More on holographic duals for QCD, towards finite chemical potential

Anastasia Golubtsova[1]

based on work with
Irina Aref'eva (MI RAS, Moscow) and Giuseppe Policastro (ENS, Paris)

arXiv:1803.06764

also work in progress with Nguyen Hoang Vu

(1) BLTP JINR, Dubna

Theory of hadronic matter under extreme conditions
January 23

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Theory of hadronic matter under extreme conditions
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Outline

DW/CFT dualities Itzhaki et. al.'98, Boonstra et. al.'98;Skenderis'99

- $AdS \Leftrightarrow DW$, CFT$\Leftrightarrow$ QFT, AdS isometry group $\Leftrightarrow$ Poincaré isometry group of DW
- a restoration of the conformal symmetry only at UV and/or IR fixed points

$$S = \int dx^5 \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] + S_{YH}.$$

The domain wall solution

$$ds^2 = e^{2\mathcal{A}(r)} \eta_{ij} dx^i dx^j + dr^2, \quad \phi = \phi(r)$$

- The scale factor $e^{\mathcal{A}}$ – measures the field theory energy scale
- The scalar field e^{ϕ} – the running coupling λ
- The β-function

$$\beta = \frac{d\lambda}{d \log E} = \frac{d\phi}{d\mathcal{A}}$$

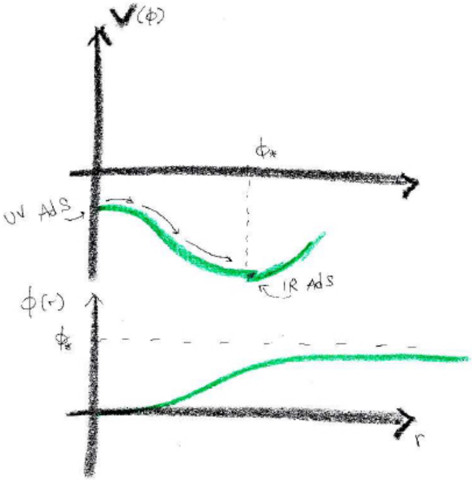

Figure: by Francesco Nitti

Improved holographic QCD Gursoy,Kiritsis' 07, Gubser'08

For asymptotically AdS UV $\quad \lambda \to 0 \quad V(\lambda) \; = \; V_0 + v_1\lambda + v_2\lambda^2 + \dots$

For confinement in the IR $\quad \lambda \to \infty \quad V(\lambda) \; \sim \; \lambda^Q (\log\lambda)^P$

- $V(\phi)$ from IHQCD model, Kiritsis et al'07'11'14'17'18

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  - :) has good behaviour in the IR-limit (can study conformal anomalies, apply to deconfined phase of QCD) Policastro'15
  - :(UV-fixed point is not the AdS

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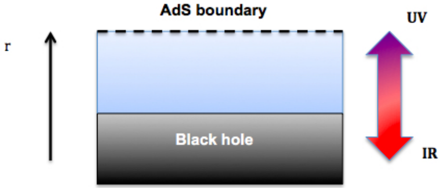
  - :(UV-fixed point is not the AdS
- $V = \sum C_i e^{k_i \phi}$, in particular, $V(\phi) = C_1 e^{k_1\phi} + C_2 e^{k_2\phi}$ – ?

RG flow at finite temperature

Thermal gas solution

$$ds^2 = e^{2A(r)}\eta_{ij}dx^i dx^j + dr^2, \quad \phi = \phi(r).$$



The black hole

$$ds^2 = e^{2A(r)}\left(-f(r)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{dr^2}{f(r)}, \quad f(r) = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}.$$

Gubser's bound for singular solutions (2000)

$$V(\phi_h) < 0, \quad V(\phi_h) \leq V(\phi_{UV}).$$

Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures $T > 300$MeV and the equation of state $\sim E = 3P$ (a traceless conformal energy-momentum tensor).

