Evolution of holographic Wilson loops in anisotropic quark-gluon plasma

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based on works with
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Outline

1 Introduction

2 Holographic Wilson Loops

3 Backgrounds with broken scale invariance

4 WL in Lifshitz-like backgrounds

The quark-gluon plasma

- QGP is created in very short time $\tau \sim 0.2$fm/c after the collision and it is anisotropic for a short time $0 < \tau < \tau < \tau_{iso}$.
- The time of locally isotropization is about $\tau \sim 2$ fm/c.

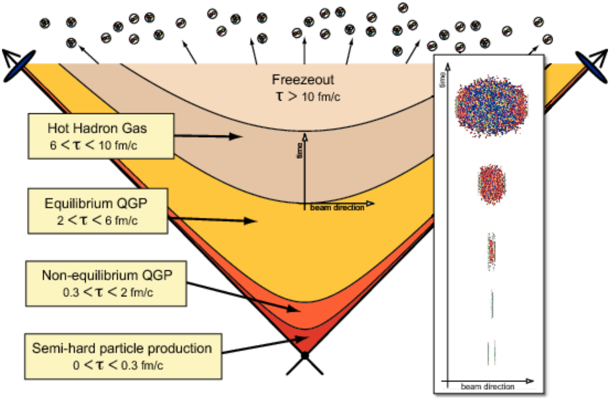

Figure: Pic. by M. Strickland

Holographic duality for QGP

- Quantum field theories with large coupling constant: long distances, strong forces
- Perturbative methods are inapplicable

SOLUTION ?
GAUGE/GRAVITY DUALITY

A correspondence between the gauge theory in D Minkowski spacetime and supergravity in $(\mathrm{D}+1)$ AAdS

't Hooft' 93, Susskind'94.

Example: The AdS/CFT correspondence

J.M. Maldacena, Adv.Theor.Math.Phys. 2, (1998).

- "top-down": supergravity theories in AdS-backgrounds
- "bottom-up": gravity theories with scalar fields, form fields in AdS. etc.

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Holographic dictionary

- $\mathrm{D} = 5$ gravity on AdS $=$ the $(\mathrm{D} = 4)$ strongly coupled theory
- $\mathrm{T} = 0$: AdS vacuum, $\mathrm{T} \neq 0$: black-hole solutions in AdS.
- $\mathrm{D} = 4$ Multiplicity in HIC $=$ BH entropy in AdS_5 Gubster et al.'08
- Thermalization time in $\mathcal{M}^{1,3}$ $=$ BH formation time in AdS^5

PROFIT?

- **Calculations in gravitational backgrounds with certain asymptotics**
- **Reduciton to classical mechanics**

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Holographic Wilson Loops

- The expectation value of WL in the fundamental representation calculated on the gravity side: Maldacena et.al.'98

$$W[C] = \langle \mathrm{Tr}_F\, e^{i\oint_C dx_\mu A_\mu} \rangle = e^{-S_{string}[C]},$$

where C in a contour on the boundary, F – the fundamental representation, S is the minimal action of the string hanging from the contour C in the bulk. The Nambu-Goto action is

$$S_{string} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}, \quad (3.1)$$

with the induced metric of the world-sheet $h_{\alpha\beta}$ given by

$$h_{\alpha\beta} = g_{MN}\partial_\alpha X^M \partial_\beta X^N, \quad \alpha, \beta = 1, 2, \quad (3.2)$$

g_{MN} is the background metric, $X^M = X^M(\sigma^1, \sigma^2)$ specify the string, σ^1, σ^2 parametrize the worldsheet.

