Yang-Mills Theory in Landau Gauge at Non-Vanishing Temperature

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TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

... work done in collaboration with Jan M. Pawlowski (Univ. Heidelberg)

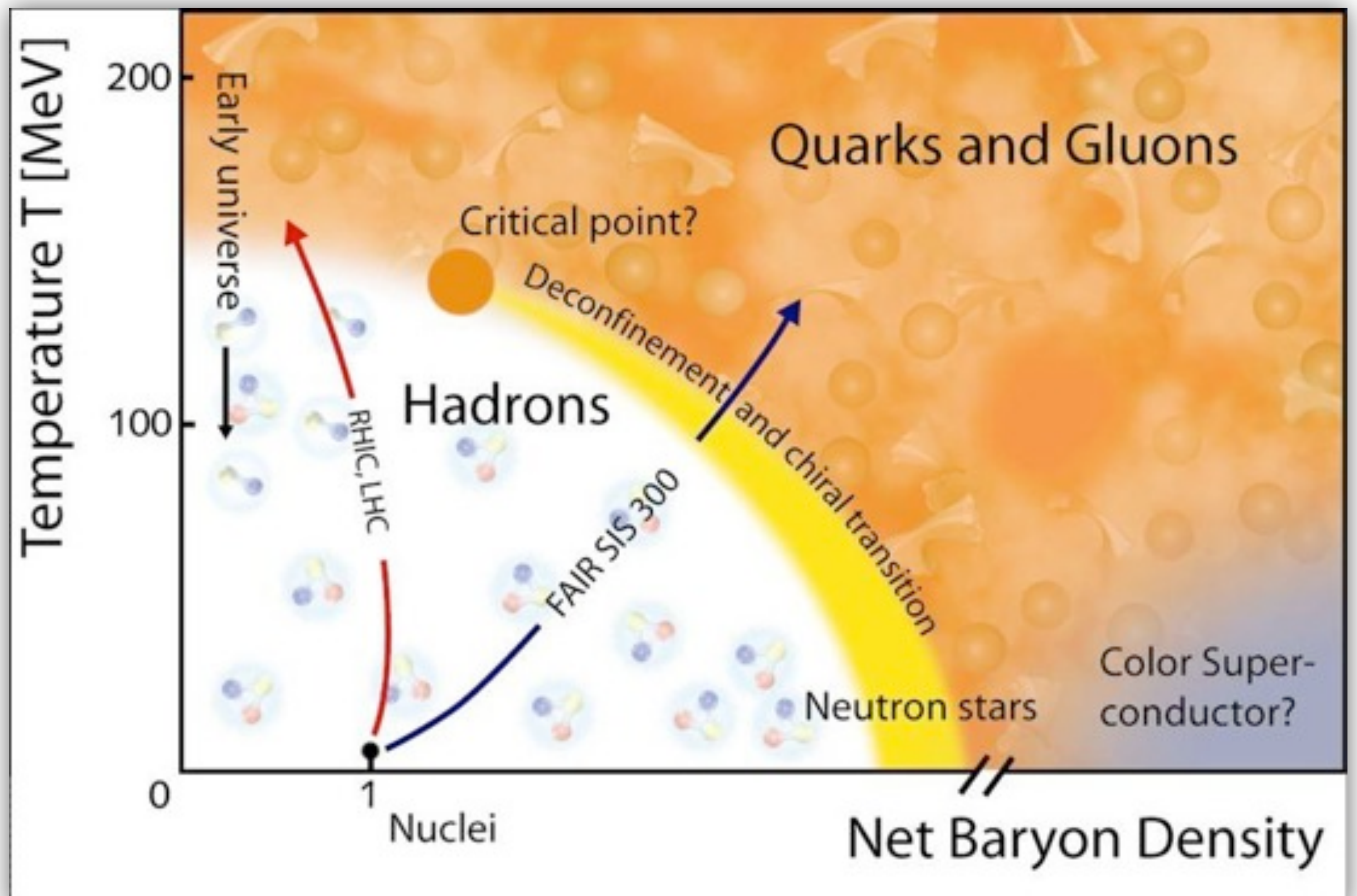
based on:

Fister, Pawlowski, arXiv: 1112.5440 [hep-ph].

Fister, Pawlowski, PoS QCD-TNT-II2011 (2011) 021 [arXiv: 1112.5429 [hep-ph]].

Motivation

ultimate goal: computation of physical observables from microscopic dynamics



credits: GSI Darmstadt

experiment:

- thermodynamic potential,
- pressure,
- entropy,
- screening masses, etc.

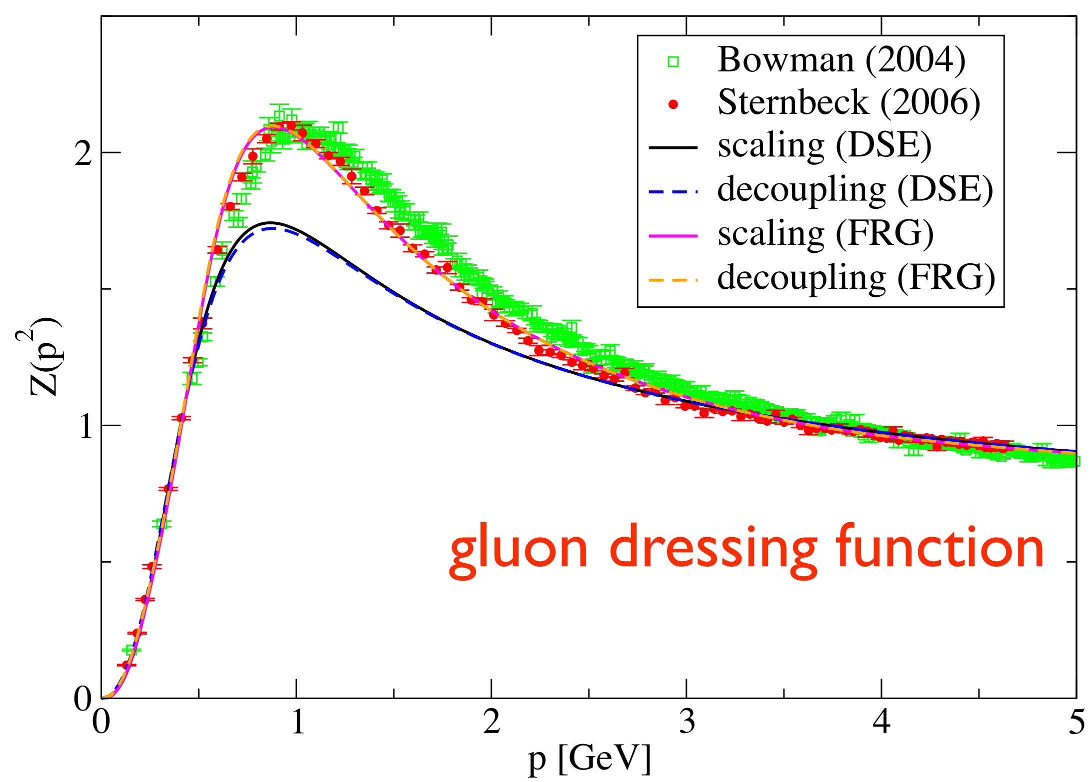
study QCD phase diagram
fully non-perturbatively:

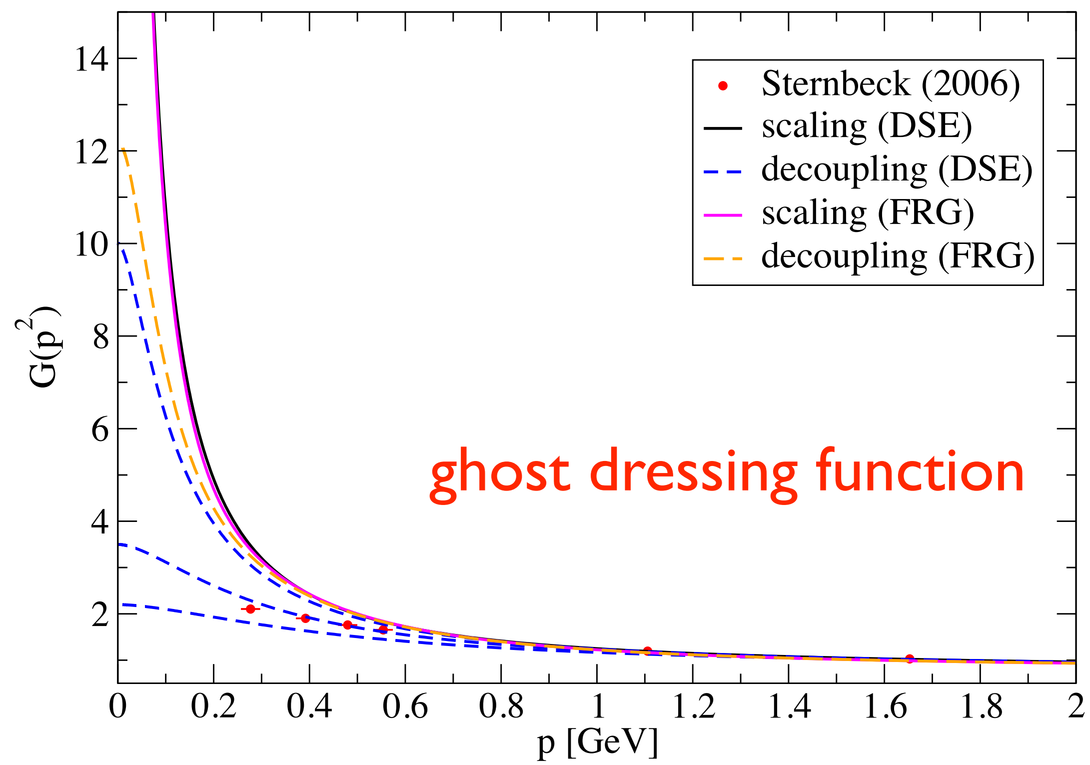
functional renormalisation group

applicable for all temperatures

Outline

- Motivation
- Yang-Mills Flow Equation
- Thermal Flow Equation
- Ghost-Gluon Vertex at Non-Vanishing Temperature
- Propagators at Non-Vanishing Temperature
- Pressure





taken from: Fischer, Maas, Pawlowski, Annals Phys. 324 (2009).

Yang-Mills Theory - Basics

Yang-Mills action: $$S_{YM} = \int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

Landau gauge: $\xi \to 0$

covariant derivative: $$D^a_{\mu\nu} = \delta^{ab} \partial_\mu + g f^{abc} A^c_\mu$$

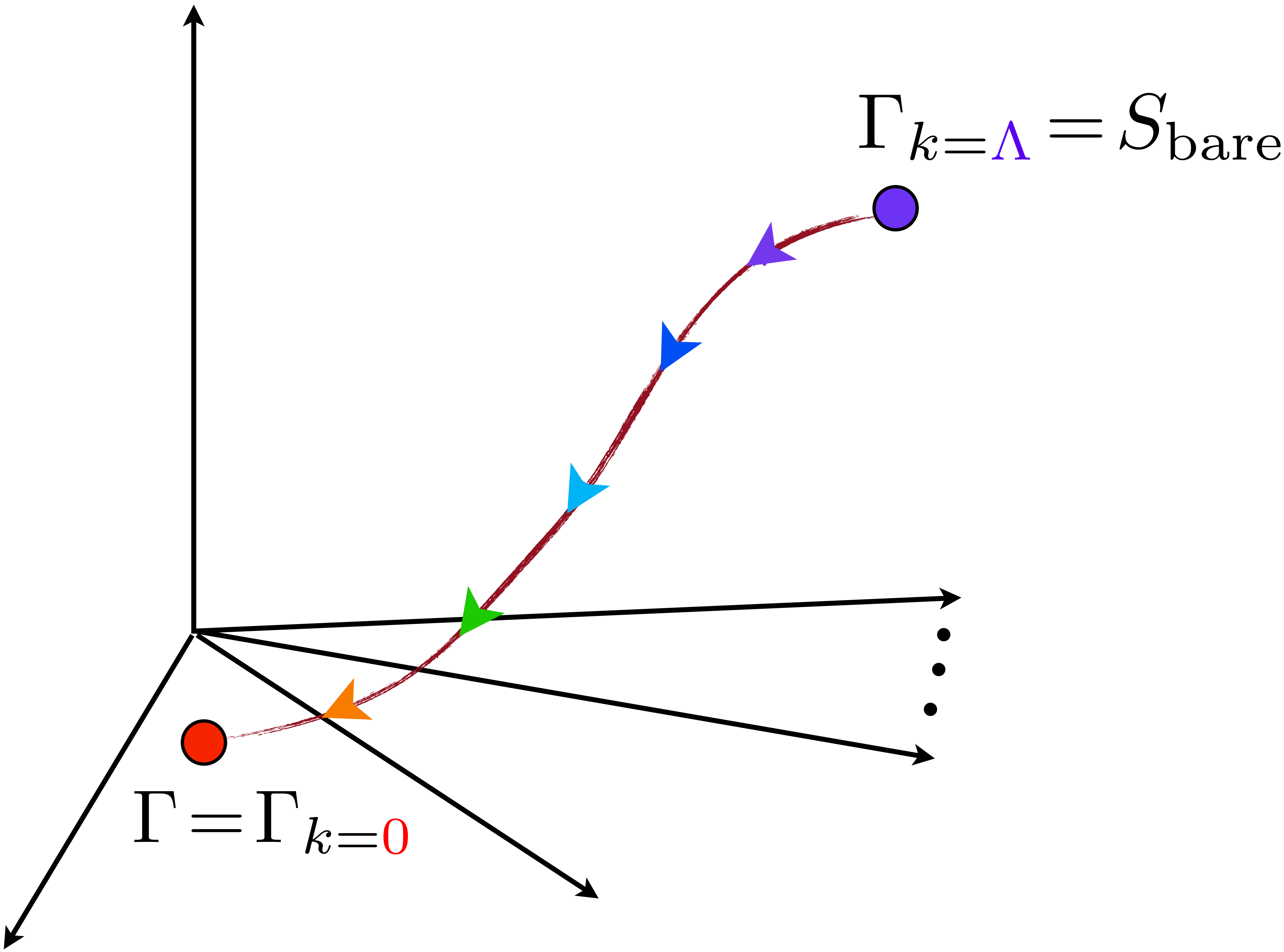
field-strength tensor: $$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

effective action $\Gamma[A, \bar{c}, c]$

$$Z[J, \eta, \bar{\eta}] \equiv e^{W[J,\eta,\bar{\eta}]} = \int \mathcal{D}A \mathcal{D}\bar{c} \mathcal{D}c \; e^{-S[A,\bar{c},c] + \int (J\cdot A + \bar{\eta}\cdot c - \bar{c}\cdot\eta)}$$

$$\Gamma[A, \bar{c}, c] = \sup_{J,\eta,\bar{\eta}} \left(\int (J \cdot A + \bar{\eta} \cdot c - \bar{c} \cdot \eta) \; - \; W[J, \eta, \bar{\eta}] \right)$$

