# Polarized distribution functions and intrinsic motion of the quarks

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# Introduction

- Presented results are based on the covariant QPM. Intrinsic motion, reflecting orbital angular momenta of quarks, is consistently taken into account. Due to covariance, transversal and longitudinal momenta appear on the same level. [*P.Z. Phys.Rev.D65, 054040(2002), D67, 014019(2003) and Eur.Phys.J. C52, 121 (2007)*].

- **In this talk:**

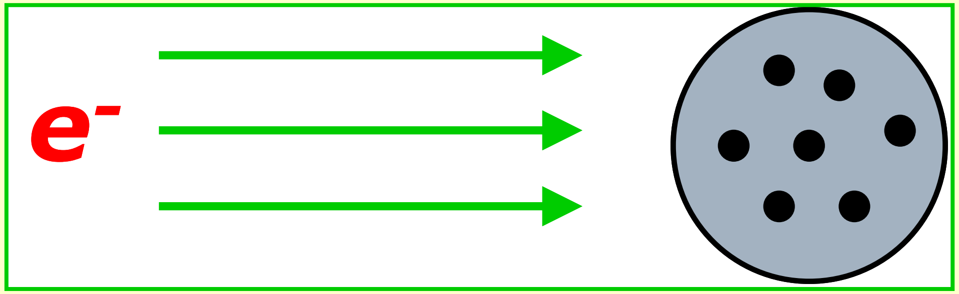
  ***Short overview of published results:***

  - $g_1$ and $g_2$ : sum rules, calculation and comparison with the data
  - Calculation related to transversity [*A.Efremov, O.Teryaev and P.Z., Phys.Rev.D70, 054018(2004) and arXiv: hep-ph/0512034*]
  - Calculation of 3D momenta distributions from structure functions

  ***New results:***

  - Relations connecting helicity, transversity and pretzelosity & predictions for experiment (SSA...)
  - Role of orbital angular momentum

# Model




$$\Delta\sigma(x, Q^2) \sim |A|^2$$

$$|A|^2 = L_{\alpha\beta} W^{\alpha\beta}$$

The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0) d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2},$$

which are expected to depend effectively on $Q^2$. These distributions measure the probability to find a quark in the state

$$u(p, \lambda\mathbf{n}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0 + m} \phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where $m$ and $p$ are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

$W^{\alpha\beta} \Rightarrow$

$$F_1(x, Q^2)$$
$$F_2(x, Q^2)$$
$$g_1(x, Q^2)$$
$$g_2(x, Q^2)$$

# Structure functions

- **Input:**

**3D distribution functions in the proton rest frame**

The distributions allow to define the generic functions $G$ and $\Delta G$:

$$G(p_0) = \sum_q e_q^2 G_q(p_0), \quad G_q(p_0) \equiv G_q^+(p_0) + G_q^-(p_0),$$

$$\Delta G(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0)$$

from which the structure functions can be obtained.

- **Result:**

**structure functions**

**($x$=Bjorken $x_B$ !)**

If one assumes $Q^2 \gg 4M^2x^2$, then:

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m + p_1 + \frac{p_1^2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0},$$

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0}$$

## $F_1$, $F_2$ - manifestly covariant form:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\,\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

## $g_1$, $g_2$ - manifestly covariant form:

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

# Comments

- In the limit of usual approach assuming *p=xP*, one gets usual relations between the structure and distribution functions like

$$F_2(x) = x\sum_q e_q^2 q(x)$$

$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left(q^+(x) - q^-(x)\right)$$

- Obtained structure functions for m→0 obey the known sum rules:

$$g_2(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y}dy,$$

which is **Wanzura - Wilczek twist-2 term** for $g_2$ approximation.

$$\int_0^1 x^\alpha\left[\frac{\alpha}{\alpha+1}g_1(x) + g_2(x)\right]dx = 0$$

For $\alpha = 2,4,6,\ldots$ relation corresponds to the **Wanzura - Wilczek sum rules**. Other special cases correspond to the **Burkhardt - Cottingham** ($\alpha = 0$) and the **Efremov - Leader - Teryaev** (ELT, $\alpha = 1$) sum rules.

***Sum rules were obtained from*:**

**1) Relativistic covariance**
**2) Spheric symmetry**
**3) One photon exchange**

- In this talk m→0 is assumed.

## Comments

Structure functions are represented by integrals from probabilistic distributions:

$$F_1, F_2 : \qquad \int G(p_0)[\ldots]\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

$$g_1, g_2 : \qquad \int \Delta G(p_0)[\ldots]\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0}.$$

**This form allows integral transforms:**

1) $g_1 \leftrightarrow g_2$ or $F_1 \leftrightarrow F_2$ (rules mentioned above were example).
2) With some additional assumptions also e.g. integral relation $g_1 \leftrightarrow F_2$ can be obtained (illustration will be given).
3) To invert the integrals and obtain $G$ or $\Delta G$ from $F_2$ or $g_1$.

## $g_1$, $g_2$ from valence quarks

Suppose that spin contribution from the sea of quark-antiquark pairs and gluons can be neglected, so proton spin is generated by valence quarks - for which *SU(6)* symmetry is assumed:

$$s_u = 4/3, \qquad s_d = -1/3,$$

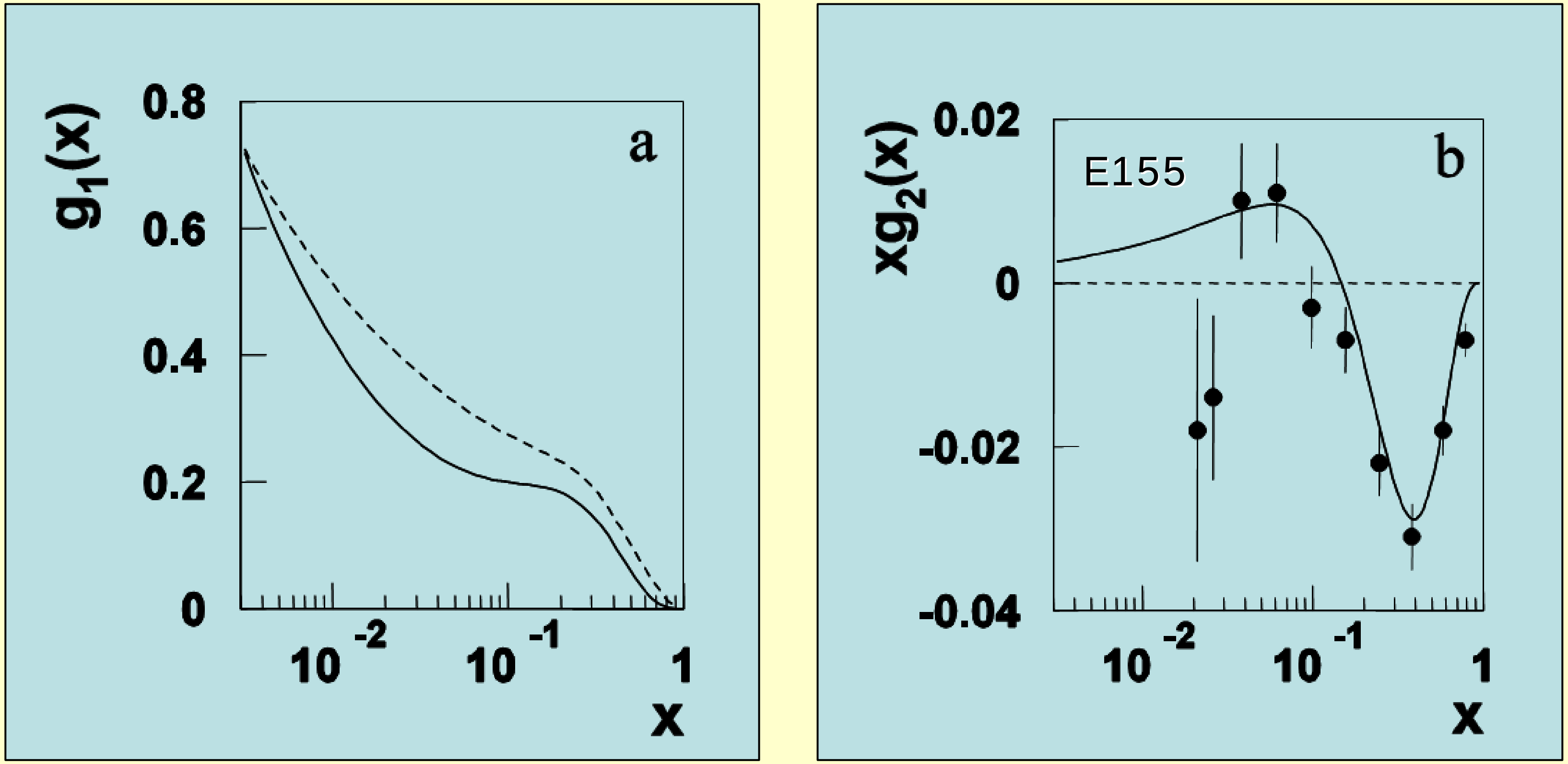
$$\Delta G_u = G_u^+ - G_u^- = \frac{2}{3} G_u, \qquad \Delta G_d = G_d^+ - G_d^- = -\frac{1}{3} G_d.$$

