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Evolution of a Quantum System: New Results.

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Motivation of work.

Currently, the study of substances in extreme conditions is relevant. The universe at the moment of explosion, quark gluon plasma, evolution of black holes, multiphoton processes, etc. In connection with these problems, the task arises of analyzing methods for describing these physical phenomena, in particular, the process of their evolution.

The content of the work.

The paper analyzes the equations of quantum theory for the probability of transitions of systems in the process of their evolution. The analysis is carried out on the basis of an extended model of methods of probability theory.

Based on this model, a new equation for the probability of quantum transitions is proposed.

For the proposed equation, the correspondence principle is fulfilled: under certain conditions, it turns into a well-known equation of quantum theory.

I. The equation for the transition probability of a quantum system in the method of path integrals, and its interpretation in probability theory.

The equation of evolution of a quantum system and the probability of quantum transitions between states.

Consider a system that is defined by the Hamiltonian

$$\hat{H} = \hat{H}_{syst} + \hat{V}_{int}(\tau), \tag{4.1}$$

where the Hamiltonian of the system has a discrete spectrum of eigenvalues (energies)

$$\hat{H}_{syst}|n\rangle = E_n|n\rangle, \tag{4.2}$$

where $n = 1, 2, 3, ...$ - the number of the quantum state; $|n\rangle$ - the vector of states;$\hat{V}_{int}(\tau)$ - interaction Hamiltonian. The interaction Hamiltonian can have the form

$$\hat{V}_{int}(\tau) = E_0 \cos(\Omega\tau). \tag{4.3}$$

We will describe the state of the system by a statistical operator $\hat{\rho}(t)$. Equation of evolution of a statistical operator:

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^{+}(t), \tag{5.1}$$

where $\hat{\rho}(t)$, $\hat{\rho}(0)$ — Statistical operators of the system, respectively, at time moment t and $t = 0$,

$$\hat{U}(t,0) = T\exp[-\frac{\imath}{\hbar}\int\limits_{0}^{t}\hat{H}(\tau)d\tau]. \tag{5.2}$$

The probability of a system transition from state $|a\rangle$ at time $t = 0$ to state $|b\rangle$ at time $t > 0$ is determined by the equation:

$$P(b,t|a,t=0) = \langle b|\hat{\rho}(t)|b\rangle = \langle b|\hat{U}(t)|a\rangle\langle a|\hat{U}^{+}(t)|b\rangle, \tag{5.3}$$

where $\langle a|\hat{\rho}(0)|a\rangle = 1$.
The kernel of the evolution operator $\langle b|\hat{U}(t)|a\rangle$ is calculated for a specific model of a quantum system.

Transition probability in coordinate representation.

Feynman (R. Feynman, A.Hibbs, Quantum mechanics and path integrals, M: World, 1968, 382 p.) presented the amplitude of quantum transitions $\langle b|\hat{U}(t)|a\rangle$ in the coordinate representation as an integral over virtual trajectories in real space:

$$\langle b|\hat{U}(t)|a\rangle = \psi(b|a) = \int_{-\infty}^{\infty} \sqrt{P_0} \exp[\imath S[b, \mathbf{x}, a]] \mathcal{D}\mathbf{x}(t), \qquad (6.1)$$

где $S[b, \mathbf{x}, a]$ - dimensionless action of a particle along a trajectory passing from point a to point b; integration is carried out over all virtual trajectories (Fig. 1).

The probability density of a particle transition is determined by the formula:

$$P(b|a) = \psi(b|a)\psi^*(b|a) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \exp[\imath(S[b;\mathbf{x},a] - S[b;\mathbf{x}',a])]\mathcal{D}\mathbf{x}(t)\mathcal{D}\mathbf{x}'(t). \qquad (7.1)$$

P_f is normalizing constant.

