Nonperturbative defects in the dense QCD

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M/ Voloshin A.G. 2008

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Plan of the talk

1. Introduction

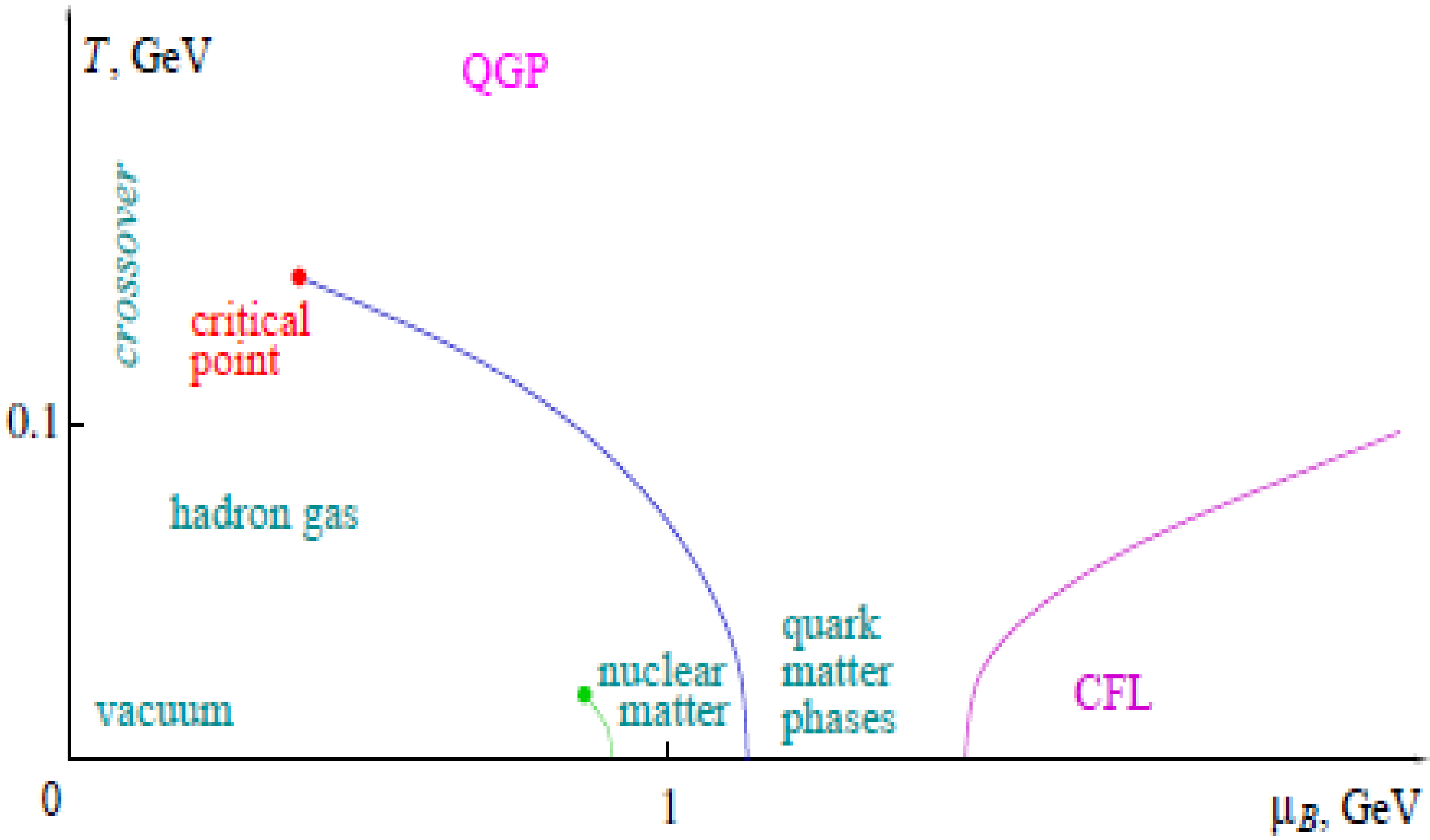
2. Nonabelian strings in the color-flavor locking phase.