Using technique of integral transforms, functions $g_1^q = \Delta q$ and $g_2^q$ can be obtained from valence quark distributions:

$$g_1^q(x) = \frac{1}{2}\left[q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^3} dy\right],$$

$$g_2^q(x) = \frac{1}{2}\left[-q_V(x) + 3x^2 \int_x^1 \frac{q_V(y)}{y^3} dy\right].$$

# $g_1$, $g_2$ from valence quarks




Calculation - solid line, data - dashed line (left) and circles (right)

- $g_1$ fit of world data by E155 Coll., Phys.Lett B **493**, 19 (2000).

# Transversity

- In a similar way also the transversity was calculated; see [*A.Efremov, O.Teryaev and P.Z., Phys.Rev.D70, 054018(2004)*]. Among others we obtained

$$\int_0^1 \delta q(x)dx = 2\int_0^1 \Delta q(x)dx$$

**- which follows from covariant kinematics!**

- Obtained transversities were used for the calculation of double spin asymmetry in the lepton pair production in proposed PAX experiment; see [*A.Efremov, O.Teryaev and P.Z., arXiv: hep-ph/0512034)*].

$$A_{TT}(y,Q^2) = \frac{\sum_q e_q^2 \delta q(x_1,Q^2)\delta q(x_2,Q^2)}{\sum_q e_q^2 q(x_1,Q^2) q(x_2,Q^2)}; \qquad x_{1/2} = \sqrt{\frac{Q^2}{s}}\exp(\pm y)$$

$$s = (P_1+P_2)^2, \qquad Q^2 = (k_1+k_2)^2, \qquad y = \frac{1}{2}\ln\frac{P_1(k_1+k_2)}{P_2(k_1+k_2)}$$

# 2004:
# Our calculation

PHYSICAL REVIEW D, VOLUME 70, 054018

**Transversity and intrinsic motion of the constituents**

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(Received 25 May 2004; published 17 September 2004)

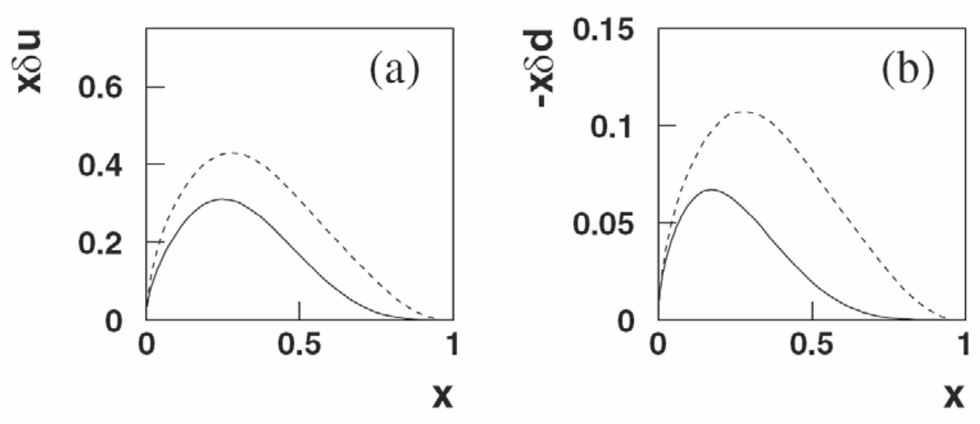
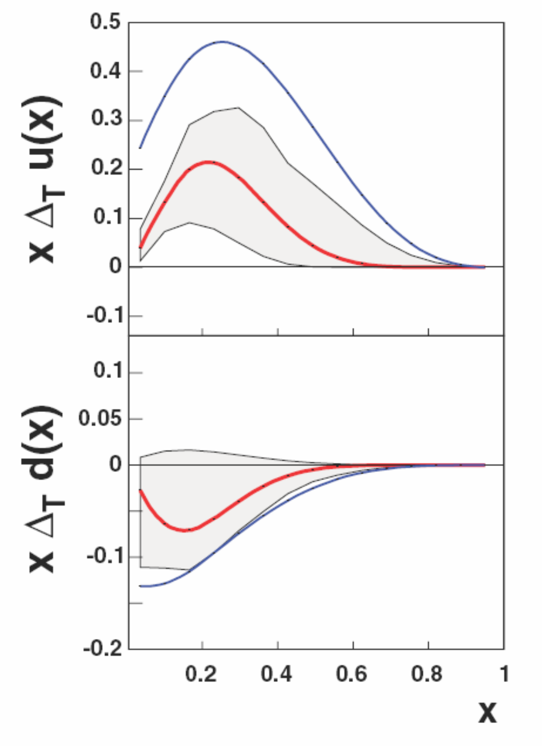



FIG. 2. Transversities of the $u$ (a) and $d$ (b) valence quarks calculated from the valence distributions (solid lines) and extracted from the experimental data on proton spin function $g_1$ (dashed lines)—the first approach; see text.

