related to ghost-ghost-gluon vertex in a (mini-) MOM scheme

$$\alpha_s(q^2) \equiv \frac{g^2(\mu)}{4\pi}\; Z(q^2,\mu^2)\cdot[J(q^2,\mu^2)]^2$$

Renormalization group invariant quantity [von Smekal, Maltman, Sternbeck ('09)].

Ten years ago: Infrared "scaling" solution of DS and FRG was the ruling paradigm
[Alkofer, Fischer, Lerche, Maas, Pawlowski, von Smekal, Zwanziger,... ('97 - '09)]
gluon and ghost dressing functions

$$Z(q^2) \propto (q^2)^{\kappa_D}\,, \quad J(q^2) \propto (q^2)^{-\kappa_G} \quad \text{for} \;\; q^2 \to 0$$

with related IR exponents for gluons and ghosts

$$\kappa_D = 2\,\kappa_G + (4-d)/2 \quad \Longrightarrow \quad \kappa_D = 2\,\kappa_G, \;\; \kappa_G \simeq 0.59 \quad \text{for} \;\; d = 4$$

It was claimed

- to hold without any truncation of the tower of DS or FRG eqs.,
- to be independent of the number of colors N_c,
- to be consistent with BRST quantization.

Running coupling:

$$\alpha_s(q^2) \to \text{const.} \quad \text{for} \;\; q^2 \to 0$$

i.e. infrared fixed point as in analytic perturbation theory (APT)

[D.V. Shirkov, I.L. Solovtsov ('97 - '02)].

Alternative : "decoupling" IR solution, was under discussion since 2005
[Boucaud et al. ('06 -'08), Aguilar et al. ('07-'08), Dudal et al. ('05-'08)]

$$\kappa_D = 1\,,\ \kappa_G = 0$$

i.e. gluon propagator and ghost dressing function becoming constant as $q^2 \to 0$

$$D(q^2) = Z(q^2)/q^2 \to \text{const.}\,, \qquad J(q^2) \to \text{const.}$$

such that

$$\alpha_s(q^2) = \frac{g^2}{4\pi}\ Z(q^2) \cdot [J(q^2)]^2 \quad \to 0 \ \text{ for } \ q^2 \to 0\,.$$

Existence has been confirmed by solving DS equations.
[Fischer, Maas, Pawlowski, Ann. Phys. 324 (2009) 2408, arXiv:0810-1987 [hep-ph]]
This has finished the debate (on the non-lattice side) which one is right,
interest shifted to criteria why this is the physically correct solution (BRST):
gaining a new understanding of the Gribov-Zwanziger picture

Claim: $J(0)$ can be chosen as an IR boundary condition.

Expect: close relation to the notorious Gribov problem.

Question: Relevance of the extreme IR behavior for phenomenology ?

IR "scaling" solution for Z, J seemed to be required by certain confinement scenarios:

- Kugo-Ojima confinement criterion [Ojima, Kugo ('78 - '79)]:
  absence of colored physical states $\Longleftrightarrow$ ghost propagator more singular
  absence of colored physical states $\Longleftrightarrow$ gluon propagator (less) singular
  .... than simple pole for $q^2 \to 0$.

- Gribov-Zwanziger confinement scenario
  [Gribov ('78), Zwanziger ('89 - ...)]:
  functional integral over gauge fields restricted to the Gribov region

$$\Omega = \Big\{ A_\mu(x) : \ \partial_\mu A_\mu = 0, \ M_{FP} \equiv -\partial D(A) \geq 0 \Big\}$$

  In the limit $V \to \infty$ the measure is accumulated at the Gribov horizon $\partial\Omega$:

  here non-trivial eigenvalues of M_{FP} approach zero: $\lambda_0 \to 0$.

$\Longrightarrow$ Ghost: $J(q^2) \ \to \ \infty$; Gluon: $D(q^2) \ \to \ 0$ for $q^2 \to 0$.

There are attempts to modify these scenarios such, that the IR "decoupling" solution can be accomodated, too. [Dudal et al. ('08 - '09), Kondo ('09)].

The Gribov problem:

- Existence of many gauge copies inside Ω.

- What are the right copies?

  Restriction inside Ω to fundamental modular region (FMR) required

$$\Lambda = \left\{ A_\mu(x) : \; F(A^g) > F(A) \;\; \text{for all} \;\; g \neq \mathbf{1} \right\},$$

  i.e. to global minimum of the Landau gauge functional $F(A^g)$?

Answer in the limit of infinite volume [Zwanziger ('04)]:

Non-perturbative quantization requires only restriction to Ω,

$$\text{i.e.} \quad \delta_\Omega(\partial_\mu A_\mu) \, \det(-\partial_\mu D^{ab}_\mu) e^{-S_{YM}[A]} \, .$$

Expectation values taken on Ω or Λ should be equal in the thermodynamic limit.

- What happens on a (finite) torus?

- How Gribov copies influence finite-size effects?

Questions to Yang-Mills theory on the lattice:

- What kind of infrared DS and FRG solutions are supported ?
- What is the influence of the fermion determinant present in full QCD ?
- Behaviour at non-zero temperature ?
- What is the influence of Gribov copies, lattice artifacts, finite-size effects ?
- Scaling, multiplicative renormalization, continuum limit ?

Lattice investigations of gluon and ghost propagators most intensively in

Adelaide: Bonnet, Leinweber, Skullerud, von Smekal, Williams, et al.;

Berlin: Burgio, E.-M. I., Müller-Preussker, Sternbeck, et al.;

Coimbra: Oliveira, Silva;

Dubna/Protvino: Bakeev, Bogolubsky, Bornyakov, Mitrjushkin;

Hiroshima/Osaka: Nakagawa, A. Nakamura, Saito, Toki, et al.;

Paris: Boucaud, Leroy, Pene, et al.;

San Carlos (São Paulo): Cucchieri, Maas, Mendes;

Tübingen: Bloch, Langfeld, Reinhardt, Watson et al.;

Utsunomiya: Furui, Nakajima.

2. How to compute Landau gauge gluon and ghost propagators on the lattice

i) Generate lattice discretized gauge fields $U = \{U_{x,\mu} \equiv e^{iag_0 A_\mu(x)} \in SU(N_c)\}$ by MC simulation from path integral:

$$Z_{\text{Latt}} = \int \prod_{x,\mu} [dU_{x,\mu}] \; (\det\, Q(\kappa, U))^{N_f} \; \exp(-S_G(U))\,,$$

standard Wilson plaquette action:

$$\begin{aligned} S_G(U) &= \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{N_c}\, \mathfrak{Re}\,\mathrm{tr}\, U_{x,\mu\nu}\right), \\ U_{x,\mu\nu} &\equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}, \qquad \beta \equiv 2N_c/g_0^2\,, \end{aligned}$$

(clover-improved or twisted mass) Dirac-Wilson fermion operator $Q(\kappa, \mu_0; U)$:

$N_f = 0$ – pure gauge case,

$N_f = 2$ – full QCD with equal bare u, d quark masses,

$a(\beta)$ – lattice spacing.

ii) Z_{Latt} is simulated with (Hybrid) MC method without gauge fixing.

iii) Fix Landau gauge for U:

$$U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\hat{\mu}}$$

standard orbits: $\{g_x\}$ periodic on the lattice;

extended orbits: $\{g_x\}$ periodic up to global $Z(N)$ transformations;

Standard (linear) definition $\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2iag_0}\left(U_{x\mu} - U^\dagger_{x\mu}\right)\Big|_{\text{traceless}}$

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4}\left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0$$

equivalent to minimizing the gauge functional

$$F_U(g) = \sum_{x,\mu}\left(1 - \frac{1}{N_c}\,\mathfrak{Re}\,\mathrm{tr}\,U^g_{x\mu}\right) = \mathrm{Min.}.$$

For uniqueness (FMR) one requires to find the global minimum

[Parrinello, Jona-Lasinio ('90), Zwanziger ('90)].

Well understood in compact $U(1)$ theory in order to get

e.g. massless photon propagator [A. Nakamura, Plewnia ('91);

Bogolubsky, Bornyakov, Mitrjushkin, Müller-Preussker, Peters, Zverev ('93-'99)].

Optimized minimization in (our) practice: simulated annealing (SA) + overrelaxation (OR)

Gribov problem: global minimum of $F_U(g)$ very hard or impossible to find.

"Best copy strategy": repeated initial random gauges

$\Longrightarrow$ best copies (bc) from subsequent SA + OR minimizations,

$\Longrightarrow$ compared with first (random) copies (fc)).

iv) Compute propagators

- Gluon propagator:

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k) \widetilde{A}^b_\nu(-k) \right\rangle \equiv \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu \ q_\nu}{q^2} \right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a} \sin \left(\frac{\pi k_\mu}{L_\mu} \right), \qquad k_\mu \in \left(-L_\mu/2, L_\mu/2\right]$$

with (cylinder and cone) cuts in order to suppress artifacts of lattice discretization and geometry.

- Ghost propagator:

$$G^{ab}(q) = \frac{1}{V^{(4)}} \sum_{x,y} \left\langle \mathrm{e}^{-2\pi i\, k\cdot(x-y)} [M^{-1}]^{ab}_{xy} \right\rangle \equiv \delta^{ab} G(q)\,.$$

$M \sim \partial_\mu D_\mu$ - Landau gauge Faddeev-Popov operator

$$M^{ab}_{xy}(U) = \sum_\mu A^{ab}_{x,\mu}(U)\,\delta_{x,y} - B^{ab}_{x,\mu}(U)\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}(U)\,\delta_{x-\hat{\mu},y}$$

$$\begin{aligned}
A^{ab}_{x,\mu} &= \quad \mathfrak{Re}\,\mathrm{tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right],\\
B^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{tr}\left[T^b T^a\, U_{x,\mu}\right],\\
C^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right], \qquad \mathrm{tr}[T^a T^b] = \delta^{ab}/2\,.
\end{aligned}$$

M^{-1} from solving

$$M^{ab}_{xy}\phi^b(y) = \psi^a_c(x) \equiv \delta^{ac}\exp(2\pi i\, k\cdot x)$$

with (preconditioned) conjugate gradient algorithm.

3. Results for gluon, ghost propagators and the running coupling in lattice quenched and full QCD at $T = 0$ (2005–2015)

- Pure gauge $N_f = 0$:
  $\beta = 5.7, 5.8, 6.0, 6.2$; $\quad 12^4, \ldots, 56^4$, $\quad aL_{max} \simeq 9.5$fm;
  huge lattices:
  $\beta = 5.7$; $\quad 64^4, \ldots, 96^4$, $\quad aL_{max} \simeq 16.3$fm.

- Full QCD $N_f = 2$:
  configurations provided by QCDSF - collaboration,
  $\beta = 5.29, 5.25$; mass parameter $\kappa = 0.135, ..., \ 0.13575$;
  $16^3 \times 32, \ \ 24^3 \times 48$.

- Results for propagators / dressing functions and α_s

$$\text{Gluon} \quad Z(q^2) \equiv q^2 D(q^2), \qquad \text{Ghost} \quad J(q^2) \equiv q^2 G(q^2)$$

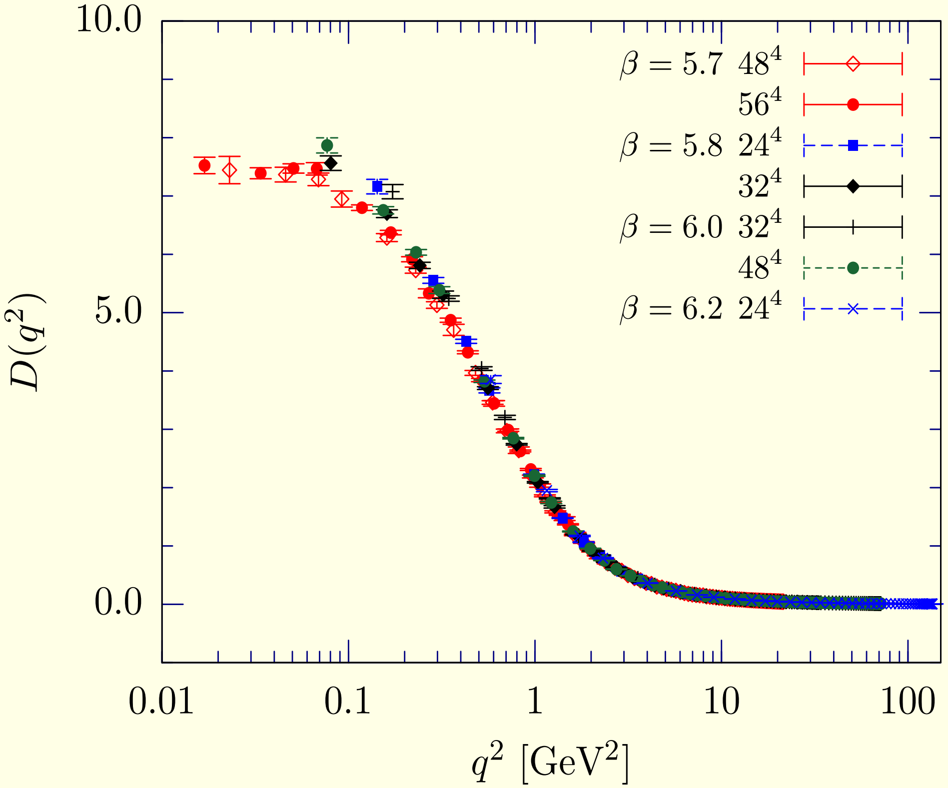
  as well as ghost-ghost-gluon vertex and Kugo-Ojima parameter.

