Meta-stable States in Quark-Gluon Plasma

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Introduction

- Lagrangian of a local $SU(N)$ gauge theory

$$\mathcal{L} = \bar{\psi} \not{D} \psi + \frac{1}{2} \mathrm{Tr}\, F^2 \tag{1}$$

$$D_\mu = \partial_\mu - igA_\mu \,, \; F_{\mu\nu} = \frac{1}{-ig} [D_\mu, D_\nu] \tag{2}$$

- Lagrangian is invariant under local $SU(N)$ gauge transformations, Λ:

$$D_\mu \longrightarrow \Lambda^\dagger D_\mu \Lambda \,, \; \psi \longrightarrow \Lambda^\dagger \psi \tag{3}$$

- Λ has the properties:

$$\Lambda^\dagger \Lambda = \mathbb{1} \,, \; \mathrm{Det}\Lambda = 1 \tag{4}$$

- Consider the transformation:

$$\Lambda_c = e^{i\phi}\,\mathbb{1} \tag{5}$$

- Becomes an element of $SU(N)$ transformations if

$$\phi = \frac{2\pi j}{N},\; j \in \{0, 1, \cdots (N-1)\} \tag{6}$$

- Defines a global transformation.

- One immediate application is in the study of finite temperature QCD.

$Z(N)$ Symmetry in Pure Gauge

- Euclidean space-time.
- At $T \neq 0$, the imaginary time extent is finite:
  $0 \to \tau \Rightarrow 0 \to \beta' = 1/T$.
- Color fields are periodic in τ:

$$A_\mu(\vec{x}, \beta') = +A_\mu(\vec{x}, 0) \tag{7}$$

- Usually, gauge transformations should be periodic.
- However, periodicity is satisfied by a group of non-trivial gauge transformations.

- Consider the following gauge transformations which are periodic up to Λ_c,

$$\Lambda(\vec{x},\beta') = \Lambda_c\,,\;\; \Lambda(\vec{x},0) = 1 \tag{8}$$

- Λ_c commutes with A's:

$$A^{\Lambda}(\vec{x},\beta') = \Lambda_c^{\dagger} A_\mu(\vec{x},\beta')\Lambda_c = A_\mu(\vec{x},\beta') = +A_\mu(\vec{x},0) \tag{9}$$

- Both the action and the boundary conditions are invariant.

- $\Lambda_c = e^{i\phi}\,\mathbb{1}$ then defines a global $Z(N)$ symmetry

  [G. 't Hooft, 1978].

- Λ_c form the center of the gauge group $SU(N)$.

Polyakov Loop and $Z(3)$ in Pure Gauge

- The expectation value of Polyakov Loop is a good order parameter for the confinement-deconfinement phase transition. [A. M. Polyakov, 1978]

$$\langle l(\vec{x})\rangle = \frac{1}{3}\,\mathrm{Tr}\,\mathbf{L}(\vec{x}) \tag{10}$$

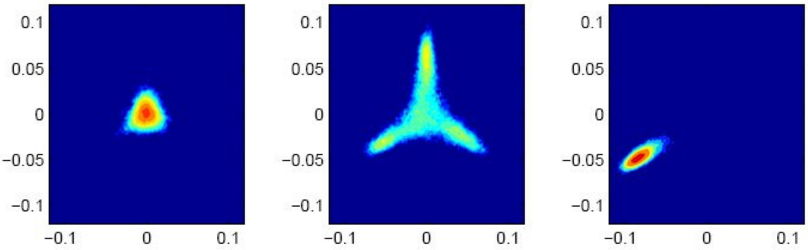
$$\text{Thermal Wilson line: } \mathbf{L}(\vec{x}) = P\,\exp\left[ig\int_0^{\beta'} A_0(\vec{x},\tau)\,d\tau\right] \tag{11}$$

  - In the confining phase ($T < T_c$), $\langle l(\vec{x})\rangle = 0$.
  - In the deconfining phase ($T > T_c$), $\langle l(\vec{x})\rangle > 0$.

