Hadron Spectra in Strong Magnetic Fields

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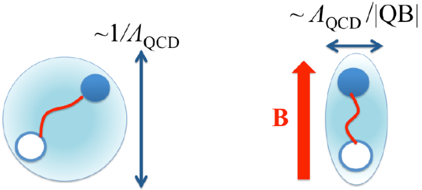
Talk by M.A. Andreichikov

QCD & QED in Strong Magnetic Field

- Strong magnetic fields - hadrons internal structure changes
- Spin and Isospin symmetries are broken
- $eB \sim \sigma \sim 10^{19}$ *Gauss* - string tension



Strong Magnetic Fields in Nature(in Gauss):

- Atomic ($l_B = 1/\sqrt{eB} = a_{Bohr}$) - $2.35 \cdot 10^9$
- Schwinger ($eB = m^2 e^3$) - $4.4 \cdot 10^{13}$
- Surface of magnetars - 10^{14}
- RHIC and LHC - $10^{18} - 10^{20}$

Systems in Mafnetic Field

- Positronium (Shabad, Usov) - Bethe-Salpeter approach
- Hydrogen (..., Khriplovich, Popov-Karnakov, Vysotsky-Godunov-Machet)
- ρ-meson (..., Mueller, Chernodub)
- Pion gas (Smilga, Agasian)
- Quark matter (Kharzeev-McLerran-Polikarpov-Zakharov..., Andreichikov-Kerbikov)
- Neutral and charged quark-antiquark systems (ρ, π) (Andreichikov-Kerbikov-Orlovsky-Simonov)
- Neutral and charged baryons (in progress)
- β-decay (Matese, O'Connel, Studenikin)
- Excitons (solid state physics) (Gorkov, Dzyaloshinskii)

The common features:

- 3d $\rightarrow$ 1d dimension reduction
- Coulomb-type interaction screening
- Sphere transforms to ellipsoid for ground state.

Papers related to the Problem:

- Yu. Simonov **"Relativistic path integral and relativistic Hamiltonians in QCD and QED"**, PRD 88, 025028 (2013)
- Yu. Simonov **"Spin interactions in mesons in strong magnetic field"**, PRD 88, 053004 (2013)
- M. Andreichikov, B. Kerbikov, V. Orlovsky, Yu. Simonov **"Meson spectrum in strong magnetic fields"**, PRD 87, 094029 (2013)
- M. Andreichikov, V. Orlovsky, Yu. Simonov **"Asymptotic Freedom in Strong Magnetic Fields"**, PRL 110, 162002 (2013)
- A. Badalian, Yu. Simonov **"Magnetic moments of mesons"**, PRD 87, 074012 (2013)
- M. Andreichikov, B. Kerbikov, V. Orlovsky, Yu. Simonov **"Neutron in strong magnetic field"**, PRD 89, 074033 (2014)
- V. Orlovsky, Yu. Simonov **"Nambu-Goldstone mesons in strong magnetic field"**, JHEP 2013:9:136 (2013)
- Yu. Simonov **"Magnetic focusing in atomic, nuclear and hadronic processes"**, arXiv:1308.5553 (2013)
- M. Andreichikov, B. Kerbikov, Yu. Simonov **"Magnetic Field Focusing of Hyperfine Interaction in Hydrogen"**, arXiv:1304.2516 (2013)
- M. Andreichikov, B. Kerbikov, Yu. Simonov **"Quark-Antiquark System in Ultra-Intense Magnetic Field"**, arXiv:1210.0227 (2012)

Plan of the Talk

- **Green's functions**: QCD & QED in magnetic field (for mesons), correlators and relativistic Hamiltonian.
- **Relativistic Hamiltonians**:
  - dynamics, confinement, magnetic moments
  - wave function factorization
  - zero modes
- **Perturbative corrections**:
  - self-energy corrections
  - color Coulomb & $q\bar{q}$ screening in nagnetic field
  - hyperfine interactions, magnetic "focusing" in Hydrgen atom and "magnetic collapse"
- **Meson mass spectrum.**
- **Baryon features (OPE, spin-isospin splitings)**
- **Neutron mass spectrum.**
- **Conclusions and discussion.**

Feynman-Fock-Schwinger Formalism:

Single quark Green's function ($\hat{A}$ - gluon field, $\hat{A}^{(e)}$ - EM field.)

$$D_i(x,y) = (m_i + \hat{\partial} - ig\hat{A} - ie_i\hat{A}^{(e)})^{-1}_{xy} = (m_i + \hat{D}^{(i)})^{-1}_{xy}$$

Feynman-Fock-Schwinger representation

$$D_i(x,y) = (m_i - \hat{D}_i)\int_0^\infty ds_i (Dz)_{xy} e^{-K_i}\Phi^{(i)}_\sigma(x,y) = (m_i - \hat{D}_i)G_i(x,y)$$

m_i - quark mass, s_i - proper time

$$K_i = m_i^2 s_i + \frac{1}{4}\int_0^{s_i} d\tau_i \left(\frac{\mathrm{d}z^{(i)}_\mu}{\mathrm{d}\tau_i}\right)^2$$

Gauge Fields in FSF Formalism::

Field-dependent term:

$$\Phi_{\sigma}^{(i)}(x,y) = P_A P_F \exp\left(ig\int_y^x A_\mu dz_\mu^{(i)} + ie_i \int_y^x A_\mu^{(e)} dz_\mu^{(i)} \right) \times$$

$$\times \exp\left(\int_0^{s_i} d\tau_i \sigma_{\mu\nu}(gF_{\mu\nu} + eB_{\mu\nu}) \right)$$

(4×4) structures for gluon and EM field:

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{H} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{H} \end{pmatrix},\ \sigma_{\mu\nu}B_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & 0 \\ 0 & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix}$$

If only magnetic field $B \neq 0$ - euclidean action - no negative eigenvalues (Stablisation theorem)

QQ Green's Function:

$$G_{q_1\bar{q}_2}(x,y) = \langle j_\Gamma(x) j_\Gamma(y) \rangle$$

And the path integral for it is ($\hat{T}$ contains gamma matrices trace):

$$G_{q_1\bar{q}_2}(x,y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz^{(1)})(Dz^{(2)}) \langle \hat{T} W_\sigma(A) \rangle_A \times$$

$$\times \exp\left(ie_1 \int_y^x A_\mu^{(e)} dz_\mu^{(1)} - ie_2 \int_y^x A_\mu^{(e)} dz_\mu^{(2)} + e_1 \int_0^{s_1} d\tau_1 (\boldsymbol{\sigma}\mathbf{B}) - e_2 \int_0^{s_2} d\tau_2 (\boldsymbol{\sigma}\mathbf{B}) \right)$$

$$\times \exp(-K_1 - K_2)$$

Gluon contribution(**Wilson loop**)
after averaging over stochastic vacuum background:

$$\langle W_\sigma(A) \rangle_A = \exp\left(-\int_0^{\tau_E} dt_E \left[\sigma |\mathbf{z}_1 - \mathbf{z}_2| - \frac{4}{3} \frac{\alpha_s}{|\mathbf{z}_1 - \mathbf{z}_2|} \right] \right)$$

Confinement + OGE (color Coulomb) (Minimal area for the Wilson loop)

Omega Formalism:

Monotonous time $t_E(\tau) = x_4 + \frac{\tau}{s} T$ provides:

$$z_4(\tau) = t_E(\tau) + \Delta z_4(\tau),\ \omega_i = \frac{T}{2s_i},\ \ T = |\mathbf{x} - \mathbf{y}|$$

Green's function with omega-variables (after averaging)

$$G_{q_1\bar{q}_2}(x, y) = \frac{T}{8\pi} \int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}} \frac{d\omega_2}{\omega_2^{3/2}} \left\langle \mathbf{x} \left| Tr(\hat{T} e^{-H_{q_1\bar{q}_2} T} \right| \mathbf{y} \right\rangle$$

Mass spectrum from Hamiltonian: (stationary point anaysis)

$$\hat{H}\psi = M_n \psi,\ \ \frac{\partial M_n(\omega_i)}{\partial \omega_i} = 0$$

Omegas are quark "dynamical" masses

Hamiltonian for $Q\bar{Q}$ Neutral Meson

$$H_{q\bar{q}} = \frac{1}{2\omega_1}(\mathbf{p}_1 - e\mathbf{A}(\mathbf{z_1}))^2 + \frac{1}{2\omega_1}(\mathbf{p}_2 + e\mathbf{A}(\mathbf{z_2}))^2 + \sigma|\mathbf{z}_1 - \mathbf{z}_2| +$$

$$+ \frac{m_1^2 + \omega_1^2 - e\boldsymbol{\sigma}^{(1)}\mathbf{B}}{2\omega_1} + \frac{m_2^2 + \omega_2^2 + e\boldsymbol{\sigma}^{(2)}\mathbf{B}}{2\omega_2}$$

Dynamics (mass & wave function) is defined by nonperturbative part:

$$H_{q\bar{q}}\Psi_n = M_n\Psi_n$$

Perturbative effects are treated as **corrections**:

$$M_{total} = M_n + \langle\Psi|V_{OGE}|\Psi\rangle + \langle\Psi|V_{SS}|\Psi\rangle + \Delta M_{SE}$$

Color Coulomb term V_{Coul}, Self-Energy V_{SE} (Wilson loop integration) and Spin-spin interaction V_{SS} are treated as perturbation.

Insight from classical equations:

$m_1 = m_2, \; q_1 = -q_2$ domain (simplest case)

$$m\frac{\mathrm{d}^2\mathbf{x}_1}{\mathrm{d}t^2} = f_{12} + q\frac{\mathrm{d}\mathbf{x}_1}{\mathrm{d}t} \times \mathbf{B};$$

$$m\frac{\mathrm{d}^2\mathbf{x}_2}{\mathrm{d}t^2} = -f_{12} - q\frac{\mathrm{d}\mathbf{x}_2}{\mathrm{d}t} \times \mathbf{B}$$

New coordinates $\mathbf{R}$, $\boldsymbol{\eta}$:

$$\mathbf{R} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}; \; \boldsymbol{\eta} = \mathbf{x}_1 - \mathbf{x}_2$$

Transformed equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(2m\frac{\mathrm{d}\mathbf{R}}{\mathrm{d}t} - q\boldsymbol{\eta} \times \mathbf{B}\right) = 0;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m}{2}\frac{\mathrm{d}\boldsymbol{\eta}}{\mathrm{d}t} - q\mathbf{R} \times \mathbf{B}\right) = f_{12}$$

Quantum case in $m_1 = m_2,\ q_1 = -q_2$ domain (ansatz):

Canonical transformation:

$$\mathbf{R} = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2};\ \boldsymbol{\eta} = \mathbf{x}_1 - \mathbf{x}_2;\ \hat{\mathbf{P}} = -i\frac{\partial}{\partial \mathbf{R}} = \hat{\mathbf{p}}_1 + \hat{\mathbf{p}}_2;\ \hat{\boldsymbol{\pi}} = -i\frac{\partial}{\partial \boldsymbol{\eta}} = \frac{\hat{\mathbf{p}}_1 - \hat{\mathbf{p}}_2}{2}$$

Hamiltonian:

$$\hat{H} = \frac{1}{4m}\left(\hat{\mathbf{P}} - \frac{1}{2}q\mathbf{B} \times \boldsymbol{\eta}\right)^2 + \frac{1}{m}\left(\hat{\boldsymbol{\pi}} - \frac{1}{2}q\mathbf{B} \times \mathbf{R}\right)^2 + V(\boldsymbol{\eta})$$

Integral of motion:

$$\hat{\mathbf{I}} = \hat{\mathbf{P}} + \frac{1}{2}q\mathbf{B} \times \boldsymbol{\eta};\ [\hat{\mathbf{I}}, \hat{H}] = 0$$

Wavefunction ansatz (Bilocal phase):

$$\Psi(\mathbf{R}, \boldsymbol{\eta}) = \phi(\boldsymbol{\eta}) e^{i\mathbf{P}\mathbf{R} - i\frac{1}{2}q(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}}$$

References:

- J.E.Avron, I.W.Herbst, B.Simon, Ann. Phys. **114**, 431 (1978)
- D.Koller, M.Malvetti, H.Pilkuhn, Phys. Lett. A **132**, 5 (1988)

Quantum case in $m_1 = m_2,\ q_1 = -q_2$ domain (solution):

Harmonic oscillator problem:

$$\frac{1}{4m}\left(\mathbf{P} - q\mathbf{B}\times\boldsymbol{\eta}\right)^2\phi - \frac{1}{m}\frac{\partial^2\phi}{\partial\boldsymbol{\eta}^2} + V(\boldsymbol{\eta})\phi = E\phi$$

External oscillator potential: $V(\boldsymbol{\eta}) = m\omega_E^2\eta^2$ – full factorization:

$$E = \Omega(n_x + n_y + 1) + \omega_E\left(n_z + \frac{1}{2}\right) + \frac{P_z^2}{4m} + \frac{P_x^2 + P_y^2}{4m}\frac{1}{1+\alpha} + \frac{\sigma}{4\beta}$$

$$\Omega = \omega_E\sqrt{(1+\alpha)};\ \alpha = \left(\frac{qB_z}{2m\omega_E}\right)^2$$

Remark:
In Coloumb potential case $V(\boldsymbol{\eta}) = -\frac{1}{|\boldsymbol{\eta}|}$ the rotational symmetry
References:

- M.Taut, Phys.Rev. A **48**, 5 (1994) (Isotropic oscillator + Coloumb)

More complicated case $m_1 \neq m_2, \; q_1 = -q_2$ domain (ansatz):

Canonical transformation:

$$\mathbf{R} = \frac{m_1 \mathbf{x}_1 + m_2 \mathbf{x}_2}{m_1 + m_2}; \;\; \boldsymbol{\eta} = \mathbf{x}_1 - \mathbf{x}_2;$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}; \;\; s = \frac{m_1 - m_2}{m_1 + m_2}$$

$$\hat{\mathbf{p}}_1 = \hat{\boldsymbol{\pi}} + \frac{\mu}{m_2}\hat{\mathbf{P}}; \;\; \hat{\mathbf{p}}_2 = -\hat{\boldsymbol{\pi}} + \frac{\mu}{m_1}\hat{\mathbf{P}}$$

