Polyakov-DSE (non-local PNJL) models: From (lattice) quark propgators to the QCD phase diagram

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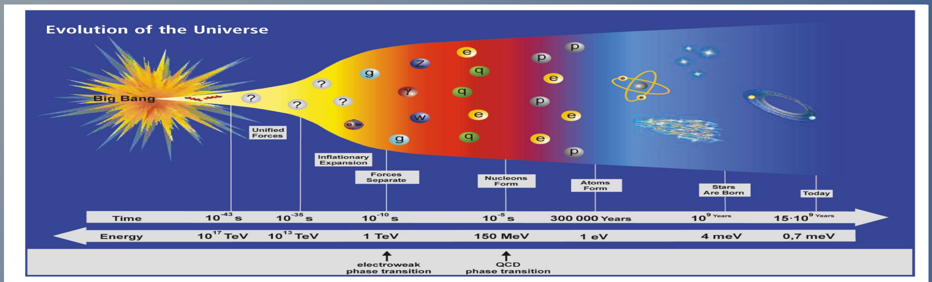
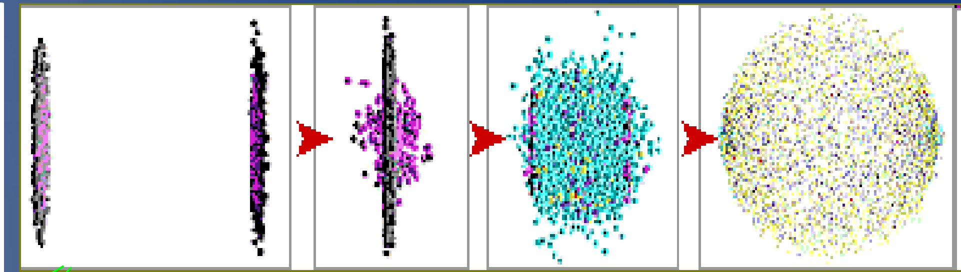
Outline

- Motivation
- The model and its parameterizations
- Thermodynamic properties and phase transitions
- Phase diagrams
- CEP determination and critical exponents
- Further developments and outlook

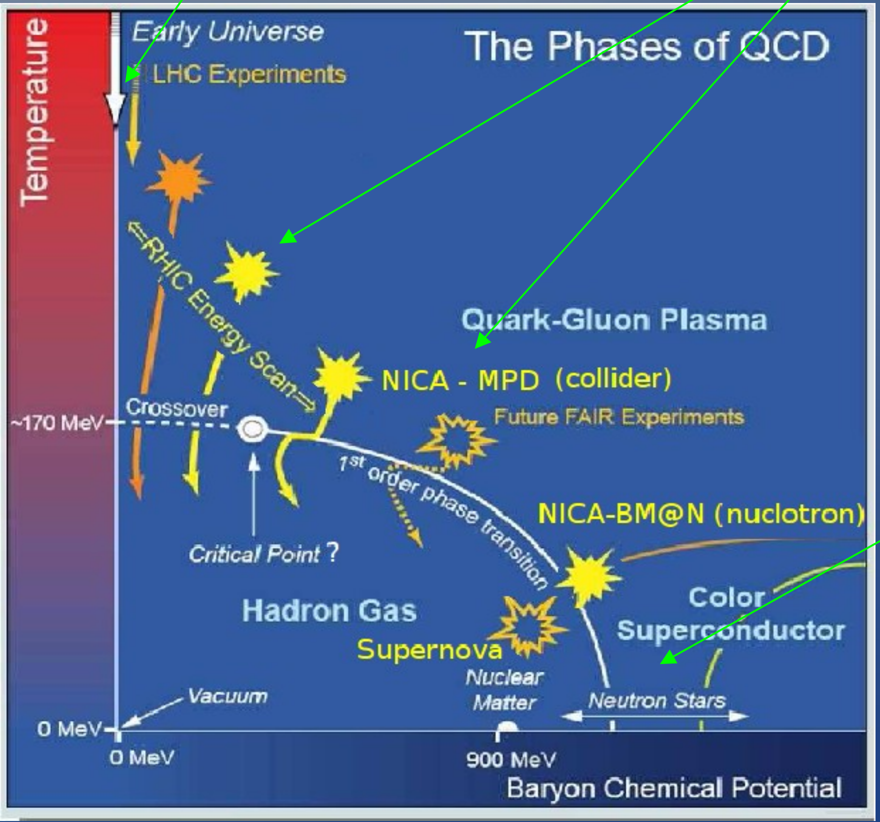
Motivation



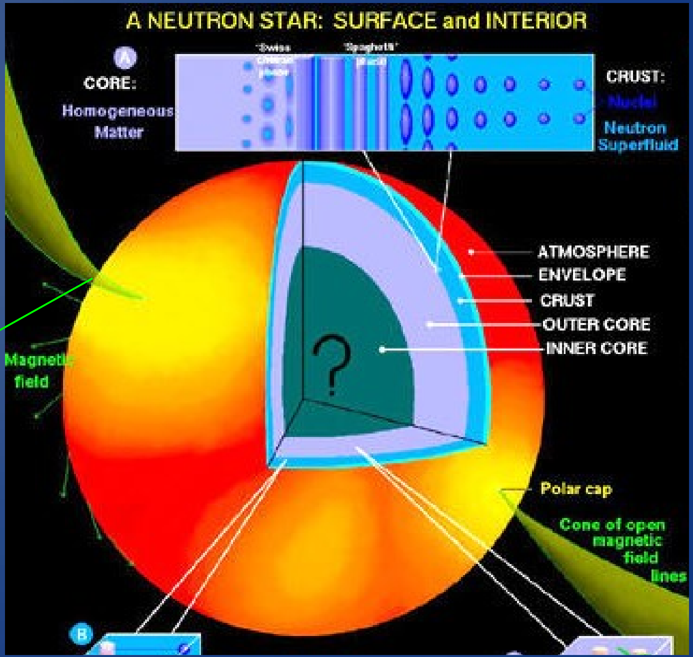
Cosmology



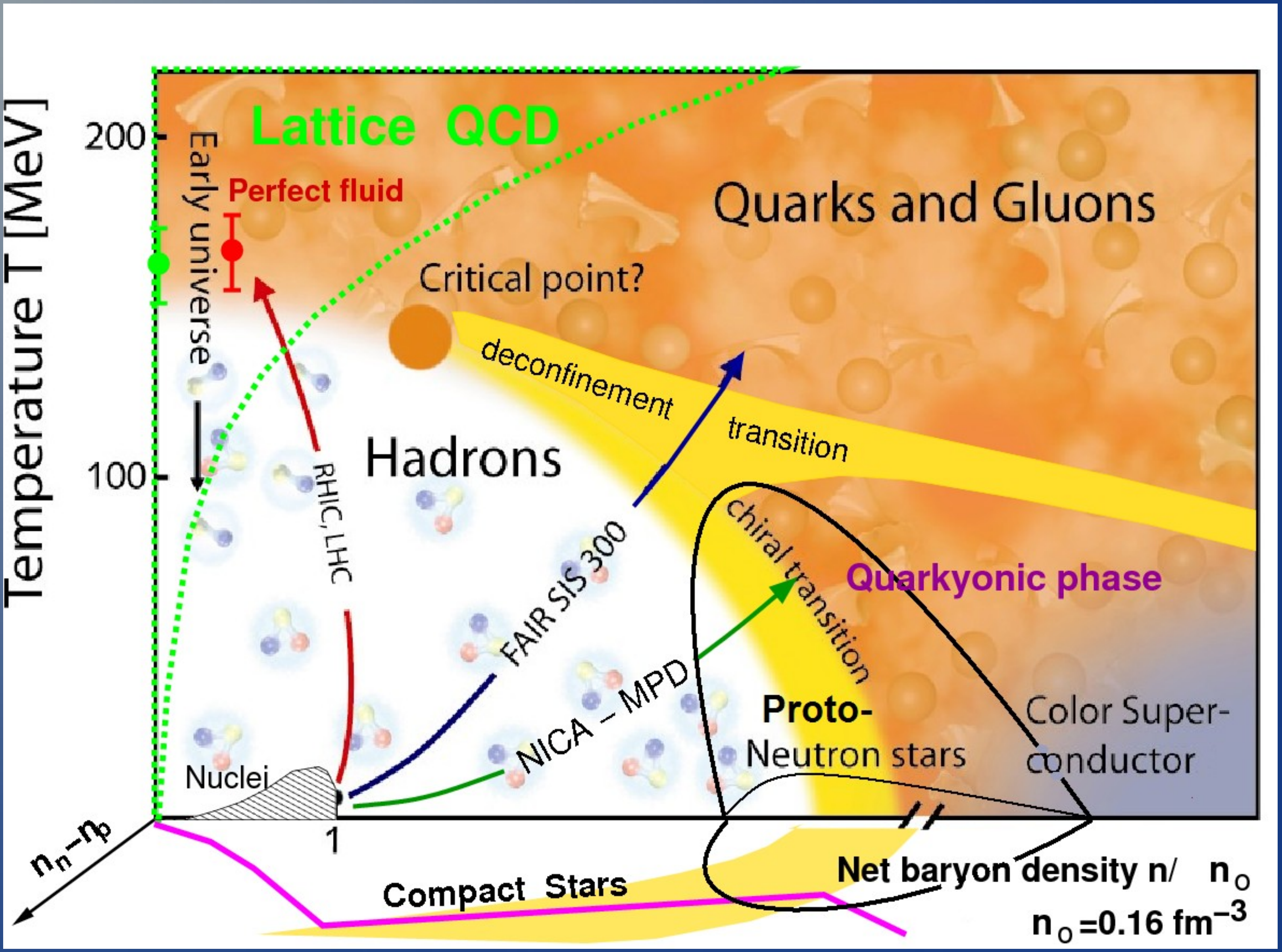
Heavy ion collisions
(RHIC, LHC, FAIR, NICA ...)



QCD Phase Diagram?



Compact star astrophysics



Lattice QCD
Temperature T [MeV]
200
100
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
chiral transition
Quarkyonic phase
Proto-
Neutron stars
Color Super-
conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o =0.16 fm^{-3}

QCD Theory and (local) NJL model

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma_\mu D^\mu - m)q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$D_\mu = \partial_\mu - ig\lambda_a A^a_\mu \quad ; \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f_{abc} A^b_\mu A^c_\nu$$

-Asymptotic freedom (high energy): almost free quarks, interactions can be determined by using perturbation theory.

-Confinement (~ 1GeV): highly non linear, confined quarks. At this range of energies mesonic properties -such as masses, coupling and decay constants, mixing angles, etc- cannot be studied directly from continuum QCD.

$$\mathcal{L}_{NJL_{local}} = \bar{q}(i\partial\!\!\!/ - m)q + g[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Non-local interactions are proposed to solve standard NJL problems:

- Absence of a confinement mechanism
- Regularization must be added to remove divergences

Non-local extended NJL models

$$\mathcal{L}_{noLoc} = \bar{q}(p)\,(\not p - m)\,q(p')\,(2\pi)^4\delta^{(4)}(p-p') + \mathcal{L}_{int\_noLoc}$$

$$\mathcal{L}_{int\_noLoc} = -\frac{G^2}{2}\left[j^a_\mu(p_1, p'_1) D^{ab}_{\mu\nu}(p_1, p_2, p'_1, p'_2) j^b_\mu(p_2, p'_2)\right]$$

Non-local color currents after Fierz transformations

$$j^a_\mu(p, p') = g(p, p')\bar{q}(p)\tau_a q(p')$$

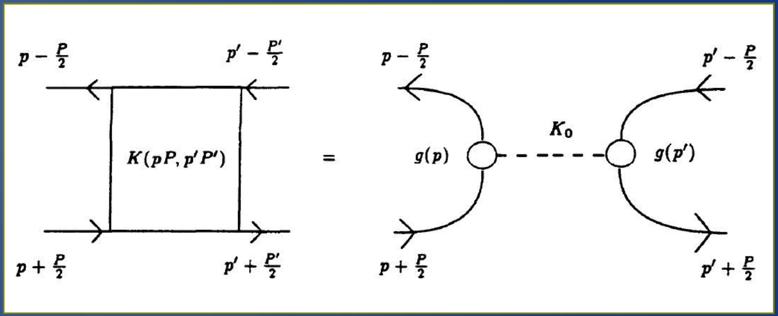
Gluon effective propagator:

$$D^{ab}_{\mu\nu}(p_1, p_2, p'_1, p'_2) = g_{\mu\nu}\delta^{ab} D(p_1, p_2, p'_1, p'_2)$$

Four point interaction Kernel

$$K(p_1, p_2, p'_1, p'_2) = g(p_1, p'_1)\,D(p_1, p_2, p'_1, p'_2)\,g(p_2, p'_2)$$

$$K(p, P; p', P') = -K_0 g(p) g(p') \delta_{P,P'}$$



The use of non-local interactions has the advantage that, with an adequate selection of the form factor(s), it could be achieved that the fermion propagator has no real mass poles. So the quarks don't appear as asymptotic states, which could be interpreted as realization of confinement in low-energy QCD.

D. Blaschke, S. Schmidt, Y. Kalinovsky. Phys. Rev. C 50, 435 (1994)

Non-local extended NJL model with WFR

$$S_E = \int d^4x \left\{ \bar\psi(x)(i\not\partial + m_q)\psi(x) - \frac{G_s}{2}\left[j_a(x) j_a(x) + j_P(x) j_P(x) \right] \right\}$$

Nonlocal currents: equivalent to separable form of gluon propagator

$$j_a(x) = \int d^4z\, g(z)\bar\psi(x+\tfrac{z}{2})\,\Gamma_a\,\psi(x-\tfrac{z}{2}) \qquad \Gamma_a = (\mathbf{1}, i\gamma_5\vec{\tau})$$

$$j_P(x) = \int d^4z\, f(z)\bar\psi(x+\tfrac{z}{2})\,\frac{i\overleftrightarrow{\not\partial}}{2\kappa_p}\,\psi(x-\tfrac{z}{2})$$

Bosonization (Hubbard-Stratonovich) and Mean Field Aproximation

$$S_E^{(MFA)} = -4N_c \int \frac{d^4p}{(2\pi)^4} \ln\left[\frac{(p)^2 + M^2(p)}{Z^2(p)} \right] + \frac{\sigma_1^2}{2G_S} + \frac{\kappa_p^2\,\sigma_2^2}{2G_S}$$

$$Z(p) = (1 - \sigma_2 f(p))^{-1}$$

$$M(p) = Z(p)(m_q + \sigma_1 g(p))$$

Dynamical mass function M(p)

Wave function renormalization Z(p)

S. Noguera, N. N. Scoccola, Phys.Rev. **D78** (2008) 114002.

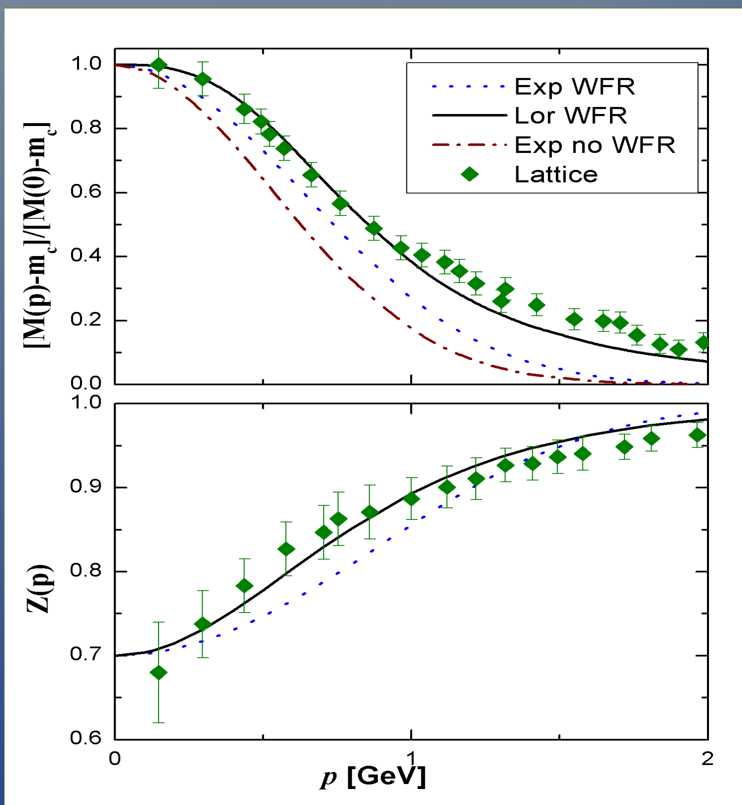
Non-local extended NJL model with WFR

Parameterization w/o WFR

Exponential (Set A)