3. Worldsheet action and monopoles.

4. Domain walls in the dense QCD

5. Decay of the domain walls. Decay of the nonabelian Strings

6. Conclusion



T, GeV
QGP
crossover
critical point
0.1
hadron gas
vacuum
nuclear matter
quark matter phases
CFL
0
1
μ_B, GeV

Color-Flavor locking phase in QCD

- The one-gluon exchange interaction amounts to the nontrivial diquark condensates of the Cooper type

$$X =< q_L q_L >, \qquad Y =< q_R q_R >$$

  with both in triplet with respect to the color and flavor groups.

- Under the flavor-color transformations from $SU(3)_L \times SU(3)_R \times SU(3)_C$ group the condensates transform as follows

$$X \to LXC^T \qquad Y \to RYC^T$$

  Hence the group $SU(3)_L \times SU(3)_R$ is broken down to the diagonal subgroup $SU(3)_{L+R}$ The abelian axial and baryonic factors $U(1)_A$ and $U(1)_B$ are spontaneously broken by the condensates yielding two Goldstone modes.

Degrees of freedom

- Hence the symmetry breaking pattern in the CFL phase implies the presence of the 18 Goldstone degrees of freedom. The eight degrees of freedom corresponding to the vector-like fluctuations are eaten by gluons via Higgs mechanism. Hence we end up with two mesons corresponding to the broken abelian symmetries and eight Goldstones corresponding to the axial-like fluctuations of the condensates X and Y.
- It is useful to construct the gauge invariant order parameter $\Sigma = XY^{+}$ which has the transformation properties similar to the ordinary chiral condensate in QCD.

- The order parameter Σ is rotated under the axial singlet $U(1)_A$ symmetry. The eight mesons are parameterized by the phase of the matrix Σ

$$\Sigma = |\Sigma| exp(i \frac{t^a \pi^a}{f_\pi})$$

where

$$|\Sigma| \propto \frac{\mu^4 \Delta^2}{g^2}$$

- The CFL mesons have to be considered as four-quark states contrary to conventional two-quark states in the zero density QCD. However let us emphasize that the mesons in CFL phase have the same quantum numbers as the mesons in the QCD at $\mu = 0$. All decays couplings of the mesons f_π are proportional to μ .

The electromagnetic $U(1)_Q$ is broken by the condensate however it is essential that the linear combination of the photon A_μ and gluon G^8_μ

$$\tilde{A} = cos\theta A + sin\theta G^8$$

remains massless and one can consider the charges with respect to $U(1)_{\tilde{Q}}$.In the CFL phase the mixing angle θ between the photon and the gluon component is small therefore the massless field is almost the original photon. The gluons turn out to be charged with respect to the massless combination of the gluon and photon $\tilde{A}$.

- The lightest Goldstone boson in this phase is isosinglet with the quantum number of $\bar{s}s$ that is can be considered as the mixture of η and η' mesons . The mass of the lightest state behaves as
$$m_{\bar{s}s}^2 \propto \frac{\Delta^2 m_u m_d}{\mu^2}$$
It total there are five mesons with masses of order m_s and four mesons with masses $\sqrt{m_{u,d} m_s}$. The Goldstone corresponding to the broken $U(1)_B$ remains massless.

Near the phase transtion is it useful to introduce the order parameter

$$\Phi^{kC} \sim \varepsilon_{ijk}\,\varepsilon_{ABC}\left(q_{\alpha}^{iA}\,q^{jB\alpha} + \bar{q}^{iA\dot{\alpha}}\,\bar{q}_{\dot{\alpha}}^{jB}\right),$$

Landau-Ginzburg description

$$\begin{aligned} S &= \int \mathrm{d}^4x \left\{ \frac{1}{4g^2}\left(F_{\mu\nu}^{a}\right)^2 + 3\,\mathrm{Tr}\,(\mathcal{D}_0\Phi)^{\dagger}(\mathcal{D}_0\Phi) \right. \\ &+ \left. \mathrm{Tr}\,(\mathcal{D}_i\Phi)^{\dagger}(\mathcal{D}_i\Phi) + V(\Phi) \right\} \end{aligned}$$

with the potential

$$V(\Phi) = -m_0^2\,\mathrm{Tr}\,(\Phi^{\dagger}\Phi) + \lambda\left(\left[\mathrm{Tr}\,(\Phi^{\dagger}\Phi)\right]^2 + \mathrm{Tr}\left[(\Phi^{\dagger}\Phi)^2\right]\right),$$

