Dual QCD Thermodynamical at Finite Temperature and Chemical Potential

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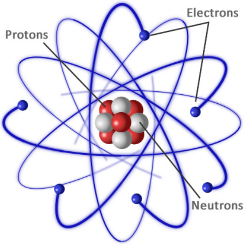
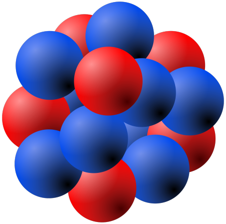
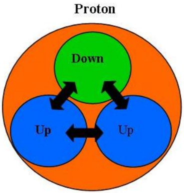
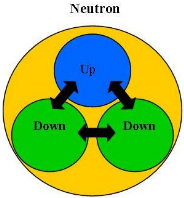
Fundamental building blocks of matter









Quantum Chromodynamics

Yang and Mills introduced the non-Abelian version of gauge theories

The QCD Lagrangian for $SU(3)$ color Yang Mills theory

$$\mathbf{L_{QCD}} = -\frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu} + \sum_{\mathbf{j}}^{\mathbf{N_F}} \overline{\psi_{\mathbf{j}}}(\mathbf{x})(\mathbf{i}\gamma^{\mu}\mathbf{D}_{\mu} - \mathbf{m_j})\psi_{\mathbf{j}}(\mathbf{x}) \qquad (1)$$

where $\psi_{\mathbf{j}}(\mathbf{x})$ are the quark fields, $\mathbf{N_F}$ is the number of quark flavors and $\mathbf{m_j}$ is the mass of the $\mathbf{j^{th}}$ quark, $\mathbf{D}_{\mu}$ is the associated covariant derivative and $\mathbf{G}_{\mu\nu}$ is the field strength tensor.

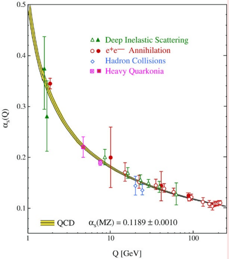
Confinement and Asymptotic Freedom

- Low energy regime $\Rightarrow$ Large distances

- High energy regime $\Rightarrow$ Short distances

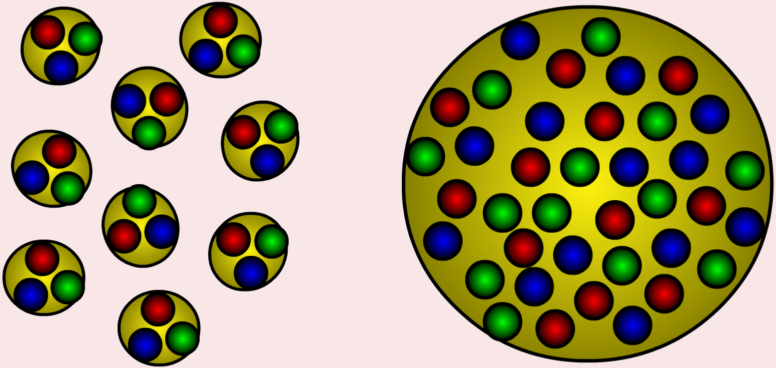
$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}n_F)\ln(\frac{Q^2}{\wedge^2_{QCD}})}, \quad (2)$$

$\wedge_{QCD}$ is the QCD scale parameter.



The Phase Diagram of QCD

In a QCD system at extremely high temperature or very high pressure the nuclear matter is expected to undergo a phase transition to a state called Quark-Gluon Plasma (QGP), identified as the deconfined dense state of matter.



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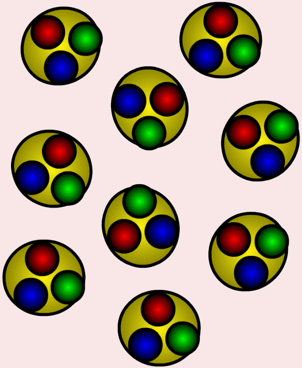
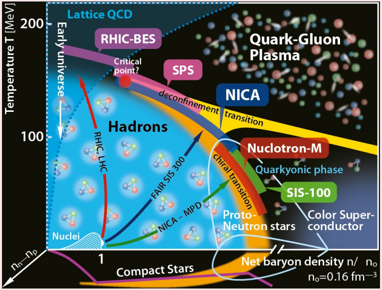
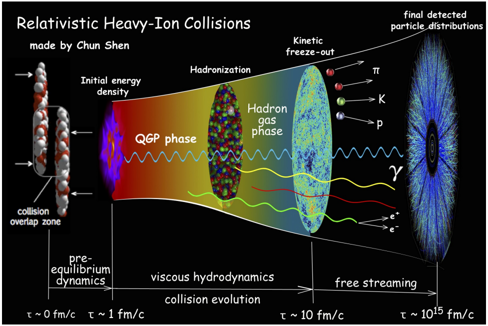