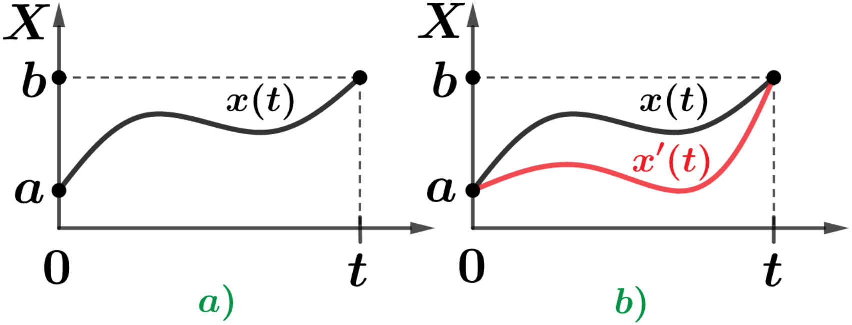


Figure 1

The equation takes the form

$$P(b_f, t|a_{in}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \cos[(S[b;\mathbf{x},a] - S[b;\mathbf{x}',a])]\mathcal{D}\mathbf{x}(t)\mathcal{D}\mathbf{x}'(t)+$$

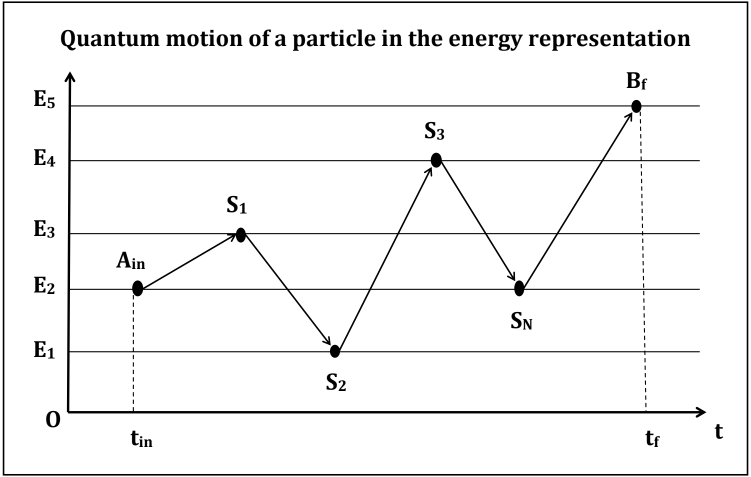
$$+\imath \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \sin[(S[b;\mathbf{x},a] - S[b;\mathbf{x}',a])]\mathcal{D}\mathbf{x}(t)\mathcal{D}\mathbf{x}'(t) \tag{8.1}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \sin[(S[b;\mathbf{x},a] - S[b;\mathbf{x}',a])]\mathcal{D}\mathbf{x}(t)\mathcal{D}\mathbf{x}'(t) = 0, \tag{8.2}$$

$$P(b|a) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_f \cos[(S[b;\mathbf{x},a] - S[b;\mathbf{x}',a])]\mathcal{D}\mathbf{x}(t)\mathcal{D}\mathbf{x}'(t). \tag{8.3}$$

Transition probability in the energy representation

The system begins to make quantum transitions between states. The quantum states of the system are determined by the equation (4.1). These transitions are shown in the figure.


Quantum motion of a particle in the energy representation
E5
E4
E3
E2
E1
O
Ain
S1
S2
S3
SN
Bf
tin
tf
t

The amplitude of quantum transitions is represented as a sum along trajectories(Biryukov A. A.,Degtyareva Y.V., Shleenkov M.A., Calculating the Probabilities of Quantum Transitions in Atoms and Molecules Numerically through Functional Integration, Bulletin of the Russian Academy of Sciences: Physics. 2018, **82**, 12, p.1565-1569.)

$$\langle b|\hat{U}|a\rangle = \sum_{n_1, n_2, ..} \langle b|\hat{U}|n_N\rangle ... \langle n_3|\hat{U}|n_2\rangle \langle n_2|\hat{U}|n_1\rangle \langle n_1|\hat{U}|a\rangle, \qquad (11.1)$$

where $\langle b| = \langle n_f|$ - state at a time $t > 0$; $|a\rangle = |n_{in}\rangle$ - state at a time $t = 0$.