First results: Gluon propagator and ghost dressing function

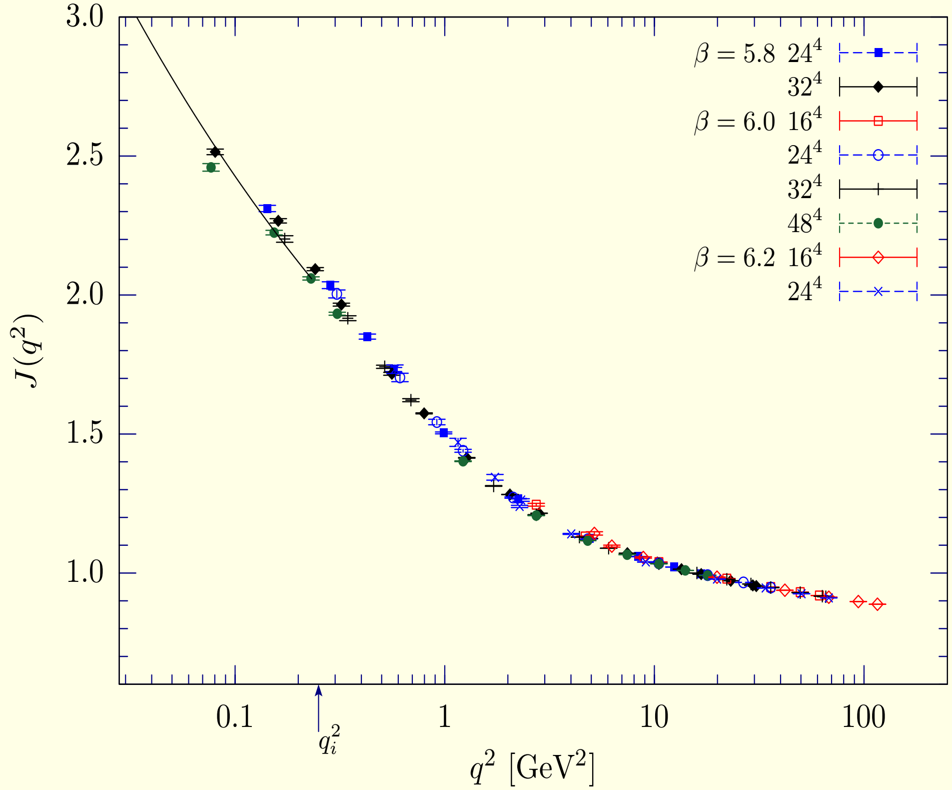
quenched QCD ($N_f = 0$), renorm. pt.: $q = \mu = 4$GeV, first OR copies

[Sternbeck, E.-M. I., Müller-Preussker, Schiller, PRD 72 (2006), Proc. IRQCD '06]

$D(q^2)$



$J(q^2)$



$\Longrightarrow$ Gluon prop. $D(q^2)$ shows plateau and not $D(q^2) \to 0$ for $q^2 \to 0$,

$\Longrightarrow$ corresponds to an effective gluon mass behaviour.

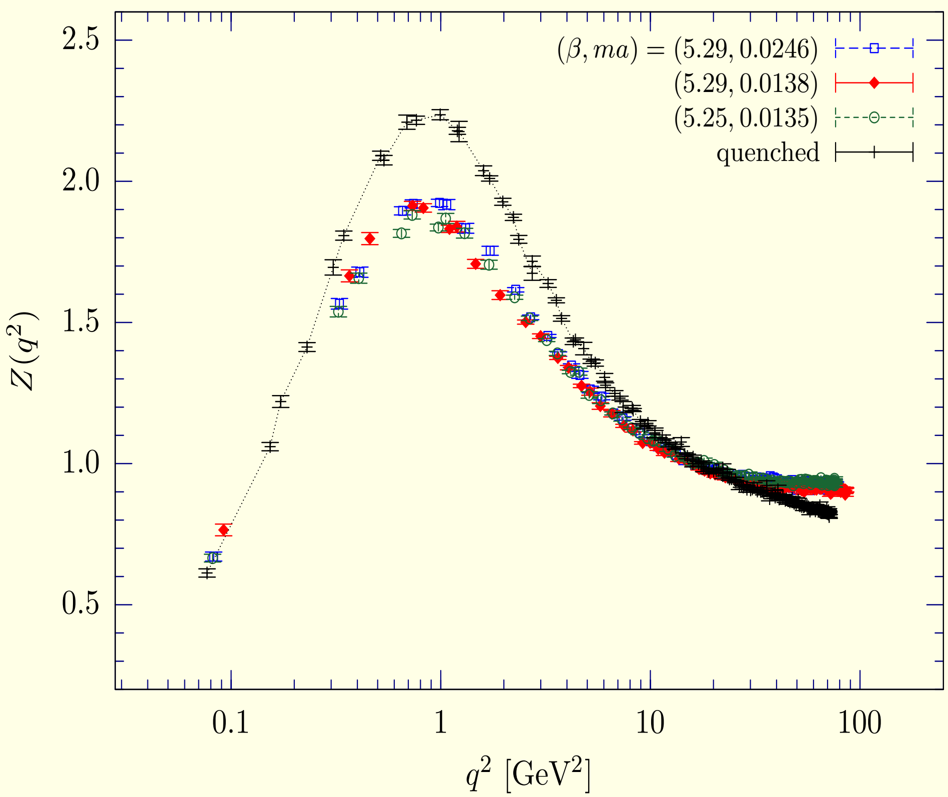
$\Longrightarrow$ Ghost dress. fct. $J(q^2)$ power-like, expon. too small for scaling solution.

Gluon and ghost dressing functions

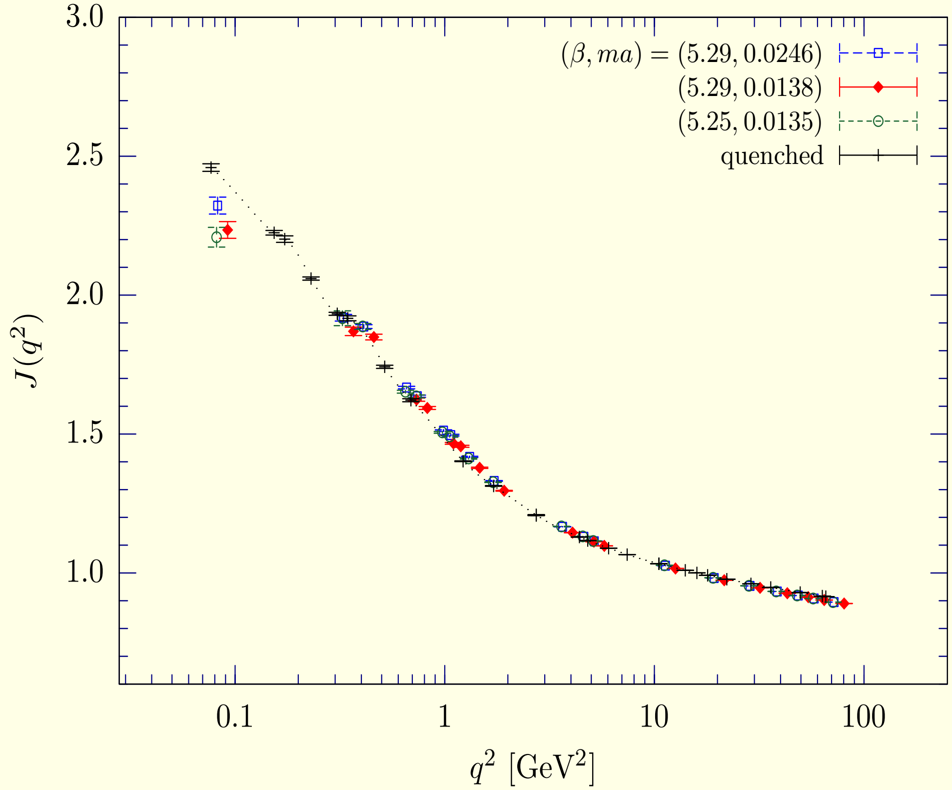
full QCD ($N_f = 2$) versus quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4\text{GeV}$

[E.-M. I., Müller-Preussker, Schiller, Sternbeck (A. DiGiacomo 70, '06)]

$Z(q^2)$



$J(q^2)$



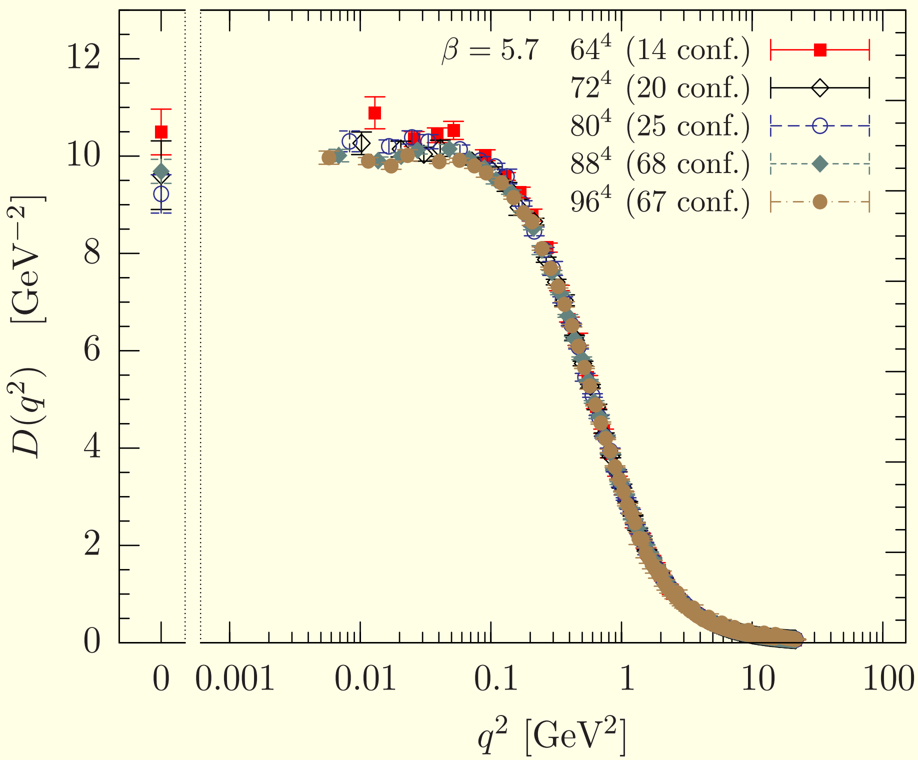
$\Longrightarrow$ Influence of virtual quark loops in $Z(q^2)$ clearly visible.

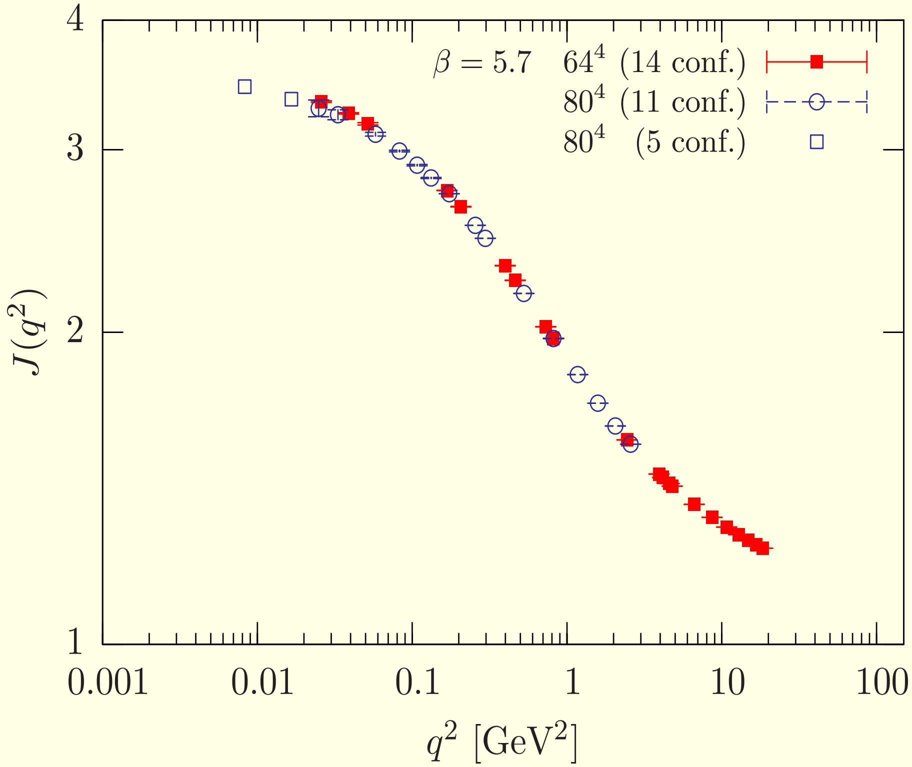
$\Longrightarrow$ No quenching effect in $J(q^2)$, since ghosts do not directly couple to quarks.

Gluon propagator and ghost dressing function on huge volumes

quenched QCD, first but long run SA + OR copies, unrenormalized

[Bogolubsky, E.-M. I., Müller-Preussker, Sternbeck, PLB 676 (2009)]





$\Longrightarrow$ Both $D(q^2)$ and $J(q^2)$ seem to tend to const..

