Quark scattering off quarks and hadrons

A. V. Friesen
in collaboration with Yu. L. Kalinovsky and V. D. Toneev

BLTP, JINR

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- to describe the spontaneous breaking of the chiral symmetry,
- to describe the properties of scalar and pseudo-scalar mesons in hot and dense matter,
- to explain dynamics of hadron matter,
- to describe the phase transitions in hadron matter.

Including of the Polyakov loop:

- simulate the confinement transition.

Nambu- Jona -Lasinio model with Polyakov loop

$$\mathcal{L}_{\mathrm{PNJL}} = \underbrace{\bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)q + G\left[\left(\bar{q}q\right)^2 + \left(\bar{q}i\gamma_5\vec{\tau}q\right)^2\right]}_{\mathrm{NJL}} - \underbrace{\mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right)}_{\text{Polyakov loop}} .$$

C. Ratti, M. Thaler, W. Weise, PRD 73, 014019 (2006)

$q = (q_u, q_d)^T$ the quark field, $\hat{m}_0 = \mathrm{diag}\,(m_u^0, m_d^0)$-the current quark mass,
$m_u^0 = m_d^0 = m_0$
$D^\mu = \partial^\mu - iA^\mu$ - the covariant derivative
$A^\mu(x) = g\mathcal{A}_a^\mu \frac{\lambda_a}{2}$, $\mathcal{A}_a^\mu$ the guage field in SU(3),
$A^\mu = \delta_0^\mu A^0 = -i\delta_4^\mu A_4$,
λ_a - the Gell-Mann matrix,
G - the four-point coupling constant.

The Polyakov loop and field Φ are determined as: $\langle \ell(\vec{x}) \rangle = \frac{1}{N_c} \mathrm{Tr}_c L(\vec{x})$,

$\Phi[A] = \langle\langle \ell \rangle\rangle$,

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right], \ \beta = 1/T$$

The effective potential

Polynomial form:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3.$$

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Logarithm form:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{1}{2}a\left(T\right)\bar{\Phi}\Phi + b\left(T\right)\ln\left[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\right],$$

$$a\left(T\right) = \tilde{a}_0 + \tilde{a}_1\left(\frac{T_0}{T}\right) + \tilde{a}_2\left(\frac{T_0}{T}\right)^2, b\left(T\right) = \tilde{b}_3\left(\frac{T_0}{T}\right)^3.$$

The parametrization of the potential

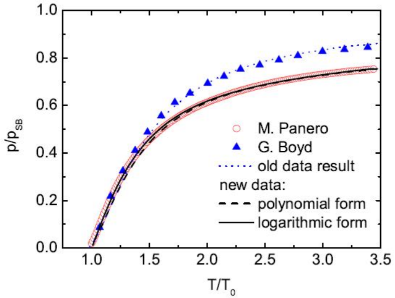


Рис. 1: The scaled pressure in the pure gauge sector as function of scaled temperature.

Old lattice data:
G. Boyd et. al. NPB 469, 419 (1996);
New one:
M. Panero PRL 103, 232001 (2009).

	$\tilde{a}_0$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{b}_3$	a_0	a_1	a_2	a_3	b_3	b_4
old	3.51	-2.47	15.2	-1.75	6.75	-1.95	2.625	-7.44	0.75	7.5
new	3.51	-5.121	20.99	-2.09	6.47	-4.62	7.95	-9.09	1.03	7.32