- The potential of the interquark interaction

$$W(T, X) = \langle \mathrm{Tr}\, e^{i\oint_{T\times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}.$$

Holographic Wilson Loops

A similar operator to probe QCD is the spatial rectangular Wilson loop of size $X \times Y$ (for large Y)

$$W(X,Y) = \langle \mathrm{Tr} e^{i \oint_{X \times Y} dx_\mu A_\mu} \rangle = e^{-\mathcal{V}(X) Y}$$

defines the so called pseudopotential $\mathcal{V}$:

$$\mathcal{V}(X) = \frac{S_{string}}{Y}.$$

The spatial Wilson loops obey the area law at all temperature, i.e.

$$\mathcal{V}(X) \sim \sigma_s X,$$

where σ_s defines the spatial string tension

$$\sigma_s = \lim_{X \to \infty} \frac{\mathcal{V}(X)}{X}.$$

Anisotropic duals

The AdS/CFT correspondence:

The Field Theory	The Gravitational Background
• the conformal group $SO(D,2)$	• the group of isometries
of a D-dimensional CFT	of AdS_{D+1}
$(t, x_i) \to (\lambda t, \lambda x_i)$, $i = 1, .., d-1$	$ds^2 = r^2 \left(-dt^2 + d\vec{x}^2_{d-1}\right) + \frac{dr^2}{r^2}$

Generalizations?

Lifshitz scaling: $t \to \lambda^{\nu} t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r,$

where ν is the Lifshitz dynamical exponent

Lifshitz metric: $ds^2 = -r^{2\nu} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2_{d-1}$

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Lifshitz-like spacetimes for holography

- Lifshitz-like metrics

$$ds^2 = r^{2\nu}\left(-dt^2 + dx^2\right) + r^2 dy_1^2 + r^2 dy_2^2 + \frac{dr^2}{r^2},$$

$$(t, x, y, r) \to (\lambda^\nu t, \lambda^\nu x, \lambda y_1, \lambda y_2, \tfrac{r}{\lambda}),$$ M. Taylor'08, Pal'09.

The 5d Lifshitz-like metrics, $z = \frac{1}{r^\nu}$

$$\text{Type} - (\mathbf{1}, \mathbf{2}) \quad ds^2 = L^2\left[\frac{\left(-dt^2 + dx^2\right)}{z^2} + \frac{\left(dy_1^2 + dy_2^2\right)}{z^{2/\nu}} + \frac{dz^2}{z^2}\right].$$

$$\text{Type} - (\mathbf{2}, \mathbf{1}) \quad ds^2 = L^2\left(\frac{\left(-dt^2 + dx_1^2 + dx_2^2\right)}{z^2} + \frac{dy^2}{z^{2/\nu}} + \frac{dz^2}{z^2}\right).$$

Type IIB SUGRA, $D3 - D7$-branes: Anisotropic QGP

$\mathcal{M} = M_5 \times X_5$: M_5 is a $5d$ Lifshitz-like metric, X_5 is an Einstein manifold.

D3-D7 system, Azeyanagi et al.'09, Mateos&Trancanelli'09

$$ds^2 = \tilde{R}^2 \left[\rho^2 \left(-dt^2 + dx^2 + dy^2\right) + \rho^{4/3} dw^2 + \frac{d\rho^2}{\rho^2}\right] + R^2 ds^2_{X_5}.$$

$$\nu = 3/2, \quad r \equiv \rho^{2/3}, \quad (t, x, y, w, \rho) \to \left(\lambda t, \lambda x, \lambda y, \lambda^{2/3} w, \frac{\rho}{\lambda}\right)$$

$\mathcal{M}_5 \times X_5$	t	x	y	r	w	s_1	s_2	s_3	s_4	s_5
D3	×	×	×		×					
D7	×	×	×			×	×	×	×	×

AdS/CFT: D3-probes in D3-background $AdS_5 \times S^5 \Rightarrow \mathcal{N} = 4$ SYM.

D7-probes in D3-background $Lif_{IR}/AdS_{5,UV} \times X_5 \Rightarrow$ deformed SYM.

Jet quenching, drag force, potentials... see Giataganas et al.'12

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Possible models

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\left(R[g] + \Lambda - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{4}e^{\lambda\phi}F^2_{(2)}\right),$$

Λ is negative cosmological constant.