Approaches to study the QCD phase diagram

- **Experiment**
- **Lattice QCD**
- **Effective theories and models**



- MIT Bag Model
- Nambu-Jona-Lasinio Model
- Hard Thermal Loop Perturbation Theory
- Hadron Resonance Gas Model

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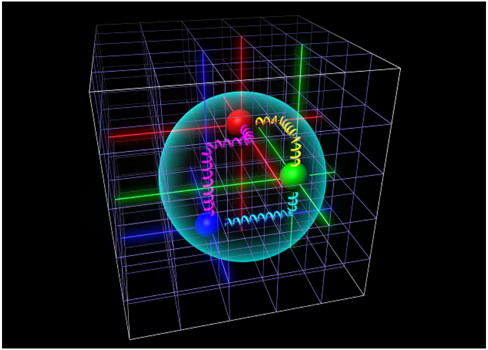

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$SU(3)$ Dual QCD Formulation

- The mathematical foundation for the dual gauge theory comes from the observation that the non-Abelian gauge symmetry allow an extra internal symmetry called magnetic symmetry which restricts and reduces the dynamical degrees of the theory.

$$D_\mu \hat{m} = 0, \ i.e. \ (\partial_\mu + g\,\mathbf{W}_\mu \times)\,\hat{m} = 0. \tag{3}$$

- The most general gauge potential which satisfies the above constraint is written as,

$$\mathbf{W}_\mu = A_\mu \hat{m} + A'_\mu \hat{m}' - g^{-1}(\hat{m} \times \partial_\mu \hat{m}) - g^{-1}(\hat{m}' \times \partial_\mu \hat{m}'), \tag{4}$$

where, A_μ and A'_μ are the Abelian component of $\mathbf{W}_\mu$ along $\hat{m}$ and $\hat{m}'$ respectively and are unrestricted by the constraint.

- The associated generalized field strength may then be written as,

$$\mathbf{G}_{\mu\nu} = (F_{\mu\nu} + B^{(d)}_{\mu\nu})\,\hat{m} + (F'_{\mu\nu} + B'^{(d)}_{\mu\nu})\,\hat{m}', \quad (5)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu,$$

$$B^{(d)}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = g^{-1}(\hat{m} \times \partial_\mu \hat{m}),$$

$$B'^{(d)}_{\mu\nu} = \partial_\mu B'^{(d)}_\nu - \partial_\nu B'^{(d)}_\mu = g^{-1}(\hat{m}' \times \partial_\mu \hat{m}'). \quad (6)$$

- The topological structure may be brought into dynamics in a dual symmetric way by imposing magnetic symmetry and the λ_3-like multiplet $\hat{m}$ may be viewed to define the mapping, $S^2_R \rightarrow SU(3)/U(1) \otimes U'(1)$, where S^2_R is the two-dimensional sphere of three dimensional space and S^2 is the group coset space fixed by $\hat{m}$.

Rotating the magnetic vector $\hat{m}$ to a fix time independent direction leads to the the value of gauge potential

$$\mathbf{W}_\mu = g^{-1}\left[\left((\partial_\mu\beta - \frac{1}{2}\partial_\mu\beta^{'})cos\alpha\right)\hat{\xi}_3 + \frac{1}{2}\sqrt{3}(\partial_\mu\beta^{'}cos\alpha)\hat{\xi}_8\right], \quad (7)$$

The associated field strength takes the form as

$$\mathbf{G}_{\mu\nu} \xrightarrow{U} (\,F_{\mu\nu} + B^{(d)}_{\mu\nu}\,)\,\hat{\xi}_3 + (\,F^{'}_{\mu\nu} + B^{'}_{\mu\nu}{}^{(d)}\,)\,\hat{\xi}_8. \quad (8)$$

