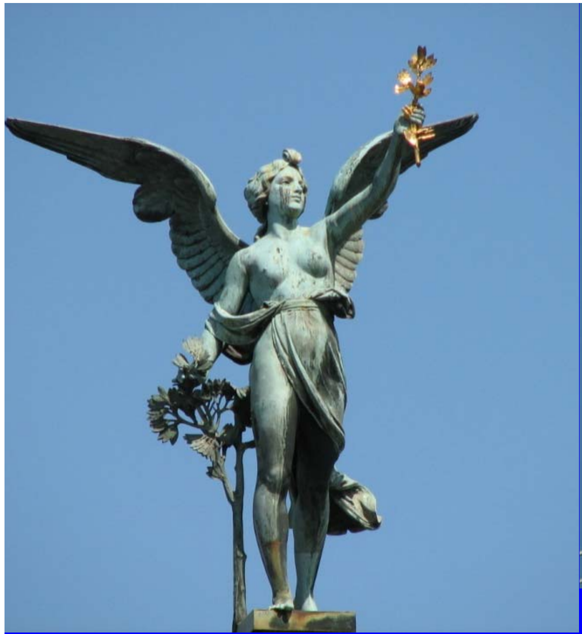
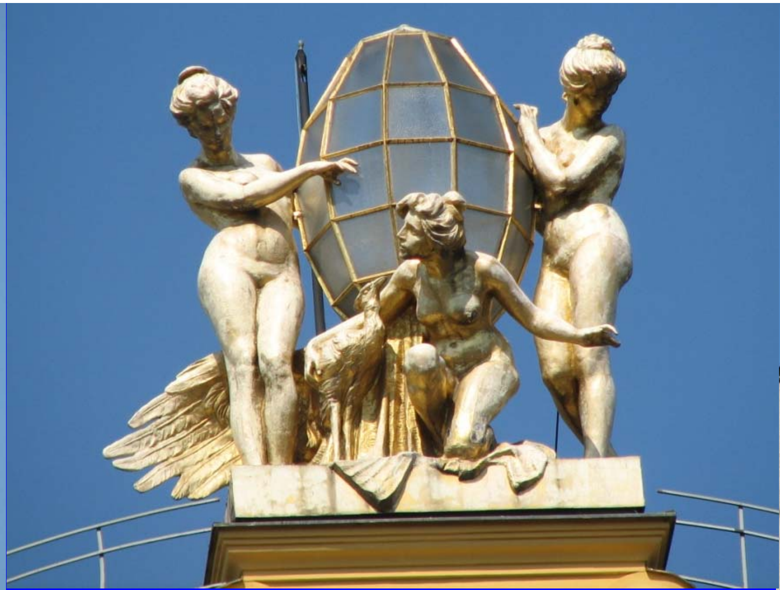
  

ASI-2008

***PRAHA -2008***

« SYMMETRY and SPIN »

