Possibilities of measurement of tensor polarizabilities of the deuteron and other nuclei in experiments with polarized beams

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OUTLINE

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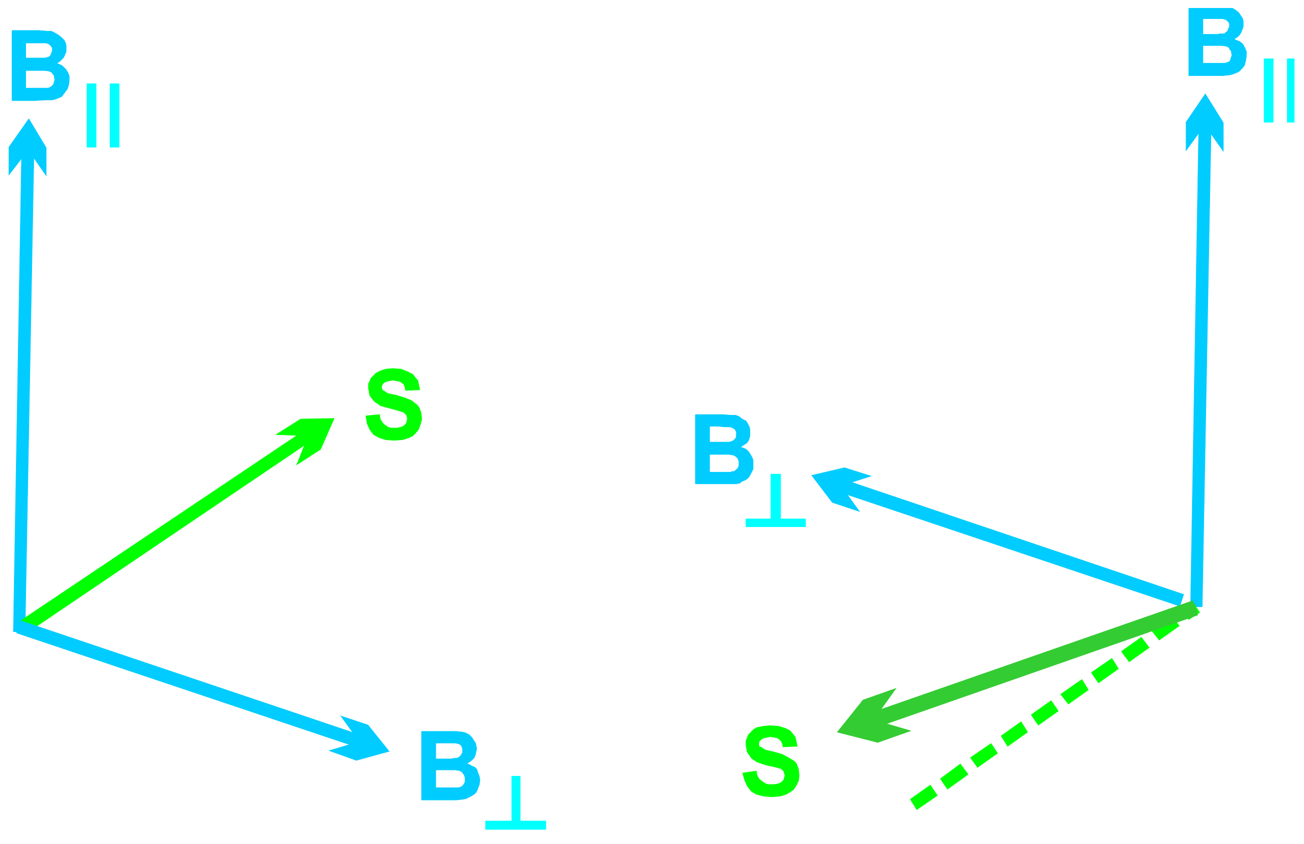
- Measurement of tensor electric polarizability of the deuteron by the resonance method
- Tensor magnetic polarizabilities of the deuteron and other nuclei in storage ring experiments
- Measurement of tensor electric and magnetic polarizabilities of the deuteron by the frozen spin method
- Summary

