Critical temperature and equation of state from $N_f = 2$ twisted mass lattice QCD

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GRADUIERTEN KOLLEG
Masse-Spektrum-Symmetrie

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tmfT-Collaboration:

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Outline

Introduction

Lattice QCD Setup

T_c and Chiral Scenarios

Thermodynamic Equation of State

Conclusions

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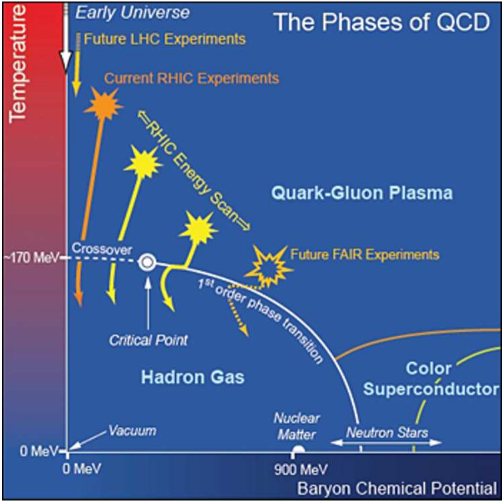
QCD Phase Diagram in the μ_B-T plane

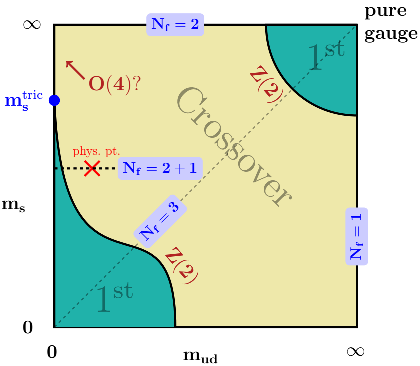


[BNL web page]

Explored by Heavy Ion Collision experiments at RHIC and LHC
in future: FAIR and NICA

QCD Phase Diagram μ_B=0

$N_f = 2+1$:



$N_f = 2$:

Argued to be in universality class of $O(4)$ 3-dim spin modell
[R. D. Pisarski, F. Wilczek, 84]

Compatible results with $O(4)$ in:
e. g. [V. Bornyakov *et al.*, 2009],
[S. Ejiri *et al.*, 2009]

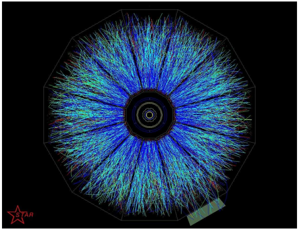
However, 1^{st} order not ruled out:
e. g. [C. Bonati *et al.*, 2012],
[C. Bonati *et al.*, 2009]

Heavy Ion Collisions

- Expansion of Quark Gluon Plasma describable by relativistic hydrodynamics

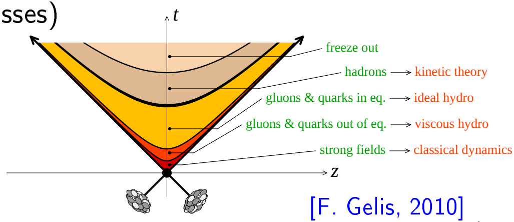
$$\dot{\epsilon} = -(\epsilon + p)(\partial \cdot u) \qquad \dot{u}^\mu = \frac{\nabla^\mu p}{\epsilon + p}$$

$$\dot{n}_B = -n_B(\partial \cdot u) \qquad \text{plus}: \ \epsilon(p)$$



[STAR Collab.]

- $\epsilon(p)$ accessible on the lattice (at small n_B)
- Most precise results from $N_f = 2 + 1$ staggered quarks (physical masses) [HotQCD, Budapest-Wuppertal]
- This work: $N_f = 2$, extension to $N_f = 2 + 1 + 1$ possible



[F. Gelis, 2010]

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Path Integral at Finite Temperature

Starting point:

Grand canonical partition function in continuum (with $k_B = 1$)
$t \to -i\tau \Rightarrow$ Euclidean path integral

$$Z(V,T,\tilde{\mu}) = \mathrm{Tr}\Big(e^{-\left(\hat{H}-\tilde{\mu}\hat{N}_q\right)/T}\Big)$$
$$= \int D\psi D\bar{\psi} DA \; e^{-\left(S_g[A]+S_f[\psi,\bar{\psi},A]\right)}$$

with gluon and quark action:

$$S_g[A] = \int_0^{1/T} d\tau \int_V d^3x \; \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$S_f[\psi,\bar{\psi},A] = \int_0^{1/T} d\tau \int_V d^3x \; \bar{\psi}\left(\not{D} + m_q - \tilde{\mu}\gamma_0\right)\psi$$