The parameters of the LG Lagrangian are

$$m_0^2 = \frac{48\pi^2}{7\zeta(3)} T_c(T_c - T), \qquad \lambda = \frac{18\pi^2}{7\zeta(3)} \frac{T_c^2}{N(\mu)},$$

The critical temperature

$$T_c \sim \frac{\mu}{(g(\mu))^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g(\mu)}\right) \ll \mu .$$

$$m_0^2 \sim T_c(T_c - T), \qquad \lambda \sim \frac{T_c^2}{\mu^2} \ll 1,$$

The Anzatz for the solution

$$\Phi(r \to \infty, \alpha) = v\,\mathrm{diag}\left(e^{i\alpha}, 1, 1\right),$$

$$A_i(r \to \infty) = \frac{\varepsilon_{ij} x^j}{r^2} \mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right),$$

Nonabelian strings-very common phenomena overlooked for the decades!

Found for the first time in SUSY QCD Hanany-Tong 2004,Konishi Et al, Shifman-Yung

Key features — tension is N times smaller than ANO strings

Very nontrivial worldsheet theory- orientational modes from

Particular pattern of the symmetry breaking on the string

G/H residual group amounts to the additional degree of freedom

A lot of indications for the existence of such strings in lattice QCD studies (Zakharov-A.G 2008)

To take into account the orientational modes introduce

$$\Phi = e^{i\alpha/3}\frac{1}{3}\left[2\phi_2+\phi_1\right] + e^{i\alpha/3}(\phi_1-\phi_2)\left(n\cdot n - \frac{1}{3}\right),$$

$$A_i = \left(n\cdot n - \frac{1}{3}\right)\varepsilon_{ij}\frac{x_j}{r^2}f(r),$$

obeying the equations

$$f'' - \frac{f'}{r} - \frac{g^2}{3}(1+2f)\,\phi_1^2 + \frac{g^2}{3}(1-f)\,\phi_2^2 = 0,$$

$$\phi_1'' + \frac{\phi_1'}{r} - \frac{1}{9}\frac{(1+2f)^2}{r^2}\phi_1 - \frac{1}{2}\frac{\partial V}{\partial \phi_1} = 0,$$

$$\phi_2'' + \frac{\phi_2'}{r} - \frac{1}{9}\frac{(1-f)^2}{r^2}\phi_2 - \frac{1}{4}\frac{\partial V}{\partial \phi_2} = 0,$$

Nonabelian strings in the dense matter first discussed in Balachandran et.at 2006, Mikhailov-A.G. 2007