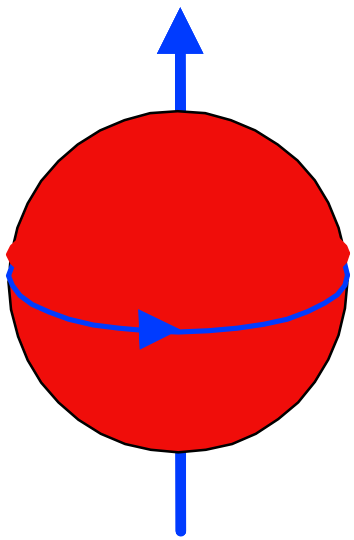
Methods of measurement of tensor polarizabilities of the deuteron and other nuclei

Methods of measurement of tensor polarizabilities of the deuteron and other nuclei have been proposed by V. Baryshevsky and co-workers:

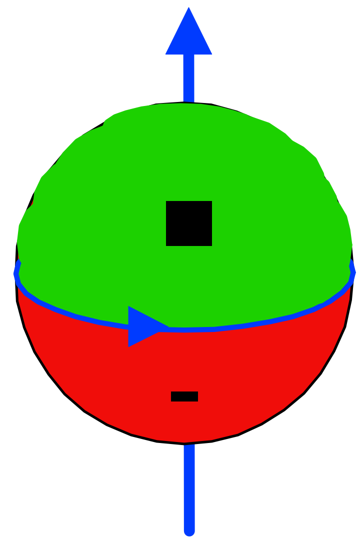
- V. Baryshevsky and A. Shirvel, hep-ph/0503214.
- V. G. Baryshevsky, STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 227–230; J. Phys. G: Nucl. Part. Phys. **35,** 035102 (2008); hep-ph/0504064; hep-ph/0510158; hep-ph/0603191.
- V. G. Baryshevsky and A. A. Gurinovich, hep-ph/0506135.



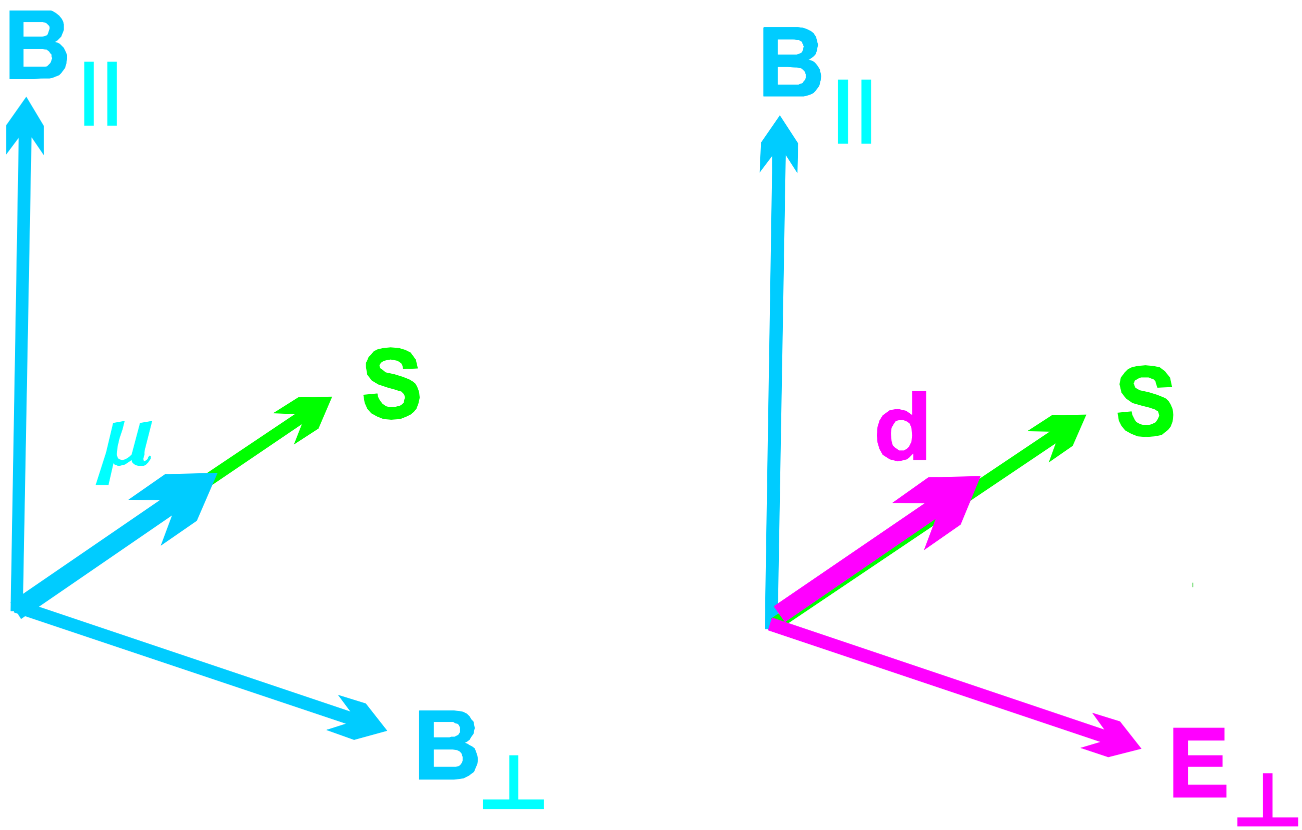
$B_{||}$
S
$B_{\perp}$
$B_{||}$
$B_{\perp}$
S