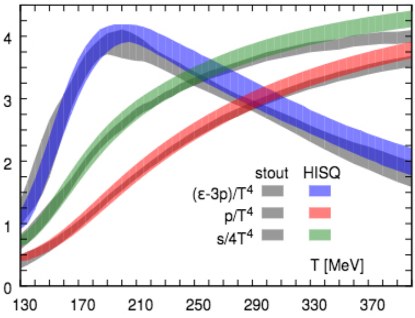


Figure: The comparison of the HISQ/tree and stout results for the trace anomaly, the pressure, and the entropy density

Pic. from Bazavov et al.'14

Outline

The action reads

$$S = \frac{1}{2\kappa^2}\int d^4x \int du\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right) - \frac{1}{\kappa^2}\int_{\partial} d^4x\sqrt{-\gamma},$$

$V(\phi) = C_1 e^{2k_1\phi} + C_2 e^{2k_2\phi}$, C_i, k_i, $i = 1, 2$ are some constants.

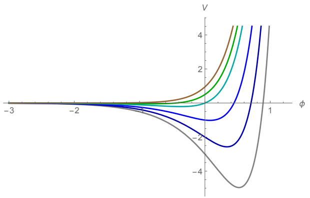


Figure: The behaviour of the potential $V(\phi)$ for $C_1 < 0$, $C_2 > 0$.

$$k_1 = k, \quad k_2 = \frac{16}{9k}.$$

The general solution

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2$$

$$\phi = -\frac{9k}{9k^2-16} \log F_1 + \frac{9k}{9k^2-16} \log F_2$$

with F_1 and F_2 given by

$$F_1 = \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1 |u - u_{01}|), \qquad \mu_1 = \sqrt{\left|\frac{3E_1}{2}\left(k^2 - \frac{16}{9}\right)\right|},$$

$$F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2 |u - u_{02}|), \qquad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\left(\left(\frac{16}{9}\right)^2 \frac{1}{k^2} - \frac{16}{9}\right)\right|},$$

where $0 < k < 4/3$ and u is positive and $u > u_{01}$.
Moreover, one has the constraint

$$E_1 + E_2 + \frac{2(\alpha^1)^2}{3} = 0, \quad E_1 < 0, \quad E_2 > 0.$$

Constraints

$$E_1 + E_2 + \frac{2(\alpha^1)^2}{3} = 0.$$

1. $\alpha^1 = 0$ Vacuum solutions, Poincaré invariant, $|E_1| = |E_2|$
2. $\alpha^1 \neq 0$ Non-vacuum ones, no Poincaré invariance $|E_1| \neq |E_2|$

- Conditions from the $V(\phi)$: $C_1 < 0, C_2 > 0, 0 < k < 4/3$.
- Constants of integration $u_{02} < u_{01}$

$$\begin{aligned} \text{left:} &\quad u < u_{02} \\ \text{middle:} &\quad u_{02} < u < u_{01} \\ \text{right:} &\quad u > u_{01} \end{aligned}$$

- The degenerate case with $u_{01} = u_{02} = u_0$,

$$\begin{aligned} \text{left:} &\quad u < u_0 \\ \text{right:} &\quad u > u_0. \end{aligned}$$

Holographic RG flow Kiritsis et al.'0812.0792

Solution in domain wall coordinates

$$ds^2 = e^{2\mathcal{A}(w)}\left(-f(w)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{dw^2}{f(w)}, \quad f = 1 \quad T = 0$$

$\phi(w)$, $\lambda = e^{\phi}$ – the running coupling, A $= e^{\mathcal{A}}$.
The β-function

$$\beta(\lambda) = \frac{d\lambda_{QFT}}{d\log E} = \frac{d\lambda}{d\mathcal{A}}$$

The X, Y-variables:

$$X(\phi) = \frac{\beta(\lambda)}{3\lambda}, \quad Y(\phi) = \frac{1}{4}\frac{g'}{\mathcal{A}'}, \quad g = \log f,$$

$$\frac{dX}{d\phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right),$$

$$\frac{dY}{d\phi} = -\frac{4}{3}\left(1 - X^2 + Y\right)\frac{Y}{X}.$$

Outline

Behaviour of solutions $u_{01} \neq 0$, $\alpha^1 = 0$

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-dt^2 + dy_1^2 + dy_2^2 + dy_3^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2,$$

$$\begin{aligned} F_1 &= \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1(u-u_{01})), \quad F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2(u-u_{02})), \\ E_1 &= -E_2, \quad E_1 < 0, \quad E_2 > 0, \quad \mu_2 = \frac{4}{3k}\mu_1. \end{aligned}$$