Impose in τ-direction periodic BC for A, anti-periodic BC for ψ

Lattice discretization $\Rightarrow$: $1/T \equiv N_\tau a$

Twisted Mass Lattice Regularization

- $N_f = 2$ Wilson twisted mass action at maximal twist:

$$S_f[U,\psi,\overline{\psi}] = \sum_x \overline{\chi}(x)\left(1 - \kappa H[U] + 2i\kappa a\mu\gamma_5\tau^3\right)\chi(x)$$

$$\psi = \frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3)\chi \quad \text{and} \quad \overline{\psi} = \overline{\chi}\frac{1}{\sqrt{2}}(1 + i\gamma_5\tau^3)$$

- Advantage: when κ tuned to critical value κ_c:

Automatic $\mathcal{O}(a)$ improvement

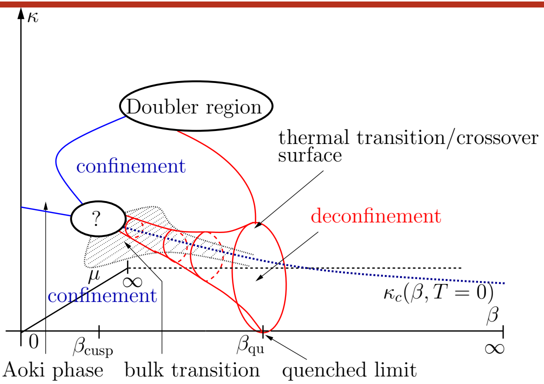
[R. Frezzotti, G.C. Rossi, 2004]

- Disadvantage: explicit flavor symmetry breaking (mostly small, but has to be checked)

- Tree level improved gauge action:

$$S_g[U] = \beta\Big(c_0\sum_P[1 - \frac{1}{3}\mathrm{ReTr}\,(U_P)] + c_1\sum_R[1 - \frac{1}{3}\mathrm{ReTr}\,(U_R)]\Big)$$

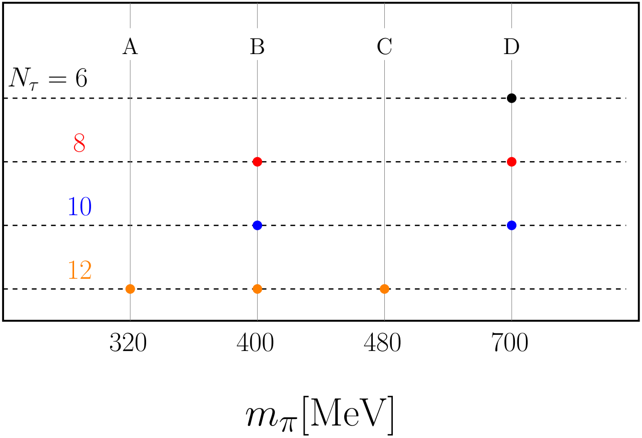
Phase Diagram & Simulation Setup



[E.-M. Ilgenfritz *et al.*, 2009]

- Simulations: β-scans parallel to $\kappa_c(\beta)$, μ adapted for $m_\pi = \text{const}$
- Piercing the "cone" at T_c
- Rely on $\kappa_c(\beta)$, $a(\beta)$ and $m_{\pi^\pm}$ from **ETM Collaboration**
  [R. Baron *et al.*, 2010]

Simulation Points



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Observables

- Polyakov-Loop L:
  Order parameter for deconfinement for $m_q \to \infty$:

$$\mathrm{Re}\,(L) = \frac{1}{3}\mathrm{Re}\left\langle \frac{1}{N_\sigma^3}\sum_{\vec{x}} \mathrm{Tr}\left(\prod_{\tau=0}^{N_\tau-1} U_0(\tau,\vec{x})\right)\right\rangle = \begin{cases} 0 & T=0 \\ >0 & T>T_c \end{cases}$$

- Chiral condensate $\langle\bar{\psi}\psi\rangle$:
  Order parameter for chiral symmetry breaking for $m_q \to 0$:

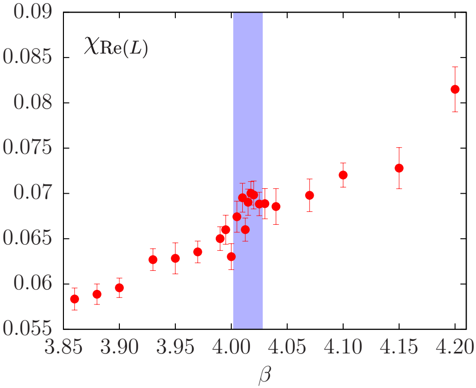
$$\langle\bar{\psi}\psi\rangle = \mathrm{Tr}\left(D^{-1}\right) = \begin{cases} >0 & T=0 \\ \to 0 & T>T_c \end{cases}$$

- Intermediate m_q: look at fluctuations (susceptibilities), expect maximum around T_c
- Renormalized Polyakov loop and chiral condensate

Signals for the Crossover: Polyakov Loop

- Polyakov loop susceptibility (no convincing signal)

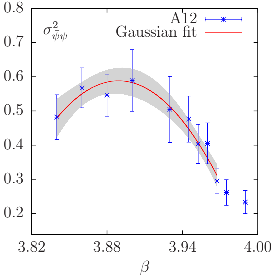
$m_\pi \approx 400$ MeV:



Disconnected Chiral Susceptibility

$$\sigma_{\langle\bar{\psi}\psi\rangle} = V/T\left(\langle\bar{\psi}\psi^2\rangle - \langle\bar{\psi}\psi\rangle^2\right)$$

$m_\pi \approx 320$ MeV:



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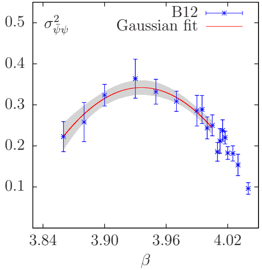
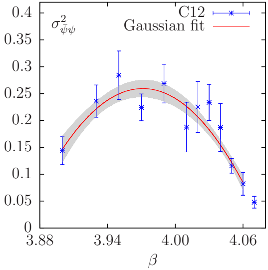
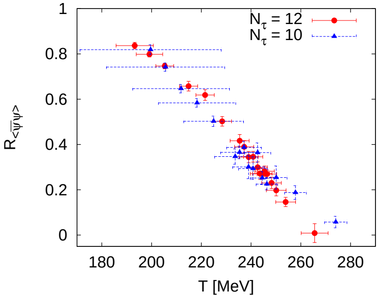


$m_\pi \approx 480$ MeV:

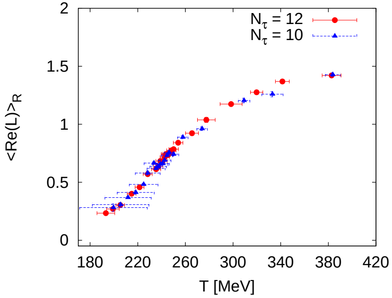


Renormalized Re(L), $\langle\bar{\psi}\psi\rangle$

$$R_{\langle\overline{\psi}\psi\rangle} = \frac{\langle\overline{\psi}\psi\rangle(T,\mu) - \langle\overline{\psi}\psi\rangle(0,\mu) + \langle\overline{\psi}\psi\rangle(0,0)}{\langle\overline{\psi}\psi\rangle(0,0)}$$



$$\langle \mathrm{Re}(L)\rangle_R = \mathrm{Re}(L)\exp\left(V(r_0)/2T\right)$$



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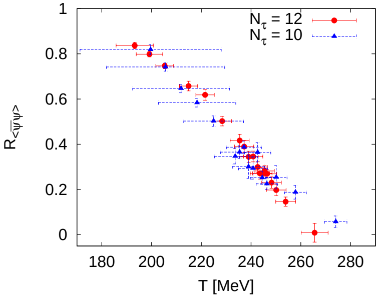
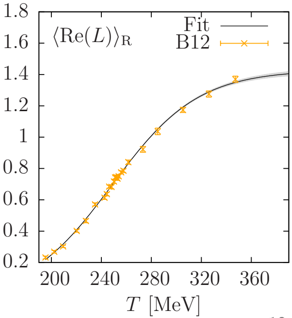


$$\langle \mathrm{Re}(L)\rangle_R = \mathrm{Re}(L)\exp\left(V(r_0)/2T\right)$$



Results

Ensemble	B12	C12	D8
T_c[MeV], $\sigma^2_{\langle\bar{\psi}\psi\rangle}$:	217(5)	229(5)	251(10)
T_c[MeV], $\langle \mathrm{Re}(L)\rangle_R$:	249(5)	258(5)	266(11)