# Spin-flip amplitude in the impact parameter representantation


Oleg V. Selyugin

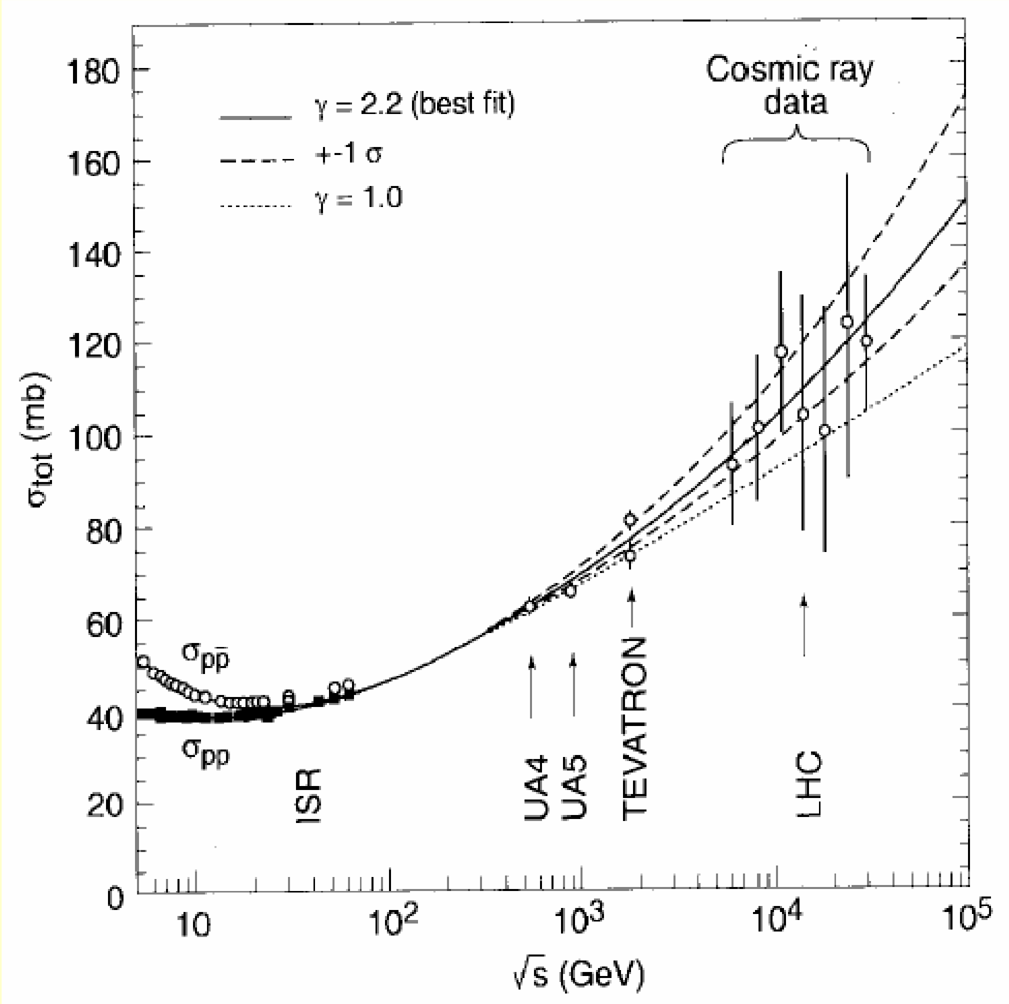
*BLTPh,JINR*

- Introduction
- Structure of hadron elastic scattering amplitude
- Non-linear equations and saturation
- Basic properties of the unitarization schemes
- b-dependence of the phases
- Analysing power in different unitarization schemes

- Summary

# Total cross-section

- Understand the asymptotic behavior of $\sigma_{tot}$



# ***Predictions*** $\sqrt{s} = 14\,TeV$

| Model | $\sigma_{\tau o \tau}$ | $\sigma_{el}/\sigma_{\tau o \tau}$ | ❒☎t =0) | B ☎t=0) |
|---|---|---|---|---|
| COMPET | 111 | | 0.11 | |
| Marsel | 103 | 0.28 | 0.12 | 19 |
| Dubna | 128 | 0.33 | 0.19 | 21. |
| Pomer.(s+h) | 150 | 0.29 | 0.24 | 21.4 |
| Serpukhov | 230 | 0.67 | | |

# Pomerons

*Regge theory*

$\sigma_{tot}(s) \sim$

*Landshoff 1984 – soft Π*

$\alpha = 0.08$

*Simple pole –* $(s/s_0)^\alpha$

*Double pole – Ln* $(s/s_0)$

*Triple pole – Ln*$^2(s/s_0)$ *+C*

*1976 BFKL (LO) - Π –* $\alpha_2 = 0.4$

*1988 HERA data (Landshoff ) – hard Π* $\alpha_2 = 0.45$

*2005 – Kovner – 7 Π –* $\alpha_\iota = 0. ..0.4. .0.8$

$$\frac{1}{2}^{+} + \frac{1}{2}^{+} \rightarrow \frac{1}{2}^{+} + \frac{1}{2}^{+} \;;$$

$$\begin{aligned}
M(s,t) = {} & T_1(s,t)\overline{u}(p_2)u(p_1)\overline{u}(k_2)u(k_1) \\
& + T_2(s,t)\overline{u}(p_2)\gamma K\, u(p_1)\overline{u}(k_2)\gamma P\, u(k_1) \\
& + T_3(s,t)\overline{u}(p_2)\gamma_5(\gamma K)u(p_1)\overline{u}(k_2)\gamma_5(\gamma P)u(k_1) \\
& + T_4(s,t)\overline{u}(p_2)\gamma_5\, u(p_1)\overline{u}(k_2)\gamma_5\, u(k_1) \\
& + T_5(s,t)[\overline{u}(p_2)(\gamma K)u(p_1)\overline{u}(k_2)u(k_1) + \overline{u}(p_2)\, u(p_1)\overline{u}(k_2)(\gamma P)u(k_1)] \\
& + T_6(s,t)[\overline{u}(p_2)(\gamma K)u(p_1)\overline{u}(k_2)u(k_1) - \overline{u}(p_2)\, u(p_1)\overline{u}(k_2)(\gamma P)u(k_1)] \\
& + T_7(s,t)\overline{u}(p_2)\gamma_5\, u(p_1)\overline{u}(k_2)\gamma_5(\gamma P)u(k_1) \\
& + T_8(s,t)\overline{u}(p_2)\gamma_5(\gamma K)u(p_1)\overline{u}(k_2)\gamma_5\, u(k_1) \qquad ;
\end{aligned}$$