# 2007:
# Extraction from the data
## (for the first time)

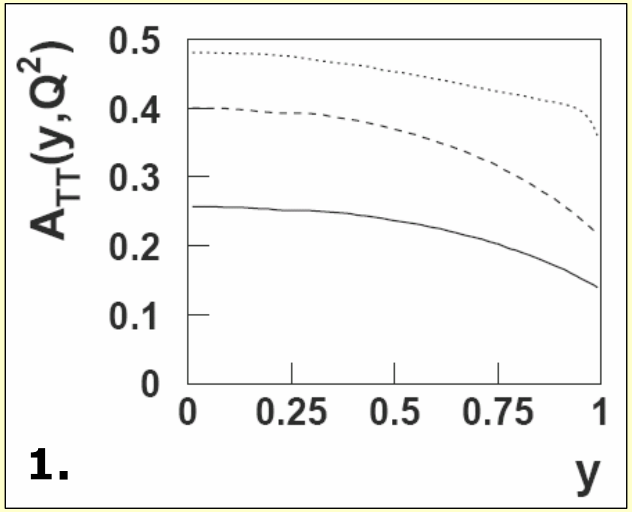



PHYSICAL REVIEW D **75,** 054032 (2007)

**Transversity and Collins functions from SIDIS and $e^+e^-$ data**

M. Anselmino,[1] M. Boglione,[1] U. D'Alesio,[2] A. Kotzinian,[3] F. Murgia,[2] A. Prokudin,[1] and C. Türk[1]

# Double spin asymmetry in PAX experiment @ FAIR



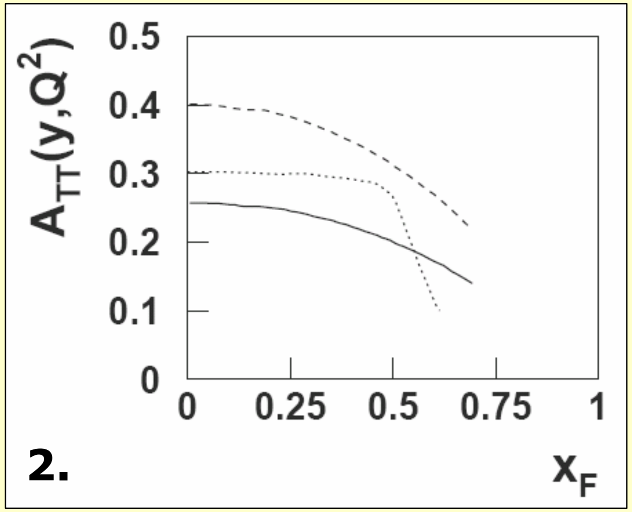




FIG. 1. Double spin asymmetry at $Q^2 = 4GeV/c$ is calculated using two transversity approaches: Interference effects are attributed to quark level only *(solid line)*. Interference effects at parton-hadron transition stage are included in addition *(dashed line)*, this curve represents upper bound only. Dotted curve corresponds to the calculation based on quark- soliton model - see text.


[19] A. V. Efremov, K. Goeke and P. Schweitzer, Eur. Phys. J. C **35** (2004) 207

[25] P.Schweitzer, D.Urbano, M.V.Polyakov, C.Weiss, P.V.Pobylitsa and K.Goeke, Phys.Rev. D **64**, 034013 (2001).


FIG. 2. Double spin asymmetry: The solid and dashed lines are the sameas in the previous figure, but here their dependence on $x_F$ is displayed.Dotted line is corresponding estimate from [18].


[18] M. Anselmino, V. Barone, A. Drago and N. N. Nikolaev, Phys. Lett. B **594** (2004) 97

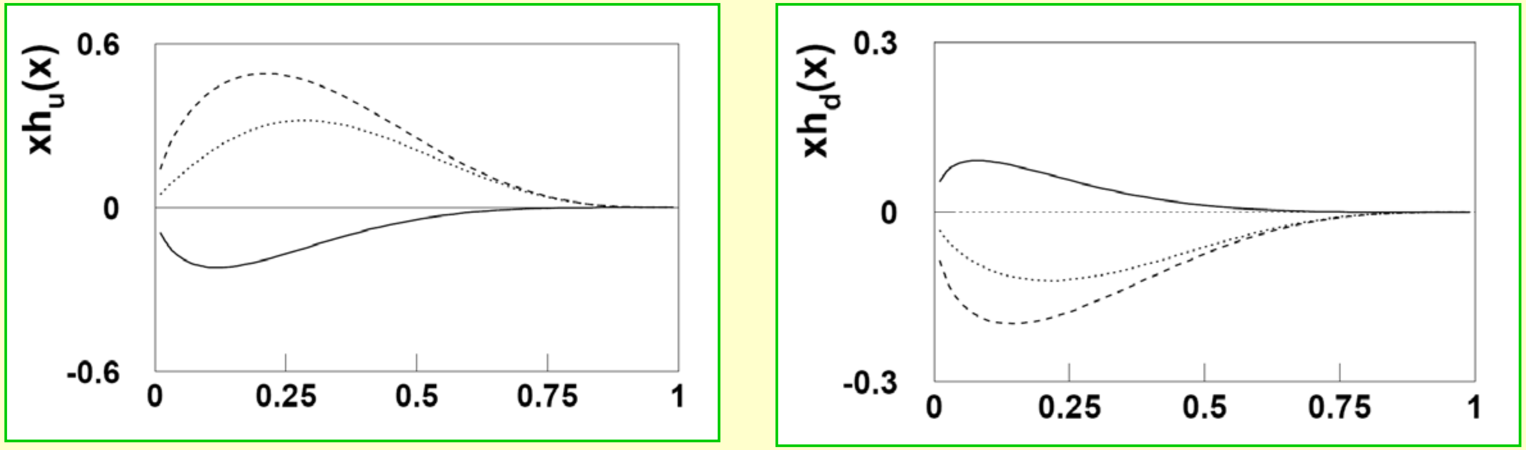
# Pretzelosity

***(PZ+Anatoly Efremov, Peter Schweitzer, Oleg Teryaev – paper is under preparation)***

$$h_{1T}^{\perp(1)}(x) = \Delta q(x) - \delta q(x) = -\Delta q_T(x) = -\int_x^1 \frac{\Delta q(y)}{y} dy$$

$$\Delta q_T(x) = 2(g_1(x) + g_2(x))$$

$$\delta q(x) = \Delta q(x) + \Delta q_T(x) = \Delta q(x) + \int_x^1 \frac{\Delta q(y)}{y} dy$$



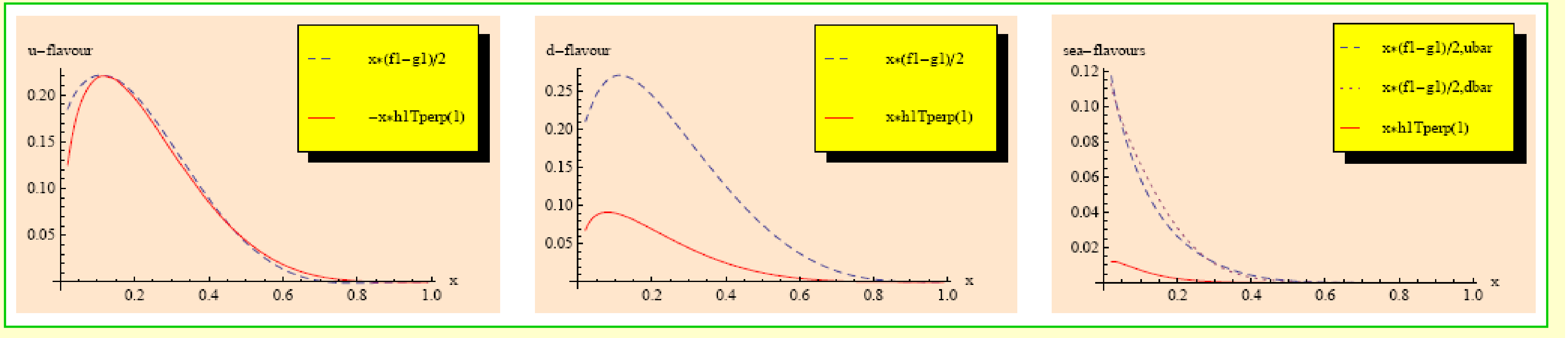

***pretzelosity (solid), transversity (dashed), helicity (dotted)***

## Test of inequality:

$$\left| h_{1T}^{\perp(1)}(x) \right| \leq \frac{1}{2}\left(q(x) - \Delta q(x)\right)$$