$\vec{\mu} = \mu \vec{\mathbf{S}}/S$



$\vec{\mathbf{d}} = d\vec{\mathbf{S}}/S$



$B_{||}$
μ
S
$B_{\perp}$
$B_{||}$
d
S
$E_{\perp}$

The tensor electric polarizability similarly affects the spin

$$V = -\frac{a_T}{g}(\mathbf{S}\cdot\mathbf{E}')^2 - \frac{b_T}{g}(\mathbf{S}\cdot\mathbf{B}')^2.$$

$\mathbf{E}'$ **and** $\mathbf{B}'$ **are the rest frame fields**

$$V = -\frac{a_T}{g}\left(bg\,B_z S_r + E_f S_f\right)^2$$

$$-b_T g\,B_z^2 S_z^2.$$

There are two resonance frequencies, $\omega \approx \omega_0$ and $\omega \approx 2\omega_0$:

$$b^2 g = b_0^2 g_0 + \left(2 + b_0^2 g_0^2\right) b_0 g_0 \cdot \Delta b_0 \cos\left(wt + j\right)$$

$$+ \frac{1}{4}\left(2 + 5 b_0^2 g_0^2 + 3 b_0^4 g_0^4\right)$$

$$\times g_0 \left(\Delta b_0\right)^2 \left\{1 + \cos\left[2\left(wt + j\right)\right]\right\}.$$

V.G. Baryshevsky and co-authors:

The tensor electric and magnetic polarizabilities of the deuteron manifest in the EDM experiment with vector-polarized deuterons and can be measured

- **The tensor electric polarizability stimulates the buildup of the vertical polarization of vector-polarized deuteron beam**
- **The tensor magnetic polarizability produces the spin rotation with two frequencies instead of one, beating and causes transitions between vector and tensor polarizations**

Measurement of tensor electric polarizability of the deuteron by the resonance method

- We use the matrix Hamiltonian for determining the evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \qquad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}.$$

The polarization vector and tensor:

$$P_i = \frac{\langle S_i \rangle}{S}, \qquad P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2S(S+1)\delta_{ij}}{2S(2S-1)},$$