Hamiltomian:

$$\hat{H} = \frac{1}{2\mu}\left(\hat{\boldsymbol{\pi}} - \frac{1}{2}\mathbf{B} \times \mathbf{R} + \frac{s}{2}\mathbf{B} \times \boldsymbol{\eta}\right)^2 + \frac{1}{2M}\left(\hat{\mathbf{P}} - \frac{1}{2}\mathbf{B} \times \boldsymbol{\eta}\right)^2 + V(\boldsymbol{\eta})$$

Integral of motion:

$$\hat{\mathbf{I}} = \hat{\mathbf{P}} + \frac{1}{2}\mathbf{B} \times \boldsymbol{\eta}$$

Wavefunction ansatz:

$$\Psi(\mathbf{R}, \boldsymbol{\eta}) = \phi(\boldsymbol{\eta}) e^{i\mathbf{P}\mathbf{R} - i\frac{1}{2}(\mathbf{B} \times \boldsymbol{\eta})\mathbf{R}}$$

References:

- J.E.Avron, I.W.Herbst, B.Simon, Ann. Phys. **114**, 431 (1978)

More complicated case $m_1 \neq m_2,\ q_1 = -q_2$ domain (solution):

$$H\phi = \frac{1}{2\mu}\left(-i\frac{\partial}{\partial\boldsymbol{\eta}} + \frac{s}{2}\mathbf{B}\times\boldsymbol{\eta}\right)^2\phi + \frac{1}{2M}\left(\mathbf{P} - \mathbf{B}\times\boldsymbol{\eta}\right)^2\phi + V(\boldsymbol{\eta})\phi = E\phi$$

In external harmonic oscillator case $V(\boldsymbol{\eta}) = \frac{1}{2}\mu\omega_E^2\eta^2$:

$$\eta'_x = \eta_x - \frac{\alpha}{1+\alpha}\frac{P_y}{B};\ \ \eta'_y = \eta_y + \frac{\alpha}{1+\alpha}\frac{P_x}{B};\ \ \eta'_z = \eta_z$$

$$\phi(\boldsymbol{\eta}) = \tilde{\phi}(\boldsymbol{\eta})e^{-i\frac{s}{2}\frac{\alpha}{1+\alpha}(P_x\eta'_x + P_y\eta'_y)}$$

After z-component factorization – Canonical two–oscillator Landau problem:

$$\frac{1}{2\mu}\left(\frac{\partial}{\partial i\eta'_x} - \frac{s}{2}B\eta'_y\right)^2\tilde{\phi} + \frac{1}{2\mu}\left(\frac{\partial}{\partial i\eta'_y} + \frac{s}{2}B\eta'_x\right)^2\tilde{\phi} + \frac{\mu\Omega^2}{2}(\eta'^2_x + \eta'^2_y)\tilde{\phi} = \left(E - \frac{P_x^2+P_y^2}{2M}\frac{1}{1+\alpha}\right)\tilde{\phi}$$

Energy:

$$E = \omega_1\left(n_1 + \frac{1}{2}\right) + \omega_2\left(n_2 + \frac{1}{2}\right) + \omega_E\left(n_z + \frac{1}{2}\right) + \frac{P_x^2+P_y^2}{2M}\frac{1}{1+\alpha} + \frac{P_z^2}{2M}$$

$$\omega_{1,2} = \frac{\sqrt{1+4\epsilon}\mp 1}{2}\omega_L;\ \ \omega_L = \frac{s}{2}\frac{B}{\mu};\ \ \epsilon = \left(\frac{\Omega}{\omega_L}\right)^2;$$

$$\alpha = \frac{B^2}{2M\mu\omega_E^2};\ \ \Omega = \omega_E\sqrt{1+\alpha};$$

Eigenvalue problem - Factorization ansatz

$$\hat{H} = \frac{1}{2\tilde{\omega}}\left(\hat{\boldsymbol{\pi}} - \frac{1}{2}\mathbf{B}\times\mathbf{R} + \frac{s}{2}\mathbf{B}\times\boldsymbol{\eta}\right)^2 + \frac{1}{2(\omega_1+\omega_2)}\left(\hat{\mathbf{P}} - \frac{1}{2}\mathbf{B}\times\boldsymbol{\eta}\right)^2 + \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma}+\gamma\right)$$

Integral of motion - **Pseudomomentum**:

$$\hat{\boldsymbol{\Lambda}} = \hat{\mathbf{P}} + \frac{1}{2}\mathbf{B}\times\boldsymbol{\eta}$$

Wave function ansatz:

$$\Psi(\mathbf{R},\boldsymbol{\eta}) = \phi(\boldsymbol{\eta})e^{i\boldsymbol{\Lambda}\mathbf{R} - i\frac{1}{2}(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}}$$

Note: confinement is treated as oscillator ($\sim$ 5% accuracy):

$$V_{conf} = \sigma\eta \to \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma}+\gamma\right);\ \frac{\partial M_n}{\partial\gamma} = 0$$

Wave Function for the Lowest State

$$\mathbf{\Lambda} = 0,\ (\mathbf{L}_\eta)_z = \left[\boldsymbol{\eta} \times \frac{\partial}{\partial i\boldsymbol{\eta}}\right]_z = 0$$

Hamiltonian for the lowest state:

$$H_0 = \frac{1}{2\tilde{\omega}}\left(-\frac{\partial^2}{\partial\boldsymbol{\eta}^2} + \frac{e^2}{4}(\mathbf{B}\times\boldsymbol{\eta})\right)^2 + \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma} + \gamma\right)$$

Lowest State Wave Function:

$$\psi(\boldsymbol{\eta}) = \frac{1}{\sqrt{\pi^{\frac{3}{2}} r_\perp^2 r_0}} \exp\left(-\frac{\eta_\perp^2}{2r_\perp^2} - \frac{\eta_z^2}{2r_0^2}\right)$$

Ellipsoid radii (contraction):

$$r_\perp = \sqrt{\frac{2}{eB}}\left(1 + \frac{4\sigma\tilde{\omega}}{\gamma e^2 B^2}\right)^{-\frac{1}{4}} \sim \frac{1}{\sqrt{eB}},\ r_0 = \left(\frac{\gamma}{\sigma\tilde{\omega}}\right)^{\frac{1}{4}} \sim \frac{1}{\sqrt{\sigma}}$$

Eigenvalues & Mass Dynamics

$$M_n(\omega_1, \omega_2, \gamma) = \varepsilon_{n_\perp, n_z} + \frac{m_1^2 + \omega_1^2 - eB\sigma_1}{2\omega_1} + \frac{m_2^2 + \omega_2^2 + eB\sigma_2}{2\omega_2}$$

$$\varepsilon_{n_\perp, n_z} = \frac{1}{2\tilde{\omega}} \left[\sqrt{(eB)^2 + \frac{4\sigma\tilde{\omega}}{\gamma}(2n_\perp + 1)} + \sqrt{\frac{4\sigma\tilde{\omega}}{\gamma}} \left(n_z + \frac{1}{2} \right) \right] + \frac{\gamma\sigma}{2}$$

For high magnetic field limit basis states are (all have **different** dynamics and ω-s):

