Inhomogeneous phases in the QCD phase diagram

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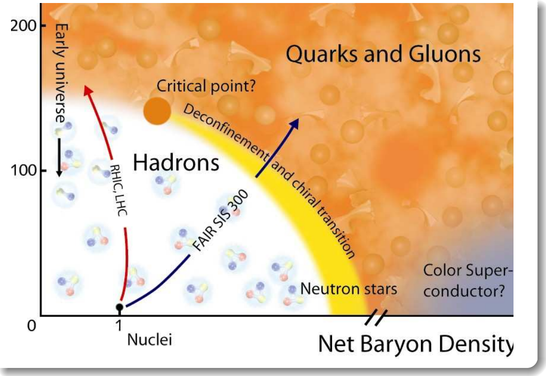
Joint Seminar “Hadron Physics” and “Theory of Hadronic Matter Under Extreme Conditions”

Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Russia, July 27, 2012

Motivation

- QCD phase diagram :



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- QCD phase diagram (NICA version):

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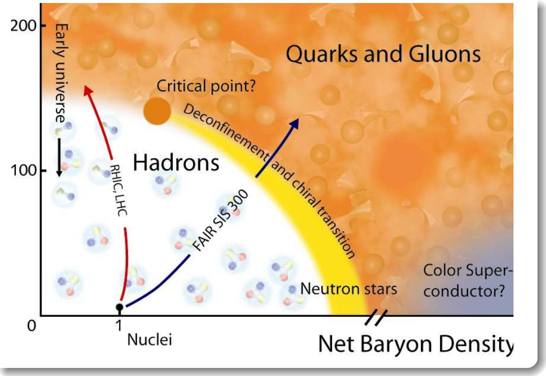

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  - critical point
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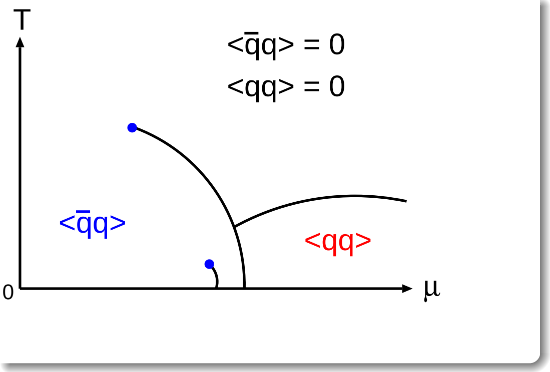

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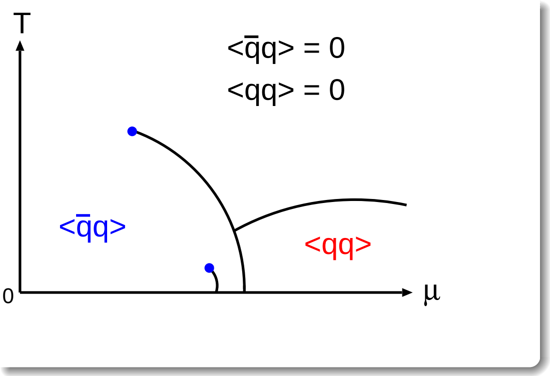

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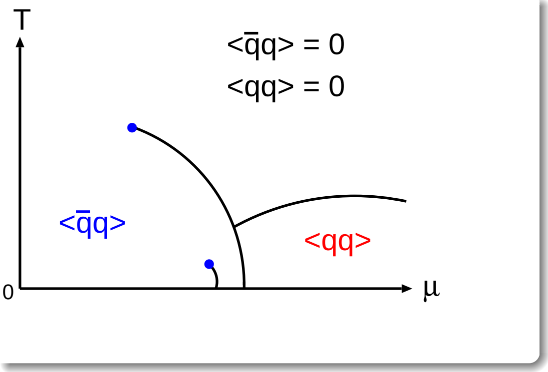

- question marks:
  - critical point
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- frequent assumption:
  $\langle\bar{q}q\rangle$, $\langle qq\rangle$ spatially constant
- How about
  inhomogeneous phases?

Are inhomogeneities important?

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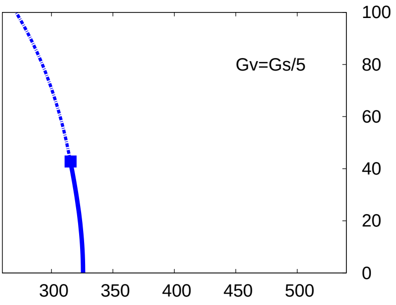
- one highlight example:

  Phase diagram in the NJL-model



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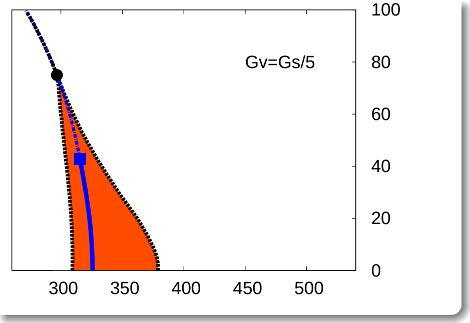

