

Microscopic Model for Chemical Freeze-Out in Heavy-Ion Collisions ¹

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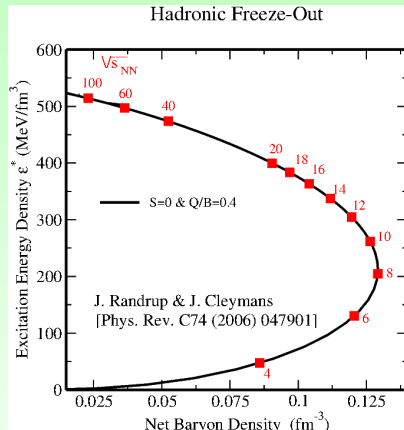
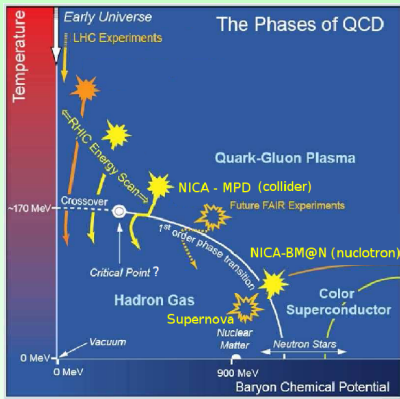
Joint Seminar HMUEC-HP @ BLTP

Dubna, May 18, 2012

¹Collaboration: J. Berdermann, J. Cleymans, D. Prorok, K. Redlich, L. Turko

...

QCD Phase Diagram & Heavy-Ion Collisions

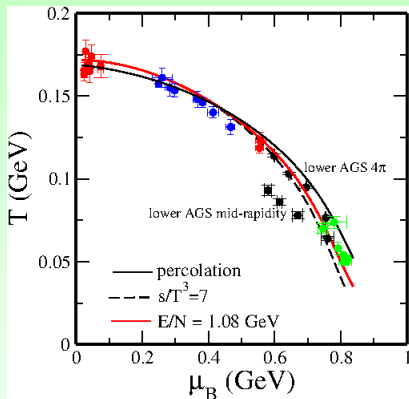


Beam energy scan (BES) programs in the QCD phase diagram

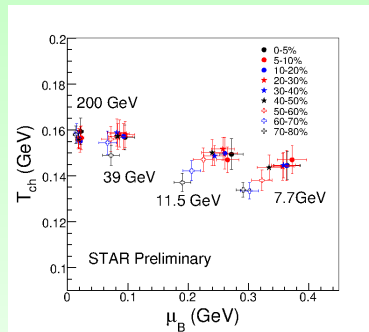
Energy density vs. baryon density at freeze-out for different $\sqrt{s_{NN}}$ (GeV)

Highest baryon densities at freeze-out shall be reached for $\sqrt{s_{NN}} \sim 8$ GeV \rightarrow QGP phase transition ?

Chemical Freeze-out in the QCD Phase Diagram



“Old” freeze-out data from RHIC (red), SPS (blue), AG (black), SIS (green).



“New” freeze-out data from STAR BES @ RHIC. Centrality dependence!

F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C73 (2006) 044905
Lokesh Kumar (STAR Collab.), arxiv:1201.4203 [nucl-ex]

Chemical freeze-out condition

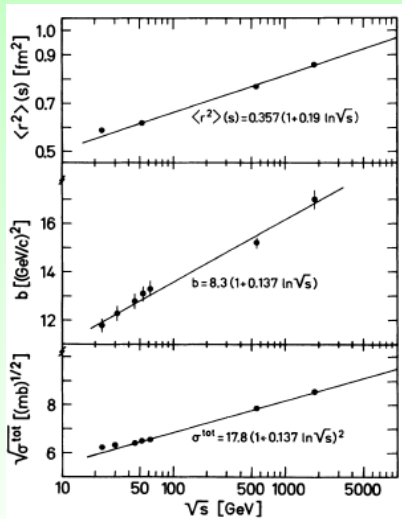
$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

$$\tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

D.B. et al., Few Body Systems (2011) arxiv:1109.5391

[hep-ph]