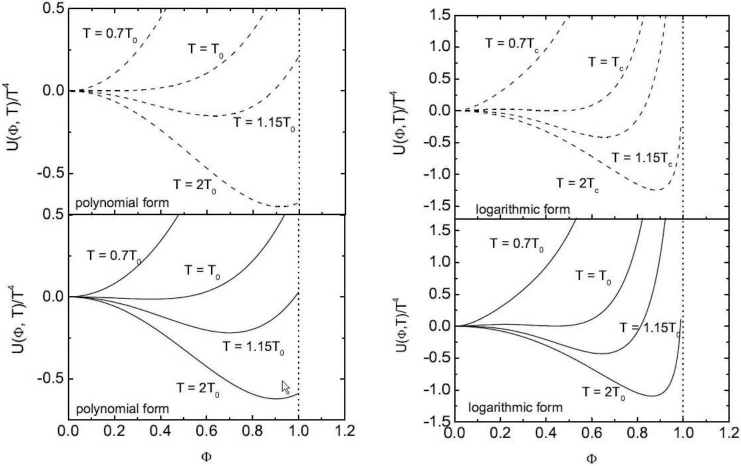


Рис. 2: The Polyakov loop effective potential $\mathcal{U}$ as a function of Φ for old (top) and new (bottom) sets of parameters.

A.V. Friesen, Yu. L. Kalinovsky, V. D. Toneev JIMPA 27, 1250013 (2012)

The gap equation

After the standard bosonisation procedure we can get the gap equation

$$m = m_0 + 4GN_cN_f \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \left[1 - f^+(E_p) - f^-(E_p)\right],$$

with modified Fermi-Dirac distribution functions:

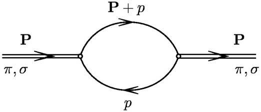
$$\begin{aligned} f^+(E_p) &= \left[\left(\Phi + 2\bar{\Phi}e^{-\beta(E_p-\mu)}\right)e^{-\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}\right]/N^+_\Phi(E_p)\,, \\ f^-(E_p) &= \left[\left(\bar{\Phi} + 2\Phi e^{-\beta(E_p+\mu)}\right)e^{-\beta(E_p+\mu)} + e^{-3\beta(E_p+\mu)}\right]/N^-_\Phi(E_p)\,, \end{aligned}$$

and

$$\begin{aligned} N^+_\Phi(E_p) &= \left[1 + 3\left(\Phi + \bar{\Phi}e^{-\beta(E_p-\mu)}\right)e^{-\beta(E_p-\mu)} + e^{-3\beta(E_p-\mu)}\right], \\ N^-_\Phi(E_p) &= \left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_p+\mu)}\right)e^{-\beta(E_p+\mu)} + e^{-3\beta(E_p+\mu)}\right]. \end{aligned}$$

Pseudo-scalar and scalar mesons

The scalar and pseudo-scalar polarization operators:



$$\Pi^{PP}_{ab}(P^2) = \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[i\gamma_5\tau^a S(p+P) i\gamma_5\tau^b S(p)\right],$$

$$\Pi^{SS}(P^2) = \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[S(p+P)S(p)\right],$$

Meson masses can be found from the equation: $1 - 2G\Pi(P^2 = M^2_{\pi,\sigma}) = 0$.

$$1 + 16GN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{M^2_\pi - 4E^2_p} \left(1 - f^+(E_p) - f^-(E_p)\right) = 0,$$

$$1 + 16GN_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \frac{E^2_p - m^2}{M^2_\sigma - 4E^2_p} \left(1 - f^+(E_p) - f^-(E_p)\right) = 0 .$$

The coupling constants can be obtained as:

$$g^{-2}_{\pi qq/\sigma qq}(T,\mu) = \frac{\partial \Pi_{\pi qq/\sigma qq}(P^2)}{\partial P^2}\Big|_{\substack{P^2=M^2_\pi \\ P^2=M^2_\sigma}} .$$

Parameters of the model

To fix the model parameters some experimental quantities are used:

- The pion decay constant $F_\pi = 0.092\mathrm{GeV}$,
- The pion mass $M_\pi = 0.139\mathrm{GeV}$

m_0 [MeV]	Λ [GeV]	G $[\mathrm{GeV}]^{-2}$	T_0 [GeV]
5.5	0.639	5.227	0.27, 0.19

The characteristic temperatures:

- T_{Mott} $(M_\pi = 2m_q)$
- T_σ^d $(M_\sigma = 2M_\pi)$

pt

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diagramm

Numerical results:

- 0.259
- 0.247

The mass spectra.