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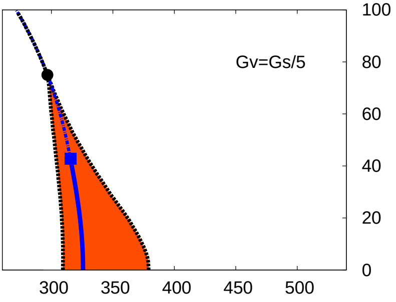

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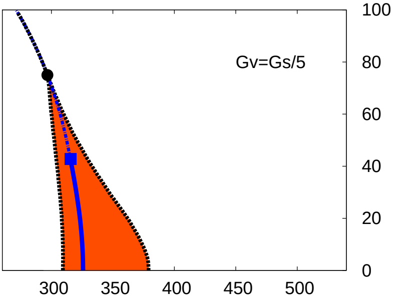

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- more details later ...

Inhomogeneous phases: (incomplete) historical overview

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- 1960s:
  - spin-density waves in nuclear matter (Overhauser)
  - crystalline superconductors (Fulde, Ferrell, Larkin, Ovchinnikov)
- 1970s – 1990s:
  - p-wave pion condensation (Migdal)
  - chiral density wave (Dautry, Nyman)
- after 2000:
  - 1+1 D Gross-Neveu model (Thies et al.)
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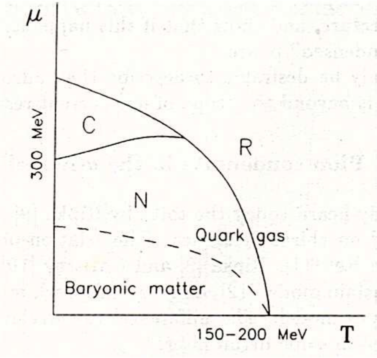

Broniowski et al. (1991)

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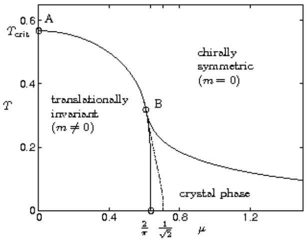

Thies, Urlichs (2003)

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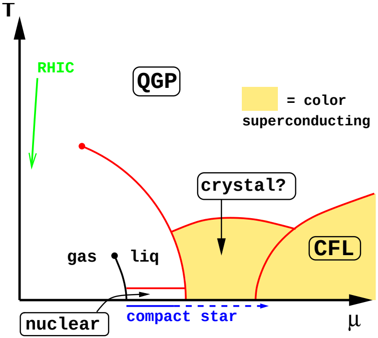

Alford (2003)

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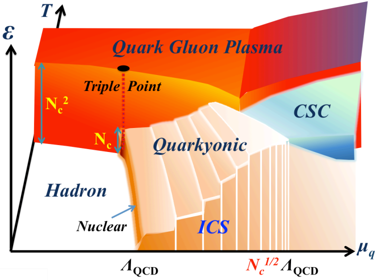

Kojo et al. (2011)

Contents

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$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

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  - retain space dependence !

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- mean-field thermodynamic potential:

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF} + \mu\bar{\psi}\gamma^0\psi)\right)$$

Mean-field model

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- mean-field Lagrangian:

$$\mathcal{L}_{MF} = \bar{\psi}(x)\,\mathcal{S}^{-1}(x)\,\psi(x) - G_S\left[S^2(\vec{x}) + P^2(\vec{x})\right]$$

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  - $\mathcal{H}_{MF}$ hermitean $\Rightarrow$ can (in principle) be diagonalized (eigenvalues E_λ)
  - $\mathcal{H}_{MF}$ time-independent $\Rightarrow$ Matsubara sum as usual

Mean-field thermodynamic potential

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- thermodynamic potential:

$$\Omega_{MF}(T,\mu;S,P) = -\frac{T}{V}\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\ \left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

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- general case: extremely difficult!

Periodic structures

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- crystal with a unit cell spanned by vectors $\vec{a}_i$, $i = 1, 2, 3$

  $\rightarrow$ periodic mass function: $M(\vec{x} + \vec{a}_i) = M(\vec{x})$

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- mean-field Hamiltonian in momentum space:

$$\mathcal{H}_{\vec{p}_m,\vec{p}_n} = \begin{pmatrix} -\vec{\sigma} \cdot \vec{p}_m \, \delta_{\vec{p}_m,\vec{p}_n} & \sum_{\vec{q}_k} M_{\vec{q}_k} \, \delta_{\vec{p}_m,\vec{p}_n+\vec{q}_k} \\ \sum_{\vec{q}_k} M^*_{\vec{q}_k} \, \delta_{\vec{p}_m,\vec{p}_n-\vec{q}_k} & \vec{\sigma} \cdot \vec{p}_m \, \delta_{\vec{p}_m,\vec{p}_n} \end{pmatrix}$$

  - different momenta coupled by $M_{\vec{q}_k}$ $\Rightarrow$ $\mathcal{H}$ is nondiagonal in momentum space!
  - $\vec{q}_k$ discrete $\Rightarrow$ $\mathcal{H}$ is still block diagonal

