Equation of state of the QCD matter in the PNJL model

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IN2P3 Les deux infinis

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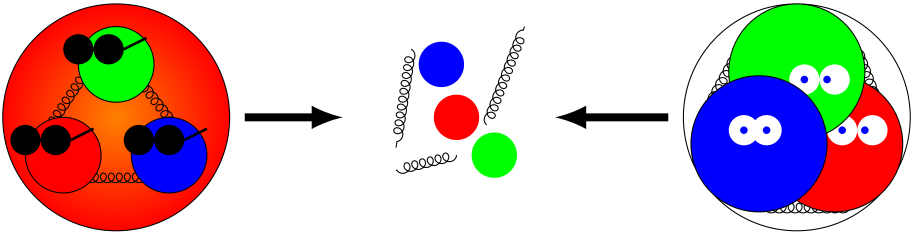
- Better understanding of the strong interaction.
- Thermodynamical (equation of state) and dynamical (cross sections) studies, to build a transport theory describing the hadronisation process.
- Study of the phase diagram of QCD.
- To go to finite μ, effective models are necessary
- PNJL provides such an approach including a first order phase transition for the chiral condensate.

Two phases predicted for QCD matter :

- Hadronic phase :
  Quarks and gluons are bound into hadrons
  This is nuclear matter, we can observe it experimentally
- QGP phase :
  Quarks and gluons are free in the medium
  We don't directly observe this phase experimentally



QCD lagrangian : life is tough

$$\mathcal{L}_{QCD} = i\delta_{ij}\bar{\psi}^i_k\gamma^\mu\partial_\mu\psi^j_k + g_s\bar{\psi}^i_k\gamma^\mu\lambda^a_{ij}A^a_\mu\psi^j_k - m_k\bar{\psi}^i_k\psi^j_k - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Perturbative approach pQCD

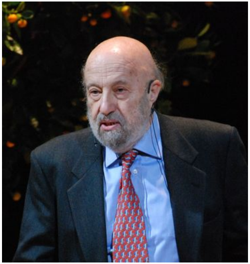
Need of a small coupling constant = large momentum transfer.

- Not usable around hadronization process yet.

Lattice approach lQCD

Static study which does not work at finite chemical potential yet.

Polyakov- extended Nambu-Jona-Lasinio (PNJL) Model

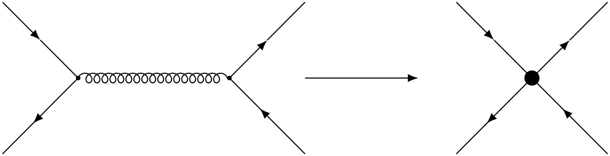


Effective model

Works only in a special domain of energy but allows finite chemical potential studies.

Contact interaction

Static approximation : no gluons propagating the interaction



Frozen gluons

$$\frac{1}{p^2 - \epsilon_g^2} = -\frac{1}{\epsilon_g^2}$$

if $p << \epsilon_g^2$

Nambu-Jona-Lasinio (NJL) Lagrangian

$$\mathscr{L}_{NJL} = \delta_{ij}\overline{\psi}^i_k(i\gamma^\mu\partial_\mu - m)\psi^j_k + G(\overline{\psi}^i_k\lambda_{ij}\psi^j_k)^2 + \text{'t Hooft term}$$

Symmetries

- Chiral symmetry $SU_L(3) \otimes SU_R(3)$
- Color symmetry $SU_c(3)$
- Flavour symmetry $SU_f(3)$

Problem

Center symmetry is missing

Confinement is not described

Free parameters

$m_q^0 = 0.0055\, GeV$

$m_s^0 = 0.134\, GeV$

$\Lambda = 0.569\, GeV$

$G = \frac{2.3}{\Lambda^2} GeV^{-2}$

$K = \frac{11}{\Lambda^5} GeV^{-5}$

Polyakov loop

Confinement is taken into consideration using an effective potential $U(\phi, \bar{\phi}, T)$, function of the Polyakov loop ϕ.