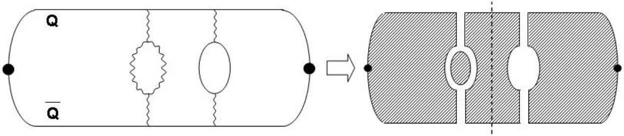
$$|++\rangle,\ |+-\rangle,\ |-+\rangle,\ |--\rangle$$

$$M_n^{++},\ M_n^{+-},\ M_n^{-+},\ M_n^{--}$$

Important role of Landau zero modes.
With $B \to \infty$, $n_\perp = n_z = 0$ (LLL):

$$M_0^{+-} \sim const,\ M_n^{++} \sim M_n^{--} \sim M_n^{-+} \sim \sqrt{eB}$$

Colour Coulomb and Asymptotic Freedom:



Gluon propagator with polarization operators

$$D(q) = \frac{4\pi}{q^2 - \frac{g^2(\mu_0^2)}{16\pi^2}(\Pi_{gl}(q) - \Pi_{q\bar{q}}(q))}$$

Standard QCD OGE potential without MF

$$V(Q) = -\frac{4}{3}\frac{4\pi\alpha_s^0}{q^2\left(1 + \frac{\alpha_s^0}{4\pi}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\ln\frac{q^2}{\mu_0^2}\right)}$$

$$\Pi_{gl}(q) = -\frac{11}{3}N_c q^2 \ln\frac{q^2}{\mu_0^2}, \quad \Pi_{q\bar{q}}(q) = -\frac{2}{3}n_f q^2 \ln\frac{q^2}{\mu_0^2}$$

Virtual $q\bar{q}$ pair at LLL in magnetic field (note, that $m^2 \sim \sigma$)

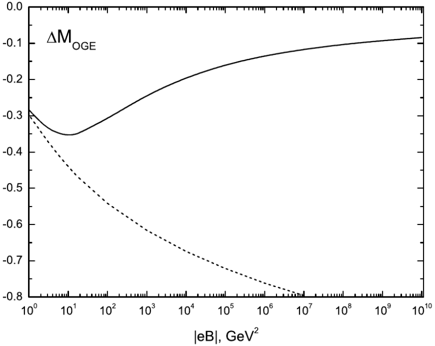
$$\frac{\alpha_s^0}{4\pi}\Pi_{q\bar{q}}(q) = -\frac{\alpha_s^0 n_f |e_q B|}{\pi}\exp\left(-\frac{q_\perp^2}{2|e_q B|}\right) T\left(\frac{q_3^2}{4m^2}\right)$$

Colour Coulomb and Asymptotic Freedom:

$$\Delta M_{OGE} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \hat{V}_{OGE} \psi^2(\mathbf{q})$$

Where screened (gluon loops and $q\bar{q}$ loops) potential is:

$$\hat{V}_{OGE} = \frac{16\pi\alpha_s}{3\left[Q^2\left(1+\frac{\alpha_s}{3\pi}\frac{11}{3}N_c \ln\frac{Q^2+M_B^2}{\mu_0^2}\right)+\frac{\alpha_s n_f |e_q B|}{\pi}\exp\left(\frac{-q_\perp^2}{2|e_q B|}\right) T\left(\frac{q_\perp^2}{4\sigma}\right)\right]}$$



Spin-dependent terms:

Quark Green function

$$S_q(x,y) = (m - D)(m^2 - D^2)^{-1}, \;\; m^2 - D^2 = m^2 - D_\mu^2 - g\sigma_{\mu\nu}F_{\mu\nu} - e\sigma_{\mu\nu}F^e_{\mu\nu}$$

Perturbation series in σF:

$$\frac{1}{m^2 - D^2 - g\sigma F} = \frac{1}{m^2 - D^2} + \frac{1}{m^2 - D^2} g\sigma F \frac{1}{m^2 - D^2} +$$

$$\frac{1}{m^2 - D^2} g\sigma F \frac{1}{m^2 - D^2} g\sigma F \frac{1}{m^2 - D^2} + \dots$$

Self-Energy contribution:

$$\Delta m^2 = -g\sigma F \frac{1}{m^2 - D^2} g\sigma F$$

For uniform magnetic field:

$$\Delta M_{SE} = -\frac{3\sigma}{2\pi\omega_i}(1 + \eta(eB))$$

Note, that for FSF representation SE and SS are given by spin-dependent correlators in Wilson loop.

Spin-Dependent Terms: Self-Energy Contribution

Spin-dependent contributions arise from $(\sigma^{(i)}F)(\sigma^{(j)}F)$ correlators.
$i = j$ - Self-Energy, $i \neq j$ - Spin-Spin.

$$\Delta M_{SE} = \sum_i \left[-\frac{3\sigma}{4\pi\omega_1} \left(1 + \eta \left(\lambda \sqrt{2e_i B + m_1^2} \right) \right) \right]$$

where $\eta(t)$ is

$$\eta(t) = i \int_0^\infty z^2 K_1(tz) e^{-z} dz$$

provides additional suppression for large dynamical quark masses ω_i

Spin-dependent terms: Spin-Spin(hyperfine) interaction:

$$V_{SS} = \frac{8\pi\alpha_s}{9\omega_1\omega_2}\delta(\mathbf{r})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

For ellipsoid wave function one has **magnetic focusing**

$$\delta(\mathbf{r}) \to \Psi^2(0) \sim eB$$

Unbounded growth! (magnetic collapse of QCD)
δ-term can't be treated in all orders by perturbation theory.
Smearing on the gluon background:

$$\delta(\mathbf{r}) \to \delta(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{\mathbf{r}^2/\lambda^2}, \ \lambda \sim 1 \ GeV^{-1}$$

(Tensor terms are neglected)

21cm line in Hydrogen

Gap between $|S=0, S_z=0\rangle$ and $|S=1, S_z=0\rangle$ in $1S$ Hydrogen state.

21cm line in H is a unique object:

- One of the best measured quantities in physics :
  $E_{hf} = 1420.405751767(1)\ MHz$ (Savely Karshenboim)
- Revolution in Radioastronomy (Purcell, 1951) - Doppler effect methods.
- Great sensivity to small deviations of parameters

Behaviour in Magnetic field

Deformation of spherical state $\rightarrow |\Psi(x)|^2$ changes $\rightarrow$ line shifts!