- follows from Soffer inequality

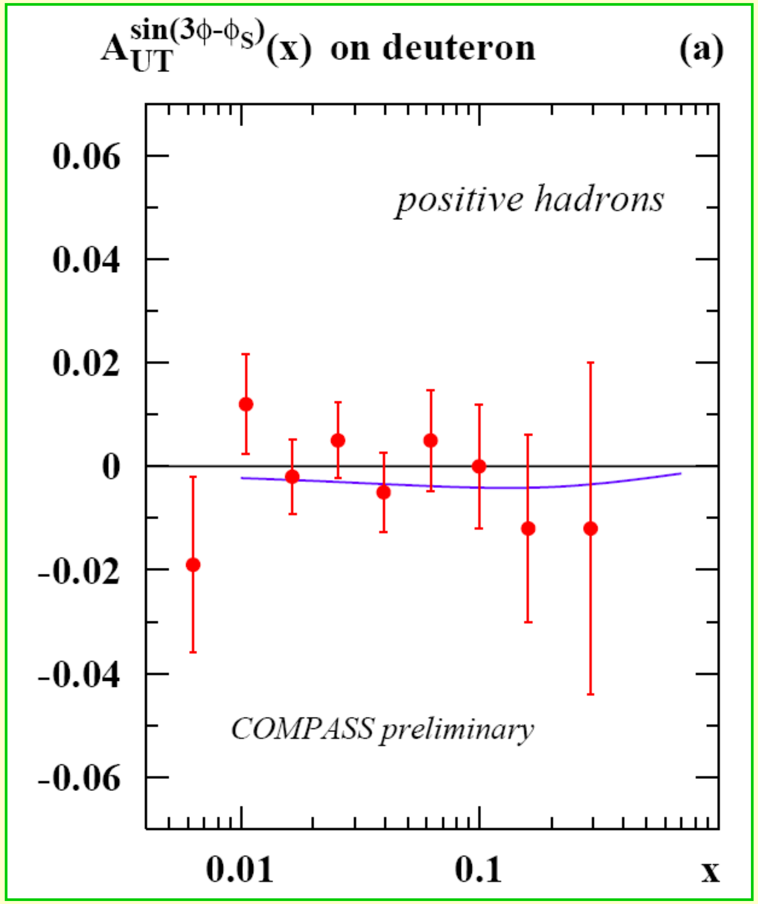
*preliminary*

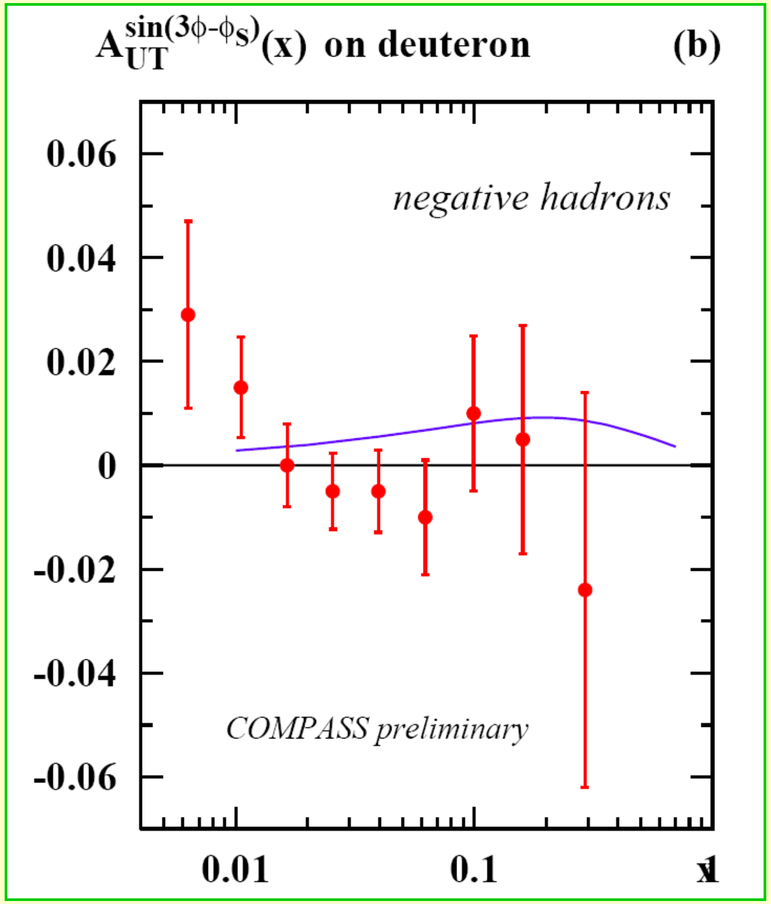



Parameterization of PDF's:

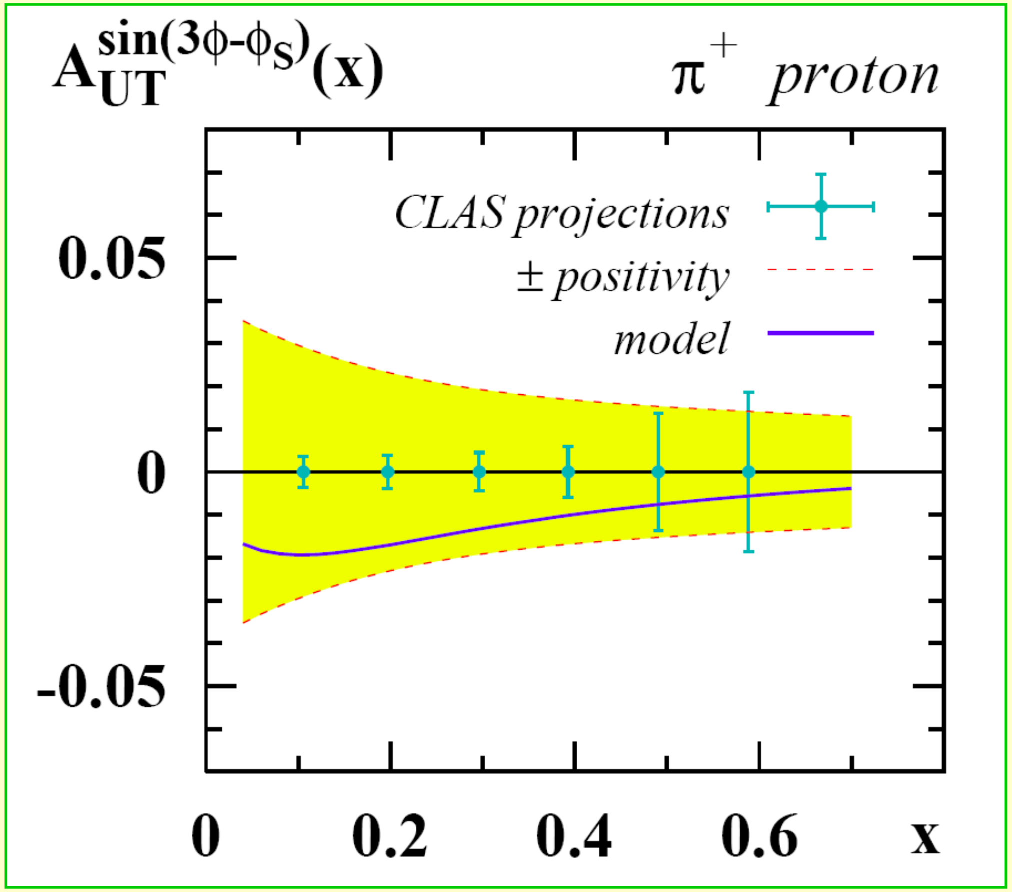

A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **4**, 463 (1998) [arXiv:hep-ph/9803445].
E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D **73**, 034023 (2006) [arXiv:hep-ph/0512114].