- Using the regular dual magnetic potential $(B_\mu^{(d)}, B_\mu^{'(d)})$ associated with the monopoles and introducing the complex scalar fields $(\phi(x), \phi^{'}(x))$ for the monopole, we obtain the dual QCD Lagrangian in the following form,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}F_{\mu\nu}^{'\,2} - \frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}B_{\mu\nu}^{'\,2} + \bar{\psi}_r\gamma^\mu[i\partial_\mu + \frac{1}{2}g(A_\mu^{(d)} + B_\mu) + \frac{1}{2\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_r + \bar{\psi}_b\gamma^\mu[i\partial_\mu + \frac{1}{2}g(A_\mu^{(d)} + B_\mu) + \frac{1}{2\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_b + \bar{\psi}_y\gamma^\mu[i\partial_\mu - \frac{1}{\sqrt{3}}g(A_\mu^{'(d)} + B_\mu^{'})]\psi_y + |(\partial_\mu + i\frac{4\pi}{g}(A_\mu + B_\mu^{(d)}))\phi|^2 + |(\partial_\mu + i\frac{4\pi\sqrt{(3)}}{g}(A_\mu^{'} + B_\mu^{'(d)}))\phi^{'}|^2 - m_0(\bar{\psi}_r\psi_r + \bar{\psi}_b\psi_b + \bar{\psi}_y\psi_y) - V. \quad (9)$$

- The confinement mechanism of the QCD vacuum can be understood in absence of color electric sources and the Lagrangian may be reduced in the following form

$$\mathcal{L}_d^{(m)} = -\frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}B_{\mu\nu}^{'}{}^2 + |(\partial_\mu + i\frac{4\pi}{g}B_\mu^{(d)})\phi|^2 + |(\partial_\mu + i\frac{4\pi\sqrt{(3)}}{g}B_\mu^{'(d)})\phi^{'}|^2 - V, \tag{10}$$

where, in relatively weak coupling near-infrared regime the use of the familiar quadratic potential for inducing the magnetic condensation of QCD vacuum is naturally desired and is given below,

$$V = \frac{48\pi^2}{g^4}\lambda(\phi^*\phi - \phi_0^2)^2 + \frac{432\pi^2}{g^4}\lambda^{'}(\phi^*\phi^{'} - \phi_0^{'2})^2, \tag{11}$$

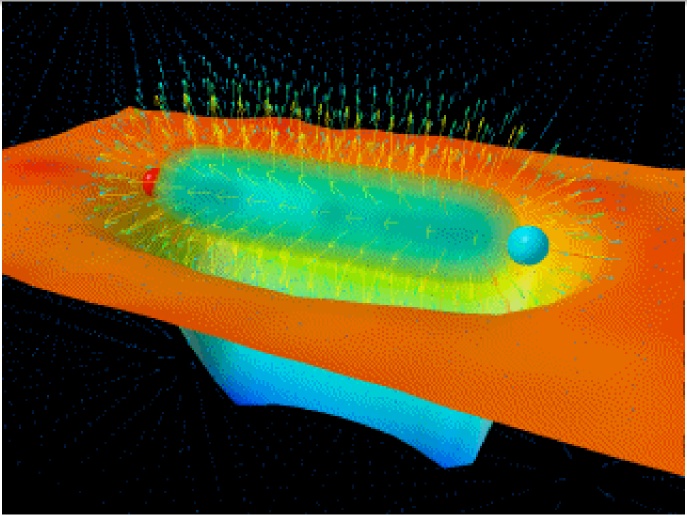
- Using the cylinderically symmetric form of the potentials, the field equations associated with the Lagrangian (8) may be expressed as given below

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\left(\rho B(\rho)\right)\right] - \frac{8\pi}{g}\left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2}B(\rho)\right)\chi^2(\rho) = 0,$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi(\rho)}{d\rho}\right) - \left[\left(\frac{n}{\rho} + (4\pi\alpha_s^{-1})^{1/2}B(\rho)\right)^2 + \frac{96\pi^2}{g^4}\lambda\left(\chi^2 - \phi_0^2\right)\right]\chi(\rho) = 0. \tag{12}$$

