Study of temperature dependence of QCD viscosity

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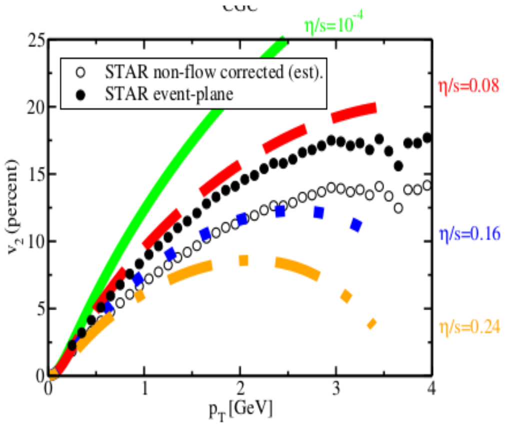
Outline:

- Introduction
- Details of the calculation
- Fitting of the data
- Backus-Gilbert method
- Conclusion

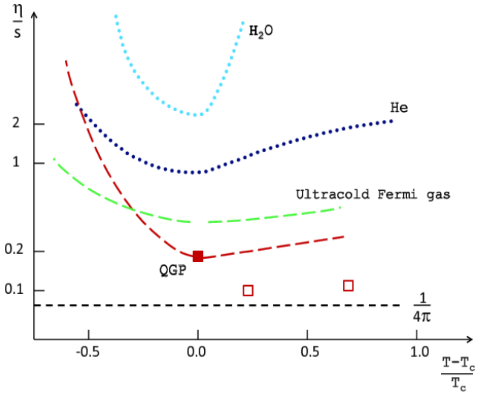


Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\phi} \sim (1 + 2v_1 cos(\phi) + 2v_2 cos^2(\phi)),\ \phi\text{-scattering angle}$$

Quark-gluon plasma is close to ideal liquid ($\frac{\eta}{s} = (1-3)\frac{1}{4\pi}$)

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)



S.Cremonini, U.Gursoy, P.Szepietowski, JHEP 1208 (2012) 167

Comparison of different liquids

Quark-gluon plasma is the most ideal liquid

Other works (SU(3) gluodynamics):

- Karsch, F. et al. Phys.Rev. D35 (1987)
- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev. D76 (2007) 101701
- H. B. Meyer, Nucl.Phys. A830 (2009) 641C-648C

Results:

- $\frac{\eta}{s} = 0.134 \pm 0.033 \; (T/T_c = 1.65, 8 \times 28^3)$
- $\frac{\eta}{s} = 0.102 \pm 0.056 \; (T/T_c = 1.24, 8 \times 28^3)$
- $\frac{\eta}{s} = 0.20 \pm 0.03 \; (T/T_c = 1.58, 16 \times 48^3)$
- $\frac{\eta}{s} = 0.26 \pm 0.03 \; (T/T_c = 2.32, 16 \times 48^3)$

SU(2) gluodynamics:

- $\frac{\eta}{s} = 0.134 \pm 0.057 \; (T/T_c = 1.2, 16 \times 32^3)$

N.Yu. Astrakhantsev, V.V. Braguta, A.Yu. Kotov, JHEP 1509 (2015) 082

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The first step:

Measurement of the correlation function:

$$C(t) = \langle T_{12}(t) T_{12}(0) \rangle$$

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The second step:

Calculation of the spectral function $\rho(\omega)$:

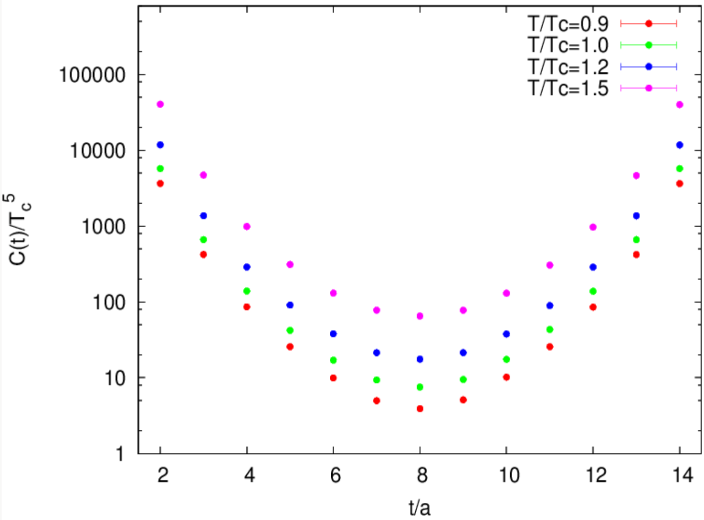
$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

Details of the calculation

- SU(3) gluodynamics
- Two-level algorithm
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.1, 1.2, 1.35, 1.5$
- Accuracy $\sim 2-3\%$ at $t = \frac{1}{2T}$
- $\langle T_{12}(x) T_{12}(y)\rangle \sim (\langle T_{11}(x) T_{11}(y)\rangle - \langle T_{11}(x) T_{22}(y)\rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285-300
- ...