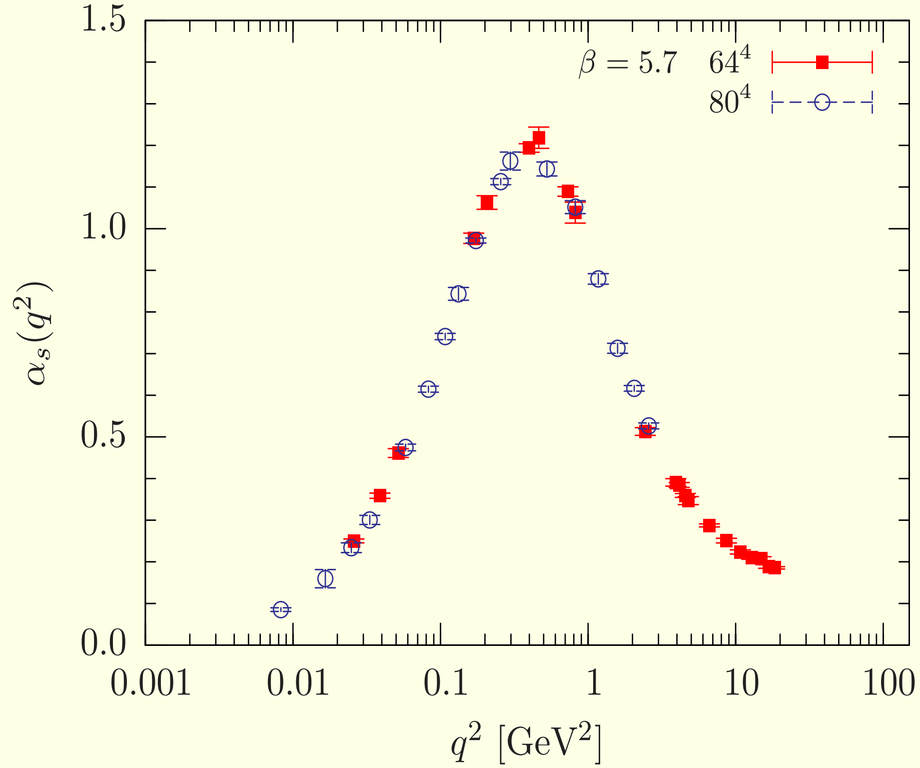
$\Longrightarrow$ Clear indication for "decoupling" solution.

$\Longrightarrow$ Here coarse lattices used. Question: continuum limit ?

Result for the running coupling on large volumes

quenched QCD, first but long run SA + OR copies, coarse lattices

[Bogolubsky, E.-M. I., Müller-Preussker, Sternbeck, PLB 676 (2009),]



- Running coupling not monotonous, $\alpha_s \to 0$ for $q \to 0$, $\Longrightarrow$ "decoupling behaviour".
- Agrees with other lattice studies, in particular for the three-gluon vertex.
- At large q^2 allows to fix $\Lambda_{\overline{MS}}$.

Finally : return to quenched SU(2) with Wilson action (known to have less good scaling behaviour etc.)

for quenched SU(2) QCD, methodical paper on gaugefixing,

$80 = 5 * 2^4$ copies per MC configuration (all flip sectors) !

gluon propagator measurement done, gauge-fixed ensembles remained

[V. G. Bornyakov, V. K. Mitrjushkin, and M. Müller-Preussker, PRD 81 (2010) 054503]

Later we returned to these ensembles for measurement of ghost dressing function

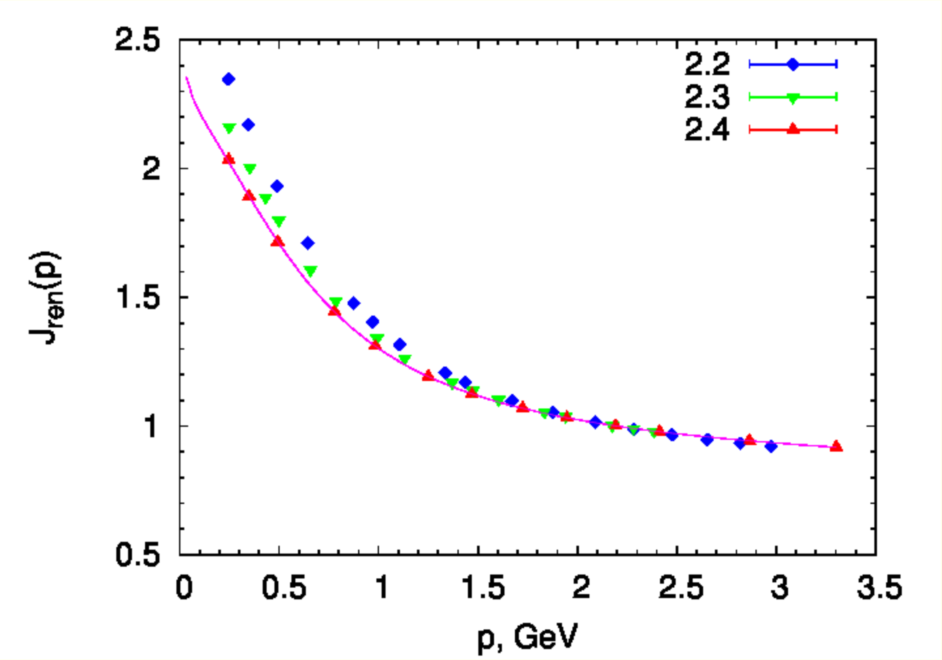
[Bornyakov, E.-M. I., Litwinski, Mitrjushkin, Müller-Preussker, arXiv:1302.5943, not published then]

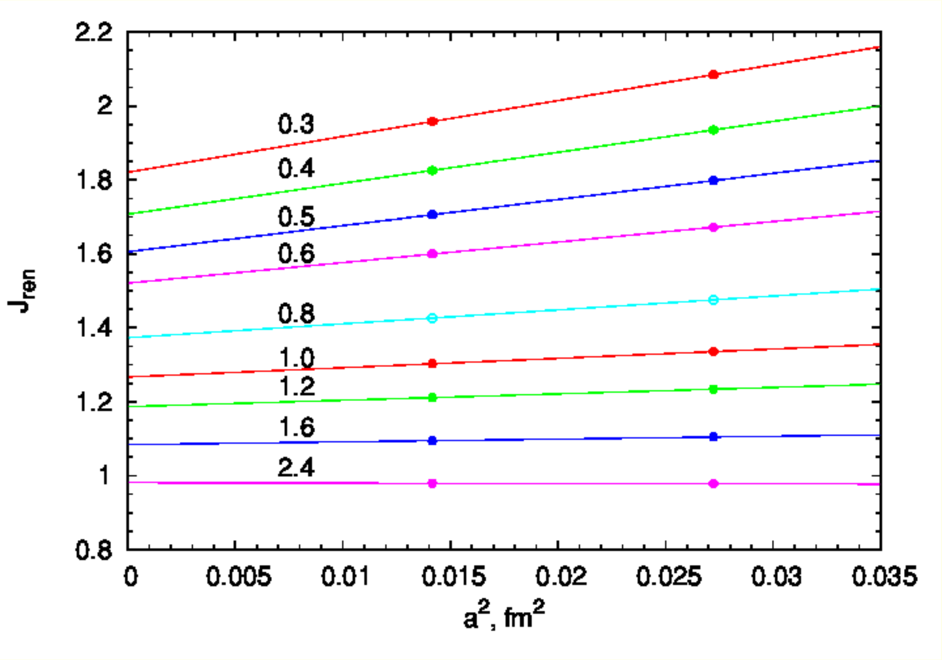
Recently: very careful investigation of continuum limit (according to present standards) $\rightarrow$ resubmitted to PRD (August 2015), accepted for publication

[Bornyakov, E.-M. I., Litwinski, Müller-Preussker, Mitrjushkin, PRD 2015 to be published]

- $aL = 3 \ldots 7$ fm (finite volume effects small !)
- lattice spacing between $a = 0.2$ fm and $a = 0.07$ fm
- fits of the continuum behavior for $p \in [0.4, 3.2]$ GeV are presented

Extrapolation $a^2 \to 0$ of the gluon dressing function at fixed momenta





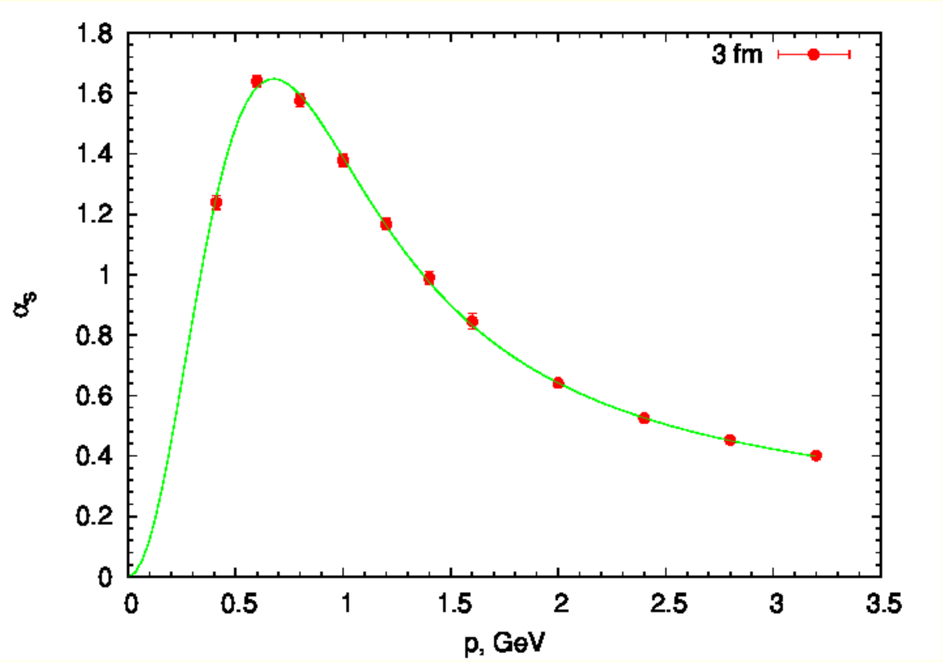
Left: Data for a linear box size of $aL = 5$ fm and three different β-values.

The fitting curve belongs to $L = 2.40$ fm.

Right: Dressing function $J_{ren}(p)$ for few selected momenta as function of a^2.

The straight lines are only to guide the eye.

Running coupling for SU(2)



The momentum dependence of the running coupling $\alpha_s(p)$ for SU(2) extracted in the continuum limit for selected momenta and $aL = 3$ fm. The curve shows a fit corresponding to the ansatz

$$f_\alpha(p) = \frac{c_1 \hat{p}^2}{1+\hat{p}^2} + \frac{c_2 \hat{p}^2}{(1+\hat{p}^2)^2} + \frac{c_3 \hat{p}^2}{(1+\hat{p}^2)^4}\,, \quad \hat{p} \equiv p/m_\alpha\,.$$

4. Systematic effects: Gribov copies, finite-volume effects, multiplicative renormalization, continuum limit

(a) Universality: gluon and ghost propagators from alternative $A_\mu(x)$ definition

Use logarithmic definition for the lattice gluon field

$$A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} = \frac{1}{i\ a\ g_0} \log\left(U_{x,\mu}\right) ,$$

minimize lattice gauge functional directly translated from continuum

$$F_U^{(\log)}[g] = \sum_{x,\mu} \frac{1}{N_c} \operatorname{tr}\left[{}^g A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} \, {}^g A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} \right] .$$

Faddeev-Popov determinant derived accordingly.

Numerical treatment differs: accelerated multigrid algorithm + preconditioning.

$\Longrightarrow$ Compare results for linear and logarithmic definition.

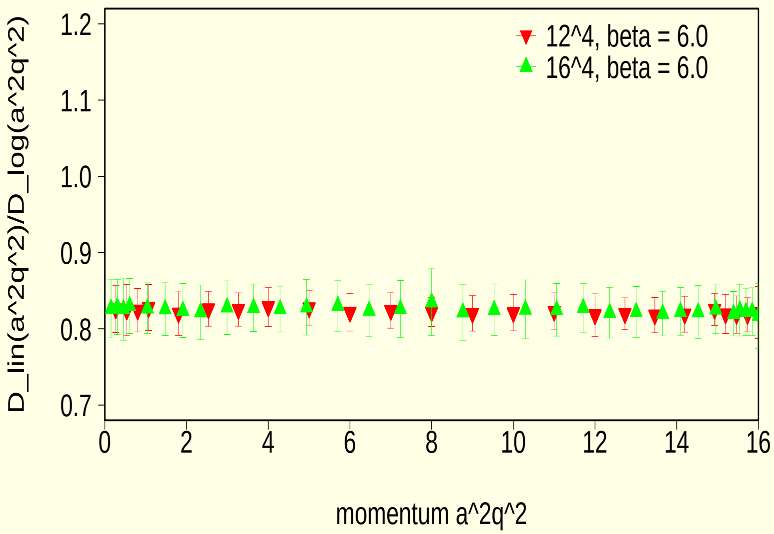
$\Longrightarrow$ Check independence of the running coupling.

$\Longrightarrow$ Compare with stochastic perturbation theory.

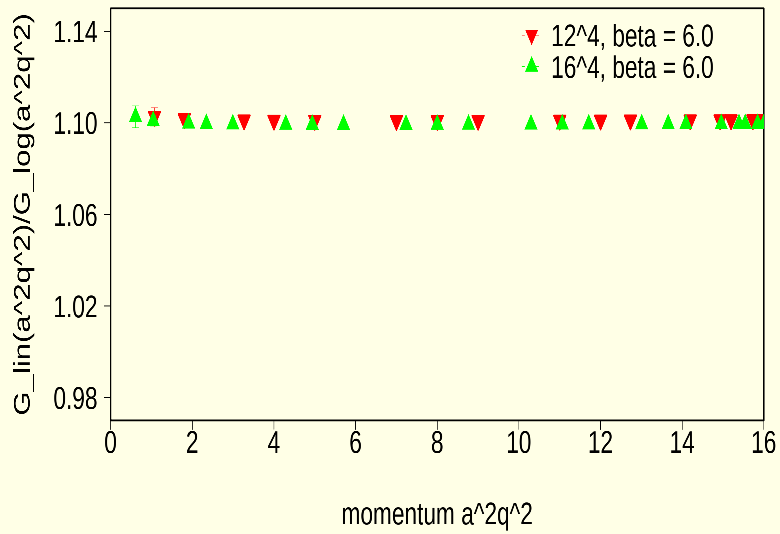
Related work: [Petrarca et al., '99; Cuchieri, Karsch, '99; Bogolubsky, Mitrjushkin, '02;...]