The Einstein equations

$$R_{mn} = -\frac{\Lambda}{3}g_{mn} + \frac{1}{2}(\partial_m\phi)(\partial_n\phi) + \frac{1}{4}e^{\lambda\phi}\left(2F_{mp}F^p_n\right) - \frac{1}{12}e^{\lambda\phi}F^2 g_{mn}.$$

The scalar field equation

$$\Box\phi = \frac{1}{4}\lambda e^{\lambda\phi}F^2, \quad \text{with} \quad \Box\phi = \frac{1}{\sqrt{|g|}}\partial_m(g^{mn}\sqrt{|g|}\partial_n\phi).$$

The gauge field

$$D_m\left(e^{\lambda\phi}F^{mn}\right) = 0.$$

Gravity duals: black brane and infalling shell

The Lifshitz-like black brane

$$ds^2 = e^{2\nu r}\left(-f(r)dt^2 + dx^2\right) + e^{2r}\left(dy_1^2 + dy_2^2\right) + \frac{dr^2}{f(r)},$$

where $f(r) = 1 - me^{-(2\nu+2)r}$. Aref'eva,AG, Gourgoulhon'16

$$F_{(2)} = \frac{1}{2}q\,dy_1 \wedge dy_2, \quad \phi = \phi(r), \quad e^{\lambda\phi} = \mu e^{4r}.$$

The Vaidya solution in Lifshitz background

$$ds^2 = -z^{-2}f(z)dv^2 - 2z^{-2}dvdz + z^{-2}dx^2 + z^{-2/\nu}(dy_1^2 + dy_2^2),$$

$f = 1 - m(v)z^{2/\nu+2}, v < 0$ – inside the shell, $v > 0$ – outside,

$$dv = dt + \frac{dz}{f(z)}, \quad z = \frac{1}{r^\nu}.$$

The Hawking temperature of the black brane:

$$T = \frac{1}{\pi}\frac{(\nu+1)}{2\nu}\,m^{\frac{\nu}{2\nu+2}}.$$

Spatial WL in Lifshitz-like backgrounds

Rectangular WL in the spatial planes xy_1(or xy_2) and y_1y_2. Possible configurations:

- a rectangular loop in the xy_1 (or xy_2) plane with a short side of the length ℓ in the longitudinal x direction and a long side of the length L_{y_1} along the transversal y_1 direction

$$x \in [0, \ell < L_x], \quad y_1 \in [0, L_{y_1}];$$

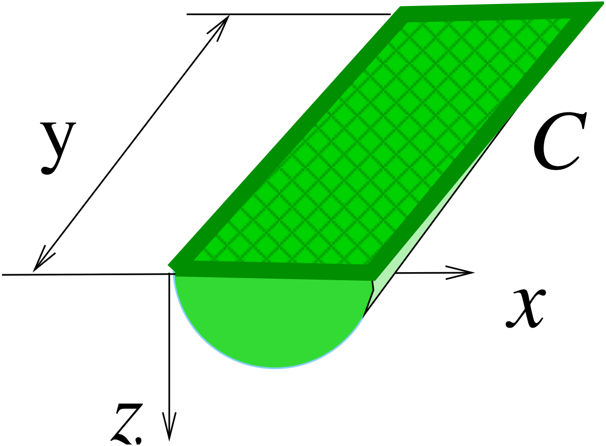
- a rectangular loop in the xy_1 plane with a short side of the length ℓ in the transversal y_1 direction and a long side of the length L_x along the longitudinal x direction:

$$x \in [0, L_x], \quad y_1 \in [0, \ell < L_{y_1}];$$

- a rectangular loop in the transversal y_1y_2 plane with a short side of the length ℓ in one of transversal directions (say y_1) and a long side of the length L_{y_2} along the other transversal direction y_2

$$y_1 \in [0, \ell < L_{y_1}], \quad y_1 \in [0, L_{y_2}].$$

Spatial WL in Lifshitz-like backgrounds



Static WL. Case 1: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(x)$, $v = v(x)$.

The renormalized Nambu-Goto action

$$S_{x,y_1(\infty),ren} = \frac{L_{y_1}}{2\pi\alpha'} \frac{1}{z_*^{1/\nu}} \int_0^1 \frac{dw}{w^{1+1/\nu}} \left[\frac{1}{\sqrt{f(z_* w) \left(1 - w^{2+2/\nu}\right)}} - 1 \right] - \frac{\nu}{z_*^{1/\nu}},$$

where $w = z/z_*$.The length scale is

$$\frac{\ell}{2} = 2z_* \int_{z_0/z_*}^1 \frac{w^{1+1/\nu}\, dw}{f(z_* w)(1 - w^{2+2/\nu})}.$$

Then pseudopotential $\mathcal{V}_{x,y_1(\infty)} = \dfrac{S_{x,y_1(\infty),ren}}{L_{y_1}}$.