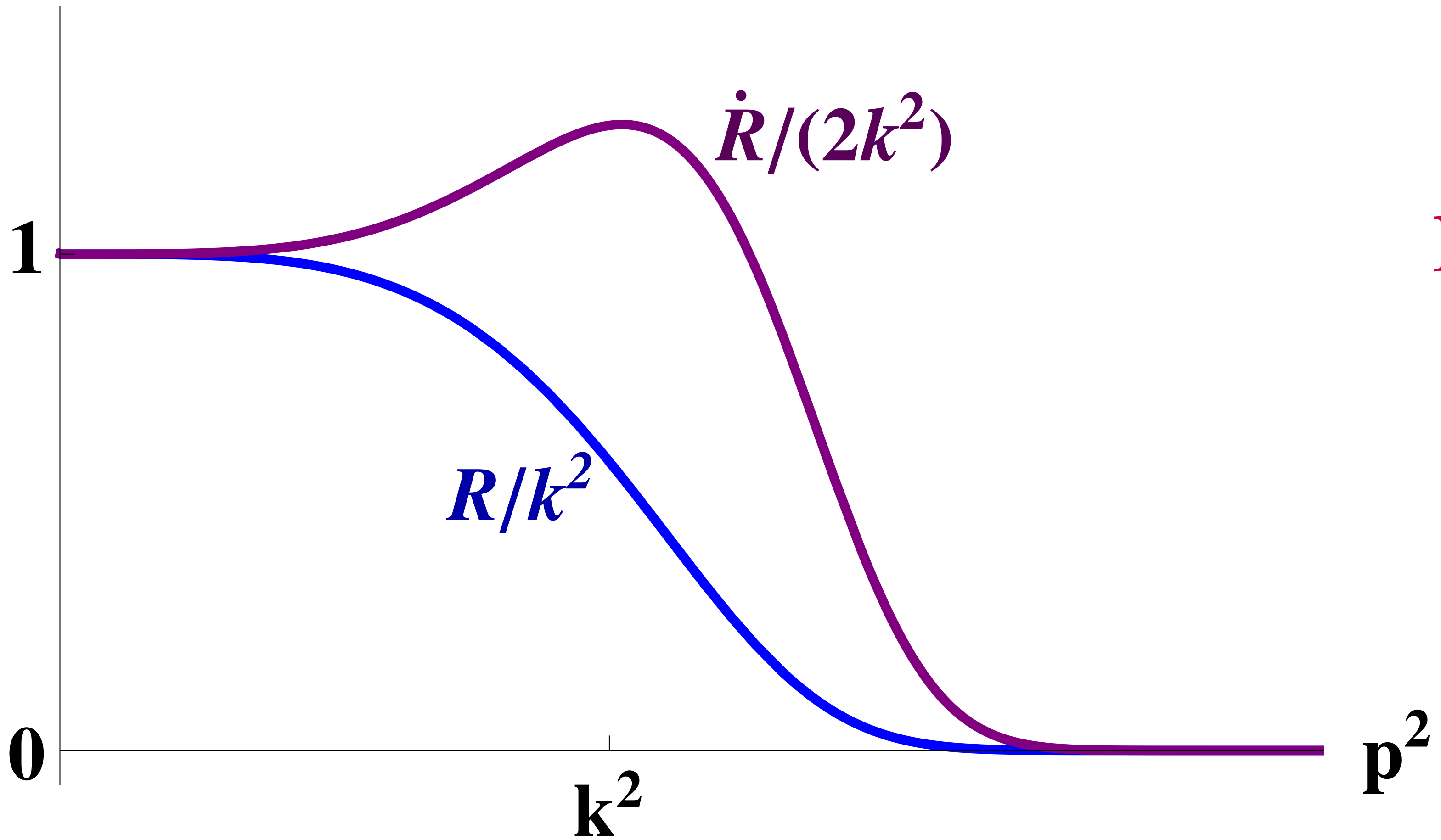
Functional Renormalisation Group (FRG)



Functional Renormalisation Group (FRG)

suppression of infrared fluctuation via a modification of the propagator

$$S_{\rm YM} \to S_{\rm YM} + \frac{1}{2}\int_p A^a_\mu R^{ab}_{k,\mu\nu}(p) A^b_\nu + \int_p \bar{c}^a R^{ab}_k(p,k) c^b$$



propagator

$$G_k[\varphi] = \frac{1}{\Gamma^{(2)}_k[\varphi] + R_k(p)}$$

k-dependent

Flow Equation (for Yang-Mills Theory)

Wetterich, Phys. Lett. B301 (1993) 90-94.

$$\partial_t \Gamma_k[A,\bar{c},c] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[A,\bar{c},c]+R_k}\;\partial_t R_k\right\} - \partial_t C_k$$

$\downarrow$

$\partial_t = k\,\partial_k$

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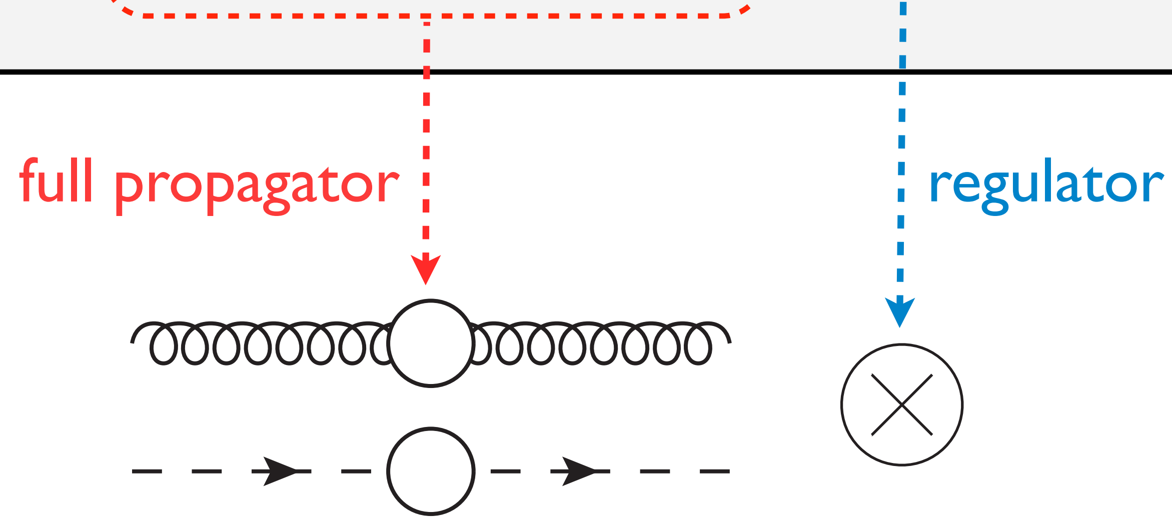
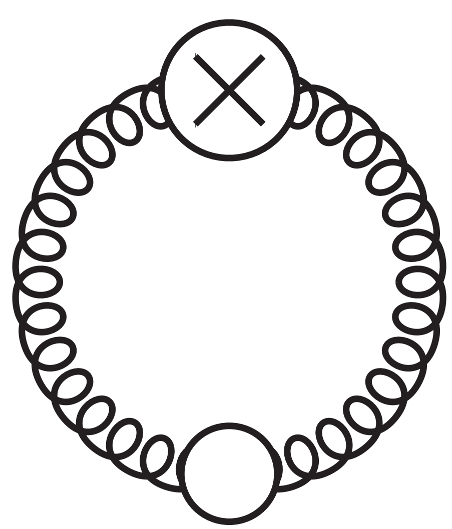
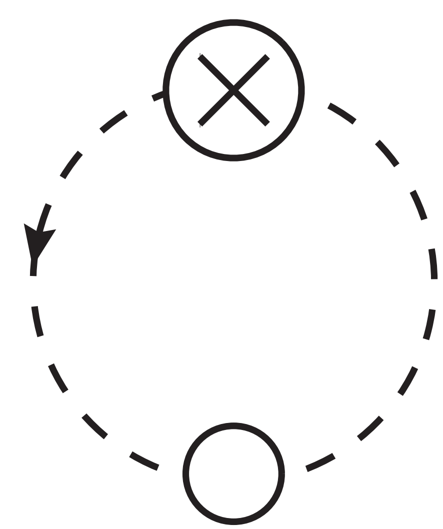
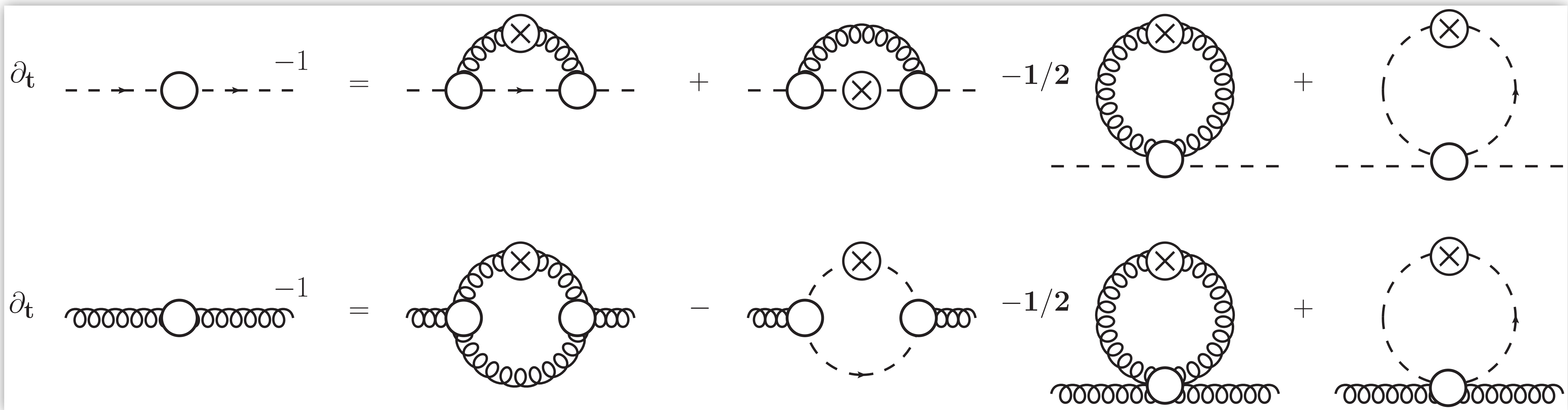


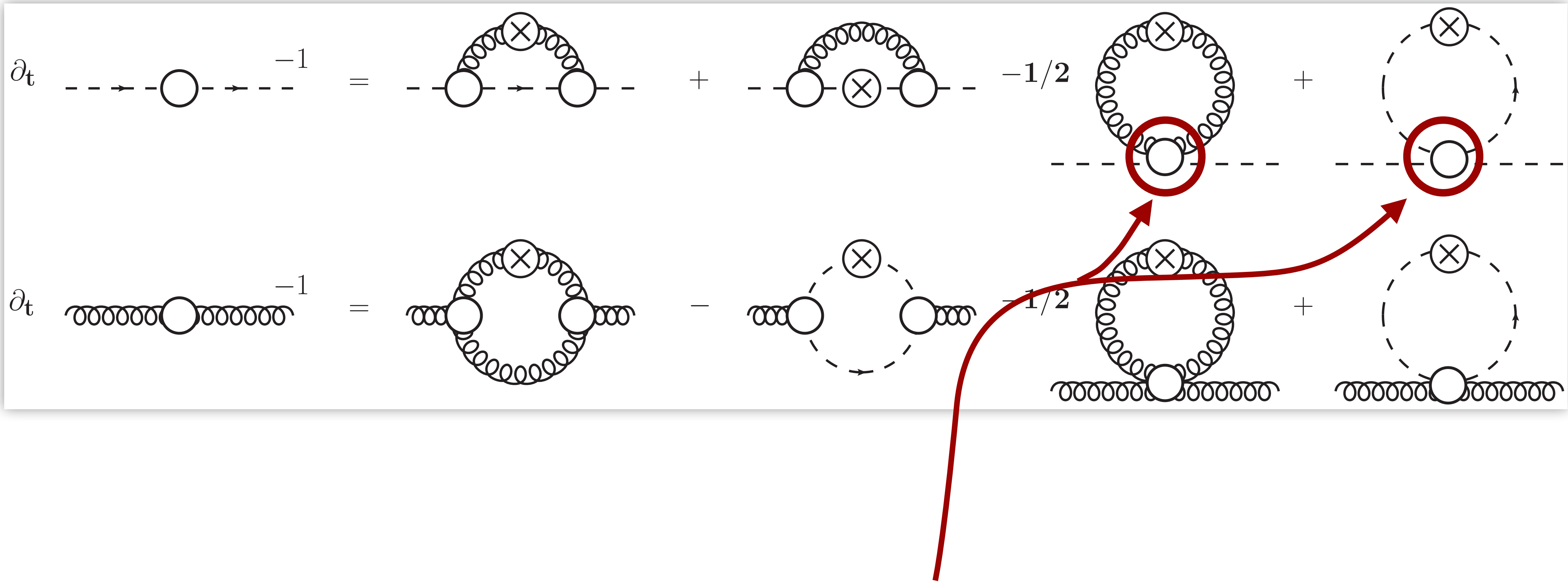
$\partial_t \Gamma[A,\bar{c},c] \;=\; \frac{1}{2}$  $-$ 

Yang-Mills Propagators

obtained from generating flow equation via functional derivation wrt the in-/out-going fields



Yang-Mills Propagators



resummation: insert Dyson-Schwinger equations for

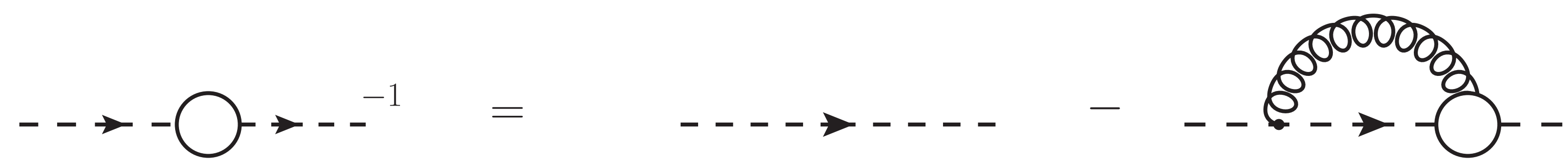
$$\Gamma^{(4)}_{\bar{c}c\bar{c}c}, \quad \Gamma^{(4)}_{\bar{c}A^2c}$$

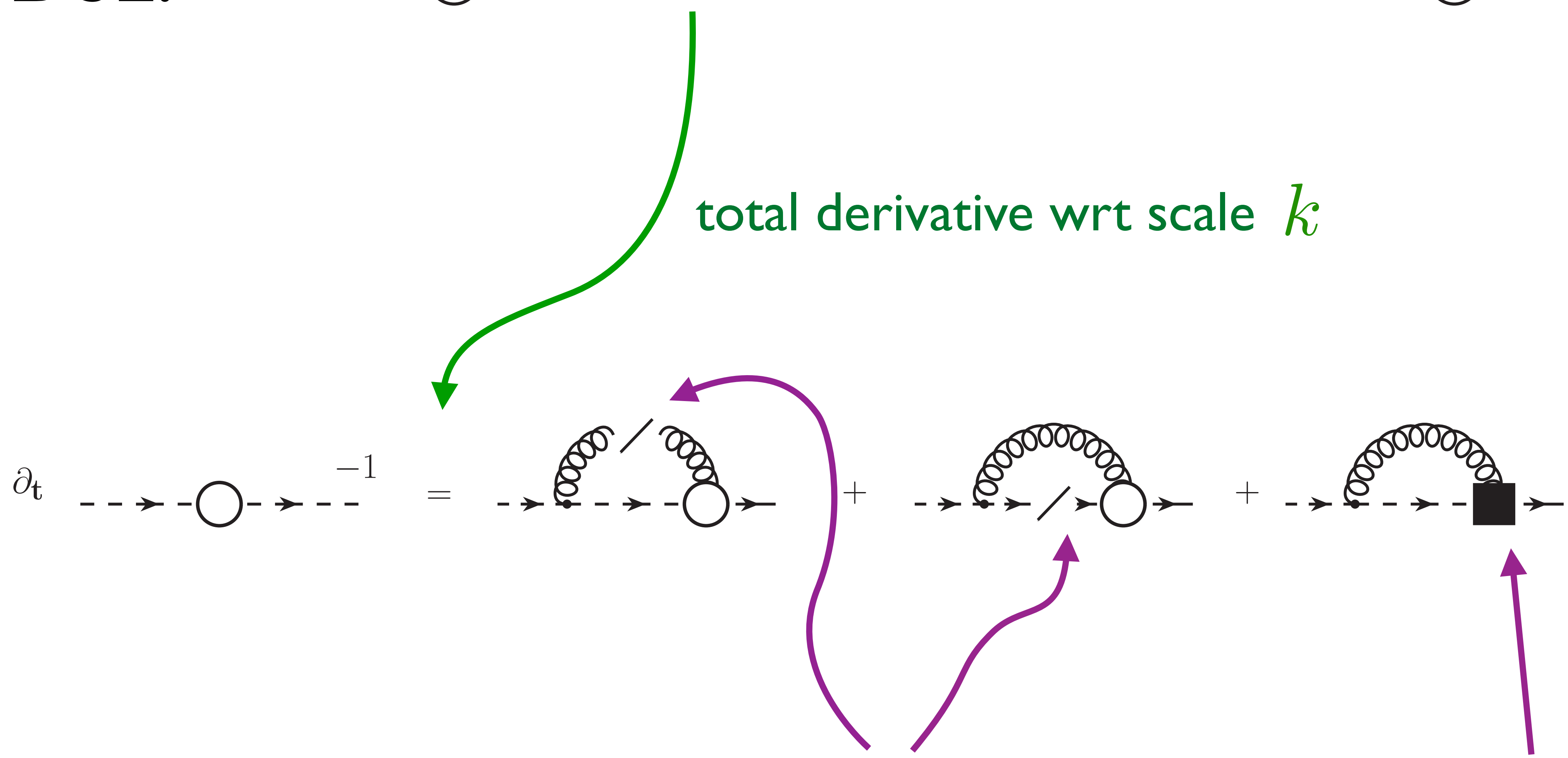
... this is equivalent to taking a derivative of the DSE