Periodic structures: minimum free energy

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$\rightarrow$ further simplifications necessary

One dimensional modulations

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- $1+1$D solutions known analytically: [M. Thies, J. Phys. A (2006)]

  $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu)$ (chiral limit), $\mathrm{sn}(\xi|\nu)$: Jacobi elliptic functions

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  - $\mathcal{H}_{CDW}$ can be diagonalized analytically
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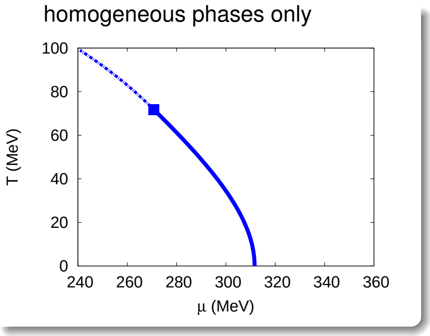
  The general problem with 1D modulations in 3+1D can be mapped to the 1 + 1 dimensional case
- 1 + 1D solutions known analytically: [M. Thies, J. Phys. A (2006)]

  $M(z) = \sqrt{\nu}\Delta \, \mathrm{sn}(\Delta z|\nu)$ (chiral limit), $\mathrm{sn}(\xi|\nu)$: Jacobi elliptic functions
- remaining task:
  - minimize w.r.t. 2 parameters: Δ, ν
  - (almost) as simple as CDW, but more powerful
  - $m \neq 0$: 3 parameters

Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]



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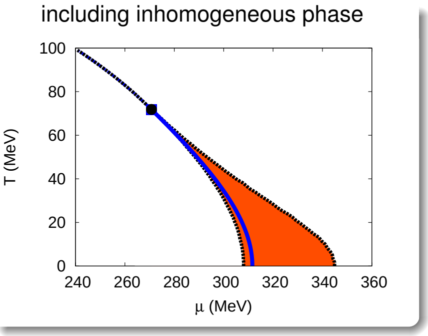

Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

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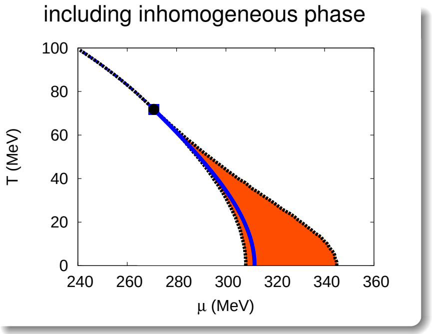

- 1st-order line completely covered by the inhomogeneous phase!
- all phase boundaries 2nd order
- critical point coincides with Lifshitz point (NJL specific)

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$

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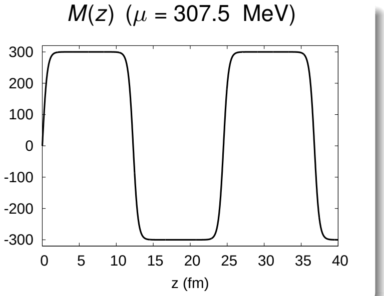

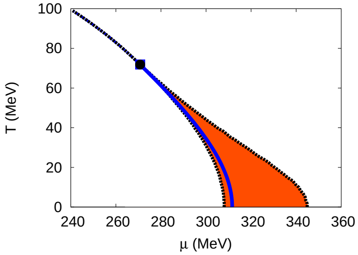

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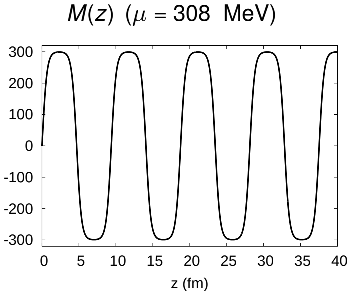

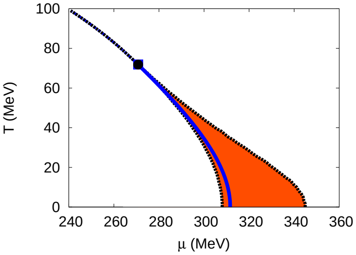

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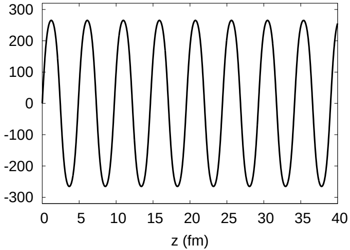
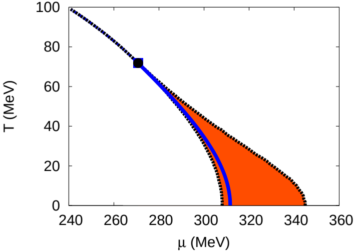
$M(z)$ ($\mu = 309$ MeV)





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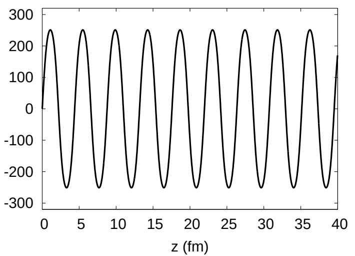