The core of the evolution operator of a quantum system can be represented as a sum along trajectories from a functional:

$$\langle b|\hat{U}|a\rangle = \sum_{n_1,n_2,..} \int_0^1 \int_0^1 \int_0^1 \int_0^1 P_f \exp[iS[b;...;n_2,\xi_2;n_1,\xi_1;a,\xi_0]]d\xi_0 d\xi_1 d\xi_2..., \tag{12.1}$$

$$S[b;...;n_2,\xi_2;n_1,\xi_1;a,\xi_0] = \sum_{k=1}^{N} S[n_k,t_k;n_{k-1},t_{k-1};\xi_{k-1}]; \tag{12.2}$$

Example (4.2)

$$S[n_k,t_k;n_{k-1},t_{k-1};\xi_{k-1}] = 2\pi(n_k - n_{k-1})\xi_{k-1} + \Omega^R_{n_k n_{k-1}}[\cos(2\pi(n_k - n_{k-1})\xi_{k-1}+$$

$$+(\Omega + \omega_{n_k,n_{k-1}})\frac{t_k + t_{k-1}}{2}) + \cos(2\pi(n_k - n_{k-1})\xi_{k-1} - (\Omega - \omega_{n_k,n_{k-1}})\frac{t_k + t_{k-1}}{2}))](t_k - t_{k-1}).$$

P_f is normalizing constant.

The probability of a quantum transition of a system can be represented as a sum along trajectories :

$$P(b|a) = \sum_{n_1,m_1=1}^{N} \int_0^1 \int_0^1 P_f \exp[i(S[b;..;n_1,\xi_1;a,\xi_0] - S[b;..;m_1,\zeta_1;a,\zeta_0])] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1, \tag{13.1}$$

The probability of a quantum transition of a system can be represented as a sum along trajectories from a real functional:

$$P(b|a) = \sum_{n_1,m_1=1}^{N} \int_0^1 \int_0^1 P_f \cos[S[b;..;n_1,\xi_1;a,\xi_0] - S[b;..;m_1,\zeta_1;a,\zeta_0] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 +$$

$$+ \sum_{n_1,m_1=1}^{N} \int_0^1 \int_0^1 P_f i \sin[S[b;..;n_1,\xi_1;a,\xi_0] - S[b;..;m_1,\zeta_1;a,\zeta_0] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1. \tag{13.2}$$

$$\sum_{n_1,m_1=1}^{N} \int_0^1 \int_0^1 P_f i \sin[S[b;..;n_1,\xi_1;a,\xi_0] - S[b;..;m_1,\zeta_1;a,\zeta_0] d\xi_0 d\xi_1 d\zeta_0 d\zeta_1 = 0. \tag{13.3}$$

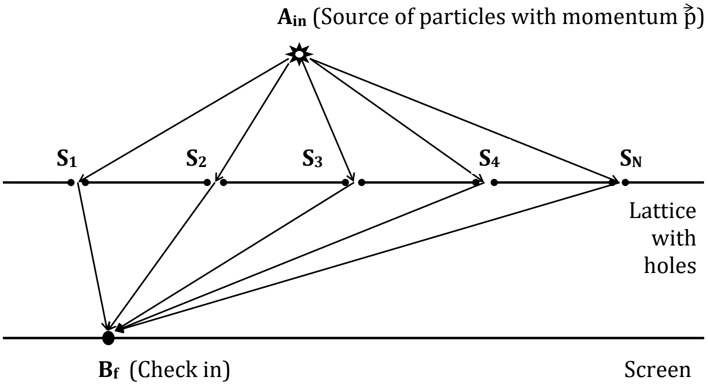
The equation is represented in the form

$$P(n_f,t_f|n_{in},t_{in})=\sum_{n_1,m_1=1}^{N}\int_0^1\int_0^1\int_0^1\int_0^1 d\xi_0 d\xi_1 d\zeta_0 d\zeta_1\times$$

$$\times P_f\cos[S[b;..;n_1,\xi_1;a,\xi_0]-S[b;..;m_1,\zeta_1;a,\zeta_0], \qquad (14.1)$$