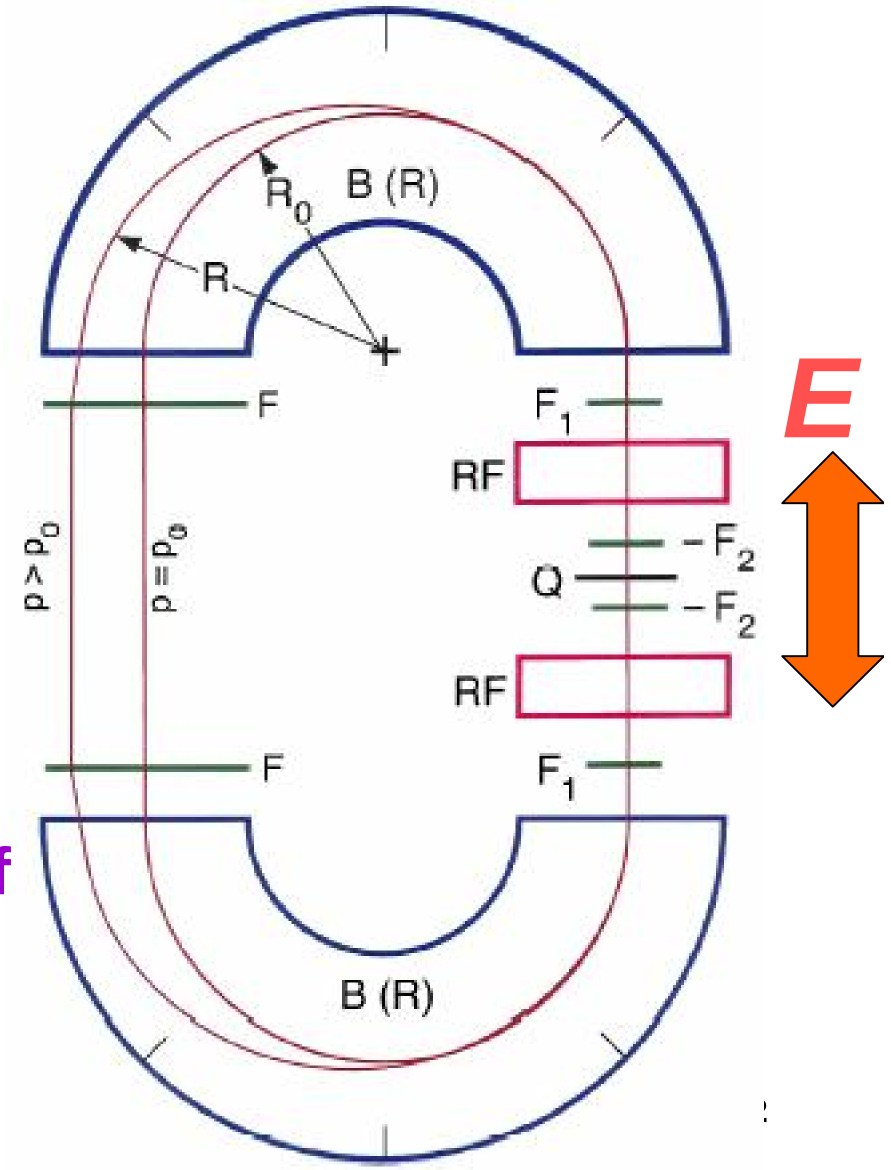
The spin matrices:

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Measurement of tensor electric polarizability of the deuteron by the resonance method

Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).