Dyson-Schwinger Approximation for the Ghost

Use: *The flow equation is the differential form of the Dyson-Schwinger equation.*

DSE:

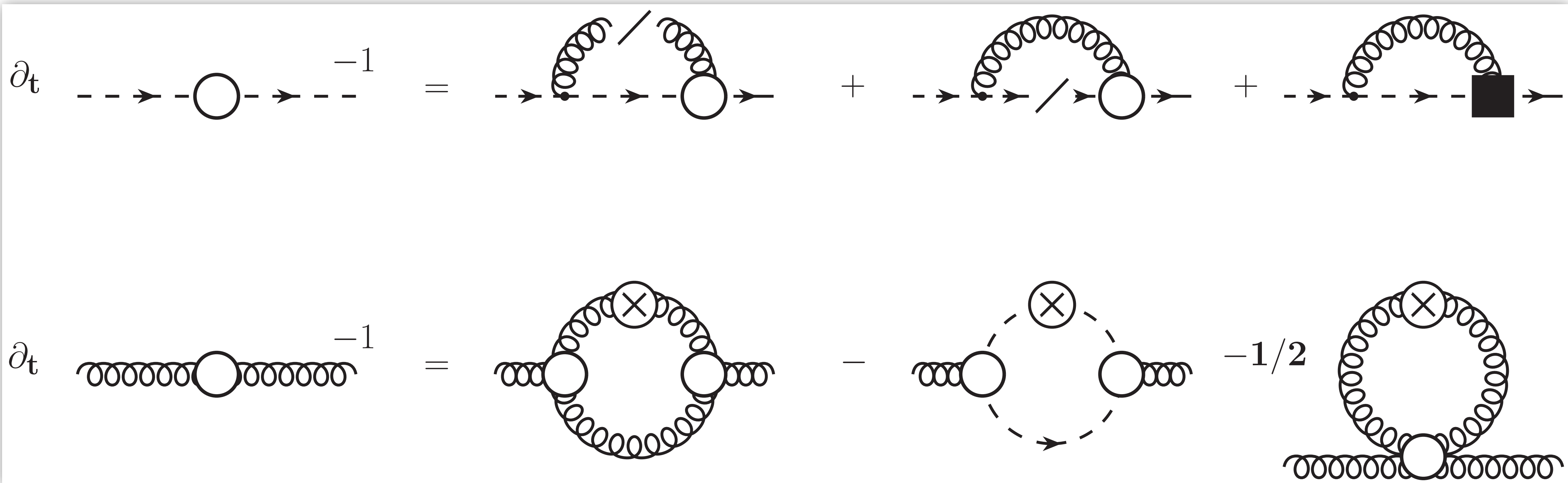


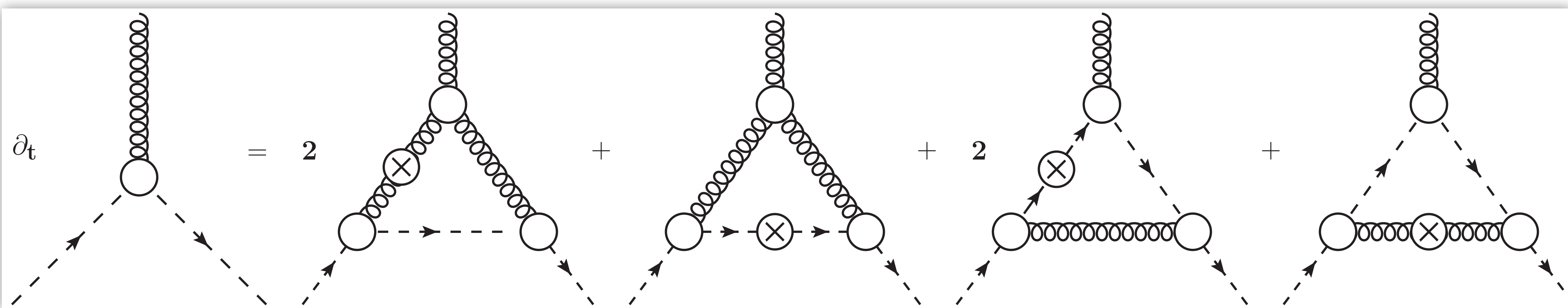


$$G[\varphi]\left(\partial_t\left(\Gamma_k^{(2)}[\varphi]+R_k\right)\right)G[\varphi]$$

$$\partial_t\Gamma^{(3)}_{\bar{c}Ac,k}$$

Truncation



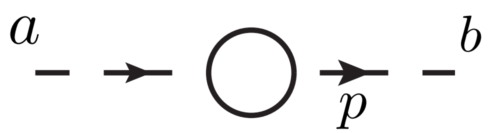


The non-triviality of the ghost-gluon vertex is crucial at finite temperature.

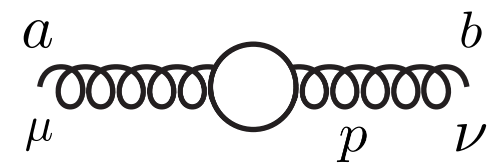
Yang-Mills Propagators - Parametrisation

zero temperature:

ghost propagator $D^{ab}_{\rm gh}(p) = -\frac{G(p)}{p^2}\delta^{ab}$



gluon propagator $D^{ab}_{{\rm gl},\mu\nu}(p^2) \; = \Pi_{\mu\nu}(p)\frac{Z(p^2)}{p^2}\delta^{ab}$



transversal projector in 4d

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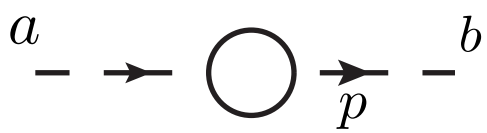

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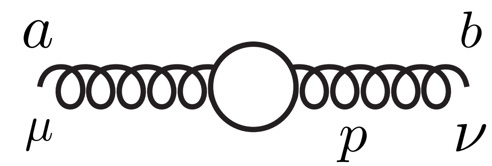


transversal projector in 4d

finite temperature (Matsubara formalism): $p_0 = 2\pi T n_p \ , \ n_p \ldots$ Matsubara modes

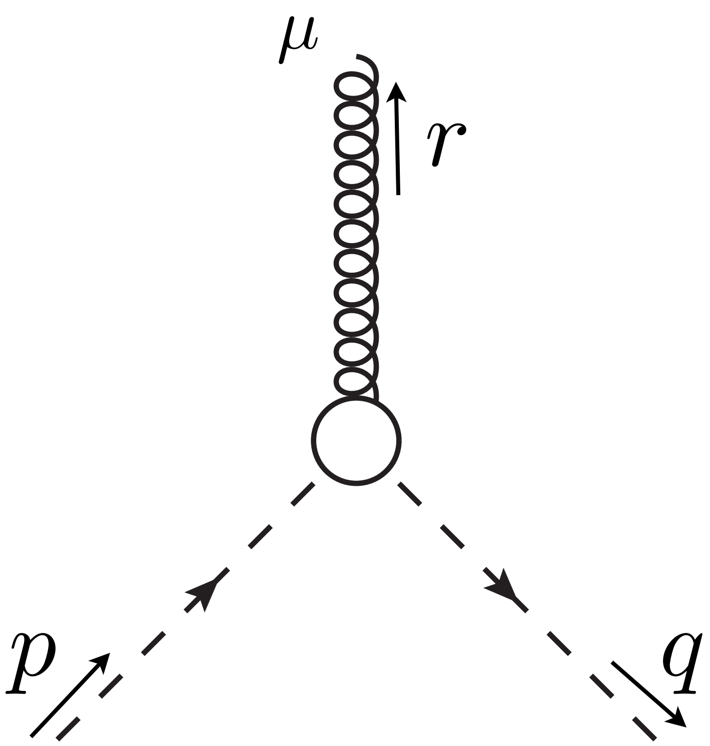
ghost propagator $D^{ab}_{\mathrm{gh}}(p_0^2, \vec{p}^{\,2}) = -\delta^{ab}\frac{G(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}}$

gluon propagator $D^{ab}_{\mathrm{gl},\mu\nu}(p_0^2, \vec{p}^{\,2}) = \delta^{ab} P^T_{\mu\nu}\frac{Z_T(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}} + \delta^{ab} P^L_{\mu\nu}\frac{Z_L(p_0^2, \vec{p}^{\,2})}{p_0^2 + \vec{p}^{\,2}}$

transversal projector in 3d subspace

longitudinal projector in 3d subspace

Ghost-Gluon-Vertex - Parametrisation



$$\Gamma^{(3)}_{\bar{c}Ac}(r;p,q) = \underbrace{i g_0 \; q_\mu}_{\text{class. vertex}} \; Z_{\bar{c}Ac}(r;p,q)$$

choose symmetric point: $p^2 = q^2 = r^2 = P^2$

$$Z_{\bar{c}Ac}(r;p,q) \rightarrow Z_{\bar{c}Ac}(P)$$

identify momentum scale
with renormalisation group scale *k*

normalisation in the infrared: $Z_{\bar{c}Ac}(0) = 1$

Flow Equation for Thermal Fluctuations

at non-vanishing temperature: quantum and thermal fluctuations

idea:

(1) calculate quantum fluctuations at zero temperature
(2) project onto thermal fluctuations and add to (1)

thermal flow:

$$\Delta\Gamma_{k,T} = \Gamma_{k,T} - \Gamma_{k,T=0}$$

Litim, Pawlowski, arXiv: hep-th/9901063.
Litim, Pawlowski, JHEP 11 (2006) 026.

advantages:

- $T \to 0$ limit trivially satisfied
- truncations for (1) and (2) may differ
- only the difference is sensitive to truncations

Flow Equation for Thermal Fluctuations

technique

(1) take zero-temperature propagator $\Gamma^{(2)}_{k=0,T=0}$ as input
(2) evolve initial condition from $k \to \Lambda :\ \Gamma^{(2)}_{k=\Lambda,T=0}$
(3) evolve propagator at finite temperature

ad (2):

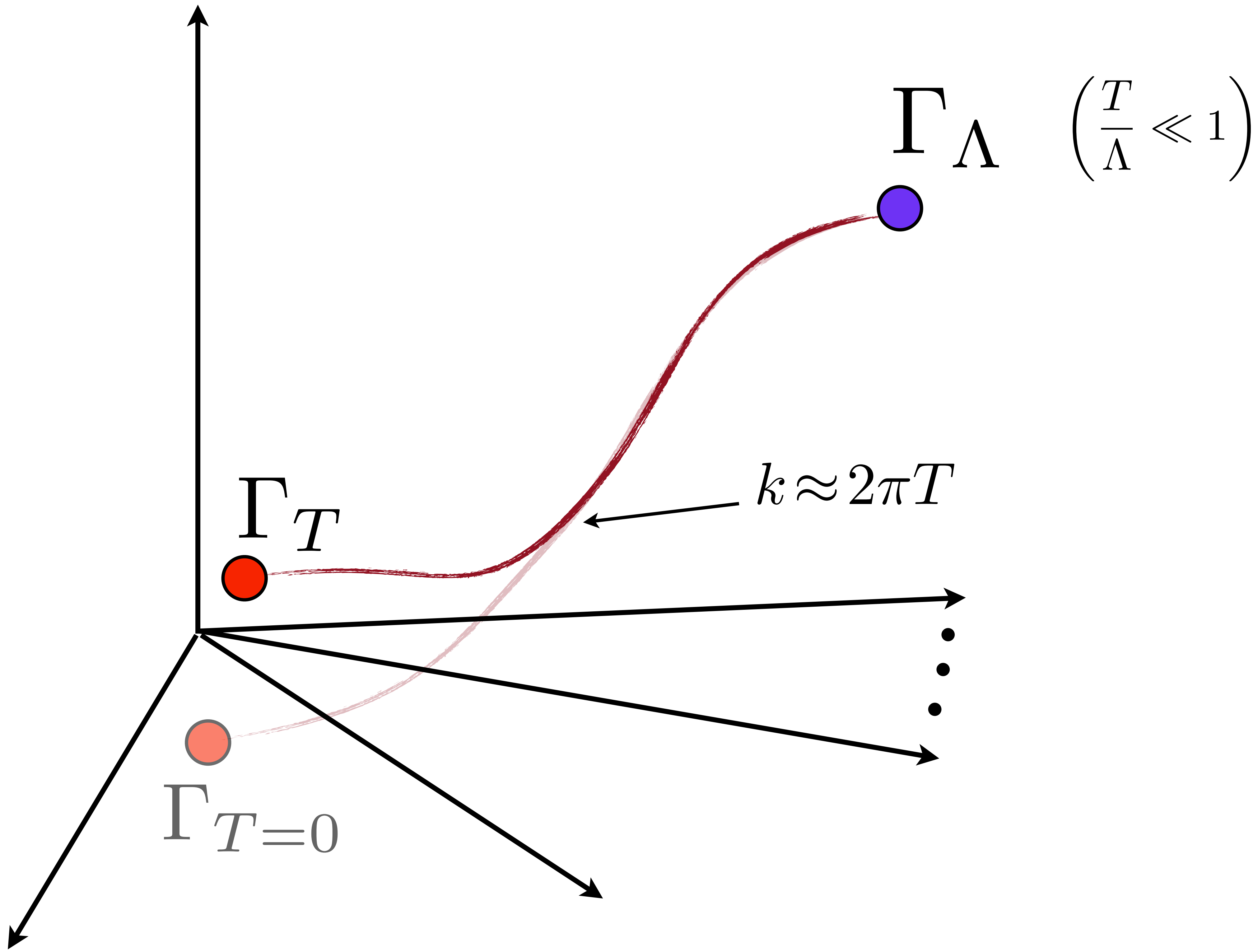
$$\Gamma^{(2)}_{k=\Lambda} = \underbrace{\Gamma^{(2)}_{k=0}}_{input} + \int_0^{\Lambda} \frac{dk'}{k'} \partial_{t'} \Gamma^{(2)}_{k',T=0}$$

UV input for temperature calculation

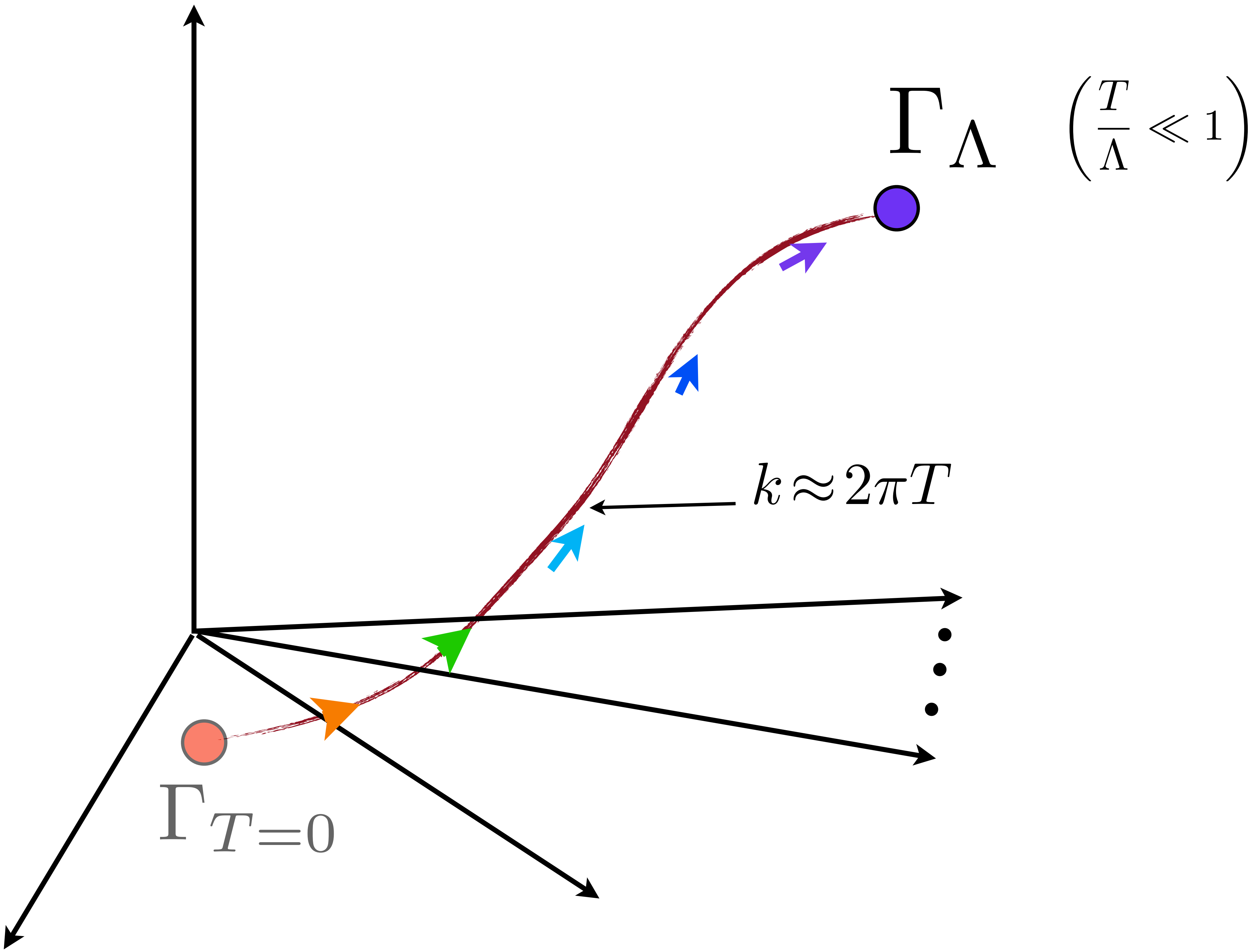
ad (3):