B. Povh, J. Hüfner, PRD 46 (1992) 990

Hadronic radii and chiral condensate

$$r_{\pi}^2(T, \mu) = \frac{3}{4\pi^2} F_{\pi}^{-2}(T, \mu) .$$

$$F_{\pi}^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu} / m_{\pi}^2 .$$

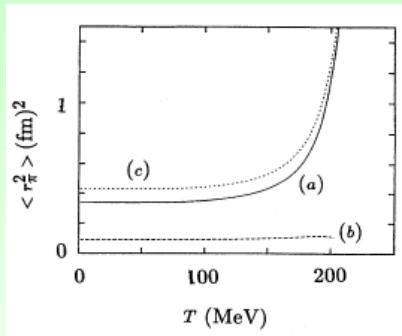
$$r_{\pi}^2(T, \mu) = \frac{3m_{\pi}^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1} .$$

$$r_N^2(T, \mu) = r_0^2 + r_{\pi}^2(T, \mu) ,$$

Expansion time from entropy conservation

$$S = s(T, \mu) V(\tau_{\text{exp}}) = \text{const}$$

$$\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu) ,$$



H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172

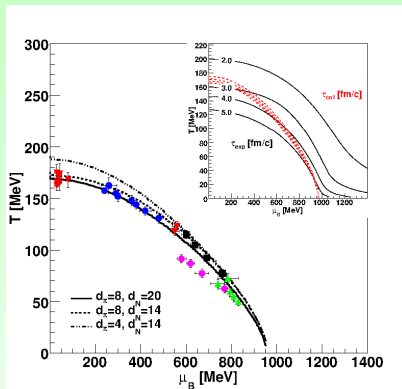
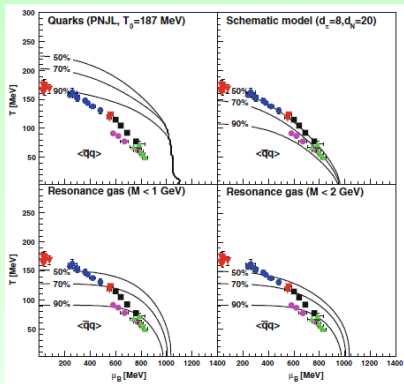
D.B., J. Berdermann, J. Cleymans, K. Redlich, Few
Body Systems (2011) [arxiv:1109.5391]

$$\begin{aligned} \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} &= 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left\{ 4N_c \int \frac{dp p^2}{2\pi^2} \frac{m}{\varepsilon_p} [f_\Phi^+ + f_\Phi^-] \right. \\ &+ \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \\ &+ \left. \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \right\} \\ &- \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)) \end{aligned}$$

S. Leupold, J. Phys. G (2006)

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate

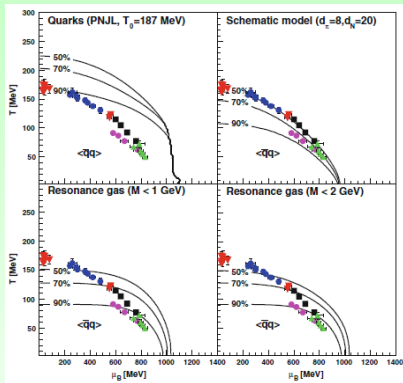


Chemical freeze-out vs. Condensate

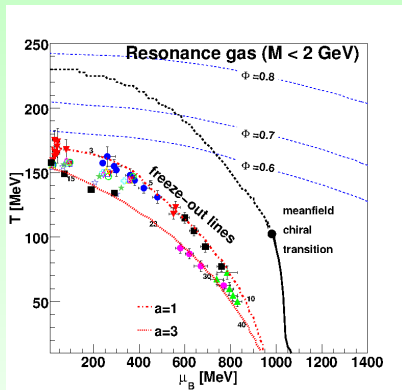
Chemical freeze-out from kinetic condition, schematic model