Signals for deconfinement and restoration of chiral symmetry at different locations in crossover region

$O(4)$ Magnetic EoS

- Near critical point of a 2^{nd} order transition: universality
- Expect data to collaps on universal curve [J. Engels & T. Mendes, 99]
- $$\langle\bar{\psi}\psi\rangle = h^{1/\delta} c\, f(d\, z) \quad \underbrace{+\, a_\tau \tau h + b_1 h + \ldots}_{\text{scaling violating terms}}$$ [S. Ejiri *et al.*, 2009]

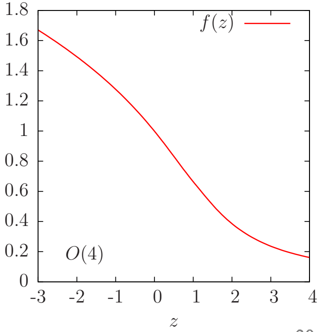
$$\tau = \beta - \beta_{\text{chiral}}, \quad h = 2\, a\, \mu, \quad z = \tau / h^{1/(\tilde{\beta}\delta)}$$

- Connection to spin model:

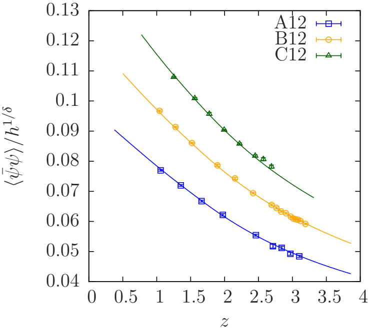
$$\langle\bar{\psi}\psi\rangle \sim M \text{ (magnetization)}$$

$$\mu \sim H \text{ (external field)}$$

$$\beta \sim T \text{ (temperature)}$$



$O(4)$ Magnetic EoS

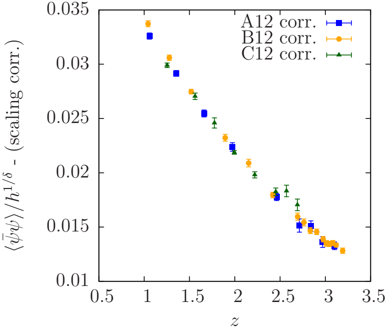


Pion Mass:

C12: 480 MeV

B12: 400 MeV

A12: 320 MeV



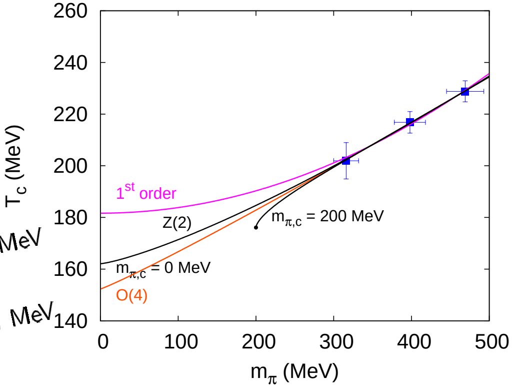
Result: $\beta_{\text{chiral}} \approx 3.76(2) \rightarrow$

$T_c(m_\pi = 0) \approx 166$ MeV

Chiral Limit Scenarios from Spin Models

- From universal function $h^{1/\delta} f(z)$ one can show:

$$T_c(m_\pi) = T_c(0) + A m_\pi^{2/(\tilde{\beta}\delta)}$$



1^{st}: $T_c(0) = 182(14)$ MeV

O(4): $T_c(0) = 152(26)$ MeV

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Integral Method

- $$\epsilon = \frac{T}{V}\frac{\partial \ln Z}{\partial \ln T}\bigg|_V \qquad p = T\,\frac{\partial \ln Z}{\partial V}\bigg|_T$$