Polyakov extended NJL Lagrangian

$$\mathscr{L}_{PNJL} = \overline{\psi}_k (i \partial\!\!\!/_\mu - m) \psi_k + G(\overline{\psi}_k \lambda_i \psi_k)^2 + \text{'t Hooft} - U(\phi, \bar{\phi}, T)$$

$$\frac{U(\phi, \bar{\phi}, T)}{T^4} = -\frac{b_2(T)}{2} \bar{\phi}\phi - \frac{b_3}{6}(\bar{\phi}^3 + \phi^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2$$

with the parameters : $b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$

a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

Still no gluons in the interaction

$U(\phi, \bar{\phi}, T)$ is a static gluon field corresponding to the $\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu}$ term in the QCD lagrangian.
The parameters are determined by fitting with the P_{YM} of lQCD.

$$\frac{U(\phi,\bar{\phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}(\bar{\phi}^3 + \phi^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2$$

with the parameters : $b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3$

a_0	a_1	a_2	a_3	b_3	b_4	T_0
6.75	-1.95	2.625	-7.44	0.75	7.5	270 MeV

- Modified quarks distributions :

$$f_\phi^+(E_i - \mu_i) = \frac{(\phi + 2\bar{\phi}\exp(\frac{-E_i-\mu_i}{T}))\exp(-\frac{E_i-\mu_i}{T}) + \exp(-3\frac{E_i-\mu_i}{T})}{1 + 3(\phi + \bar{\phi}\exp(-\frac{E_i-\mu_i}{T}))\exp(-\frac{E_i-\mu_i}{T}) + \exp(-3\frac{E_i-\mu_i}{T})}$$

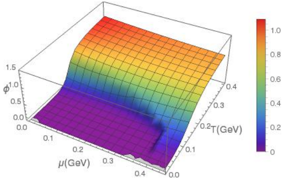
- For $\phi = \bar{\phi} = 0$, "poor man's nucleon" : $E_N = 3E, \mu_N = 3\mu$
- Leads to quarks suppression below T_c.

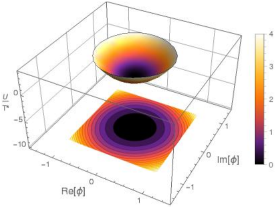
Hung-Ming Tsai and Berndt Müller 2009, J. Phys. G : Nucl. Part. Phys. 36 075101

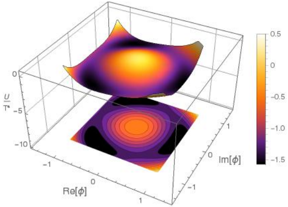
Hadrons and hadronic matter in chiral quarks model, David Blaschke, Dubna 2011

PNJL model

- T_0, critical temperature of center symmetry breaking.
- Below T_0 ,the Polyakov loop is 0. The center symmetry is not broken. The quarks are confined.
- Above T_0, the Polyakov loop is not zero. The center symmetry is broken. The quarks are deconfined.

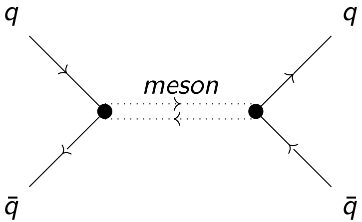






Quark-antiquark bound states

In NJL, degrees of freedom are quarks. Mesons need to be build from quark-antiquarks bound states



Propagator of the mesons

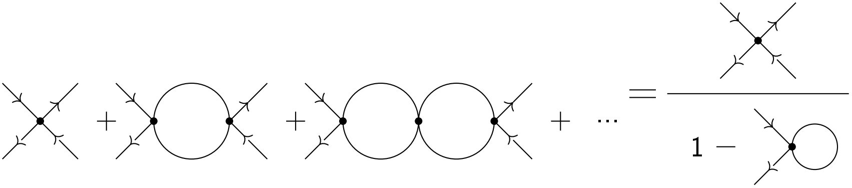
$$iU(k^2) = \Gamma \frac{-ig_m^2}{k^2 - m^2} \Gamma$$

Mesons masses

The mass is given by the poles : $m = k$

Equation of Bethe-Salpeter

$$iU(k^2) = \Gamma \frac{2ig_m^2}{1-2g_m^2\Pi(k^2)}\Gamma$$



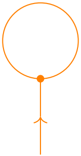
Mesons masses

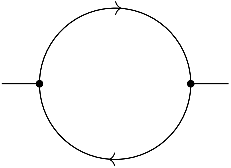
By analogy, the mass is given by the poles :

$$1 - 2G^2\Pi(k^2 = m^2) = 0$$

Polarisation function

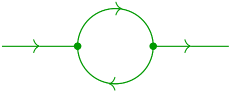
$$\Pi(p^2) = -TN_c \sum_n \int \frac{d^3p}{(2\pi)^3} Tr(i\gamma_5 S(p) i\gamma_5 S(p-k))$$