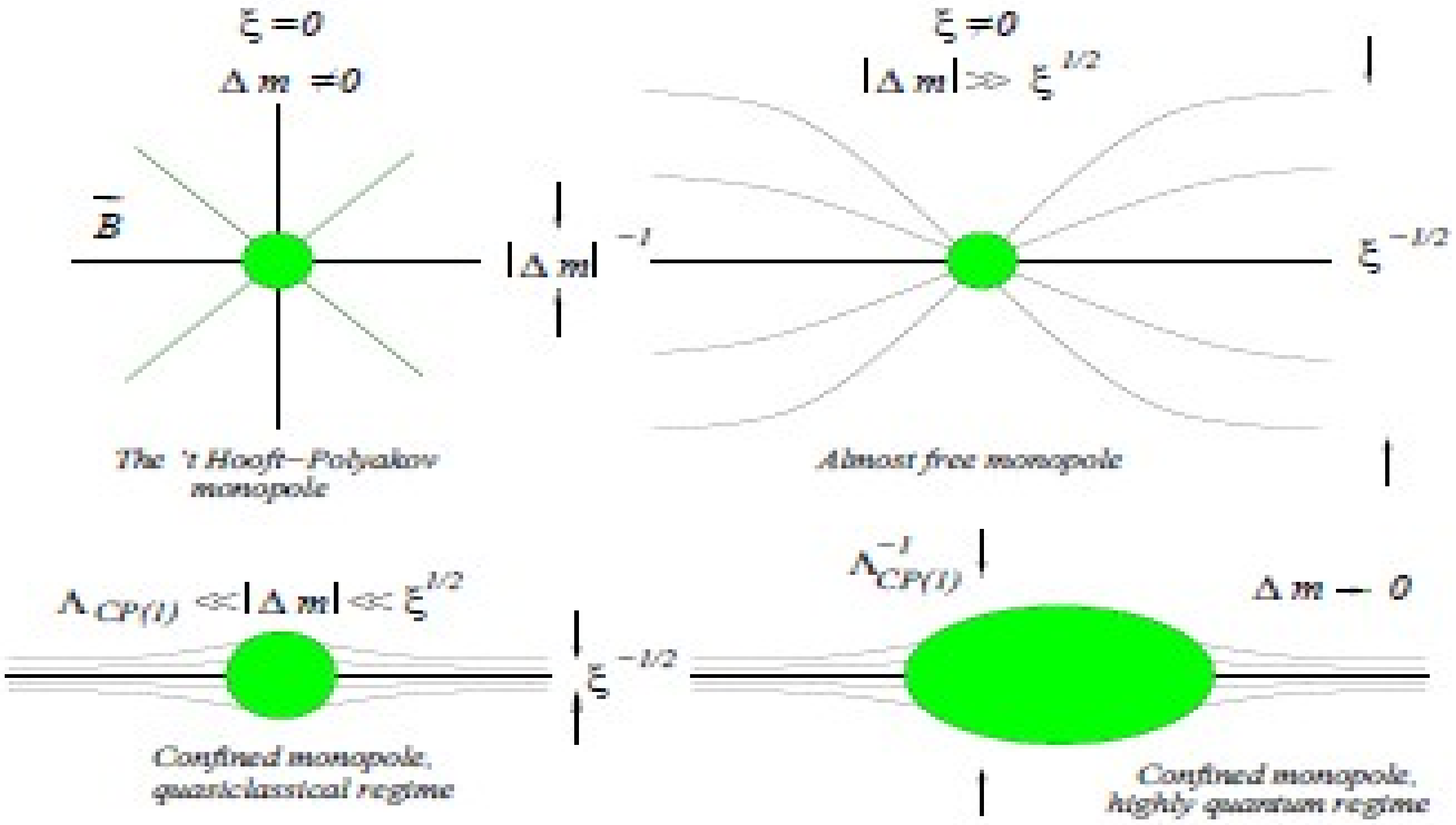
The tension of the string diverges logarithmically

$$S_{NC} = T_0 \int d^4x\, \mathcal{L}_{NC}\,, \qquad T_0 = 2\pi v^2 \ln(L m_0)\,,$$

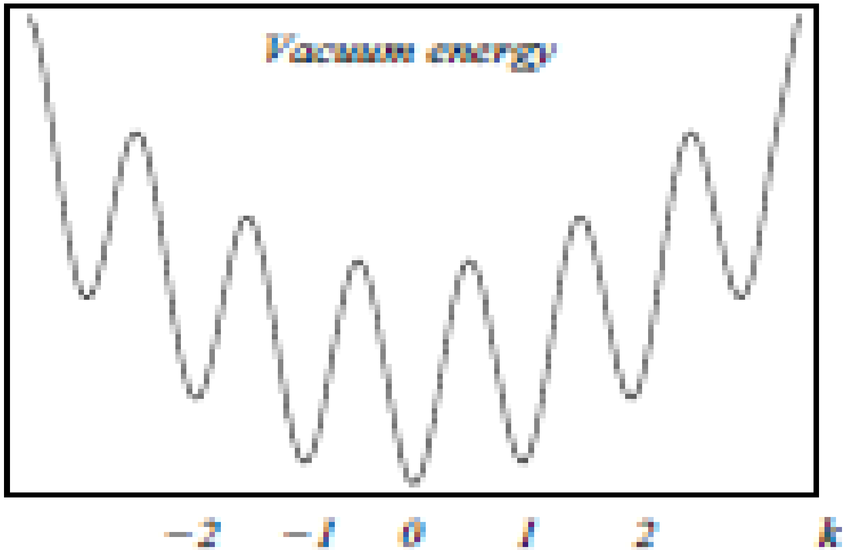
$$S_{CP(2)} = 2\beta \int dt\, dx_3 \left\{ 3 \left| \mathcal{D}_0 n^A \right|^2 + \left| \mathcal{D}_3 n^A \right|^2 \right\}\,,$$