  But: $V = N_\sigma^3\ a$
  $T = \frac{1}{N_\tau\ a}$

- Trace anomaly (interaction measure):

$$I = \epsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a}$$

- Starting point for $p(T)$ and $\epsilon(T)$ via

$$\frac{I}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) \qquad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} d\tau \frac{\epsilon - 3p}{\tau^5}\bigg|_{\text{LCP}}$$

  on lines of constant physics (LCP)

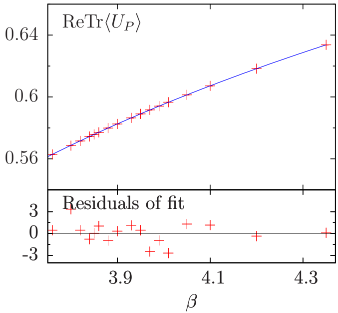
Trace Anomaly

- $$\begin{aligned}\frac{I}{T^4} &= \frac{\epsilon - 3p}{T^4} = -\frac{T}{T^4\ V}\left\langle \frac{d\ln Z}{d\ln a}\right\rangle_{\text{sub}} \\ &= N_\tau^4 \left(a\frac{d\beta}{da}\right)\left(\frac{c_0}{3}\langle \mathrm{Re\,Tr}\,U_P\rangle_{\text{sub}} + \frac{c_1}{3}\langle \mathrm{Re\,Tr}\,U_R\rangle_{\text{sub}}\right. \\ &\left. + \frac{\partial\kappa_c}{\partial\beta}\langle\bar{\chi}H[U]\chi\rangle_{\text{sub}} - \left(2a\mu\frac{\partial\kappa_c}{\partial\beta} + 2\kappa_c\frac{\partial(a\mu)}{\partial\beta}\right)\langle\bar{\chi}i\gamma_5\tau^3\chi\rangle_{\text{sub}}\right)\end{aligned}$$

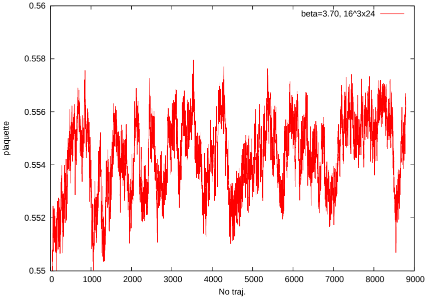
- Starting point for $p(T)$ and $\epsilon(T)$ by integral method
- Subtracted expectation values: $\langle\ldots\rangle_{\text{sub}} \equiv \langle\ldots\rangle_{T>0} - \langle\ldots\rangle_{T=0}$
  $\rightarrow$ interpolations for $T = 0$ data
- Preliminary results for $m_\pi \approx 400$ MeV and $m_\pi \approx 700$ MeV

$T = 0$ Subtractions & Interpolations

example: plaquette:



$\beta = 3.70$, $a\mu = 0.009$, $m_\pi \approx 400$ MeV

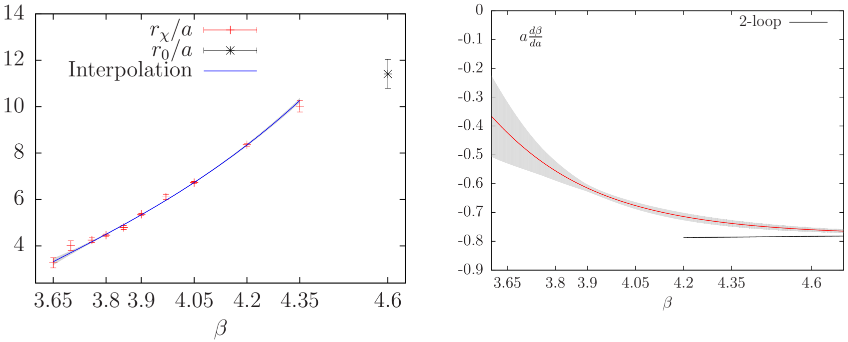


β-Function

[M. Cheng et al.: Phys.Rev. D77:014511, 2008]