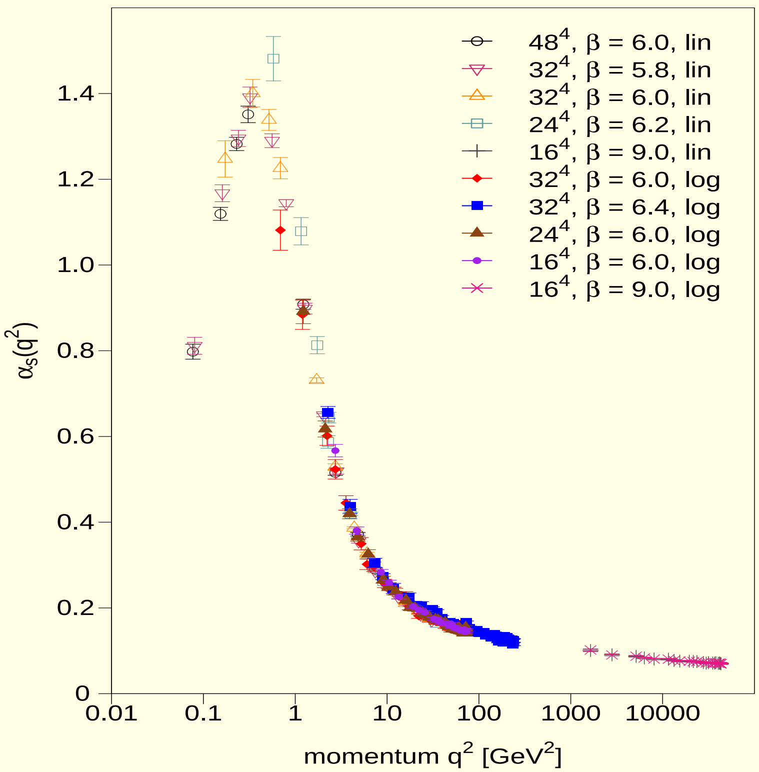
Linear definition results vs. logarithmic definition, $\beta = 6.0$

Gluon propagator ratio



Ghost propagator ratio





Running coupling

$\Rightarrow$ multipl. renormalizability confirmed.

$\Rightarrow$ $\alpha_s(q^2)$ in given MOM scheme approx. renorm. independent.

[E.-M. I., Menz, Müller-Preussker, Schiller, Sternbeck, PRD (2010); arXiv:1010.5120 [hep-lat]]

Monte Carlo vs. numerical stochastic perturbation theory (NSPT)

NSPT with Langevin technique allows for higher loop perturbation theory.

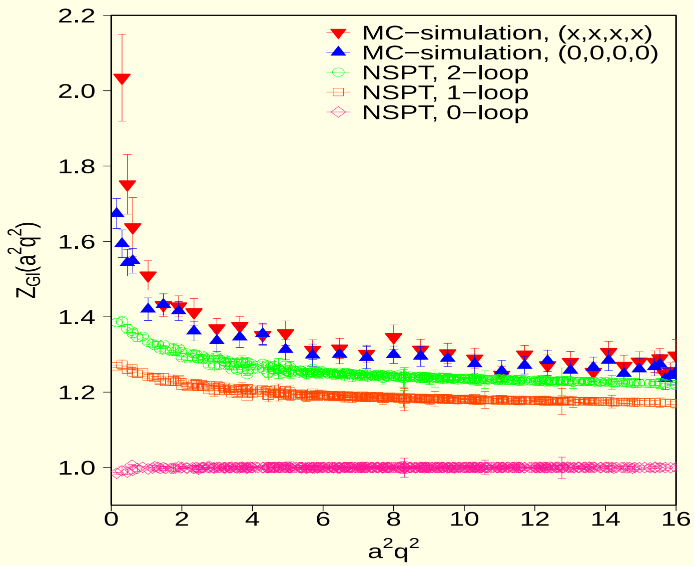
Logarithmic definition for A_μ is natural.

[di Renzo, E.-M. I., Perlt, Schiller, Torrero, '09 - '10]

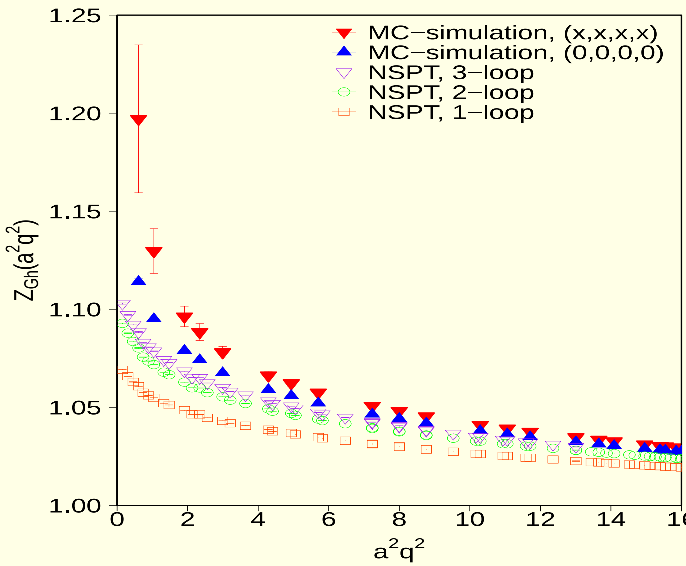
Compare arbitrary Polyakov loop sectors (x, x, x, x) with real sector $(0, 0, 0, 0)$.

Here: 16^4, large $\beta = 9.0$ for both approaches.

Gluon dressing fct.



Ghost dressing fct.



$\Rightarrow$ Nice consistency, approach to full result can be checked !

(b) Gribov copy effects and continuum limit in pure $SU(2)$ gauge theory

[Bakeev, Bogolubsky, Bornyakov, Burgio, E.-M. I., Mitrjushkin, Müller-Preussker ('04 - '09)]

Improved gauge fixing $\implies$ getting 'close' to the FMR:

- Simulated annealing (SA):

  Find g's randomly with statistical weight:

$$W \propto \exp\left(-\frac{F_U(g)}{T_{SA}}\right).$$

Let "temperature" T_{SA} slowly decrease.

In practice SA clearly wins for large lattice sizes.

(Over)relaxation (OR) has to be applied subsequently in order to reach

$$(\partial \mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) < \epsilon \quad \text{for all} \quad x\,.$$

- $\mathbb{Z}(N_c)$ flips:

  Gauge functional $F_U(g)$ minimized by enlarging the gauge orbit with respect to $\mathbb{Z}(N_c)$ non-periodic gauge transformations:

$$g(x + L\hat{\nu}) = z_\nu g(x)\,, \quad z_\nu \in \mathbb{Z}(N_c)\,.$$

  For $SU(N_c)$ the N_c^4 different sectors of Polyakov loop averages are combined.

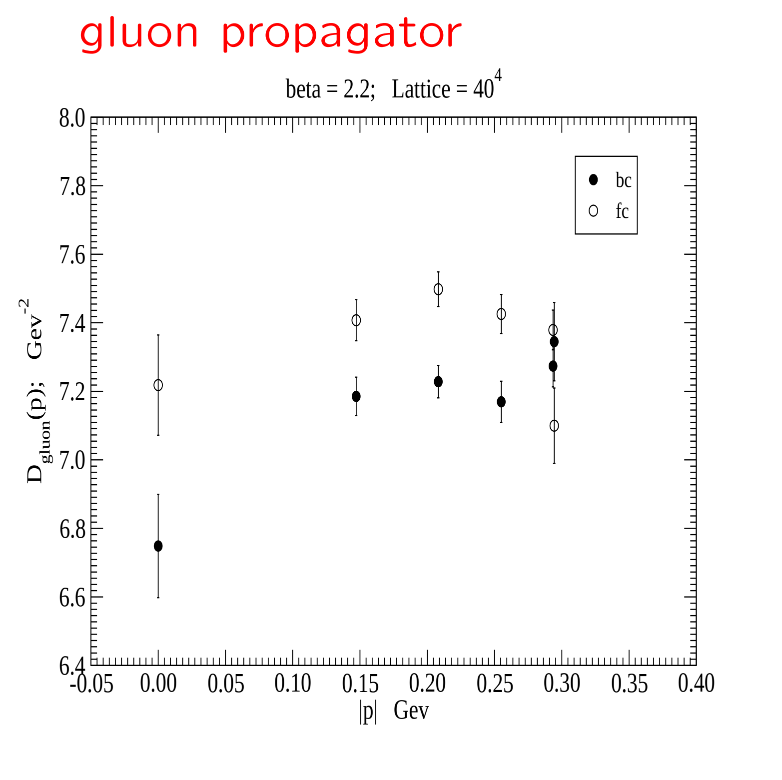
In order to view Gribov copy effects we compare:

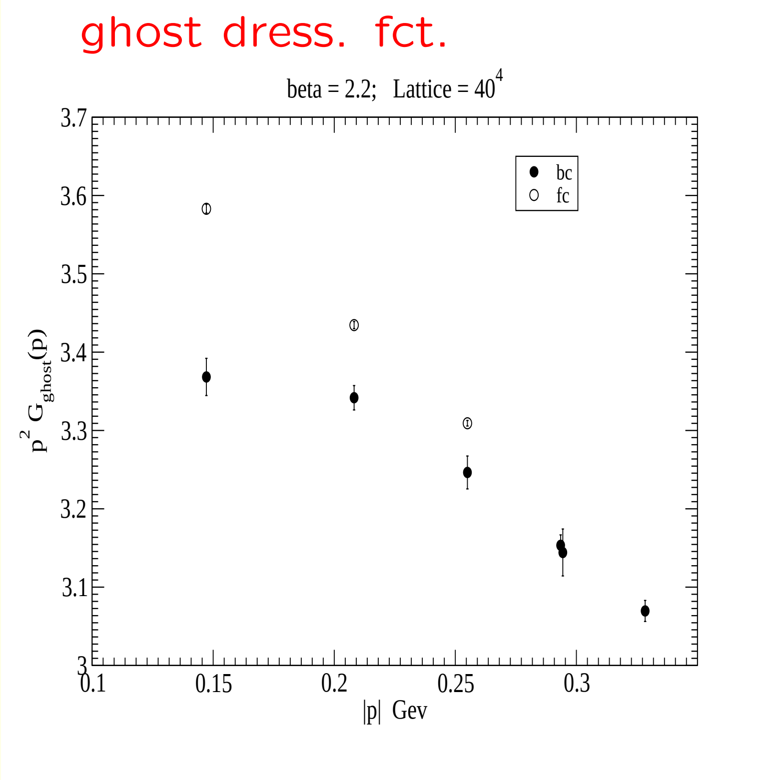
- first (random) copy from simulated annealing "fc SA",
- best copy from $\mathbb{Z}(2)$ flips + simulated annealing "bc FSA",

  compare typically 5 copies in each of the $2^4 = 16$ Polyakov loop sectors.

Gluon propagator and ghost dressing fct.: fc SA versus bc FSA

[Bornyakov, Mitrjushkin, Müller-Preussker, PRD 79 (2009), arXiv:0812.2761 [hep-lat]]





$\Longrightarrow$ Gribov copies important for both gluon and ghost !

$\Longrightarrow$ The closer to the global minimum (FMR), the weaker the 'singularity'

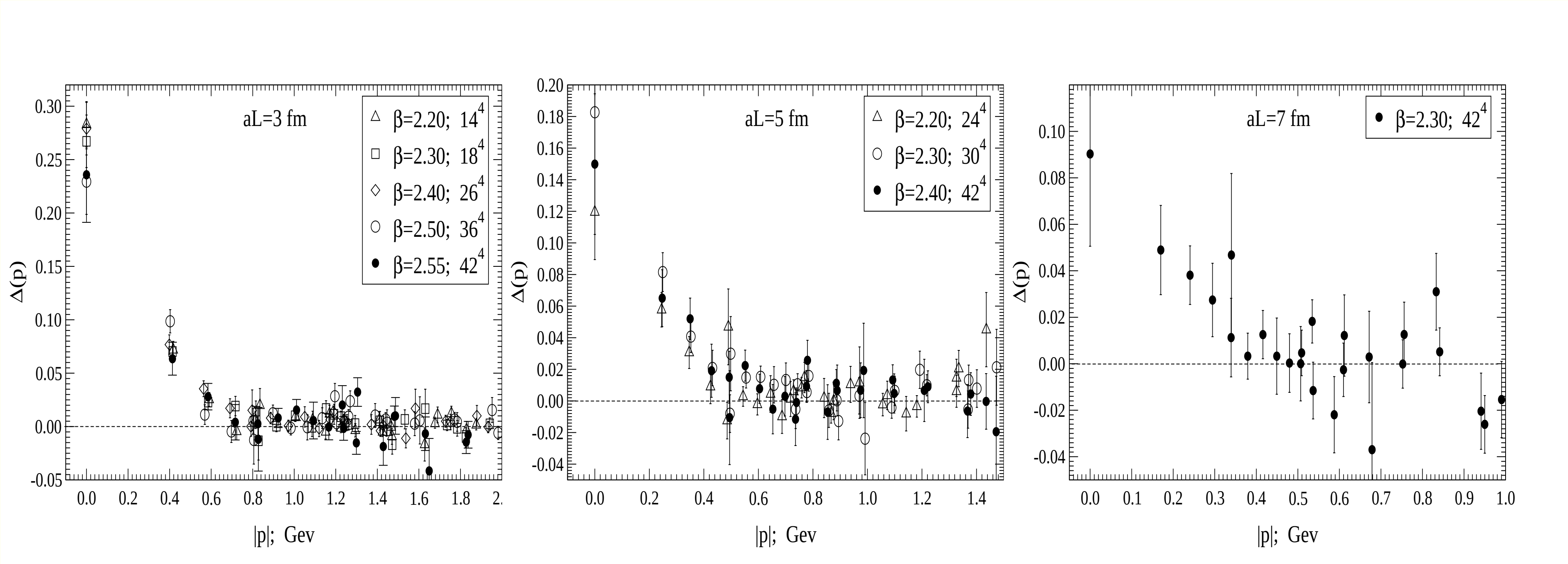
of the ghost dressing fct., the lower the IR values of the gluon propagator.