Resonance Method of Electric-Dipole-Moment Measurements in Storage Rings

Oscillating longitudinal electric field stimulates an oscillation of particle velocity in resonance with spin precession

Initial matrix Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} E_0 + w_0 + \mathrm{A} + \mathrm{B} & 0 & \mathrm{A} \\ 0 & E_0 + 2\mathrm{A} & 0 \\ \mathrm{A} & 0 & E_0 - w_0 + \mathrm{A} + \mathrm{B} \end{pmatrix},$$

$$A = a_0 + a_1 \cos(\omega t + \varphi) + a_2 \cos[2(\omega t + \varphi)],$$

$$B = b_0 + b_1 \cos(\omega t + \varphi) + b_2 \cos[2(\omega t + \varphi)],$$

$$a_0 = -\tfrac{1}{2}\alpha_T B_z^2 \gamma_0 \left[\beta_0^2 + \tfrac{1}{4}\left(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4\right)(\Delta\beta_0)^2\right],$$

$$a_1 = -\tfrac{1}{2}\alpha_T B_z^2 \left(2 + \beta_0^2\gamma_0^2\right)\beta_0\gamma_0 \cdot \Delta\beta_0,$$

$$a_2 = -\tfrac{1}{8}\alpha_T B_z^2 \left(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4\right)\gamma_0(\Delta\beta_0)^2,$$

$$b_0 = -\beta_T B_z^2 \gamma_0 \left[1 + \tfrac{1}{4}\left(1 + 3\beta_0^2\gamma_0^2\right)\gamma_0^2(\Delta\beta_0)^2\right],$$

$$b_1 = -\beta_T B_z^2 \beta_0 \gamma_0^3 \cdot \Delta\beta_0,$$

$$b_2 = -\tfrac{1}{4}\beta_T B_z^2 \left(1 + 3\beta_0^2\gamma_0^2\right)\gamma_0^3(\Delta\beta_0)^2$$

Connection between the polarization vector and tensor and the spin amplitudes:

$$P_\rho = \frac{1}{\sqrt{2}}(C_1 C_0^* + C_1^* C_0 + C_0 C_{-1}^* + C_0^* C_{-1}),$$

$$P_\phi = \frac{i}{\sqrt{2}}(C_1 C_0^* - C_1^* C_0 + C_0 C_{-1}^* - C_0^* C_{-1}),$$

$$P_z = (C_1 C_1^* - C_{-1} C_{-1}^*),$$

$$P_{\rho\rho} = \frac{3}{2}(C_1 C_{-1}^* + C_1^* C_{-1} + C_0 C_0^*) - \frac{1}{2},$$

$$P_{\phi\phi} = -\frac{3}{2}(C_1 C_{-1}^* + C_1^* C_{-1} - C_0 C_0^*) - \frac{1}{2},$$

$$P_{\rho\phi} = i\frac{3}{2}(C_1 C_{-1}^* - C_1^* C_{-1}).$$

Vector- polarized beam (initial horizontal polarization):

$$\boldsymbol{P}(0) = \sin\theta\cos\psi\,\boldsymbol{e}_\rho + \sin\theta\sin\psi\,\boldsymbol{e}_\phi + \cos\theta\,\boldsymbol{e}_z$$

$$P_{\rho\rho} = \tfrac{1}{2}\left[3\sin^2(\theta)\cos^2(\psi) - 1\right],$$

$$P_{\phi\phi} = \tfrac{1}{2}\left[3\sin^2(\theta)\sin^2(\psi) - 1\right], \quad P_{\rho\phi} = \tfrac{3}{4}\sin^2(\theta)\sin(2\psi)$$

Θ and ψ are spherical angles

Tensor- polarized beam (initial horizontal polarization and $S_l = 0$):

$$P(0) = 0, \quad P_{\rho\rho}(0) = 1 - 3\sin^2\theta\cos^2\psi,$$

$$P_{\phi\phi}(0) = 1 - 3\sin^2\theta\sin^2\psi, \quad P_{zz}(0) = 1 - 3\cos^2\theta,$$

$$P_{\rho\phi}(0) = -\frac{3}{2}\sin^2\theta\sin(2\psi),$$

$$P_{\rho z}(0) = -\frac{3}{2}\sin(2\theta)\cos\psi, \quad P_{\phi z}(0) = -\frac{3}{2}\sin(2\theta)\sin\psi.$$