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k=\Lambda} + \int_{\Lambda}^{k} \frac{dk'}{k'} \partial_{t'} \Gamma^{(2)}_{k',T}$$

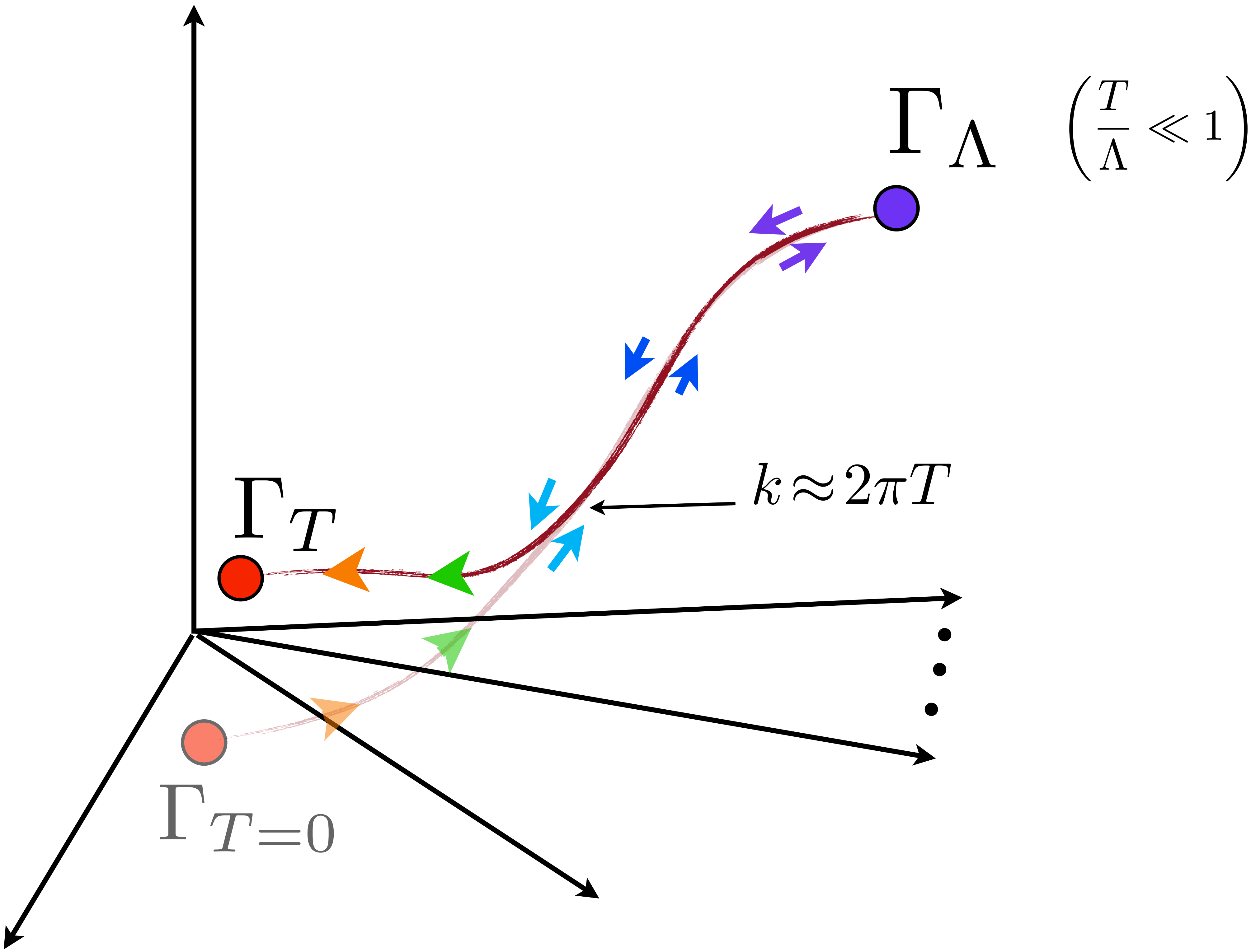
Flow at T=0 vs T>0



Solving the Flow Equation at T=0



Solving the Flow Equation at T>0



Ghost-Gluon Vertex

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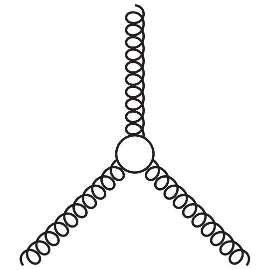
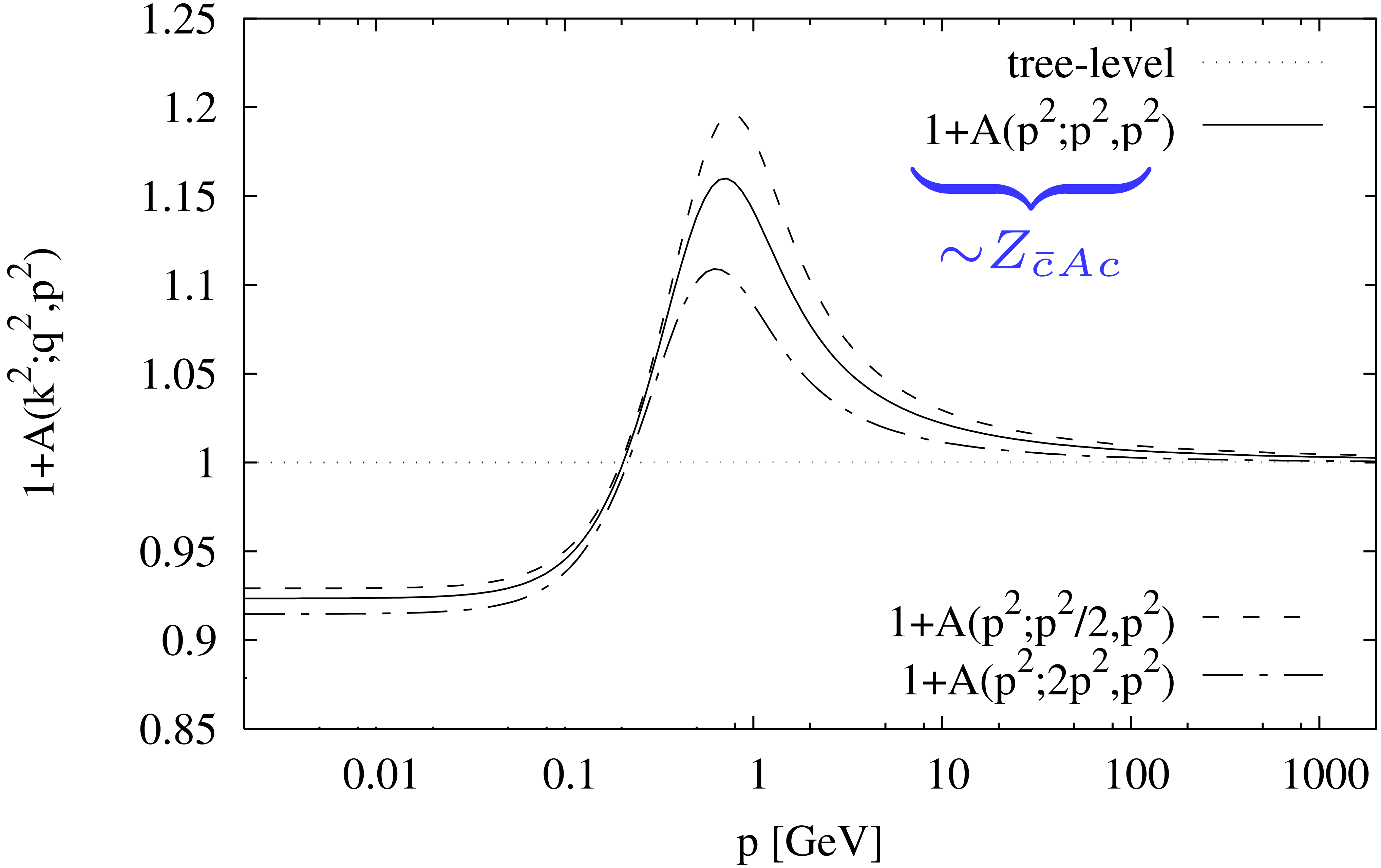
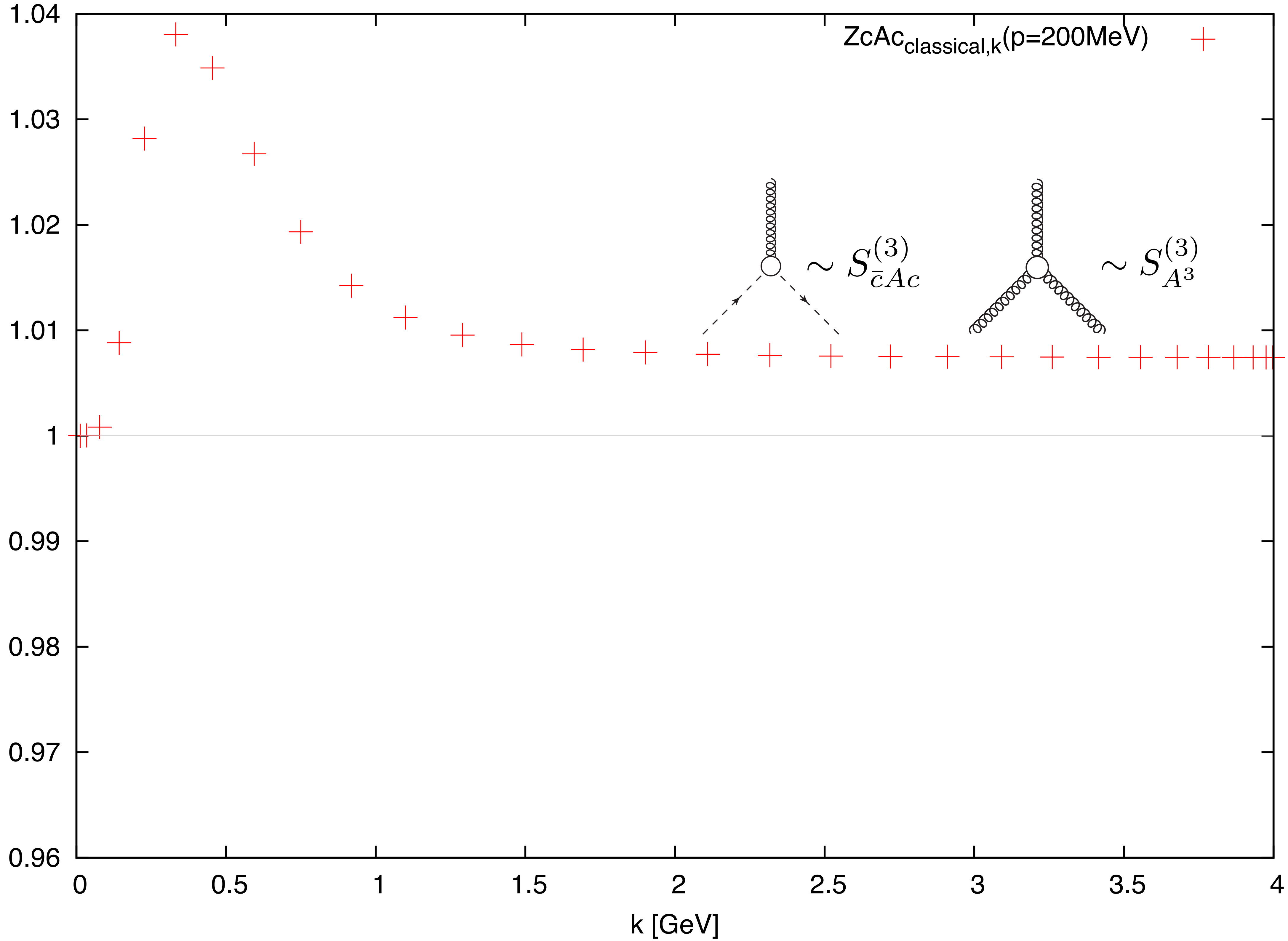