# SSA for COMPASS:



# SSA prediction for CLAS@JLab:



## Momentum distributions from structure function $F_2$

Deconvolution of $F_2$ :

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0} \Rightarrow$$

$$G(p) = \sum_q e_q^2 G_q(p) = \frac{1}{\pi M^3 x^2}\left(\frac{2F_2(x)}{x} - F_2'(x)\right); \qquad x = \frac{2p}{M} \Rightarrow$$

$$P_q(p) = 4\pi p^2 M G_q(p) = -x^2\left(\frac{q(x)}{x}\right)' = q(x) - xq'(x).$$

**Remarks:**

- $G$ measures in $d^3p$, $P$ in the $dp/M$
- $p_{max}=M/2$ – due to kinematics in the proton rest frame, $\Sigma p=0$
- Self-consistency test: $\left(\frac{q(x)}{x}\right)' \leq 0.$

## Momentum distributions in the proton rest frame

***Input q(x)*** ⇒

***MRST LO 4GeV²***

***$q_{val}=q-\bar{q}$***

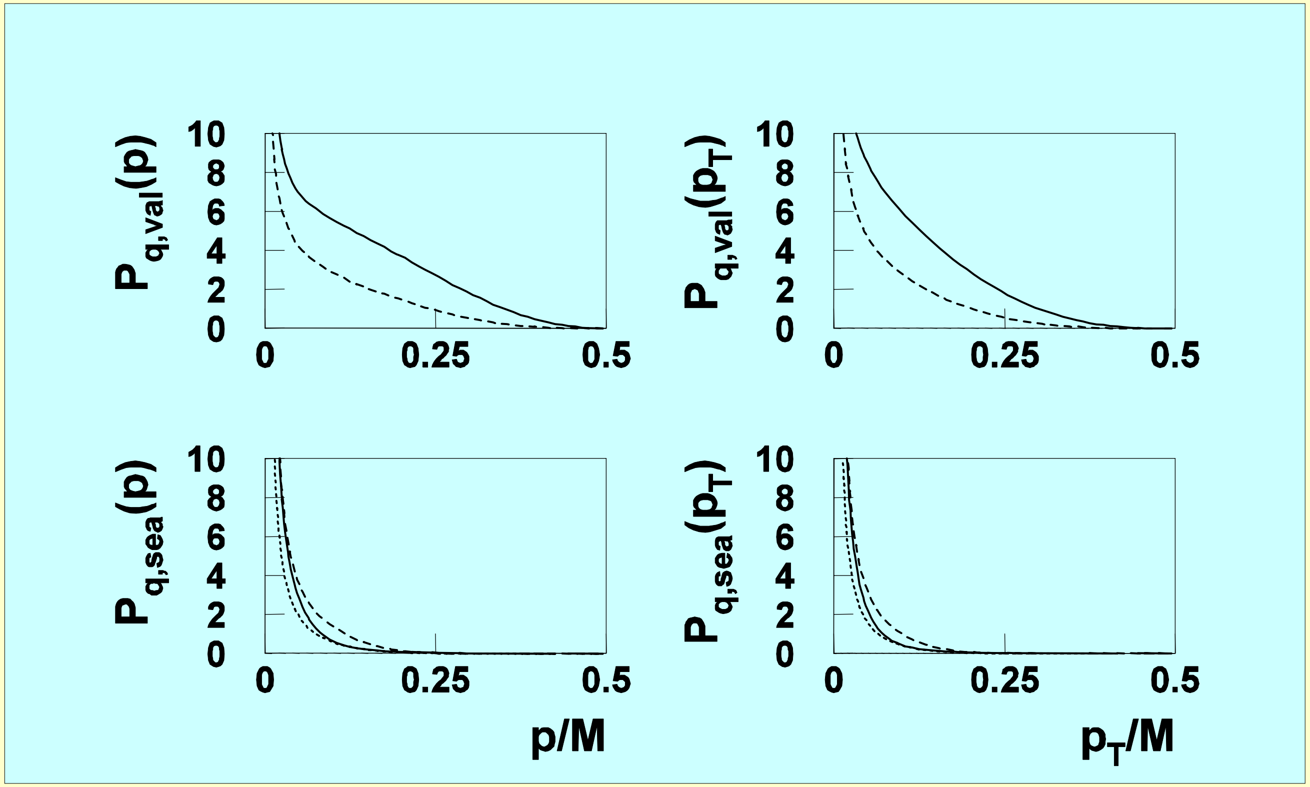



FIG. 1: The quark momentum distributions in the proton rest frame: the $p$ and $p_T$ distributions for valence quarks $P_{q,val} = P_q - P_{\bar{q}}$ and sea quarks $P_{\bar{q}}$ at $Q^2 = 4GeV^2$. Notation: $u, \bar{u}$ - solid line, $d, \bar{d}$ - dashed line, $\bar{s}$ - dotted line.

***$<p_{val}>=0.11$ (0.083) GeV/c for u (d) quarks***

## Momentum distributions from structure function $g_1$

Deconvolution of $g_1$ :

$$\Delta q(x) = \int \Delta G_q(p_0)\left(m + p_1 + \frac{p_1^2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0} \Rightarrow$$

$$\Delta P_q(p) = 4\pi p^2 M \Delta G_q(p) = 3\Delta q(x) + 2\int_x^1 \frac{\Delta q(y)}{y}dy - x\Delta q'(x)$$

***Since $G=G^+ + G^-$ and $\Delta G=G^+ + G^-$*** $\Rightarrow$

$$G_q^{\pm}(p) = \frac{1}{2}(G_q(p) \pm \Delta G_q(p))$$

$$P_q^{\pm}(p) = \frac{1}{2}(P_q(p) \pm \Delta P_q(p))$$

$d^3p$ $\qquad$ $dp/M$

***... obtained from $F_2$ ,$g_1$ and represent distribution of quarks with polarization ±.***

## Distribution functions $f^{\pm}(x)$

$$q(x) = Mx \int G_q(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$f_q^{\pm}(x) = Mx \int G_q^{\pm}(p)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

Let us note: $f_q^{+} + f_q^{-} = q$ but $f_q^{+} - f_q^{-} \neq \Delta q$ !!

(equality takes place only in non-covariant IMF approach)