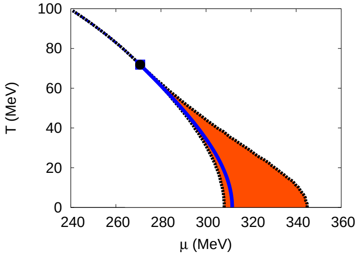

Mass functions and density profiles ($T = 0$)

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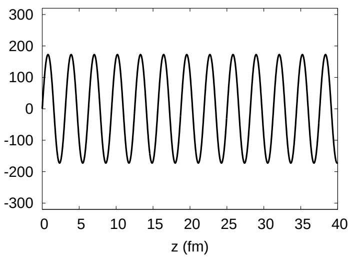
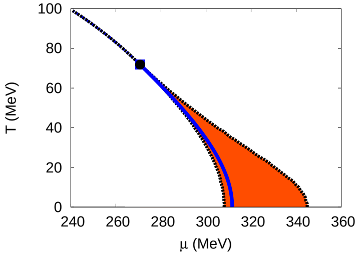
$M(z)$ (μ = 320 MeV)





Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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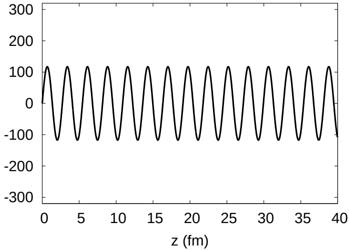

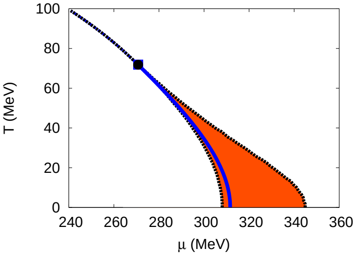

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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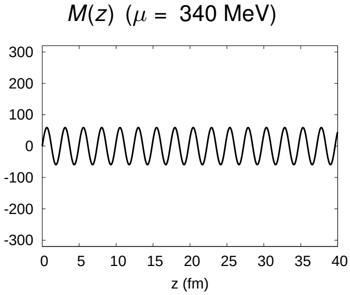

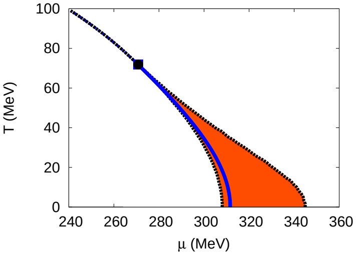

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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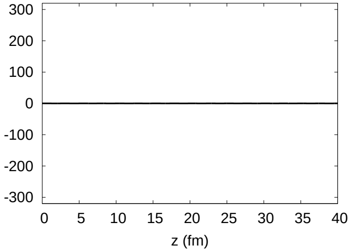
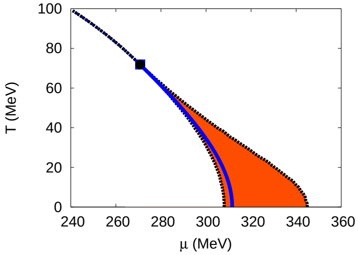
$M(z)$ ($\mu =$ 345 MeV)





Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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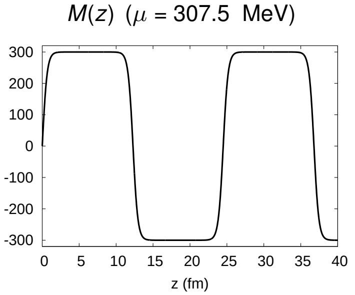

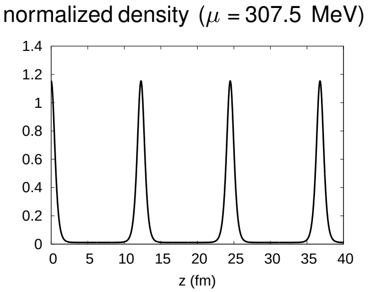

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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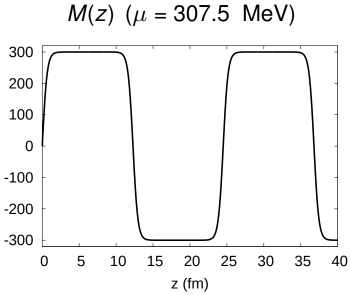

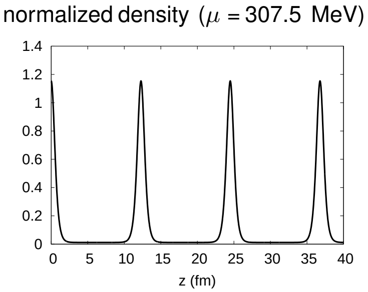

- Quarks reside in the chirally restored regions, cf. bag model!

