Self-consistent resolution of the particles with structure scattering problem by unitarity's conserving

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References

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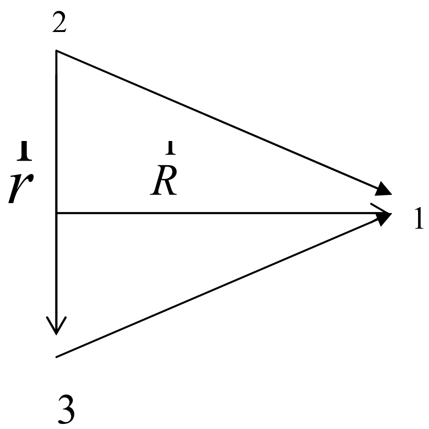
This report presents a method of Shrodinger equation for scattering a particle on the two-body bound system's solution and the complex potential's reconstruction using the special form for the conserving unitarity wave function. It is sequential of [1]

1. Schrodinger equation for scattering a particle on the two-body bound system with the complex potential

We start from Schrodinger equation for scattering a particle 1 with the mass m_1 and the momentum $\vec{k}$ in c.m. on the two particles bound system (2, 3) with masses (m_2, m_3)



In c.m. by using Jacobi coordinates $\vec{r} = \{x, y, z\} = \vec{r}_2 - \vec{r}_3$, $\vec{R} = \{X, Y, Z\} = \vec{r}_1 - \vec{R}_{23}$, where $\vec{R}_{23} = \dfrac{m_2\vec{r}_2 + m_3\vec{r}_3}{m_2 + m_3}$, Schrodinger equation can be written in the form

$$[-\frac{\hbar}{2M}\Delta_{\vec{R}} - \frac{\hbar}{2\mu}\Delta_{\vec{r}} + V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r}) + V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r}) + V_{23}(\vec{r})]\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r}) . \qquad (1)$$

μ and M are reduced masses in the systems of particles (2,3) and [1,(2,3)] accordingly. The potentials V_{12} and V_{13} are in principle complex but V_{23} is the pure real functions.
Well known methods of the Shrodinger equation with complex potential's solution as method of partial waves or eikonal do not conserve the norm of function.

The Eq. (1) depends on two variables $\vec{r}$ and $\vec{R}$.

Suppose the bound system of particles (2, 3) as $|\vec{R}| \to \infty$ is described by the equation

$$[-\frac{\hbar}{2\mu}\Delta_{\vec{r}} + V_{23}(\vec{r})]\Phi_0(\vec{r}) = E_0\Phi_0(\vec{r}) \qquad (2)$$

$\Phi_0(\vec{r})$ is the ground state normalized solution of this equation

at energy E_0. Choose the wave function $\Psi(\vec{r},\vec{R})$ in the optical spirit

$$\psi(\vec{R},\vec{r}) = e^{if_k(\vec{R},\vec{r})}\Phi_0(\vec{r}) \qquad (3)$$

where $f_k(\vec{R},\vec{r})$ is <u>pure real</u> function. Substituting eq.(3) in eq.(1), differentiating $\Psi(\vec{r},\vec{R})$ in $\vec{r}$ we get

$$\{-\frac{\hbar}{2M}\Delta_{\vec{R}} - \frac{\hbar}{2\mu}\Delta_{\vec{r}} f(\vec{R},\vec{r}) - i\frac{\hbar}{\mu}\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r})\cdot\vec{\nabla}_{\vec{r}}\ln\Phi_0(\vec{r}) +$$
$$V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r}) + V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r}) + \frac{\hbar}{2\mu}[\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r})]^2\}\psi(\vec{R},\vec{r}) = \qquad (4)$$
$$(E - E_0)\psi(\vec{R},\vec{r})$$

We can consider the equation (4) as Schrodinger equation

$$\{-\frac{\hbar}{2M}\Delta_{\vec{R}} + V(\vec{R},\vec{r})\}\psi(\vec{R},\vec{r}) = (E - E_0)\psi(\vec{R},\vec{r}) \qquad (5)$$

with complex potential

$$V(\vec{R},\vec{r}) = v(\vec{R},\vec{r}) + iu(\vec{R},\vec{r}), \qquad (6)$$

where the real part is

$$v(\vec{R},\vec{r}) = \mathrm{Re}V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r}) + \mathrm{Re}V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r}) + \frac{\hbar}{2\mu}[\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r})]^2 \qquad (7)$$

and imaginary one

$$u(\vec{R},\vec{r}) = \mathrm{Im}\,V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r}) + \mathrm{Im}\,V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r})$$
$$-\frac{\hbar}{\mu}\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r}) \cdot \vec{\nabla}_{\vec{r}} \ln \Phi_0(\vec{r}) - \frac{\hbar}{2\mu}\Delta_{\vec{r}} f(\vec{R},\vec{r}) \quad (8)$$

The wave function $f_k(\vec{R},\vec{r})$ is included in full function (3). From (4) and (5) we see that information about the system (2,3) is contained only in the imaginary part of potential.