- Under $Z(3)$ transformation, $\langle l(\vec{x})\rangle$ transforms as a field with charge one

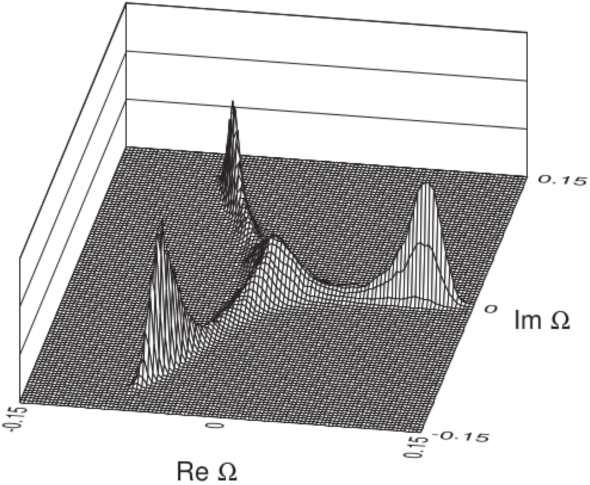
$$\langle l(\vec{x})\rangle \quad \longrightarrow \quad \Lambda_c \langle l(\vec{x})\rangle = e^{2\pi i j/3}\langle l(\vec{x})\rangle \tag{12}$$

- For $T > T_c$, $Z(3)$ symmetry is broken spontaneously.
  - Gives rise to 3 degenerate phases or vacua.
  - Separated by domain walls.
  - Creation of $Z(3)$ bubbles.
- For $T < T_c$, $Z(3)$ symmetry is restored.
- $\langle l(\vec{x})\rangle$ is an order parameter for studying $Z(3)$ symmetry.



$T < T_c$ $\quad$ $T \approx T_c$ $\quad$ $T > T_c$

Y. Iwasaki, *et. al*, 1992.



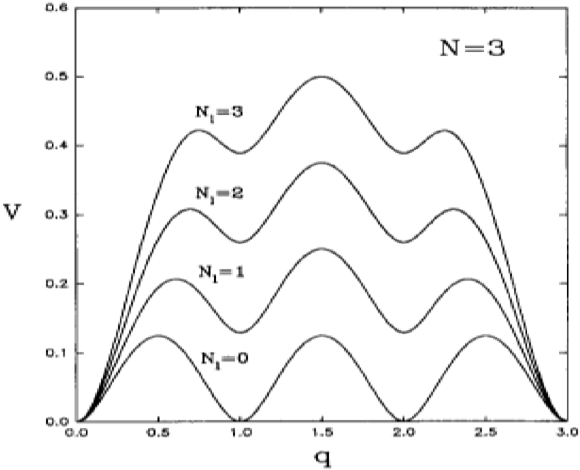
With Fermions

- Quarks fields are anti-periodic: $\psi(\vec{x}, \beta') = -\psi(\vec{x}, 0)$
- Under $Z(N)$:

$$\psi^{\Lambda}(\vec{x}, \beta') = \Lambda_c^{\dagger}\psi(\vec{x}, \beta') = e^{-i\phi}\psi(\vec{x}, \beta') \neq -\psi(\vec{x}, 0) \qquad (13)$$

- The full theory is not invariant under the $Z(N)$ transformations.
- The fermion fields break the $Z(N)$ symmetry explicitly except $j = 0$.
- Leads to one stable vacua for which $\langle l(\vec{x}) \rangle$ is real ($j = 0$)

- Others (Two for $SU(3)$) are (possible) meta-stable states.



- One loop effective potential calculation. [Dixit and Ogilvie, 1991]

- $Z(3)$ meta-stable states are expected to appear near T_c. [Dixit and Ogilvie, 1991; O. Machtey and B. Svetitsky, 2010]

- They are expected to play important roles both in the context of heavy-ion collision and in the early Universe.
  [J. Ignatius *et. al* 1992; B. Layek *et. al*, 2006; U. S. Gupta *et. al*, 2011]

- It is necessary to do the full QCD calculation from first principles.

Brief Introduction to Lattice

- Starts with QCD action in continuum

$$S[\psi, \bar{\psi}, A] = \int d^4x \overline{\psi}(\gamma^\mu D^\mu + m)\psi + \frac{1}{2}\mathrm{Tr}\int d^4x F^{\mu\nu}F_{\mu\nu} \quad (14)$$

- Euclidean rotation is performed: $t \rightarrow -i\tau$. Time extent: $aN_\tau = \frac{1}{T}$

- Use of Path Integral formalism. Fermions are represented by Grassman variables.

- Partition Function: $Z = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S[\psi,\bar{\psi},A]}$

- A_μ's are replaced by gauge links $\Rightarrow U_\mu(x) \equiv e^{igA_\mu(x+a\hat{\mu}/2)}$

- Covariant derivative:

$$D_\mu\psi(x) \rightarrow \frac{1}{2a}\left[U_\mu(x)\psi(x+a_\mu) - U_\mu^\dagger(x-a_\mu)\psi(x-a_\mu)\right]$$

- Partition Function: $Z = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}e^{-S} = \int \mathcal{D}U[\det M]e^{-S_g}$

- Coupling constant: $\beta = 2N_c/g^2$. Lattice spacing decreases with increasing β.