Biryukov A. A.,Degtyareva Y.V., Shleenkov M.A., Calculating the Probabilities of Quantum Transitions in Atoms and Molecules Numerically through Functional Integration, Bulletin of the Russian Academy of Sciences: Physics. 2018, **8**2, 12, p.1565-1569.

Diffraction of particles



Drawing 1. Particle diffraction

The amplitude of the particle transition from the source a_{in} to the point $_f$ of the screen is represented by the expression

$$\psi(b|a)=\sum_{n=1}^{N}\sqrt{P_f}\exp[\imath S[b;n,a]], \tag{16.1}$$

Where
$S[b;n,a]=\omega t-\mathbf{k}(\mathbf{r}_{an}+\mathbf{r}_{nb})$ — Dimensionless action of a particle along line passing through the points $\mathbf{a},\mathbf{n},\mathbf{b}$; $\mathbf{r}_{an}$ - line from source to the coordinates of the center of the hole of the diffraction grating; $\mathbf{r}_{nb}$—line from the coordinates of the center of the hole of the diffraction grating to the point on the screen ; $\mathbf{k}$—the wave vector of the particle; $\sqrt{P_f}$ -is the amplitude of the wave function.
The probability of a particle transition is determined by the formula:

$$P(b|a)=\psi(b|a)\psi^*(b|a)=\sum_{m,n=1}^{N}P_f\exp[\imath(S[b;n,a]-S[b;m,a])]. \tag{16.2}$$

P_f is normalizing constant.

The equation takes the form

$$P(b|a) = \sum_{m,n=1}^{N} P_f \cos[(S[b;n,a] - S[b;m,a])] +$$

$$+\imath \sum_{m,n=1}^{N} P_f \sin[(S[b;n,a] - S[b;m,a])]. \quad (17.1)$$

$$\sum_{m,n=1}^{N} P_f \sin[(S[b;n,a] - S[b;m,a])] = 0 \quad (17.2)$$

Therefore

$$P(b|a) = \sum_{m,n=1}^{N} P_f \cos[(S[b;n,a] - S[b;m,a])]. \quad (17.3)$$

Probabilistic interpretation of the equation for the transition probability

We draw a conclusion from the analysis of the equations (8.3),(14,1),(17.3), that are built on the basis of the analysis of specific quantum processes. The equation for the transition probability of a quantum system between states $|a\rangle$ and $|b\rangle$ has the form

$$P(b|a) = \sum_{m,n=1}^{N} P_f \cos[(S[b;n;a] - S[b;m;a])], \tag{17.3}$$

where $S[b;n;a]$ - action along a virtual random trajectory in the space of state symbols between states $|a\rangle$ and $|b\rangle$;the symbol $\sum$ means summation or integration over all possible pairs of virtual trajectories. The expression (17.3) allows one to calculate the quantum transition probability outside the perturbation theory method. The expression allows performing calculations using the Monte Carlo method.

We represent the equation for the transition probability in the form

$$P(b|a) = \sum_{m,n=1}^{N} g_{n,m} P(S[b;n;a], S[b;m;a]), \tag{19.1}$$

$$where\ P(S[b;n;a], S[b;m;a]) = P_f \mid \cos[S[b;n;a] - S[b;m;a] \mid,$$

$$g_{n,m} = \cos[S[b;n;a] - S[b;m;a]| \cos[S[b;n;a] - S[b;m;a] \mid^{-1}$$

is the function takes the value +1 or -1, or 0 depending on the action values $S[b;n;a], S[b;m;a]$. We represent the equation in the form

$$P(b|a) = \sum_{n=1}^{N} P(b;n;a) + \sum_{m \neq n=1}^{N} g_{n,m} P(S[b;n;a], S[b;m;a]), \tag{19.2}$$

where $P(S[b;n;a], S[b;m;a])$- the probability of a couple of joint events. They are characterized by numbers $S[b;n;a], S[b;m;a]$. Summation is carried out for all pairs of joint trajectories.