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate



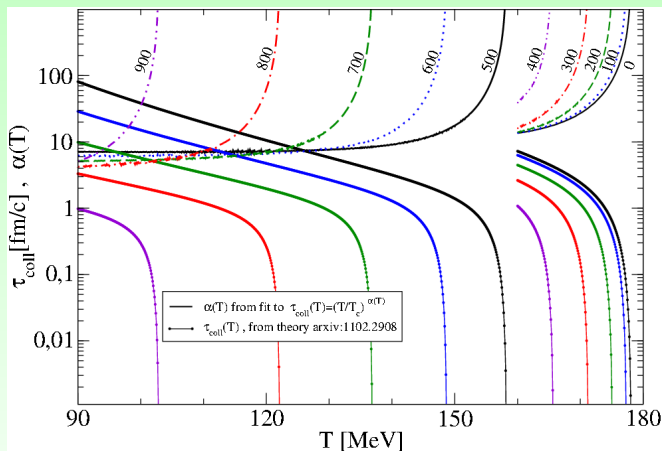
Chemical freeze-out vs. Condensate



Chemical freeze-out from kinetic condition, $a \sim$ inverse system size

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2012)

Strong T-Dependence of (inelastic) Collision Time

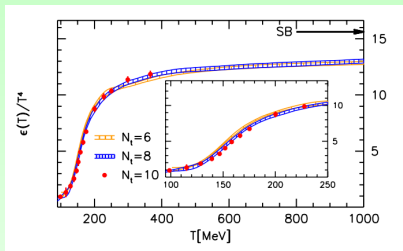


See: C. Blume in: NICA White Paper (2012)

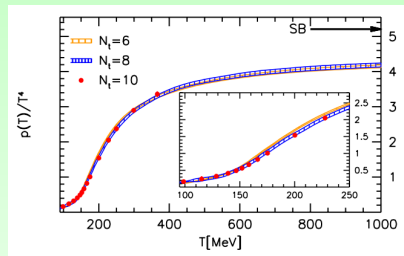
C. Wetterich, P. Braun-Munzinger, J. Stachel, PLB (2004)

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2012)

- The model works astonishingly well!
- Improvements are plenty:
 - Hadron mass formulae, e.g. from holographic QCD ...
 - Spectral functions - generalized Beth-Uhlenbeck
 - Thermodynamics ... hydrodynamics .
- Beyond freeze-out towards the deconfined phase: Mott-Hagedorn model



The energy density normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices.



The pressure normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices.

S. Borsanyi *et al.* "The QCD equation of state with dynamical quarks," JHEP **1011**, 077 (2010)

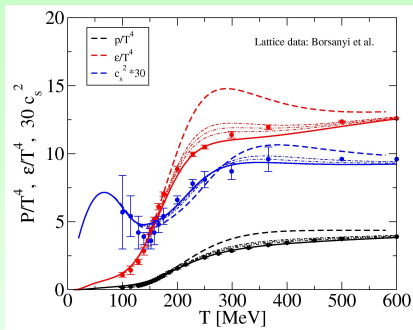
The energy density per degree of freedom with the mass M

$$\begin{aligned} \varepsilon(T, \mu_B, \mu_S) &= \sum_{i: m_i < m_0} g_i \varepsilon_i(T, \mu_i; m_i) \\ &+ \sum_{i: m_i \geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2) A(M, m_i) \varepsilon_i(T, \mu_i; M), \end{aligned}$$