Correlation functions



Spectral function

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0$, $\rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega\to\infty}^{NLO} = \frac{1}{10}\frac{d_A}{(4\pi)^2}\omega^4\left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  $\sim$ 90% of the total contribution $t = 1/2/T$
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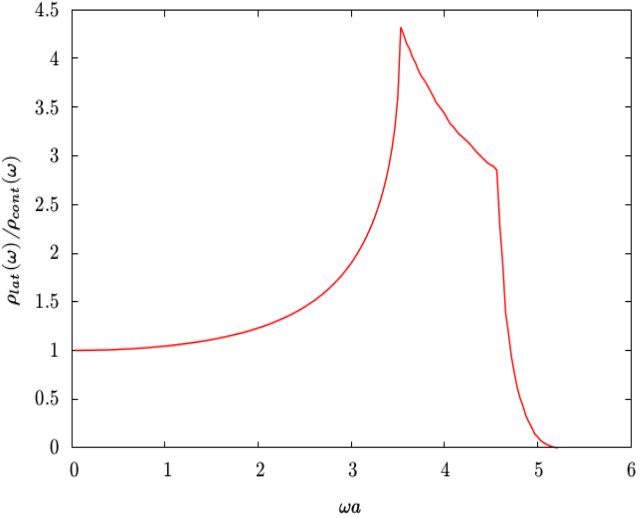
Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

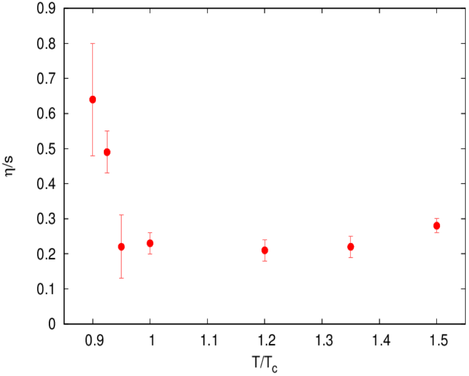
Lattice spectral function ρ_{lat}

Takes into account discretization errors in temporal direction



Spectral function

$$\rho_1(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$
$$\chi^2/dof \sim 1,\ A \sim 1,\ \omega_0/T \sim 7 - 8$$



Two additional ansatzs:

- $\rho_2(\omega) = \frac{1}{2}B\ \omega(1 + \tanh[\gamma(w_0 - w)]) + \frac{1}{2}A\rho_{lat}(\omega)(1 + \tanh[\gamma(w - w_0)])$
- $\rho_3(\omega) = B\ \omega(1 + C\omega^2)\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$

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- Asymptotic freedom works well from $\omega > 3$ GeV
- Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV
  $\Rightarrow$ Main source of uncertainty in the fitting procedure

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Conclusion

Accuracy improvement leads to improvement of our knowledge about UV part of spectral function but not about IR one

Backus-Gilbert method for the spectral function

- Problem: find $f(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega f(\omega) K(x_i,\omega), \quad K(x_i,\omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{f}(\bar{\omega})$ ($\delta(\bar{\omega},\omega)$ - resolution function):

$$\tilde{f}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega},\omega) f(\omega)$$

- Let us expand $\delta(\bar{\omega},\omega)$ as

$$\delta(\bar{\omega},\omega) = \sum_i b_i(\bar{\omega}) K(x_i,\omega) \quad \tilde{f}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

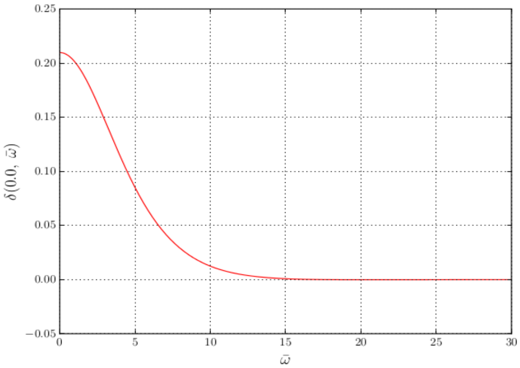
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i,\omega)(\omega - \bar{\omega})^2 K(x_j,\omega), R_i = \int d\omega K(x_i,\omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

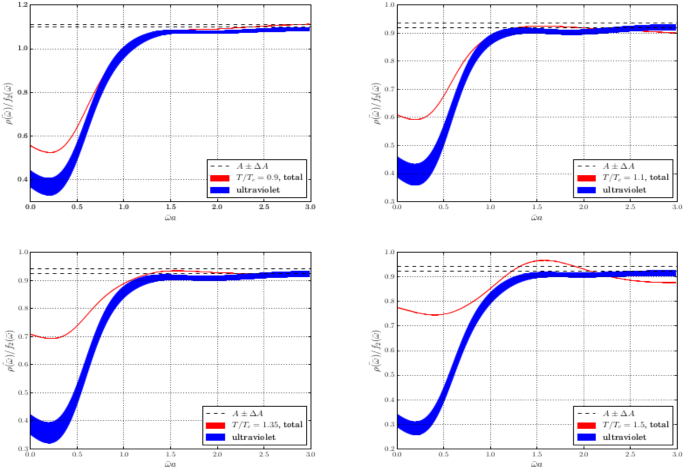
Resolution function $\delta(0,\omega)$ ($T/T_c = 1,\ \lambda = 0.001$)