# Regge limit

*for fixed* **t**

*The crossing matrix factorized in the limit* $s \to \infty;$

*the Regge-pole contributions to the helicity amplitudes in the s-chanel*

$$\Phi^B_{\lambda_3\lambda_4\lambda_1\lambda_2}(s,t) = \sum_i g^i_{\lambda_3\lambda_1} g^i_{\lambda_4\lambda_2} [\sqrt{-t}\,]^{|\lambda_3-\lambda_1|+|\lambda_4-\lambda_2|} (\frac{s}{s_0})^{\alpha_i} (1 \pm \exp[-i\pi\alpha_i])$$

$$\Phi^B_{++--}(s,t) = \Phi^B_{+--+}(s,t) \to 0$$

*when exchanged Regge poles have natural parity*

$$\Phi^B_{++++}(s,t) = \Phi^B_{+-+-}(s,t).$$

# Impact parameter representation

$$M(s,t) = \frac{ip}{2\pi}\int_0^\infty \Gamma(s,b)\; e^{-i\vec{q}\vec{b}}\; d^2b$$

$$M(s,t) = \frac{1}{2\pi}\int_0^\infty b\; J_0(b\,q)\;\Gamma(s,b)\,db$$

## Unitarity in impact parameter representation

$$S\,S^{+} = 1$$

$$\mathrm{Im} < p_1, p_2, out \,|\, T \,|\, p_1, p_2, in > = \frac{(2\pi)^4}{2} \sum_{\gamma} d\gamma\, \delta(\sum_{1}^{2} p_r - (\sum_{1}^{n} q_r) |\, T_{\gamma\alpha} |^2$$

$$T(s,t) = is \int_{0}^{\infty} b\, db\, J_0(bq)\, \Gamma(b,s)$$

$$\left|\Gamma(s,b)\right| \leq 1;$$

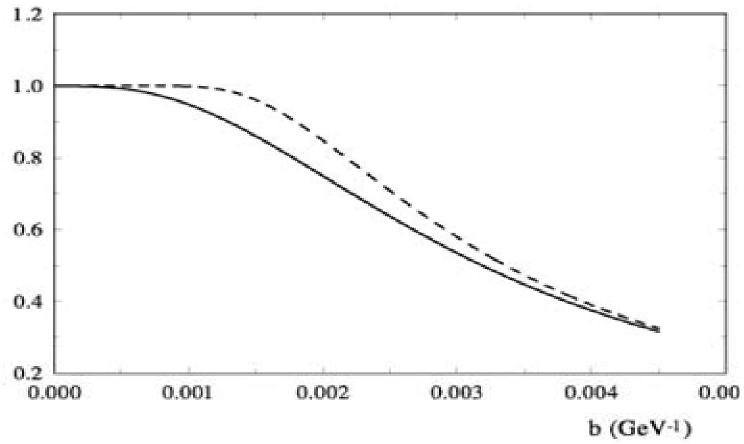
$$\mathrm{Im}\,\Gamma(s,b) = |\,\mathrm{Im}\,\Gamma(s,b)\,|^2 + |\,\mathrm{Re}\,\Gamma(s,b)\,|^2 + g_{inel}$$