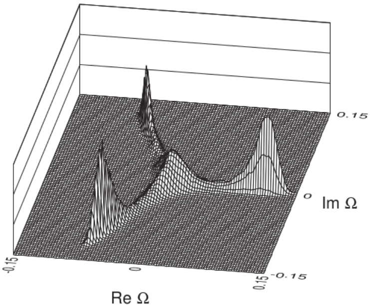
- Pure Gauge: $\det M = 1$

- Polyakov Loop: $\langle l(\vec{x})\rangle = \frac{1}{N_c}\mathrm{Tr}\left[\prod_{\tau=0}^{N_\tau} U_4(\vec{x},\tau)\right]$

- Translational invariance: $L = \frac{1}{V}\sum_{\vec{x}} l(\vec{x})$.

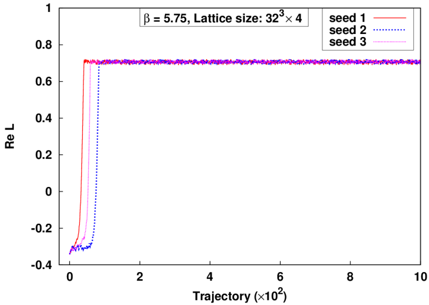
Observing Meta-stable States in Lattice

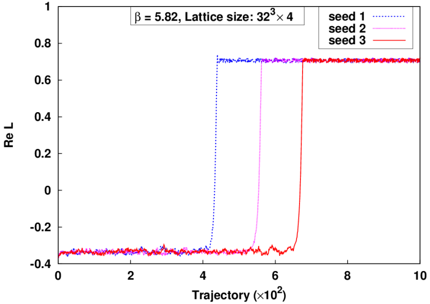
- The initial configuration can not arbitrary.
- Perform a pure gauge calculation on a lattice near the critical temperature ($\beta = \beta_c = 5.6925$). [G. Boyd, *et al*, 1995]

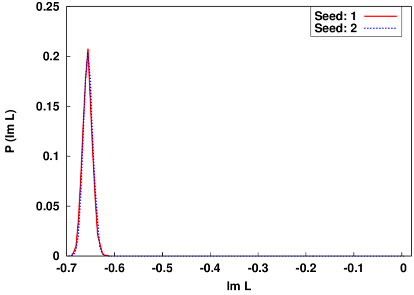


Numerical Parameters and Studies

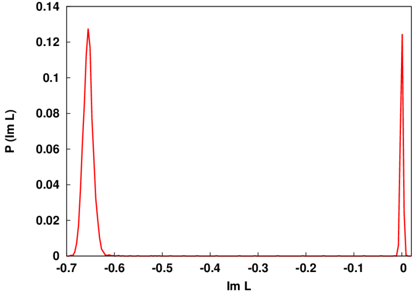
- Select one configuration each for the the following cases:
  - $\mathrm{Re}\, L \ll 0$ and $\mathrm{Im}\, L \gg 0$, i.e. $\theta \simeq 2\pi/3$.
  - $\mathrm{Re}\, L \ll 0$ and $\mathrm{Im}\, L \ll 0$, i.e. $\theta \simeq -2\pi/3$.
  - $\mathrm{Re}\, L \gg 0$ and $\mathrm{Im}\, L \sim 0$, i.e. $\theta = 0$.
- Alternatively, use an initial configuration with all the temporal links on a fixed time slice set to $e^{\pm i2\pi/3}\mathbb{1}$.
- MILC code is used.
- Various Lattice sizes: $V = 16^3, 24^3, 32^3$ and $N_\tau = 4, 6$.
- $am_{u,d} = 0.01$; $5.2 \leq \beta \leq 6.0$; $N_f = 2, 3$.
- Each gauge configuration after 10 trajectories has been analyzed.
- For each β, we have collected 3000 gauge configurations.


β = 5.75, Lattice size: 32³× 4
seed 1
seed 2
seed 3
Re L
Trajectory (×10²)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10

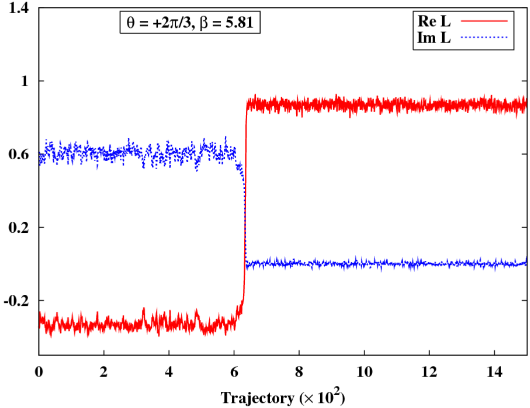

β = 5.82, Lattice size: 32³× 4
seed 1
seed 2
seed 3
Re L
Trajectory (×10²)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10