was dropped

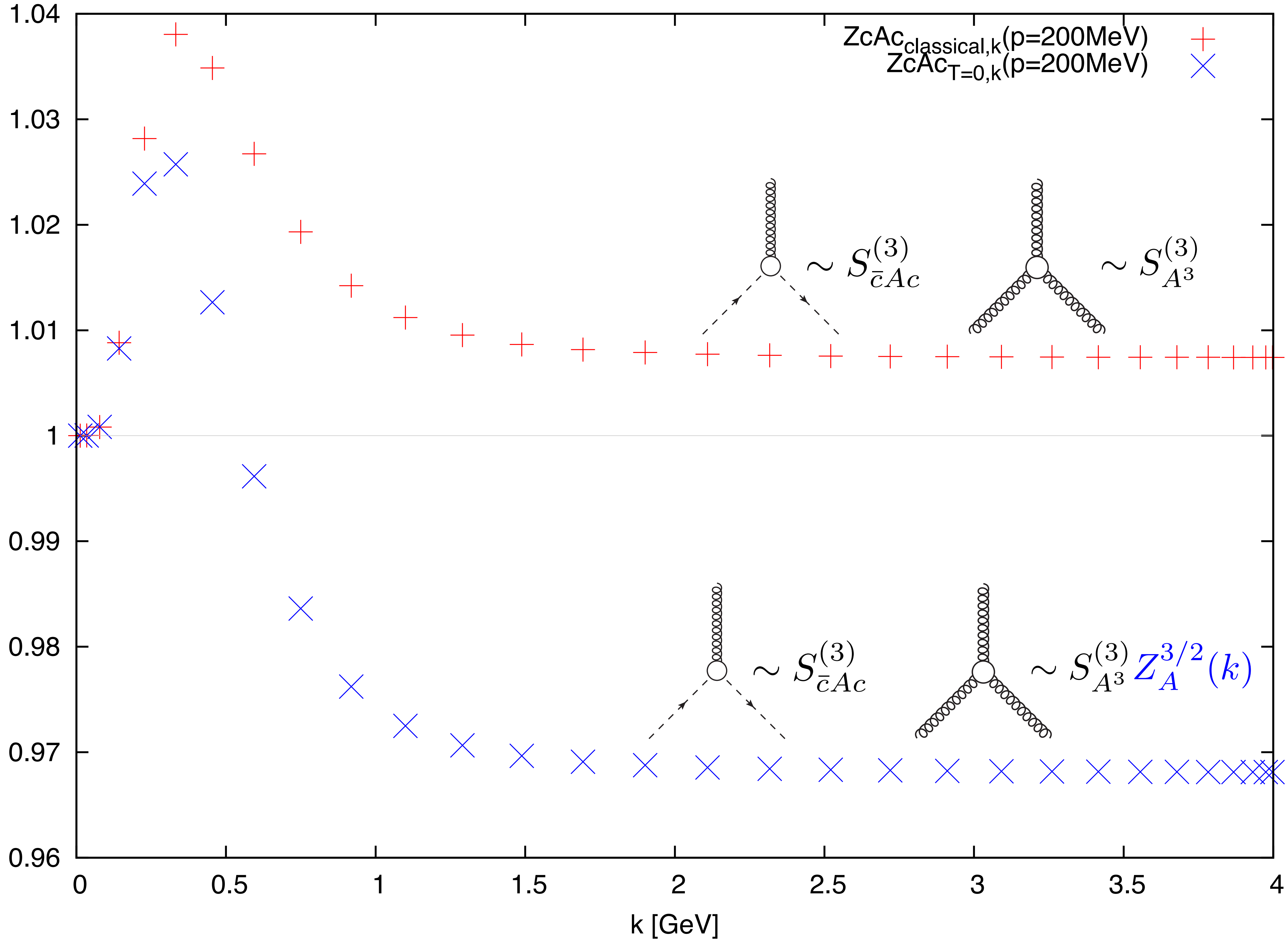


W. Schleifenbaum, A. Maas, J. Wambach, R. Alkofer, Phys. Rev. **D72** (2005) 014017.

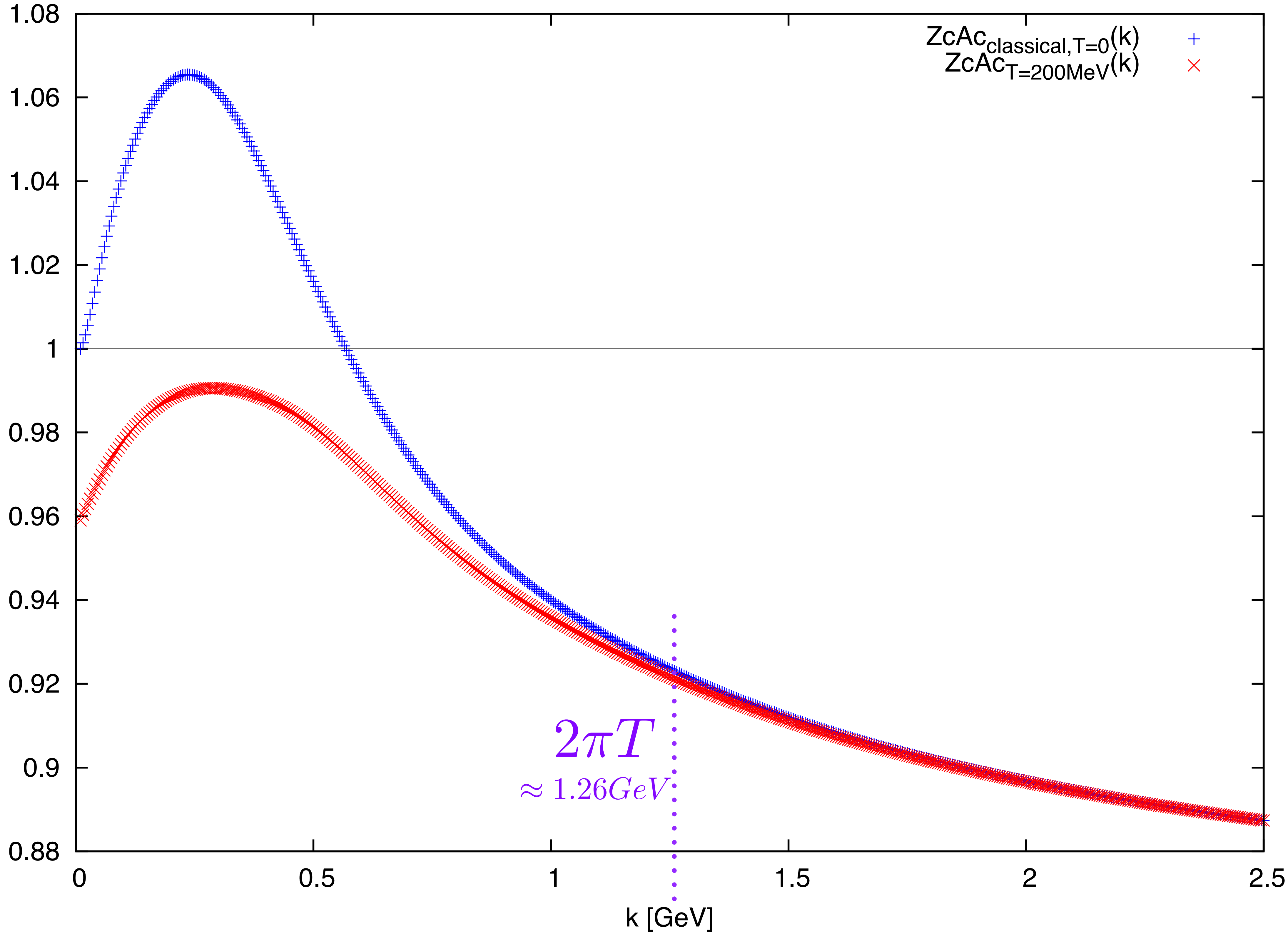
Ghost-Gluon Vertex



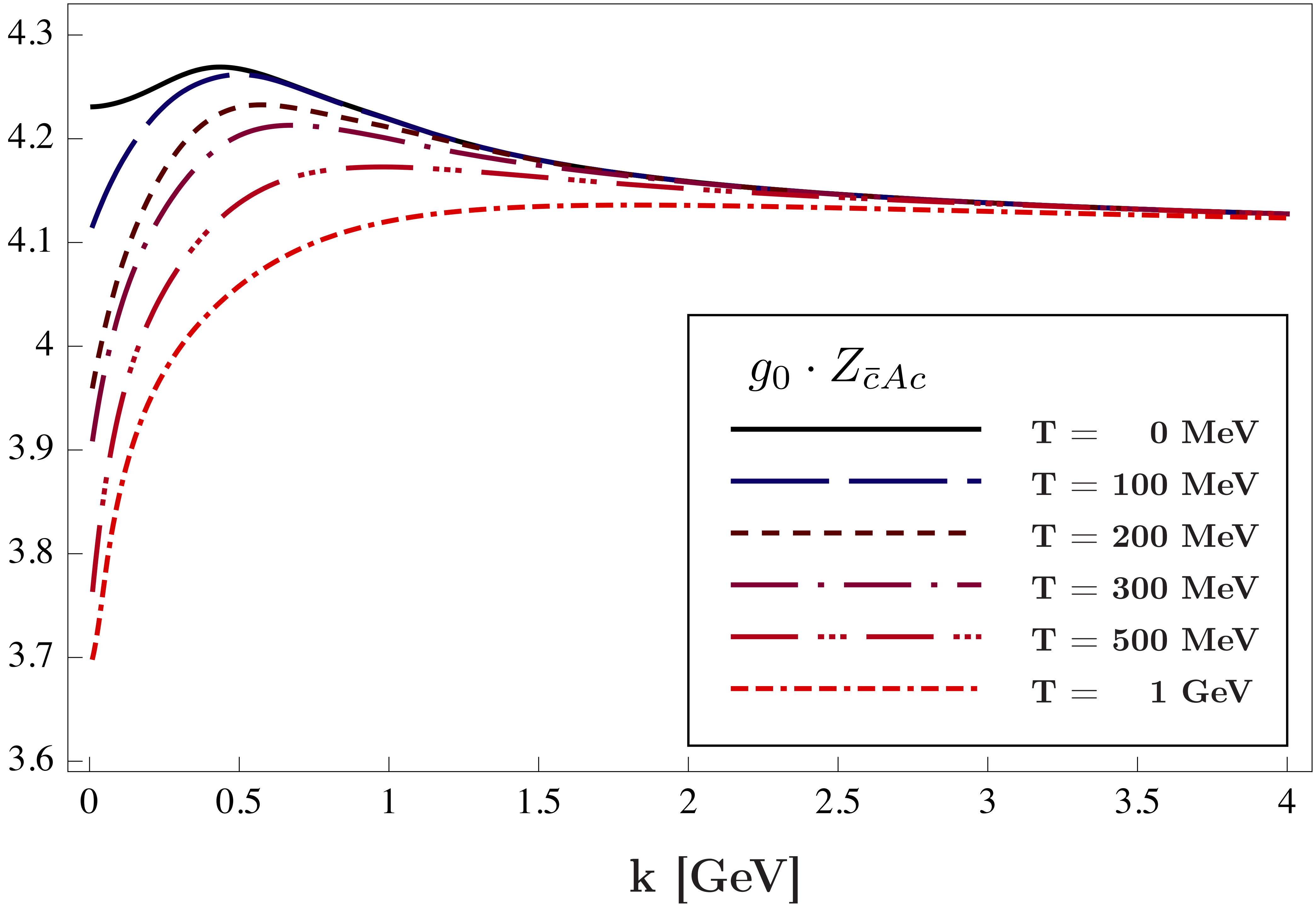
Ghost-Gluon Vertex



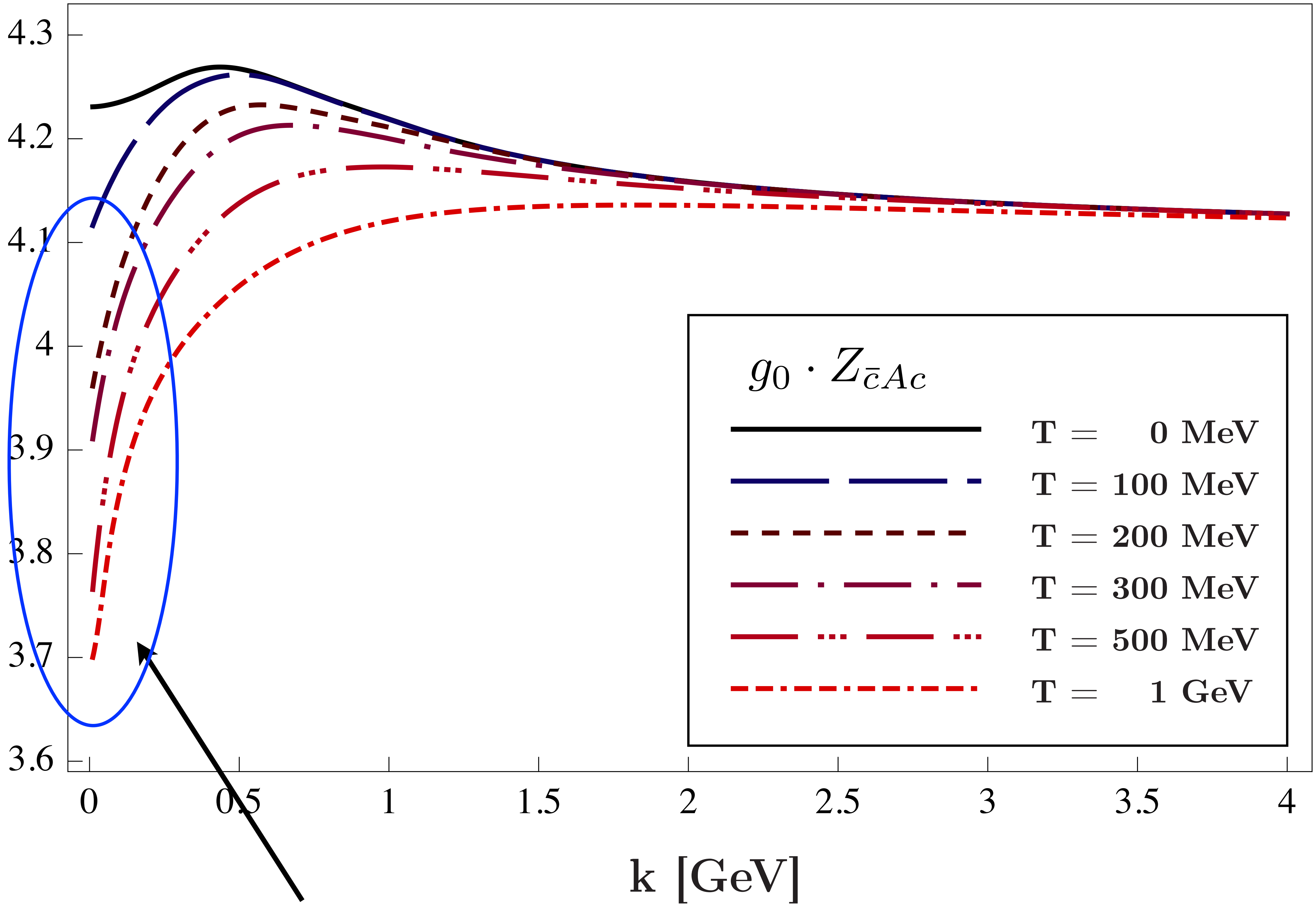
Ghost-Gluon Vertex



Ghost-Gluon Vertex

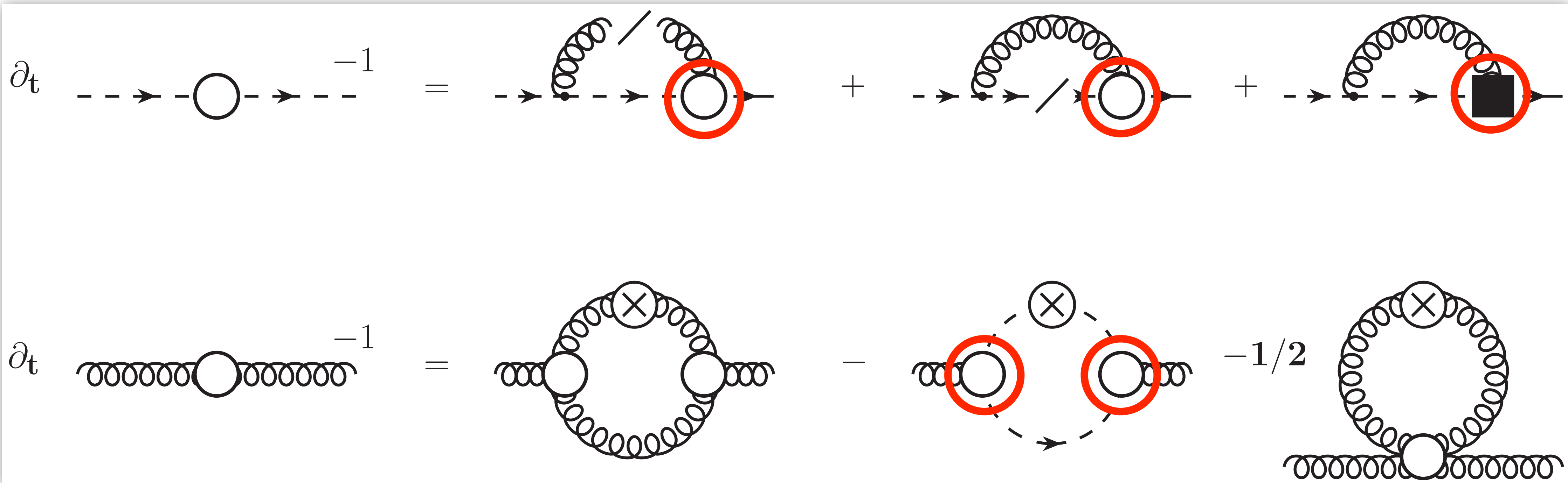


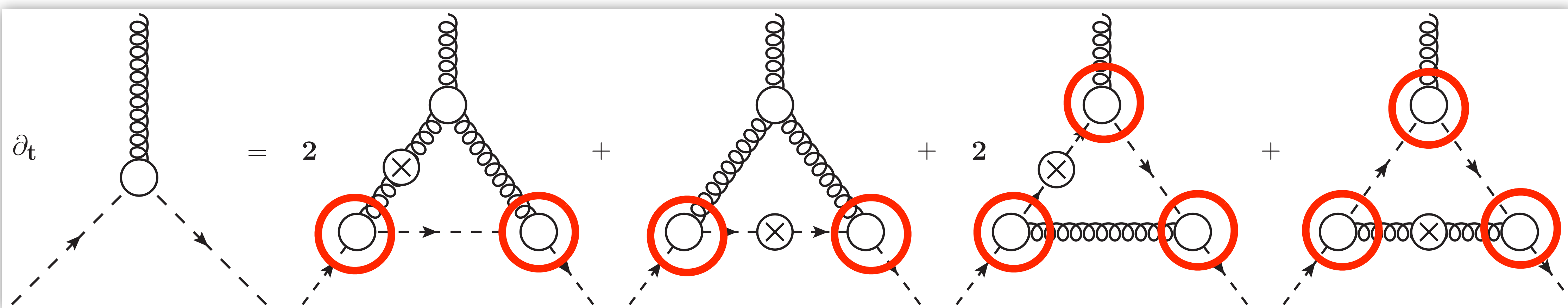
Ghost-Gluon Vertex



stabilisation effect together with ghost propagator

Stabilisation of Ghost-Sector

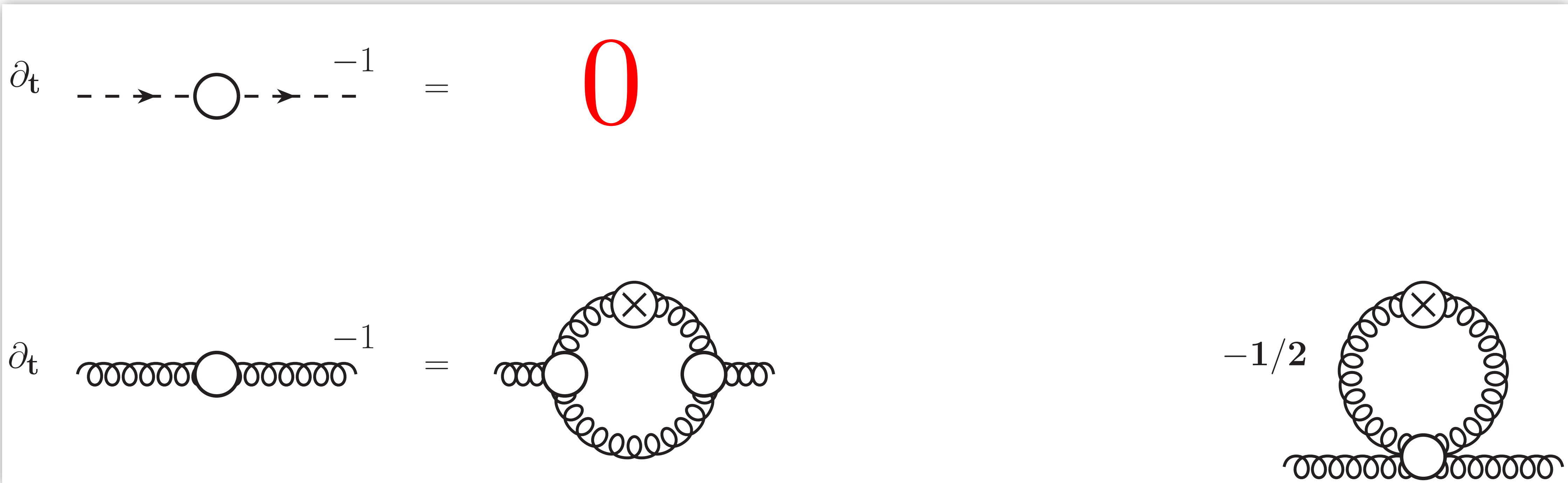


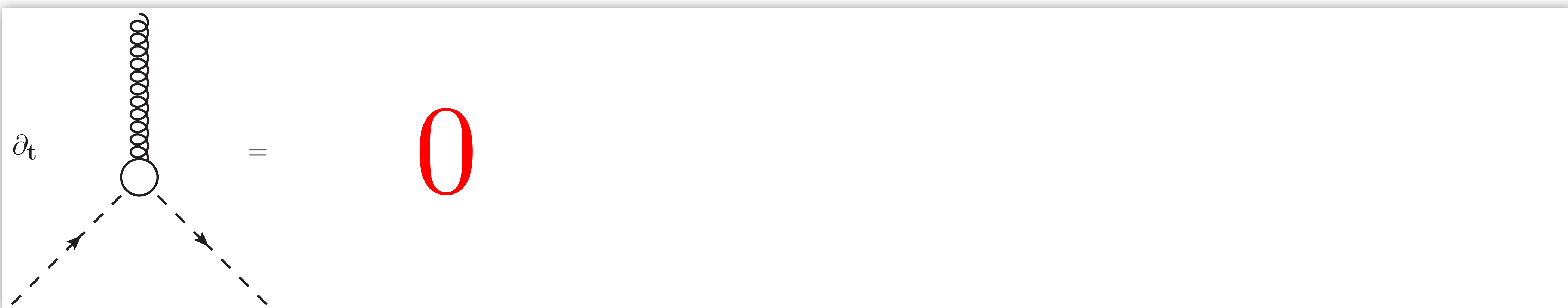


The non-triviality is crucial at finite temperature.