# Saturation bound

$$T(s,t) = \frac{1}{2\pi}\int_0^\infty b\ J_0(bq)\,[1-e^{-\chi(s,b)}]\,db$$

$$\chi(s,b) = 2\pi\int_0^\infty q\ J_0(bq)\ M_B(s,q)\,dq$$

$$\chi(s,b) = -\frac{1}{2k}\int_{-\infty}^{\infty} dz V[\sqrt{z^2+b^2}]$$



# Non-linear equation (K-matrix)

$$N[y] = \Gamma(s,0);$$

$$\frac{dN}{dy} = \Delta N(1-N);$$

$$y = \ln(s/s_0);$$

$$N[y] = \frac{s^{\Delta y} f(b)}{1+s^{\Delta y} f(b)};$$

$$N[y] = \frac{i\,\chi(s,b)}{1+\chi(s,b)};$$

$$\Gamma(s,b) = \frac{K(s,b)}{1-i\,K(s,b)};$$

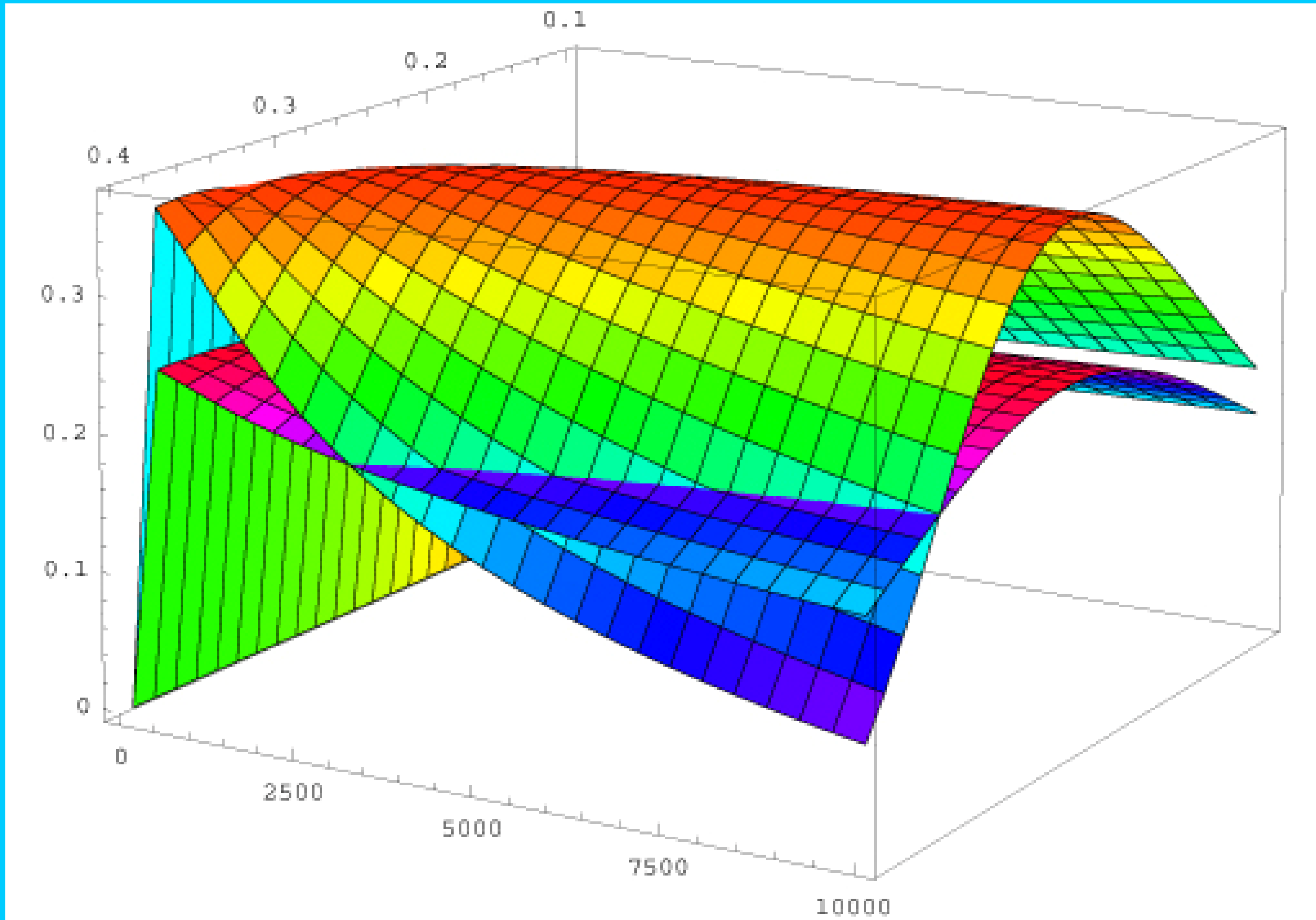
# Non-linear equation (eikonal)