Experimental Studies:

Harvard maser 1 mHz up to 1000 G

Hyperfine Interaction - Interaction of Two Dipoles

$$B = 3\frac{(\boldsymbol{\mu_e}\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\boldsymbol{\mu_e}}{r^3} + \frac{8\pi}{3}\boldsymbol{\mu_e}\delta(\mathbf{r})$$

Energy is:

$$H = -(\boldsymbol{\mu}_p \cdot \mathbf{B}_e)$$

And the final equation is:

$$\hat{H}_{hf} = \frac{\alpha g_p}{mM}\left[\frac{8\pi}{3}|\Psi(0)|^2 + \int d^3\mathbf{r}|\Psi(\mathbf{r})|^2\left(\frac{3(\mathbf{s}_e\cdot\mathbf{r})(\mathbf{s}_p\cdot\mathbf{r})}{r^5} - \frac{(\mathbf{s}_e\cdot\mathbf{s}_p)}{r^3}\right)\right]$$

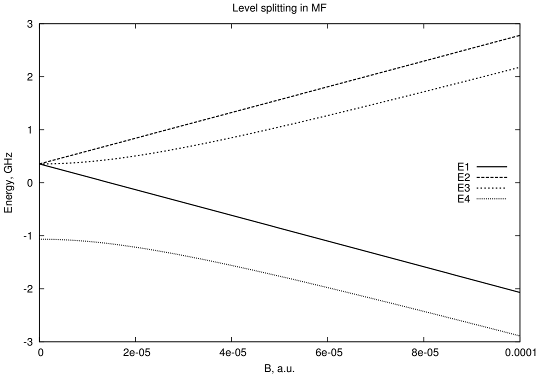
Tensor term is zero at $B = 0$
Estimate for the energy is:

$$E_{hf} \sim \alpha^2\frac{m}{M}R_\infty$$

Classic Zeeman effect

$$\hat{H}_{spin} = A(\boldsymbol{\sigma}_e \cdot \boldsymbol{\sigma}_p) + \frac{e}{2m}(\boldsymbol{\sigma}_e \cdot \mathbf{B}) - \frac{eg}{2M}(\sigma_p \cdot \mathbf{B})$$

$$A = \frac{2\pi}{3}\frac{\alpha g}{mM}|\Psi(0)|^2 = \frac{2}{3}\alpha^4 g \frac{m}{M} m$$



From $|S=0\rangle$, $|S=1\rangle$ to $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Hydrogen atom: variational approach

Hamitonian for LLL $\hat{l}_z|\psi_0\rangle = 0$:

$$\hat{\mathcal{H}} = -\frac{1}{2m}\left(\Delta_\perp + \frac{\partial^2}{\partial z^2}\right) + \omega \hat{l}_z + \frac{m\omega^2\rho^2}{2} - \frac{\alpha}{\sqrt{\rho^2+z^2}} + \mu_B \sigma_z B$$

Trial wavefunction:

$$\psi_0(\mathbf{x}) = \sqrt{A} e^{-\frac{x^2+y^2}{2r_\perp^2} - \frac{z^2}{2r_z^2}}, \quad A = (\pi^{3/2} r_\perp^2 r_z)^{-1}$$

Variational procedure:

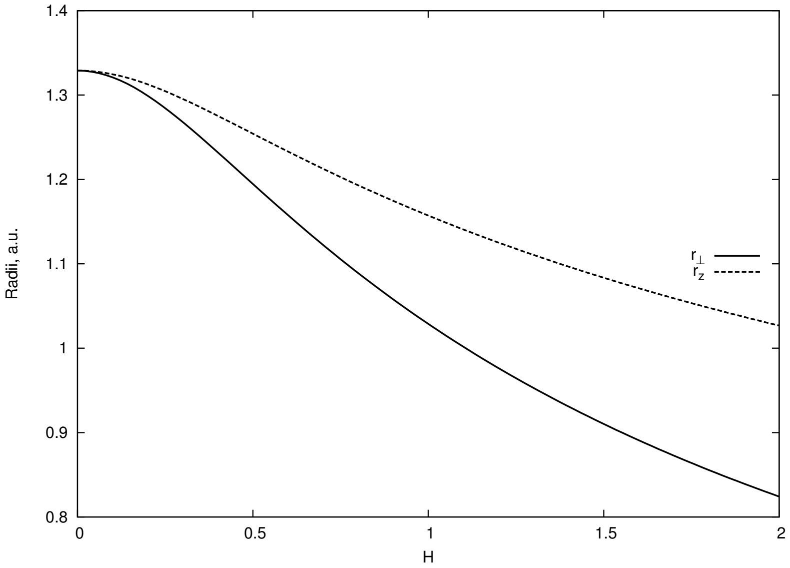
$$E_0(r_\perp, r_z) = \langle \psi_0 | \hat{H}(r_\perp, r_z) | \psi_0 \rangle, \frac{\partial E_0}{\partial r_\perp} = \frac{\partial E_0}{\partial r_z} = 0$$

Final analytical expression:

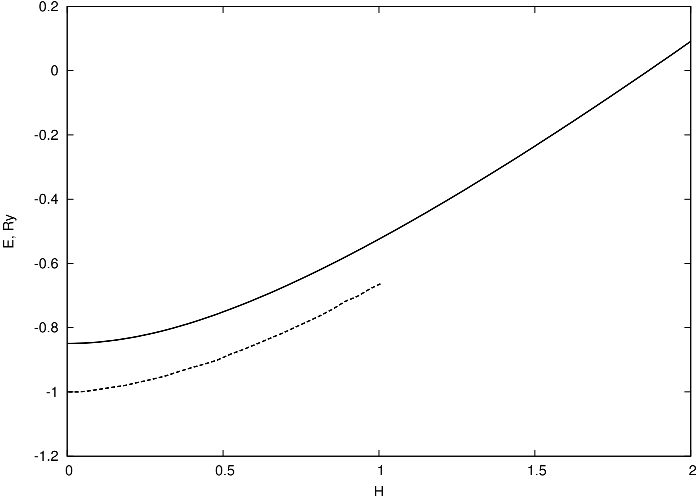
$$E_0(r_\perp, r_z) = \frac{1}{2mr_\perp^2}\left(1 + \frac{\beta^2}{2}\right) + \frac{m\omega^2 r_\perp^2}{2} - \frac{\alpha\beta}{r_\perp\sqrt{\pi(1-\beta^2)}} \ln \frac{1+\sqrt{1-\beta^2}}{1-\sqrt{1-\beta^2}}$$

The same expression was derived from the path integral with gaussian smearing in xy-plane with parameter $1/(eB)^{1/2}$ (M. Bachmann, H. Kleinert, A. Peltser, PRA62, 52509/1-21 (2000))

Hydrogen atom: radii



Hydrogen atom: ground state energy



dashed curve - H. Praddaude, Phys. Rev. A6, 1321 (1972)

Hydrogen atom: tensor part of the HF matrix element

How to work with tensor and delta parts simultaneously(Biot-Savart):

$$\mathbf{A} = \frac{\boldsymbol{\mu} \times \mathbf{r}}{r^3}, \ V_{SS} = \boldsymbol{\mu}\mathbf{B}$$

$$V_{SS} = 2 g_e g_p \mu_B^p \mu_B^e \left[- \left(\frac{\hat{s}_p^x \hat{s}_e^x}{r_\perp^2} I_1 + \frac{\hat{s}_p^y \hat{s}_e^y}{r_\perp^2} I_2 + \frac{\hat{s}_p^z \hat{s}_e^z}{r_z^2} I_3 \right) + (\hat{\mathbf{s}}_e \cdot \hat{\mathbf{s}}_p) \left(\frac{I_1}{r_\perp^2} + \frac{I_2}{r_\perp^2} + \frac{I_3}{r_z^2} \right) \right]$$

These integrals could be calculated analytically:

$$I_1 = I_2 = \int \frac{x^2 \psi^2}{r^3} d^3x, \ I_3 = \int \frac{z^2 \psi^2}{r^3} d^3x$$

The final expression depends on $r_\perp \sim \frac{1}{\sqrt{H}}, \ r_z \sim \frac{1}{lnH}$:

$$V_{SS} = 2 g_e g_p \mu_B^p \mu_B^e \ \left[(F_1(H) + F_2(H))(\hat{\mathbf{s}}_e \cdot \hat{\mathbf{s}}_p) + (F_1(H) - F_2(H)) \hat{s}_e^z \hat{s}_p^z \right]$$