Weaker resonance at the frequency $\omega \approx \omega_0$

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}\left(1 - \cos\left(2\omega' t\right)\right)\right] P_z(0)$$
$$+\frac{2\mathcal{E}_0}{3\omega'}\left\{\frac{1}{2}[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{\omega_0 - \omega}{\omega'}\cos\left(2\varphi\right)\left(1 - \cos\left(2\omega' t\right)\right) - \sin\left(2\varphi\right)\sin\left(2\omega' t\right)\right]\right.$$
$$\left.+P_{\rho\phi}(0)\left[\frac{\omega_0 - \omega}{\omega'}\sin\left(2\varphi\right)\left(1 - \cos\left(2\omega' t\right)\right) + \cos\left(2\varphi\right)\sin\left(2\omega' t\right)\right]\right\}, \quad \mathcal{E}_0 = \frac{a_2}{2},$$

Stronger resonance at the frequency $\omega \approx 2\omega_0$

$$P_z(t) = \left[1 - \frac{4\mathcal{E}_0'^2}{\omega''^2}\left(1 - \cos\left(\omega'' t\right)\right)\right] P_z(0)$$
$$+\frac{2\mathcal{E}_0'}{3\omega''}\left\{[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{2\omega_0 - \omega}{\omega''}\cos\left(\varphi\right)\left(1 - \cos\left(\omega'' t\right)\right) - \sin\left(\varphi\right)\sin\left(\omega'' t\right)\right]\right.$$
$$\left.+2P_{\rho\phi}(0)\left[\frac{2\omega_0 - \omega}{\omega''}\sin\left(\varphi\right)\left(1 - \cos\left(\omega'' t\right)\right) + \cos\left(\varphi\right)\sin\left(\omega'' t\right)\right]\right\}, \quad \mathcal{E}_0' = \frac{a_1}{2},$$

Final vertical beam polarization for initial horizontal vector polarization and ω≈ω_0:

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}[1 - \cos(2\omega' t)]\right]\cos(\theta)$$

$$+\frac{\mathcal{E}_0}{2\omega'}\sin^2(\theta)\left\{\frac{\omega_0 - \omega}{\omega'}\cos[2(\psi - \varphi)][1 - \cos(2\omega' t)]\right.$$

$$\left. + \sin[2(\psi - \varphi)]\sin(2\omega' t)\right\}$$

Final vertical beam polarization for initial horizontal tensor polarization and ω≈$2\omega_0$:

$$P_z(t) = -\frac{2\mathcal{E}_0'}{\omega''}\sin^2(\theta)\left\{\frac{2\omega_0 - \omega}{\omega''}\cos(2\psi - \varphi)[1 - \cos(\omega'' t)]\right.$$

$$\left. + \sin(2\psi - \varphi)\sin(\omega'' t)\right\}.$$

Theoretical data

Tensor electric polarizability of deuteron:

$\alpha_T = -6.2 \times 10^{-41}$ cm^3 — J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. **A644**, 221 (1998).

$\alpha_T = -6.8 \times 10^{-41}$ cm^3 — X. Ji and Y. Li, Phys. Lett. **B591**, 76 (2004).

$\alpha_T = 3.2 \times 10^{-41}$ cm^3 — J. L. Friar and G. L. Payne, Phys. Rev. C **72**, 014004 (2005).

Tensor magnetic polarizability of deuteron:

$\beta_T = 1.95 \times 10^{-40}$ cm^3 — J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. **A644**, 221 (1998).