$$\frac{dN}{dy} = (-Ln[1-N])\ (1-N);$$

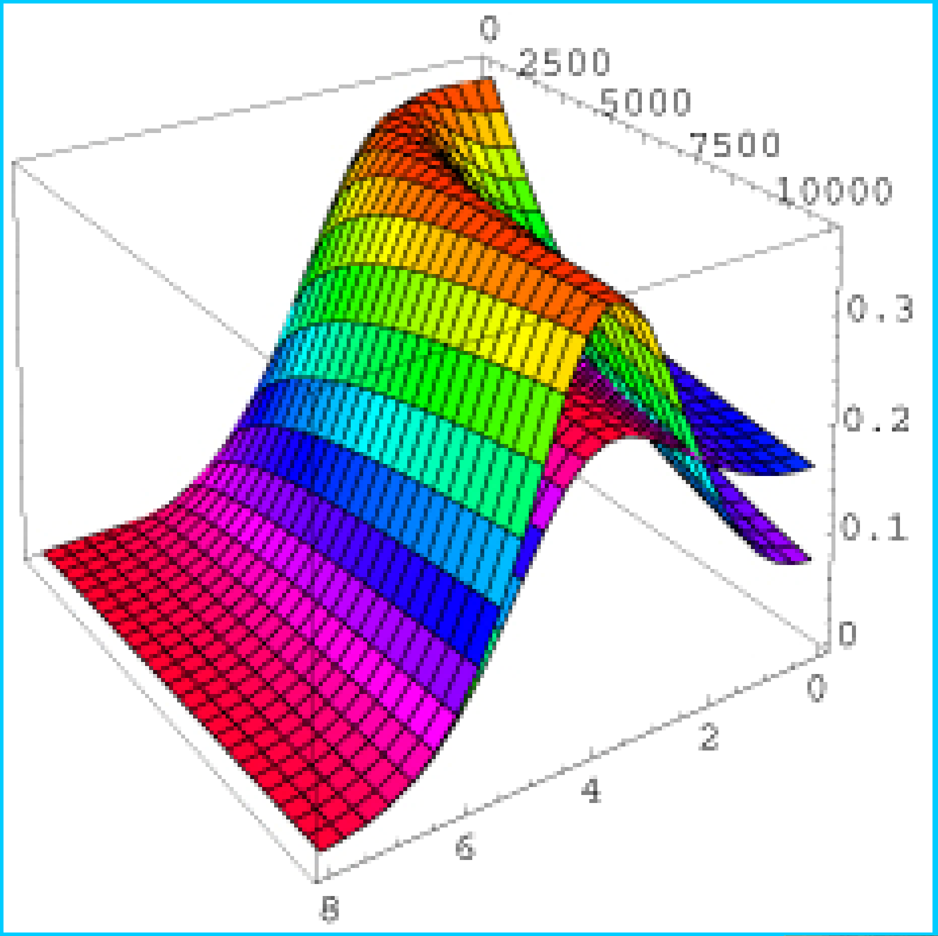
$$N(s,b) = 1 - \exp[i\,\chi(s,b)];$$

# Eikonal and K-matrix

$\frac{dN}{dy}$



# Eikonal and K-matrix



# Interpolating form of unitarization

$$G(s,b) = 1-(1+\chi(s,b)/\gamma)^{-\gamma};$$

$$\gamma = 1; \qquad G(s,b) = 1-\frac{1}{(1+\chi(s,b))} = \frac{\chi(s,b)}{1+\chi(s,b)};$$

$$\gamma \to \infty; \qquad G(s,b) = 1-Exp(-\chi(s,b);$$

$$\frac{dN}{dy} = \gamma\,(1-[1-N]^{1/\gamma})\,(1-N);$$

# Normalization and forms of unitarization

$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\,db \frac{1}{2}\,(1-\exp[-\chi(s,b)])$$

$$\chi(s,b) = 2\;\delta(s,b);$$

$$\sigma_{tot}(s) = 4\pi \int_0^\infty b\,db\;(1-\exp[-\chi(s,b)])$$

$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\,db\;\frac{U(s,b)}{1+U(s,b)};$$

$$U(s,b) = \delta(s,b) = \chi(s,b)/2;$$

$$\sigma_{tot}(s) = 4\pi \int_0^\infty b\,db\;\frac{\chi(s,b)}{1+\chi(s,b)/2};$$

Low energies $\chi_{eik}(s,b) = \chi_{U-m}(s,b) = F_{Born}(s,t);$

# Inelastic cross sections

eikonal

$$\sigma^{eik}{}_{inel}(s) = 2\pi\int_0^\infty b\,db\ \ (1-\exp[-2\chi(s,b)])$$

U-matrix

$$\frac{dN_u}{dy} = \Delta N_u\,(1-N_n / c_u); \qquad c_u = 2;$$

$$\sigma^{U-m}{}_{inel}(s) = 2\pi\int_0^\infty b\,db\ \frac{\chi(s,b)}{(1+\chi(s,b)/2)^2};$$

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \to 1;$$

# Asymptotic features of unitarization schemes

**Low energies**

$$G_{eik}(s,b) \approx \chi(s,b);$$

$$G_{U-m}(s,b) \approx \chi(s,b);$$

**High energies**

$$G_{eik}(s, b \le b_{sat}) = 1;$$

$$G_{U-m}(s, b \le b_{sat}) = 2;$$

# Effect of antishadowing

*S. Troshin (hep-ph/0701241 v4  June 4*

*« Black  Disk  Limit is a direct consequence of the exponential unitarization with an extra assumption on the pure imaginary nature of the phase shift »*