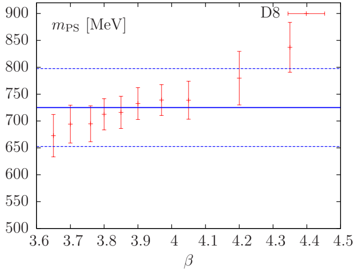
$$\left(a\frac{d\beta}{da}\right) = -\left(\frac{r_\chi}{a}\right)\left(\frac{d\left(\frac{r_\chi}{a}\right)}{d\beta}\right)^{-1}$$

$$\left(\frac{r_\chi}{a}\right)(\beta) = \frac{1+n_0 R(\beta)^2}{d_0(a_{2L}(\beta)+d_1 R(\beta)^2)} \qquad R(\beta) = \frac{a_{2L}(\beta)}{a_{2L}(3.9)} \qquad r_0 = 0.420(15)\ \mathrm{fm}$$

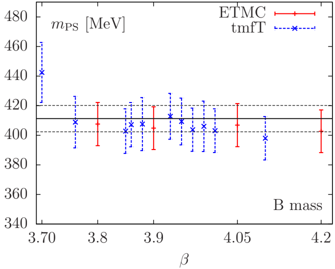


Lines of Constant Physics, constant m_π

$m_\pi \approx 700$ MeV: presently fulfilled up to 10 %

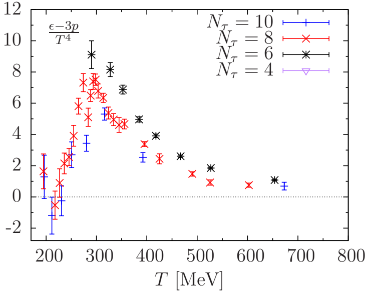


$m_\pi \approx 400$ MeV:

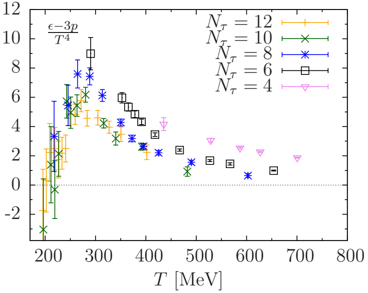


Trace anomaly - Lattice Artifacts

$m_\pi \approx 700$ MeV:



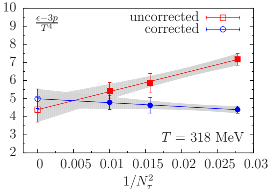
$m_\pi \approx 400$ MeV:



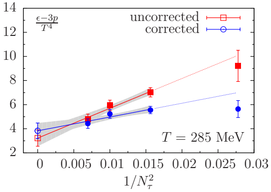
Trace Anomaly, Tree Level Corrections

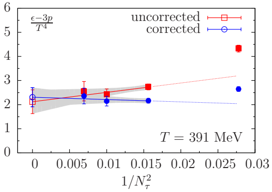
- Observe large lattice artifacts in $\frac{I}{T^4}$
- Lattice pressure in the free limit p^L_{SB} known: [P. Hegde *et al.*, 2008]
- Twisted mass action: [O. Philipsen & L. Zeidlewicz, 2010]
- Corrected by division by $p^L_{\mathrm{SB}}/p^C_{\mathrm{SB}}$ [S. Borsanyi *et al.*, 2010]

$m_\pi \approx 700$ MeV:



$m_\pi \approx 400$ MeV:





Tree Level Corrections (closer look), SB (Free) Limit

Lattice:

$$\frac{p_F^L(\mu)}{T^4} = 3\int_{[0,2\pi)^3} \frac{d^3k}{(2\pi)^3}\frac{1}{N_t}\sum_{n=0}^{N_t-1} \ln \mathrm{Det}\left(|G(k)|^2 + (a\mu)^2\right)$$

$$G(k) = (am) + 2r\sum_\mu \sin^2\left(\frac{ak_\mu}{2}\right) + i\sum_\mu \gamma_\mu \sin(ak_\mu)$$

Continuum:

$$\frac{p_F^C(\mu/T)}{T^4} = 2\frac{7}{8}\frac{\pi^2}{90}g(\mu/T)$$

$$g(\mu/T) = \frac{360}{7\pi^4}\int_{\mu/T}^{\infty} dx\; x\sqrt{x^2-(\mu/T)^2}\,\ln(1+e^{-x})$$

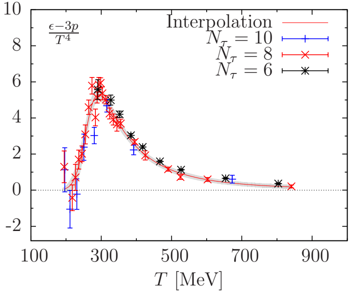
Correction:

N_τ	4	6	8	10	12
$p^L_{\mathrm{SB}}/p^C_{\mathrm{SB}}$	2.586	1.634	1.265	1.134	1.084

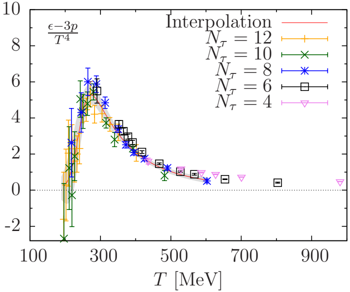
.