For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{x,y_1(\infty)}(\ell,\nu) \underset{\ell\sim 0}{\sim} -\frac{\mathcal{C}_1(\nu)}{\ell^{1/\nu}}.$$

For large ℓ

$$\mathcal{V}_{x,y_1(\infty)}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,1}(\nu)\,\ell.$$

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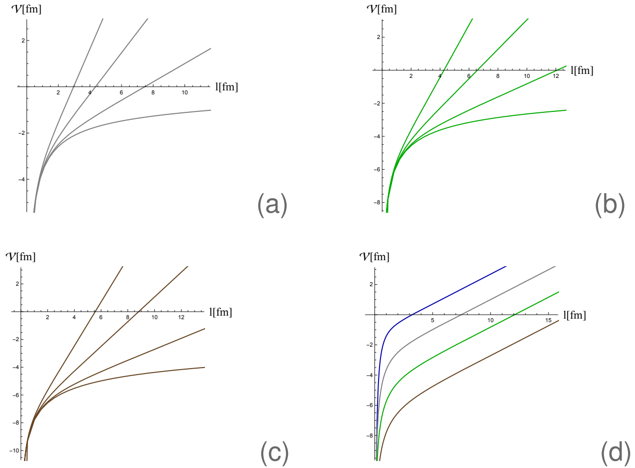

Figure: $\mathcal{V}_{x,y_{1}(\infty)}$ as a function of ℓ, $\nu = 2, 3, 4$((a),(b),(c)). The temperature $T = 30, 100, 150, 200$ MeV (from down to top) for all. In (d): $\mathcal{V}_{x,y_{1}(\infty)}$ for $\nu = 1, 2, 3, 4$ (from top to down) at $T = 100$ MeV.

Static WL. Case 2: $\sigma^1 = x$, $\sigma^2 = y_1$, $z = z(y_1)$, $v = v(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1, x_{(\infty)}, ren} = \frac{L_x}{2\pi\alpha'}\frac{1}{z_*}\int_{z_0/z_*}^{1}\frac{dw}{w^2}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{2+2/\nu}\right)}}-1\right]-\frac{1}{z_*}.$$

The length scale is

$$\ell = 2z_*^{1/\nu}\int_0^1 \frac{w^{2/\nu}dw}{f(z_* w)\left(1-w^{2+2/\nu}\right)}$$

The pseudopotential $\mathcal{V}_{y_1, x_{(\infty)}} = \dfrac{S_{y_1, x_{(\infty)}, ren}}{L_x}$.

For small ℓ – the deformed Coulomb part

$$\mathcal{V}_{y_1, x_{(\infty)}} \underset{\ell\sim 0}{\sim} -\frac{\mathcal{C}_2(\nu)}{\ell^{\nu}}.$$

For large ℓ

$$\mathcal{V}_{y_1, x_{(\infty)}}(\ell, \nu) \underset{\ell\to\infty}{\sim} \sigma_{s,2}(\nu)\,\ell.$$

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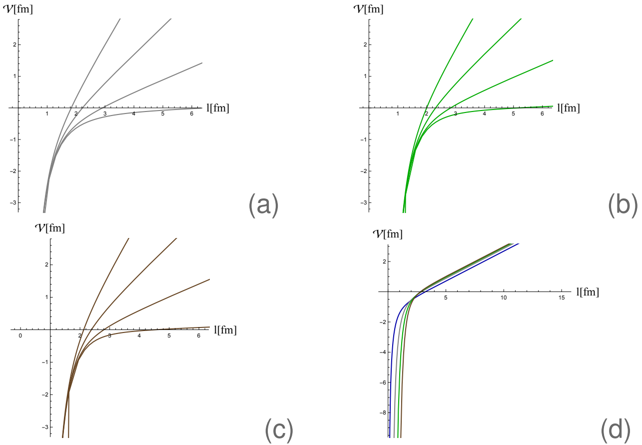