Hydrogen atom: tensor part of the HF matrix element (continue)

$$F_1(H) = \frac{1}{\sqrt{\pi} r_\perp^2 r_z} \left[\frac{2}{1-\beta^2} - \frac{\beta^2}{(1-\beta^2)^{3/2}} \ln \frac{1+\sqrt{1-\beta^2}}{1-\sqrt{1-\beta^2}} \right], \quad (1)$$

$$F_2(H) = \frac{2}{\sqrt{\pi} r_z^3} \left[-\frac{2}{1-\beta^2} + \frac{1}{(1-\beta^2)^{3/2}} \ln \frac{1+\sqrt{1-\beta^2}}{1-\sqrt{1-\beta^2}} \right]. \quad (2)$$

At $H \to 0$, $\beta \to 1$, $r_\perp = r_z = r$ one obtains:

$$F_1 = F_2 = F = \frac{4}{3\sqrt{\pi}} r^{-3} = \frac{4\pi}{3} |\Psi(0)|^2. \quad (3)$$

Finally, we have following asymptotics:$(H \gg 1)$

$$\beta \sim \frac{\ln H}{\sqrt{H}}, \; F_1 \sim H \ln H, \; F_2 \sim \sqrt{H} \ln^2 H. \quad (4)$$

Hydrogen atom: Hamiltonian diagonalization with tensor part

Full spin Hamiltonian:

$$\hat{H}_{ss} = 2g_e g_p \mu_B^p \mu_B^e \left[(F_1(H) + F_2(H))(\hat{\boldsymbol{\sigma}}_e \cdot \hat{\boldsymbol{\sigma}}_p) + (F_1(H) - F_2(H))\hat{\sigma}_e^z \hat{\sigma}_p^z \right] + g_p \mu_N \hat{\sigma}_p^z B - \mu_B \hat{\sigma}_e^z B$$

The point of interest is splitting between $|a\rangle = |S = 1, S_z = 0\rangle$ and $|b\rangle = |S = 0, S_z = 0\rangle$ levels.

$$\nu = E_a - E_b = \Delta E_{hfs} \sqrt{\gamma^2 + \left(\frac{2\mu_B B}{\Delta E_{hfs}}\right)^2 \left(1 + g\frac{m}{m_p}\right)^2}, \tag{5}$$

where

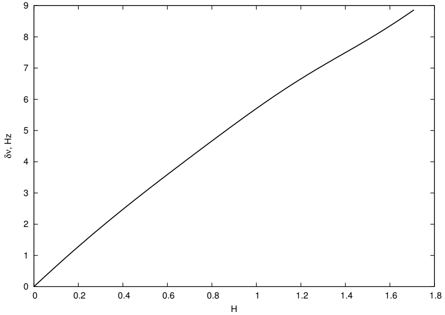
$$\Delta E_{hfs} = \frac{32\pi}{3} g_p \mu_B \mu_N |\Psi(0)|^2 \tag{6}$$

is classic hyperfine splitting,

$$\gamma = \frac{F_1 + F_2}{2F}$$

"Magnetic Focusing" term, for $H = 0, \ \gamma = 1$

Hydrogen atom: levels, splitting, effect



$$\delta\nu \simeq \alpha^6 \left(\frac{m}{m_p}\right) m(H \ln^2 H) \simeq 10^{-6}(H \ln^2 H)\ MHz,\ H \gg 1 \tag{7}$$

$$\delta\nu \simeq \Delta E_{hfs} \left(1 - \frac{r_\perp^2}{r_z^2}\right),\ H \ll 1\ (B < 100\ G) \tag{8}$$

Spin States Mixing for Meson

For $B = 0$:

$|S=0,\ m=0\rangle,\ |S=1,\ m=0,\pm1\rangle$

For $eB \gg \sigma$:

$|++\rangle,\ |+-\rangle,\ |-+\rangle,\ |--\rangle$

$$M_{11} = M_0^{+-} + \Delta M_{pert} - \langle a_{SS}^{+-}\rangle;\ M_{22} = M_0^{-+} + \Delta M_{pert} - \langle a_{SS}^{-+}\rangle$$

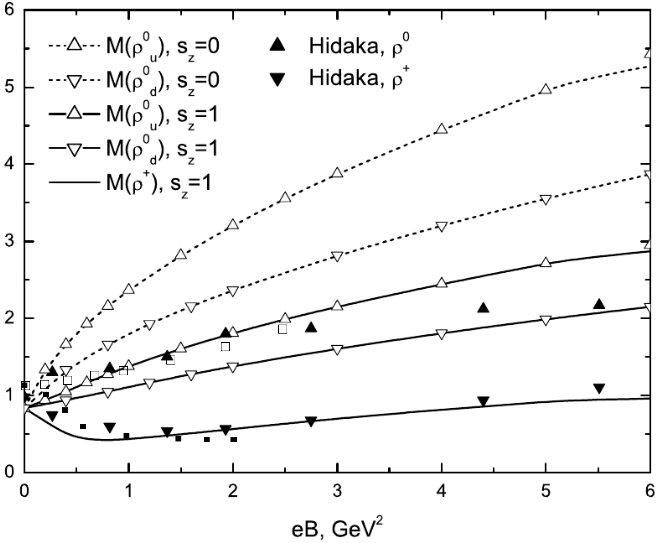
$$E_{1,2} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\left(\frac{M_{22} - M_{11}}{2}\right)^2 + 4a_{ss}^{+-}a_{ss}^{-+}}$$

$$E_3 = M_0^{++} + \Delta M_{pert} + a_{SS}^{++}$$

$$E_4 = M_0^{--} + \Delta M_{pert} + a_{SS}^{--}$$

Indices $+$ and $-$ are related to ω_i^{+-} etc.
$(a_{SS} = \langle\Psi|V_{SS}|\Psi\rangle)$

Meson Spectrum in Magnetic Field vs Lattice



Black squares - $\rho^0(s_z = 0)$, white squares $\rho^0(s_z = 1)$ (ITEP Lattice).