# Momentum distributions in the proton rest frame

*2) q(x) & Δq(x)*

*MRST & LSS LO 4GeV²*

$$f_q(x) = f_q^+(x) + f_q^-(x)$$

$$\Delta f_q(x) = f_q^+(x) - f_q^-(x)$$

**Remark:**
$x\Delta f_q(x)$ are similar to $xq_{val}(x)$

⇒ spin contribution comes dominatly from valence region

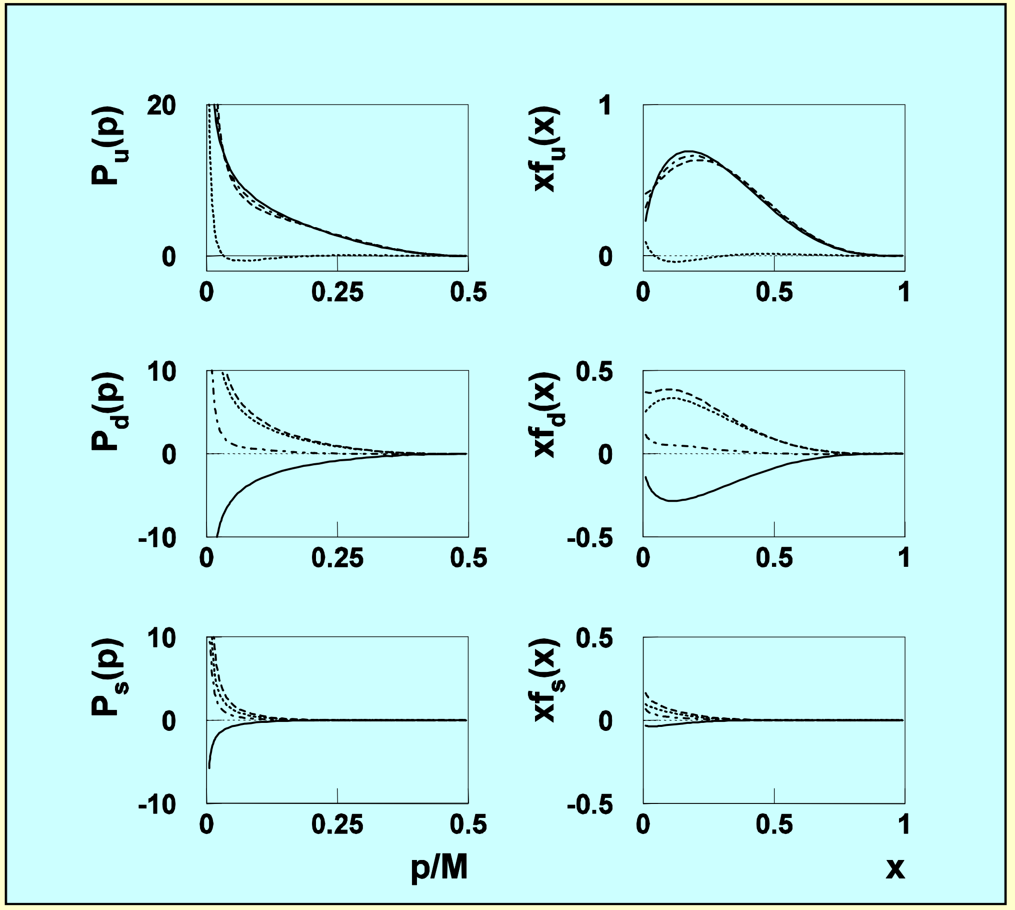



FIG. 2: Probability distributions $\Delta P_q, P_q, P_q^+, P_q^-$ of $u, d, s$ quarks (left) and related structure functions $\Delta f_q, f_q, f_q^+, f_q^-$ (right) are represented by the solid, dashed, dash-and-dot and dotted lines.

# Intrinsic motion and angular momentum

- Forget structure functions for a moment…
- Total angular momentum consists of ***j=l+s***.
- In relativistic case ***l,s*** are not conserved separately, only ***j*** is conserved. So, we can have pure states of ***j*** ***(j²,j_z)*** only, which are represented by the bispinor spherical waves:

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\boldsymbol{\omega}) \\ i^{-\lambda}\sqrt{p_0-m}\,\Omega_{j\lambda j_z}(\boldsymbol{\omega}) \end{pmatrix},$$

where $\boldsymbol{\omega} = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ ($l$ defines the parity) and

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\,Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j-j_z}{2j}}\,Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j - \frac{1}{2},$$

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\,Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j+j_z+1}{2j+2}}\,Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j + \frac{1}{2}.$$

# j=1/2

For $j = j_z = 1/2$ and $l = 0$ :

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}}\begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta\exp(i\varphi) \end{pmatrix} \end{pmatrix}.$$

For the superposition

$$\Psi(\mathbf{p}) = \int a_k \psi_{kjlj_z}(\mathbf{p})dk; \qquad \int a_k^\star a_k dk = 1$$

the average spin contribution to the total angular momentum is calculated as

$$\langle s\rangle = \int \Psi^\dagger(\mathbf{p})\Sigma_z\Psi(\mathbf{p})d^3p; \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_z & \cdot \\ \cdot & \sigma_z \end{pmatrix}.$$

# Spin and orbital motion

$$\langle s\rangle = \int a_p^\star a_p \frac{(p_0+m)+(p_0-m)\left(\cos^2\theta-\sin^2\theta\right)}{16\pi p^2 p_0} d^3p$$

$$= \frac{1}{2}\int a_p^\star a_p\left(\frac{1}{3}+\frac{2m}{3p_0}\right)dp.$$

$$\langle l\rangle = \frac{1}{3}\int a_p^\star a_p\left(1-\frac{m}{p_0}\right)dp, \qquad \langle l\rangle + \langle s\rangle = 1/2$$

$$\Gamma_1 = \int_0^1 g_1(x)dx = \frac{1}{2}\int \Delta G(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p$$

**$\langle s\rangle, \Gamma_1$: two ways, one result**

**-covariant approach is a common basis**

# Comments

- for fixed *j=1/2* both the quantities are almost equivalent: *more kinetic energy (in proton rest frame) generates more orbital motion and vice versa.*

$$\langle l \rangle = \frac{1}{3}\int a_p^\star a_p\left(1-\frac{m}{p_0}\right)dp = \frac{1}{3}\int a_p^\star a_p \frac{E_{kin}}{p_0}dp,$$

$$\langle E_{kin} \rangle = \int a_p^\star a_p E_{kin} dp; \qquad E_{kin} = p_0 - m$$

- $\langle s \rangle, \Gamma_1$ are controlled by the factor $\left(\frac{1}{3}+\frac{2m}{3p_0}\right)$, two extremes:

•*massive and static quarks* $(m \simeq p_0) \Rightarrow \langle s \rangle = j = 1/2$ *and* $\langle l \rangle = 0$

•*massless quarks* $(m \ll p_0) \Rightarrow \langle s \rangle = 1/6$ *and* $\langle l \rangle = 1/3$

-this scenario is clearly preferred for quarks with effective mass on scale of *MeV* and momentum of $10^2$ *MeV*.

- important role of the intrinsic quark orbital motion emerges as a direct consequence of the covariant approach

# Proton spin

**Second scenario (chiral limit):**

$$S = \Delta\Sigma, \qquad L = 2\Delta\Sigma; \qquad \Delta\Sigma = \sum_q \int_0^1 \Delta q(x)dx.$$