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\left(\rho B'(\rho)\right)\right] - \frac{8\pi\sqrt{3}}{g}\left(\frac{n'}{\rho} + \frac{4\pi\sqrt{3}}{g}B'(\rho)\right)\chi'^2(\rho) = 0,$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi'(\rho)}{d\,\rho}\right) - \left[\left(\frac{n'}{\rho} + \frac{4\pi\sqrt{3}}{g}B'(\rho)\right)^2 + \frac{864\pi^2}{g^4}\lambda'(\chi'^2 - \phi_0'^2)\right]\chi'(\rho) = 0. \tag{13}$$



- Utilizing the asymptotic solutions $B(\rho)=-\frac{ng}{4\pi\rho}[1+F(\rho)]$ and $B^{'}(\rho)=-\frac{n^{'}g}{4\sqrt{3}\pi\rho}[1+G(\rho)]$, the energy per unit length of the resulting flux tube configuration may be derived in the following form,

$$k_{(n,n')}=2\pi\int_0^\infty \rho d\rho\left[\frac{n^2g^2}{32\pi^2\rho^2}\left(\frac{dF}{d\rho}\right)^2+\frac{n^2}{\rho^2}F^2(\rho)\chi^2(\rho)+\left(\frac{d\chi}{d\rho}\right)^2+3\lambda\alpha_s^{-2}(\chi^2-\phi_0^2)^2\right]+2\pi\int_0^\infty \rho d\rho\left[\frac{n^{'2}g^2}{96\pi^2\rho^2}\left(\frac{dG}{d\rho}\right)^2+\frac{n^{'2}}{\rho^2}G^2(\rho)\chi^{'2}(\rho)+\left(\frac{d\chi^{'}}{d\rho}\right)^2+27\lambda^{'}\alpha_s^{-2}(\chi^{'2}-\phi_0^{'2})^2\right], \tag{14}$$

where $F(\rho)\overset{\rho\to\infty}{\longrightarrow}C\sqrt{\rho}exp\Big(-m_B\rho\Big)$ and $G(\rho)\overset{\rho\to\infty}{\longrightarrow}C^{'}\sqrt{\rho}exp\Big(-m_B^{'}\rho\Big)$.

- Incorporating color reflection invariance the masses of the magnetic glueballs is estimated by evaluating the string tension $k_{(n,n')}$ of the resulting flux tube written as, $k_{(n,n')} = \frac{1}{2\pi\alpha'} = \gamma_{(n,n')}\phi_0^2$,

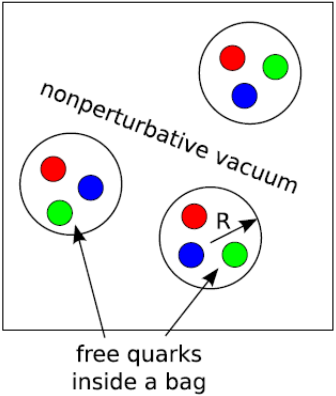
γ	α_s	$\bar{m}_\phi(GeV)$	$\bar{m}_B(GeV)$	$\lambda_{QCD}^{(d)}(fm)$	$\xi_{QCD}^{(d)}(fm)$	$\kappa_{QCD}^{(d)}$
5.617	0.25	1.20	1.75	0.57	0.83	0.69
6.828	0.24	1.69	1.62	0.61	0.59	0.99
8.093	0.23	2.17	1.52	0.65	0.46	1.42
9.833	0.22	2.90	1.41	0.70	0.34	2.05

Table: The masses of vector and scalar glueball as functions of α_s.

Hadronic bag and dual QCD vacuum

- The ground state hadron are spherically symmetric and quarks are confined to a sphere of finite size.
- A model of hadronic bag was identified describing the typical phase structure of QCD.
- The hadron energy in its confined phase is expressed as,

$$E_h = BV + \frac{C}{R_h}. \qquad (15)$$



- **A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V.F. Weisskopf, Phys. Rev. D 9 (1974) 3471.**

- B represent the bag pressure which is derived from dual QCD formalism and expressed in the following form,

$$B_{su3}^{1/4} = \left(\frac{\lambda}{\pi^2}\right)^{1/4} \frac{\bar{m}_B}{4}, \tag{16}$$

- Inside the bag, positive contribution to energy +B and negative contribution to pressure -B inside the bag.
- Outside, the bag, negative contribution to energy -B and positive contribution to pressure +B outside the bag.