Neutral Baryon Hamiltonian and Factorization

Neutral baryon - $e_1 = e_2 = -e_3/2$ (Isospin broken i.e. $|ddu\rangle$)
Spin configurations in strong B are $|\pm\pm\pm\rangle$.

$$B \to \infty: \ |--+\rangle \sim const(zero\ mode),\ other\ |\pm\pm\pm\rangle \sim \sqrt{eB}$$

Factorization is possible only for $\omega_1 = \omega_2 = \omega$ (only $|++\pm\rangle$ & $|--\pm\rangle$ states).
Baryon dynamics is defined by the hamiltonian:

$$H_{qqq} = \sum_{i=1}^{3} \frac{(\mathbf{p}^{(i)} - e_i \mathbf{A}^{(i)})^2 + m_i^2 + \omega_i^2 + e_i \boldsymbol{\sigma}_i \mathbf{B}}{2\omega_i} + V_{conf}$$

$$V_{conf} = \sigma \sum_{i=1}^{3} |\mathbf{z}^{(i)} - \mathbf{z}_Y| \simeq 3\frac{\sigma}{2}\gamma + \frac{\sigma}{2\gamma} \sum (\mathbf{z}^{(i)} - \mathbf{R})^2$$

where $\mathbf{z}_Y$ - String junction (Torricelli) point ($\mathbf{z}_Y = \mathbf{R}$ for simplicty)

Jacobi coordinates

$$\mathbf{R} = \frac{1}{\omega_+} \sum \omega_i \mathbf{z}^{(i)}$$

$$\boldsymbol{\eta} = \frac{1}{\sqrt{2}} (\mathbf{z}^{(2)} - \mathbf{z}^{(1)})$$

$$\boldsymbol{\xi} = \sqrt{\frac{\omega_3}{2\omega_+}} (\mathbf{z}^{(1)} + \mathbf{z}^{(2)} - 2\mathbf{z}^{(3)})$$

Canonical conjugate momenta:

$$\mathbf{P} = -i \frac{\partial}{\partial \mathbf{R}}, \ \mathbf{q} = -i \frac{\partial}{\partial \boldsymbol{\xi}}, \ \boldsymbol{\pi} = -i \frac{\partial}{\partial \boldsymbol{\eta}}$$

Psudomomentum (Integral of Motion)($\hat{\mathbf{F}} \Psi = \mathbf{P} \Psi$):

$$\hat{\mathbf{F}} = i \frac{\partial}{\partial \mathbf{R}} - i \frac{e}{4} \sqrt{\frac{2\omega_+}{\omega_3}} (\mathbf{B} \times \boldsymbol{\xi})$$

Factorization Ansatz:

$$\Psi(\mathbf{R}, \boldsymbol{\xi}, \boldsymbol{\eta}) = \phi(\boldsymbol{\xi}, \boldsymbol{\eta}) e^{i\mathbf{P}\mathbf{R} + i\frac{e}{4}\sqrt{\frac{2\omega_+}{\omega_3}}(\mathbf{B} \times \boldsymbol{\xi})\mathbf{R}}$$

Baryon Hamiltonian - Jacobi Coordinates:

$$\mathbf{P}=0,\ L_z^{\xi}=0,\ L_z^{\eta}=0$$

$$\boldsymbol{\xi}\times\frac{\partial}{\partial i\boldsymbol{\xi}}=\mathbf{L}^{\xi},\ \boldsymbol{\eta}\times\frac{\partial}{\partial i\boldsymbol{\eta}}=\mathbf{L}^{\eta}$$

Hamiltonian:

$$H_{qqq}=-\frac{1}{2\omega}(\Delta_{\xi}+\Delta_{\eta})+\frac{1}{2\omega}\left(\frac{eB}{4}\right)^2\left(\frac{\omega_+^2}{\omega_3^2}(\boldsymbol{\xi}_\perp)^2+(\boldsymbol{\eta}_\perp)^2\right)+\frac{e\mathbf{B}}{4\omega}\left(\frac{\omega_3-2\omega}{\omega_3}\mathbf{L}^{\xi}+\mathbf{L}^{\eta}\right)$$

$$+\sum\frac{m_i^2+\omega_i^2+e_i\boldsymbol{\sigma}_i\mathbf{B}}{2\omega_i}+V_{conf}$$

Confinement term:

$$V_{conf}=\frac{\sigma\gamma}{2}+\frac{\sigma}{2\gamma}\left(\frac{\omega_3^2+2\omega^2}{\omega_+\omega_3}\boldsymbol{\xi}^2+\boldsymbol{\eta}^2\right)$$

Mass spectrum and Wave Function for $|--+\rangle$:

$$\frac{M_0}{\sqrt{\sigma}}=\Omega_{\xi\perp}+\Omega_{\eta\perp}+\frac{1}{2}(\Omega_{\xi z}+\Omega_{\eta z})+\frac{3\sqrt{\sigma}\gamma}{2}+\frac{m_d^2+\omega^2-(e/2)B}{\omega\sqrt{\sigma}}+\frac{m_u^2+\omega_3^2-eB}{2\omega_3\sqrt{\sigma}}$$

Wave function (ξ & η full separation):

$$\Psi(\boldsymbol{\eta},\boldsymbol{\xi})=\psi_1(\xi_\perp)\psi_2(\xi_z)\psi_3(\eta_\perp)\psi_4(\eta_z)$$

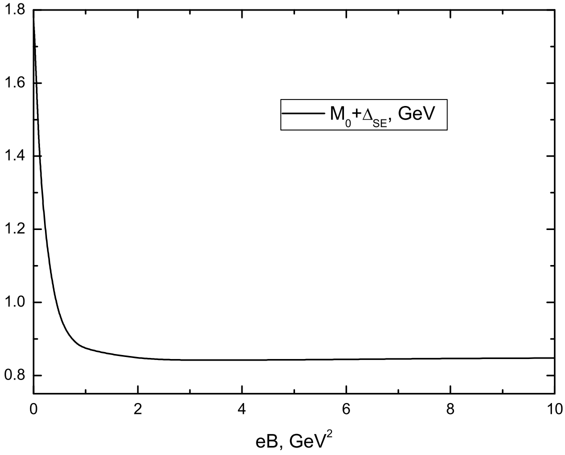
$$\psi_1(\xi_\perp)=\frac{1}{\sqrt{\pi r_{\xi\perp}^2}}\exp\left(-\frac{\xi_\perp^2}{2r_{\xi\perp}^2}\right),\quad \psi_2(\xi_z)=\frac{1}{(\pi r_{\xi z}^2)^{1/4}}\exp\left(-\frac{\xi_z^2}{2r_{\xi z}^2}\right)$$

$$\psi_3(\eta_\perp)=\frac{1}{\sqrt{\pi r_{\eta\perp}^2}}\exp\left(-\frac{\eta_\perp^2}{2r_{\eta\perp}^2}\right),\quad \psi_4(\eta_z)=\frac{1}{(\pi r_{\eta z}^2)^{1/4}}\exp\left(-\frac{\eta_z^2}{2r_{\eta z}^2}\right)$$

In strong $B\to\infty$ asymptotics:

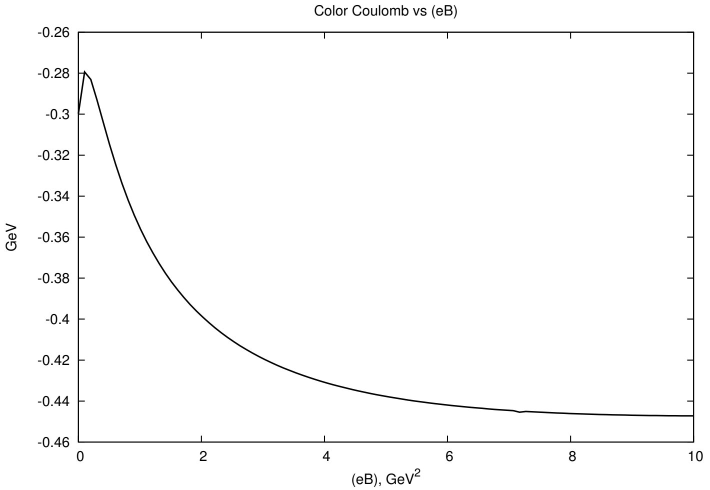
$$r_{\xi\perp}\sim r_{\eta\perp}\sim\frac{1}{\sqrt{eB}},\quad r_{\xi z}\sim r_{\eta z}\sim\frac{1}{\sqrt{\sigma}}$$

Dynamical Mass M_0 + Self-Energy Correction for $|--+\rangle$



$eB > 2$ GeV - saturation, zero mode works.