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

$$f(p) = 0 \quad , \quad \sigma_2 = 0$$



Parameterizations with WFR

Exponential (Set B)

$$f(p) = e^{-(p^2/\Lambda_1^2)}$$

$$g(p) = e^{-(p^2/\Lambda_0^2)}$$

Lattice adjusted Lorentzian (Set C)

$$f(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} f_z(p)$$

$$g(p) = \frac{1+\alpha_z}{1+\alpha_z f_z(p)} \frac{\alpha_m f_m(p) - m_q \alpha_z f_z(p)}{\alpha_m - m_q \alpha_z}$$

$$f_m(p) = [1+(p^2/\Lambda_0^2)^{3/2}]^{-1} \quad , \quad f_z(p) = [1+(p^2/\Lambda_1^2)]^{-5/2}$$

$$\alpha_z = -0.3 \quad , \quad \alpha_m = 309\ MeV$$

M.B. Parapily et al., PRD 73 (2006) 054504

S. Noguera, N. N. Scoccola, Phys.Rev. **D78** (2008) 114002

Nonlocal PNJL model: including the Polyakov loop

We incorporate the Polyakov loop using covariant derivative

$$D_\mu \equiv \partial_\mu - i A_\mu$$

Assuming that quarks move into a color gauge field

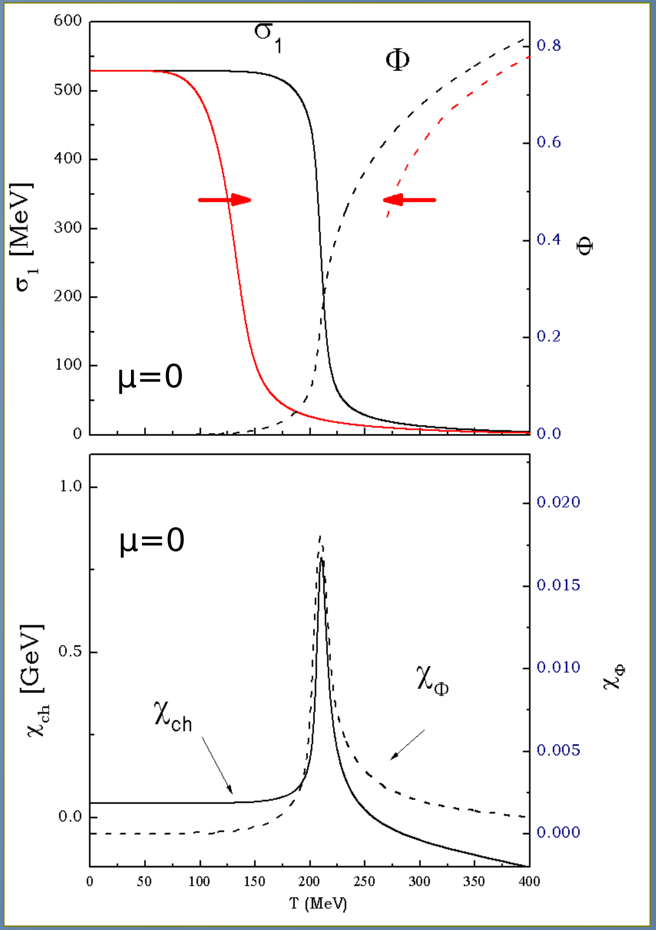
$$\phi = i A_0 = i g \delta_{\mu 0} G_a^\mu \lambda^a / 2$$

And the traced Polyakov loop (parameter of the confinement) results: $\Phi = \frac{1}{3} \mathrm{Tr}\ \exp(i\phi)$

Using the named Polyakov gauge, the matrix ϕ has diagonal representation: $\phi = \phi_3 \lambda_3 + \phi_8 \lambda_8$

In order to keep Ω_{MFA} real valued :

$$\phi_8 = 0 \ ; \quad \Phi = \frac{1}{3}[1 + 2\cos(\phi_3 / T)]$$



D.B., M. Buballa, A. Radzhabov, M.K. Volkov, Phys. At. Nucl. **71** (2008) 1981; PRD **83** (2011) 116004

Non-local extended PNJL model at finite T and μ

Matsubara and imaginary time formalisms

$$p^2 \to \omega_p^2 = (\omega_n - i\mu)^2 + \vec{p}^{\,2} \quad \text{with} \quad \omega_n = (2n+1)\pi T$$

$$\int \frac{d^4 p}{(2\pi)^4} g(p) \to 2T \sum_{n=0}^{\infty} \int \frac{d^3 \vec{p}}{(2\pi)^3} g(\omega_n - i\mu, \vec{p})$$

Polyakov loop

$$\mu \to \mu_c = \mu - i\phi_c$$

$$N_c \to \sum_c \quad \text{with} \quad \phi_c = +\phi_3, 0, -\phi_3 \quad \text{for } r, g, b$$

$$\Omega_{MFA}(T,\mu) = -4T \sum_c \sum_n \int \frac{d^3 \vec{p}}{(2\pi)^3} \ln\left[\frac{\rho_{nc}^2 + M^2(\rho_{nc}^2)}{Z^2(\rho_{nc}^2)}\right] + \frac{\sigma_1^2}{2G_S} + \frac{\varkappa_p^2 \sigma_2^2}{2G_S} + \mathcal{U}(\Phi, T)$$

$$\rho_{nc}^2 = \left[(2n+1)\pi T - i\mu + \phi_c\right]^2 + \vec{p}^{\,2}$$

$$\mathcal{U}(\Phi,T) = \left[-\tfrac{1}{2} a(T)\Phi^2 + b(T)\ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4)\right] T^4$$

$a(T)$, $b(T)$ fitted to lattice QCD results.

S. Roessner, C. Ratti and W. Weise, Phys. Rev. D 75 (2007) 034007
G.A. Contrera, M. Orsaria, N.N. Scoccola, Phys. Rev. D 82 (2010) 054026

Regularization

Ω_{MFA} turns to be divergent and needs to be regularized. We use

$$\Omega^{reg}_{MFA} = \Omega_{MFA} - \Omega_{free} + \Omega^{reg}_{free} + \Omega_0$$

where:
- Ω_{free} is obtained from Ω_{MFA} by setting $\sigma_1=\sigma_2=0$.
- Ω_0 is a constant fixed by the condition $\Omega^{reg}_{MFA} = 0$ at $T=\mu=0$
- Ω^{reg}_{free} is the regularized expression for the thermodynamical potential in the absence of fermion interactions. It is given by

$$\Omega^{reg}_{free} = -4T \int \frac{d^3\vec{p}}{(2\pi)^3} \sum_c \left[\ln\left(1+\exp\left[-\frac{E_p+\mu+i\phi_c}{T}\right]\right) + \ln\left(1+\exp\left[-\frac{E_p-\mu-i\phi_c}{T}\right]\right)\right]$$

with $E_p = \sqrt{\vec{p}^2 + m^2}$

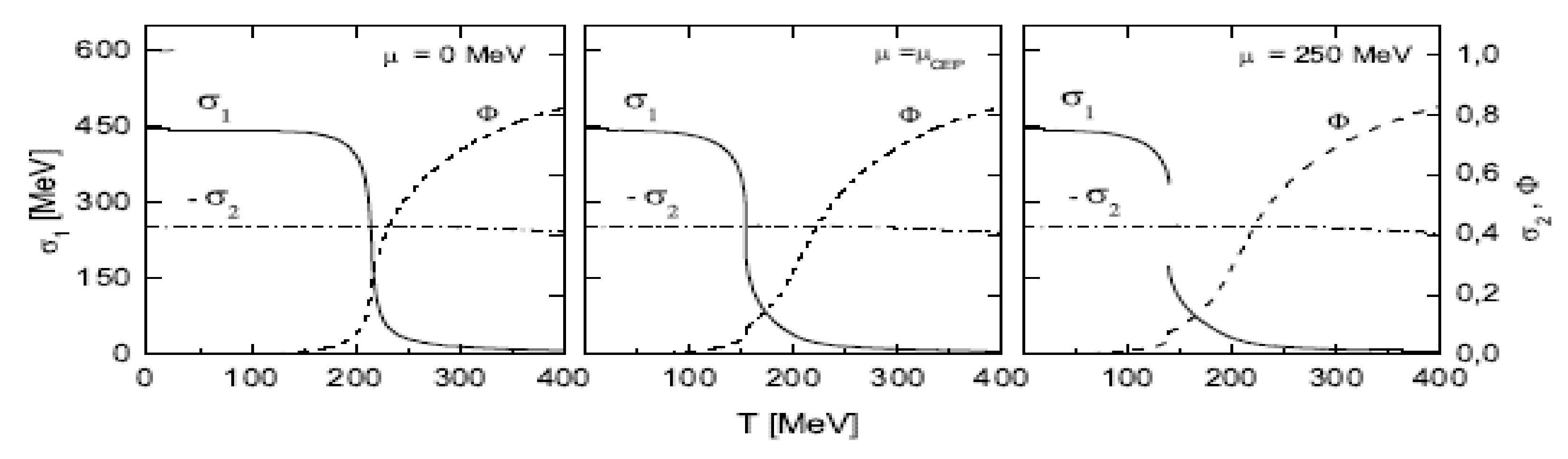
D.Gómez Dumm and N.N. Scoccola, Phys. Rev. C **72** (2005) 014909.

What could be determined with $\Omega^{reg}_{MFA}(T,\mu)$?

- Mean field values $\sigma_{1,2}$ and Φ at a given T and μ

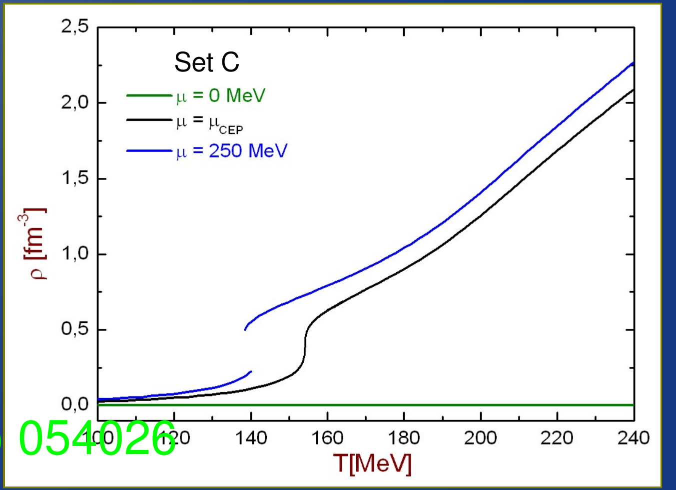
$$\left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\sigma_1}\right|_{T,\mu} = \left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\sigma_2}\right|_{T,\mu} = \left.\frac{\partial\Omega^{reg}_{MFA}}{\partial\Phi}\right|_{T,\mu} = 0$$

"gap" equations



- Quark condensate $\langle\bar{q}q\rangle$; quark density ρ

$$\langle\bar{q}q\rangle = \frac{\partial\Omega^{reg}_{MFA}}{\partial m} \quad ; \quad \rho = -\frac{\partial\Omega^{reg}_{MFA}}{\partial\mu}$$



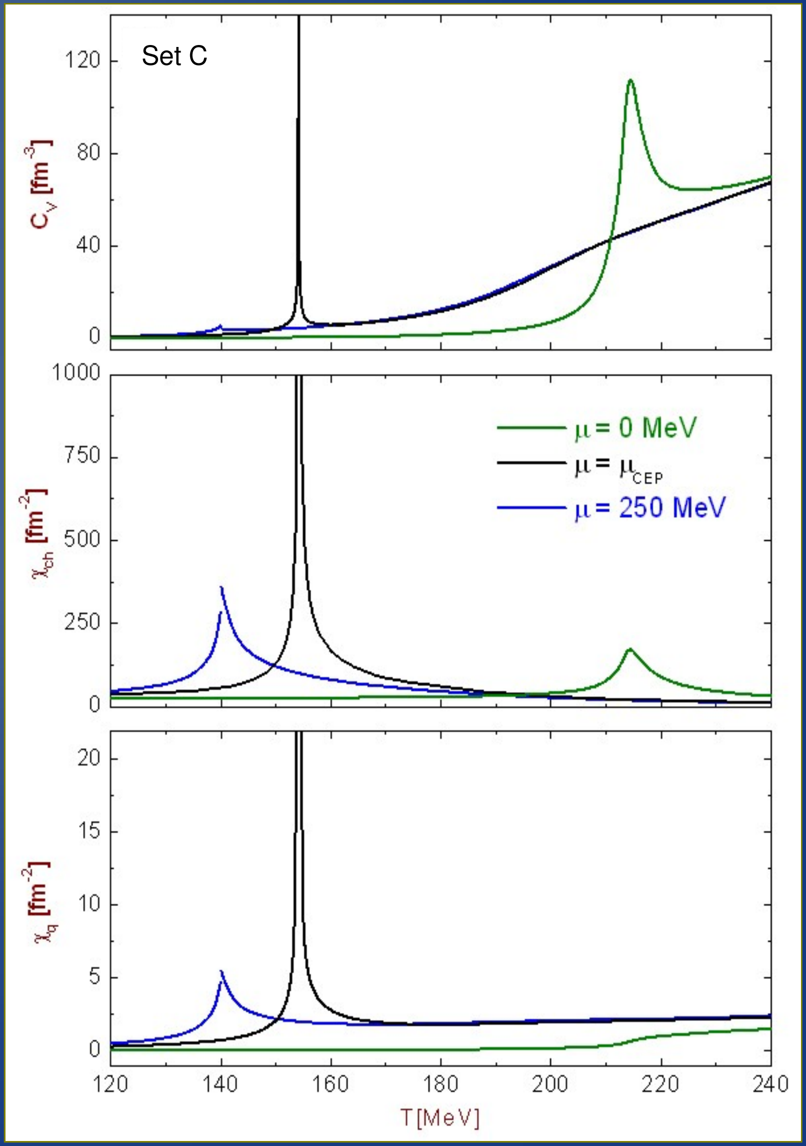
G. Contrera, M. Orsaria, N.N. Scoccola, PRD 82 (2010) 054026

What could be determined with $\Omega_{MFA}^{reg}(T,\mu)$?

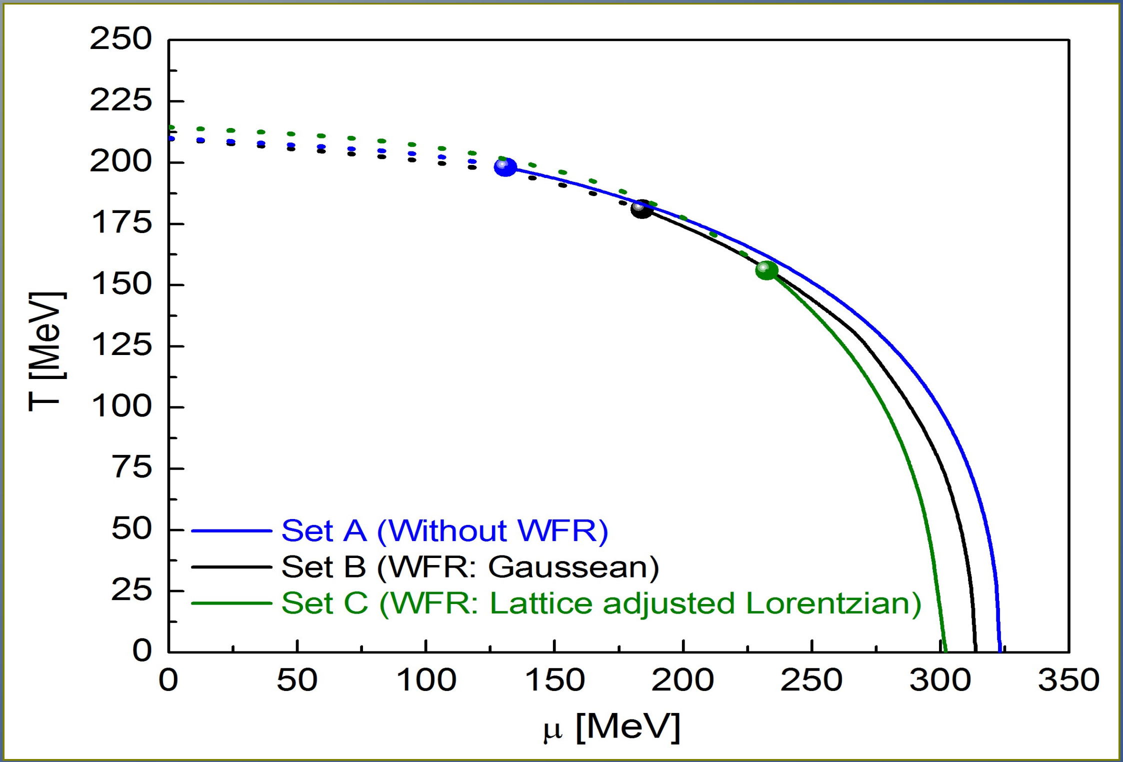
- Specific heat: $c_v = -T \dfrac{\partial^2 \Omega_{MFA}^{reg}}{\partial T^2}$