The pressure, energy density and entropy density for hadron and plasma phase for $SU(3)$ dual QCD vacuum is given as,

$$P_\pi = 3 \times \frac{\pi^2}{90} T^4, \qquad P_p^{su3} = \frac{37}{90}\pi^2 T^4 + T^2\mu_q^2 + \frac{\mu_q^4}{3\pi^2} - B_{su3}.$$

$$\epsilon_\pi = 3 \times \frac{\pi^2}{30} T^4, \qquad \epsilon_p^{su3} = \frac{37}{30}\pi^2 T^4 + 3\mu_q^2 T^2 + \frac{3\mu_q^4}{2\pi^2} + B_{su3}$$

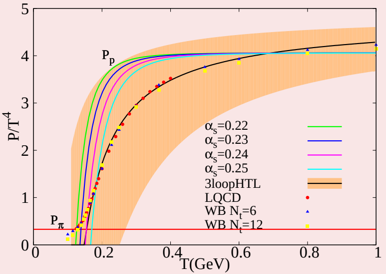
$$s_\pi = 2 \times \frac{\pi^2}{15} T^4, \qquad s_p^{su3} = \frac{74}{45}\pi^2 T^3 + 2T^2\mu_q^2$$

The critical temperature of QGP-phase transition for $SU(3)$ dual QCD vacuum given by,

$$T_{c(\pi)\ su3}^{QGP} = \left(\frac{45}{17\pi^2}\right)^{1/4} B_{su3}^{1/4} \approx 0.72 B_{su3}^{1/4}.$$

- Critical parameters for QGP-hadron phase transition for different values of strong couplings in the infrared sector of $SU(3)$ dual QCD.

α_s	$T_c^0(GeV)$	$(\mu_c, T_c^\mu)(GeV)$
0.22	0.170	(0.48, 0.068)
0.23	0.154	(0.43, 0.065)
0.24	0.138	(0.39, 0.063)
0.25	0.125	(0.33, 0.055)



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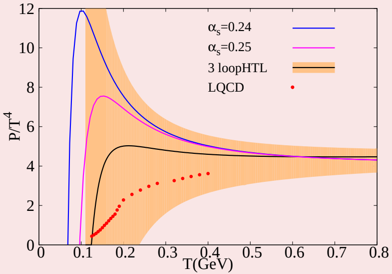
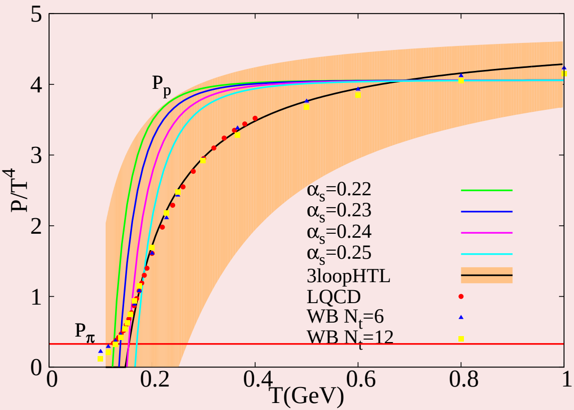


Variation of Normalized pressure for QGP phase at $\mu_B = 0$ and $\mu_B = 0.4GeV$



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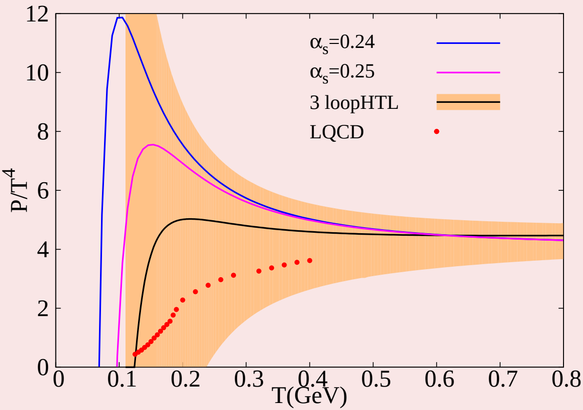
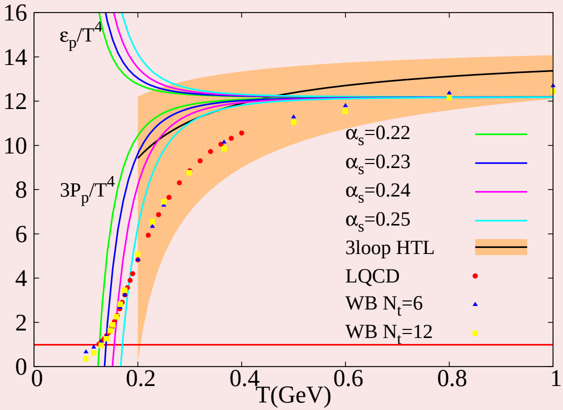