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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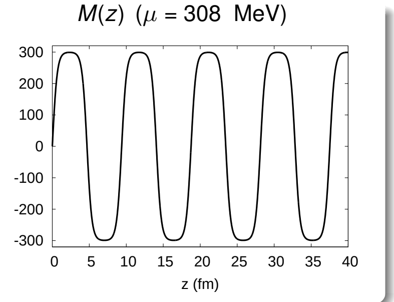

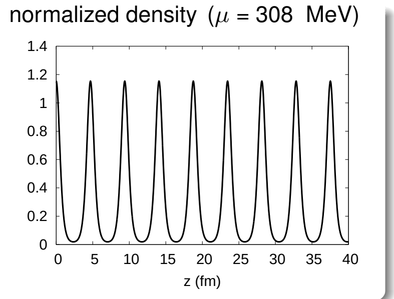

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Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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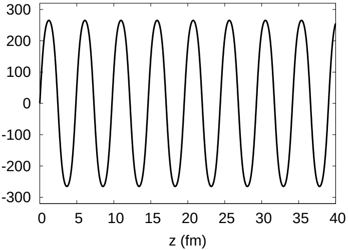
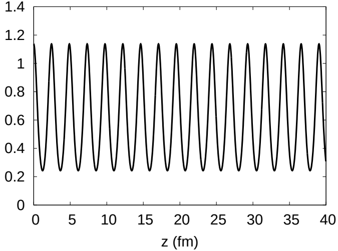
$M(z)$ ($\mu = 309$ MeV)



normalized density ($\mu = 309$ MeV)



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Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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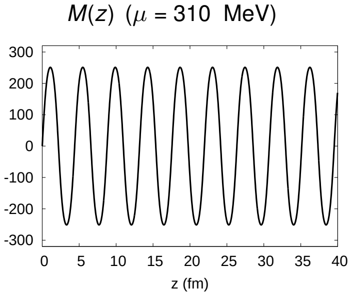

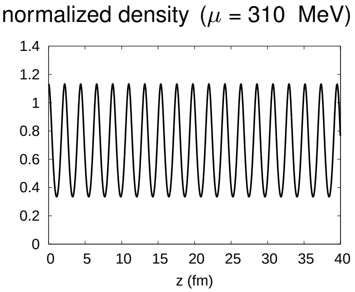

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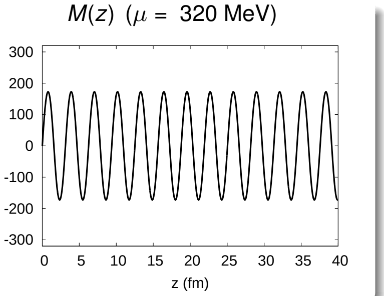

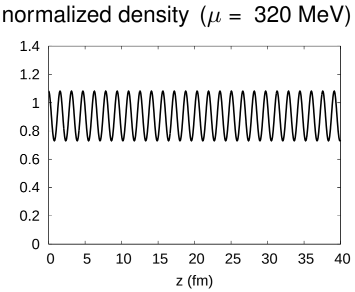

- Quarks reside in the chirally restored regions, cf. bag model!
- Density gets smoothened with increasing μ and T.

Mass functions and density profiles ($T = 0$)

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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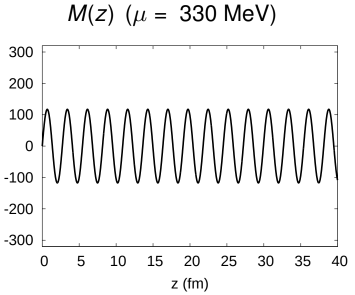

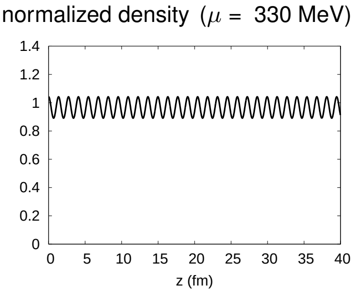

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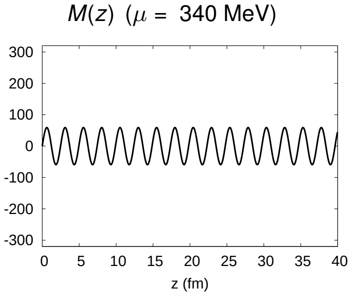

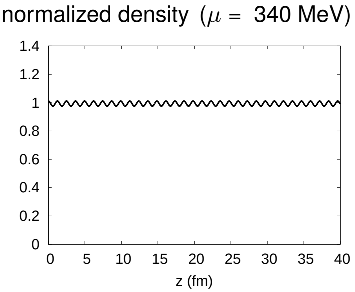

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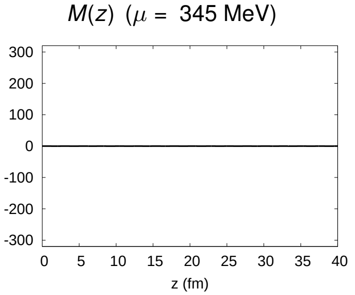