A vanishing ghost-gluon vertex stops the ghost-flow.

Stabilisation of Ghost-Sector





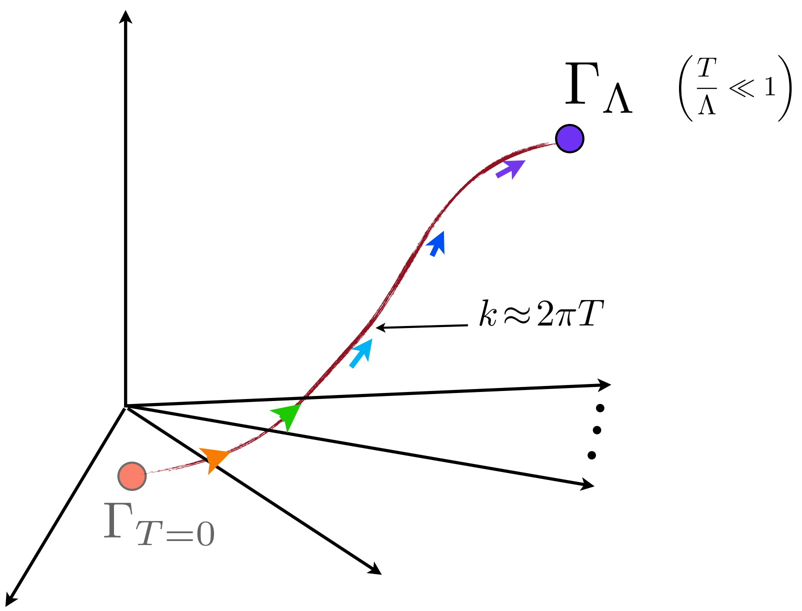
The non-triviality is crucial at finite temperature.

A vanishing ghost-gluon vertex stops the ghost-flow.

Numerics

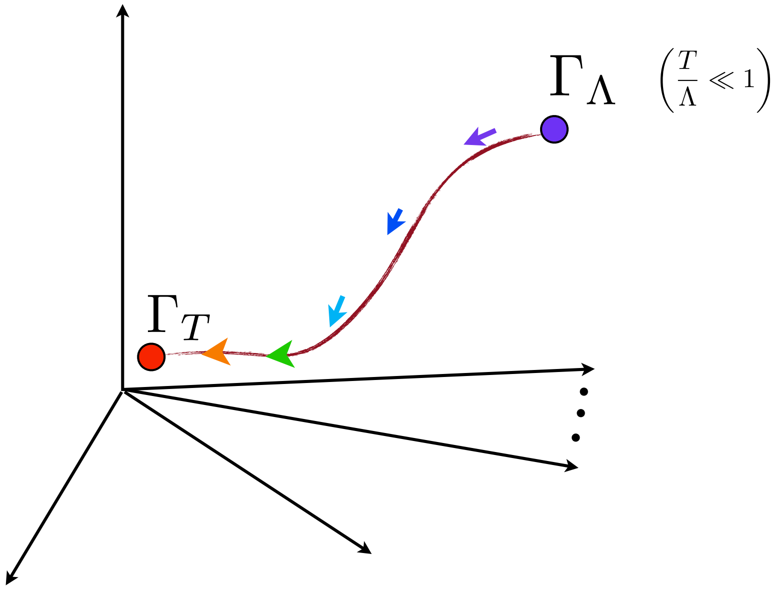
initial condition: *Iteration*

$$\Gamma^{(n)}_{k,i+1} = \Gamma^{(n)}_{k=0,i} + \int_0^k \frac{dk'}{k'} \mathrm{Flow}^{(n)}_{i+1}\left(\Gamma^{(n)}_{k,i}, \mathrm{Flow}^{(n)}_i\right)$$



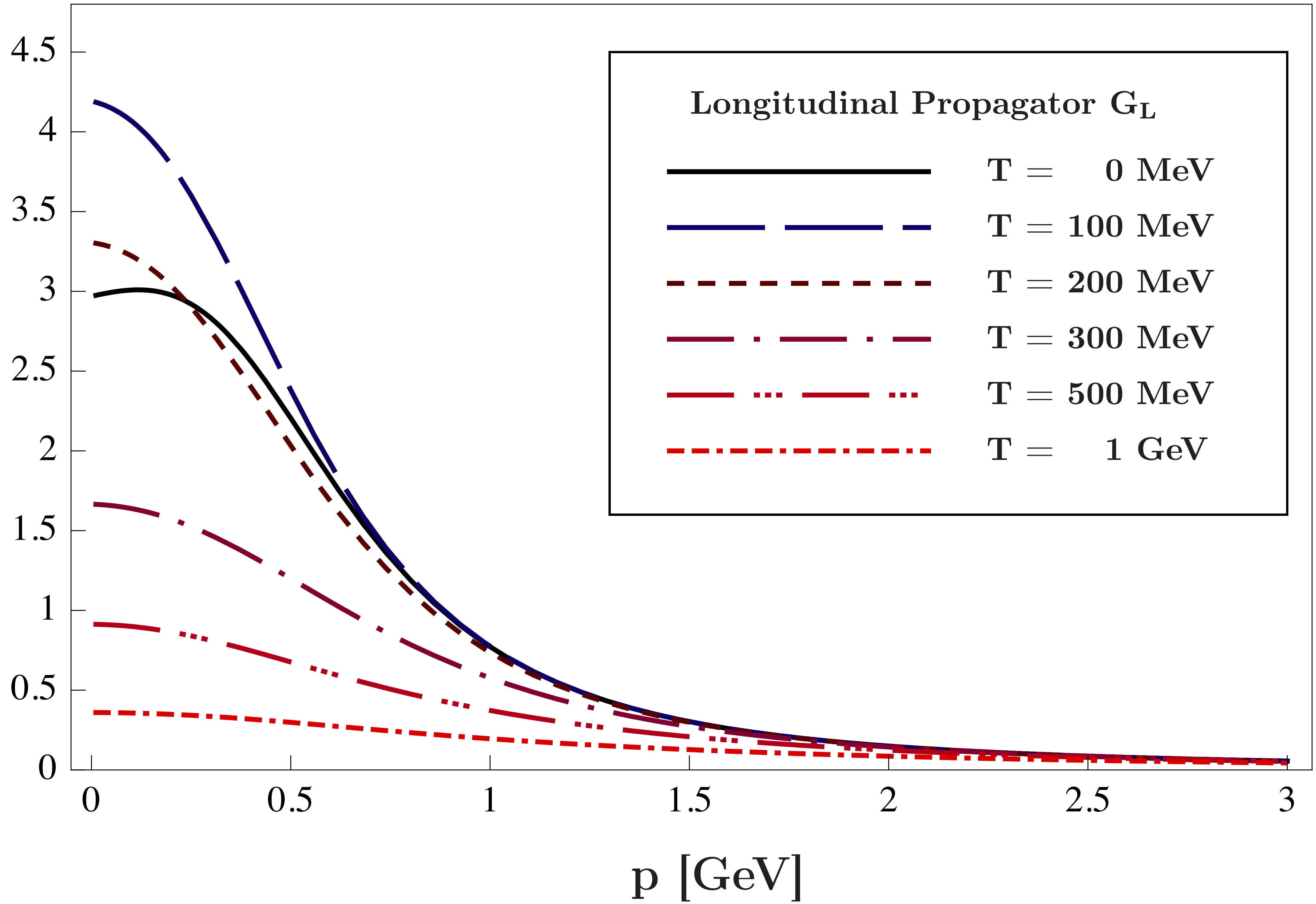
finite temperature: *Evolution*

$$\Gamma^{(n)}_{k_{i-1}} = \Gamma^{(n)}_{k_i} + \frac{k_{i-1} - k_i}{k_i} \mathrm{Flow}^{(n)}_{k_i}$$