The dilaton

$$\phi = \frac{9k}{9k^2-16} \log \frac{F_2}{F_1}$$

and its potential

$$V = C_1 e^{2k\phi} + C_2 e^{32\phi/(9k)} = C_1 \left(\frac{F_2}{F_1}\right)^{\frac{18k^2}{9k^2 16}} + C_2 \left(\frac{F_2}{F_1}\right)^{\frac{32}{9k^2-16}} .$$

The dilaton

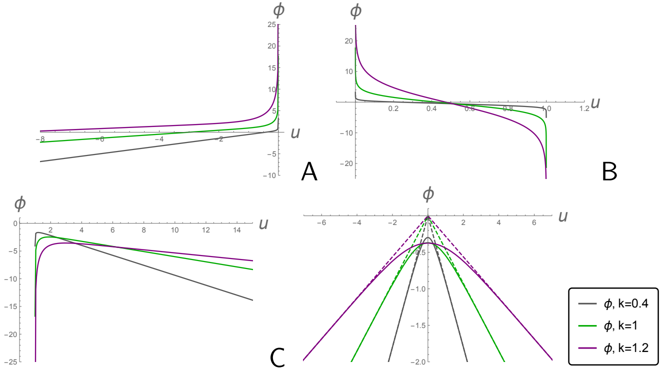


Figure: A) the dilaton for $u < u_{02}$,B) the dilaton for $u_{02} < u < u_{01}$ C) the dilaton for $u > u_{01}$ D) $u > u_{01}$, $u_{01} = u_{02}$. For all $E_1 = E_2 = -1$, $C_1 = -C_2 = -1$, $k = 0.4, 1, 1.2$.

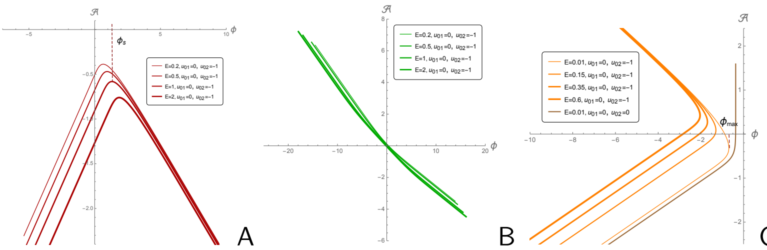


Figure: The behaviour of $\mathcal{A}$ as a function of ϕ for the vacuum solutions, we fix values of $u_{01} = 0$ and $u_{02} = -1$ and varying $|E_1| = |E_2|$, denoted as E: **A)** the dilaton for $u < u_{02}$,B) the dilaton for $u_{02} < u < u_{01}$, C) by organge $u > u_{01}$, $u_{01} = 1$; by the brown curve $u_{01} = u_{02}$ and $E = 0.01$.

Boundaries for $u_{01} \neq u_{02}$

The middle solution $u_{02} < u < u_{01}$ (conformally flat)

- IR $u \to u_{02} + \epsilon$ $ds^2 \sim z^{\frac{18k^2}{64-9k^2}} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
  $z \sim \frac{64-9k^2}{4(16-9k^2)} (u - u_{02})^{\frac{64-9k^2}{4(16-9k^2)}}$, $\phi \sim -\frac{36k}{64-9k^2} \log z \to +\infty.$
- UV $u \to u_{01} - \epsilon$ $ds^2 \sim z^{\frac{8}{9k^2-4}} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
  $\phi \sim \frac{9k}{4-9k^2} \log z \to -\infty$, $z \sim \frac{16-9k^2}{9k^2-4} (u - u_{01})^{\frac{4-9k^2}{16-9k^2}}$.

The right solution $u > u_{01}$ (conformally flat)

- UV $u \to u_{01} + \epsilon$ the same as at $u \to u_{01} + \epsilon$
- IR $u \to +\infty$ $ds^2 \sim z^{2/3} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
  $\phi \sim \log z \to -\infty$

Boundaries: $u_{01} = u_{02} = u_0$, $\alpha^1 = 0$

- In the UV $u \to u_0$ we obtain the AdS-spacetime

$$ds^2 \sim \frac{1}{z^2}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim 4u^{1/4}.$$

The dilaton is constant in the UV

$$\phi = \frac{9k}{16 - 9k^2} \log \frac{3k}{4} + \frac{9k}{2(16 - 9k^2)} \log \left| \frac{C_1}{C_2} \right| .$$