Corrected Trace Anomaly & Interpolation

$m_\pi \approx 700$ MeV:



$m_\pi \approx 400$ MeV:



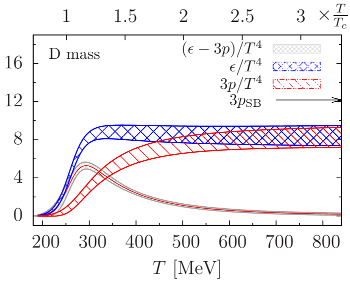
Interpolation for corrected $I\,/\,T^4$ [S. Borsanyi *et al.*, 2010] :

$$\frac{I}{T^4} = \exp\left(-h_1\bar{t} - h_2\bar{t}^2\right) \cdot \left(h_0 + \frac{f_0\left\{\tanh f_1\bar{t} + f_2\right\}}{1 + g_1\bar{t} + g_2\bar{t}^2}\right)$$

Pressure and Energy Density

$$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} d\tau \, \frac{\epsilon - 3p}{\tau^5}\bigg|_{\mathrm{LCP}}$$

$m_\pi \approx 700$ MeV:



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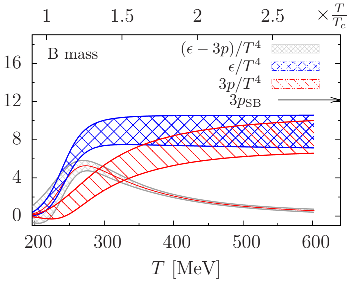

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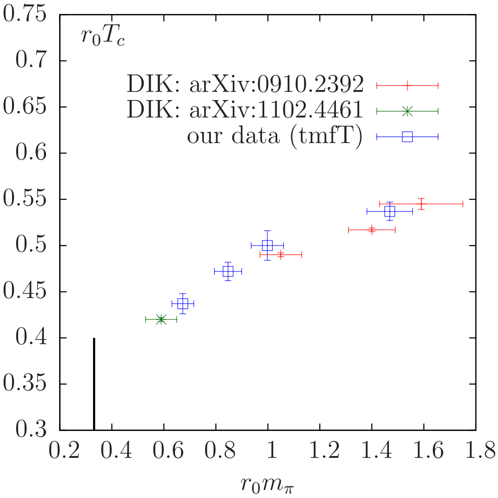
Conclusions & Outlook

- Conclusions:
  - T_c for pion masses in the range 300 - 700 MeV
  - Chiral limit so far inconclusive
  - $O(4)$ scaling compatible up to scaling violation terms
  - Thermodynamic equation of state for two pion masses
  - Have to further improve precision at $T > 0$ and $T = 0$
- Outlook:
  - $N_f = 2 + 1 + 1$

спасибо

Comparison with DIK-Collaboration

(G. Schierholz et. al) from $\sigma_{\langle\bar{\psi}\psi\rangle}$:



Renormalization of Re(L)

$$\left\langle \mathrm{Tr}\hat{L}^\dagger(\vec{x})\mathrm{Tr}\hat{L}(\vec{0})\right\rangle = \mathrm{e}^{-\frac{F_{\bar{q}q}(\vec{x},T)-F_0(T)}{T}} \underset{T\to 0}{\longrightarrow} \mathrm{e}^{-\frac{V(|\vec{x}|)}{T}}$$

Renormalization of $\langle\bar{\psi}\psi\rangle$

$$\langle\overline{\psi}\psi\rangle_{ren} = Z_{\mathrm{P}} \left(\langle\bar{\chi} i \gamma_5 \tau^3 \chi\rangle_{bare} + \frac{\mu \;\, c_{\mathrm{P}}(\beta)}{a^2} \right) + \ldots$$