$\Longrightarrow$ $D(q^2 \to 0) = 0$? Would contradict DS and FRG eqs. !

Gribov copy sensitivity for the gluon propagator bc FSA versus fc SA

$$\Delta(p) = \frac{D^{fc}(p) - D^{bc}(p)}{D^{bc}(p)}$$

[Bornyakov, Mitrjushkin, Müller-Preussker, PRD 81 (2010), arXiv:0912.4475 [hep-lat]].

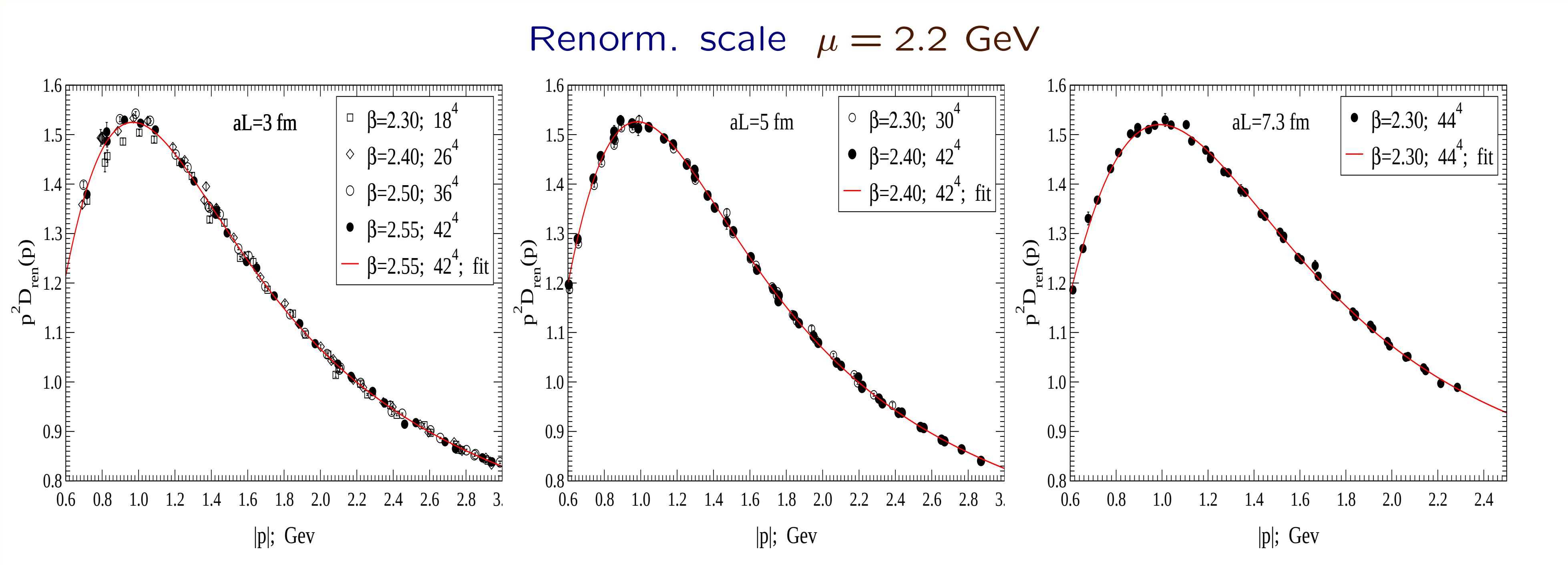


Gribov copy effect: $\Rightarrow$ at low momenta, appr. independent of lattice spacing,

$\Rightarrow$ weakens with increasing physical volume [Zwanziger ('04)].

Finite-volume and cont. limit results for renormalized gluon dressing fct. bc FSA

[Bornyakov, Mitrjushkin, Müller-Preussker, PRD 81 (2010), arXiv:0912.4475 [hep-lat]].



$\Longrightarrow$ For $\beta \geq 2.40$, $p > 0.6$MeV renormalized data fall on top of each other.

$\Longrightarrow$ Contin. limit reached, good fits available.

$\Longrightarrow$ Curves for different linear sizes $3, 5, 7$ fm nicely agree.

$\Longrightarrow$ Analogous result for the ghost dressing fct. available.

$\Longrightarrow$ Resulting MOM-scheme $\alpha_s(q^2)$ is approx. renorm-invariant,

i.e. Z factors for ghost and gluon dressing functions nicely cancel each other.

5. Results for gluon, ghost propagators and the running coupling in lattice quenched and full QCD at $T > 0$ (2010 – ?)

Temperature dependence from $T \equiv 1/aL_\tau, \quad L_\tau \ll L_\sigma$.

Separate time and space components, Matsubara frequency $\omega \sim q_4$.

Transverse (magnetic) gluon propagator:

$$D_T \sim \langle \sum_{i=1}^{3} A_i^a(q) \; A_i^a(-q) - \frac{q_4^2}{\vec{q}^2} \; A_4^a(q) \; A_4^a(-q) \rangle$$

Longitudinal (electric) gluon propagator:

$$D_L \sim (1 + \frac{q_4^2}{\vec{q}^2}) \; \langle \; A_4^a(q) \; A_4^a(-q) \; \rangle$$

$T > T_c \implies$ spontaneous $Z(3)$ symmetry breaking.

Polyakov loop average $\langle L \rangle$ takes values in 3 sectors.

Real sector = "physical" sector.

[See Cucchieri, Karsch, '00; Bogolubsky, Mitrjushkin, '02; Fischer, Maas, Mueller, '10; ....]

Our investigations:

- quenched QCD, fixed scale approach

  [Aouane, Bornyakov, E.-M. I., Mitrjushkin, Müller-Preussker, Sternbeck, PRD 85, 034501 (2012)]

  $L_\sigma^3 \times L_\tau, \quad L_\sigma = 48, \quad L_\tau = 4, 6, \ldots, 18$ varies,

  $a = a(\beta = 6.337) \simeq 0.055$ fm fixed,

  $\Longrightarrow \quad T_c \simeq 1/(L_\tau \cdot a) = 1/(12a)$.

- full QCD with $N_f = 2$ twisted mass fermions

  [tmfT Coll.: Burger, E.-M. I., Lombardo, Müller-Preussker, Philipsen, Urbach, Zeidlewicz et al., '09 - '12]

  [Aouane, Burger, E.-M. I., Müller-Preussker, Sternbeck, PRD 87, 114502 (2013)]

  $L_\sigma = 32, \quad L_\tau = 12$, vary $a = a(\beta) \leq 0.09$ fm

  at fixed $\quad m_\pi = \ 320,\ 400,\ 480$ MeV.

Main aim: provide input for DS and FRG equations in terms of fit formulae valid within non-perturbative range $[\ 0.4 \text{ GeV}, 3 \text{ GeV}\ ]$.

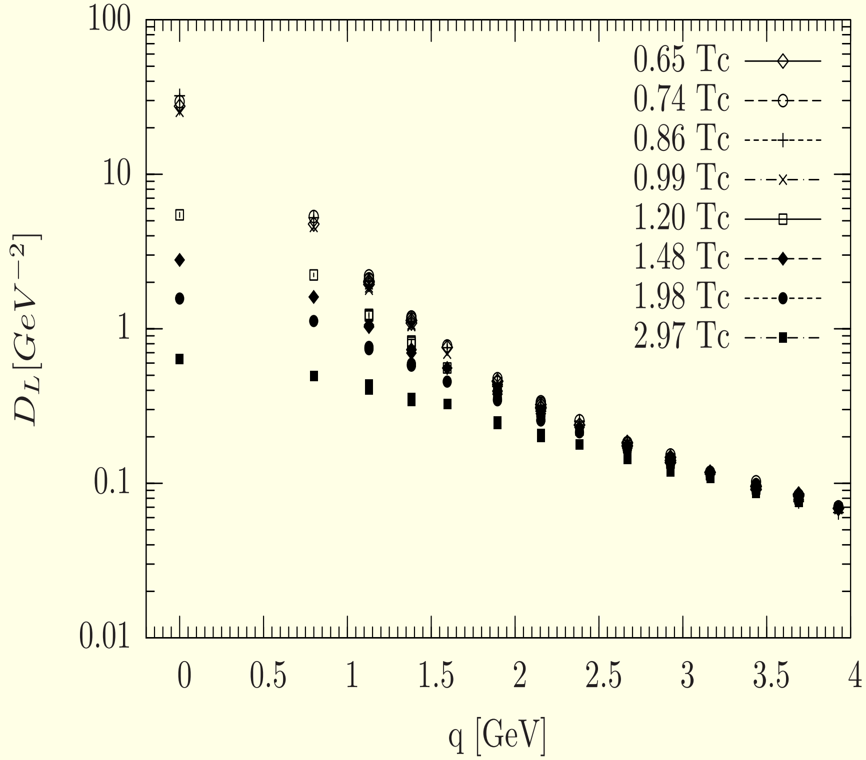
$$Z_{fit}(q) = q^2 \frac{c\ (1 + d\, q^{2n})}{(q^2 + r^2)^2 + b^2}, \qquad J_{fit}(q) = \left(\frac{f^2}{q^2}\right)^k + \frac{h\ q^2}{q^2 + m_{gh}^2}$$

$$n = 1,\ b = 0 \qquad\qquad m_{gh} = 0$$

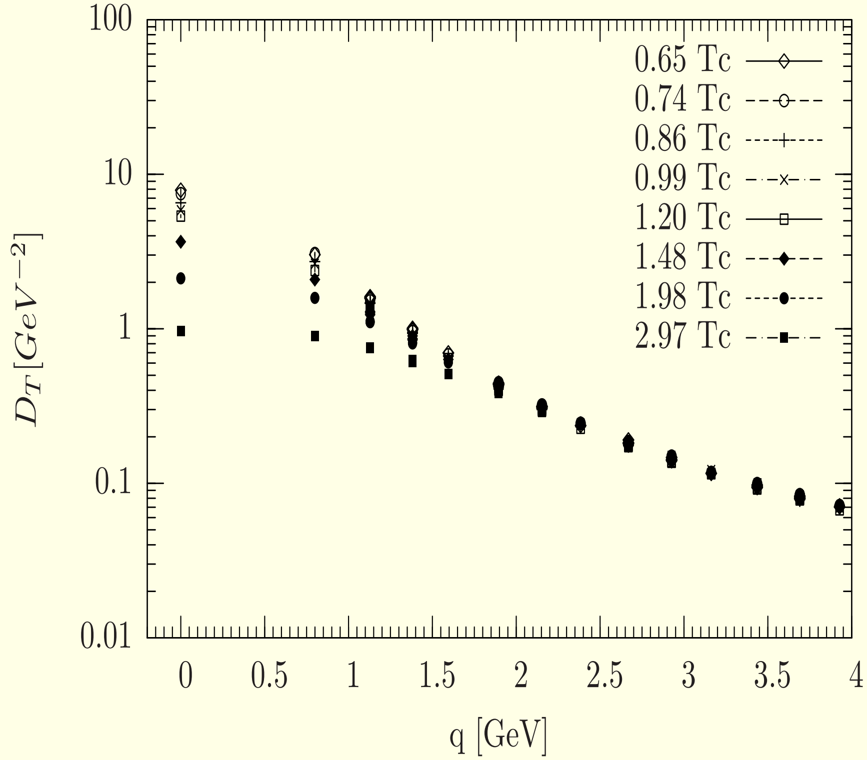
Quenched QCD:

renormalized gluon propagator for $T > 0$, $q_4 = 0$, renorm. scale $\mu = 5$ GeV

longitudinal gluon propagator



transversal gluon propagator



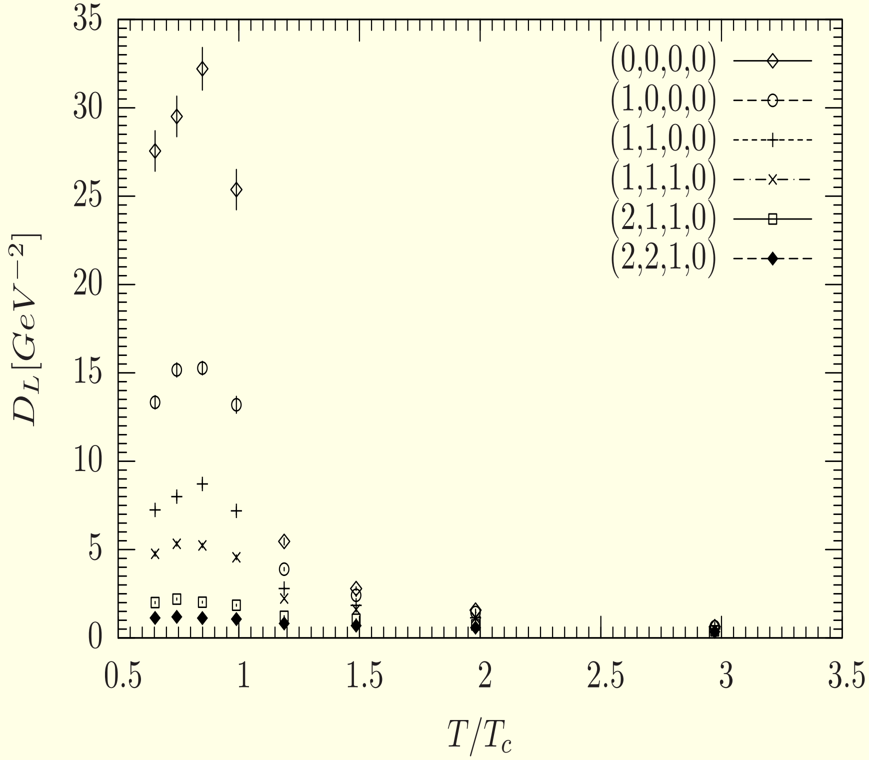
$\Longrightarrow$ Gluon propagator depends on T at low momenta.