- In the IR $u \to +\infty$ we obtain the conformally flat spacetime

$$ds^2 \sim z^{2/3}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim e^{-\frac{3\mu_1 u}{4+3k}}.$$

The dilaton in the IR

$$\phi \sim \log z \to -\infty$$

Boundaries: $u_{01} = u_{02}$, $\alpha^1 = 0$

- In the UV $u \to 0$ we obtain the AdS-spacetime

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The dilaton in the IR

$$\phi \sim \log z \to -\infty$$

RG equations at $T = 0$

The domain wall coordinates $dw = F_1^{\frac{16}{9k^2-16}} F_2^{\frac{9k^2}{16-9k^2}} du$.
The running coupling

$$\lambda = e^{\phi} = \left(\frac{F_2}{F_1}\right)^{\frac{9k}{9k^2-16}}.$$

The energy scale

$$\mathsf{A} = e^{\mathcal{A}} = F_1^{\frac{4}{9k^2-16}} F_2^{\frac{9k^2}{4(16-9k^2)}}.$$

The X-function

$$X = \frac{1}{3}\left(\frac{F_2}{F_1}\right)^{\frac{9k}{16-9k^2}} \frac{\lambda'}{\mathcal{A}'}.$$

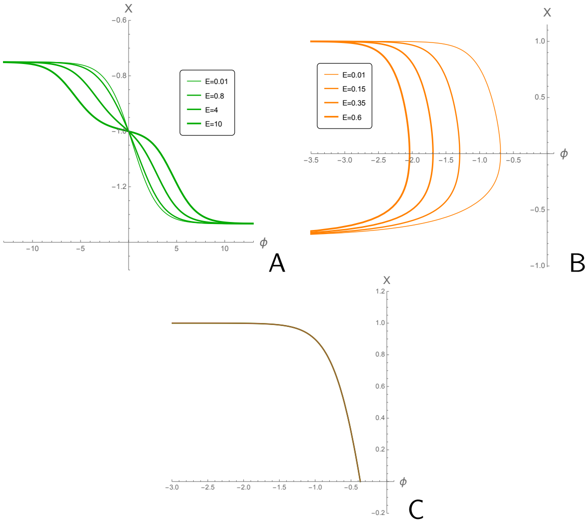

Figure: X on ϕ A) $u_{02} < u < u_{01}$, B) $u > u_{01} \neq 0$ C) $u > u_{01} = u_{02} = 0$

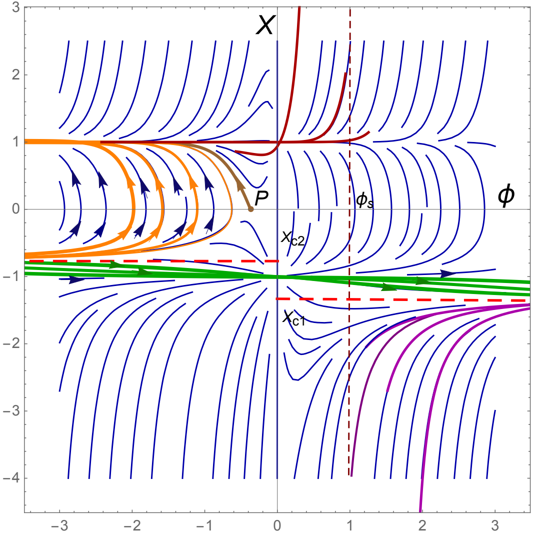

Figure: X on ϕ as a dynamical system with imposed figures of analytic flows

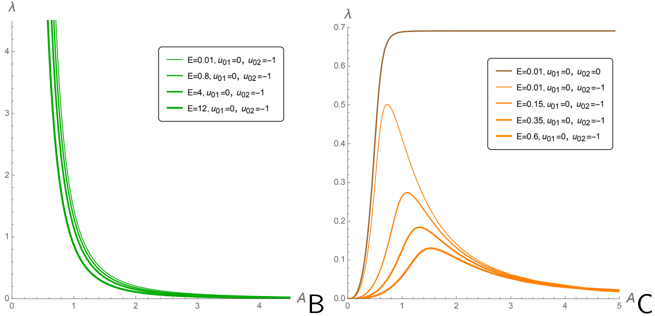

Figure: The dependence of the coupling constant on the energy A on the dilaton plotted using the solutions for $\mathcal{A}$ and ϕ: A) the branch $0 < u < u_{01}$; B) the branch $u > u_{01}$. For all plots $k = 1$, $C_1 = -2$, $C_2 = 2$, different curves on the same plot corresponds to the different values of $|E_1| = |E_2|$, labeled as E on the legends and different u_{01} and u_{01} also indicated on the legends.