Variation of energy density, entropy density and speed of sound for QGP



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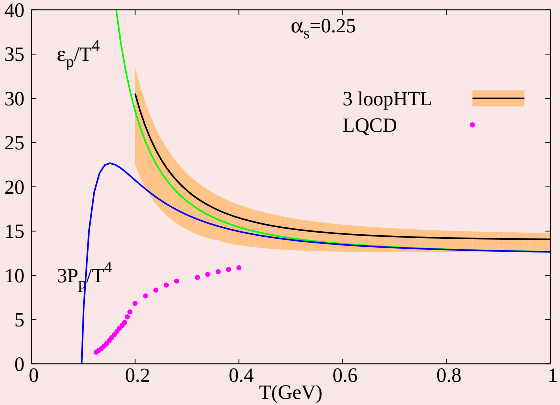

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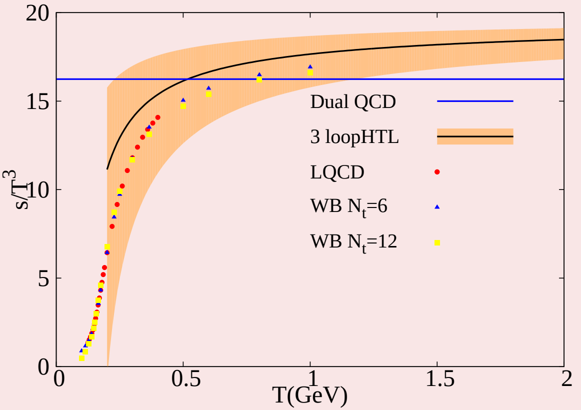

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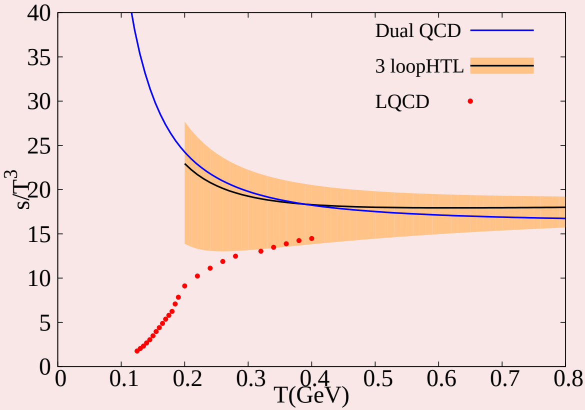

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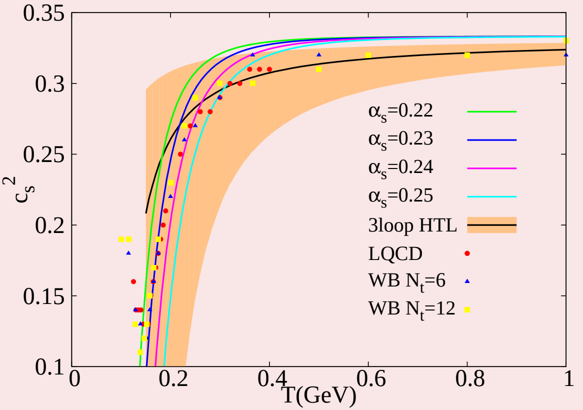