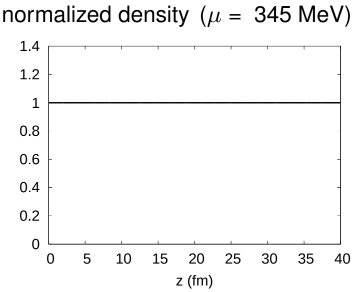

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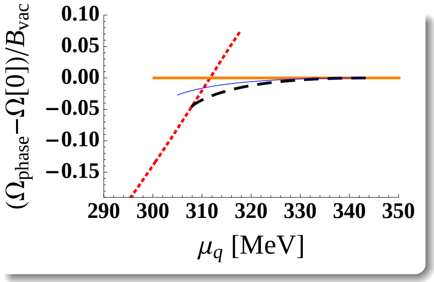
Free energy difference

[D. Nickel, PRD (2009)]



- homogeneous chirally broken
- solitons
- chiral density wave:
  $M_{CDW}(z) = \Delta\, e^{iqz}$

- soliton phase favored, when it exists
- $\delta\Omega_{soliton} \approx 2\delta\Omega_{CDW} \quad \Rightarrow$ CDW never favored

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field:
  - $\bar{\psi}\gamma^\mu\psi \to \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
  - $\langle\bar{\psi}\gamma^3\psi\rangle$ possible for inhomogeneous phases, but neglected

Including vector interactions

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  - $\langle\bar{\psi}\gamma^3\psi\rangle$ possible for inhomogeneous phases, but neglected
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\,n(\vec{x})$ “shifted chemical potential”

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
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- further approximation: $n(\vec{x}) \to \langle n\rangle = const. \quad \Rightarrow \quad \tilde{\mu} = const.$

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field:
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  - $\langle\bar{\psi}\gamma^3\psi\rangle$ possible for inhomogeneous phases, but neglected
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\, n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \to \langle n\rangle = const. \quad \Rightarrow \quad \tilde{\mu} = const.$
  - questionable in the inhomogeneous phase at low μ and T
  - ok near the restored phase (including the Lifshitz point)

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- additional vector term: $\mathcal{L}_V = -G_V(\bar\psi\gamma^\mu\psi)^2$
- additional mean field:
  - $\bar\psi\gamma^\mu\psi \to \langle\bar\psi\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
  - $\langle\bar\psi\gamma^3\psi\rangle$ possible for inhomogeneous phases, but neglected
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu \;=\; \mathcal{H}_{MF}|_{G_V=0} - \tilde\mu(\vec{x})$
  - $\tilde\mu(\vec{x}) = \mu - 2G_V\, n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \to \langle n\rangle = const. \quad\Rightarrow\quad \tilde\mu = const.$
  - questionable in the inhomogeneous phase at low μ and T
  - ok near the restored phase (including the Lifshitz point)
  - advantage: known analytic solutions can still be used

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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  - advantage: known analytic solutions can still be used
  - additional parameter: $\tilde{\mu}$, fixed by constraint $\frac{\partial\Omega_{MF}}{\partial\tilde{\mu}} = 0$

Phase diagram

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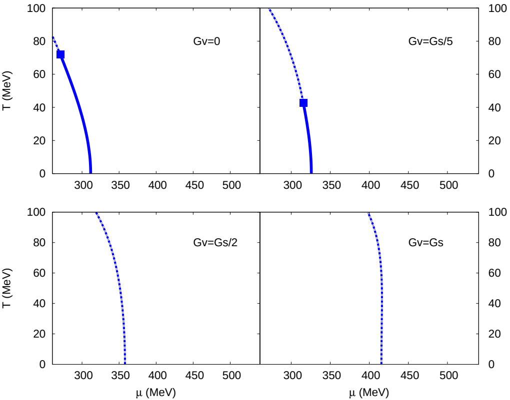

- homogeneous phases: strong G_V-dependence of the critical point

Phase diagram

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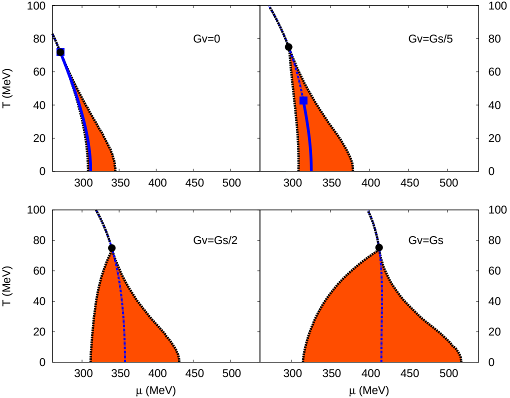

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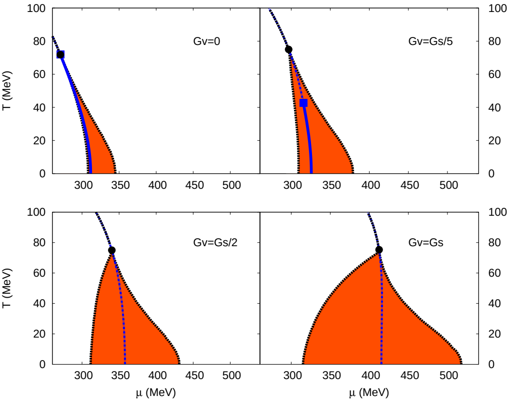
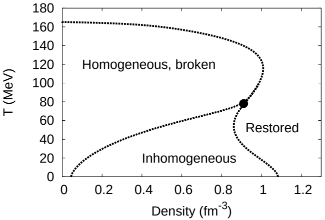


T-$\langle n \rangle$ phase diagram:



- independent of G_V!