Outline

Non-vacuum solutions, black holes

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} \sum_{i=1}^{3} dy_i^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2.$$

$$\phi = -\frac{9k}{9k^2-16} \ln F_1 + \frac{9k}{9k^2-16} \ln F_2,$$

$$F_1 = \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1(u-u_{01})),\ F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2(u-u_{02})),$$

$$\mu_1 = \sqrt{|\frac{3E_1}{2}|} \sqrt{\frac{16}{9} - k^2}, \quad \mu_2 = \sqrt{|\frac{3E_2}{2}|} \frac{4}{3k} \sqrt{\frac{16}{9} - k^2} = \frac{4}{3k} \sqrt{\frac{E_2}{E_1}} \mu_1.$$

$$E_1 + E_2 + \frac{2}{3}(\alpha^1)^2 = 0.$$

Dilaton at boundaries $\alpha^1 \neq 0$

- $u_{01} \neq u_{02}$
  - $u \to u_{01} + \epsilon$
  $\phi_{u \to u_{01} - \epsilon} \sim -\frac{9k}{16-9k^2} \log \left[\sqrt{\frac{C_2 E_1}{C_1 E_2}} \frac{\sinh(\mu_2\,(u_{01}-u_{02}))}{\mu_1\,\epsilon} \right] \to -\infty.$
  - $u \to +\infty$ $\phi_{u\to\infty} \sim -\frac{9k}{16-9k^2} \left[(\mu_2 - \mu_1)\, u + \frac{1}{2} \log |\frac{C_2 E_1}{C_1 E_2}| \right].$
- $u_{01} = u_{02}$

$$\phi|_{u\to\pm\infty} \sim -\frac{9k}{9k^2 - 16} \left[(\mu_2 - \mu_1)\, u + \frac{1}{2} \log |\frac{C_2 E_1}{C_1 E_2}| \right]$$

$$\phi|_{u\to 0} \sim \frac{9k}{16 - 9k^2} \left[\log \left(\frac{\mu_2}{\mu_1} \right) + \frac{1}{2} \log |\frac{C_1 E_2}{C_2 E_1}| \right].$$

- $\mu_1 \quad = \quad \mu_2, \quad E_2 = \frac{6k^2(\alpha^1)^2}{16 - 9k^2}.$
- $\mu_1 \quad > \quad \mu_2$

Black hole solution

The absence of conic singularity

- $\mu_2 = \mu_1$
- $\frac{4}{3\mathcal{C}^{3/2}}\alpha^1\beta = 2\pi$, Hawking temperature: $\frac{1}{\beta} = T = \frac{2}{3\pi}\frac{|\alpha^1|}{\mathcal{C}^{3/2}}$.

The black hole, $u = +\infty$ is the horizon

$$
\begin{aligned}
ds^2 &= \mathcal{C}\,\mathcal{X}\Big(-e^{-2\mu u}dt^2 + d\vec{y}^2\Big) + \mathcal{C}^4\,\mathcal{X}^4 e^{-2\mu u}du^2,\\
\mathcal{X} &= (1-e^{-2\mu u})^{-\frac{8}{16-9k^2}}\,(1-e^{-2\mu(u-u_{02})})^{\frac{9k^2}{2(16-9k^2)}},\\
\mathcal{C} &\equiv 2^{\frac{16}{(16-9k^2)}}\,(3\mu)^{\frac{1}{2}}\,|C_1|^{\frac{8}{2(9k^2-16)}}\left(\frac{C_2}{k}e^{-2\mu u_{02}}\right)^{\frac{9k^2}{4(16-9k^2)}}(16-9k^2)^{-\frac{1}{4}}.
\end{aligned}
$$

$$
\phi = \frac{9k}{9k^2-16}\log\left[\sqrt{\left|\frac{E_1C_2}{E_2C_1}\right|}\frac{\sinh(\mu(u-u_{02}))}{\sinh(\mu u)}\right].
$$