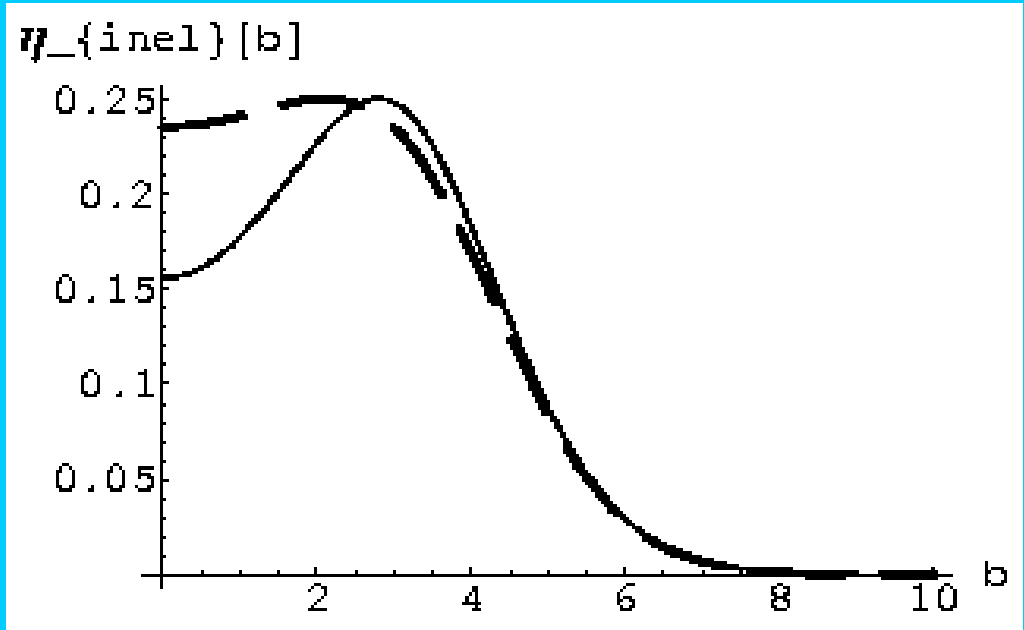
*Renormalized eikonal*

$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\, db \ (1-\exp[-\delta(s,b)])$$

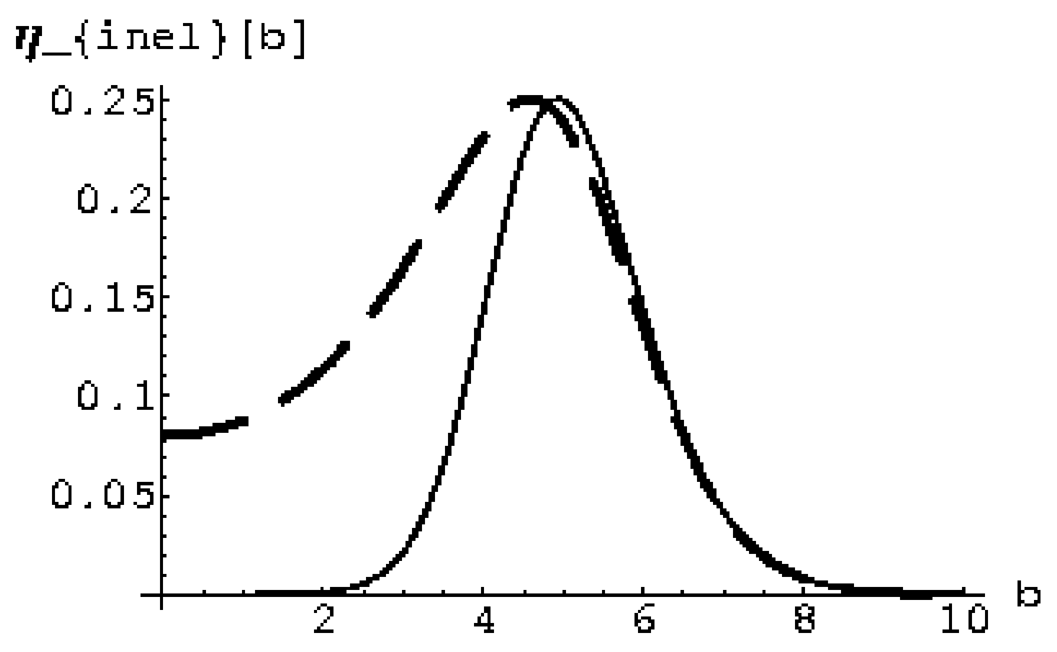
$$\sigma_{inel}(s) = 8\pi \int_0^\infty b\, db \ \mathrm{e}^{-\delta(s,b)} (1-\mathrm{e}^{-\delta(s,b)})$$