X. Ji and Y. Li, Phys. Lett. **B591**, 76 (2004).

Estimated expirimental sensitivity of the resonance method

Tensor electric polarizability of deuteron:

$\delta\alpha_T \sim 10^{-45} \div 10^{-44}\ \mathrm{cm}^3$

(initial tensor-polarized beam)

A. J. Silenko, Phys. Rev. C **75**, 014003 (2007).

Tensor magnetic polarizabilities of the deuteron and other nuclei in storage ring experiments

Tensor magnetic polarizability of the deuteron can be measured without both a resonance field and a restriction of spin rotation

Only vertical magnetic field is used

A. J. Silenko, Phys. Rev. C **77**, 021001(R) (2008).

$$A = -\frac{1}{2} a_T b^2 g B_z^2,$$

$$B = -b_T g B_z^2.$$

When the direction of the initial tensor polarization is defined by the spherical angles *θ* and *ψ*,

$$P_r(t) = \sin\left(2q\right)\sin\left(w_0 t + y\right)\sin\left(bt\right),$$

$$P_f(t) = -\sin\left(2q\right)\cos\left(w_0 t + y\right)\sin\left(bt\right),$$

$$P_z(t) = 0,$$

$$b = B - A = -\left(b_T - \frac{1}{2}a_T b^2\right) g B_z^2.$$

When *t ~ 1000 s*, *$P_{\|}$ ~ 1 %.*
This polarization is measurable.

Measurement of tensor electric and magnetic polarizabilities of the deuteron by the frozen spin method

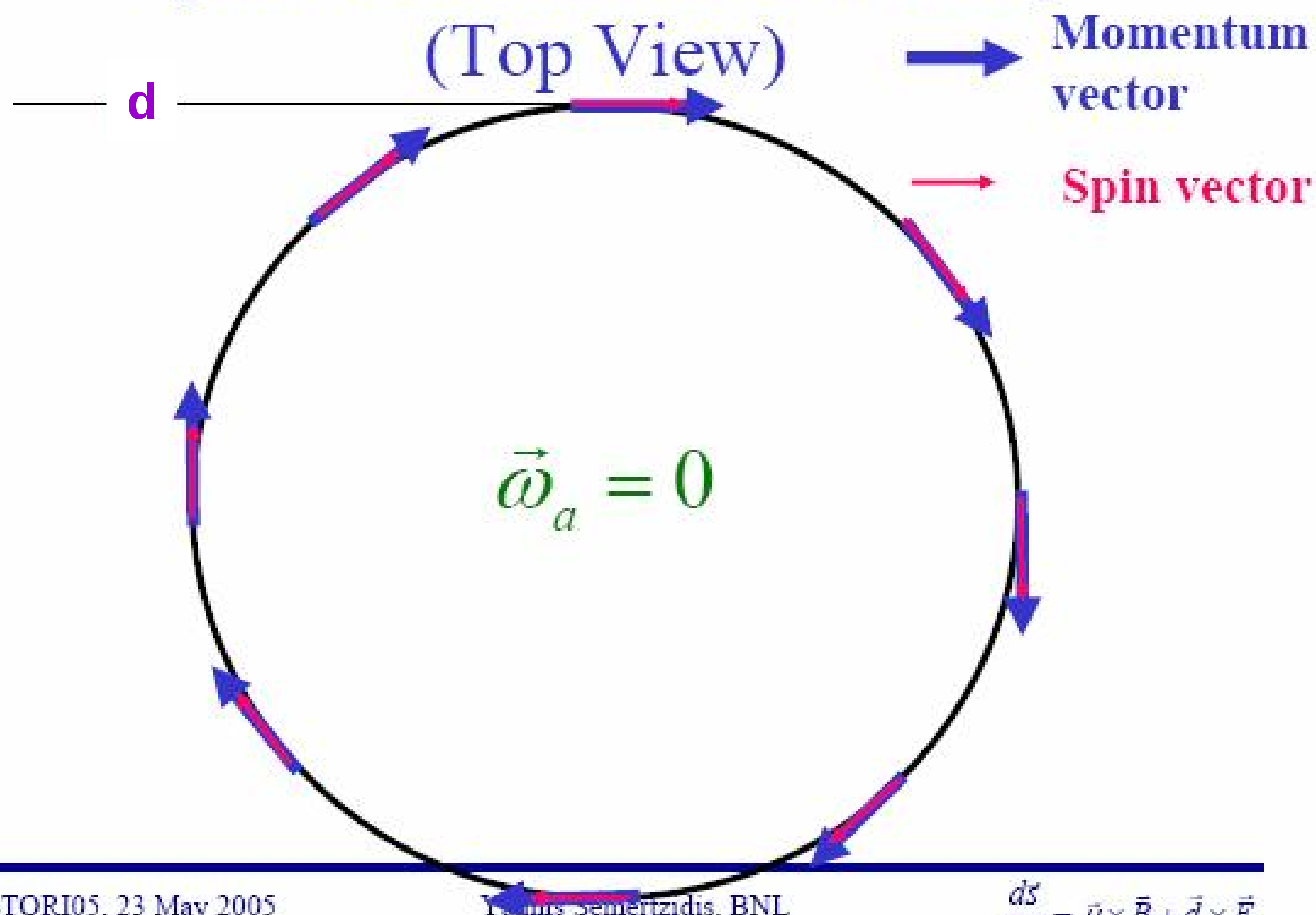
$$\boldsymbol{\omega}_a = -\frac{e}{m}\left[a\mathbf{B} - \frac{ag}{g+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{B}) + \left(\frac{1}{g^2-1} - a\right)\boldsymbol{\beta}\times\mathbf{E}\right].$$