Consider the asymptotic case $|\vec{R}| >> |\vec{r}|$. We can expand in Maclaurin series the function $f_k(\vec{R},\vec{r})$ with respect to $\vec{r} = \{x, y, z\}$. For example this expansion up to third partial derivations with account the symmetry is

$$f(\vec{R},\vec{r}) = f(\vec{R}) + C(\vec{R})(x + y + z) + \frac{1}{2!}[B(\vec{R})(x^2 + y^2 + z^2)$$
$$+ 2\widetilde{B}(\vec{R})(xz + xy + yz)] + \frac{1}{3!}[D(\vec{R})(x^3 + y^3 + z^3) +$$
$$4\widetilde{D}(\vec{R})(x^2 y + xy^2 + y^2 z) + 18\widetilde{\widetilde{D}}(\vec{R})xyz] \quad (9)$$

where

$f(\vec{R},\vec{r})\Big|_{\vec{r}=0} = f(\vec{R})$, $f_x'(\vec{R},\vec{r})_{\vec{r}=0} = f_y'(\vec{R},\vec{r})_{\vec{r}=0} = f_z'(\vec{R},\vec{r})_{\vec{r}=0} = C(\vec{R})$,

$f_{xx}''(\vec{R},\vec{r})_{\vec{r}=0} = f_{yy}''(\vec{R},\vec{r})_{\vec{r}=0} = f_{zz}''(\vec{R},\vec{r})_{\vec{r}=0} = B(\vec{R})$,

$f_{xy}''(\vec{R},\vec{r})_{\vec{r}=0} = f_{xz}''(\vec{R},\vec{r})_{\vec{r}=0} = f_{yz}''(\vec{R},\vec{r})_{\vec{r}=0} = \widetilde{B}(\vec{R})$,

$f_{xxx}'''(\vec{R},\vec{r})_{\vec{r}=0} = f_{yyy}'''(\vec{R},\vec{r})_{\vec{r}=0} = f_{zzz}'''(\vec{R},\vec{r})_{\vec{r}=0} = D(\vec{R})$,

$$f'''_{xxy}(\vec{R},\vec{r})_{\vec{r}=0} = f'''_{yyx}(\vec{R},\vec{r})_{\vec{r}=0} = f'''_{zzx}(\vec{R},\vec{r})_{\vec{r}=0} = f'''_{xxz}(\vec{R},\vec{r})_{\vec{r}=0} = f'''_{yyz}(\vec{R},\vec{r})_{\vec{r}=0}$$
$$= f'''_{zzy}(\vec{R},\vec{r})_{\vec{r}=0} = \widetilde{D}(\vec{R})$$

$$f'''_{xyz}(\vec{R},\vec{r})_{\vec{r}=0} = \widetilde{\widetilde{D}}(\vec{R})$$

We obtain from Shrodinger equation (4) the system of two equations regards to the functions $f(\vec{R})$, $C(\vec{R})$, $B(\vec{R})$, $\widetilde{B}(\vec{R})$, $D(\vec{R})$, $\widetilde{D}(\vec{R})$, for example taking into consideration in those equations all members with (x, y, z) coordinate power up to one

$$\begin{aligned} & -i\frac{\hbar}{2M}[\Delta_{\vec{R}} f(\vec{R}) + \Delta_{\vec{R}} C(\vec{R})(x+y+z))] - \\ & i\frac{\hbar N}{2\mu a^2}[C(\vec{R})(x+y+z)] - i\frac{\hbar}{2\mu}[B(\vec{R}) + D(\vec{R})(x+y+z) + \\ & \frac{4}{3}\widetilde{D}(\vec{R})(x+y+z)] + i\,\mathrm{Im}(V_{12}+V_{13}) = 0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} & \frac{\hbar}{2M}[(\vec{\nabla}_{\vec{R}} f(\vec{R})^2 + 2\vec{\nabla}_{\vec{R}} f(\vec{R})\vec{\nabla}_{\vec{R}} C(\vec{R})(x+y+z)] + \\ & \frac{\hbar}{2\mu}[3C^2(\vec{R}) + C(\vec{R})(B(\vec{R}) + \widetilde{B}(\vec{R}))(x+y+z) + \\ & \mathrm{Re}(V_{12}+V_{13})] = (E-E_0) \end{aligned} \tag{11}$$