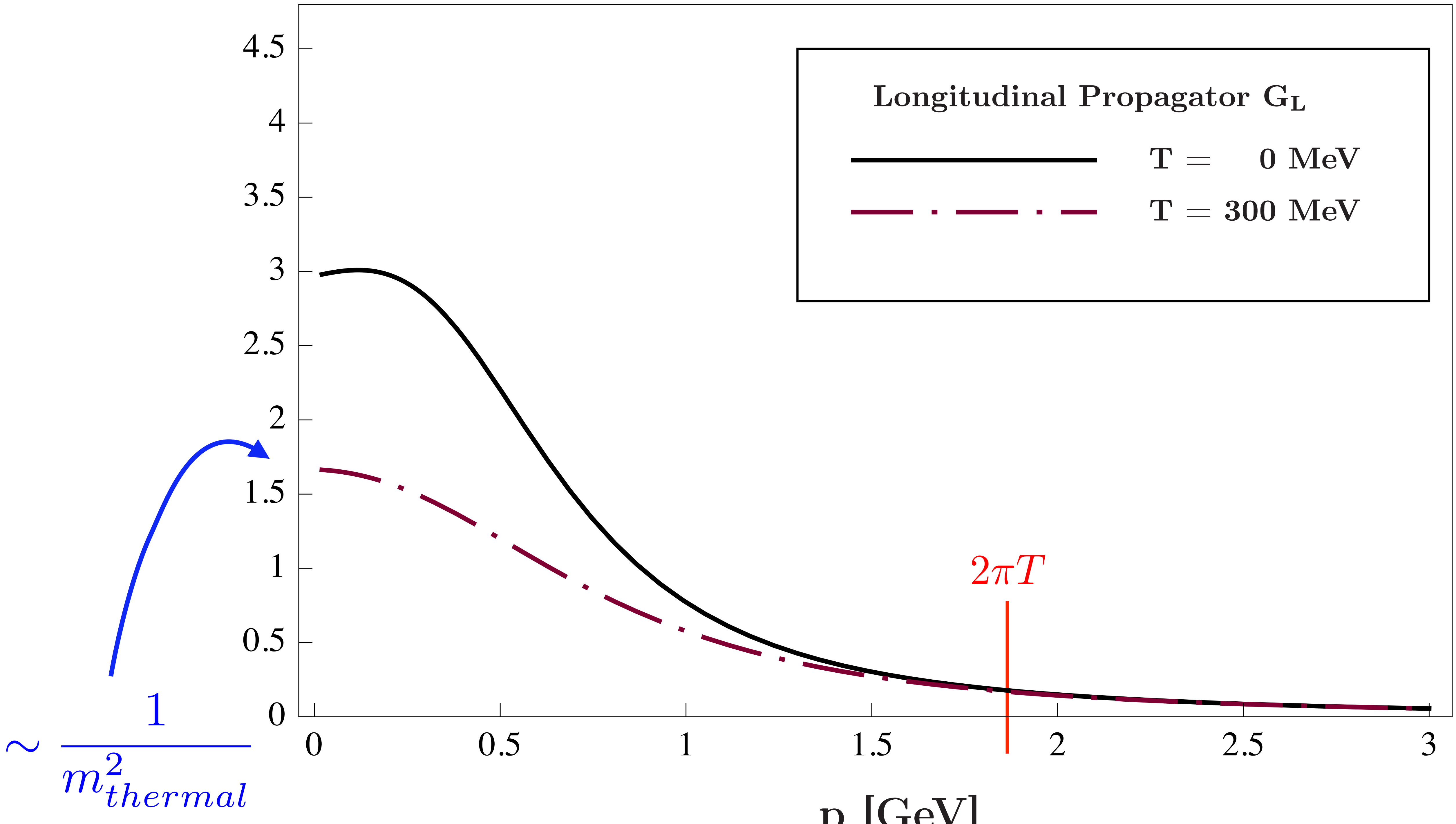


Propagators

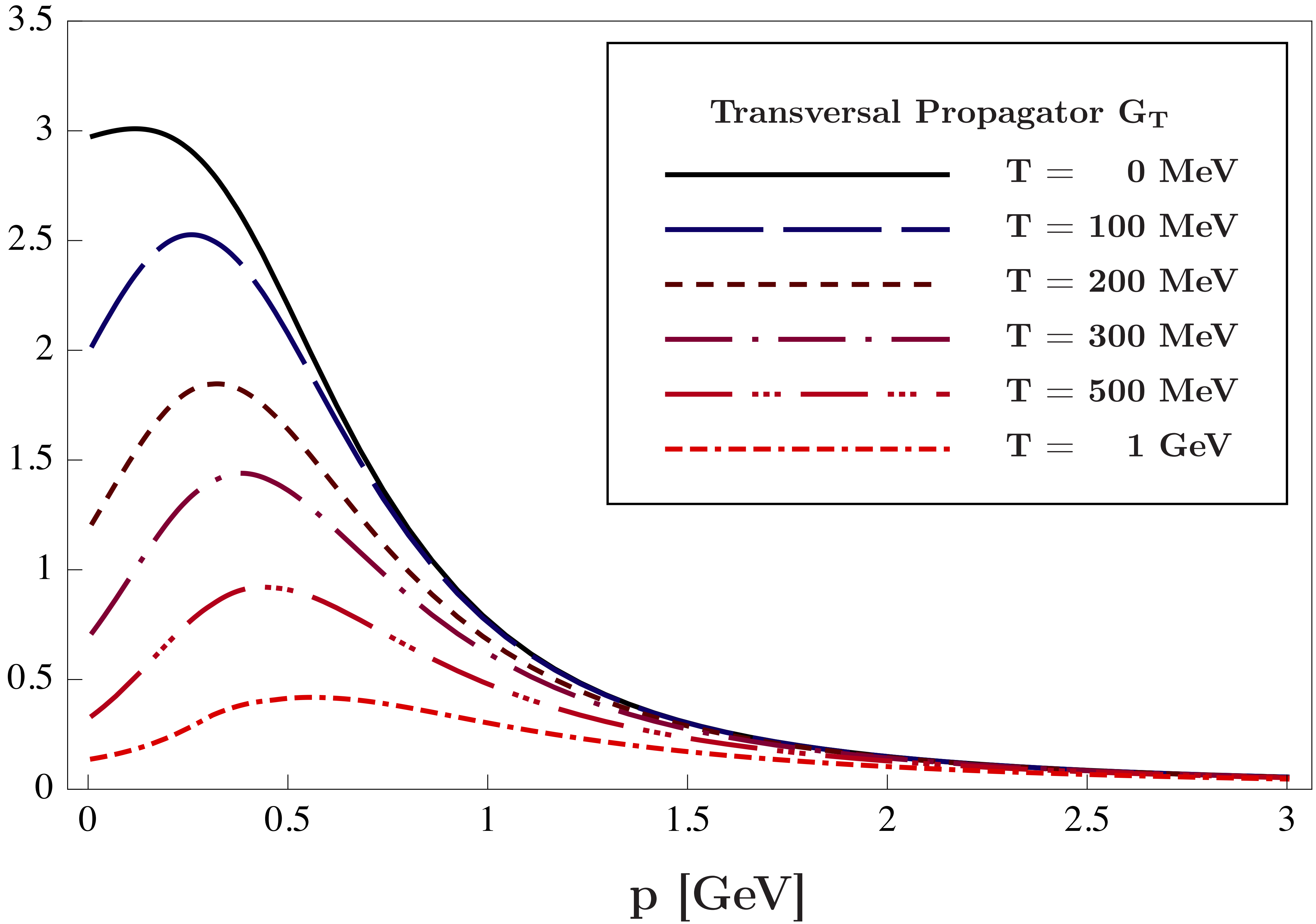
Longitudinal Gluon-Propagator



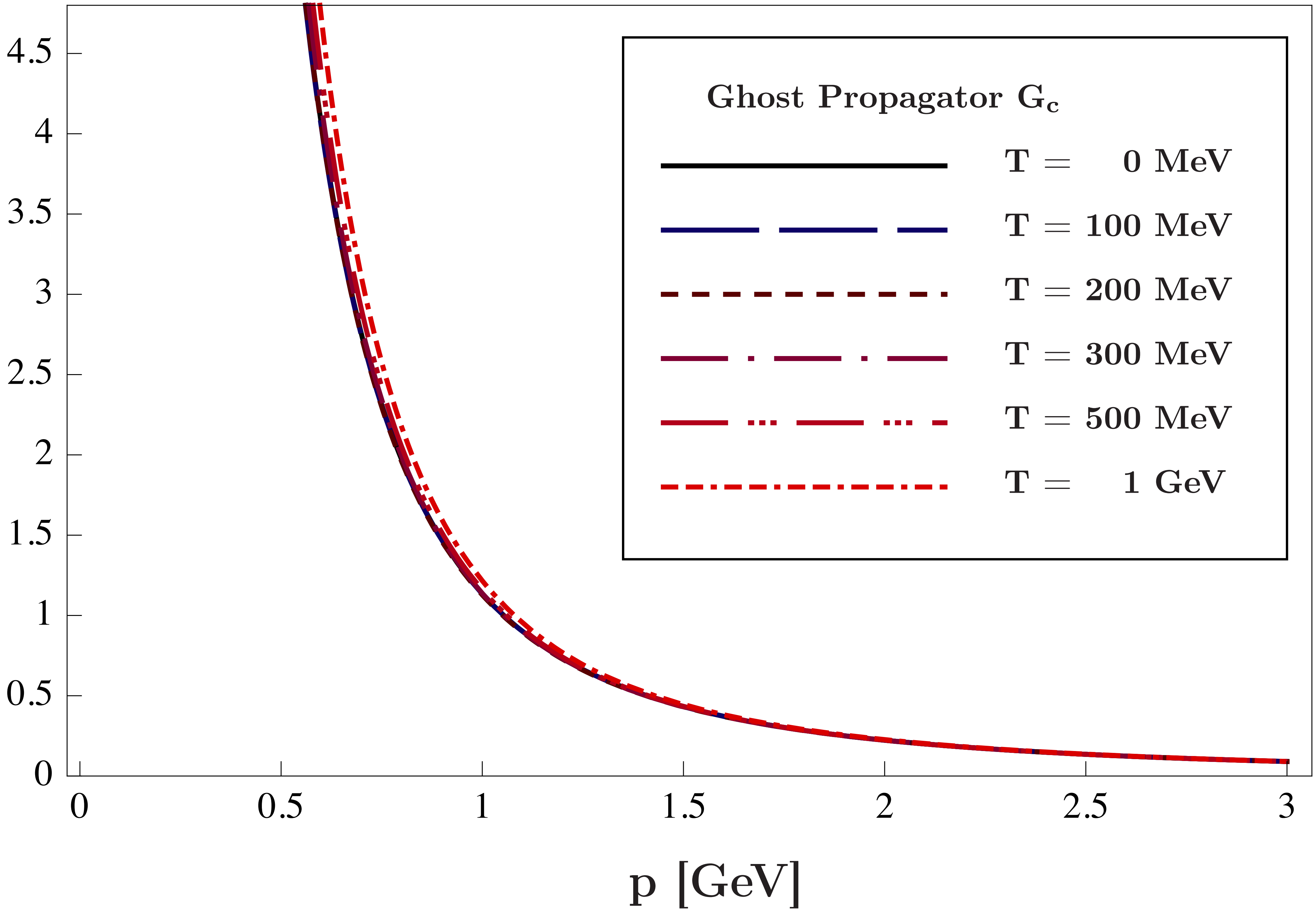
Longitudinal Gluon-Propagator



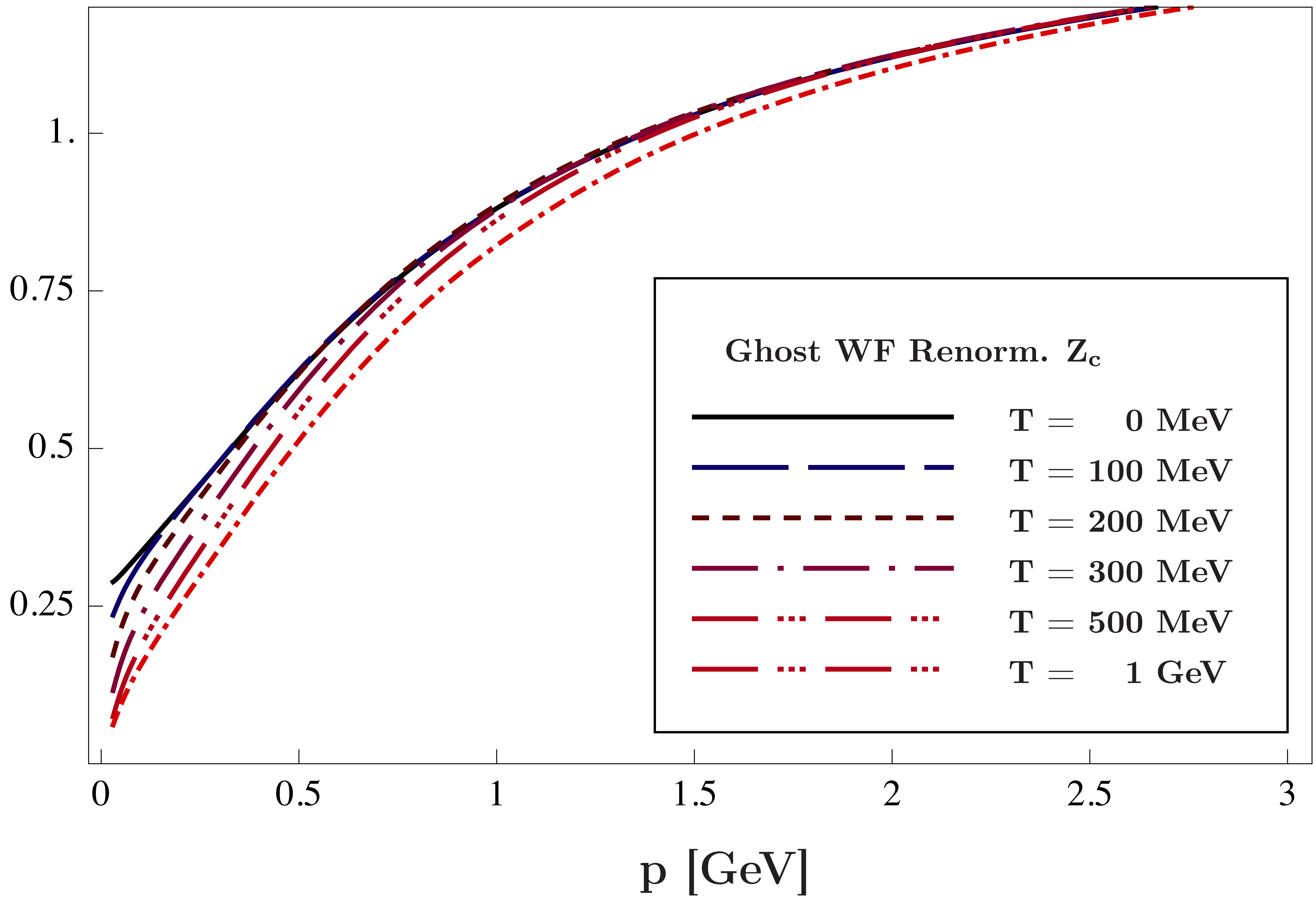
Transverse Gluon-Propagator



Ghost-Propagator

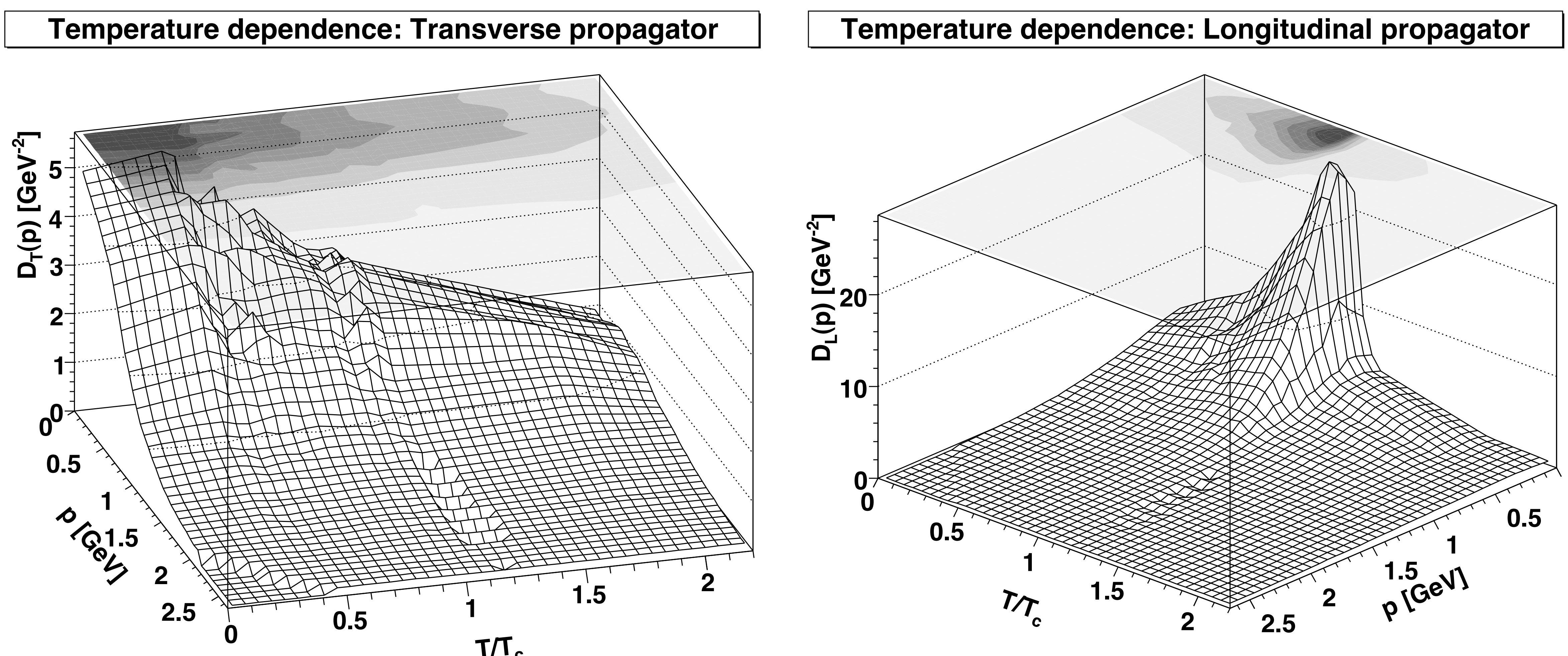


Ghost Wave-Function Renormalisation



Lattice Results

Cucchieri, Maas, Mendes, Phys. Rev. D 75, 076003 (2007).
Fischer, Maas, Mueller, Eur. Phys. J. C 68, 165 (2010).
Bornyakov, Mitrjushkin, arXiv: 1103.0442 [hep-lat].
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Mueller-Preussker, Sternbeck, arXiv: 1108.1735 [hep-lat].



0.5

A. Maas, arXiv: 0911.0348 [hep-lat].

Pressure

preliminary

Pressure

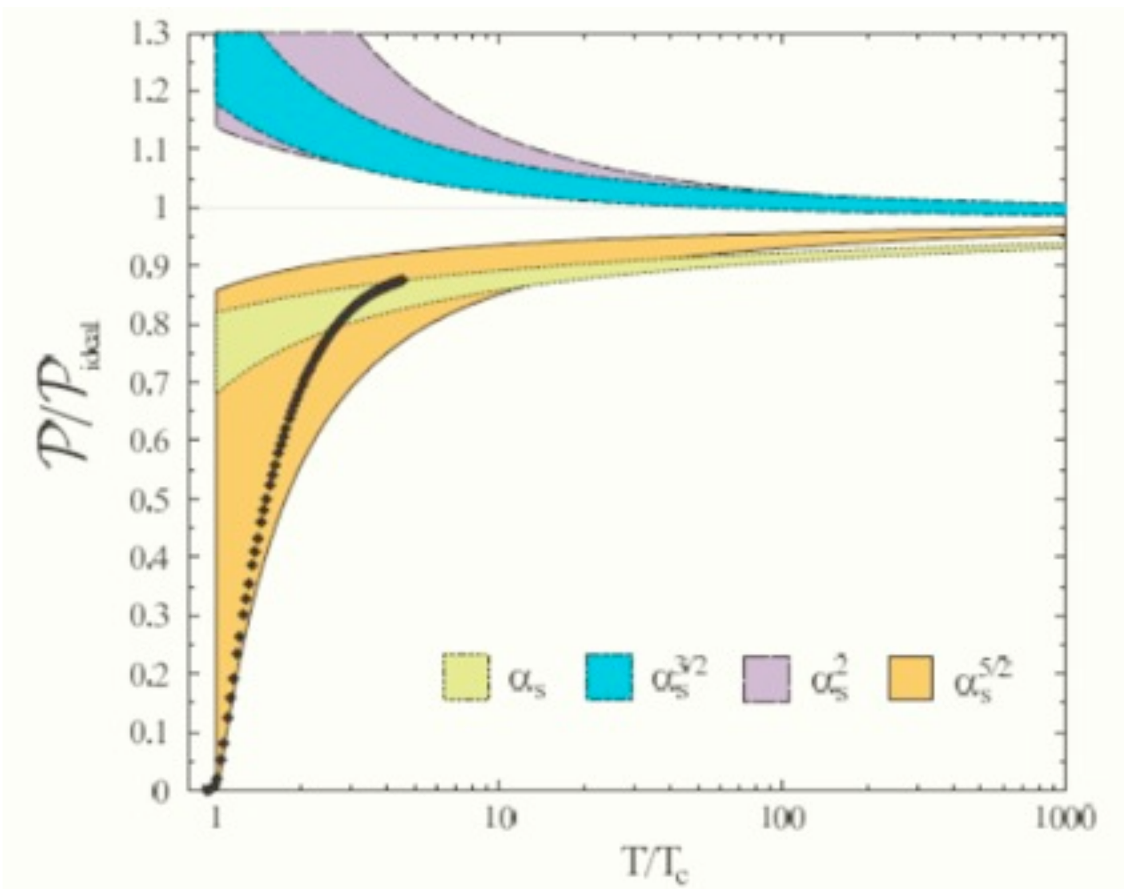
the thermal pressure is the effective action evaluated on the EoM, normalisation to zero in the vacuum

$$p_k\left(\bar{A}\right) = -\Delta\Gamma_{k,T}(\bar{A}) \qquad = -\left(\Gamma_{k,T} - \Gamma_{k,T=0}\right)$$