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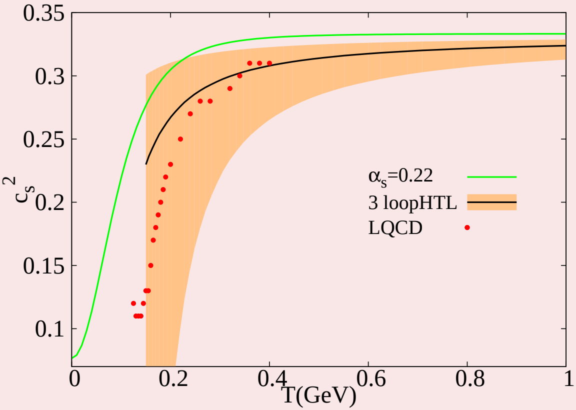
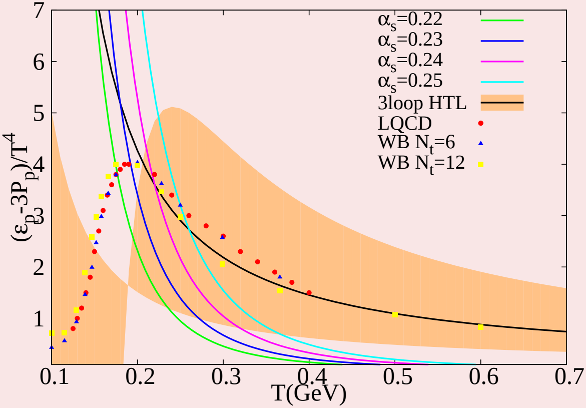


Variation of trace anomaly for QGP



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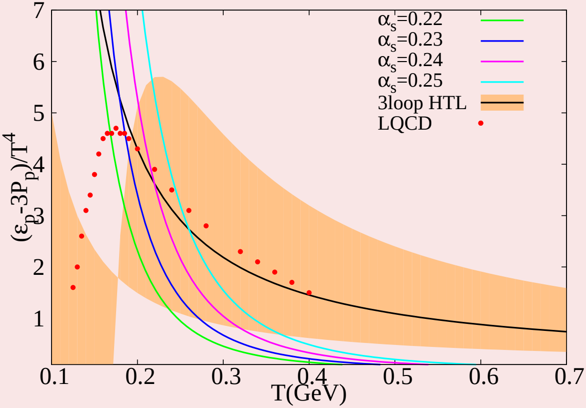

Variation of free energy for QGP phase for $SU(3)$ dual QCD vacuum

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0.22	0.170	2.8504	0.02844	0.09421
0.23	0.154	3.1929	0.02653	0.08534
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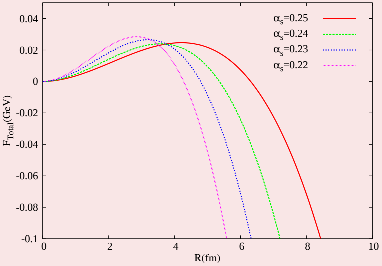

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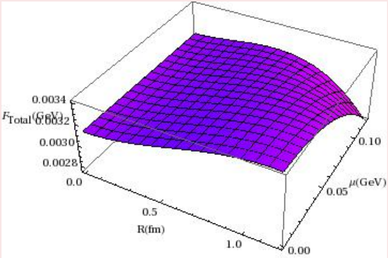

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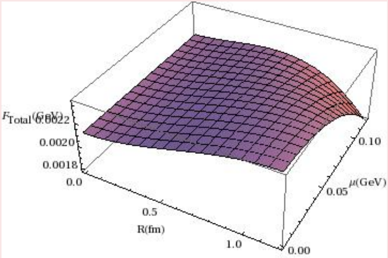

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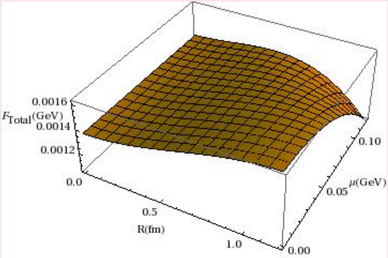

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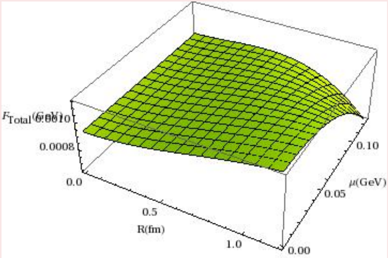

The variation of chemical potential with temperature and Bag constant for $SU(3)$ dual QCD