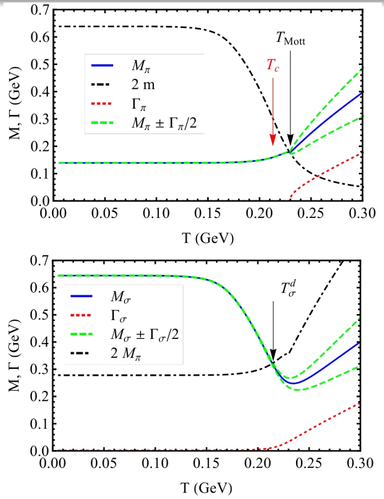


Рис. 3: Mass spectra at $\mu = 0$ GeV.

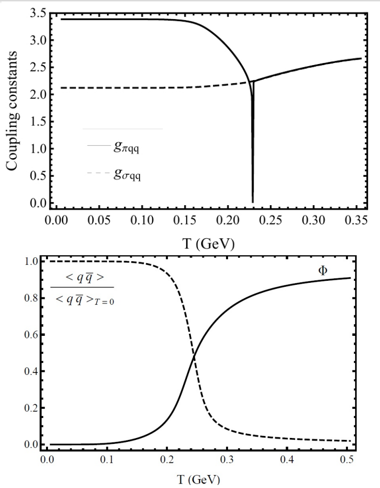


Рис. 4: The couplings and the order parameters.

The grand potential

The grand potential in the mean field approximation:

$$\Omega(\Phi, \bar{\Phi}, \mathrm{m}, \mathrm{T}, \mu) \;=\; \mathcal{U}\left(\Phi, \bar{\Phi}; \mathrm{T}\right) + \mathrm{G}\langle \bar{\mathrm{q}}\mathrm{q}\rangle^2 + \Omega_{\mathrm{q}} \;,$$

with the quark part:

$$\Omega_{\mathrm{q}} = -2\mathrm{N_c N_f} \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\mathrm{E_p} - 2\mathrm{N_f T} \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} \left[\ln \mathrm{N}^{+}_{\Phi}(\mathrm{E_p}) + \ln \mathrm{N}^{-}_{\Phi}(\mathrm{E_p})\right] \;.$$

If Ω is known, the basic thermodynamic quantities can be obtained:

$$\begin{aligned}
\mathrm{p} &= -\frac{\Omega}{\mathrm{V}}, \\
\mathrm{s} &= -\left(\frac{\partial\Omega}{\partial \mathrm{T}}\right)_{\mu}, \\
\varepsilon &= -\mathrm{p} + \mathrm{Ts} + \mu\,\mathrm{n}, \\
\mathrm{n} &= -\left(\frac{\partial\Omega}{\partial \mu}\right)_{\mathrm{T}},
\end{aligned}$$

Numerical results

Lattice: A. Ali Khan PRD 64, 074510 (2001)

$T_c^{pol} = 0.239(\text{new}), 0.253(\text{old}),\ T_c^{log} = 0.23(\text{new}), 0.234(\text{old})$

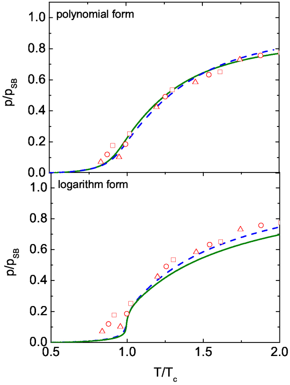


Рис. 5: The scaled pressure.

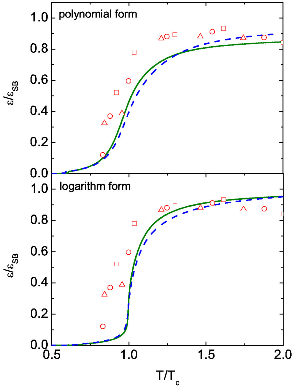


Рис. 6: The scaled energy density.