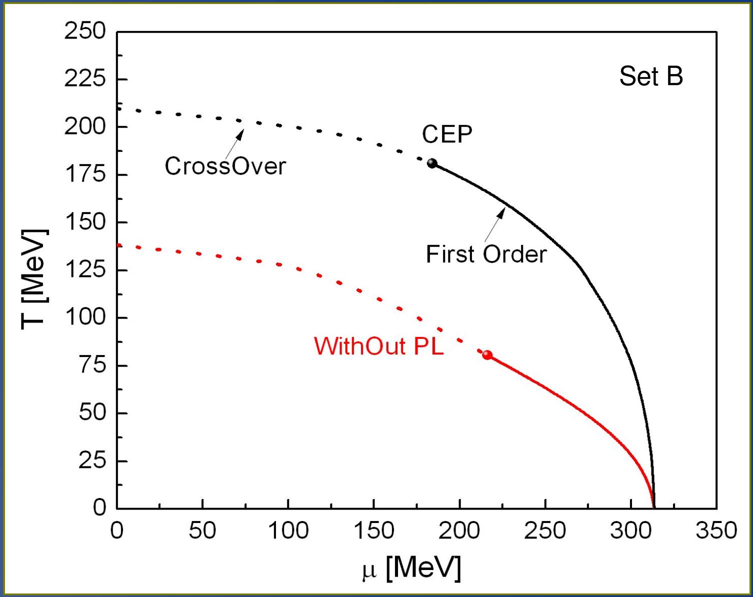
- Chiral susceptibility: $\chi_{ch} = \dfrac{\partial \langle \bar{q}q \rangle}{\partial m}$

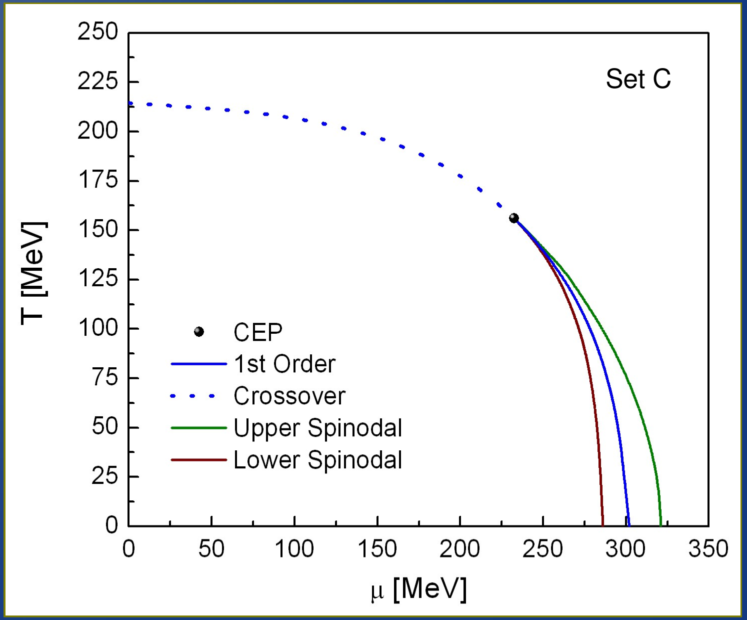
- Quark number susceptibility: $\chi_q = \dfrac{\partial \rho}{\partial \mu}$

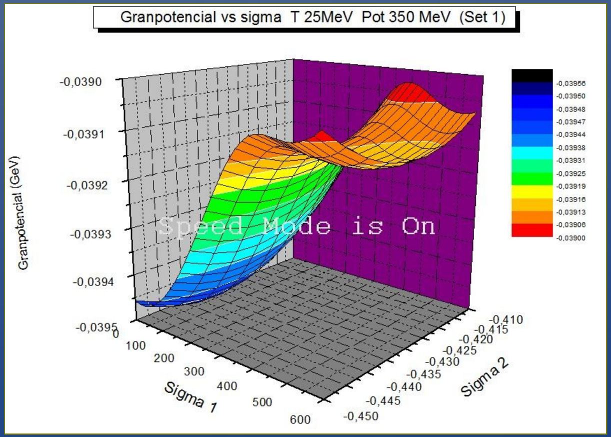


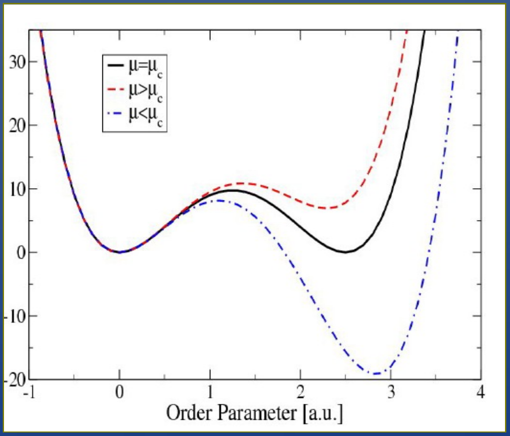
Phase diagrams



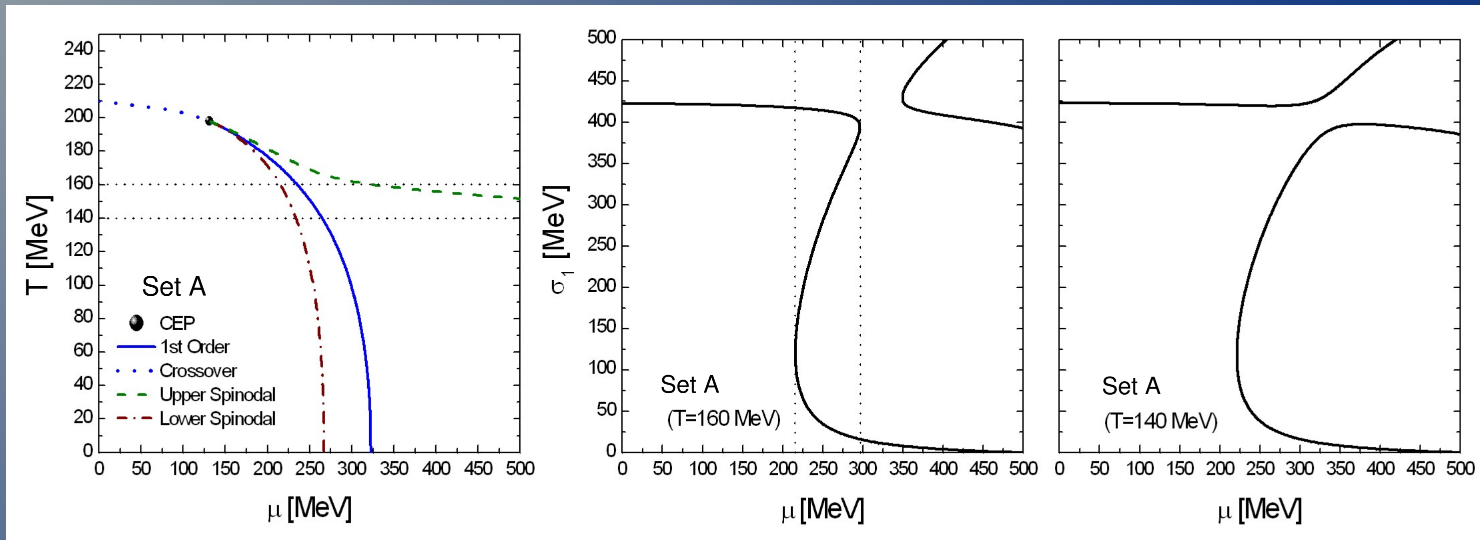


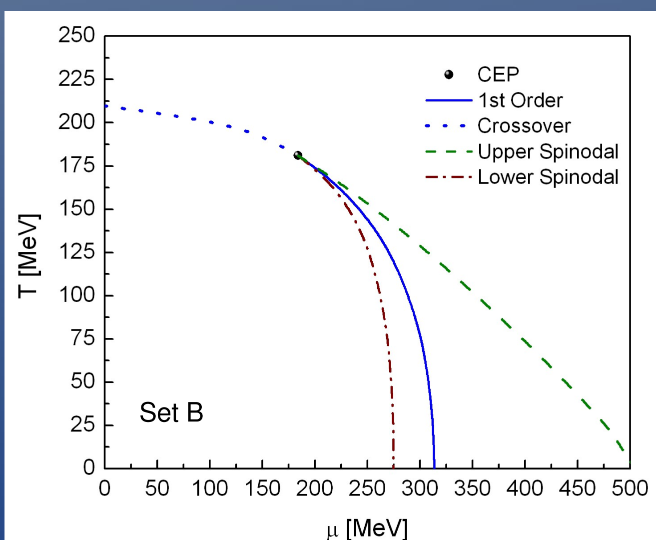


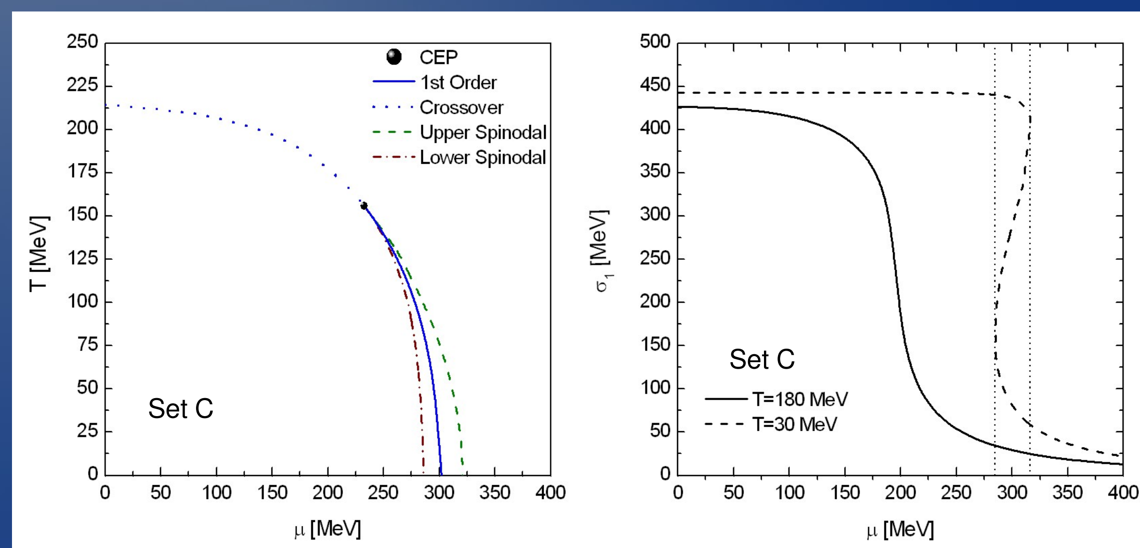




Phase diagrams: Spinodal differences



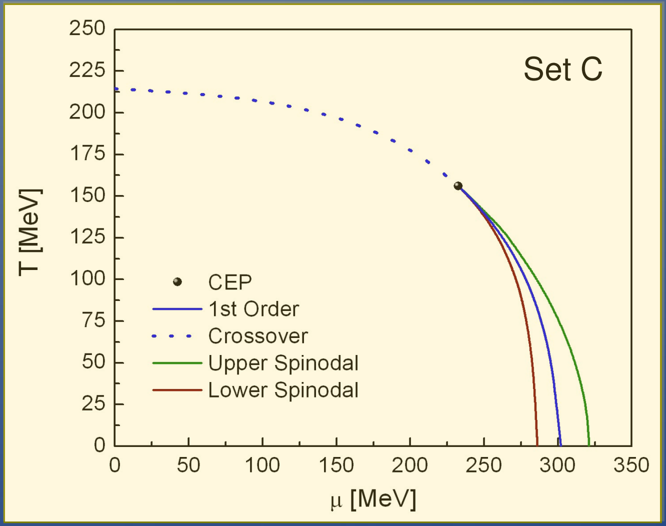


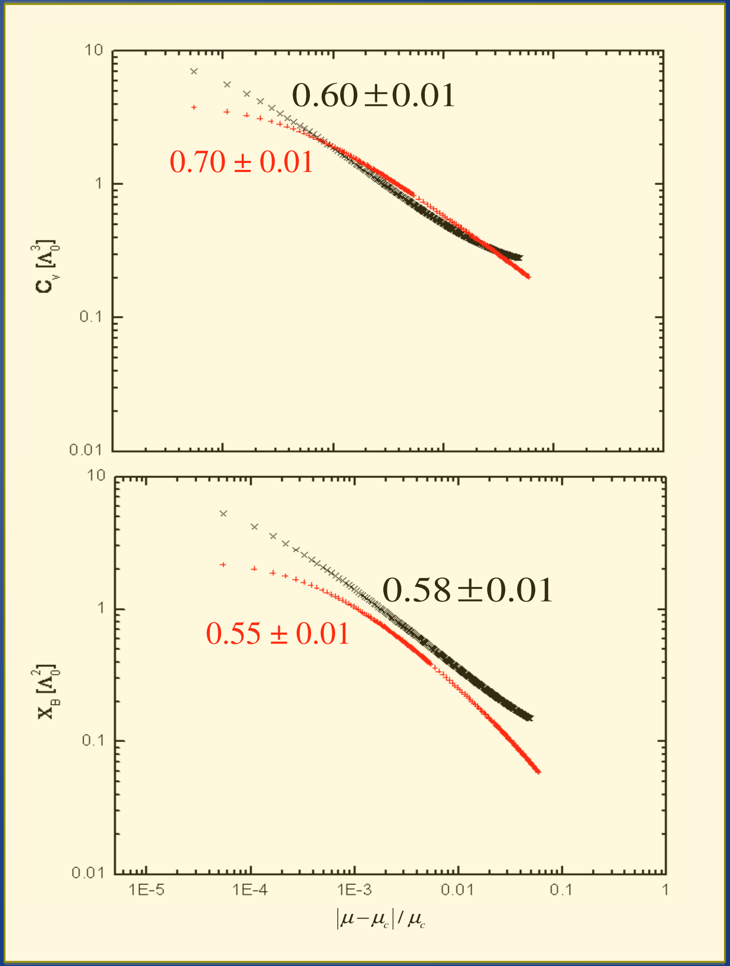


CEP verification: Critical exponents (first attempts)

$$\chi_{ch} = \left|\frac{h-h_c}{h_c}\right|^{-\gamma_{ch}} \qquad \chi_q = \left|\frac{h-h_c}{h_c}\right|^{-\gamma_q} \qquad c_v = \left|\frac{h-h_c}{h_c}\right|^{-\alpha}$$

where $|h - h_C|$ is the distance to the critical point in the (μ,T) plane and γ_{ch}, γ_q and α are the corresponding critical exponents.





For trajectories which are not tangential to the critical line, they should be $\gamma_{ch} = \gamma_q = \alpha = 2/3$.

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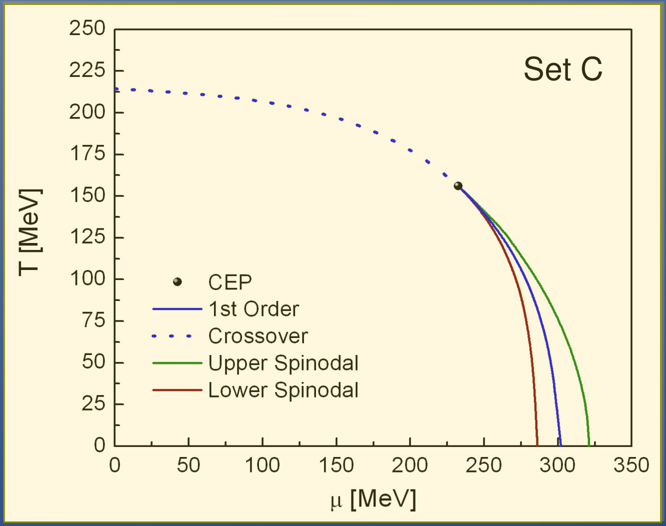
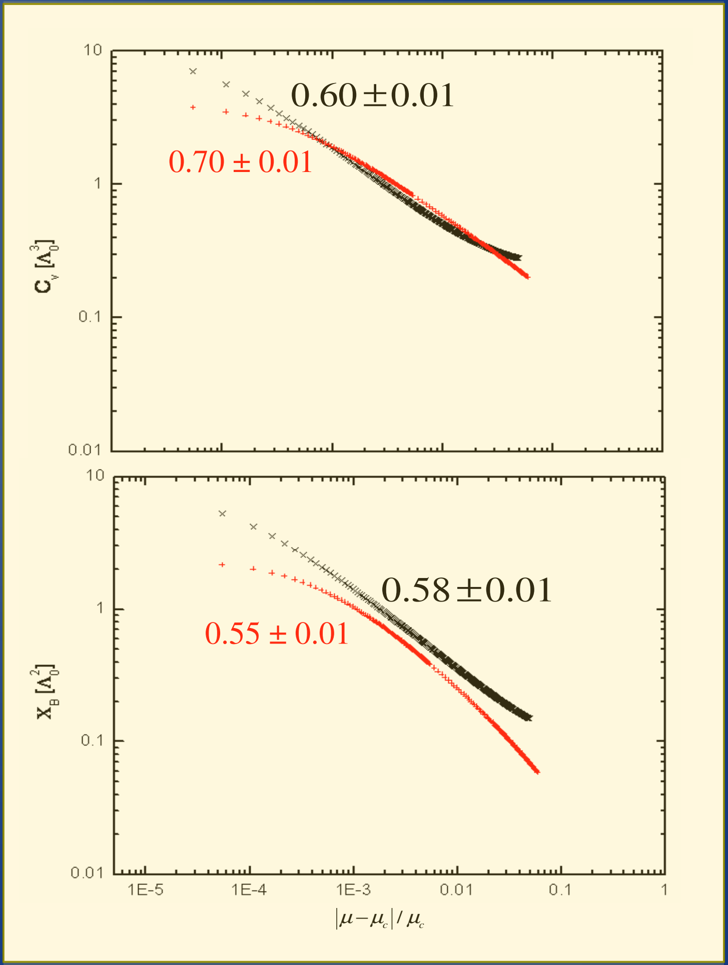




For trajectories which are not tangential to the critical line, they should be $\gamma_{ch} = \gamma_q = \alpha = 2/3$.