Special case: 5d AdS-Schwarzschild solution

$u_{01} = u_{02}$

$$ds^2 = \mathcal{C}\left(1-e^{-2\mu u}\right)^{-\frac{1}{2}}\left(-e^{-2\mu u}dt^2 + d\vec{y}^2\right) + \mathcal{C}^4\left(1-e^{-2\mu u}\right)^{-2}e^{-2\mu u}du^2,$$

$$\mu = -\frac{4}{3}\alpha^1, \quad \mathcal{C} = (2\sqrt{2})^{1/2}\left(\frac{C_1}{E_1}\right)^{\frac{4}{9k^2-16}}\left(\frac{C_2}{E_2}\right)^{\frac{9k^2}{4(16-9k^2)}}.$$

The dilaton

$$\phi = \frac{9k}{2(16-9k^2)}\log\left|\frac{C_1E_2}{C_2E_1}\right|.$$

The curvature

$$R = -\frac{5\mu^2}{\mathcal{C}^4}.$$

$z = z_h\left(1-e^{-2\mu u}\right)^{\frac{1}{4}}$, $\mathcal{C} = z_h^{-2}$, $ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + d\vec{y}^2 + \frac{dz^2}{f(z)}\right)$,

$f = 1-\left(\frac{z}{z_h}\right)^4$.

Holographic RG-flow at finite T

The scale factor of the domain wall for the finite temperature case is

$$\mathcal{A} = \frac{1}{2}\log(\mathcal{C}) + \frac{1}{2}\log(\mathcal{X})$$

the energy scale A reads

$$\mathrm{A} \equiv e^{\mathcal{A}} = \mathcal{C}^{\frac{1}{2}}\mathcal{X}^{\frac{1}{2}},$$

The change of the coordinate

$$dw = \mathcal{C}^2\,\mathcal{X}(u)^2 e^{-2\mu u}du.$$

$\lambda = e^{\phi}$ – the running coupling, The β-function

$$\beta(\lambda) = \frac{d\lambda}{d\mathcal{A}}$$

The X, Y-variables:

$$X(\phi) = \frac{\beta(\lambda)}{3\lambda}, \quad Y(\phi) = \frac{1}{4}\frac{g'}{\mathcal{A}'}, \quad g = \log f,$$

$$f = e^{-2\mu u}.$$

Holographic RG-flow at finite T

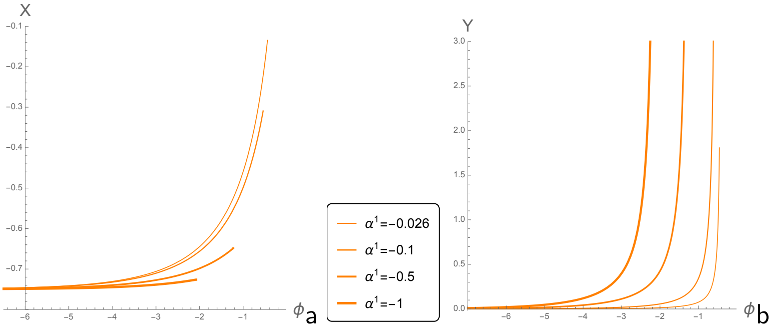

Figure: a) The dependence of the scalar function X on ϕ. b) The dependence of the scalar function Y on ϕ. For both plots $\alpha^1 < 0$, $u > u_{01}$, $C_1 = -C_2 = -2$, $k = 1$, $u_{01} = 0$, $u_{02} = -1$.

The running coupling λ on the energy scale

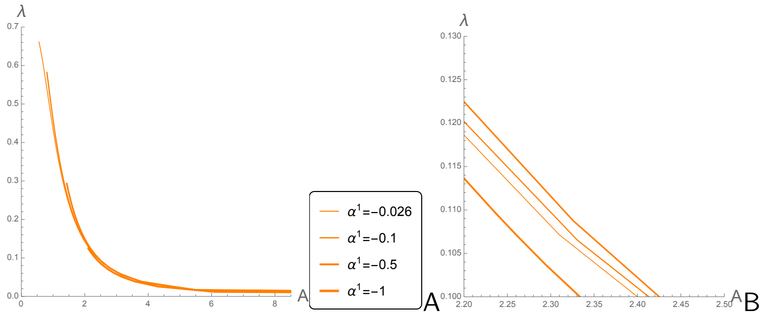

Figure: A) The dependence of λ on the energy scale A= $e^{\mathcal{A}}$. In all cases constants that the potential is fixed with $k = 1$, $C_1 = -2$, $C_2 = 2$ and we vary α^1, $u_{01} = 0$ and $u_{02} = -1$. B) A zoomed region of **A)**.