The phase diagram

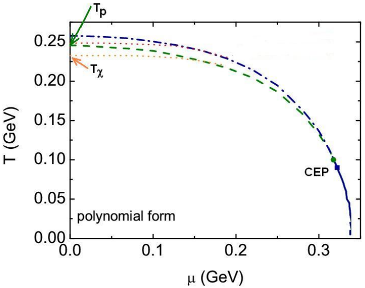


Рис. 7: The phase diagram in SU(2) PNJL.

The characteristic temperatures:

- T_χ - the crossover $\dfrac{\partial < q\bar{q} >}{\partial T}$ ($T_\chi = 0.245$ GeV)
- T_p - the deconfinement transition $\dfrac{\partial \Phi}{\partial T}$ (T_p =0.233 GeV)
- $T_c = \dfrac{T_\chi + T_p}{2}$ ($T_c = 0.239$ GeV)
- T_{CEP} ($T_{CEP} = 0.09$, $\mu_{CEP} = 0.322$ GeV)

back

The quark-quark scattering

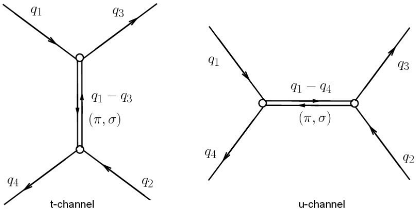


Рис. 8: Feynman diagrams of quark-quark scattering.

The amplitude of qq- scattering in π and σ - meson channels $(\Gamma_\pi = (\mathrm{i}\gamma_5) \cdot \mathrm{g}_{\pi\mathrm{qq}},\ \Gamma_\sigma = 1 \cdot \mathrm{g}_{\sigma\mathrm{qq}})$

$$-\mathrm{i}T_t = \bar{u}(q_3)\Gamma_\pi u(q_1)\frac{1}{(q_1-q_3)^2 - M_\pi^2}\bar{u}(q_4)\Gamma_\pi u(q_2) + \bar{u}(q_3)\Gamma_\sigma u(q_1)\frac{1}{(q_1-q_3)^2 - M_\sigma^2}\bar{u}(q_4)\Gamma_\sigma u(q_2),$$

$$-\mathrm{i}T_u = \bar{u}(q_4)\Gamma_\pi u(q_1)\frac{1}{(q_1-q_4)^2 - M_\pi^2}\bar{u}(q_3)\Gamma_\pi u(q_2) + \bar{u}(q_4)\Gamma_\sigma u(q_1)\frac{1}{(q_1-q_4)^2 - M_\sigma^2}\bar{u}(q_3)\Gamma_\sigma u(q_2).$$

The total quark-quark amplitude is $\mathrm{T_{qq}} = \frac{1}{4\mathrm{N_c^2}} \sum_{\mathrm{c}} |\mathrm{T_t} + \mathrm{T_u}|^2$,

$$
\begin{aligned}
|\mathrm{T_t}|^2 &= \left(|\mathrm{D_t^\sigma}|^2(\mathrm{t} - 4\mathrm{m}^2)^2 + |\mathrm{D_t^\pi}|^2\mathrm{t}^2\right), \\
|\mathrm{T_u}|^2 &= \left(|\mathrm{D_u^\sigma}|^2(\mathrm{u} - 4\mathrm{m}^2)^2 + |\mathrm{D_u^\pi}|^2\mathrm{u}^2\right), \\
\mathrm{T_t T_u^*} &= -\frac{1}{2\mathrm{N_c}}(\mathrm{D_t^\sigma D_u^\sigma}(\mathrm{tu} + 4\mathrm{m}^2(\mathrm{u}+\mathrm{t}) - 16\mathrm{m}^2) - \mathrm{D_t^\sigma D_u^\pi u}(\mathrm{t} - 4\mathrm{m}^2) \\
&\quad -\mathrm{D_t^\pi D_u^\sigma t}(\mathrm{u} - 4\mathrm{m}^2) + \mathrm{D_t^\pi D_u^\pi tu}),
\end{aligned}
$$