CEP determination: Set of equations

In the finite quark mass regime the CEP determination can be performed by solving the following set of coupled equations:

$$\left.\frac{\partial \Omega^{reg}_{MFA}}{\partial \sigma_1}\right|_{T,\mu} = \left.\frac{\partial \Omega^{reg}_{MFA}}{\partial \sigma_2}\right|_{T,\mu} = \left.\frac{\partial \Omega^{reg}_{MFA}}{\partial \Phi}\right|_{T,\mu} = 0$$

$$\frac{d^2\Omega^{reg}_{MFA}(\mu, T, \Phi, \sigma_2)}{d\sigma_1^2} = 0 \qquad \frac{d^3\Omega^{reg}_{MFA}(\mu, T, \Phi, \sigma_2)}{d\sigma_1^3} = 0$$

$$2\sigma_2' \frac{\partial^2\Omega}{\partial\sigma_1\partial\sigma_2} + 2\Phi' \frac{\partial^2\Omega}{\partial\sigma_1\partial\Phi} + \frac{\partial^2\Omega}{\partial\sigma_1\partial\sigma_1} + (\sigma_2')^2 \frac{\partial^2\Omega}{\partial\sigma_2^2} + 2\sigma_2'\Phi' \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi} + (\Phi')^2 \frac{\partial^2\Omega}{\partial\Phi^2} = 0$$

with

$$\sigma_2' = \frac{\partial\sigma_2}{\partial\sigma_1} = \frac{\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^2\Omega}{\partial\sigma_1\partial\sigma_2} - \frac{\partial^2\Omega}{\partial\sigma_1\partial\Phi}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}}{\left(\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)^2 - \frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^2\Omega}{\partial\Phi^2}} \qquad \Phi' = \frac{\partial\Phi}{\partial\sigma_1} = \frac{\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^2\Omega}{\partial\sigma_1\partial\Phi} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^2\Omega}{\partial\sigma_1\partial\sigma_2}}{\left(\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)^2 - \frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^2\Omega}{\partial\Phi^2}}$$

Y. Hatta, T. Ikeda . Phys. Rev. D 67 (2003) 014028
G.A. Contrera, M. Orsaria, N.N. Scoccola, Phys. Rev. D 82 (2010) 054026

$$\frac{d^3\Omega}{d\sigma_1^3} = 3\sigma_2'' \frac{\partial^2\Omega}{\partial\sigma_1\partial\sigma_2} + 3(\sigma_2')^2 \frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2^2} + 6\sigma_2'\Phi' \frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2\partial\Phi} + 3\Phi'' \frac{\partial^2\Omega}{\partial\sigma_1\partial\Phi} + 3(\Phi')^2 \frac{\partial^3\Omega}{\partial\sigma_1\partial\Phi^2}$$

$$+3\sigma_2' \frac{\partial^3\Omega}{\partial\sigma_1^2\partial\sigma_2} + 3\Phi' \frac{\partial^3\Omega}{\partial\sigma_1^2\partial\Phi} + \frac{\partial^3\Omega}{\partial\sigma_1^3} + 3\sigma_2''\Phi' \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi} + 3(\sigma_2')^2\Phi' \frac{\partial^3\Omega}{\partial\sigma_2^2\partial\Phi} + (\sigma_2')^3 \frac{\partial^3\Omega}{\partial\sigma_2^3}$$

$$+3\sigma_2'\Phi'' \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi} + \sigma_2'(\Phi')^2 \frac{\partial^3\Omega}{\partial\sigma_2\partial\Phi^2} + 3\sigma_2'\sigma_2'' \frac{\partial^2\Omega}{\partial\sigma_2^2} + 3\Phi'\Phi'' \frac{\partial^2\Omega}{\partial\Phi^2} + (\Phi')^3 \frac{\partial^3\Omega}{\partial\Phi^3}$$

$$= 0$$

$$\sigma_2'' = \left[\left(\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)^2 - \frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^2\Omega}{\partial\Phi^2}\right]^{-1} \left[2\Phi'\left(\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2\partial\Phi} - \frac{\partial^3\Omega}{\partial\sigma_1\partial\Phi^2}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)\right.$$

$$+2\Phi'\sigma_2'\left(\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^3\Omega}{\partial\sigma_2^2\partial\Phi} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_2\partial\Phi^2}\right) + 2\sigma_2'\left(\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2^2} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2\partial\Phi}\right)$$

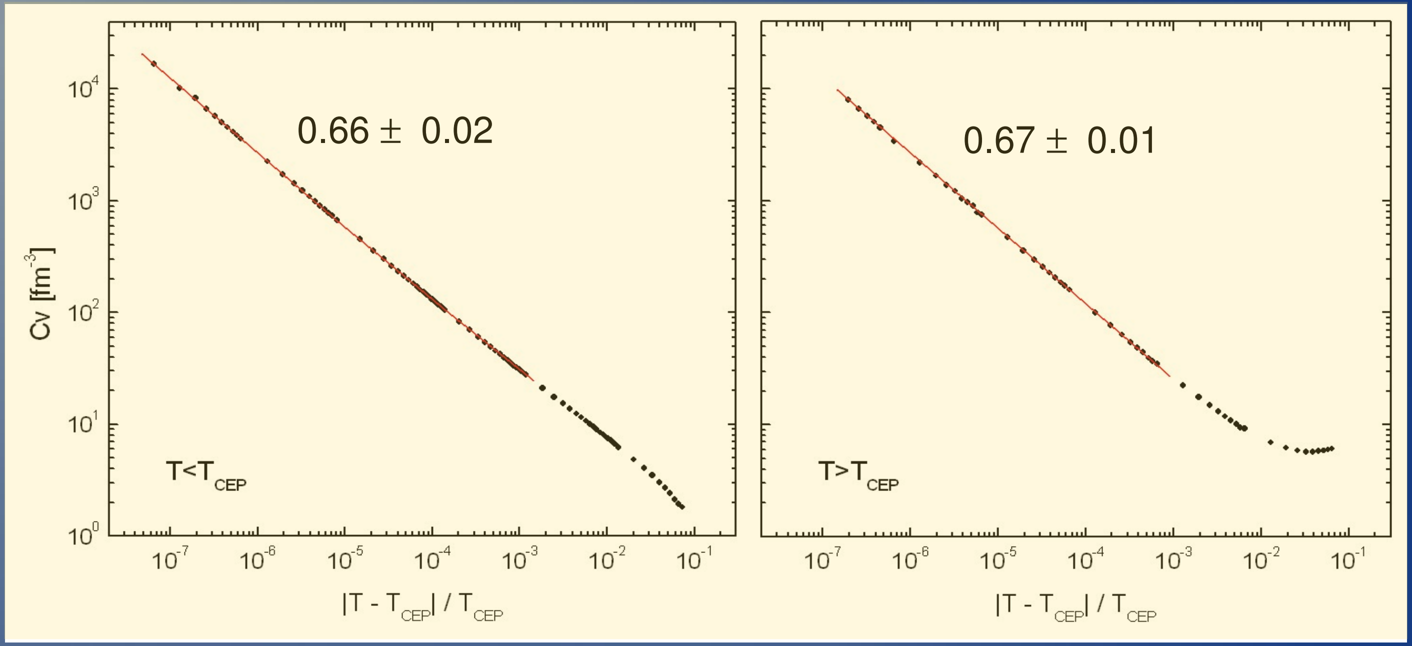
$$\left.+\left(\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^3\Omega}{\partial\sigma_1^2\partial\sigma_2} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_1^2\partial\Phi}\right) + (\sigma_2')^2\left(\frac{\partial^3\Omega}{\partial\sigma_2^3}\frac{\partial^2\Omega}{\partial\Phi^2} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_2^2\partial\Phi}\right) + (\Phi')^2\left(\frac{\partial^2\Omega}{\partial\Phi^2}\frac{\partial^3\Omega}{\partial\sigma_2\partial\Phi^2} - \frac{\partial^3\Omega}{\partial\Phi^3}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)\right]$$

$$\Phi'' = \left[\left(\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)^2 - \frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^2\Omega}{\partial\Phi^2}\right]^{-1} \left[2\Phi'\left(\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^3\Omega}{\partial\sigma_1\partial\Phi^2} - \frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2\partial\Phi}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)\right.$$

$$+2\Phi'\sigma_2'\left(\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^3\Omega}{\partial\sigma_2\partial\Phi^2} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_2^2\partial\Phi}\right) + 2\sigma_2'\left(\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2\partial\Phi} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_1\partial\sigma_2^2}\right)$$

$$\left.+\left(\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^3\Omega}{\partial\sigma_1^2\partial\Phi} - \frac{\partial^3\Omega}{\partial\sigma_1^2\partial\sigma_2}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right) + (\sigma_2')^2\left(\frac{\partial^3\Omega}{\partial\sigma_2^2\partial\Phi}\frac{\partial^2\Omega}{\partial\sigma_2^2} - \frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\frac{\partial^3\Omega}{\partial\sigma_2^3}\right) + (\Phi')^2\left(\frac{\partial^2\Omega}{\partial\sigma_2^2}\frac{\partial^3\Omega}{\partial\Phi^3} - \frac{\partial^3\Omega}{\partial\sigma_2\partial\Phi^2}\frac{\partial^2\Omega}{\partial\sigma_2\partial\Phi}\right)\right]$$

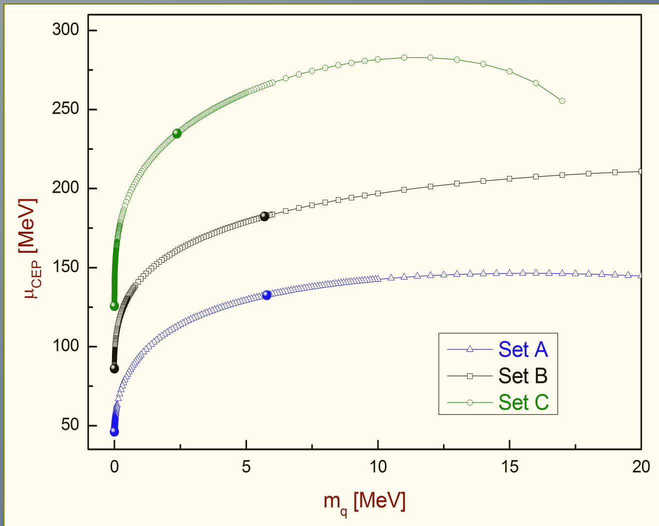
CEP verification: finite quark mass

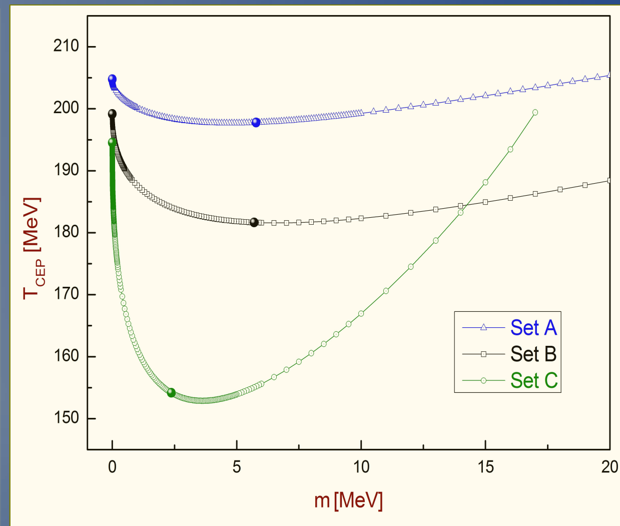


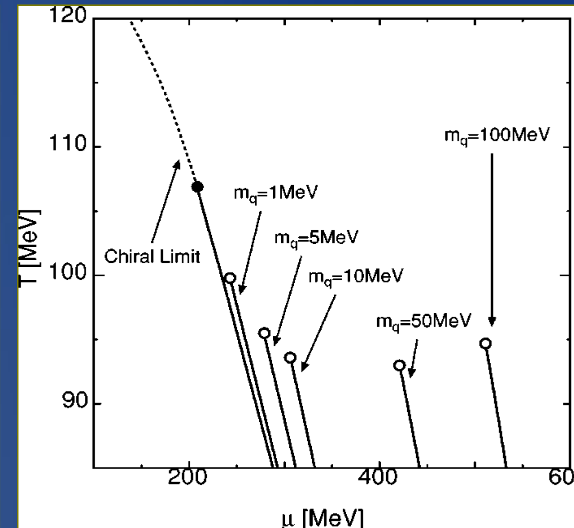
	Set A	Set B	Set C
$T_c(0)$	210.0	209.8	214.5
μ_{CEP}	132.5	182.3	234.8
T_{CEP}	197.8	181.6	154.2
$\mu_c(0)$	321.5	311.6	298.1

	γ_{ch}	γ_q	α
$\mu \rightarrow$	0.67(1)	0.67(1)	0.66(1)
$\mu \leftarrow$	0.66(1)	0.66(1)	0.67(1)
$T \uparrow$	0.67(1)	0.67(1)	0.66(2)
$T \downarrow$	0.66(1)	0.66(1)	0.67(1)
MF exponent	2/3	2/3	2/3

CEP verification: more results !

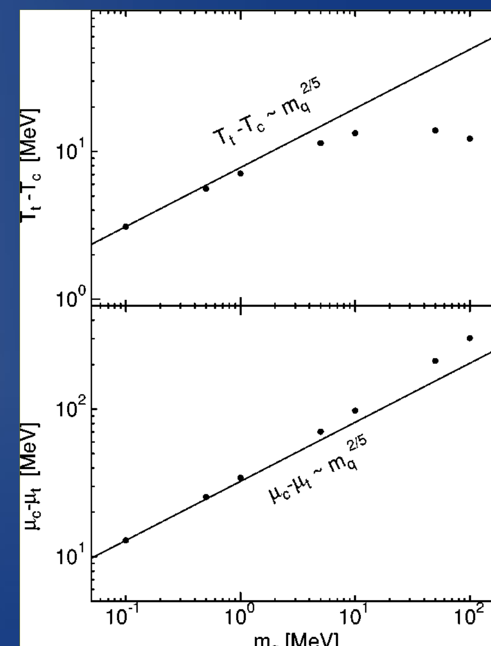


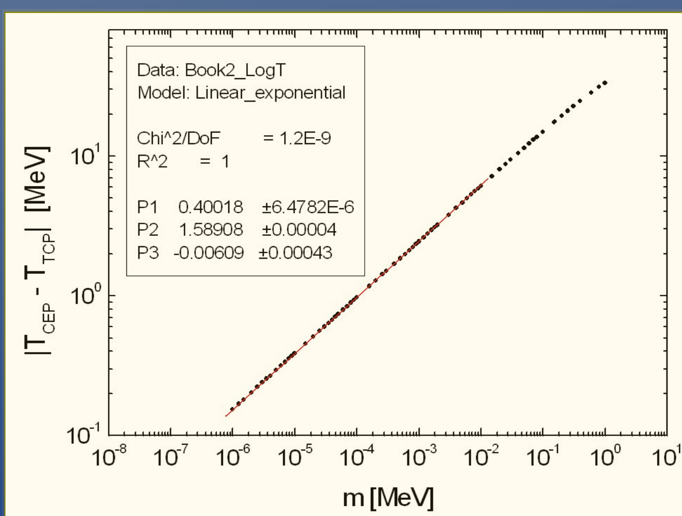


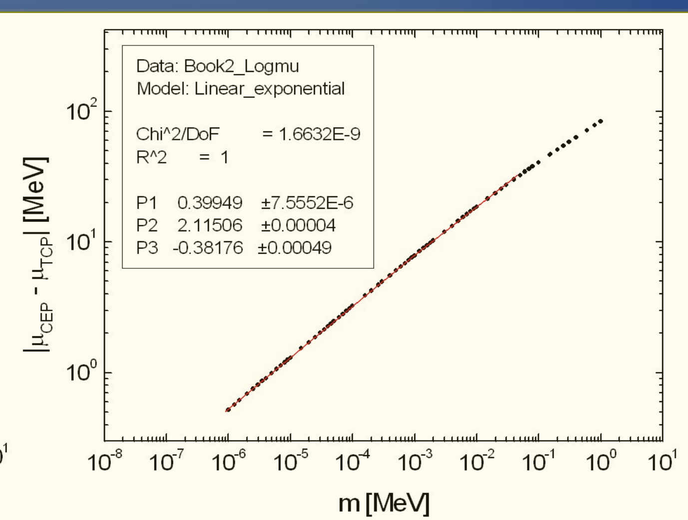


$$\Delta T_{CEP} = T_{CEP}(m) - T_{TCP} = -c\ m^{2/5} + O(m^{4/5})$$

$$\Delta \mu_{CEP} = \mu_{CEP}(m) - \mu_{TCP} = +d\ m^{2/5} + O(m^{4/5})$$







Y. Hatta, T. Ikeda . Phys. Rev. D 67 (2003) 014028

Chemical potential in the Polyakov-loop potentials

$T_0 = 208$ MeV which corresponds to 2 flavors case. Then, following Ref [1], we used also a polinomial ansatz for $\mathcal{U}$ given by

$$\mathcal{U}_2(\Phi, T) = \left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}\Phi^4\right] T^4 \tag{10}$$

with the temperature-dependent coefficient

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 \tag{11}$$

and the following set of parameters, $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, and $b_4 = 7.5$.