Let us replace random numbers in equations (19.1),(19.2) with random events: $a - A_{in}; b - B_f; S[b;n;a] - S_{fni}$. Let's write the equations for random events

$$P(B_f \mid A_{in}) = \sum_{n=1}^{N} P(S_{fni}) + \sum_{n \neq m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}), \tag{20.1}$$

$$P(B_f \mid A_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}), \tag{20.2}$$

where $P(S_{fni} \cap S_{fmi})$ - probability of random joint events. The equations are presented in diagrammatic form in Figure 4.
Equations (20.1), (20.2) show that the quantum process takes place in the space of random events (trajectories) that are pairwise compatible.

FIGURE 3

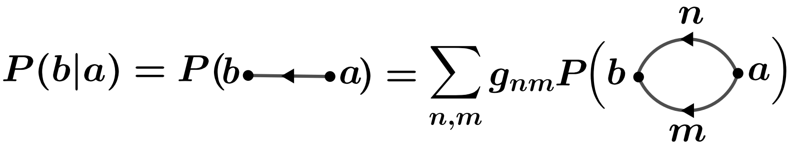


Figure 3

Hypothesis.

We believe that the quantum process is realized in the space of random events (virtual trajectories), which are all compatible with each other. We prove that the transition probability of a quantum system satisfies the equation:

$$P(b \mid a) = \sum_{n,m=1}^{N} g_{nm} P(S[b;n;a], S[b;m;a]) + 4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[b;n;a], S[b;m;a], S[b;k;a]) +$$

$$+ \ldots + 2^{N-1} g_{12\ldots N} P(S[b;1;a], S[b;2;a], \ldots, S[b;N;a]). \tag{22.1}$$

$$g_{n,m} = +1 \; or \; -1, or \; 0, n \neq m; \; g_{n,n} = +1, n = m; \; g_{nmk} = +1 \; or \; -1, or \; 0.$$

Where $P(S[n_{in};n;n_f], S[n_{in};m;n_f], S[n_{in};k;n_f])$ - the probabilities of compatibility of three trajectories. The equation is constructed in the theory of stochastic processes in the space of joint events.

Biryukov A. A.,Equations of quantum theory in the space of randomly joint quantum events. EPJ Web of Conferences 222, year 2019, pages 03005, https://doi.org/10.1051/epjconf/201922203005

Model of the space of joint random events with symmetric difference and sum of events

KOLMOGOROV'S AXIOMS:

INITIAL CONCEPTS:

Ω — a set of elementary events ω, space of elementary events.

$\mathfrak{F}$ — a set of subsets from Ω, set of casual events A, B, C, ...

AXIOM I:

$\mathfrak{F}$ — is algebra of sets (exist $A \cup B, A \cap B, A \setminus B, ...$, which belong $\mathfrak{F}$).

AXIOM II:

The probability of an event (a set measure) is entered $P(A) \geq 0$.

AXIOM III:

$P(\Omega) = 1$.

AXIOM IV:

not joint events if follows $A \cap B = 0, P(A + B) = P(A) + P(B)$.

Joint events in Kolmogorov's axiomatics

Definition: two events S_1, S_2 are joint, if $S_1 \cap S_2 \neq 0$;
probability of association of joint events

$$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2) \qquad (25.1)$$

Definition: events $S_1, S_2, ..., S_N$ are joint, if $S_n \cap S_m \neq 0, S_n \cap S_m \cap S_k \neq 0, ..., S_1 \cap S_2 \cap ... \cap S_N \neq 0$, where $n, m, k, ... = 1, 2, ..., N$.
Probability of association of N joint events

$$P(\bigcup_{n=1}^{N} S_n) = \sum_{n=1}^{N} P(S_n) - \sum_{n<m=1}^{N} P(S_n \cap S_m) + \sum_{n<m<k=1}^{N} P(S_n \cap S_m \cap S_k) + \ldots + (-1)^{N-1} P(S_1 \cap ... \cap S_N). \qquad (25.2)$$

Rozanov Yu. A.,Lectures on probability theory, 1968,Science,Moscow,p.120.