$$a = \frac{g-2}{2}, \qquad \boldsymbol{\beta}\cdot\mathbf{B} = 0, \qquad E_r = \frac{ab_f g^2}{1 - ab^2 g^2} B_z.$$

The spin orientation relatively the momentum direction remains unchanged!

F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, Phys. Rev. Lett. **93**, 052001 (2004).

Spin Precession in EDM Ring
(Top View)



STORI05, 23 May 2005 Y... Semertzidis, BNL

$$\frac{ds}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}$$

Correction to the Hamilton operator

$$V = -a_T g\left(b_f B_z + E_r\right)^2 S_r^2$$

$$-b_T g\left(B_z + b_f E_r\right)^2 S_z^2.$$

$$V = -\frac{g}{\left(1-ab^2 g^2\right)^2} B_z^2 \left[a_T (1+a)^2 b^2 S_r^2 + b_T S_z^2\right].$$

$$A = -a_T \frac{(1+a)^2 b^2 g}{2\left(1-ab^2 g^2\right)^2} B_z^2,$$

$$B = -b_T \frac{g}{\left(1-ab^2 g^2\right)^2} B_z^2.$$

When the direction of the initial tensor polarization is defined by the spherical angles *θ* and *ψ*,

$$P_r(t) = \sin(2q)\left[bt \cdot \sin(w_0 t + y) + \frac{A}{w_0}\sin(w_0 t)\sin y\right],$$

$$P_f(t) = \sin(2q)\left[-bt \cdot \cos(w_0 t + y) + \frac{A}{w_0}\sin(w_0 t)\cos y\right],$$

$$P_z(t) = -\frac{2A}{w_0}\sin^2 q \,\sin(w_0 t)\sin(w_0 t + 2y),$$

$$b = B - A = -\frac{g}{\left(1 - ab^2 g^2\right)^2} B_z^2 \left[b_T - \frac{1}{2} a_T (1+a)^2 b^2\right].$$

For the tensor electric polarizability of the deuteron, the resonance method and the frozen spin method provide the experimental sensitivity of the same order of magnitude

The use of the frozen spin method improves the experimental conditions of measurement of tensor magnetic polarizability of the deuteron

Summary

- Spin dynamics caused by the tensor electric and magnetic polarizabilities of the deuteron and other nuclei in storage rings is calculated
- Potential for the measurement of the tensor polarizabilities of the deuteron and other nuclei in experiments with polarized beams is investigated
- For the tensor electric polarizability of the deuteron, the resonance method and the frozen spin method provide the experimental sensitivity of the order of $10^{-45} \div 10^{-44}$ cm^3
- The use of the frozen spin method provides the measurement of the tensor magnetic polarizability of the deuteron

Thank you for attention