In a more recent work [3] it has been proposed the following μ-dependent logarithmic potential:

$$\mathcal{U}_3(\Phi, T, \mu) = (a_0 T^4 + a_1 \mu^4 + a_2 T^2 \mu^2)\Phi^2 + a_3 T_0^4 \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4), \tag{12}$$

where the parameters are $a_0 = 1.85$, $a_1 = 1.44\text{x}10^{-3}$, $a_2 = 0.08$, $a_3 = 0.40$.

In the same way, we propose a polynomial potential which include this μ-dependence. So we have:

$$\mathcal{U}_4(\Phi, T) = \left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}\Phi^4\right] T^4 \tag{13}$$

with the T-μ-dependent coefficient.

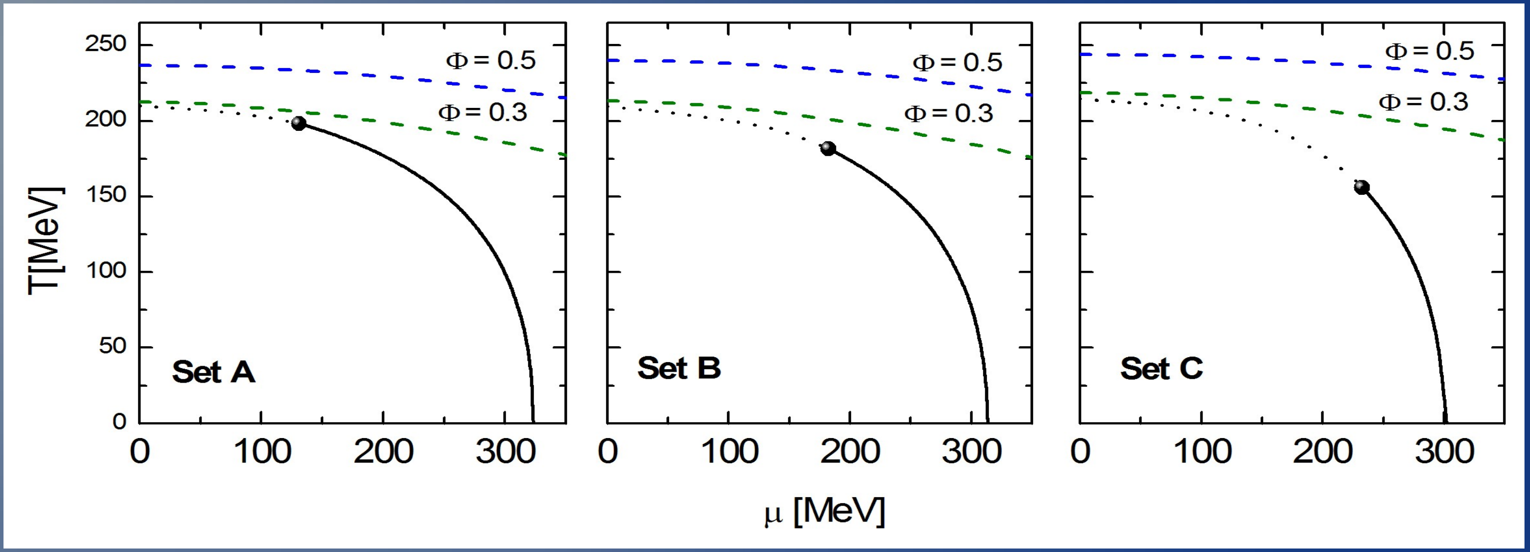
$$b_2(T, \mu) = [a_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4] + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 \tag{14}$$

and the same set of parameters as in the polinomial case, but adding the μ related ones $c_2 = 30/(7\pi^2) = 0.4342$ and $c_4 = 15/(7\pi^4) = 0.02199$

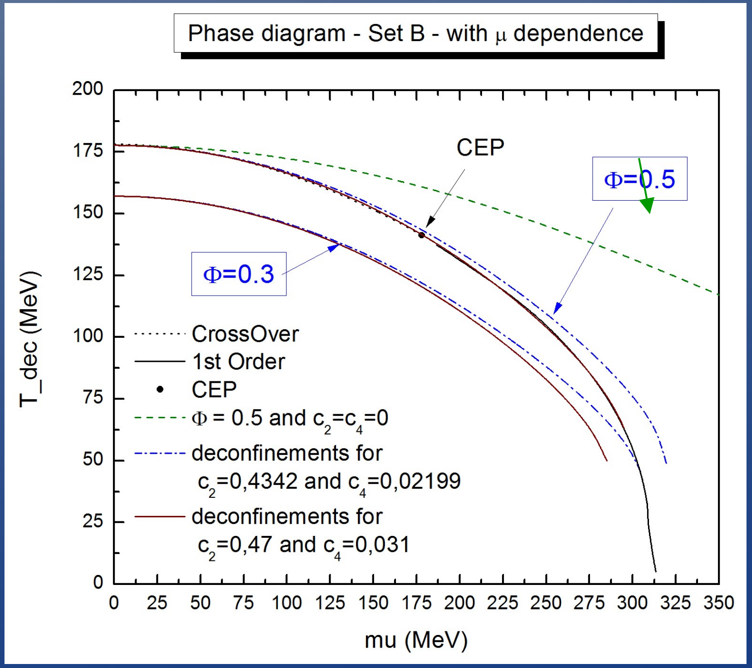
[1] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73, 014019 (2006)
[3] V. A. Dexheimer and S. Schramm. Phys.Rev.C 81, 045201 (2010)

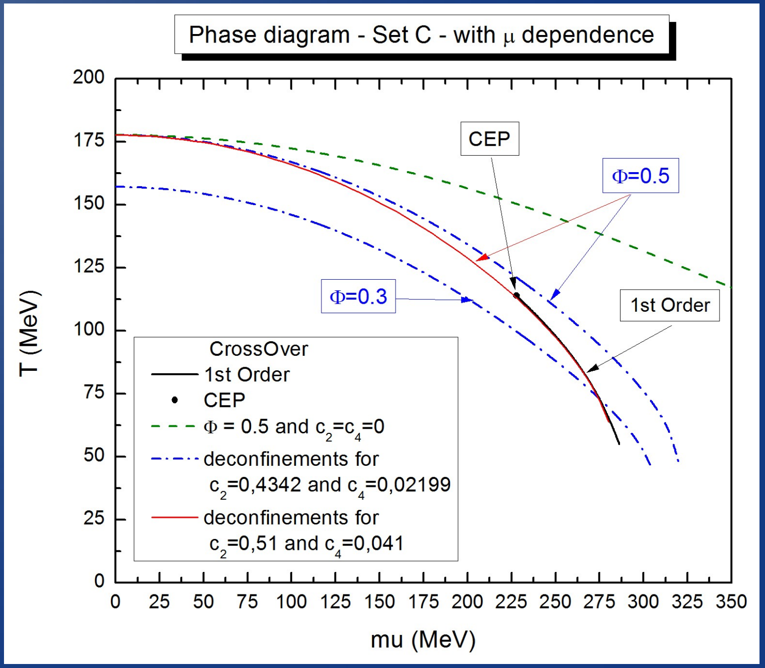
Fresh results: Phase diagrams w. Chiral & Polyakov lines

T_0=270 MeV
and
RRW potential

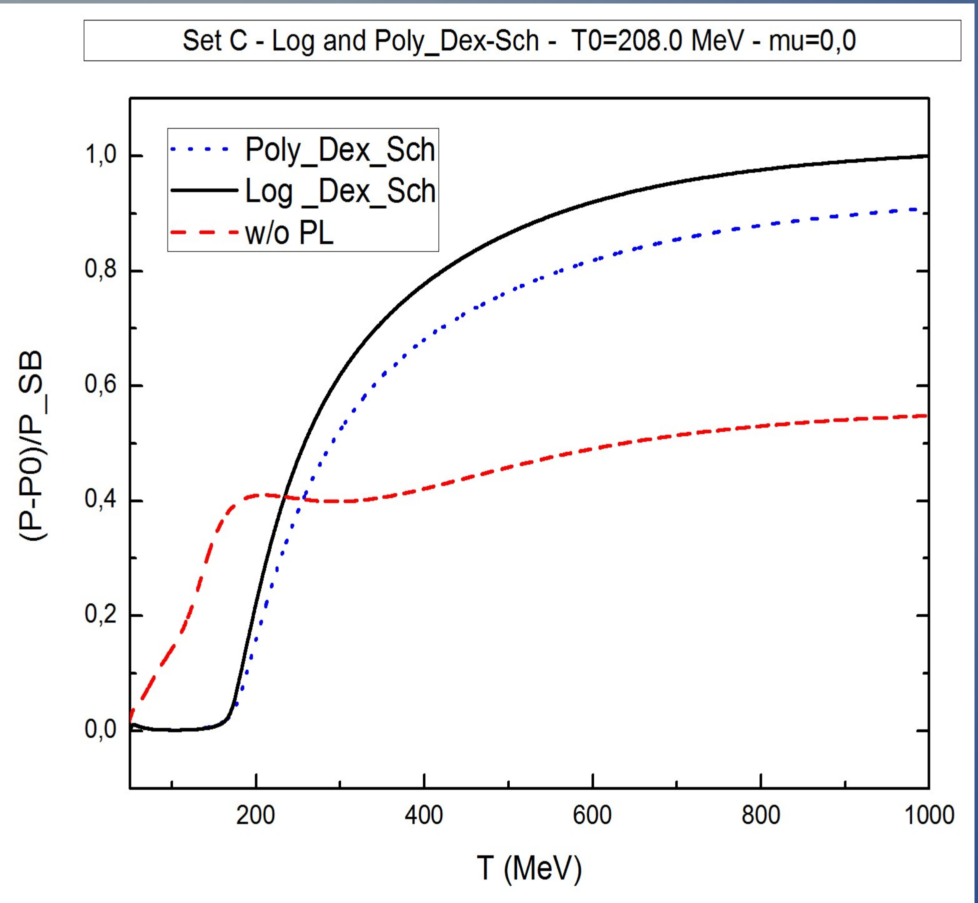


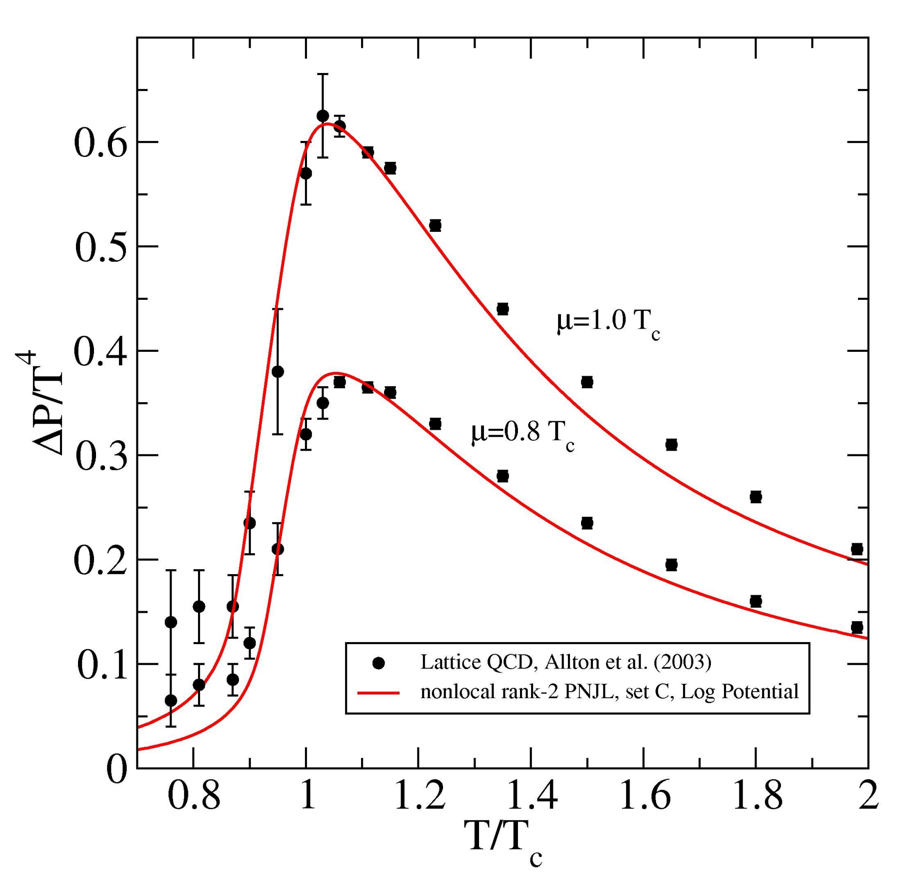
T_0=208 MeV
and
μ-dependent
polynomial
potential





Preliminary results: Pressure & Pressure difference





The pressure difference ΔP=P(T,μ)-P(T,0) is very sensitive to the formfactor par
and in particular to the vector meson coupling which is important at nonzero μ.