Pole

$$1 - 2G^2\Pi(k) = 0$$



$$\Pi^{Ps}_{ff'}(k_0, \vec{k}) = -\frac{N_c}{4\pi^2}[A(m_f, \mu_f, T) + A(m'_f, \mu - f', T)$$

$$+[(m_f - m_{f'})^2 - (k_0 + \mu_f - \mu_{f'})^2 + \vec{k}^2] B_0(\vec{k}, m_f, \mu_f, m'_f, \mu'_f, k_0)]$$

Limitations of the model

- Gluons don't participate in the interaction : low energy approximation.
- 4-point interactions are non renormalizable : need of a cut-off.

In fact, PNJL simply consider gluons as a static background field creating a pressure on the medium.

We cannot expect this model to work beyond T $> 2.5\,T_c$ where transverse gluons are expected to contribute significantly.

Equation of State

Partition function

As always in statistical physics, we need the partition function :
$Z = Tr[\exp -\beta(H - \mu N)] = \exp(-\beta\Omega)$

Grand potential

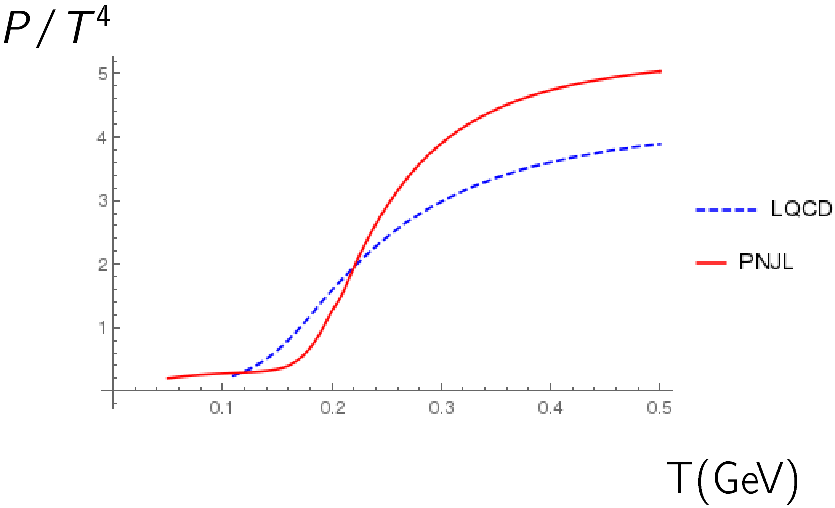
The grand potential is calculated from the partition function.

$$\Omega = -2\int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_p$$

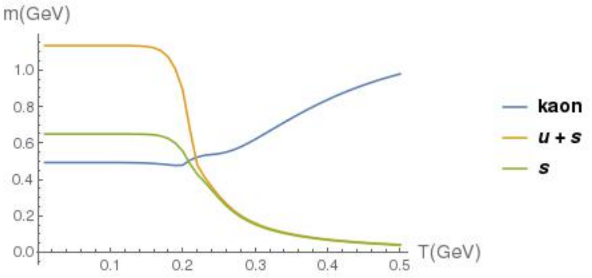
$$+ + 2T\int_0^\infty (\ln[1+\exp(-\beta(E_p-\mu))] + \ln[1+\exp(-\beta(E_p+\mu))]$$

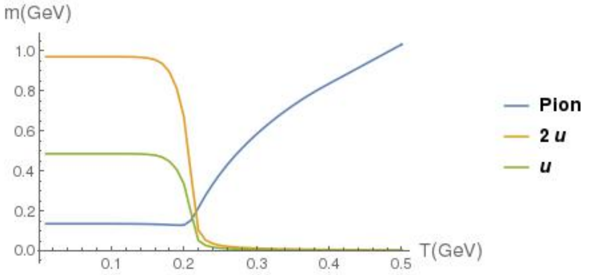
$$+2G\sum_k < \bar{\psi}_k\psi_k >^2 -4K\Pi_i < \bar{\psi}_k\psi_j > +U_{PNJL})$$

Non interactive fermion gas + PNJL mean field


P/T^4
5
4
3
2
1
0.1
0.2
0.3
0.4
0.5
LQCD
PNJL
T(GeV)

Grand potential



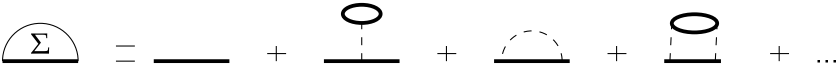


Don't forget mesons !