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Static WL. Case 3: $\sigma^1 = y_1$, $\sigma^2 = y_2$, $z = z(y_1)$, $\nu = \nu(y_1)$

The renormalized Nambu-Goto action

$$S_{y_1 y_{2(\infty)},ren} = \frac{L_{y_2}}{2\pi\alpha'}\frac{1}{z_*^{1/\nu}}\int_{\frac{z_0}{z_*}}^{1}\frac{dw}{w^{1+1/\nu}}\left[\frac{1}{\sqrt{f(z_* w)\left(1-w^{4/\nu}\right)}}-1\right]-\frac{\nu}{z_*^{1/\nu}}.$$

The length scale is

$$\ell = z_*^{1/\nu}\int\frac{dw}{w^{1-3/\nu}\sqrt{f(z_* w)\left(1-w^{4/\nu}\right)}}.$$

The pseudopotential $\mathcal{V}_{y_1,y_{2(\infty)}} = \dfrac{S_{y_1,y_{2(\infty)}}}{L_{y_2}}$.

For small ℓ –

$$\mathcal{V}_{y_1,y_{2(\infty)}} \underset{\ell\to 0}{\sim} -\frac{\mathcal{C}_3(\nu)}{\ell}.$$

For large ℓ

$$\mathcal{V}_{y_1,y_{2(\infty)}}(\ell,\nu) \underset{\ell\to\infty}{\sim} \sigma_{s,3}(\nu)\,\ell.$$

Static WL.Case 3: $\sigma^1 = y_1,\ \sigma^2 = y_2,\ z = z(y_1),\ \nu = \nu(y_1)$

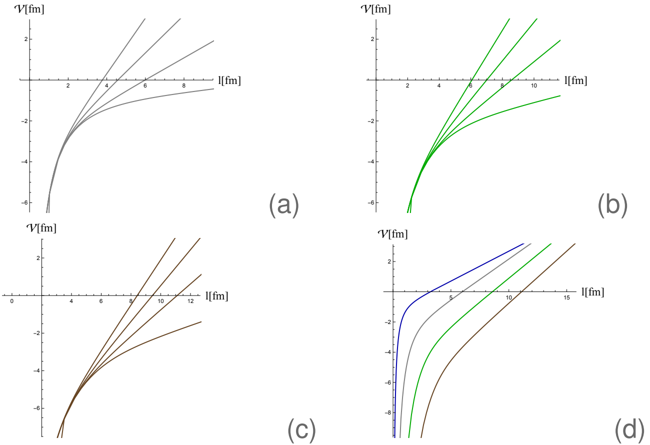

Figure: $\mathcal{V}_{y_1, y_{2(\infty)}}$ as a function of ℓ for $\nu = 2, 3, 4$ ((a),(b),(c), respectively). We take $T = 30, 100, 150, 200$ MeV from down to top, for (a),(b) and (c). In (d) $\mathcal{V}_{y_1, y_{2(\infty)}}$ for $\nu = 1, 2, 3, 4$ (from left to right) at $T = 100$ MeV.

Static WL. Spatial string tension

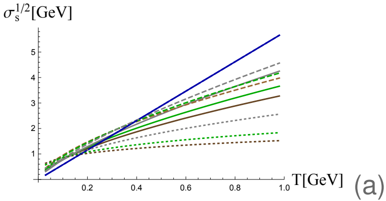

Figure: The dependence of the spatial string tension $\sqrt{\sigma_s}$ on orientation and temperature. The solid lines corresponds to the rectangular Wilson loop with a short extent in the x-direction, while the dashed lines correspond to a short extent in the y-direction. The dotted lines correspond to the rectangular Wilson loop in the transversal y_1y_2 plane. (a) Blue line corresponds to $\nu = 1$, gray lines correspond to $\nu = 2$, green lines correspond to $\nu = 3$ and the brown ones correspond to $\nu = 4$. (b) The spatial string tension $\sqrt{\sigma_s}$ for different orientations for $\nu = 4$.