Symmetric difference joint events in Kolmogorov's axiomatics

Symmetric difference of two joint events

$$S_1^- = S_1 \setminus (S_1 \cap S_2), \ , \ S_2^- = S_2 \setminus (S_1 \cap S_2), \ P(S_1^- \cup S_2^-) = P(S_1) + P(S_2) - 2P(S_1 \cap S_2). \quad (26.1)$$

Symmetric difference of N joint events

$$P(\bigcup_{n=1}^{N} S_n^-) = \sum_{n=1}^{N} P(S_n) - 2 \sum_{n<m=1}^{N} P\big(S_n \cap S_m\big) +$$

$$+4 \sum_{n<m<k=1}^{N} P\big(S_n \cap S_m \cap S_k\big) + \ldots + (-2)^{N-1} P(S_1 \cap S_2 \cap \ldots \cap S_N). \quad (26.2)$$

Prohorov A. B., Ushakov V.G., Ushakov N.G., Problems in the theory of probability, 1986, Science,Moscow,p.328.

In addition to Kolmogorov's axioms. Let's define quantum joint events.

Two events S_1^q, S_2^q are quantum joint, if $S_1^q \cap S_2^q \neq 0$, probability of association of events

$$P(S_1^q \cup S_2^q) = P(S_1^q) + P(S_2^q) + P(S_1^q \cap S_2^q) \tag{27.1}$$

Symmetric sum of two quantum joint events is a postulate

$$S_1^+ = S_1^q \cup (S_1^q \cap S_2^q), \quad , \quad S_2^+ = S_2^q \cup (S_1^q \cap S_2^q) \tag{27.2}$$

$$P(S_1^+ \cup S_2^+) = P(S_1^q) + P(S_2^q) + 2P(S_1^q \cap S_2^q) \tag{27.3}$$

Symmetric association of N quantum events S_n^+ is sum:

$$P\left(\bigcup_{n=1}^{N} S_n^+\right) = \sum_{n=1}^{N} P(S_n^q) + 2 \sum_{n<m=1}^{N} P\big(S_n^q \cap S_m^q\big) -$$

$$-4 \sum_{n<m<k=1}^{N} P\big(S_n^q \cap S_m^q \cap S_k^q\big) + \ldots + (-2)^{N-1} P(S_1^q \cap S_2^q \cap \ldots \cap S_N^q). \tag{27.4}$$

The uniform equation for a symmetric difference and symmetric sums of random joint events

Let's designate joint events S_n, S_n^q one symbol S_n.
Let's designate events S_n^-, S_n^+ one symbol $\tilde{S}$.
These designations give the chance to write down the equations for $P(\bigcup_{n=1}^N S_n^-)$, $P(\bigcup_{n=1}^N S_n^+)$ in the form of one equation

$$P(\bigcup_{n=1}^{N} \tilde{S}_n) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_n \cap S_m) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_n \cap S_m \cap S_k) + \ldots + 2^{N-1} g_{12\ldots N} P(S_1 \cap S_2 \cap \ldots \cap S_N), \qquad (28.1)$$

$$g_{n,m} = +1 \; or \; -1, or 0, g_{nmk} = -g_{n,m}, \ldots$$

The equation for probability of transition between the random joint events

The equation for for probability of transition between the random not joint events