Below T_{Mott}, mesons are lighter than their constituants. They are stable, present in the medium and contribute to the pressure.

Mesonic fluctuations

To add mesons, we need to go beyond the mean field approximation, to the next to leading order in the $\frac{1}{N_c}$ expansion and consider ring diagrams.

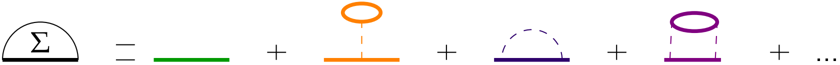


E. Quack and S. P. Klevansk, Phys rev C, V49, nb6 (1994) '‘t Hooft scaling : $g\bar{\psi}A_\mu\psi \rightarrow gN_c\bar{\psi}\frac{A_\mu}{N_c}\psi$ with $gN_c = cst$

$$g^{2l}N_c^k \equiv (gN_c)^{2l}{N_c}^{k-2l}$$

k is the number of fermion lines and l is the number of interaction lines.

$$iS_\Sigma(p) = iS(p)\Big(\boxed{O(1)O(N_c)} + \boxed{O((gN_c)^2)O(1)} + \boxed{O((gN_c)^2)O(\frac{1}{N_c})} + \boxed{O((gN_c)^4)O(\frac{1}{N_c})} + \dots\Big)$$



Beth-Uhlenbeck approach

- We want the grand potential associated to this last diagram.
- Evaluating the Matsubara sum :

$$\Omega_M^{(0)} = \frac{g_M}{2}\int\frac{d^3p}{(2\pi)^3}\int_0^\infty d\omega\left(1+\frac{1}{\exp(\beta(\omega-\mu_M))-1}+\frac{1}{\exp(\beta(\omega+\mu_M))-1}\right)$$
$$\times\ln\left[\frac{1-2K\Pi(\omega-\mu_M+i\epsilon,p)}{1-2K\Pi(\omega-\mu_M-i\epsilon,p)}\right]$$

- Beth and Uhlenbeck :

  Express the 2nd virial coefficient of the Kamerlingh-Onnes equation of state for non ideal gas in terms of two body scattering phase shift.

The same analogy can be done here.

The amplitude of the exchanged meson appears in the expression of the S-Matrix for quark-antiquark scattering channel.

$$S(E,\vec{p}) = \exp(2i\delta(E,\vec{p}))$$

Pole

$$1 - 2G^2\Pi(k) = 0$$

Where δ is :

$$\delta(E,\vec{p}) = -\frac{1}{2i}\ln\left[\frac{1-2G\Pi(\omega-\mu_M-i\epsilon,\vec{p})}{1-2G\Pi(\omega-\mu_M+i\epsilon,\vec{p})}\right]$$

Poles of the amplitude of the exchanged mesons.

Hüfner J. and al ann phys, 234, 225-244 (1994)

Blashke D. and al, arXiv :1305.3907v3 (2014)

Torres Rincon J., Aichelin J., Phys rev C, 96, 0425205 (2017)

Mesonic grand potential

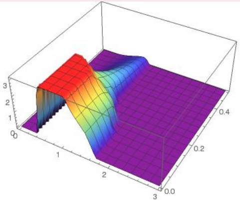
$$\Omega_M = -\frac{g_M}{8\pi^3}\int dp p^2 \int \frac{ds}{\sqrt{s+p^2}}\left[\frac{1}{\exp(\beta(\sqrt{s+p^2}-\mu)-1)}+\frac{1}{\exp(\beta(\sqrt{s+p^2}+\mu)-1)}\right]\delta_M$$

Phase shift : the physics

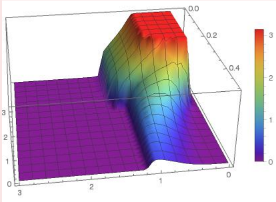
The phase shift depends on the mesons masses

$$\delta_M = -Arg[1-2K_M\Pi_M]$$

Kaon



Pion



Traditional PNJL - Before

One of the parameter is $T_0 = 270 MeV$, the critical temperature for confinement.
This is the pure Yang-Mills critical temperature.