$\Longrightarrow$ Longitudinal component most sensitive.

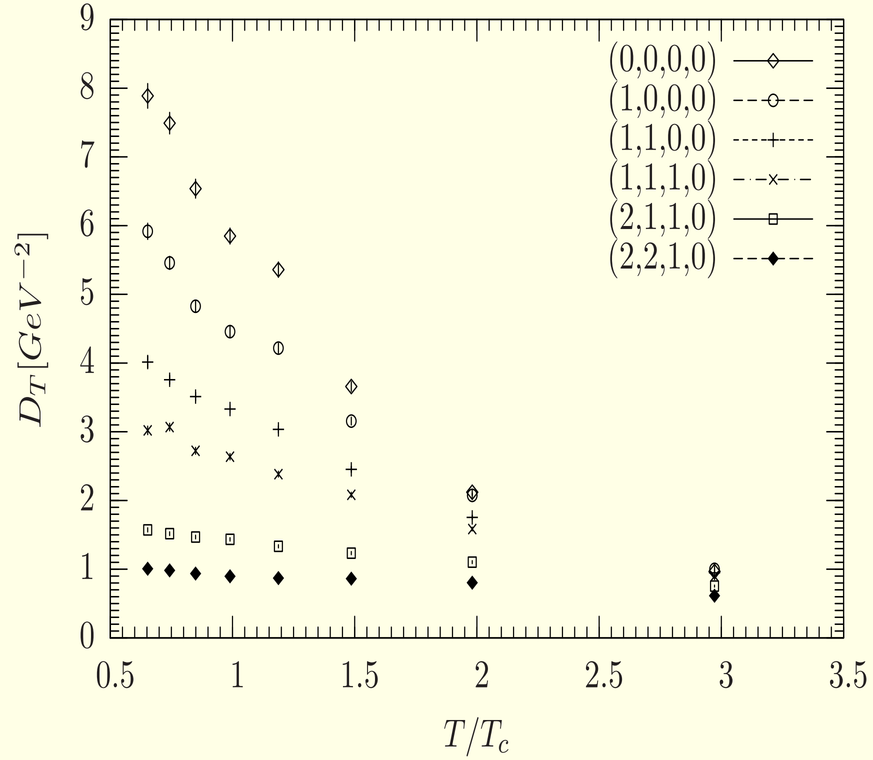
$\Longrightarrow$ Not shown: Ghost propagator less T-dependent.

Renormalized propagator vs. T at lowest fixed momenta

longitudinal gluon propagator



transversal gluon propagator



$\Longrightarrow$ Longitudinal propagator – indicator for the 1^{st} order transition.

Systematic effects studied at $T =$ 0.86 T_c, 1.20 T_c

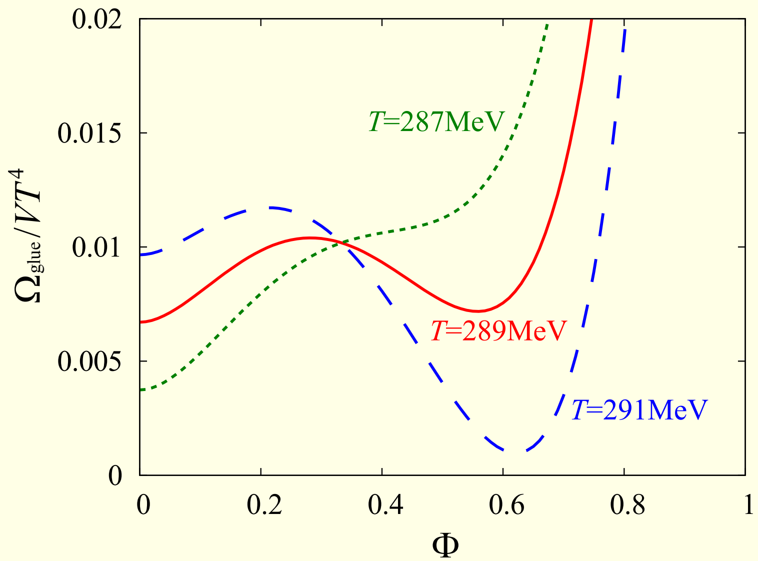
$\Longrightarrow$ finite size, Gribov copy effects turn out small,

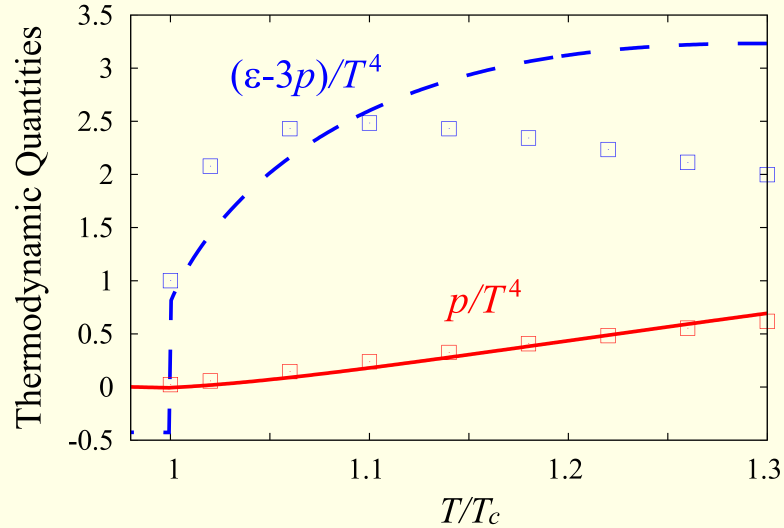
$\Longrightarrow$ continuum limit well reached at $a = 0.055$ fm.

Order parameter and EoS of pure Yang-Mills theory

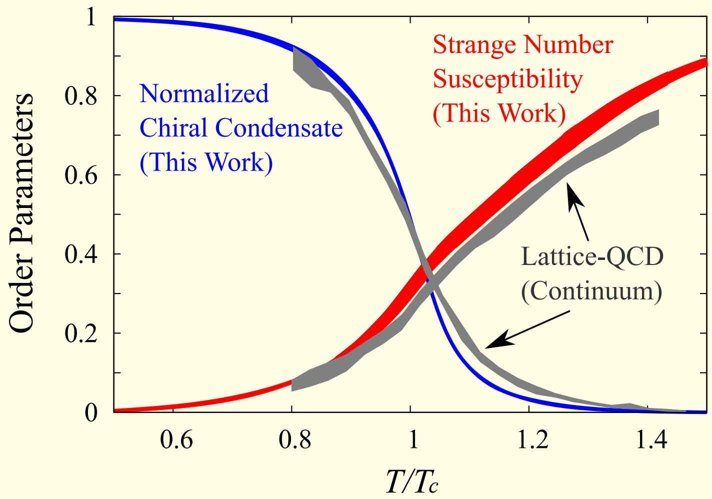
Transition temperature and rise of pressure are successfully, the trace anomaly less successfully reconstructed from our T-dependent propagator data !

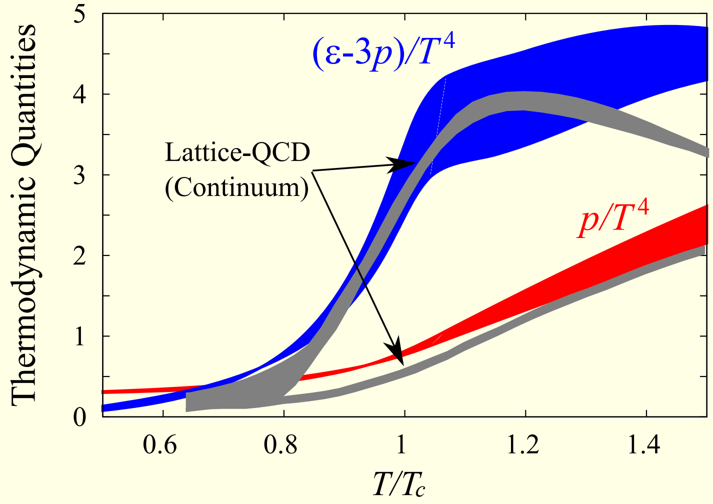
[Fukushima and Kashiwa, Phys. Lett. B723 (2013) 360, arXiv:1206.0685].





In the same paper, based on schematic lattice propagators of full QCD, the “order parameters” and the EoS of full QCD have been presented :





What about propagators for full QCD ?

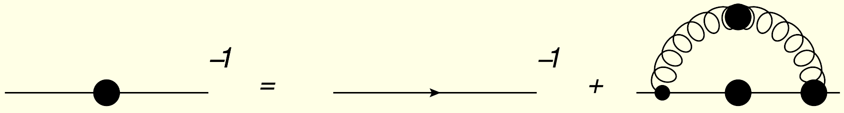
Can one obtain non-quenched propagators from the quenched ones without actually doing the non-quenched lattice simulation ?
How good can DSE predict what will be measured on the lattice in a full-QCD simulation ?

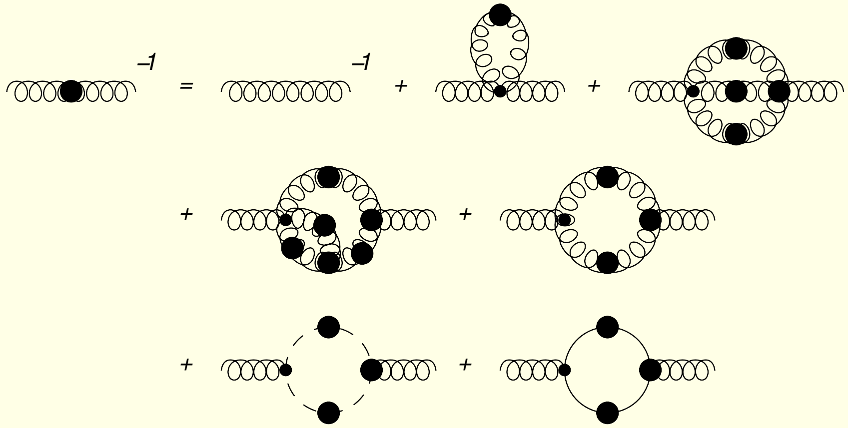
[Fischer and Luecker, Phys. Lett. B718 (2013) 1036, arXiv:1206.5191 and arXiv:1306.6022].

"Progagators and phase structure of $N_f = 2$ and $N_f = 2+1$ QCD"

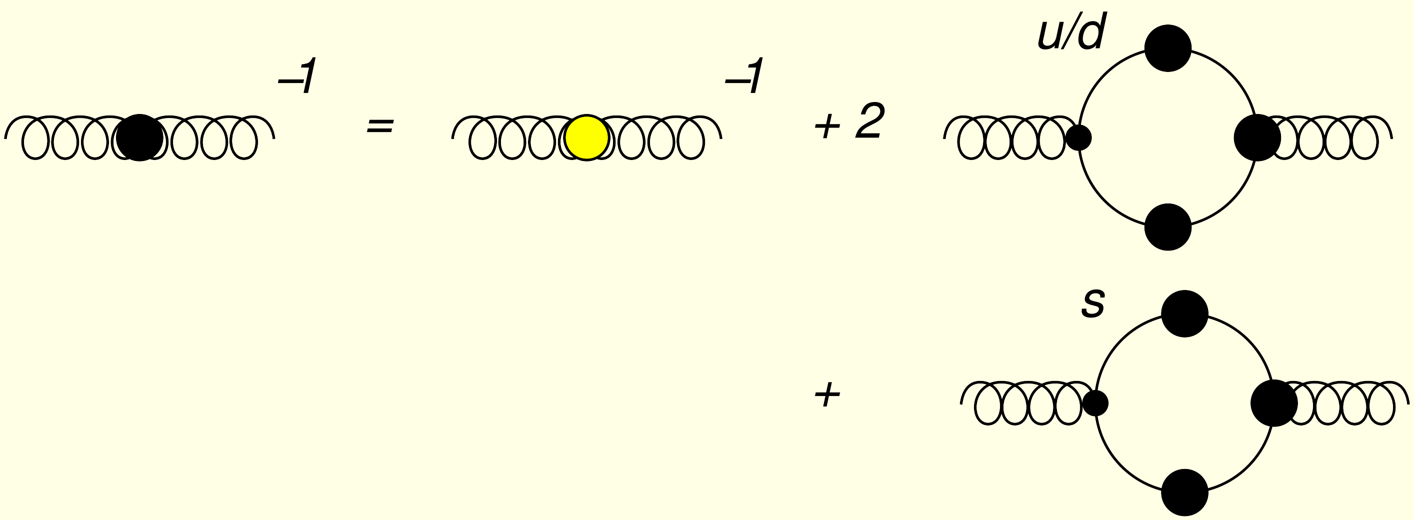
The full set of Dyson-Schwinger equations was used to predict the T-dependence of full QCD propagators from the quenched ones, in dependence on m_π as a parameter to characterize the would-be non-quenched simulations.