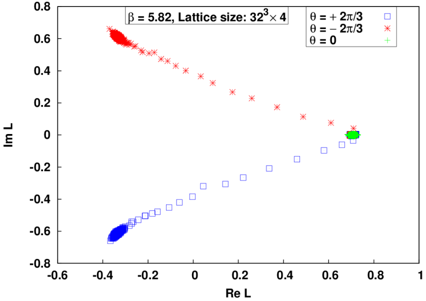


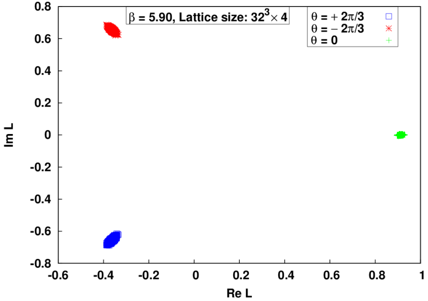
Seed: 1
Seed: 2
P (Im L)
Im L
0.25
0.2
0.15
0.1
0.05
0
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

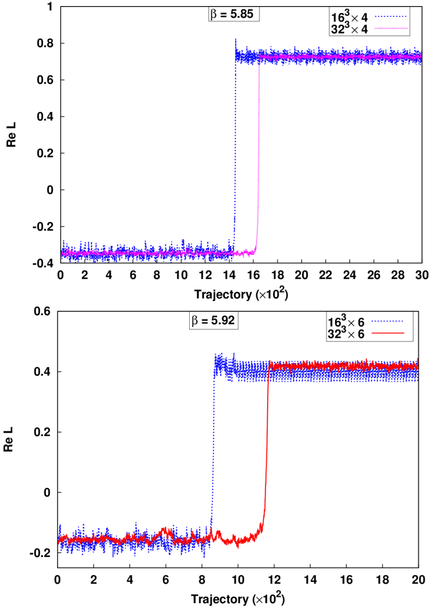


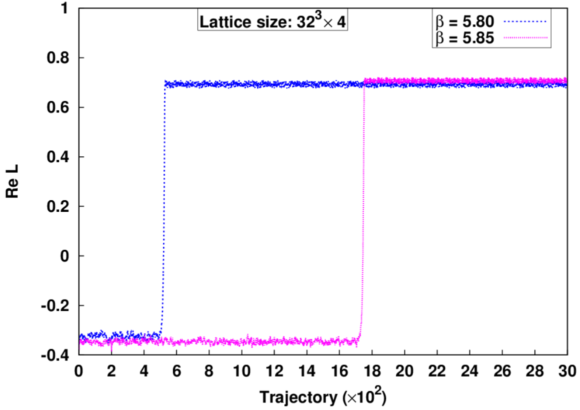
P (Im L)
Im L
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0


θ = +2π/3, β = 5.81
Re L
Im L
1.4
1
0.6
0.2
-0.2
0
2
4
6
8
10
12
14
Trajectory (× 10^2)


β = 5.82, Lattice size: 32³× 4
θ = + 2π/3
θ = − 2π/3
θ = 0
Im L
Re L
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


β = 5.90, Lattice size: 32³× 4
θ = + 2π/3
θ = − 2π/3
θ = 0
Im L
Re L
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


β = 5.85
16^3 × 4
32^3 × 4
Re L
Trajectory (×10^2)
β = 5.92
16^3 × 6
32^3 × 6
Re L
Trajectory (×10^2)


Lattice size: 32³× 4
β = 5.80
β = 5.85
Re L
Trajectory (×10²)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30

Determining Temperature

- $\beta_m \simeq 5.80$.

- Use β-function to determine the corresponding temperature. Calibration formula of the MILC collaboration [T. Blum, *et. al*, 1995] at $T = 0$:

$$\begin{aligned} am_\rho &= 0.72 - 2.25(\beta - 5.45) + 1.75(\beta - 5.45)^2 + 7.75\, am_q \\ &+ 10.01\, am_q(\beta - 5.45) - 20.08\, (am_q)^2 \end{aligned} \tag{15}$$

- However, $5.2 < \beta < 5.6$ in the above β-function study.

- Assumption is that the corresponding temperature is will not change drastically.

- Using $m_\rho = 770$ MeV from experiments, we find $T_m \simeq 750$ MeV.

Summary

- We find that the meta-stable states start to appear after $T_m = 750$ MeV.
- Far away from T_c as contrary to previous studies.
- No $Z(N)$ domain structure below T_m.
- More rigorous studies with higher statistics are necessary.
- β-function study has to be extended to the relevant range.

Thank you!!