Quarks are here too ! - Better

Slight change in the critical temperature. We use the reduced temperature to quantify it.

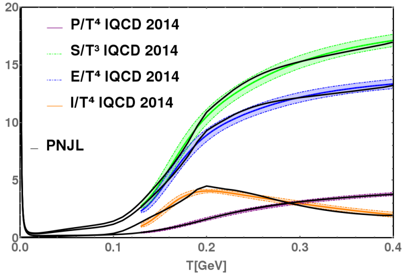
$T^{eff} = \frac{T - T_c}{T_c} \rightarrow T^{eff}_{YM} \simeq 0.57 T^{eff}_{rs}$ *https ://arxiv.org/abs/1302.1993, Haas and al.*

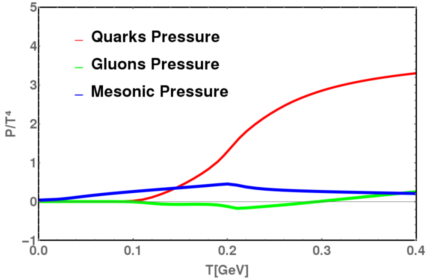
This rescale the critical temperature to $T_0 = 190 MeV$

Not enough ! - Our personal touch

In addition to this rescaling we consider that the parameter T_0 depends on the temperature : $T_0(T)$.

Equation of state at zero μ





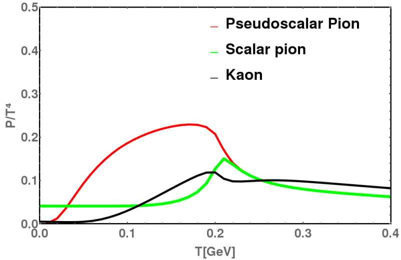
PRELIMINARY

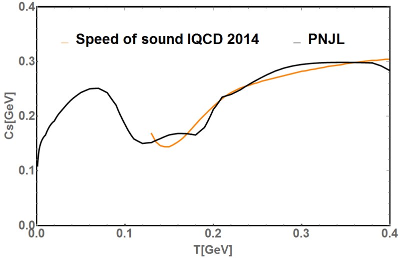
https ://arxiv.org/abs/1407.6387v2, HotQCD Collaboration

We reproduce lattice results ! ! !

We have an effective model based on a lagrangian that shares QCD symmetry and match lattice results.
This is an effective theory, no sign problem, we can expand to finite chemical potential.

Equation of state at zero μ





Mesonic contributions to the pressure

As expected, Mesons contribute only at low temperature.

Critical temperature

Minimum of speed of sound : localisation of the cross over region.

Lattice at finite μ

Lattice can handle Taylor expansion around zero chemical potential.

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \dots$$

The κ coefficient is the second order derivative of our function :

$$\kappa = \left. \frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2} \right|_{\mu_B=0}$$

"On the critical line of 2+1 flavor QCD" Cea, Cosmai, Papa

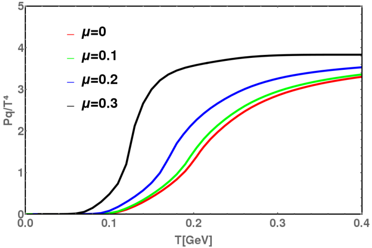
Our critical temperature

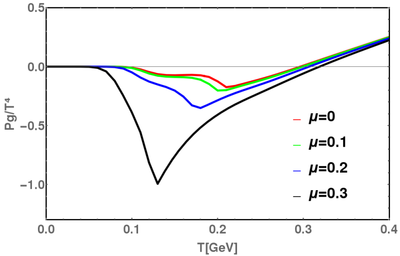
At $\mu_B = 0$, we get the critical temperature : $T_c = 198 MeV$