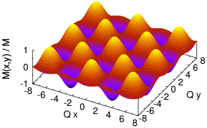
- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Two-dimensional modulations

Two-dimensional modulations

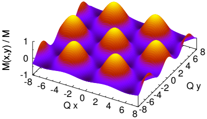
- consider two shapes:
  - square lattice ("egg carton")

    $$M(x, y) = M \cos(Qx) \cos(Qy)$$



  - hexagonal lattice

    $$M(x, y) = \frac{M}{3} \left[2 \cos(Qx) \cos\left(\frac{1}{\sqrt{3}} Qy\right) + \cos\left(\frac{2}{\sqrt{3}} Qy\right) \right]$$



- minimize both cases numerically w.r.t. M and Q

Two-dimensional modulations: results

[S. Carignano, M.B., arXiv:1203.5343]

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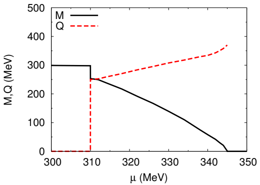
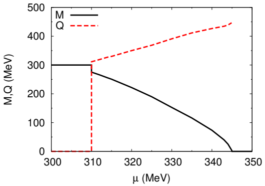
- amplitudes and wave numbers:
  - egg carton:



  - hexagon:



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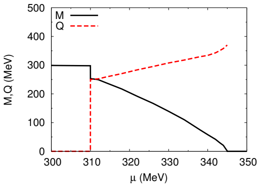

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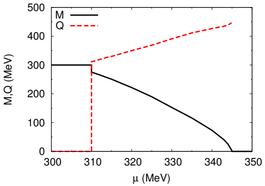
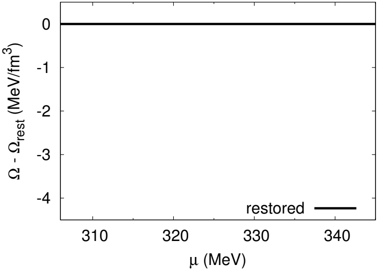


free-energy gain at T = 0:



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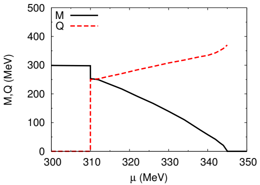

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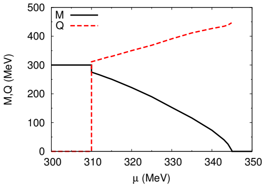

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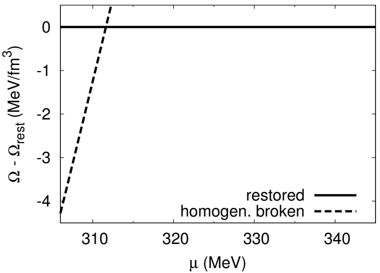

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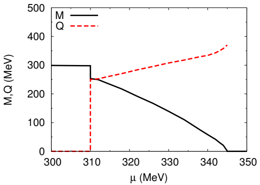

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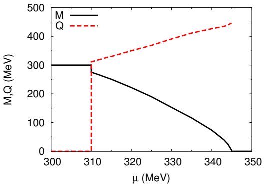

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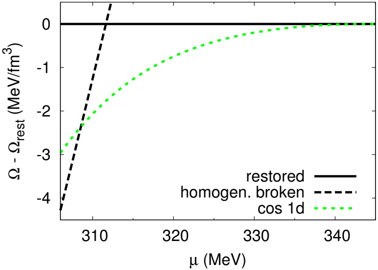

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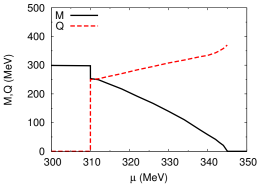

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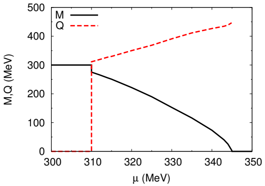

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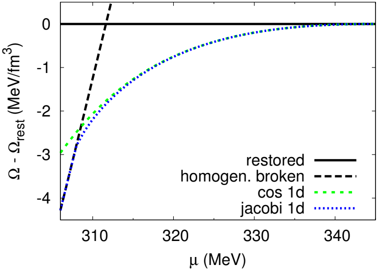

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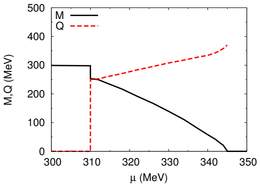

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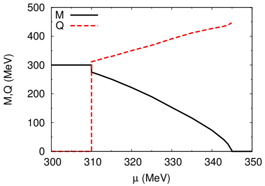

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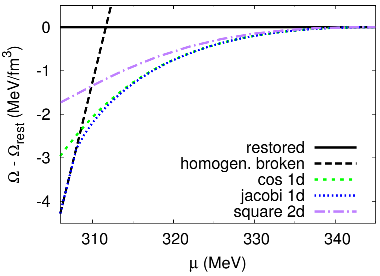