the mesons propagators

$$
\mathrm{D_t^{(\sigma,\pi)}} = \frac{\mathrm{g^2_{(\sigma,\pi)qq}}}{\mathrm{t} - \mathrm{M^2_{(\sigma,\pi)}}}, \quad \mathrm{D_u^{(\sigma,\pi)}} = \frac{\mathrm{g^2_{(\sigma,\pi)qq}}}{\mathrm{u} - \mathrm{M^2_{(\sigma,\pi)}}}.
$$

There are two independent types of quark-quark scattering reactions:

$\mathrm{uu} \to \mathrm{uu}(\mathrm{dd} \to \mathrm{dd})$ include both scattering channels,

$\mathrm{ud} \to \mathrm{ud}(\mathrm{du} \to \mathrm{du})$ include only u-channel,

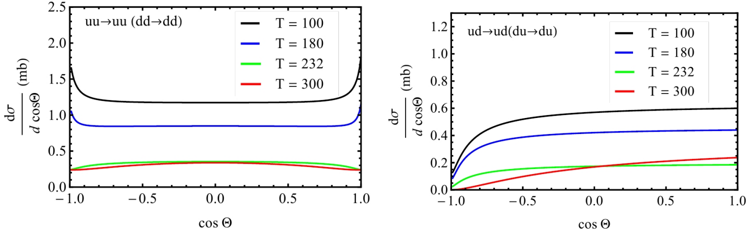


Рис. 9: The differential qq - cross-section.

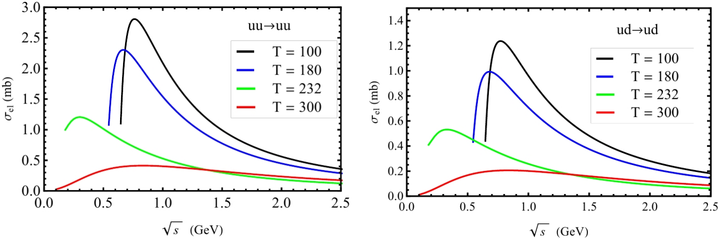


Рис. 10: The total qq - cross-section.

The quark-antiquark scattering

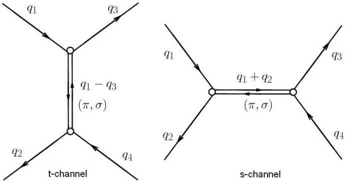


Рис. 11: The Feynman diagrams.

$$|T_t|^2 = \left(|D_t^\sigma|^2(t-4m^2)^2 + |D_t^\pi|^2 t^2\right),$$

$$|T_s|^2 = \left(|D_s^\sigma|^2(s-4m^2)^2 + |D_s^\pi|^2 s^2\right),$$

$$T_t T_s^* = -\frac{1}{2N_c}(D_t^\sigma D_s^\sigma(ts + 4m^2(s+t) - 16m^2)$$

$$D_t^\sigma D_s^\pi s(t-4m^2) - D_t^\pi D_s^\sigma t(s-4m^2) + D_t^\pi D_s^\pi ts$$

$$D_t^{(\sigma,\pi)} = \frac{g^2_{(\sigma,\pi)qq}}{t - M^2_{(\sigma,\pi)}}, \quad D_s^{(\sigma,\pi)} = \frac{g^2_{(\sigma,\pi)qq}}{s - M^2_{(\sigma,\pi)}}.$$