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WL in time-dependent backgrounds.Case 1

$$S_{x,y_{1(\infty)}} = \frac{L_y}{2\pi\alpha'}\int \frac{dx}{z^{1+1/\nu}}\sqrt{1 - f(z,v)v'^{\,2} - v'z'}, \quad \prime \equiv \frac{d}{dx}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{(\nu+1)}{\nu z}(1 - fv'^2 - 2v'z'), \\ z'' &= -\frac{\nu+1}{\nu}\frac{f}{z} + \frac{\nu+1}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2}fv'^2\frac{\partial f}{\partial z} - v'z'\frac{\partial f}{\partial z}, \\ &+ 2\frac{(\nu+1)}{\nu z}fv'z'. \end{aligned}$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{x,y_{1(\infty)}} = \frac{S_{x,y_{1(\infty)},ren}}{L_{y_1}}$$

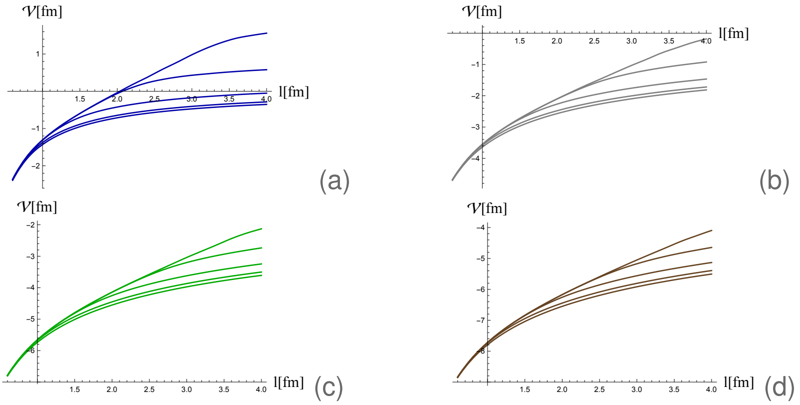

Figure: $\mathcal{V}_{x,y_{1}(\infty)}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to time $t = 0.1, 0.5, 0.9, 1.4, 2$ (from down to top, respectively).

$$\delta\mathcal{V}_1(x,t) = \mathcal{V}_{x,y_{1(\infty)}}(x,t) - \mathcal{V}_{x,y_{1(\infty)}}(x,t_f).$$

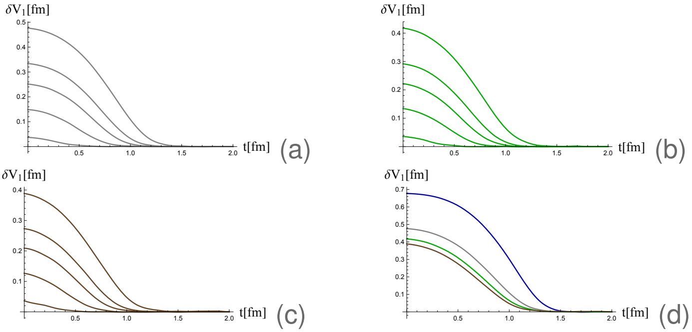

Figure: The time dependence of $-\delta\mathcal{V}_1(x,t)$, for different values of the length ℓ, $\nu = 2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 0.7, 1.2, 1.5, 1.7, 2$ (from down to top, respectively). In (d) we have shown $-\delta\mathcal{V}_1(x,t)$ as a function of t at $\ell = 2$ for $\nu = 1,2,3,4$ (from top to down).

WL in time-dependent backgrounds.Case 2

$$S_{y_1,x_{(\infty)}} = \frac{L_x}{2\pi\alpha'}\int dy_1 \frac{1}{z^2}\sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(z,v)(v')^2 - 2v'z'\right)}, \quad ' \equiv \frac{d}{dy_1}.$$

The corresponding equations of motion are

$$v'' = \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{\nu+1}{\nu z}\left(z^{2-2/\nu} - \frac{2\nu}{(1+\nu)}fv'^2 - 2v'z'\right),$$

$$z'' = -\frac{\nu+1}{\nu}fz^{1-2/\nu} + \frac{2(\nu-1)z'^2}{\nu} + \frac{2}{\nu}\frac{f^2v'^2}{z} - \frac{1}{2\nu}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2\nu}f\frac{\partial f}{\partial z}v'^2$$
$$- z'v'\frac{\partial f}{\partial z} + \frac{4}{z}fz'v'.$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1,x_{(\infty)}} = \frac{S_{y_1,x_{(\infty)},ren}}{L_{y_1}}.$$