- Width of the resolution function $\omega/T \sim 4$
- Hydrodynamical approximation works up to $\omega/T < \pi$
- Problem: large contribution from ultraviolet tail ($\sim 50\%$)

Model for the ultraviolet contibution

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

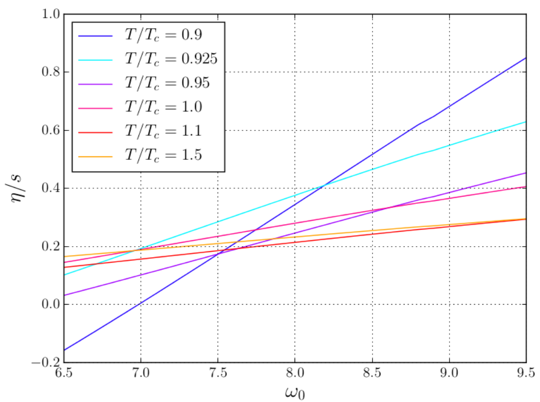


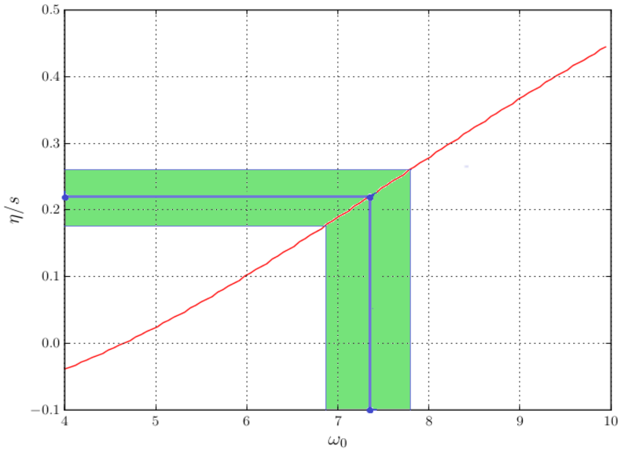
Solution:

- Take ultraviolet contribution in the form:

$$\rho_{ultr} = A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

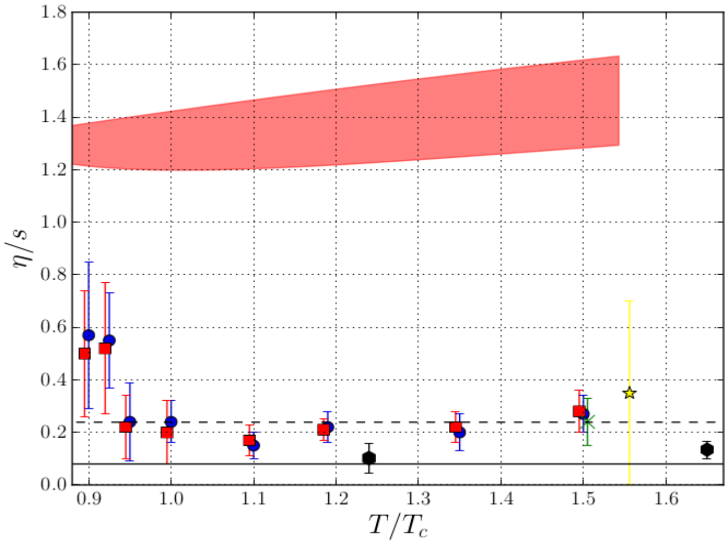
- Determine the value of the A BG method
- Subtract ultraviolet contribution and obtain η/s as a function of ω_0

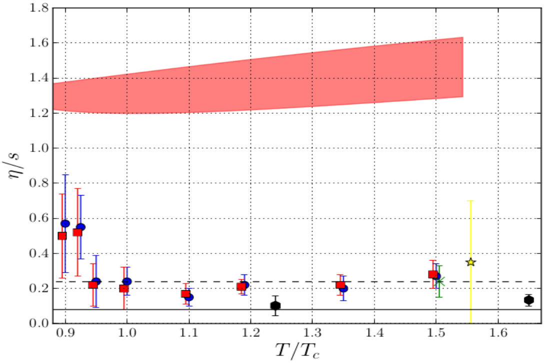




- For $T/T_c = 1.$ $\omega_0/T = 7.33 \pm 0.47$
- $\eta/s = 0.22 \pm 0.04$

Our results





Conclusion:

- We calculated η/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Applied fitting procedure and Backus-Gilbert method for the SF
- η/s is close to N=4 SYM and in agreement with experiment
- Large deviation from perturbative results