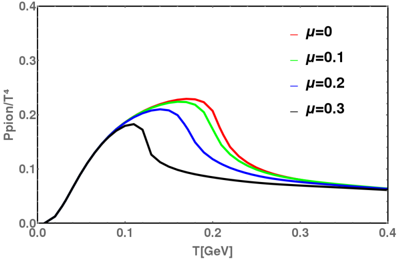
Our coefficient

The corresponding κ coefficient is : $\kappa = 0.018$

At finite μ



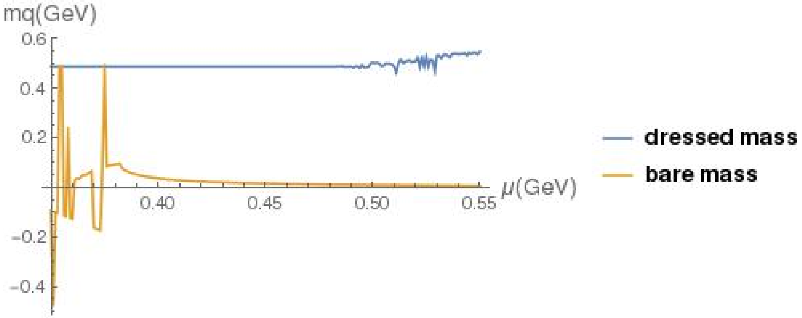




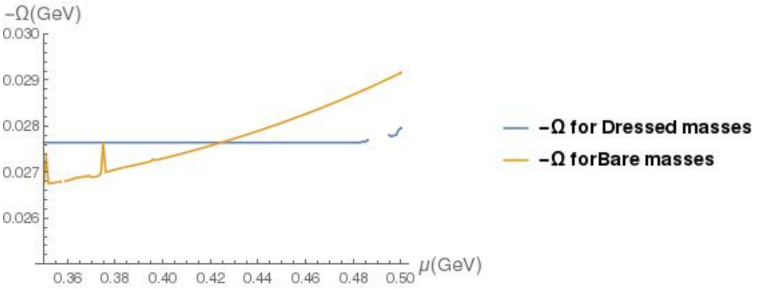
Finite μ

- Results before the first order transition.
- The transition gets sharper though.
- Mesons are vanishing.

- To determine the critical chemical potential, we first computing the two solutions for bare and dressed quarks mass.
- The solutions does not match, meaning that we have a first order transition



To determine precisely the value of μ_{crit}, we use the same process but for the grand potential.



Critical chemical potential

The value obtained is 0.425 GeV for T=0.

Conclusion :

- PNJL : effective model to study the phase diagram at finite μ.

PNJL + T0(T) + Pressure beyond mean field (mesons)

$$=$$

✓ Lattice equation of state at $\mu = 0$.

✓ Lattice equation of state at $\mu \simeq 0$.

- 1st order phase transition not available yet (but soon !).

✓ But the transition gets sharper when μ increases.

✓ First order transition localized at $\mu = 0.425$ GeV at T $= 0$.

What's next ?

- Pressure beyond mean field, but chiral condensate calculated in mean field. *E. Quack and S. P. Klevansk, Phys rev C, V49, nb6 (1994)*

  O(10%) and 16% for the masses

- Expand results at higher μ, beyond the phase transition.

- Localize the Critical End Point CEP.

- Apply our equation of state to Neutron Star.

Thank you for your attention ! !

Sign problem

- Partition function : $Z = \int \mathscr{D}_U \mathscr{D}_{\bar{\psi}} \mathscr{D}_{\psi} \exp(-S)$
- With the action :
  $S = \int d^4x \bar{\psi}(\gamma_\nu(\partial_\nu + iA_\nu) + \mu\gamma_4 + m)\psi = \int d^4x \bar{\psi} M \psi$
- μ appears as an A_4 imaginary quadrivector and :
  $M = \gamma_\nu \partial_\nu + i\gamma_\nu A_\nu + \mu\gamma_4 + m$
- We then have :
  $M^\dagger(\mu) = M(-\mu^*)$
- The action is now complex. It can be seen using the hermiticity of the γ_5 matrix. M hermiticity valide at $\mu = 0$ and but not for finite μ.

$U_A(1)$ anomaly

- Classical action invariant $\rightarrow$ symmetry.
- Quantum action not invariant $\rightarrow$ symmetry broken.
- Symmetry broken by quantum fluctuation : Anomalies !

S matrix

$S(p, E) = \exp(2i\delta(\vec{p}, E)) = \frac{F_J(\vec{k}, E^*)}{F_J(\vec{k}, E)}$

The zeroes of the Jost function are the poles of the S-matrix.

- S-matrix has a pole at k = $+i\kappa$: Bound states have exponentially decaying solutions.
- Poles in the lower half plane can be written as k = -i$\kappa + \gamma$
  - γ= 0, resonances
  - γ = 0, antibound or virtual states.