The worldsheet action

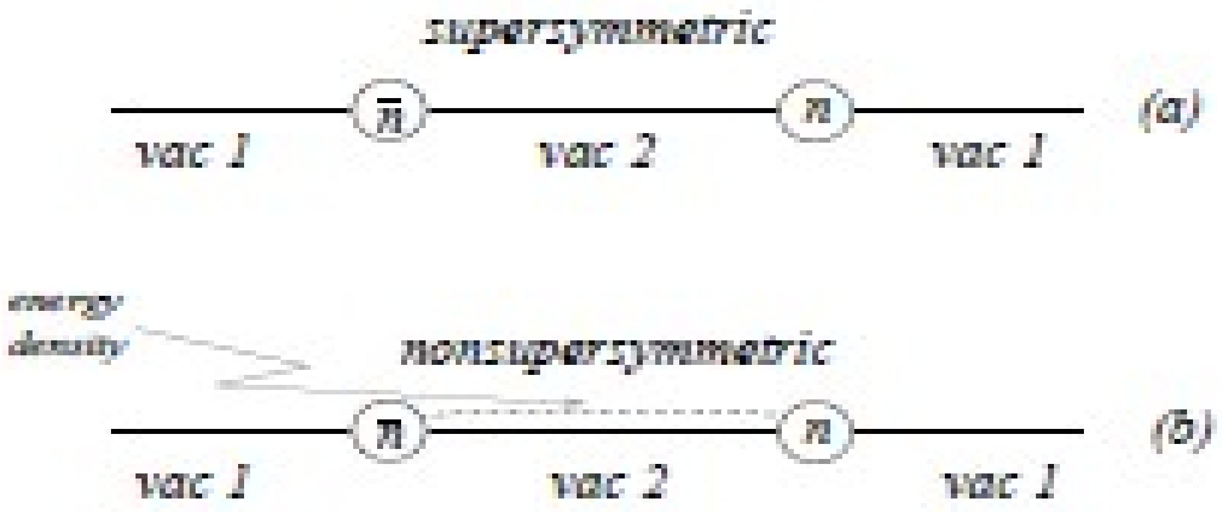
$$\beta = \frac{2\pi}{g^2} \int_0^\infty r dr \left\{ \left(\frac{d}{dr} \rho(r) \right)^2 + \frac{1}{r^2} f^2 (1-\rho)^2 + g^2 \left[\frac{\rho^2}{2} (\phi_1^2 + \phi_2^2) + (1-\rho)(\phi_2 - \phi_1)^2 \right] \right\}.$$



Tong 2004, Shifman-Yung 2004



$$E_k \sim N\,\Lambda^2_{CP}\left\{1 + \text{const}\left(\frac{2\pi k}{N}\right)^2\right\}\,, \quad k = 0\ldots, N-1\,.$$



$$4\pi \times \mathrm{diag}\,\frac{1}{2}\,\{\,1,\,-1,\,0\}\ ,$$

Shifman-Yung-A.G. 2004

The simplest case with one nonzero mass

$$m^u = m^d = 0 \qquad m^s \neq 0$$

$$\delta V(\Phi) = \epsilon \left\{ \Phi_u^\dagger \Phi^u + \Phi_d^\dagger \Phi^d \right\} ,$$

$$\epsilon = \frac{48\pi^2}{7\zeta(3)} \frac{m_s^2}{4\mu^2} T_c^2 \ln \frac{\mu}{T_c} .$$

$$\langle \Phi \rangle = \mathrm{diag}\left(v_u, v_u, v_s\right) ,$$

$$v_u^2 = \frac{m_0^2 - 2\epsilon}{8\lambda}, \qquad v_s^2 = \frac{m_0^2 + 2\epsilon}{8\lambda} .$$

The mass of the strange quark induces the potential

$$V_{CP} = \omega \int dt\, dx_3\, (3n_3^2 - 1)\,,$$

$$\omega = \frac{2\pi}{3}\,\epsilon \int_0^\infty r dr\, (\phi_2^2 - \phi_1^2) \sim \epsilon\, \frac{v^2}{m_0^2}\,,$$

If all quarks are massive

$$m_u = m_d \ll m_s.$$

$$m_\pi \sim \sqrt{m_u m_s}\, \frac{T_c}{\mu}\,.$$

One can introduce the topological term on the string worldsheet

$$\mathcal{L}_\theta = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu A^\nu = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu\left(n_a\partial^\nu n^a\right) .$$

The direct calculation shows that the bulk and worldsheet topological terms coincide

$$\theta_{3+1} = \theta_{1+1} ,$$

The topological terms in the dense QCD can be seen by the nonabelian string!