The event $A_{in}(t_{in})$ is realized at the moment time t_{in}. The event $B_f(t_f)$ is realized at the moment $t_f > t_{in}$. One of the events $S_n, n = 1, 2, ...N$, is realized at the moment t $(t_f > t > t_{in})$. Equation for events:

$$B_f \cap A_{in} = B_f \cap (\bigcup_{n=1}^{N} S_n) \cap A_{in} = \bigcup_{n=1}^{N} (B_f \cap S_n \cap A_{in}) = \bigcup_{n=1}^{N} S_{fni} \tag{30.1}$$

The probability of transition from an event $A_{in}(t_{in})$ to an event $B_f(t_f)$ is defined equation

$$P(B_f \cap A_{in}) = P(B_f \cap (\bigcup_{n=1}^{N} S_n) \cap A_{in}) = P(\bigcup_{n=1}^{N} (B_f \cap (S_n) \cap A_{in})) = \sum_{n=1}^{N} P(B_f \cap S_n \cap A_{in}) = \sum_{n=1}^{N} P(S_{fni}) \tag{30.2}$$

It is possible to present in a look of Markov equation.

$$P(B_f \mid A_{in}) = \sum_{n=1}^{N} P(B_f \mid S_n) P(S_n \mid A_{in}) \tag{30.3}$$

It is possible to write down in a look $B_f \cap S_n \cap A_{in} = S_{fni}$, $P(f \mid i) = \sum_{n=1}^{N} P(S_{fni})$

The equation for probabilities of transition in space of the random joint events

The equation for the random joint events we will write down in a look

$$B_f \cap A_{in} = B_f \cap (\bigcup_{n=1}^{N} \tilde{S}_n) \cap A_{in} = \bigcup_{n=1}^{N} (B_f \cap \tilde{S}_n \cap A_{in}) = \bigcup_{n=1}^{N} \tilde{S}_{fni}. \tag{31.1}$$

We will designate $B_f \cap \tilde{S}_n \cap A_{in} = \tilde{S}_{fni}$, $B_f \cap S_n \cap A_{in} = S_{fni}$.
It is possible to present the offered equation in a look

$$P(B_f \mid A_{in}) = P(\bigcup_{n=1}^{N} \tilde{S}_{fni}) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots +$$
$$+2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \tag{31.2}$$
$$g_{n,m} = +1 \; or \; -1, or 0.$$

The equation for probabilities of transition in space of the random joint events

It is possible to present the offered equation in a look

$$P(B_f \mid A_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \qquad (32.1)$$

$$g_{n,m} = +1 \; or \; -1, or \; 0, n \neq m; \; g_{n,n} = +1, n = m; \; g_{nmk} = +1 \; or \; -1, or \; 0.$$

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Interpretation of the stochastic process equation for joint events

Model of physical system

Let's consider system which can stay in states $n = 1, 2..., N$.
The system eventually makes transitions between states.
The transition of the system from the state n_{in} at the time t_{in} to the state n_f at the time $t_f > t_{in}$ is carried out along a certain random trajectory determined by the set of numbers $n_{in}, n_1, n_2, ...n_f$.
Action of system is determined for each trajectory:

$$S[n_{in}, n_1, n_2, ..., n_N, n_f]$$

.
Action of system we shall present as:$S[n_{in}, n, n_f]$ where $n(n_1, n_2, ..., n_N)$ a multiindex.

Interpretation of the stochastic process equation for joint event

To events B_f, A_{in} we put in conformity of the states $n_f = b, n_{in} = a$.
To events S_{fni} we put in conformity the action of system $S[n_f; n; n_{in}] = S[b; n; a]$.
The equation in space of the states of system becomes

$$P(b \mid a) = \sum_{n,m=1}^{N} g_{nm} P(S[b;n;a], S[b;m;a]) +$$

$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[b;n;a], S[b;m;a], S[b;k;a]) + \ldots +$$

$$+2^{N-1} g_{12\ldots N} P(S[b;1;a], S[b;2;a], ..., S[b;N;a]). \quad (35.1)$$

$$g_{n,m} = +1 \; or \; -1, or \; 0, n \neq m; \;\; g_{n,n} = +1, n = m; \;\; g_{nmk} = +1 \; or \; -1, or \; 0.$$