Full Dyson-Schwinger equations for the quark and the gluon propagator





Truncated gluon Dyson-Schwinger equation relating the quenched and the non-quenched gluon propagator (for u, d and eventually s quarks) (yellow insert = quenched non-pert. gluon propagator)



A by-product of this study : quark propagator at $T \neq 0$ (was not yet studied by us for twisted mass at $T \neq 0$)

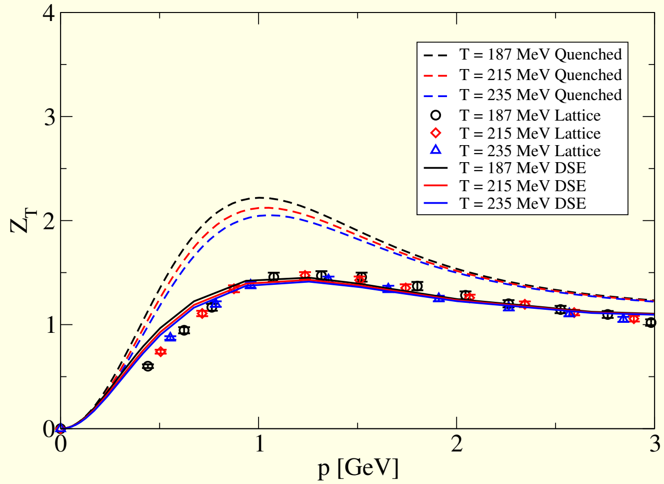
The quark propagator will perhaps be measured in future finite-T simulations (now with $N_f = 2 + 1 + 1$).

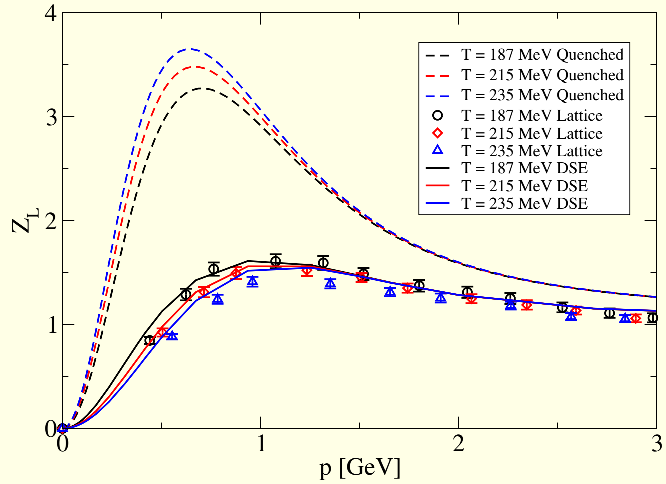
Will be interesting to compare with DSE predictions !

Our quenched propagator data used as input and the DSE prediction for $N_f = 2$, compared with our non-quenched data. The pion mass is $m_\pi = 316$ MeV as in our twisted mass simulation.

[Fischer and Luecker, Phys. Lett. B718 (2013) 1036, arXiv:1206.5191 and arXiv:1306.6022].

Left: transversal propagator, right: longitudinal propagator



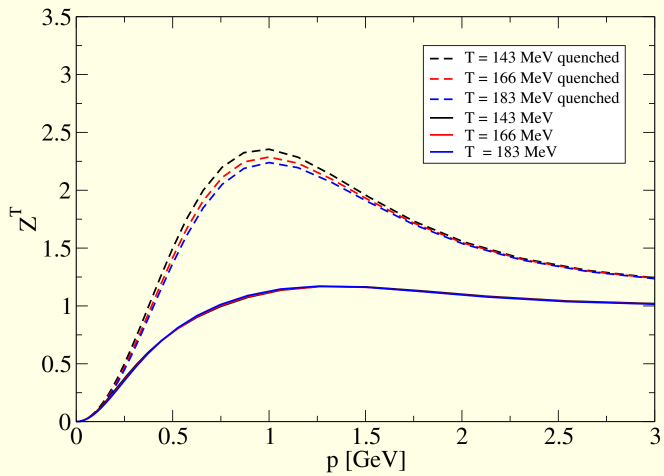


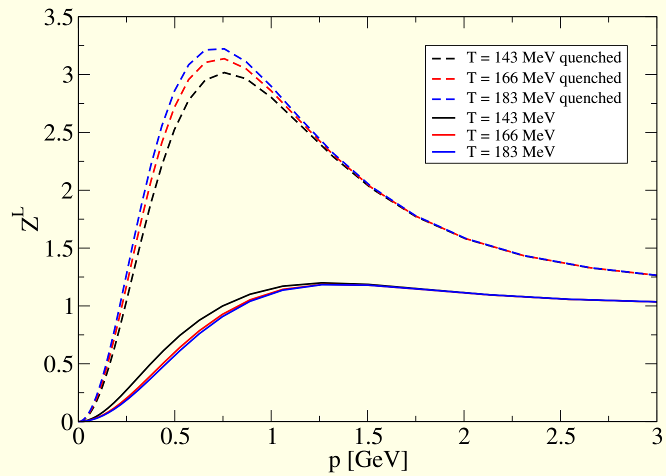
Our quenched propagator data used as input and the DSE prediction for $N_f = 2 + 1$. The pion mass is the physical one.

[Fischer, Luecker and Welzbacher, PRD 90 (2014) 034022, arXiv:1405.4762]

“Phase structure of three and four flavor QCD”

Left: transversal propagator, right: longitudinal propagator



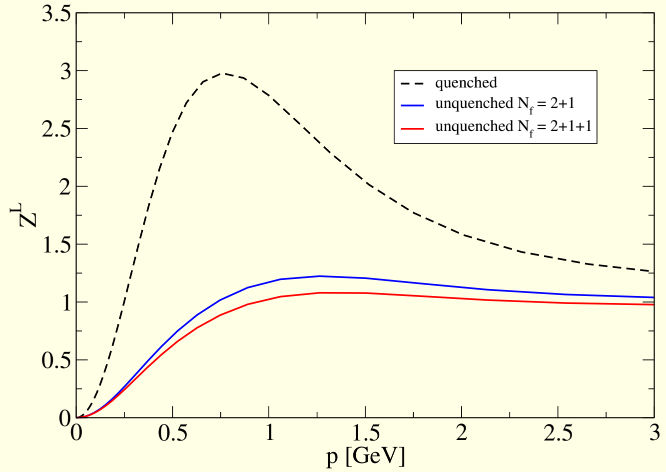


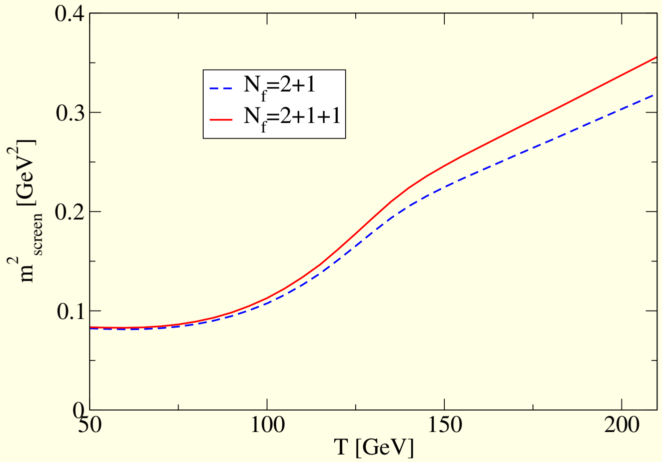
Our quenched propagator data used as input and the DSE prediction for $N_f = 2+1+1$ at $T = 135$ MeV and for physical quark masses.

[Fischer, Luecker and Welzbacher, PRD 90, 034022 (2014), arXiv:1405.4762]

“Phase structure of three and four flavor QCD”

Left: longitudinal propagator for $N_f = 2+1$ und $N_f = 2+1+1$, right: gluon screening mass as function of T.





Phys. Rev. D 87 (2013) 114502, arXiv:1212.1102

"Landau gauge gluon and ghost propagators from lattice QCD with $N_f = 2$ twisted mass fermions at finite temperature"

R. Aouane, F. Burger, E.-M. I., M. Müller-Preussker, A. Sternbeck

has provided the unquenched propagators for twisted mass ensembles of the tmfT collaboration,

in continuum parametrizations ready for comparison with DSE predictions in the momentum ranges :

- 0.4 $GeV < q <$ 3.0 GeV for the gluon propagators (perfect !) fitting parameter b^2 in the Grivov-Stingl fit is compatible with zero (no splitting in complex conjugate poles is visible in this momentum range !)

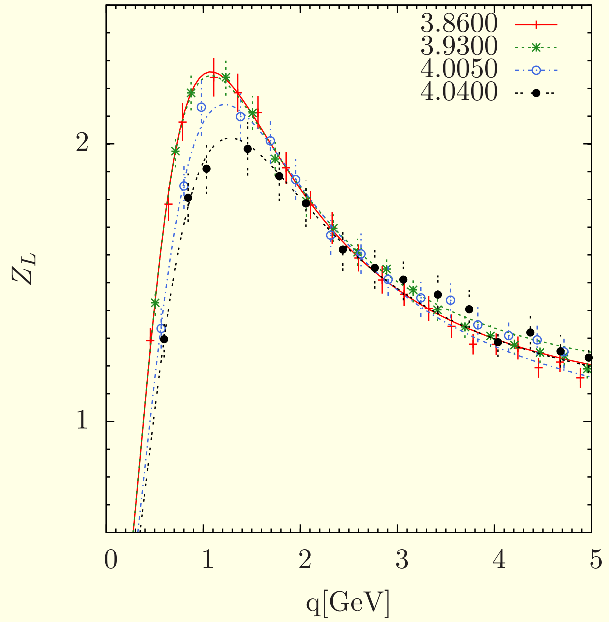
- 0.4 $GeV < q <$ 4.0 GeV for the ghost propagator (less good fit correct within few percent, a mass term m_{gh} wouldn't help),

Full QCD:

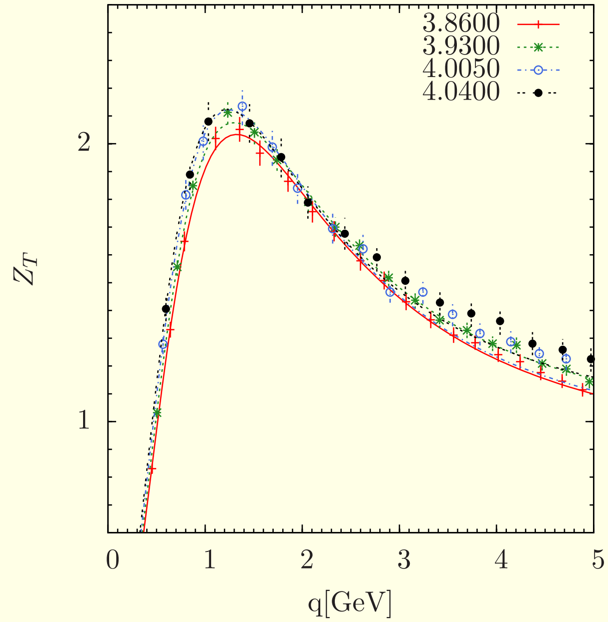
bare gluon and ghost dressing functions within the crossover range,

$q_4 = 0$, $m_\pi \simeq 400$ GeV; fits in 0.4 GeV $\leq q \leq 3.0$ GeV.

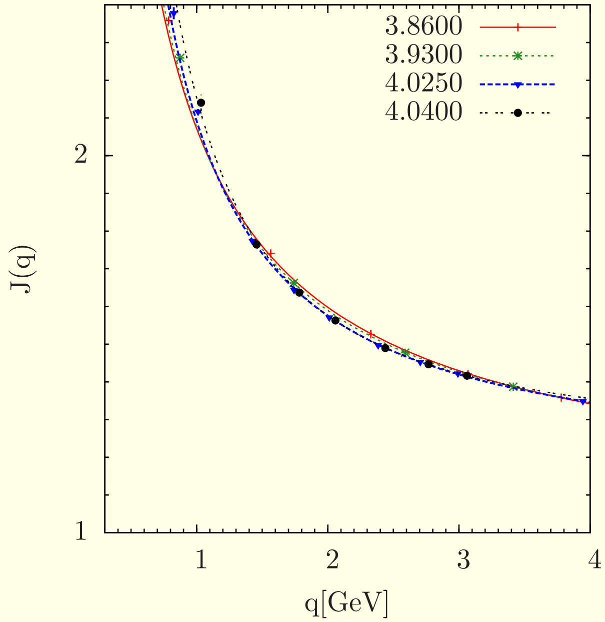
long. gluon



trans. gluon



ghost



$\Longrightarrow$ Smooth behaviour of all propagators $\leftrightarrow$ crossover

$\Longrightarrow$ Longitudinal component again most sensitive.

$\Longrightarrow$ Ghost propagator weakly T-dependent.