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \to 1;$$

# 2 TeV



# 14 TeV



# Corresponding phases

$$f_l(s) = \frac{i}{2}(1 - \sqrt{1 - 4\eta_l(s)}\,), \quad |f_l| \leq \frac{1}{2}$$

$$f_l(s) = \frac{i}{2}(1 + \sqrt{1 - 4\eta_l(s)}\,), \quad \frac{1}{2} \leq |f_l| \leq 1.$$

$$\chi_{U-m}(s,b) = 2\ tanh\{\frac{1}{2}\chi_{eik}(s,b)\};$$

$$\chi_{eik}(s,b) = log\{\frac{1 + \chi_{U-m}(s,b)/2}{1 - \chi_{U-m}(s,b)/2}\};$$

# Renormalization of the U-matrix -→ K-matrix

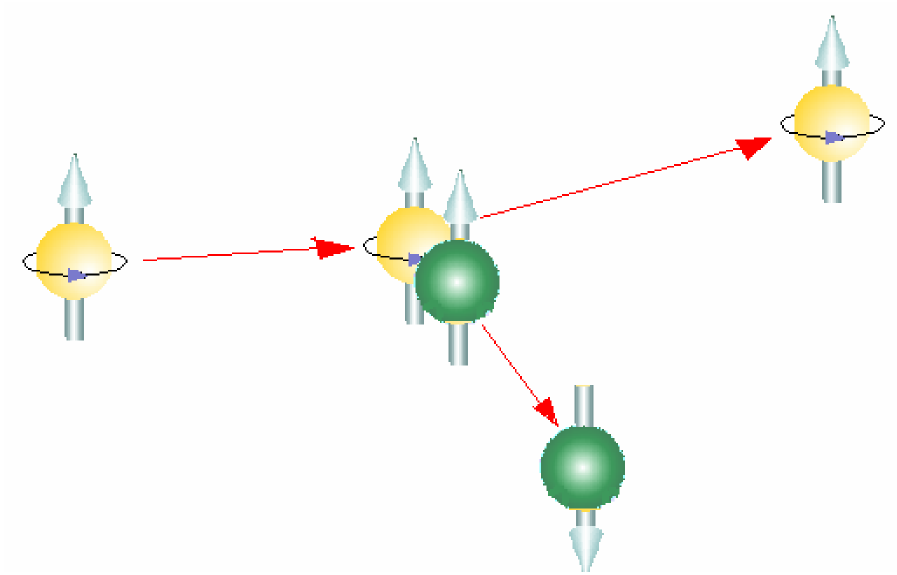
$$S = \frac{1 - \chi(s,b)/2}{1 + \chi(s,b)/2}; \qquad G(s,b) = 1 - S = \frac{\chi(s,b)}{1 + \chi(s,b)/2};$$

$$S = \frac{1}{1 + \chi(s,b)}; \qquad G(s,b) = 1 - S = \frac{\chi(s,b)}{1 + \chi(s,b)};$$

$$\sigma_{tot}(s) = 4\pi \int_0^\infty b\, db\, \frac{\chi(s,b)}{1 + \chi(s,b)};$$

$$G_{inel}(s,b) = \frac{\chi^2(s,b) + \chi(s,b)/2}{(1 + \chi(s,b))^2}$$

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} = \frac{1}{2};$$



*RHIC beams +internal targets ≡*

*fixed target mode*

*$\sqrt{s} \sim 14$ GeV*

# Analysing power

$$A_N = \frac{\sigma^{\Uparrow} - \sigma^{\Downarrow}}{\sigma^{\Uparrow} + \sigma^{\Downarrow}}$$

$$\frac{d\sigma}{dt} = \pi \left| e^{i\alpha\varphi} F_C(t) + F_N(s,t) \right|^2$$

$$A_N \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \operatorname{Im}[F_{nfl} F_{fl}^*]$$

$$A_N \frac{d\sigma}{dt} = \frac{4\pi}{s^2} |F_{nfl}||F_{fl}^*| \sin(\varphi_1 - \varphi_2)$$

$$M(s,t) = \frac{ip}{2\pi} \int_0^\infty \Gamma(s,b)\; e^{-i\vec{q}\vec{b}}\, d^2 b$$

$$T(s,t) = \frac{1}{2\pi} \int_0^\infty b\; J_0(bq)\; [1 - e^{-\hat{\chi}(s,b)}] db$$

$$\widehat{\chi}(s,b) = \chi_c(s,b) - i\chi_{LS}(s,b)(\sigma_1 + \sigma_2) \bullet (b \times \widehat{l}\,)$$