Free energy through the holographic on-shell action

$$\frac{I_{reg}^{on-shell}}{\beta V_3} = -\left(6\mathcal{A}'(u) + \frac{f'(u)}{f(u)}\right)|_{u=\epsilon}\,.$$

The expansion of $\mathcal{A}$ near $u \sim 0$ $\mathcal{A} \sim -\frac{4}{16-9k^2}\log u + \mathcal{A}_0 + \mathcal{A}_1 u + \ldots$, with

$$\begin{aligned}
\mathcal{A}_0 &= \frac{1}{2}\log\mathcal{C} - \frac{4}{16-9k^2}\log(2\mu) + \frac{9k^2}{4(16-9k^2)}\log(1-e^{2\mu u_{02}}),\\
\mathcal{A}_1 &= \frac{4\mu}{16-9k^2} + \frac{9k^2}{2(16-9k^2)}\frac{\mu}{e^{-2\mu u_{02}}-1}.
\end{aligned}$$

$$\frac{I_{reg}}{\beta V_3} = \frac{1}{16-9k^2}\left(\frac{24}{\epsilon} + \mu\Big(8 - 18k^2 - \frac{27k^2}{e^{-2\mu u_{02}}-1}\Big)\right).$$

The counterterm (Papadimitriou'11)

$$I_{ct} = -\frac{8\gamma}{3}\int d^4x\sqrt{h}e^{k\phi}\,.$$

The asymptotics of ϕ is given by $\phi \sim \frac{9k}{16-9k^2}\log u + \phi_0 + \phi_1 u + \ldots$

$$\phi_0 = -\frac{9k}{16-9k^2}\log\left(\frac{4}{3k}\sqrt{\frac{C_2}{C_1}}\frac{\sinh(-\mu u_{02})}{\mu}\right),$$
$$\phi_1 = -\frac{9k}{16-9k^2}\mu\coth(-\mu u_{02}).$$

$$\mathcal{L}_{ct} = -\frac{24}{16-9k^2}(\frac{1}{\epsilon}+4\mathcal{A}_1+k\phi_1)(1-\mu\epsilon) = -\frac{24}{16-9k^2}\frac{1}{\epsilon}+o(\epsilon)\,.$$

The renormalized action is then

$$\frac{I_{ren}}{\beta V_3} = \frac{I_{reg}+I_{ct}}{\beta V_3} = \frac{1}{2}\left(\mu - \frac{27k^2}{16-9k^2}\sqrt{\Lambda^2+\mu^2}\right),$$

where the UV scale is $\frac{\mu}{\Lambda} = \sinh(-\mu u_{02})$. The difference between the FE of the black brane solution and the FE of the vacuum at $\mu = 0$, is

$$\mathcal{F} \sim -\frac{1}{2}\left(\mu - \frac{27k^2}{16-9k^2}(\sqrt{\Lambda^2+\mu^2}-\Lambda)\right).$$

Free energy through black hole thermodynamics

$$d\mathcal{F} = -s dT.$$

The black brane entropy density reads

$$s = \frac{V_3}{4}\,\mathcal{C}^{\frac{3}{2}},$$

$$s\,T = \frac{V_3}{2\pi}\,\mu, \quad \mathcal{F} = -\int s\,dT = -\frac{V_3}{2\pi}\int_0^{\mu} \frac{\mu'}{T}\frac{dT}{d\mu'}d\mu'.$$

The temperature

$$T = \frac{2}{3\pi Q^{3/2}}|\frac{3}{4}\mu|^{1/4} e^{\frac{27k^2}{4(16-9k^2)}\,u_{02}\,\mu}, \quad T = \frac{\sqrt{2}}{3^{3/4}\pi Q^{3/2}}\mu^{1/4} e^{-\frac{27k^2}{4(16-9k^2)}\operatorname{arcsinh}(\frac{\mu}{\Lambda})}.$$

$$\mathcal{F} = -\frac{V_3}{8\pi}\left(\mu - \frac{27k^2}{16-9k^2}(\sqrt{\Lambda^2+\mu^2} - \Lambda)\right),$$

$u_{02} \to 0$, $\Lambda \to 0$ the free energy $\mathcal{F} = -\frac{V_3}{8\pi}\mu$.