In the same way, for quark-antiquark scattering we have a similar situation:

$$u\bar{u} \to u\bar{u}(d\bar{d} \to d\bar{d}) \text{ includes both channels,}$$

$$u\bar{d} \to u\bar{d}(d\bar{u} \to d\bar{u}) \text{ includes only t-channel}$$

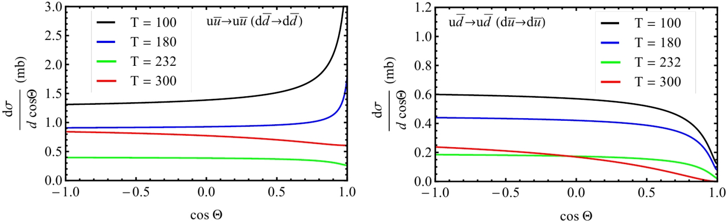


Рис. 12: The differential q$\bar{\text{q}}$ - cross-section.

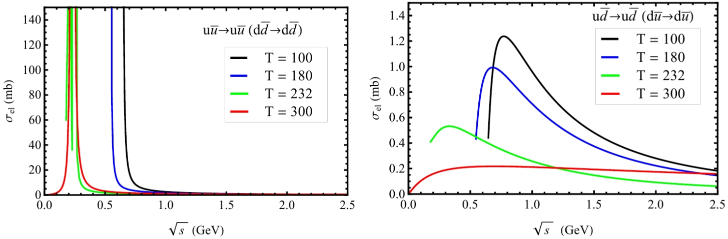


Рис. 13: The total q$\bar{\text{q}}$ - cross-section.

The quark-hadron elastic scattering

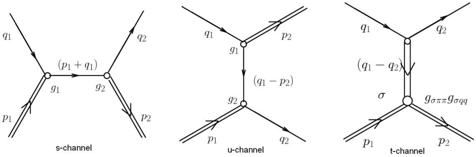


Рис. 14: The Feynman diagrams quark-hadron scattering.

$$-\,iT_1 = \overline{u}(q_2)g_2(i\gamma_5)\frac{((\widehat{q}_1+\widehat{p}_1)+m)}{s-m^2}(i\gamma_5)g_1u(q_1),\;\; iT_1^* = -iu(q_2)g_2(i\gamma_5)\frac{((\widehat{q}_1+\widehat{p}_1)+m)}{s-m^2}(i\gamma_5)g_1\overline{u}(q_1)\;.$$

$$-\,iT_2 = \overline{u}(q_2)g_2(i\gamma_5)\frac{((\widehat{q}_1-\widehat{p}_2)+m)}{u-m^2}(i\gamma_5)g_1u(q_1),\;\; iT_2^* = u(q_2)g_2(i\gamma_5)\frac{((\widehat{q}_1-\widehat{p}_2)+m)}{u-m^2}(i\gamma_5)g_1\overline{u}(q_1).$$

$$-\,iT_3 = \overline{u}(q_2)1u(q_2)\frac{1}{t-M_\sigma^2}g_{\sigma\pi\pi}g_{\sigma qq},\;\; iT_3^* = \overline{u}(q_2)(\gamma_0 1\gamma_0)u(q_2)\frac{1}{t-M_\sigma^2}g_{\sigma\pi\pi}g_{\sigma qq}.$$

The total elastic cross-section $|\mathrm{T}|^2 = \mathrm{f_c} \sum_{\mathrm{c}} |\mathrm{T_1} + \mathrm{T_2} + \mathrm{T_3}|^2$, with

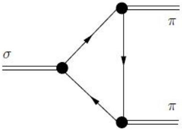
$$
\begin{aligned}
\mathrm{T_1 T_1^*} &= \mathrm{N_c K_s^2 [M_\pi^4 - (s - m^2)(u - m^2)]}, \\
\mathrm{T_1 T_2^*} &= \mathrm{T_2 T_1^* = N_c K_s K_u [-M_\pi^4 + (s - m^2)(u - m^2)]}, \\
\mathrm{T_1 T_3^*} &= \mathrm{T_3 T_1^* = N_c K_s K_t [m(s - u)]}, \\
\mathrm{T_2 T_2^*} &= \mathrm{N_c K_u^2 [M_\pi^4 - (s - m^2)(u - m^2)]}, \\
\mathrm{T_3 T_2^*} &= \mathrm{T_2 T_3^* = -N_c K_u K_t [m(s - u)]}, \\
\mathrm{T_3 T_3^*} &= \mathrm{N_c K_t^2 (4m^2 - t)}\ ,
\end{aligned}
$$