1. Monopoles in the dense QCD are in highly quantum regime!

2. The worldsheet theory of the nonabelian string is strongly coupled!

3. The mass of the monopole localized at the string is of order worldsheet nonperturbative scale

4. They are confined!Only monopole-antimonopole pairs.

Shifman-Yung-A.G. 2011

Domain walls in the dense QCD(Son,Stephanov,Zhitnitsky ..

- The effective potential for the lightest state is generated by masses and by instantons and reads as

$$V(\phi_{\bar{s}s}) = m^2_{\bar{s}s}(1 - cos\phi_{\bar{s}s}) + cm_s\mu\Delta^2 cos(\phi_{\bar{s}s} - \theta)$$

  where the second term is induced by the instantons and is suppressed at large μ. The form of the potential implies the existence of the domain walls of the axion type which interpolate between $\phi_{\bar{s}s} = 0$ at $z = -\infty$ and $\phi_{\bar{s}s} = 2\pi$ at $z = \infty$ (Son, Stephanov,Zhitnitsky 02). The tension of the metastable domain wall is proportional to the density and the BCS gap.
- There are also strings which are analogue of the "axion" string

- Consider first the domain walls built from π^0 mesons in the following parametrization

$$\sigma = f_\pi \cos\chi \cos(\theta) \qquad \pi^1 = f_\pi \sin\chi \cos(\phi)$$

$$\pi^0 = f_\pi \cos\chi \sin(\theta) \qquad \pi^3 = f_\pi \cos\chi \sin(\phi)$$

- The π^0 wall solution reads as

$$\chi = 0 \qquad \theta = 4 \arctan e^{m_\pi z}$$

If we consider the small fluctuations around the wall the are two bound states around wall solution.

- The first discrete mode corresponds to the zero mode while the second

$$E^2 = k_x^2 + k_y^2 - 3m_\pi^2$$

amounts to instability
- In the magnetic field the unstable discrete mode gets shifted to

$$E^2 = (2n+1)eB - 3m_\pi^2$$

that is above the critical value

$$B_0 = \frac{3m_\pi^2}{e} = 1.0 \times 10^{19}\, G$$

the wall is locally stable.

- The tension of the wall is

$$T = 8f_\pi^2 m_\pi$$

and the anomalous density of the baryonic charge at the wall

$$\rho_{Bar} = \frac{eB}{2\pi}$$

Hence the energy per the baryon number at the wall in

$$\frac{E}{N_{Bar}} = \frac{16\pi f_\pi^2 m_\pi}{eB}$$

- The stack of the meson domain walls is energetically favorable with respect to the nuclear matter above

$$B_1 = \frac{16\pi f_\pi^2 m_\pi}{e m_N} = 1.1 \times 10^{19} G$$

- Sunmmarizing ; walls are not topological since can be unwound inside $SU(2)$ flavor group and are absolutely unstable in the absence of the magnetic field. At $B > B_0$ the wall becomes locally unstable while at $B > B_1$ it is favorable to organize the nuclear matter as the stack of the pionic domain walls (Son, Stephanov 08).
- The density of the baryon charge has the form

$$\rho_{bar} \propto B \qquad Q_{bar} = \int d^3x((U^{-1}dU)^3 + U^{-1}dUF)$$

  which can be though about as the liquid of the Skyrmions.
- Note that for infinite wall the second term in the baryon density works while for the finite wall the density follows from the first one (Son, Stephanov 08).

- The key feature of the decay of the wall in the dense QCD - the current along the effective string at the boundary of the hole
- The current along the string is determined from the CS term and identification of the chemical potential with the flavor gauge field (Son, Zhitnitsky 05)

$$J_{string} \propto \mu$$

- The effect of current along the hole boundary is twofold. Not all charges escape the wall and the magnetic field is partially screened
- There is also induced angular momentum on the wall in the dense QCD (Son, Zhitnitsky 05)

$$M \propto \mu$$

Conclusions

1. There are nonabelian strings in the CFL phase

2. There are confined monopoles localized at the Nonabelian string

3. There are stable domain walls in the external Magnetic field

4. The decaying domain walls and strings provide the pattern of CME