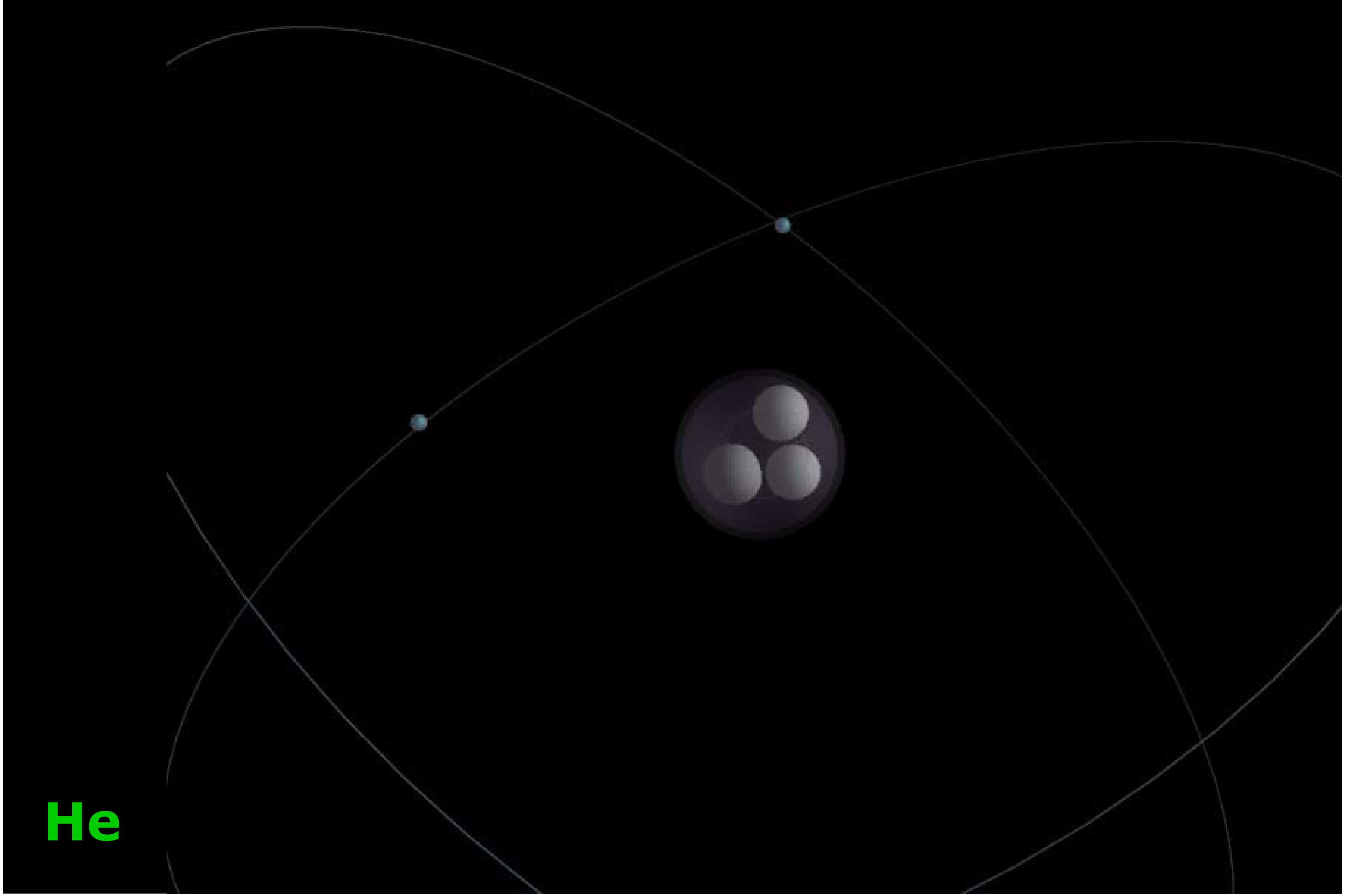
implies great contribution of quark orbital angular momentum. In this way a room for gluon contribution can be rather sensitive to the longitudinal polarization:

*For $\Delta\Sigma \approx 1/3$, 0.3 and 0.2 gluon contribution represents 0, 10 and 40%. Recent data from COMPASS and HERMES are close to $\Delta\Sigma \approx 1/3$, in this way rather small gluon contribution is preferred – which does not contradict existing experimental data.*

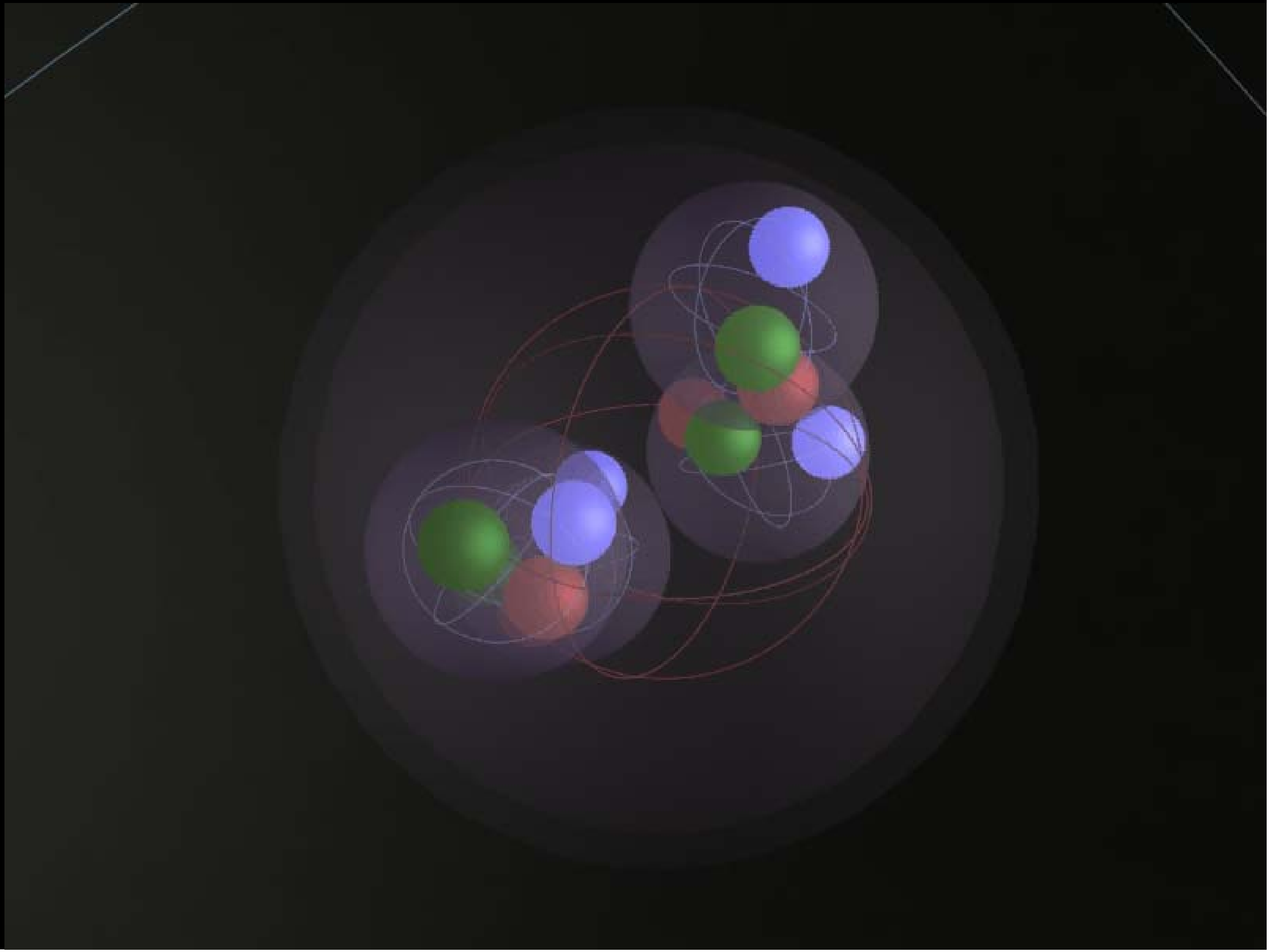
Orbital motion of quarks would well fit to other motions inside atom like orbital motion of electrons...