We can use Maclaurin series up to the first degrees of vector's $\vec{r}$ coordinates for potentials $V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r})$ and $V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r})$

Choosing the wave function $\Phi_0(\vec{r})$ for example in Gauss form

$$\Phi_0(\vec{r}) = Ne^{-\frac{r^2}{2a^2}} = \frac{a^{-3}}{(\sqrt{\pi})^{3/2}} e^{-\frac{r^2}{2a^2}}, \tag{12}$$

Suppose $\mathrm{Im}(V_{12}+V_{13}) = U_0(\vec{R}) + U_{01}(\vec{R})(x+y+z)$

and $\mathrm{Re}(V_{12}+V_{13})=V_0(\vec{R})+V_{01}(\vec{R})(x+y+z)$

Equating the coefficients before the same degrees of vector's $\vec{r}$ coordinates we obtain

$$f(\vec{R})=\vec{k}\vec{R}, \qquad |k|^2=\frac{2M}{\hbar}(E-E_0), \qquad C(\vec{R})=\sqrt{-\frac{2\mu V_0}{3\hbar}},$$

$$V_0<0, \qquad B(\vec{R})=\frac{2\mu U_0}{\hbar}, \qquad \widetilde{B}(\vec{R})=-\frac{2\mu}{\hbar}(V_{01}+U_0),$$

$$D(\vec{R})+\frac{4}{3}\widetilde{D}(\vec{R})=\frac{2\mu}{\hbar}[U_{01}+\frac{\hbar N}{2\mu a^2}\sqrt{\frac{2\mu V_0}{3\hbar}}]$$

$U_0(\vec{R}),U_{01}(R),V_0(\vec{R}),V_{01}(\vec{R})$ are wells as for example

$U_0(\vec{R})=\begin{cases} U_0, in|x|<x_0,|y|<y_0,|z|<z_0 \\ 0, out \end{cases}$ Those satisfy to the energy independence's reservation of potentials and functions $f(\vec{R})$, $C(\vec{R})$, $B(\vec{R})$, $\widetilde{B}(\vec{R})$ and so on and there asymptotic.

<u>Then the norm of the function (3) $\Psi(\vec{r},\vec{R})$ will be just the same as $e^{i\vec{k}\vec{R}}$</u>

2. Lippmann - Shwinger type equation for T-matrix of scattering the particle on bound two-body system

In principle the scattering process's probability defined the integral

$$\begin{aligned} &\int d^3 \mathrm{R} e^{i\vec{k}\vec{R}} \int d^3 r \Phi_n^*(\vec{r}) e^{-if_{\vec{k}'}(\vec{R},\vec{r})} \Phi_0(\vec{r}) = \\ &\int d^3 \mathrm{R} e^{i\vec{k}\vec{R}} \langle \Phi_n(\vec{r}) | e^{if_{\vec{k}'}(\vec{R},\vec{r})} | \Phi_0(\vec{r}) \rangle \end{aligned} \qquad (13)$$

$\Phi_n(\vec{r})$ is the final state function of the system (2,3).

Multiplying the Eq. (5) to the right on Φ_n^*. Assume that $\Phi_0(\vec{r})$ is the unique solution of Eq (2). Then (5) will be

$$\begin{aligned} &\{-\frac{\hbar}{2M}\Delta_{\vec{R}} + \overline{V(\vec{R},\vec{r}_c)}\} \langle \Phi_n | e^{if(\vec{R},\vec{r})} | \Phi_0 \rangle = \\ &(E - E_0) \langle \Phi_n | e^{if(\vec{R},\vec{r})} | \Phi_0 \rangle \end{aligned}, \qquad (14)$$

In equation (14) $\overline{V(\vec{R},\vec{r}_c)}$ is the potential function at some value $\vec{r} = \vec{r}_c = \{x_c, y_c, z_c\}$.

Then we can posses the solution of (14) on the assumptions scattering problem regards to the coordinate $\vec{R}$ that is as two-body problem.