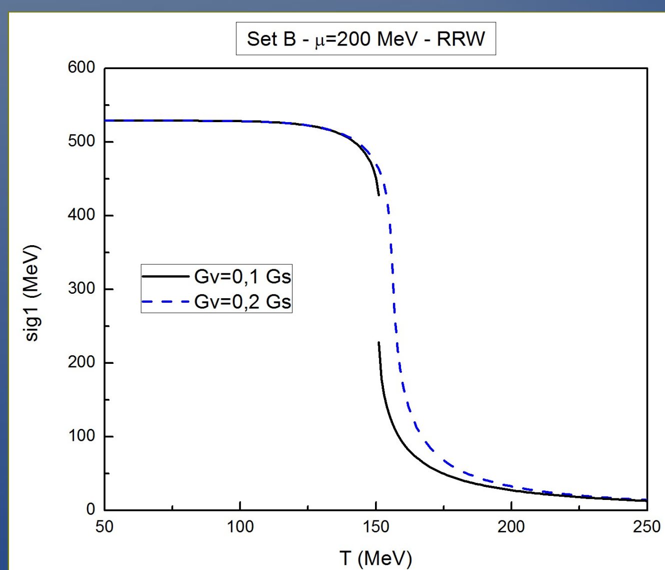
Development : vector interactions + color neutrality

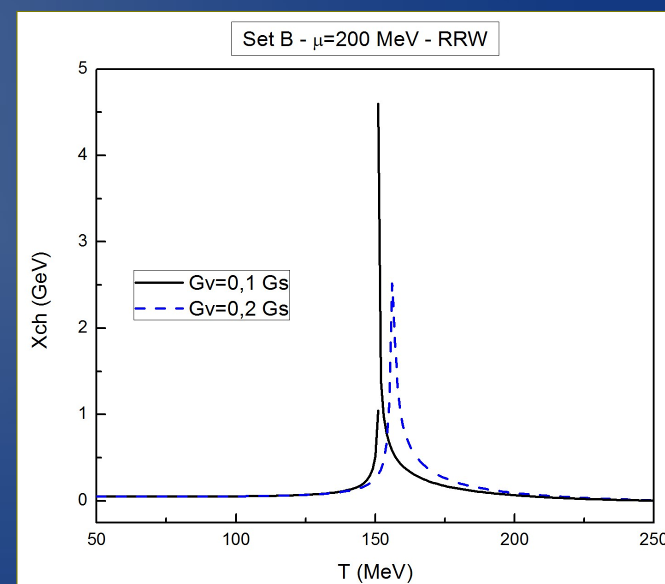
$$\Omega_{MFA}(T,\mu) = -\frac{4T}{\pi^2}\sum_c \sum_n \int \frac{d^3\vec{p}}{(2\pi)^3} \ln\left[\frac{q_{nc}^2 + M^2\left(\rho_{nc}^2\right)}{Z^2\left(\rho_{nc}^2\right)}\right] + \frac{\sigma_1^2}{2G_S} + \frac{\grave{u}_p^2\,\sigma_2^2}{2G_S} - \frac{\omega_V^2}{2G_V} + \mathrm{U}\left(\Phi,\Phi^*,T\right)$$

$$\rho_{nc}^2 = [\omega_n - i\mu_c + \phi_c]^2 + \vec{p}^{\,2} \longrightarrow \begin{array}{l} \phi_c = \lambda_3\phi_3 + \lambda_8\phi_8 \\ \mu_c = \mu I + \lambda_3\mu_3 + \lambda_8\mu_8 \end{array}$$

$$q_{nc}^2 = \left[\omega_n - i\mu_c^r + \phi_c\right]^2 + \vec{p}^{\,2} \longrightarrow \mu_c^r = \mu_c - \omega_V\, g(\rho_{nc}^2)$$

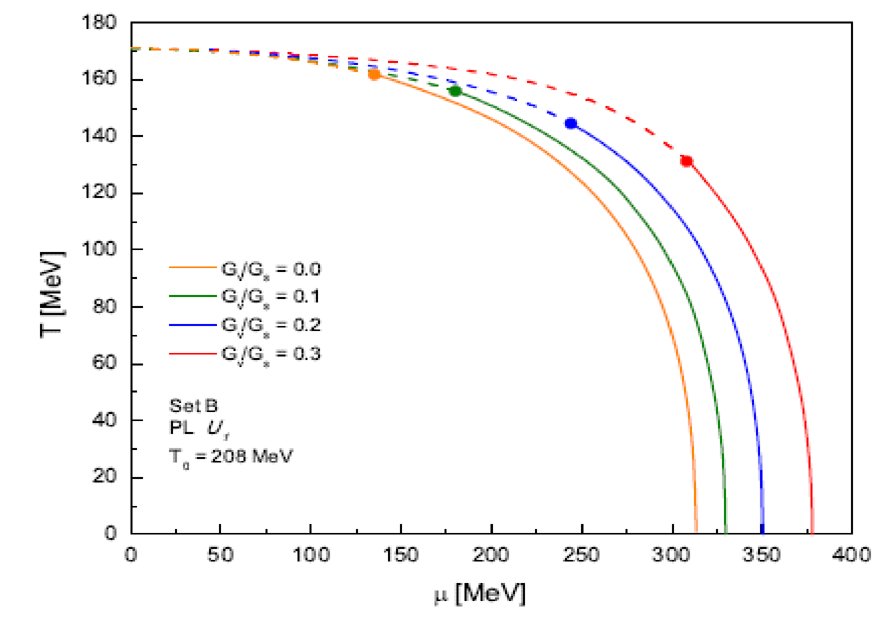
$$\begin{array}{l} \phi_8 = 0 \\ \mu_3 = 0 \\ \mu_8 = 0 \end{array} \longrightarrow$$

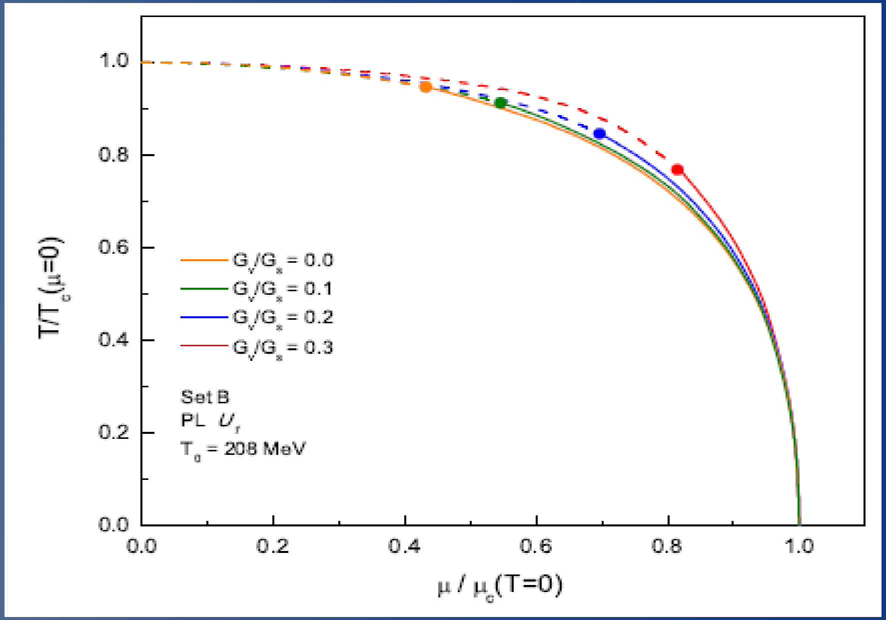


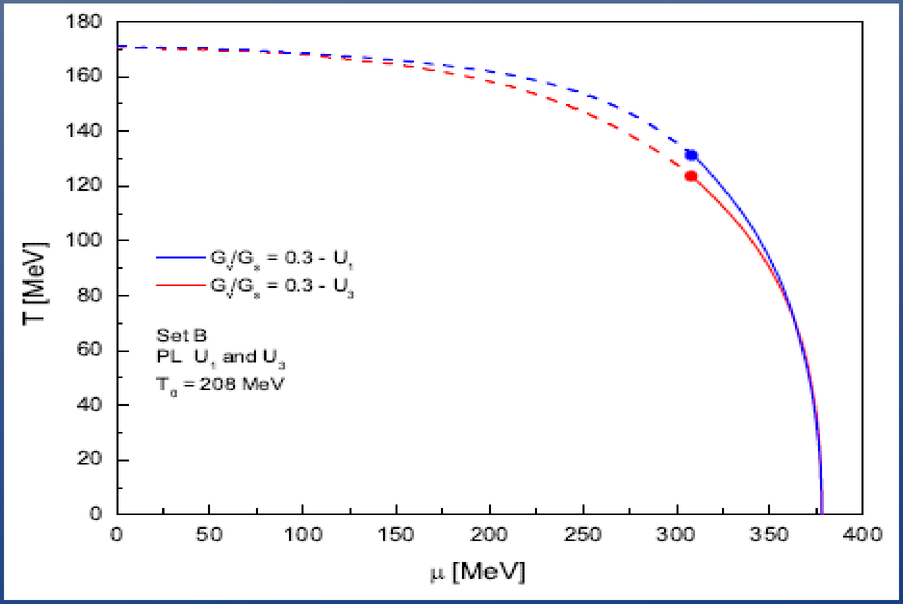


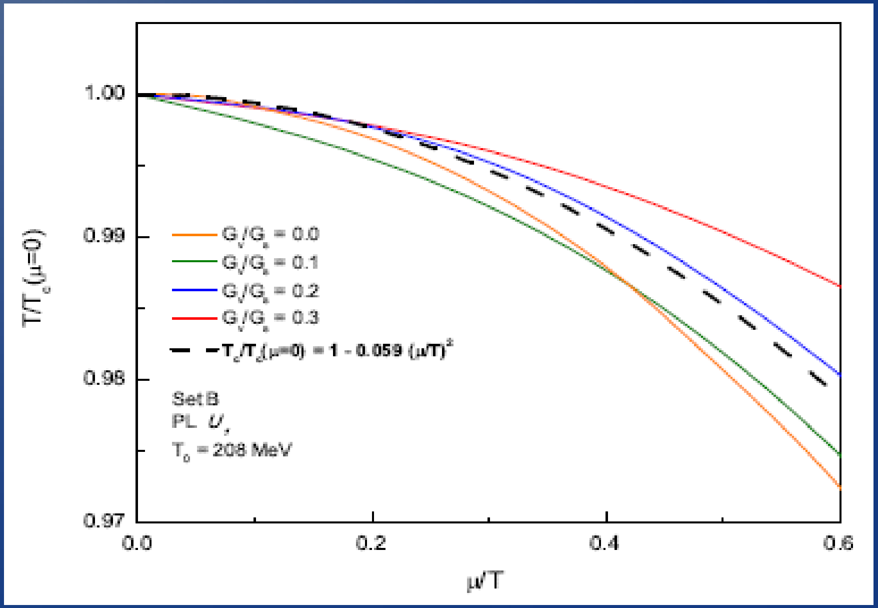
D. Gómez Dumm, D.B. Blaschke, A.G. Grunfeld, N.N. Scoccola, Phys.Rev.D **78**, 114021 (2008).
K. Fukushima, Phys.Rev.D **77**, 114028 (2008).
C. Sasaki, B. Friman, K. Redlich, Phys.Rev.D **75**, 054026(2007).

Vector coupling effects on Phase diagram and CEP

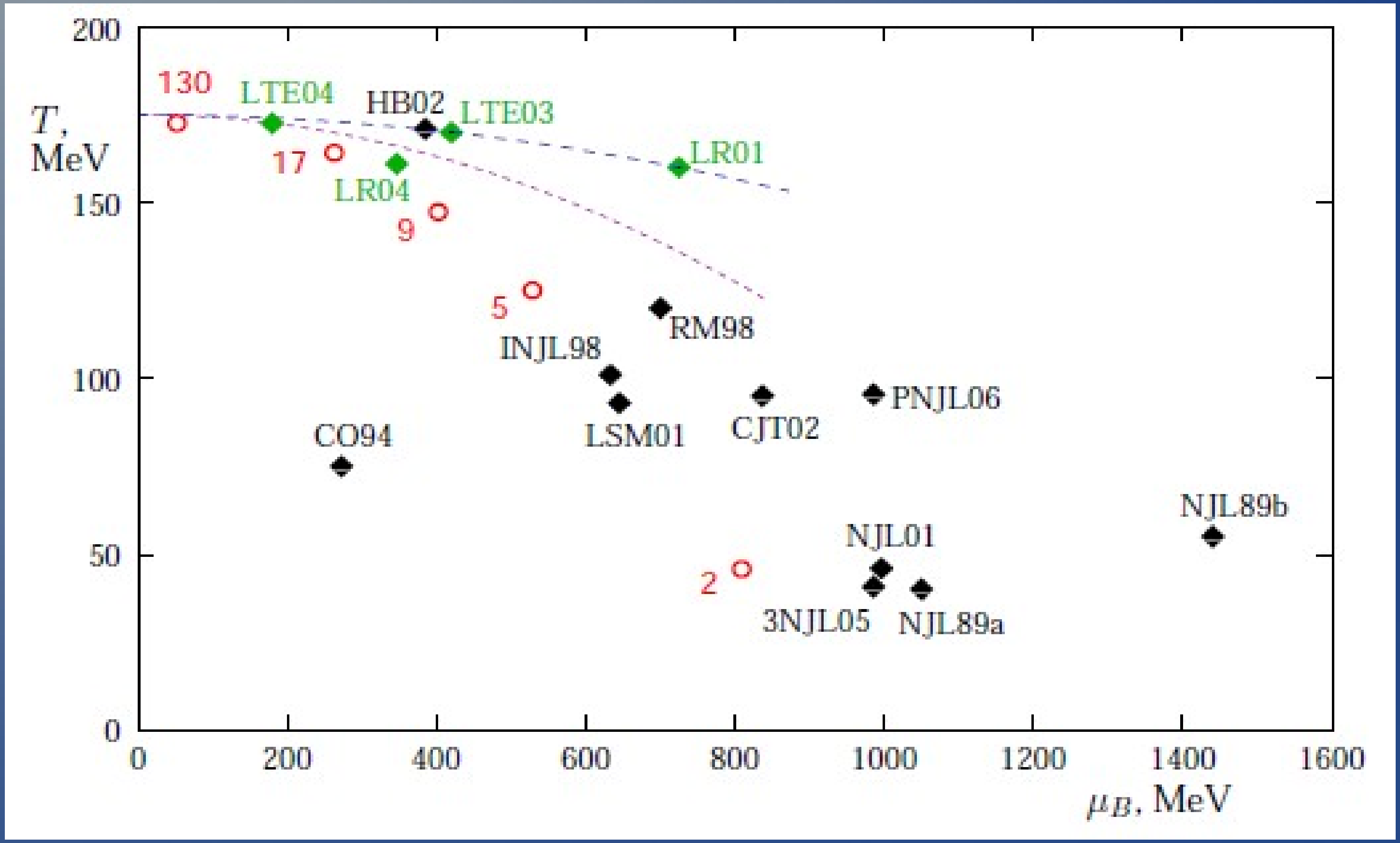








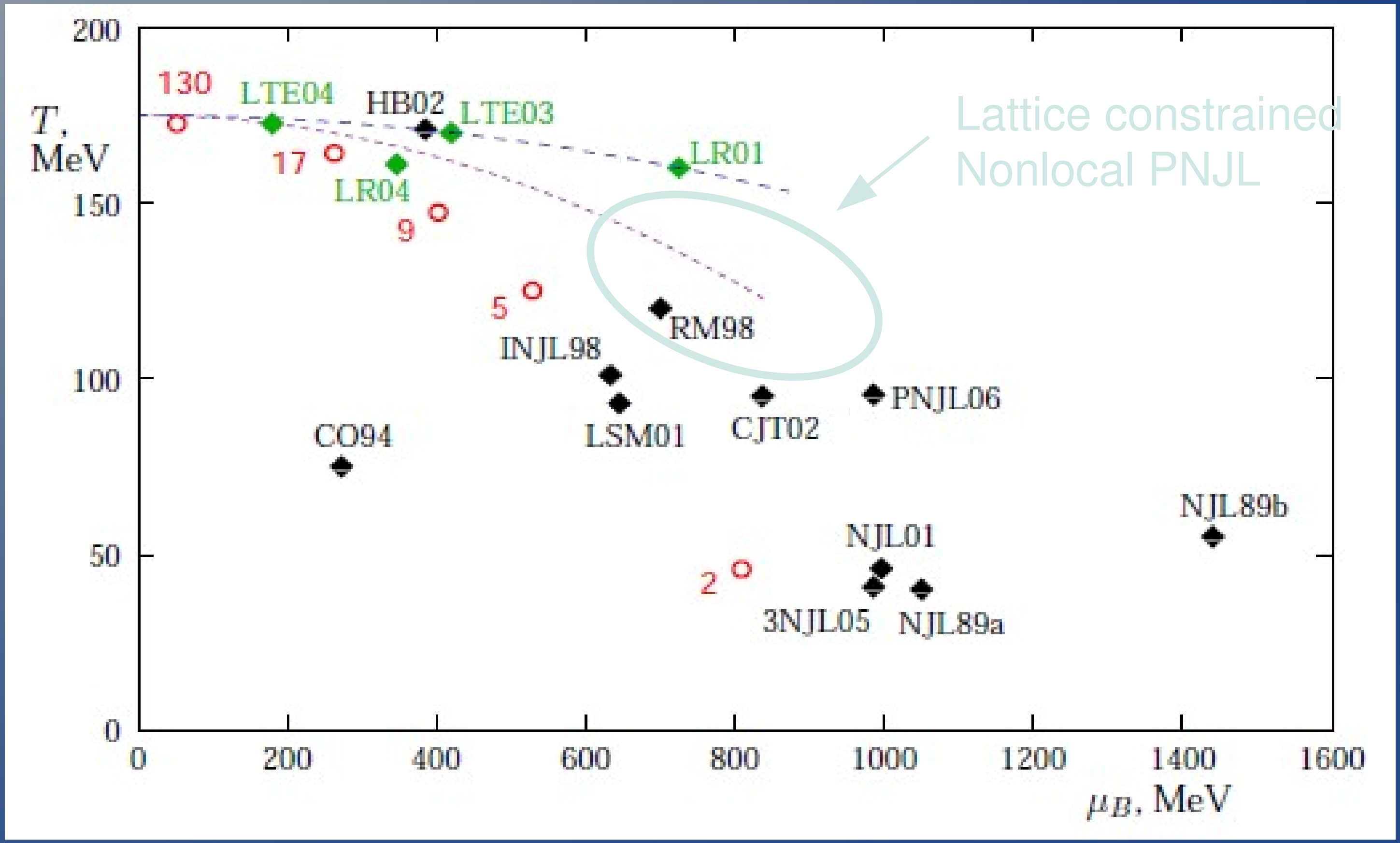
Constrain P-DSE models with lattice data: a solution for the Stephanov problem ?



M. Stephanov, PoS LAT2006, 024 (2006)

Unconstrained models predict a “skymap” of critical points in the QCD ph

Constrain P-DSE models with lattice data: a solution for the Stephanov problem ?