He

**...or like orbital motion of nucleons**



# Orbital motion of quarks



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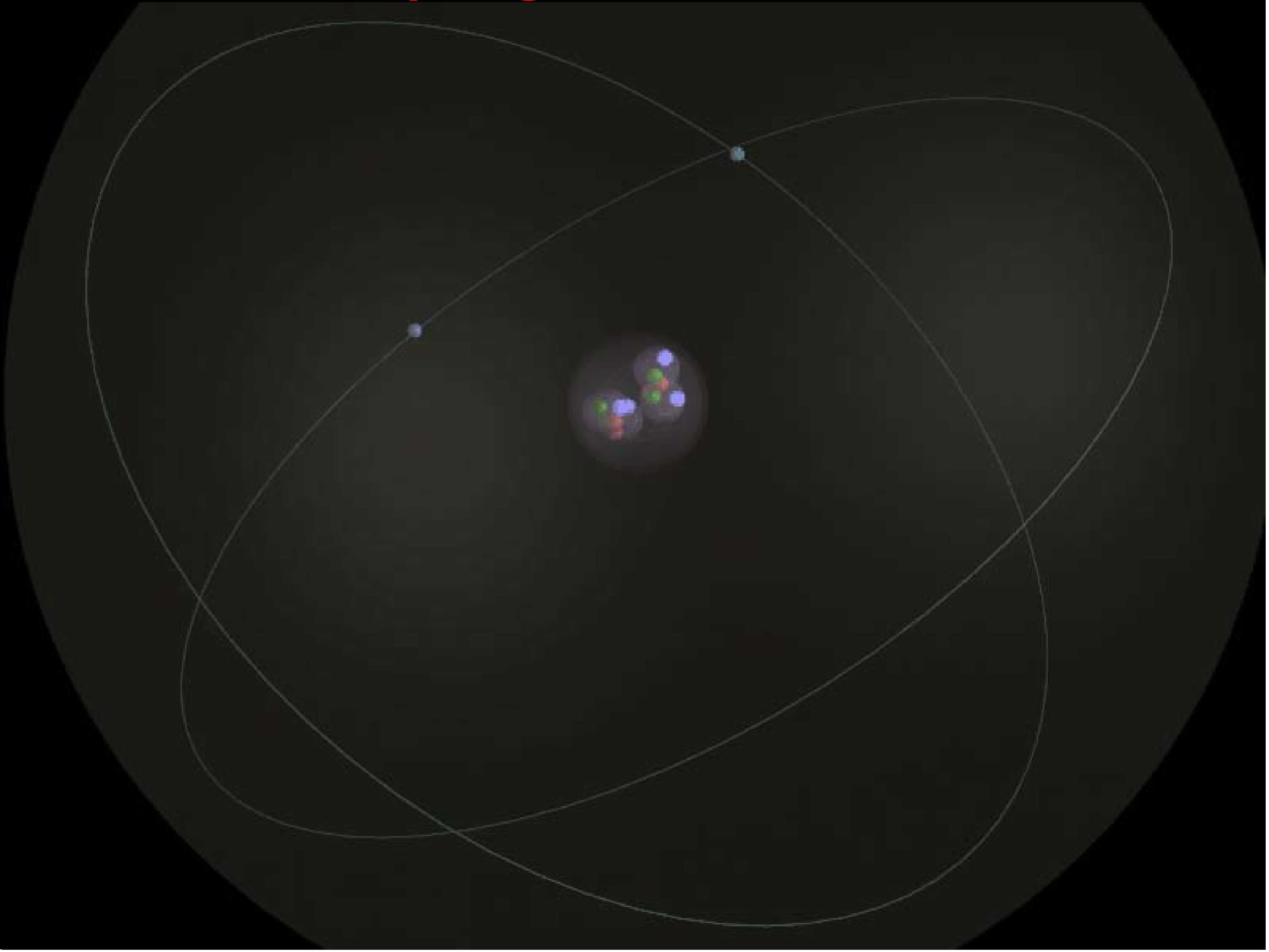
# Orbital motion of everything...



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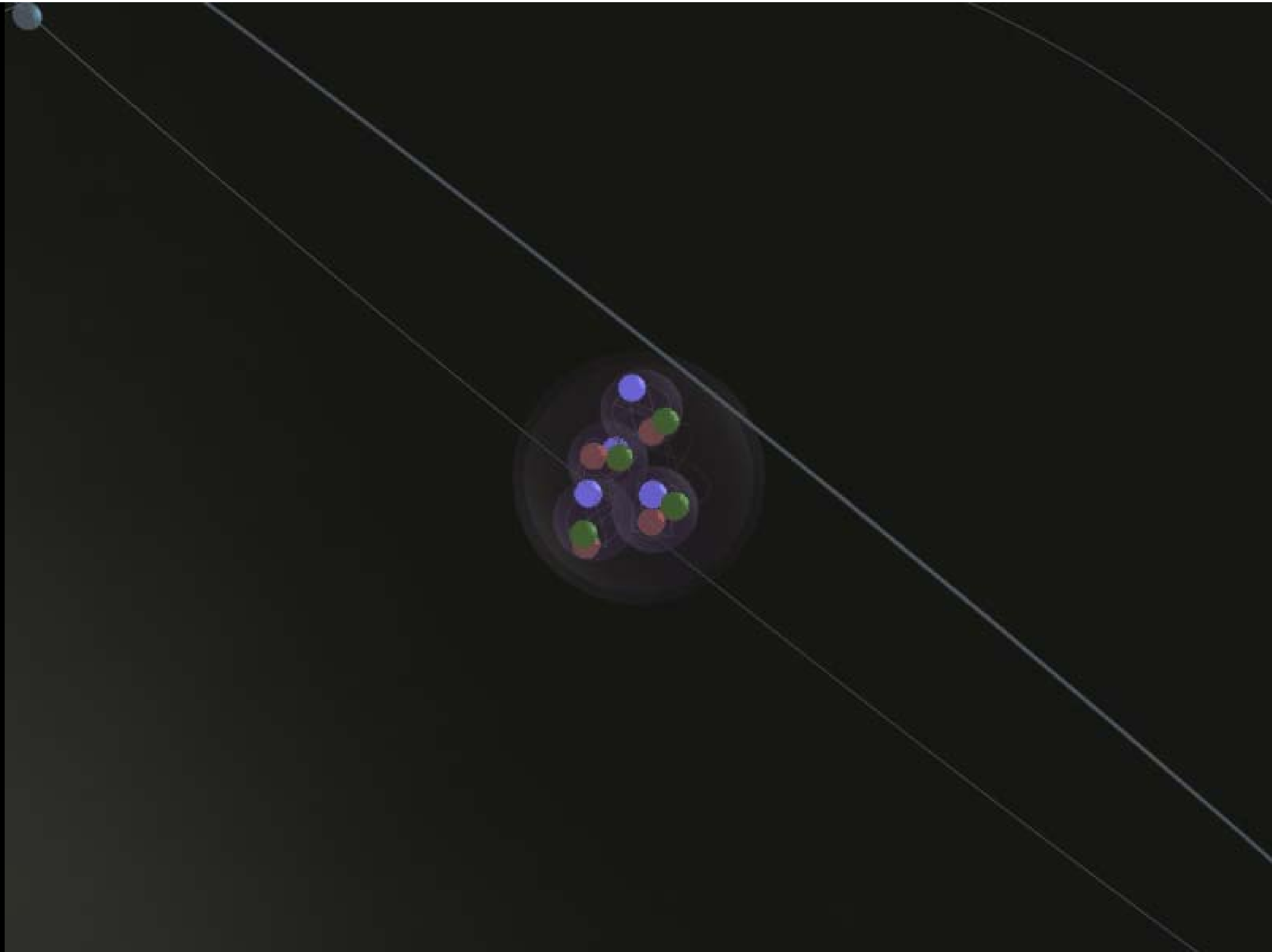
# Summary

Covariant version of QPM involving quark orbital motion was studied.

**Main new results:**

- In chiral limit model suggests mutual dependence of helicity, transversity and pretzelosity, the last two distributions can be obtained from helicity. Predictions for the SSA.
- Model allows to calculate 3D quark momenta distributions (in proton rest frame) from the structure functions.
- Important role of quark orbital motion, which follows from covariant approach, was pointed out. Orbital angular momentum can represent as much as 2/3 $j$. Then the spin function $g_1$ is reduced correspondingly.



# Thank you!