Spectral function

$$\begin{aligned} A(M, m) &= N_M \frac{\Gamma \cdot m}{(M^2 - m^2)^2 + \Gamma^2 \cdot m^2}, \\ \Gamma(T) &= C_\Gamma \left(\frac{m}{T_H} \right)^{N_m} \left(\frac{T}{T_H} \right)^{N_T} \exp \left(\frac{m}{T_H} \right) \end{aligned}$$

Hagedorn resonance gas: hadrons with finite widths



$$P(T) = T \int_0^T dT' \frac{\epsilon(T')}{T'^2}.$$

N_m in the range from $N_m = 2.5$ (dashed line) to $N_m = 3.0$ (solid line).

$$C_\Gamma = 10^{-4}$$

$$N_T = 6.5$$

$$T_H = 165 \text{ MeV}$$

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H} \right)^{N_m} \left(\frac{T}{T_H} \right)^{N_T} \exp \left(\frac{m}{T_H} \right)$$

D. Blaschke & K. Bugaev, *Fizika B* **13**, 491 (2004); *PPNP* **53**, 197 (2004)

State-dependent hadron resonance width

$$A_i(M, m_i) = N_M \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2},$$

$$\Gamma_i(T) = \tau_{\text{coll},i}^{-1}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T n_j(T)$$

D. B., J. Berdermann, J. Cleymans, K. Redlich, PPN 8, 811 (2011)
[arXiv:1102.2908]

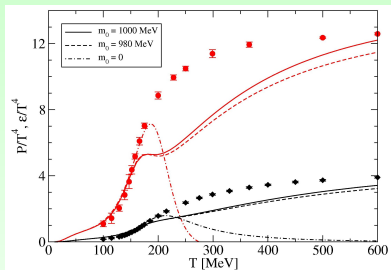
For pions (mesons)

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1}; \quad \langle \bar{q}q \rangle_T = 304.8 [1 - \tanh(0.002 T - 1)]$$

For nucleons (baryons)

$$r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu); \quad r_0 = 0.45 \text{ fm pion cloud.}$$

Mott-Hagedorn resonance gas



Quarks and gluons are missing!

Mott-Hagedorn resonance

gas: Pressure and energy density for three values of the mass threshold

$m_0 = 1.0$ GeV (solid lines)

$m_0 = 0.98$ GeV (dashed lines)

and

$m_0 = 0$ (dash-dotted lines)

Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\text{PNJL, MF}}(T)$ by including perturbative corrections

$$P(T) = P_{\text{HRG}}^*(T) + P_{\text{PNJL, MF}}(T) + P_2(T) ,$$

$$P_{\text{HRG}}^*(T) = \frac{P_{\text{HRG}}(T)}{1 + (P_{\text{HRG}}(T)/(aT^4))^\alpha} ,$$

with $a = 2.7$ and $\alpha = 1.8$.

Quark and gluon contributions

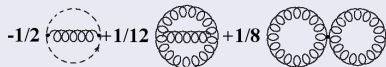
$$P_2(T) = P_2^{\text{quark}}(T) + P_2^{\text{gluon}}(T)$$