M. Stephanov, PoS LAT2006, 024 (2006)

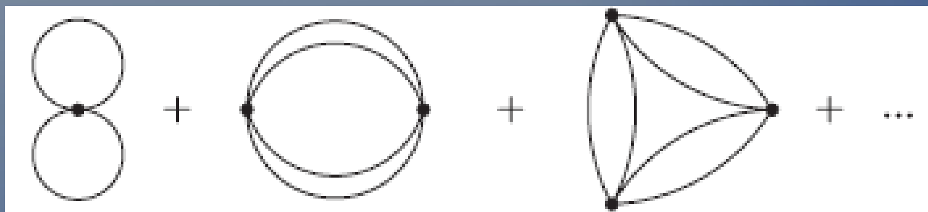
Unconstrained models predict a “skymap” of critical points in the QCD ph
Lattice constrained Polyakov-DSE (nonlocal PNJL) models favor a much

Development : Beyond mean Field - Mesons

$$\Omega = i\,\mathrm{Tr}\ln(S^{-1}) + i\,\mathrm{Tr}(\Sigma S) + \Psi(S) + U(\Phi, \bar{\Phi}) - \Omega_0$$

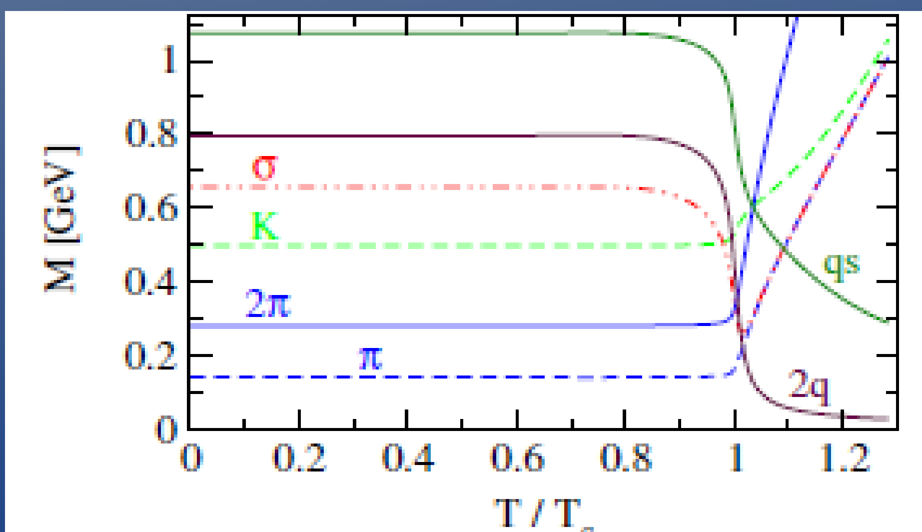
$$\Psi_{\mathrm{glasses}} = -\sum_{M=\pi,\sigma} \frac{G}{2}[-\mathrm{Tr}(\Gamma^M iS)]^2$$

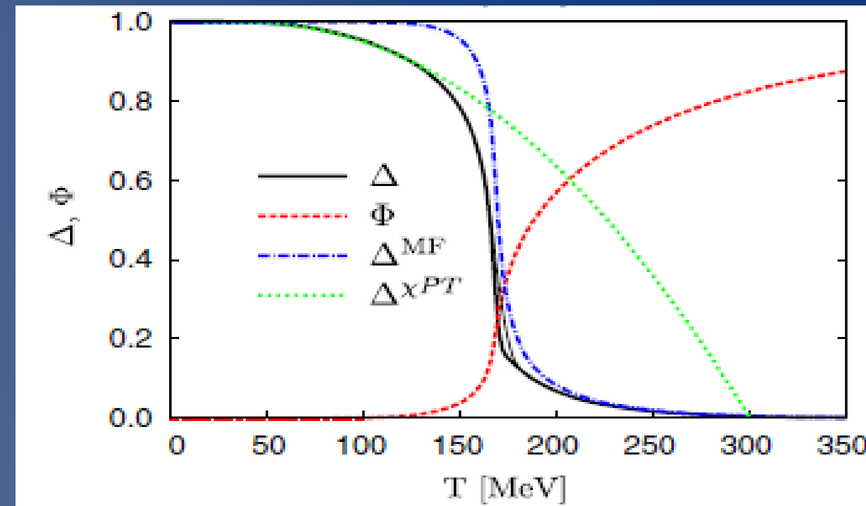
(MF)

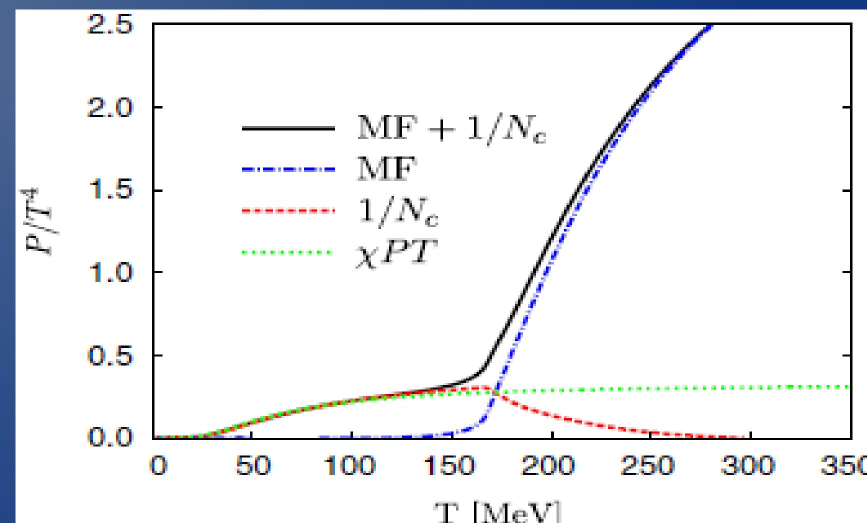


$$\Psi_{\mathrm{ring}} = -\sum_{\mathrm{M}=\pi,\sigma} \frac{d_M}{2} i\,\mathrm{Tr}\ln[1 - G\Pi^{\mathrm{M}}]$$

(Mesons)

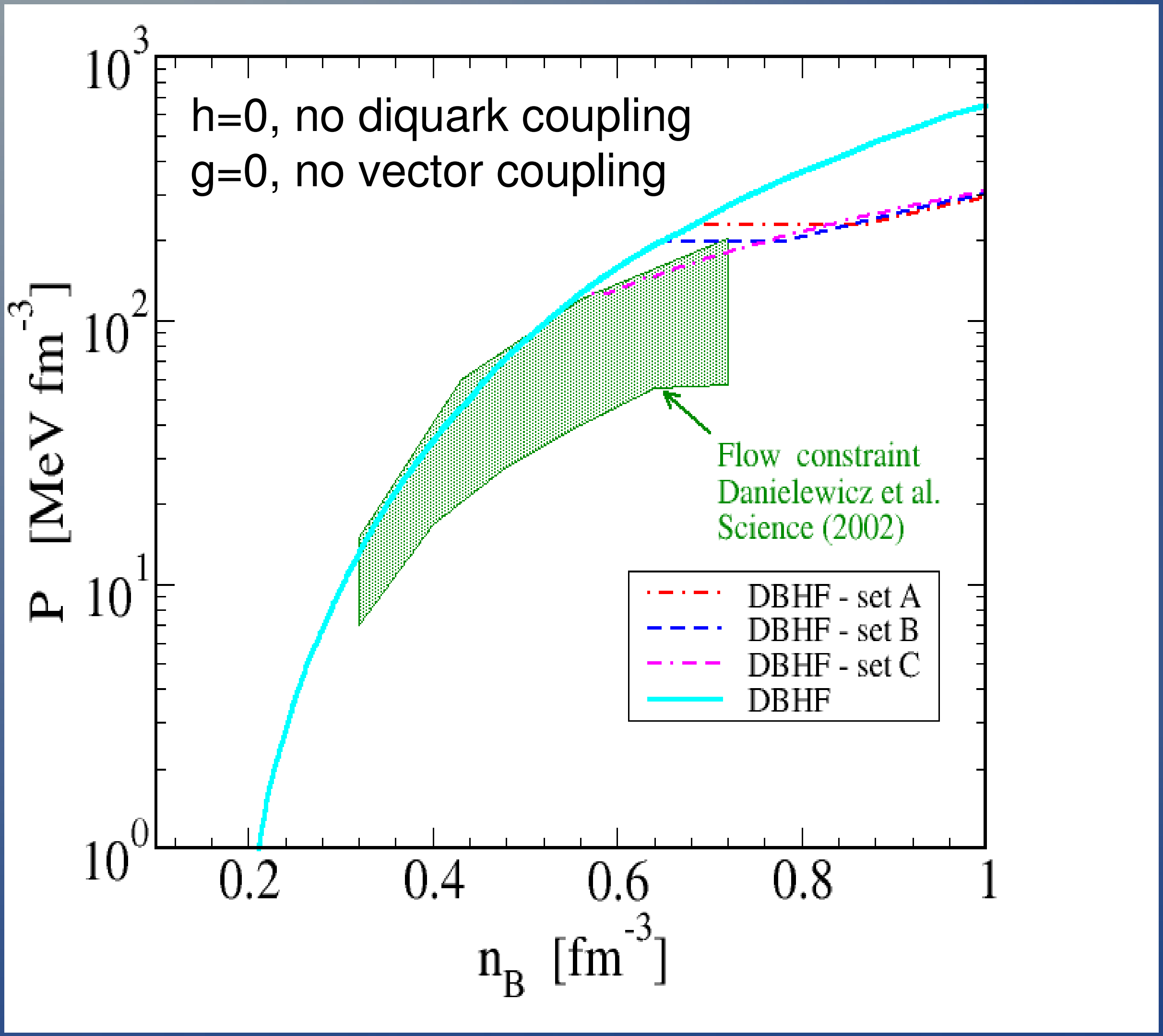


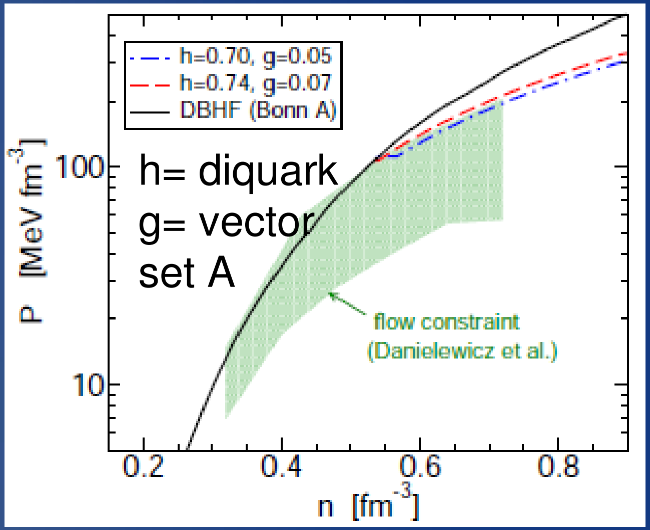


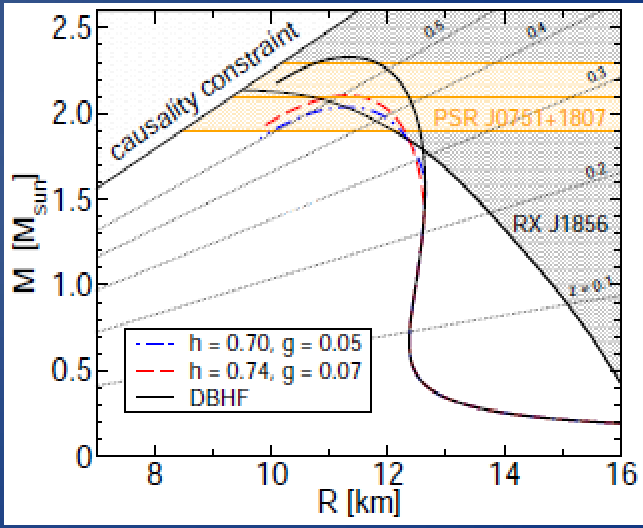


A. Radzhabov et al., PRD 83 (2011) 1
D. Horvatic et al., PRD 84 (2011) 016

T=0, Hybrid Equation of State







D.B., S. Benic, G. Contrera et al., in preparation (2012)

D.B., D. Gomez Dumm, A. Grun
T. Klahn, N. Scoccola, PRC75 (

Conclusions - Outlook

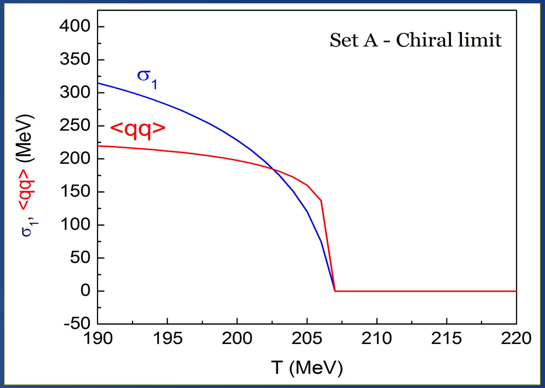
- The Polyakov-DSE (nonlocal PNJL) model provides a model framework which is suitable to
  address lattice QCD data for:
  - quark propagator (vacuum and finite T, μ ; quark mass and flavor dependence, ...)
  - order parameters (chiral condensate, traced Polyakov loop, ...)
  - equation of state (dependences on T, μ, μ_I ; quark masses …)
  - meson properties (masses, spectral functions, correlation functions, …)

- It allows to extrapolate to the "Terra Incognita" of the phase diagram and predict its structure
- Microscopically based EoS for the exotic QCD degrees of freedom region (HIC, CS, SN)

Further development needed to include consistently the hadronic phase of QCD by go

Supplementary material

CEP determination: Landau expansion



In the chiral limit (m=0), the condensate $\langle \bar{q}q \rangle$ vanishes at $T = T_c(\mu)$, around this point we can perform

$$\Omega_{MFA}^{(reg)}(\mu, T, \Phi, \sigma_2, \langle \bar{q}q \rangle) = \hat{\Omega} + A\langle \bar{q}q \rangle^2 + C\langle \bar{q}q \rangle^4 + O\left(\langle \bar{q}q \rangle^6\right)$$

Where the coefficients $\hat{\Omega}$, A and C are functions of μ, T, Φ and σ_2, given by

$$A(\mu, T, \Phi, \sigma_2) = \frac{1}{4\, S_{11}^2}\left(\frac{1}{8\, G_S} - S_{21}\right) \qquad C(\mu, T, \Phi, \sigma_2) = \frac{S_{42}}{128\, S_{11}^4} - \frac{S_{32}}{32\, S_{11}^5}\left(\frac{1}{8\, G_S} - S_{21}\right)$$

$$\hat{\Omega}(\mu, T, \Phi, \sigma_2) = \frac{8\,T}{\pi^2}\sum_{n_c}\sum_{n}\int \frac{d^3\vec{p}}{(2\pi)^3}\ln Z(\rho_{n,\vec{p}}^{c}) + \frac{\kappa_p^2\sigma_2^2}{2\, G_S} + U(\Phi, T) + \Omega_{(reg)}^{free} + \Omega_0.$$

with

$$S_{jk} = S_{jk}(\mu, T, \Phi, \sigma_2) = \sum_{n_c}\int_{p,n} g^j(\rho_{n,\vec{p}}^{c})\left(\frac{Z^2(\rho_{n,\vec{p}}^{c})}{(\rho_{n,\vec{p}}^{c})^2}\right)^k$$

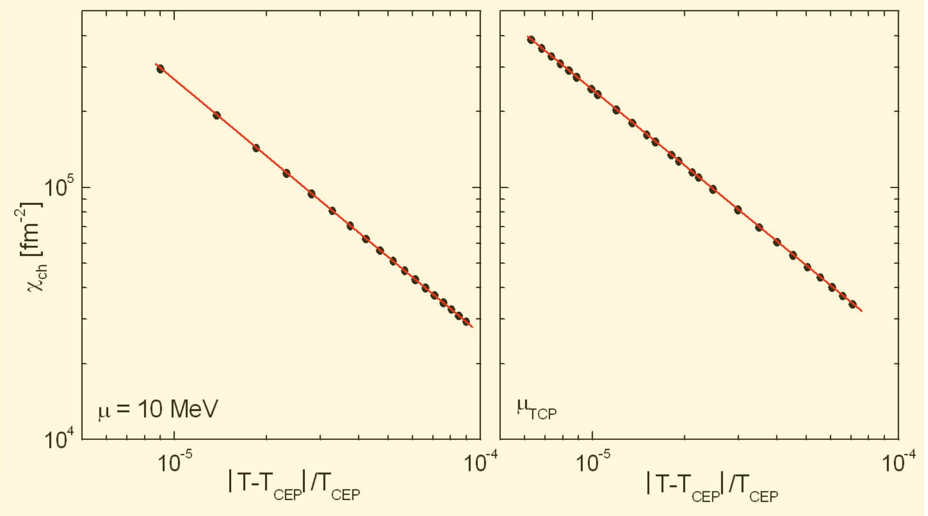
We observed that

- C>0 ⟶ 2nd order transition at A=0.
- C=0 ⟶ the transition stops to be 2nd order.
- C<0 ⟶ 1st order phase transition.