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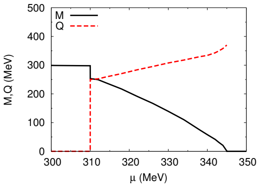

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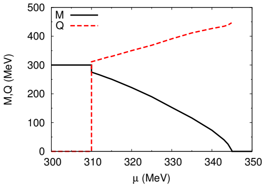

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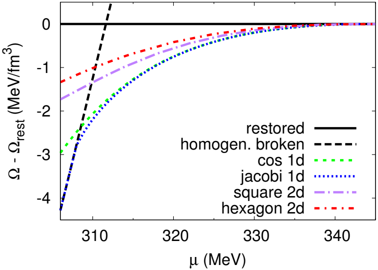

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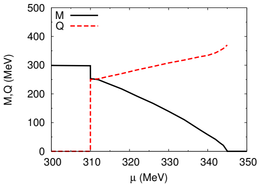

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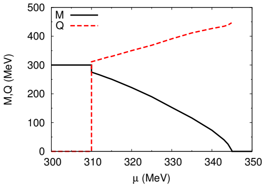

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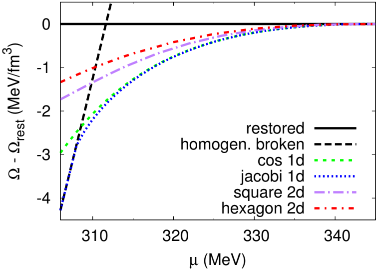

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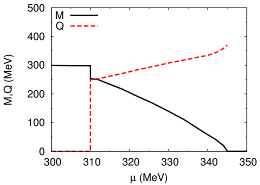

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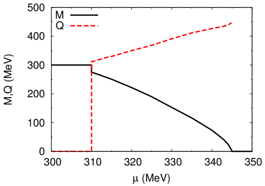

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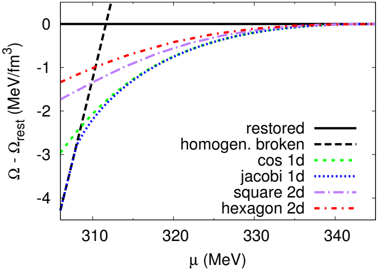

- 2D not favored in this regime
- more Fourier components:

$$M(x, y) = \sum_{n=1}^{3} M_n \cos(nQx)\cos(nQy)$$

no effect!

Two-dimensional modulations: further results

[S. Carignano, M.B., arXiv:1203.5343]

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- rectangular lattice:

  $M(x, y) = M \cos(Q_x x) \cos(Q_y y)$

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[S. Carignano, M.B., arXiv:1203.5343]

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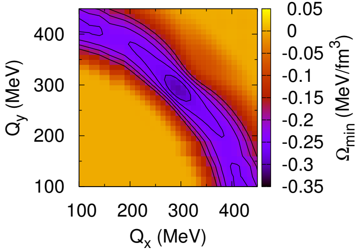

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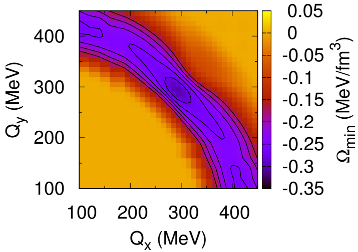

$\Rightarrow$ “egg carton” local minimum

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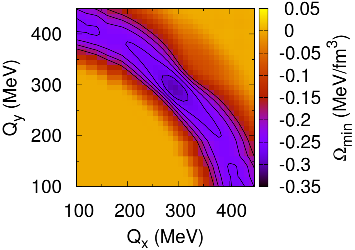
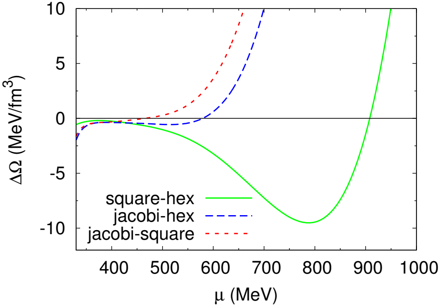


$\Rightarrow$ "egg carton" local minimum

- higher chemical potentials



- 450 MeV $< \mu <$ 900 MeV: egg carton favored
- $\mu >$ 900 MeV: hexagon favored

Conclusions

- Inhomogeneous phases must be considered!

Conclusions

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- NJL model with one- and two-dimensional modulations of $\langle\bar{q}q\rangle$:
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  - inhomogeneous phase rather stable w.r.t. vector interactions
  - number susceptibility always finite (for $G_V > 0$)
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  - competition with color superconductivity must be taken into account

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- experimental signatures?
  - to be worked out ...

Collaborators

TECHNISCHE UNIVERSITÄT DARMSTADT

Dominik Nickel
(INT Seattle $\rightarrow$ Siemens)

Stefano Carignano
(TU Darmstadt)

Daniel Nowakowski
(TU Darmstadt)