WL in time-dependent backgrounds.Case 2

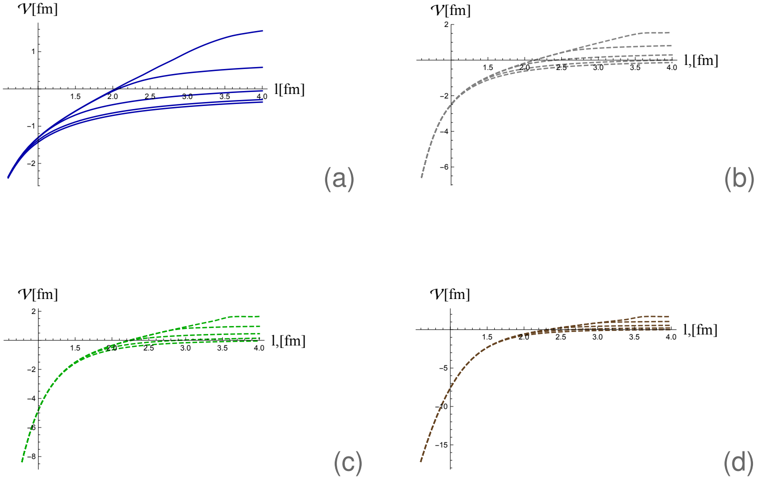

Figure: $\mathcal{V}_{y_1 x_\infty}$ as a function of ℓ at fixed values of t for $\nu = 1, 2, 3, 4$ ((a),(b),(c),(d), respectively). Different curves correspond to $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top. In (**??**) we take $M = 1$.

WL in time-dependent backgrounds.Case 2

$$\delta\mathcal{V}_{y_1,x_{(\infty)}}(x,t) = \mathcal{V}_{y_1,x_{(\infty)}}(x,t) - \mathcal{V}_{y_1,x_{(\infty)}}(x,t_f).$$

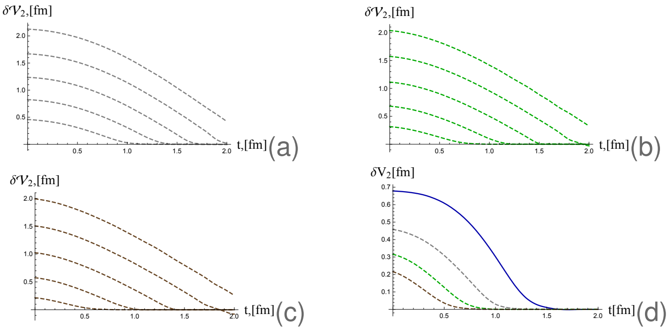

Figure: The time dependence of $-\delta\mathcal{V}_{y_1,x_{(\infty)}}(x,t)$ for different values of the length ℓ, $\nu = 2,3,4$ ((a),(b),(c), respectively). Different curves correspond to $\ell = 2, 2.5, 3, 3.5, 4$ (from down to top, respectively). In (d) $-\delta\mathcal{V}_2(x,t)$ as a function of t at $\ell = 2$ for $\nu = 1,2,3,4$ (from top to down, respectively).

WL in time-dependent backgrounds.Case 3

$$S_{y_1,y_{2,(\infty)}} = \frac{L_{y_2}}{2\pi\alpha'}\int dy_1 \frac{1}{z^{1+1/\nu}}\sqrt{\left(\frac{1}{z^{2/\nu-2}} - f(v')^2 - 2v'z'\right)}.$$

The corresponding equations of motion are

$$\begin{aligned} v'' &= \frac{1}{2}\frac{\partial f}{\partial z}v'^2 + \frac{2}{z\nu}\left(z^{2-2/\nu} - \frac{\nu+1}{2}fv'^2 - 2v'z'\right), \\ z'' &= -\frac{2}{\nu}fz^{1-2/\nu} + 2\frac{\nu-1}{\nu}\frac{z'^2}{z} + \frac{\nu+1}{\nu z}f^2v'^2 - \frac{1}{2}\frac{\partial f}{\partial v}v'^2 - \frac{1}{2}f\frac{\partial f}{\partial z}v'^2 \\ &- z'v'\frac{\partial f}{\partial z} + \frac{2(\nu+1)}{\nu z}fv'z'. \end{aligned} \quad (5.1)$$