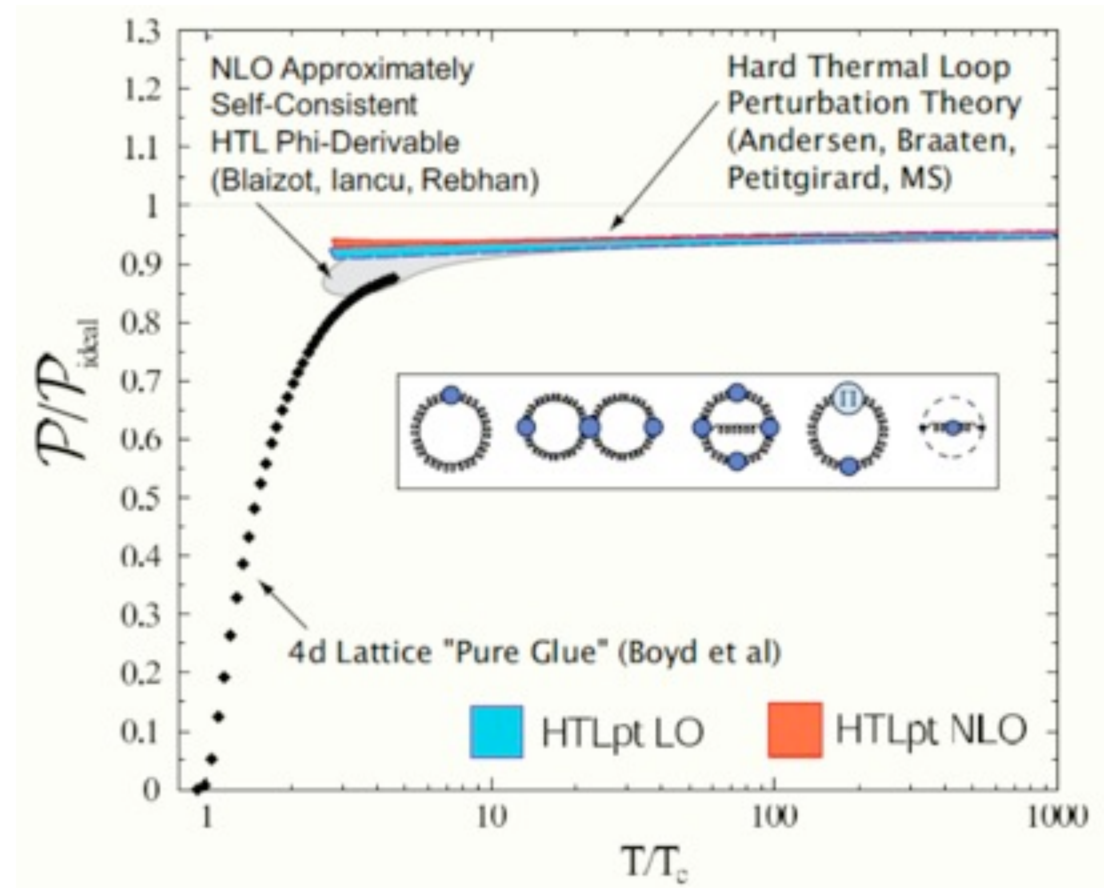
projection onto physical subspace:

- one chromoelectric mode
- one chromomagnetic mode

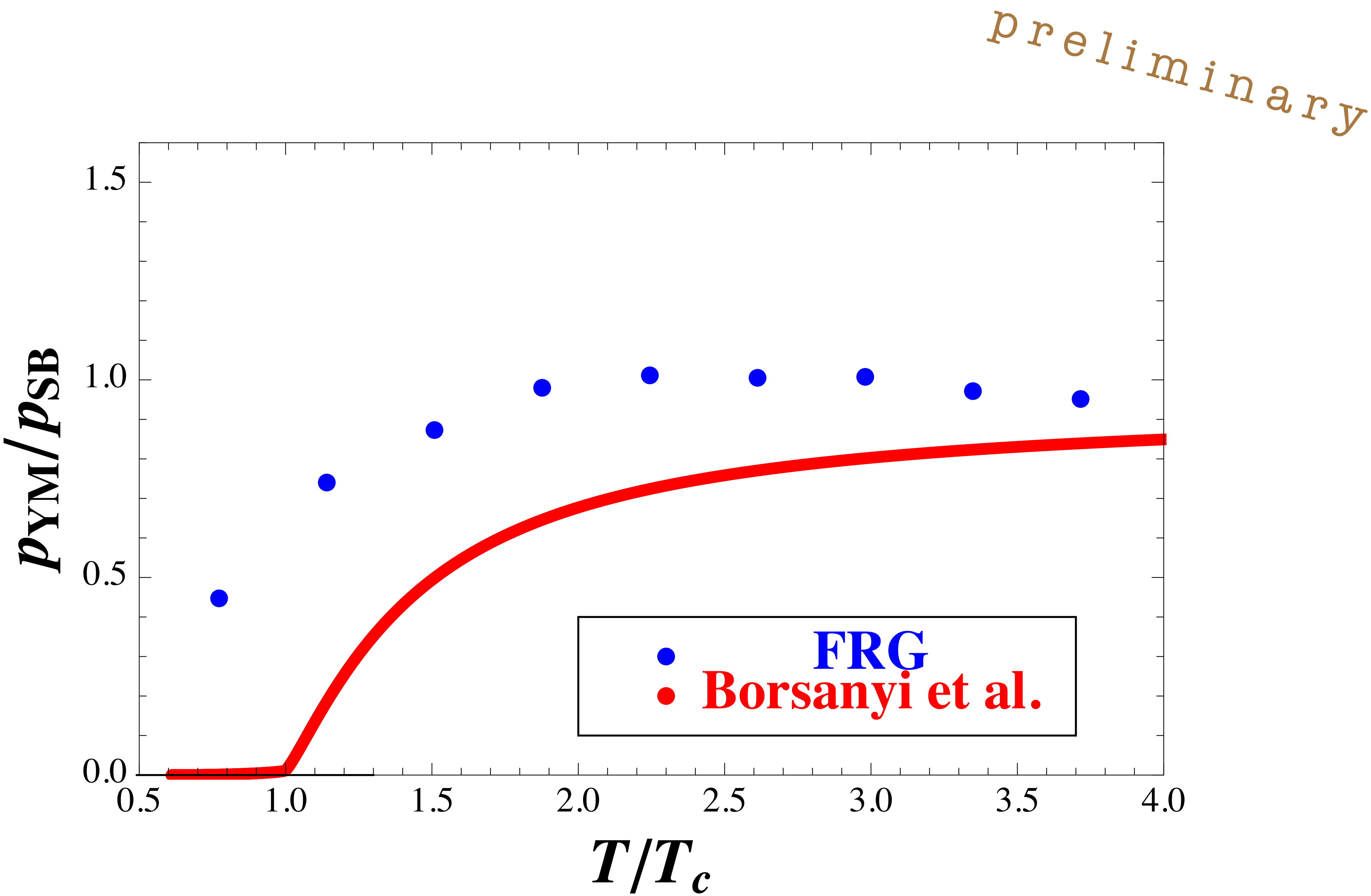
Pressure from Other Methods



Andersen, Strickland, Su,
Phys. Rev. Lett. 104 (2010).

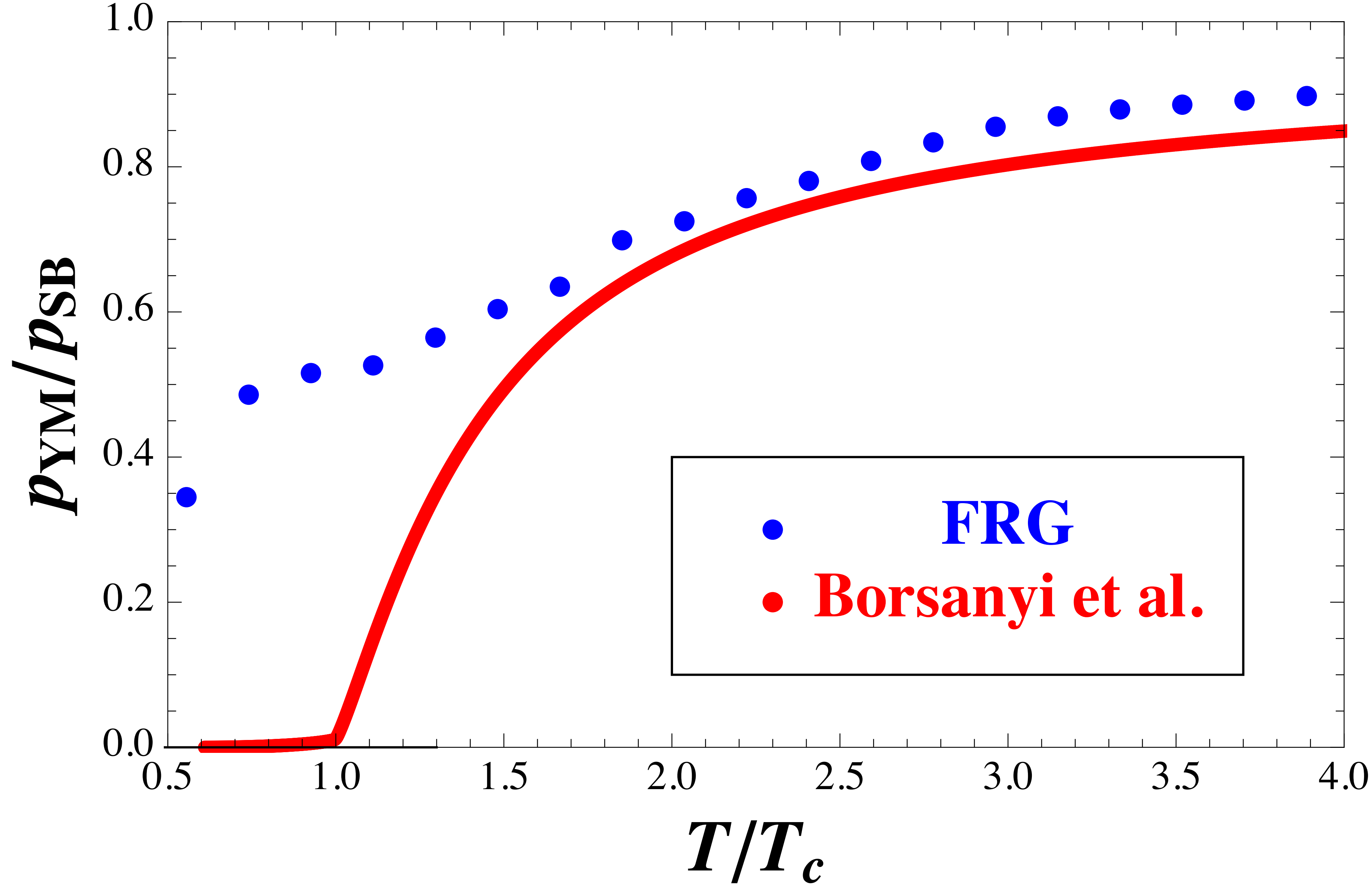


Pressure with T=0 props

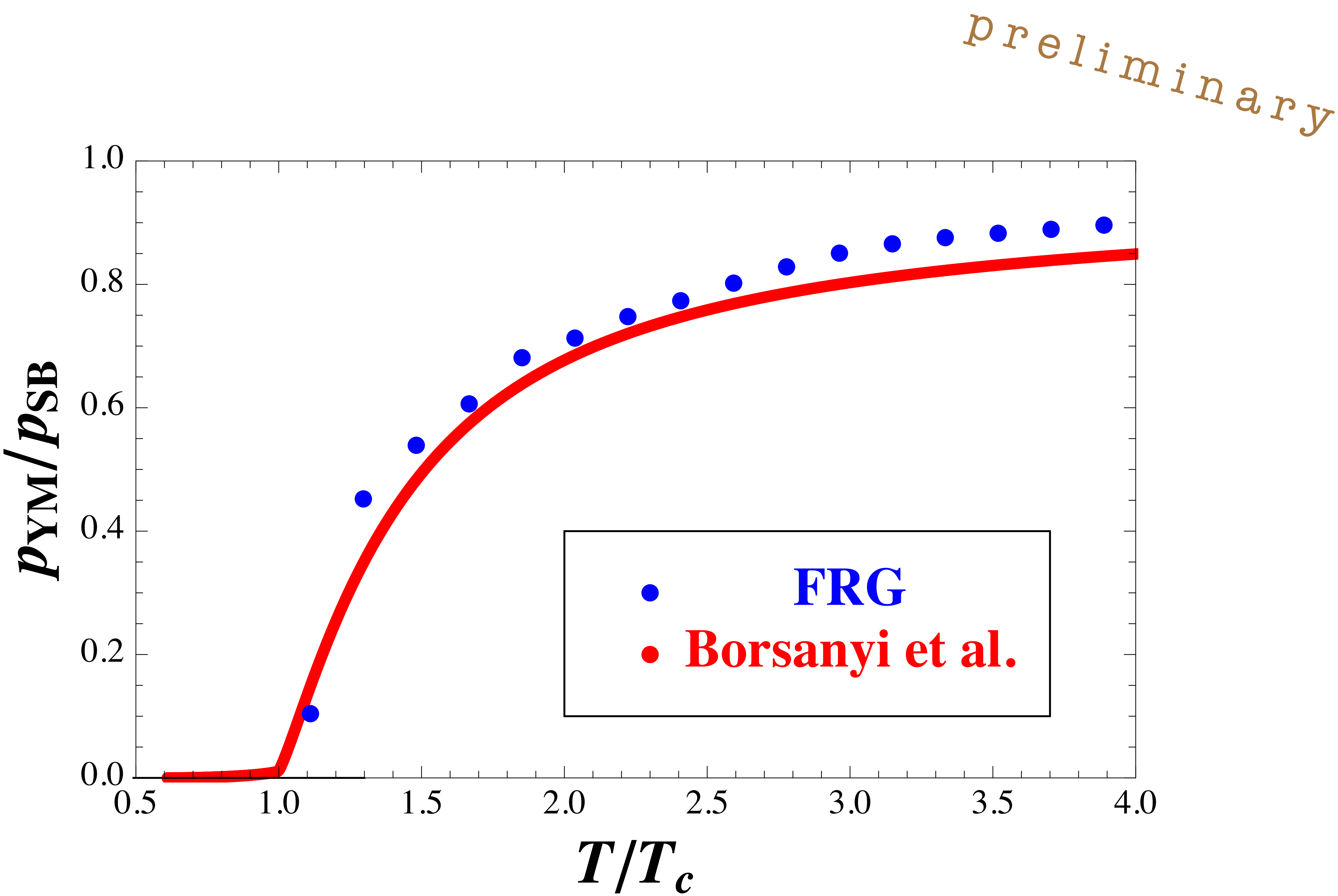


Pressure without Polyakov Loop

preliminary



Pressure with Polyakov Loop



Summary / Outlook

motivation: Yang-Mills propagators and thermodynamics, QCD

idea: introduce thermal flow equation and add to zero temperature part

$$\Gamma^{(2)}_{k,T} = \Gamma^{(2)}_{k,0} + \Delta\Gamma^{(2)}_{k,T}$$

for propagators
for pressure

flow at finite temperature

zero temperature result („input“)

(purely) thermal flow

have seen:
- ghost-gluon vertex
  mild variation, suppressed with temperature
- temperature-dependent, non-perturbative propagators
  chromoelectric and chromomagnetic propagators suppressed
- first results for pressure
  drop-off for low temperatures, Polyakov loop and T-dep. propagators crucial

outlook:
- thermodynamic quantities
- couple to full QCD calculation

J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684 (2010) 262-267.
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, Phys. Rev. Lett. 106 (2011) 022002.