FIGURE 4

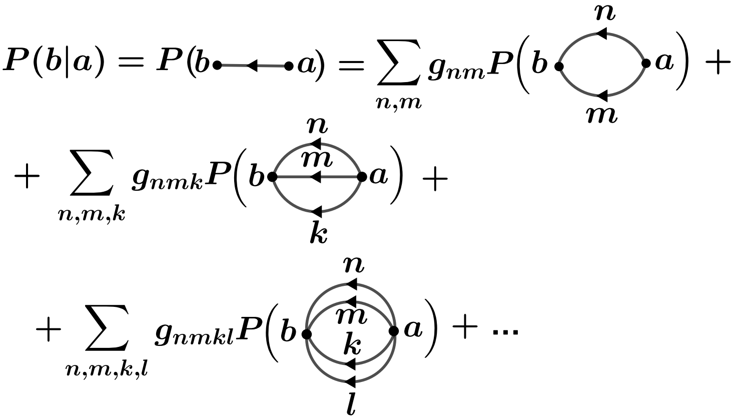

Figure 4

Probabilities of transition for in pairs joint events

The equation in space of the states of system becomes

$$P(b \mid a) = \sum_{n,m=1}^{N} g_{nm} P(S[b;n;a], S[b;m;a]) \tag{37.1}$$

.

$$g_{n,m} = +1 \ or \ -1, or \ 0, n \neq m; \ g_{n,n} = +1, n = m$$

Probabilities of transition for in pairs joint events

We do a postulate

$$P(S[b;n;a],S[b;m;a]) = g_{nm}P_f \mid \cos[S_{fni} - S_{fmi}] \mid \tag{38.1}$$

$$g_{nm} = \cos[S_{fni} - S_{fmi}] | \cos[S_{fni} - S_{fmi}] |^{-1} \tag{38.2}$$

Therefore

$$P(b \mid a) = \sum_{n,m=1}^{N} P_f \cos[S_{fni} - S_{fmi}]. \tag{38.3}$$

Results

1. The system of axioms of probability theory is supplemented by a new axiom on joint quantum random events. A symmetric sum of events is introduced.
2. In the new set of general events we have the equation for stochastic process

$$P(B_f \mid A_i) = \sum_{n=1}^{N} P(S_{fni}) + 2 \sum_{n<m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S_{fni} \cap S_{fmi} \cap S_{fki}) + \ldots + 2^{N-1} g_{12\ldots N} P(S_{f1i} \cap S_{f2i} \cap \ldots \cap S_{fNi}), \quad (39.1)$$

3. The equation in space of the states of system becomes

$$P(b \mid a) = \sum_{n,m=1}^{N} g_{nm} P(S[b;n;a], S[b;m;a]) +$$
$$+4 \sum_{n<m<k=1}^{N} g_{nmk} P(S[b;n;a], S[b;m;a], S[b;k;a]) + \ldots +$$
$$+2^{N-1} g_{12\ldots N} P(S[b;1;a], S[b;2;a], \ldots, S[b;N;a]). \quad (39.2)$$

Results

4. For the case when events in the process are only joint in pairs

$$P(S_{fni} \cap S_{fmi} \cap S_{fki}) = 0, \;\; P(S_{f1i} \cap S_{f2i} \cap ... \cap S_{fNi}) = 0$$

The equation has the form
$$P(B_f \cap A_i) = \sum_{n=1}^{N} P(S_n) + 2 \sum_{n>m=1}^{N} g(S_n, S_m) P(S_n \cap S_m) \tag{40.1}$$

The equation in space of the states of system becomes

$$P(b \mid a) = \sum_{n,m=1}^{N} P_f \cos[S[b; n; a] - S[b; m; a]]. \tag{40.2}$$

This equation describes real quantum processes (particle diffraction, quantum transitions, and others).

Thanks for your attention!