The boundary conditions $z(\pm\ell) = 0$, $v(\pm\ell) = t$. The initial conditions $z(0) = z_*$, $v(0) = v_*$, $z'(0) = 0, v'(0) = 0$. The pseudopotential is

$$\mathcal{V}_{y_1,y_{2,(\infty)}}(t,\ell) = \frac{S_{y_1,y_{2,(\infty)},ren}}{L_{y_2}}.$$

WL in time-dependent backgrounds.Case 3

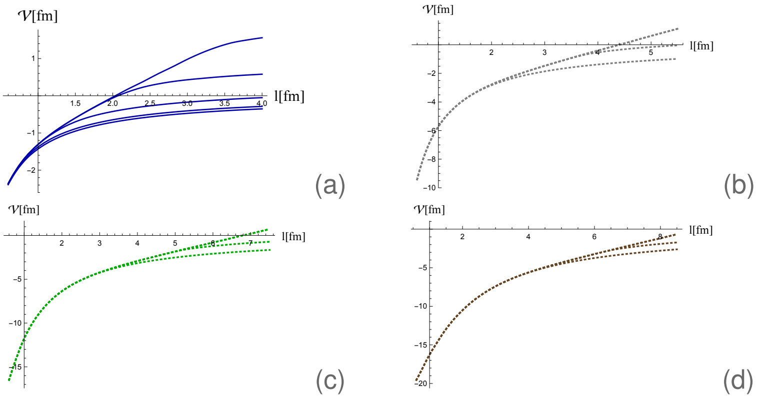

Figure: $\mathcal{V}_{y_1,y_{2,(\infty)}}(l,t)$ as a function of the length ℓ at fixed values of t, $\nu = 1,2,3,4$ ((a),(b),(c),(d)). (a): we take $t = 0.1, 0.5, 0.9, 1.4, 2$ from down to top, respectively; for plots (b),(c),(d): $t = 0.4, 1.5, 2.5, 3.34, 4$ from down to top, respectively.

WL in time-dependent backgrounds.Case 3

$$\delta\mathcal{V}_{y_1,y_{2,(\infty)}}(x,t) = \mathcal{V}_{y_1,y_{2,(\infty)}}(x,t) - \mathcal{V}_{y_1,y_{2,(\infty)}}(x,t_f).$$

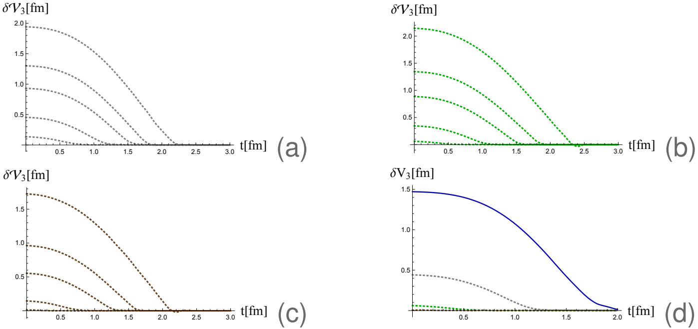

Figure: $-\delta\mathcal{V}_{y_1,y_{2,(\infty)}}(x,t)$ on t for different ℓ, $\nu = 2,3,4$ ((a),(b),(c)). (a): $l = 2.2, 3, 3.85, 4.4, 5.2$ from top to down; (b): $l = 3, 4.1, 5.2, 6, 7.1$ from top to down; (c): $l = 3.4, 4.6, 5.9, 6.8, 8$ from top to down. In (d): $-\delta\mathcal{V}_3(x,t)$ as a function of t at $\ell = 3$ for $\nu = 1,2,3,4$ (from top to down, respectively).

Summary and Outloook

Open questions

1. Time-like Willson loops
2. Jet quenching
3. Isotropization, RG-flow $AdS_3 \times \mathrm{R}^2$?
4. "Top-down" model?

Thank you for your attention!