Quark and gluon contributions

$P_2^{\text{quark}}(T)$



$P_2^{\text{gluon}}(T)$



Total perturbative QCD
correction

$$P_2 = -\frac{8}{\pi} \alpha_s T^4 (I_\Lambda^+ +$$

$$\frac{3}{\pi^2} ((I_\Lambda^+)^2 + (I_\Lambda^-)^2))$$

$$\xrightarrow{\Lambda/T \rightarrow 0} -\frac{3\pi}{2} \alpha_s T^4$$

where

$$I_\Lambda^\pm = \int_{\Lambda/T}^{\infty} \frac{dx x}{e^x \pm 1}$$

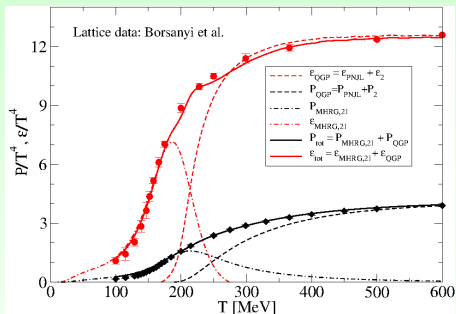
· Energy corrections

$$\varepsilon_2(T) = T \frac{dP_2(T)}{dT} - P_2(T) .$$



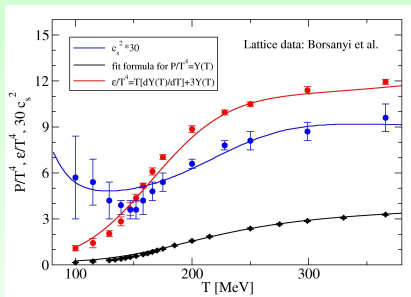
Quarks, gluons and hadron resonances

$$P_{\text{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3 p}{(2\pi)^3} \int dM A_i(M, m_i) T \ln \left\{ 1 + \delta_i e^{-[\sqrt{p^2 + M^2} - \mu_i]/T} \right\},$$



- Quark-gluon plasma contributions are described within the improved PNJL model with α_s corrections.
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.

Quarks, gluons and hadron resonances II



- Contribution restricted to the region around the chiral/deconfinement transition 170-250 MeV
- Fit formula for the pressure

$$P = aT^4 + bT^{4.4} \tanh(cT - d),$$

$$a = 1.0724, \quad b = 0.2254, \\ c = 0.00943, \quad d = 1.6287$$

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.
- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

To do

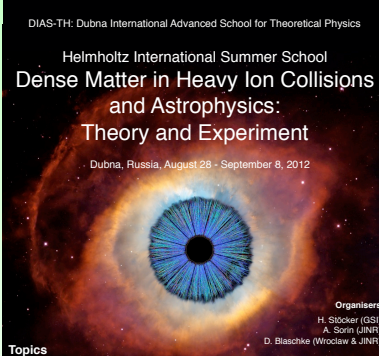
- Join hadron resonance gas with quark-gluon model.
- Calculate kurtosis and compare with lattice QCD.
- Spectral function for low-lying hadrons from microphysics (PNJL model ...).

Invitation to upcoming events

DIAS-TH: Dubna International Advanced School for Theoretical Physics

Helmholtz International Summer School
**Dense Matter in Heavy Ion Collisions
and Astrophysics:
Theory and Experiment**

Dubna, Russia, August 28 - September 8, 2012



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A. Sorin (JINR)
D. Blaschke (Wrocław & JINR)

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dm12@theor.jinr.ru
<http://theor.jinr.ru/>


Topics

- Equation of state & QCD phase transitions
- Transport properties in dense QCD matter
- Hadronization & freeze-out in heavy ion collisions (HIC)
- Astrophysics of compact stars (CS)
- Simulations of dense QCD, HIC and CS
- Experiments and observational programs



The New Physics of **Compact Stars: CompStar School 2012**

Equation of State for Compact Star Interiors and Supernovae



Zadar, Croatia
September 24 - 28, 2012

Topics
Theory for hot and dense QCD and nuclear matter
Heavy- ion collision physics
Astrophysics of compact stars and supernovae

Lecturers

- David Blaschke** *Review of EoS and Compact Stars*
[University of Wrocław, IFB Dubna]
- Fiorella Burgio** *Theoretical approaches towards a modern nuclear EoS*
[INFN Catania]
- Pavel Danielewicz** *Heavy ion experiments and EoS constraints*
[Michigan State University]
- Tobias Fischer** *EoS and neutrino transport for supernova explosions*
[University of Basel]
- Mariano Mendez** *Observations of masses and radii of compact stars*
[University of Groningen]
- Bernd-Jochen Schaefer** *Aspects of the QCD phase diagram and the EoS*
[University of Gießen]

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webpage: <http://andrei.laha.be/india2012/>

Registration deadline 06. 07. 2012.