Color Coulomb Correction with Screening for Baryon



Hyperfine - Spin-Spin +OPE Corrections

Spin-Spin

$$V_{SS} = b(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + d(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3) + d(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3)$$

Where b & d are:

$$b = \frac{8\alpha_s}{9\omega_1\omega_2}\langle\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle, \;\; d = \frac{8\alpha_s}{9\omega_{1,2}\omega_3}\langle\delta(\mathbf{r}_{1,2} - \mathbf{r}_2)\rangle$$

Delta-function are smeared as for mesons
Average over the lowest state $|--+\rangle$:

$$\Delta M_{SS} = \langle - - + | V_{SS} | - - + \rangle = b - 2d$$

$$\Delta M_{SS} \simeq 50 \;\; MeV \;\; for \;\; B = 0$$

(Instead of $\Delta M_{SS} \simeq 300 \;\; MeV$
for $|S = 1/2, m = -1/2\rangle$ and $|S = 3/2, m = -1/2\rangle$)

One-Pion -Exchange Correction

$$V_{OPE}^{ij} = \frac{4\pi g^2}{\omega_i \omega_j} \left[\frac{(\boldsymbol{\sigma}_i \cdot \mathbf{k})(\boldsymbol{\sigma}_j \cdot \mathbf{k})}{k^2 + m_{\pi^+}^2} 2\tau_+^i \tau_-^j + \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{k})(\boldsymbol{\sigma}_j \cdot \mathbf{k})}{k^2 + m_{\pi^-}^2} 2\tau_-^i \tau_+^j + \right.$$

$$\left. \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{k})(\boldsymbol{\sigma}_j \cdot \mathbf{k})}{k^2 + m_{\pi^0}^2} \tau_3^i \tau_3^j \right] \left(\frac{\Lambda^2}{k^2 + \Lambda^2} \right)^2$$

Nuclear forces change signs and magnitudes:

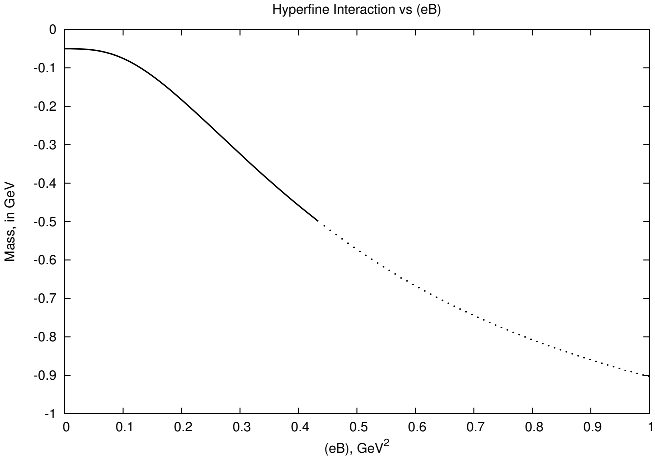
- Spin violation
- Isospin violation
- Pion masses changes with B $m_{\pi^\pm} \sim \sqrt{eB}, \ m_{\pi^0} \sim const$

For neutron in $B > \sigma$: $|ddu\rangle$, $|--+\rangle$ coupling constant:

$$\alpha_{hf} = \alpha_{ss} + \alpha_{ope}$$

(only π^0 exchange in $B \to \infty$ limit)

Hyperfine Correcion: Results



Solid - calculation, Dotted - perturbation theory is inoperable
"Magnetic QCD collapse" during perturbation theory(magnetic "focusing")

Spin State Mixing for $B < \sigma$(Isospin $|ddu\rangle$)

To calculate spin splittings, i.e. $n - \Delta^0$, diagonalize full Hamiltonian

$$H_\sigma = b(\sigma_1 \cdot \boldsymbol{\sigma}_2) + d(\sigma_2 \cdot \boldsymbol{\sigma}_3) + d(\sigma_1 \cdot \boldsymbol{\sigma}_3) - c_3\sigma_{3z} + c(\sigma_{1z} + \sigma_{2z})$$

$$c = \frac{eB}{4\omega}, \; c_3 = \frac{eB}{2\omega_3}$$

Basis for $m = -1/2$ spin projection:

$$\Psi_s = \alpha|--+\rangle + \frac{\beta}{\sqrt{2}}(|+--\rangle + |-+-\rangle)$$

Splitting is:

$$M_\pm = \frac{E_{11} + (b-2d)_{11} + E_{22} + b_{22}}{2} \pm$$

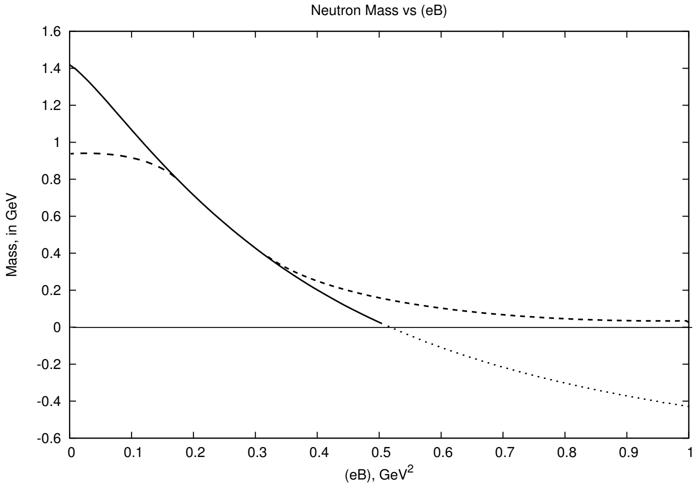
$$\pm\sqrt{\frac{(E_{22} + b_{22} - E_{11} - (b-2d)_{11})^2}{4} + 8d_{12}d_{21}}$$

Important:

b_{ij} and d_{ij} contain different ω's due to different dynamics of $|-+-\rangle$, $|+--\rangle$ and $|--+\rangle$

Only $|--+\rangle$ mass is falls with eB, masses of the other states grow!

Hypothetical Mass of Neutron in Magnetic Field



Solid - calculations, Dashed $0 - 0.15\ GeV$ - mixing of states, Dashed $> 0.35\ GeV$ - behaviour according stabilization theorem, Dotted - perturbation theory is inoperable

Conclusions

What's done:

- Relativistic path integral formalism was adopted for QCD + QED in strong magnetic fields .
- Meson and baryon spectra in magnetic field were obtained.
- Inoperability of perturbation theory for the Fermi-contact-like interactions in strong magnetic fields was formulated(revisited)
- Meson magnetic moments were calculated
- Hamiltonians with C.M. factorization for neutral 2- and 3-body systems were obtained.
- OGE screeining with $q\bar{q}$-loops was considered.

What's next?

- Stochastic EM fields in condensed matter.
- Thermodynamics and phase transitions in strong magnetic field
- Nuclear forces and nuclear forces in magnetic field
- ...