$$
\mathrm{K_s} = \frac{\mathrm{g_{\pi qq}^2}}{\mathrm{s - m^2}}, \quad \mathrm{K_u} = \frac{\mathrm{g_{\pi qq}^2}}{\mathrm{u - m^2}}, \quad \mathrm{K_t} = \frac{\mathrm{g_{\sigma\pi\pi} g_{\sigma qq}}}{\mathrm{t - M_\sigma^2}},
$$

The reactions types:

$$
\begin{aligned}
&(\mathrm{u}\pi^0 \to \mathrm{u}\pi^0, \mathrm{d}\pi^0 \to \mathrm{d}\pi^0), \\
&(\mathrm{u}\pi^0 \to \mathrm{d}\pi^+ \Leftrightarrow \mathrm{d}\pi^+ \to \mathrm{u}\pi^0, \mathrm{u}\pi^- \to \mathrm{d}\pi^0 \Leftrightarrow \mathrm{d}\pi^0 \to \mathrm{u}\pi^-), \\
&(\mathrm{u}\pi^- \to \mathrm{u}\pi^-, \mathrm{d}\pi^+ \to \mathrm{d}\pi^+).
\end{aligned}
$$

Decay $\sigma \to \pi\pi$ and coupling strength $g_{\sigma\pi\pi}$



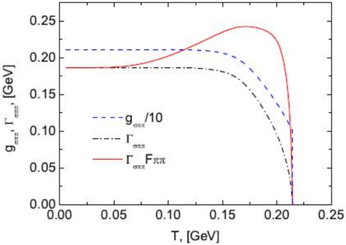
A. Friesen et al. Phys. Part.Nucl. Let. 9, 1 (2012)

$$g_{\sigma\pi\pi}(T,\mu) = 2 g_{\sigma qq} g^2_{\pi qq} A_{\sigma\pi\pi}(T,\mu)$$

$$\Gamma_{\sigma\to\pi\pi} = \frac{3}{2}\,\frac{g^2_{\sigma\pi\pi}}{16\pi\ M_\sigma}\sqrt{1-\frac{4M_\pi^2}{M_\sigma^2}}\ .$$

$$A_{\sigma\pi\pi} = 2mN_cN_f \int \frac{d^3q}{(2\pi)^3}\,\frac{F_E}{2E_q}\cdot\frac{(q\cdot p)^2 - (2M_\sigma^2 + 4M_\pi^2)(q\cdot p) + M_\sigma^2/2 - 2M_\sigma^2 E_q^2}{(M_\sigma^2 - 4E_q^2)((M_\pi^2 - 2(q\cdot p))^2 - M_\sigma^2 E_q^2)}$$

$$F_E = (1 - f^+(E_p) - f^-(E_p))$$



$g_{\sigma\pi\pi} = 2.0^{+0.3}_{-0.9}$ GeV

$\Gamma_{\sigma\pi\pi} = 0.282^{+0.07}_{-0.05}$ GeV
(BES, PRD 65, 097505 (2002); hep-ex/0104050)

$\Gamma_{\sigma\pi\pi} = 0.324^{+0.04}_{-0.042}$ GeV
(E791, Fermilab, PRD 68, 014011 (2003))