D. Gómez Dumm, N.N. Scoccola, Phys.Rev. C 72, 014909 (2005).

CEP determination: Chiral limit

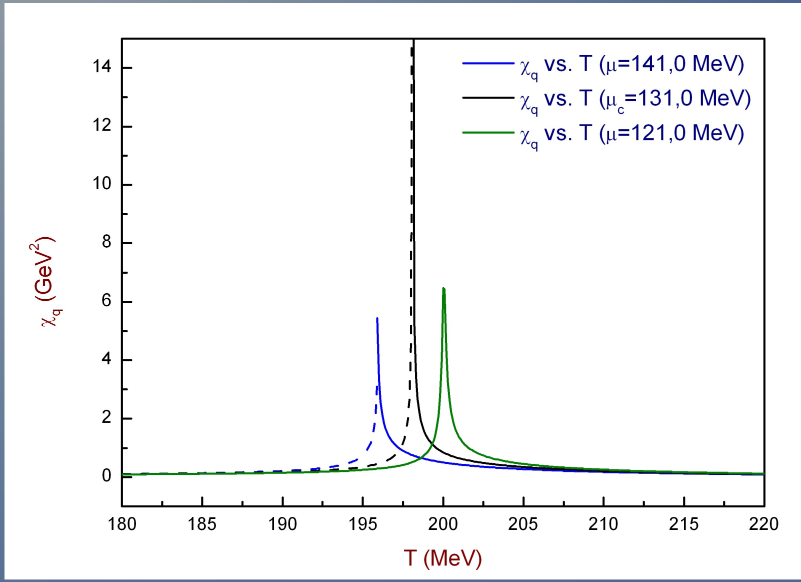
	Set A	Set B	Set C
$T_c(0)$	206.6	205.9	209.7
μ_{TCP}	46.2	86.1	125.7
T_{TCP}	204.8	199.2	194.6
$\mu_c(0)$	297.6	285.5	268.2

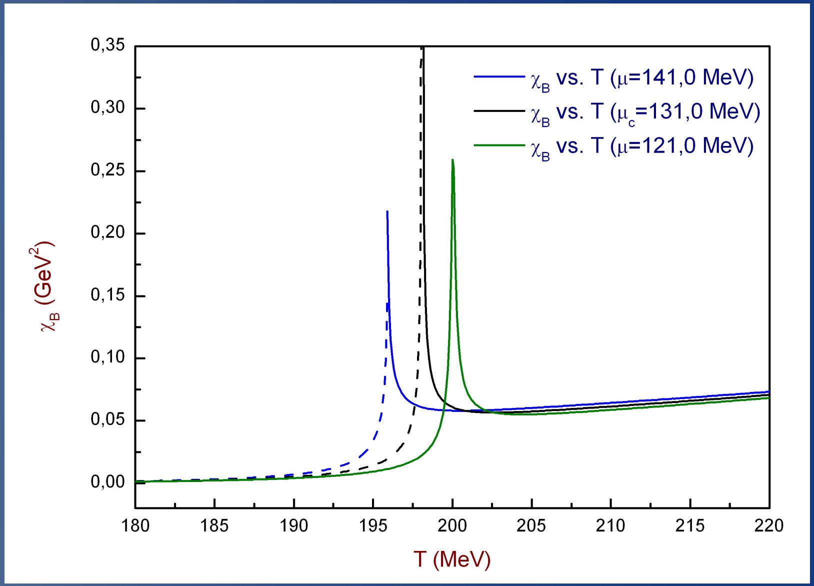


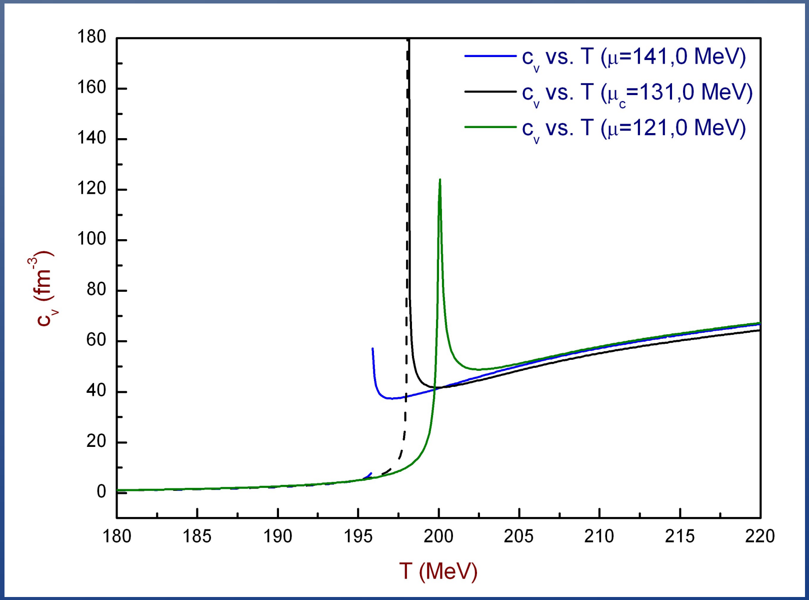
Chiral limit results

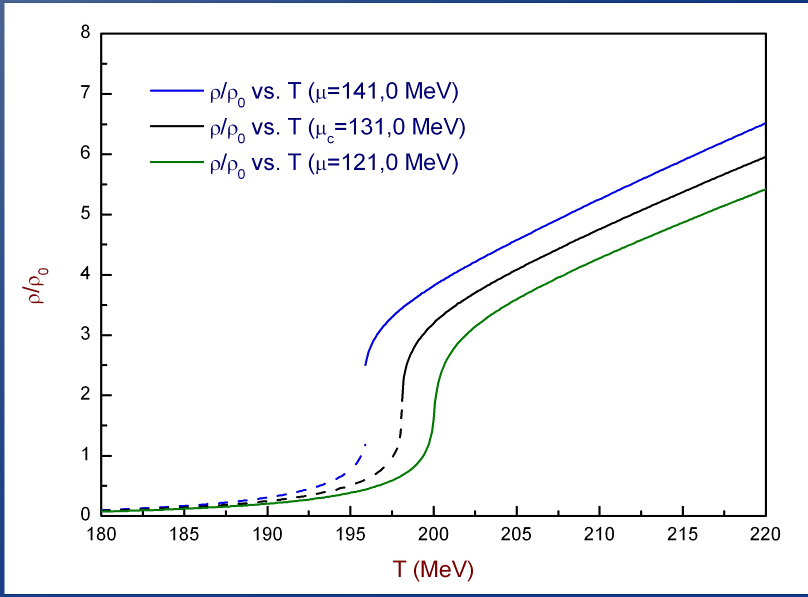
	Set C	γ_{ch}	γ_q	α
Point in	$\mu \rightarrow$	1.00(1)	0.00(1)	0.00(1)
2nd order	$T \uparrow$	1.00(1)	0.00(1)	0.00(1)
critical line	MF exponent	1	0	0
	$\mu \rightarrow$	1.00(1)	0.51(1)	0.50(1)
TCP	$T \uparrow$	1.00(1)	0.51(1)	0.50(1)
	MF exponent	1	1/2	1/2

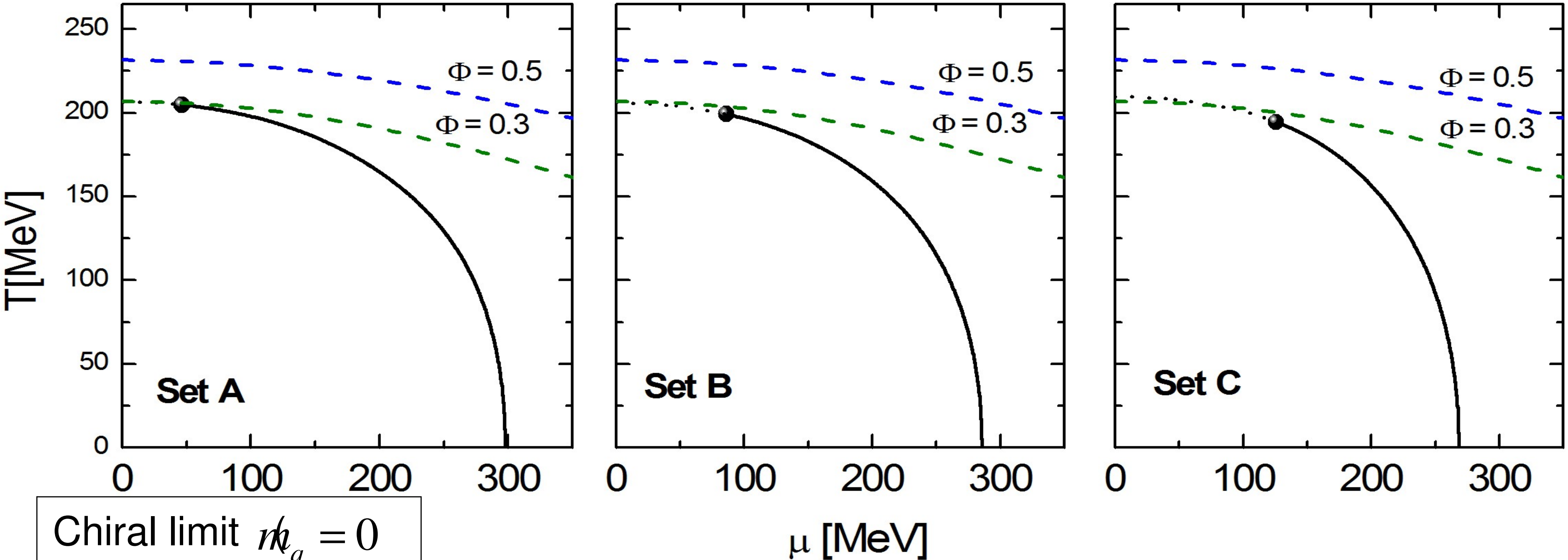
Phase transition thermodynamics



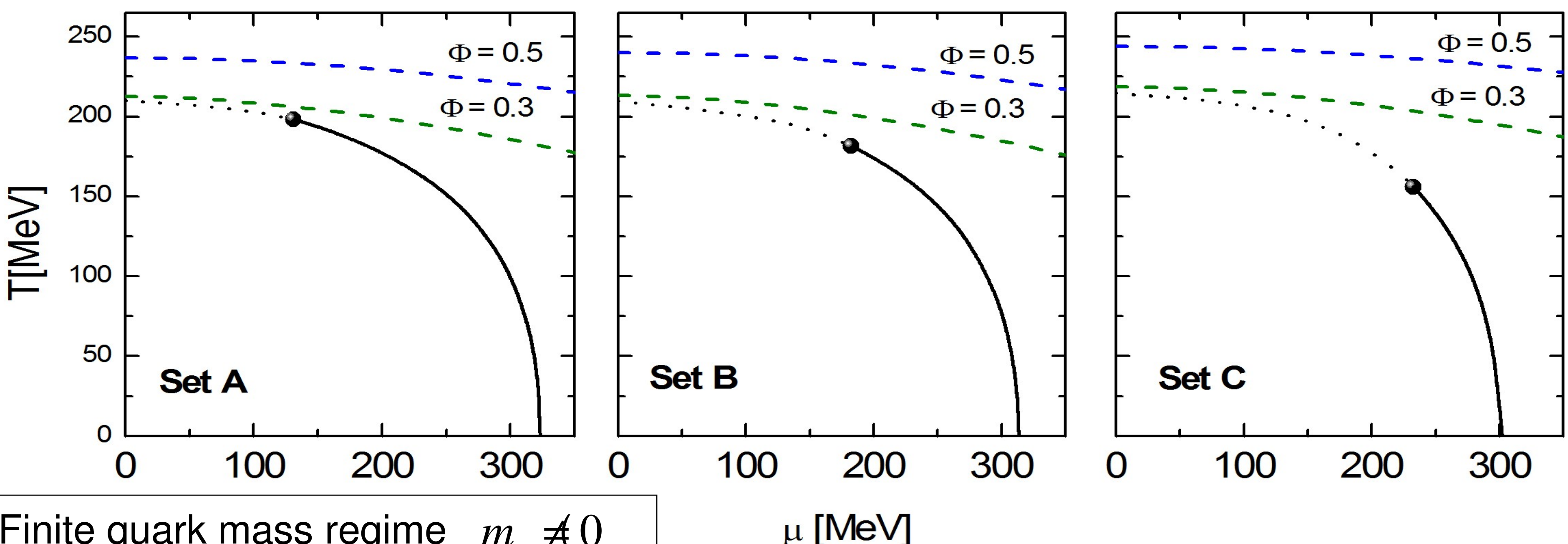








Chiral limit ($m_q = 0$)

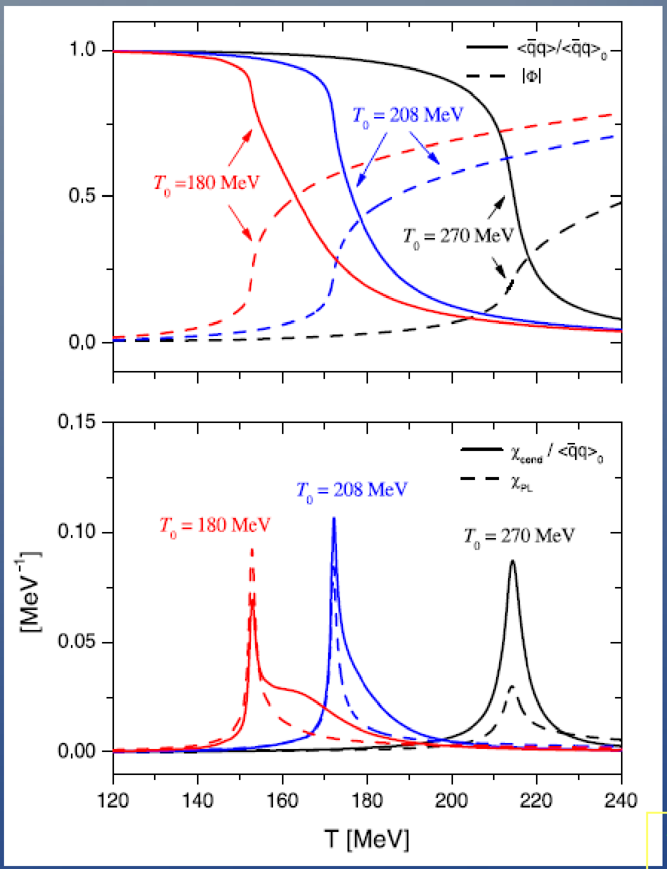


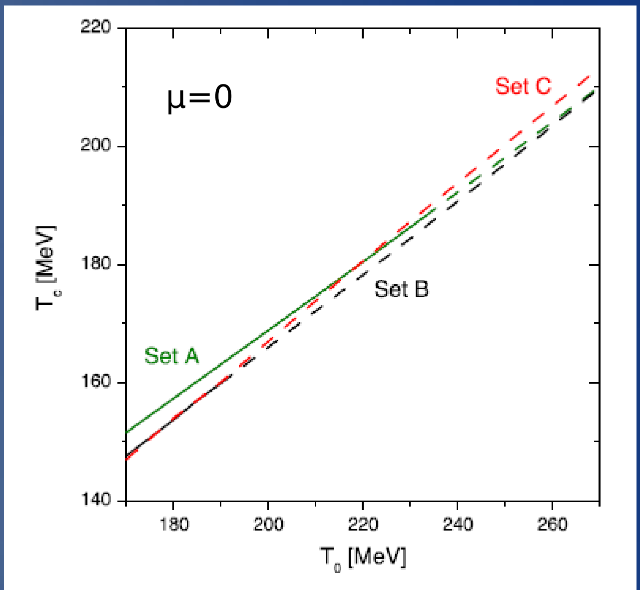
Finite quark mass regime ($m_q \neq 0$)

New improvements to the model: Adjustment of T_0

$$\mathcal{U}(\Phi,\Phi^*,T)=\left\{-\frac{1}{2}a(T)\Phi\Phi^*+b(T)\ln\left[1-6\Phi\Phi^*+4\Phi^3+4(\Phi^*)^3-3(\Phi\Phi^*)^2\right]\right\}T^4,$$

$$a(T)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2,\qquad b(T)=b_3\left(\frac{T_0}{T}\right)^3.$$





The values of a_i and b_3 are fitted [9] to lattice QCD results, which in absence of dynamical quarks lead to a deconfinement temperature $T_0 \approx 270$ MeV. However, as mentioned above, it has been argued [23] that in the presence of light dynamical quarks this value has to be modified accordingly, e.g. $T_0 \simeq 208$ MeV for $N_f = 2$ and $T_0 \simeq 180$ MeV for $N_f = 3$. Effects of this change in T_0 will be

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