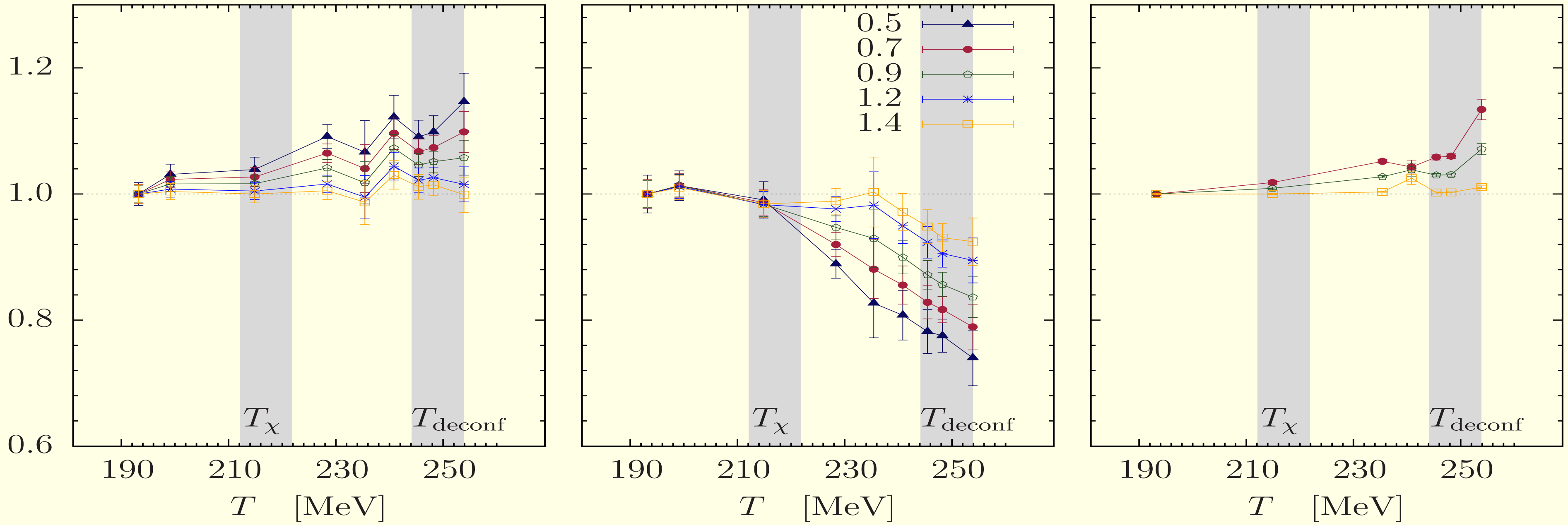
$\Longrightarrow$ Not shown: $m_\pi \simeq 320, 480$ GeV look similar.

Renormalized propagator ratios vs. T at fixed lowest non-zero momenta.

Renorm. scale $\mu = 2.5$ GeV; T_{min} = smallest available T.

$$R_{T,L}(q,T) = D^{ren}_{T,L}(q,T)/D^{ren}_{T,L}(q,T_{\text{min}}), \qquad R_G(q,T) = G^{ren}(q,T)/G^{ren}(q,T_{\text{min}})$$

trans. gluon long. gluon ghost



T_χ, T_{deconf} – pseudocritical chiral and deconfinement temperature

[tmfT Collaboration: F. Burger et al., arXiv:1102.4530 (2011), revised PRD 87 (2013) 074508]

$\Longrightarrow$ Characteristic low-momentum behaviour in crossover region.

$\Longrightarrow$ To be used as input for DS (or FRG) equations to predict $\langle\bar{\psi}\psi\rangle$ etc.

5. Conclusion and outlook

- Lattice results support "decoupling solution" as long as we assume approach

$$F_U(g) \to \text{Global Min.}$$

  Alternative: find copies with lowest non-trivial FP eigenvalue.

  $\Longrightarrow$ Ghost dressing fct. gets IR-enhanced, gluon prop. slightly suppressed.

  [Sternbeck, Müller-Preussker, arXiv:1211.3057 [hep-lat]]

- Gribov effects turn out to be important for the IR asymptotic behavior. For pure LGT simulated annealing + $\mathbb{Z}(N)$ flips ("bc FSA") provides (decoupling) solution with weak finite-size effects.

- Continuum limit can be consistently reached within the non-perturbatively and phenomenologically important range around 1GeV.

- Full QCD results will allow to tune DS and FRG truncations and provide input into Bethe-Salpeter or Faddeev Eqs.

- Basic debate "scaling" versus "decoupling" solution still continues, but without strong consequences for phenomenological applications !

- Longitudinal and transversal progagators at $T > 0$ are important indicators for the phase structure !

Alternative approach to solve the Gribov problem ?

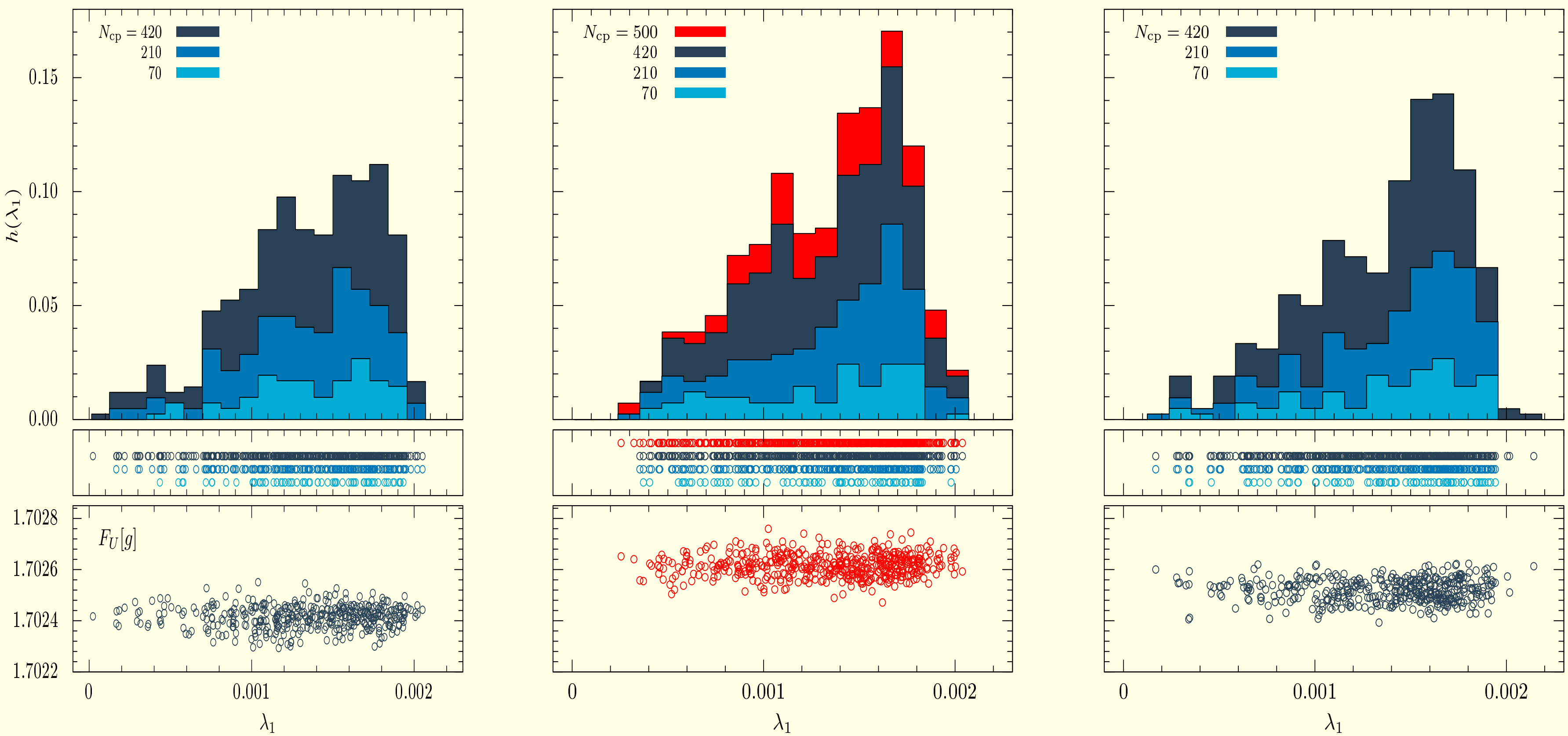
[Sternbeck, Müller-Preussker, Phys. Lett. B726 (2013) 396, arXiv:1211.3057]

Search for the copy with the smallest first non-trivial eigenvalue λ_1 of the FP operator.

- Study distributions for λ_1.
- Study correlation λ_1 with gauge functional $F_U[g]$.
- Find gluon and ghost propagators for lowest-λ_1 copies (“lc”).
- Compare with “fc” and “bc” results related to random and best $F_U[g]$ copies.

Here $SU(2)$ pure gauge theory: $\beta = 2.30$, lattice size 56^4.

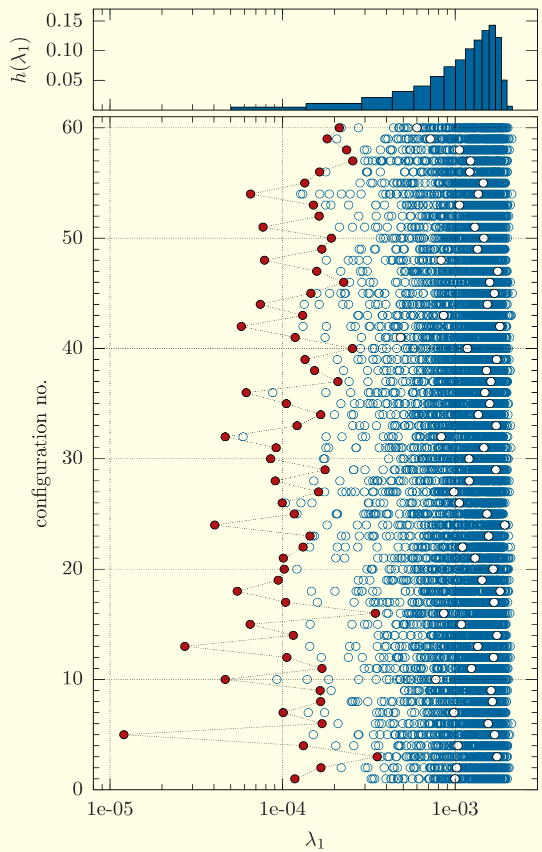
Distributions for single gauge field configurations for various numbers of copies:



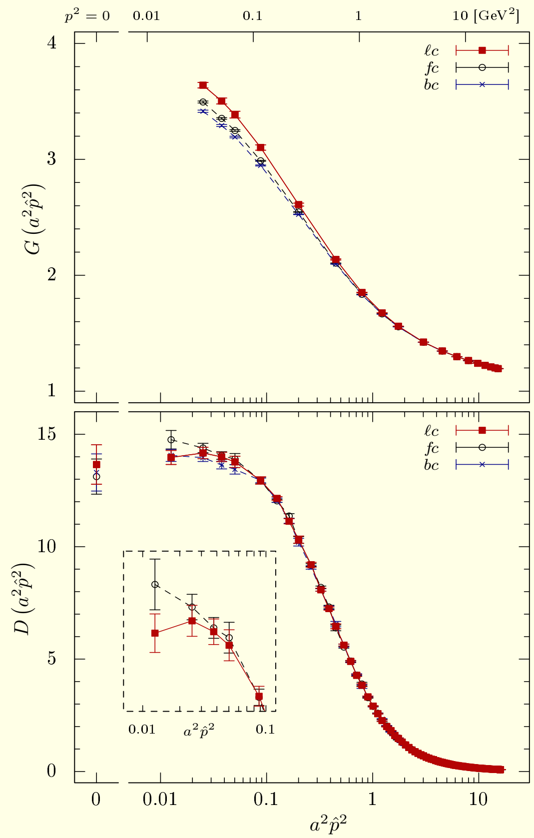
$\Longrightarrow$ Need many copies in order to see the tail at smallest λ_1.

$\Longrightarrow$ No correlation seen between λ_1 and $F_U[g]$.

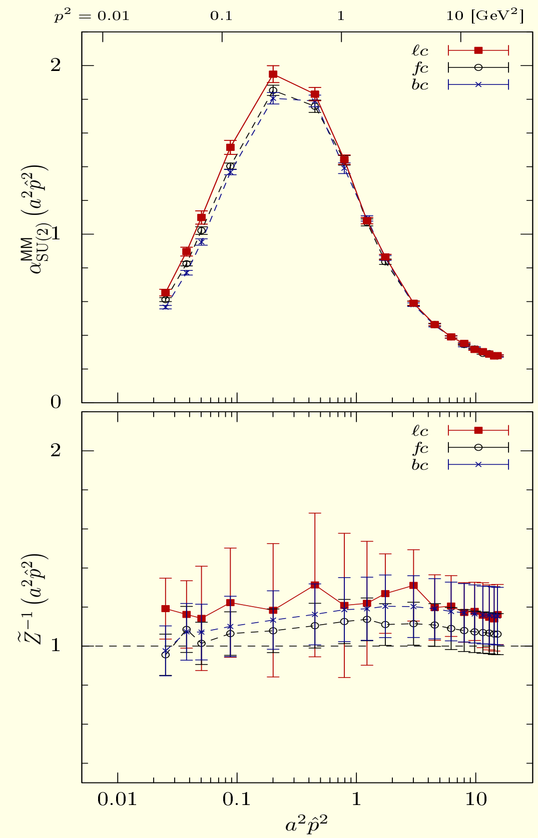
λ_1 distribution



Ghost dress. fct.
Gluon propagator



running coupling $\alpha_s(p)$
ghost-gluon-vertex fct.



$\Longrightarrow$ Ghost dress. fct. slightly more IR singular, gluon prop. slightly suppressed.

$\Longrightarrow$ Effect still small, however it works in the right direction.

$\Longrightarrow$ Can the IR scaling solution be reached on the lattice ? Remains open.