Corresponding equation (14) Lippmann – Shwinger operator equation will be

$$T = \overline{V} + \overline{V} G_0(E - E_0 + i\varepsilon) T . \qquad (15)$$

The Green function $G_0(E - E_0 + i0)$ is defined as usually

$$G_0(E - E_0 + i0) = (E - E_0 - H_0 + i0)^{-1} \qquad (16)$$

Formula (15) include the Hamiltonian

$$H_0 = -\frac{\hbar}{2M}\Delta_{\vec{R}} \qquad (17)$$

Let the initial and final asymptotic state wave functions of three particles system be

$$\phi_0(\vec{R},\vec{r}) = e^{i\vec{p}_0\vec{R}}$$

and

$$\phi_n(\vec{R},\vec{r}) = e^{i\vec{p}_0'\vec{R}} \qquad (18)$$

Then we can calculate all the quantum mechanics characteristics using T-matrix elements

$$\begin{aligned}&\langle \vec{p}_0' | T | \vec{p}_0 \rangle = \langle \vec{p}_0' | \bar{V} | \vec{p}_0 \rangle \\ &+ \langle \vec{p}_0' | \bar{V} G_0(E - E_0 + i\varepsilon) T | \vec{p}_0 \rangle\end{aligned}. \qquad (19)$$

For elastic scattering when $\Phi_n(\vec{r}) = \Phi_0(\vec{r})$ ratio (14) changes

$$\begin{aligned}&\langle \vec{p}_0' | T | \vec{p}_0 \rangle = \langle \vec{p}_0' | \bar{V} | \vec{p}_0 \rangle \\ &+ \langle \vec{p}_0' | \bar{V} G_0(E - E_0 + i\varepsilon) T | p_0 \rangle\end{aligned}. \qquad (20)$$

Introduce the optical potential

$$V_{opt.} = \overline{V(\vec{R},\vec{r}_c)} = v_{opt.} + iu_{opt.}, \qquad (21)$$

where

$$\begin{aligned}&v_{opt.}(\vec{R}) = v(\vec{R},\vec{r}_c) = \\ &\left.\{V_{12}(\vec{R} - \frac{\mu}{m_2}\vec{r}) + V_{13}(\vec{R} + \frac{\mu}{m_3}\vec{r}) + \frac{\hbar}{2\mu}[\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r})]^2\}\right|_{\vec{r}=\vec{r}_c}\end{aligned} \qquad (22)$$

and

$$\begin{aligned}&u_{opt.}(\vec{R}) = u(\vec{R},\vec{r}_c) = \\ &\left.\{-\frac{\hbar}{\mu}\vec{\nabla}_{\vec{r}} f(\vec{R},\vec{r}) \cdot \vec{\nabla}_{\vec{r}} \ln \Phi_0(\vec{r}) - \frac{\hbar}{2\mu}\Delta_{\vec{r}} f(\vec{R},\vec{r})\}\right|_{\vec{r}=\vec{r}_c}\end{aligned} \qquad (23)$$

Then the integral Lippmann-Schwinger type equation for two - body scattering problem is

$$\begin{aligned}&\left\langle \vec{p}_0' \middle| T \middle| \vec{p}_0 \right\rangle = \left\langle \vec{p}_0' \middle| V_{opt.} \middle| \vec{p}_0 \right\rangle + \\ &\left\langle \vec{p}_0' \middle| V_{opt.} \middle| \vec{k} \right\rangle \left\langle \vec{k} \middle| G_0(E - E_0 + i\varepsilon) \middle| \vec{k}' \right\rangle \left\langle \vec{k}' \middle| T \middle| \vec{p}_0 \right\rangle\end{aligned} \quad (24)$$

In the right hand T-matrix elements satisfy to the optical theorem. Note the T-matrix elements as the solutions of integral Lippmann-Schwinger Eq. (24) must satisfy optical theorem.
undetermined parameters x_c, y_c, z_c. we can define by comparing experimental data with the theoretical values using the amplitude in the mathematical eikonal method [2-6] with parameter $\beta = 1$ and solving the system of three equations in three variables x_c, y_c, z_c

$$\frac{4\pi}{p_0} \mathrm{Im}\, F(0) = \sigma_{tot}, \quad \frac{1}{t} \ln \frac{|F(z)|^2}{|F(0)|^2} = b_{\exp}, \quad \frac{\mathrm{Re}\, F(0)}{\mathrm{Im}\, F(0)} = \alpha_{\exp}.$$

3. Conclusion

We obtain:
1. the wave function of a particle on bound system's scattering that satisfies unitarity and remolded potential;

2. the integral two - body Lippmann-Schwinger type equation for this case that satisfies unitarity too and T-matrix elements can be find by the mathematical eikonal method [2-6]