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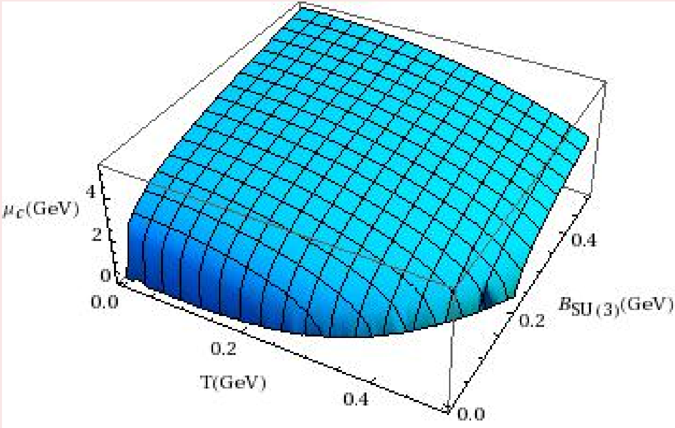
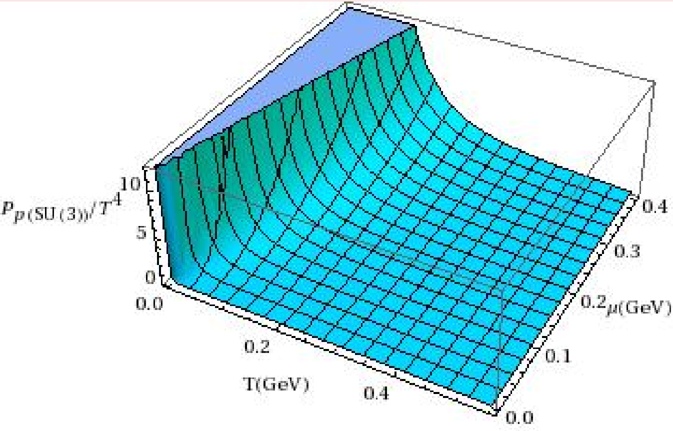


Variation of scaled pressure difference, quark number density and susceptibility in the $T-\mu$ plane.



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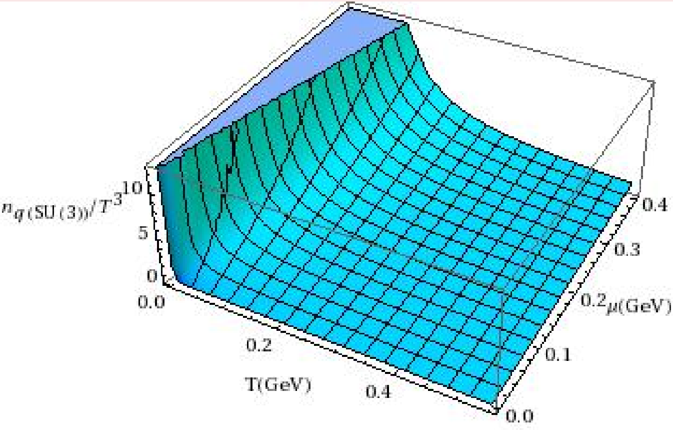

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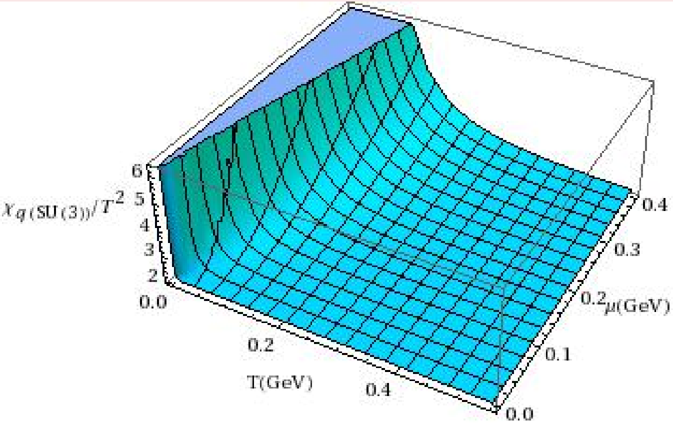

Critical parameters for QGP phase in $T-\mu$ plane with Hard thermal loop perturbation theory and Lattice QCD

- Field Decomposition formulation for $SU(3)$ color gauge and its mass spectrum have been analyzed.
- Within the frame of hadronic Bag, HTLpt and LQCD, the equation of state and critical parameters for QGP phase transition are investigated for non-vanishing μ_B.
- Normalized pressure for QGP phase in the $T-\mu_B$ plane.
- Variation of energy density, entropy density and speed of sound for QGP
- Variation of Trace Anomaly, conformal measure and free energy for QGP phase.
- Variation of Chemical potential with temperature and hadronic Bag Constant
- Variation of scaled pressure difference, quark number density and susceptibility in the $T-\mu_B$ plane.

THANK YOU