Free energy

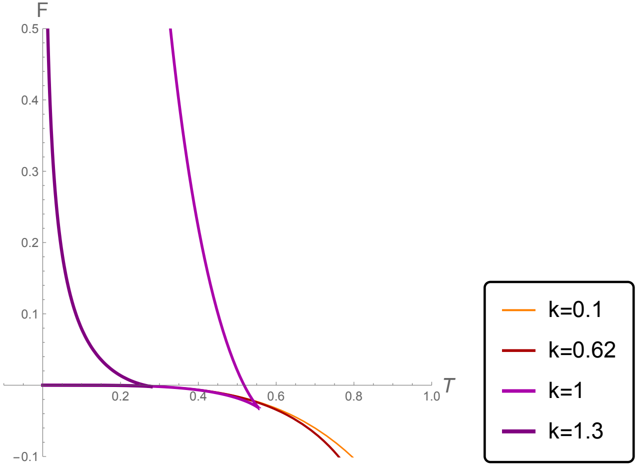

Figure: The dependence of the free energy F on the temperature T for the different shapes of the potential (different k, $C_1 = -2$, $C_2 = 2$).

Outline

Holographic Wilson loops

The expectation value of WL calculated on the gravity side:

$$W[C] = e^{-S_{string}[C]}.$$

The action of the string is the so-called Nambu-Goto action

$$S_{NG} = \int d\sigma^1 d\sigma^2 e^{\frac{2\phi}{3}} \sqrt{|g|},$$

g is the determinant of the induced metric on the worldsheet and σ^1 and σ^2 are coordinates on the worldsheet.

$$g_{\alpha\beta} = G_{MN} \partial_\alpha X^M \partial_\beta X^N,$$

M, N are indices of the embedded space X^N, i.e. $M, N = 0, \ldots, 4$, α, β are indices on the string worldsheet σ^α, $\alpha, \beta = 0, 1$.
We parametrize the worldsheet as

$$\sigma^0 = t, \quad \sigma^1 = y^1, u = u(y^1)$$

Holographic Wilson loops

$$S_{NG} = T \int dx_1 \left(\frac{F_2}{F_1}\right)^{\frac{6k}{9k^2-16}} \sqrt{e^{2\mathcal{A}}(e^{2\mathcal{A}} + e^{8\mathcal{A}} u'^2)}.$$

The equation of motion for the turning point

$$u'^{\,S} = \pm e^{\frac{2\phi}{3}} \frac{e^{-3\mathcal{A}}\sqrt{e^{4\mathcal{A}} - c^2 e^{-\frac{4\phi}{3}}}}{c}$$

hence the distance between quarks is

$$\frac{\ell^S}{2} = \int du \frac{c e^{3\mathcal{A}}}{e^{\frac{2\phi}{3}}\sqrt{e^{4\mathcal{A}} - c^2 e^{-\frac{4\phi}{3}}}}.$$

For the Nambo-Goto action we have

$$S^S_{NG} = T \int du \frac{e^{7\mathcal{A}+\frac{4\phi}{3}}}{\sqrt{e^{4\mathcal{A}+\frac{4\phi}{3}} - c^2}}.$$

Outline

The bottom line

Done

- Vacuum and non-vacuum holographic RG-flows (new solutions) were constructed
- Holographic running coupling mimic QCD
- Holographic RG flows can have AdS fixed points.
- Studies of RG flow based on analytic exact solutions

The bottom line

Done

- Vacuum and non-vacuum holographic RG-flows (new solutions) were constructed
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?

- The form of the potential is the same as in the more complicated model with chemical potential (see Aref'eva&Rannu'18)
- Analysis of confinement-deconfinement phase transition (Polchinski-Strassler model?).
- Holographic c-theorem?

Thank you for attention!