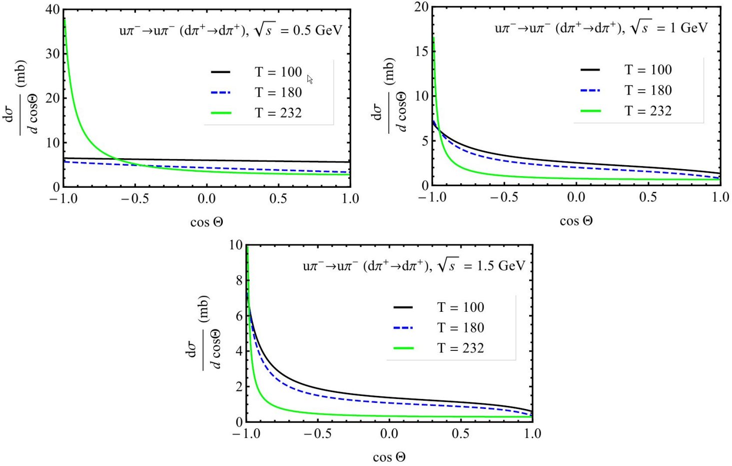


Рис. 15: The differential cross section.

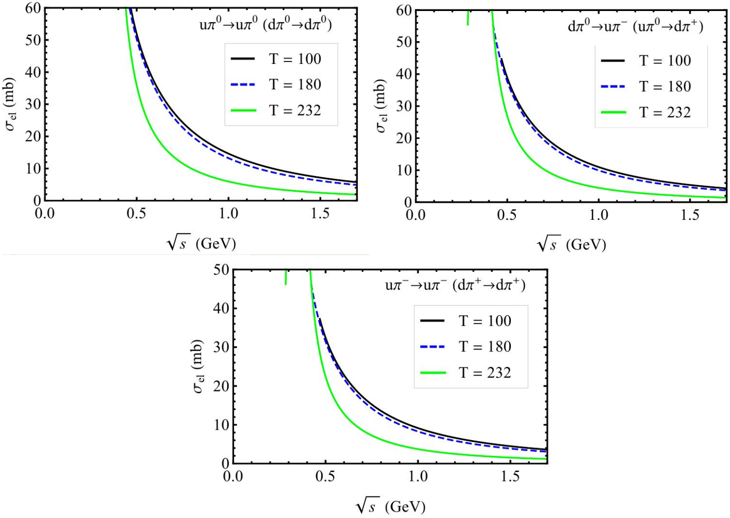


Рис. 16: The total cross section.

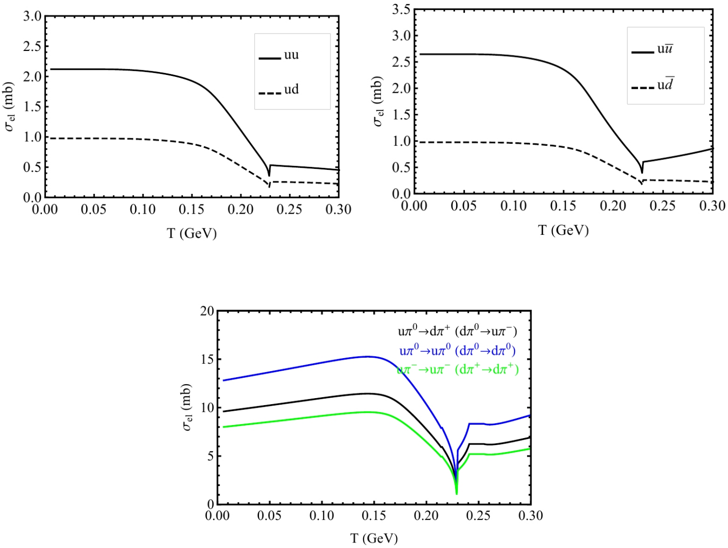


Рис. 17: The temperature dependence of total cross sections.

Conclusion

- In the context of NJL model with Polyakov loop the differential and total cross sections of the elastic quark-quark, quark-antiquark and quark-hadron scattering.
- The obtained results can be used at investigation of the transitions from hadron phase to quark-gluon phase.