# *Eikonal case*

$$F_{nf}(s,t) = ip\int_0^\infty b\ J_0(bq)\ [1 - e^{-\chi_c(s,b)}]db$$

$$F_{sf}(s,t) = p\int_0^\infty b^2\ J_1(bq)\,\chi_{LS}\ e^{-\chi_c(s,b)}\,db$$

$$F^{B}_{\lambda_3\lambda_4\lambda_1\lambda_2}(p,q) = U_{\lambda_3\lambda_4\lambda_1\lambda_2}(p,q) + i\frac{\pi}{8}\sum_{\lambda'\lambda''} d\Omega_{\hat{k}} U_{\lambda_3\lambda_4\lambda'\lambda''}(p,k) F^{B}_{\lambda_3\lambda_4\lambda'\lambda''}(k,q)$$

$$\Gamma_1(s,b) = \frac{(u_1 + u_1^2 - u_2^2)(1 + u_3 + u_4) - 2(1 + 2u_1 - 2u_2)u_5^2}{(1 + u_1 - u_2)(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2}$$

$$\approx \frac{u_1}{1 + u_1};$$

$$\Gamma_5(s,b) = \frac{u_5}{(1 + u_1 + u_2)(1 + u_3 + u_4) - 4u_5^2} \approx \frac{u_5}{(1 + u_1)^2};$$

$$F_{sf}(s,t) = p\int_0^\infty b^2 \; J_1(bq)\frac{u_5}{(1 + u_1)^2} db$$

## *K-matrix*

$$\Gamma(s,b)=\frac{\chi_c+\sigma\chi_{sf}}{(1+\chi_c+\sigma\chi_{sf})}=1-\frac{1}{(1+\chi_c+\sigma\chi_{sf})}=1-\frac{(1+\chi_c)-\sigma\chi_{sf}}{(1+\chi_c)^2-(\sigma\chi_{sf})^2}$$

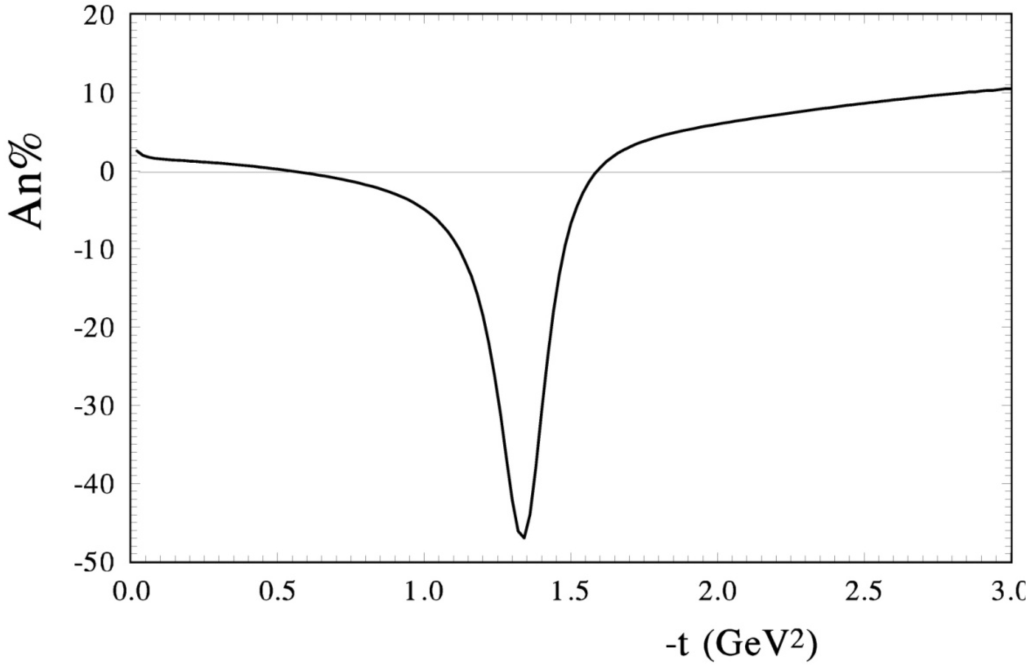
$$=\frac{\chi_c}{1+\chi_c}+\sigma\frac{\chi_{sf}}{(1+\chi_c)^2}\ ;$$

## *U-matrix*

$$\Gamma(s,b)=\frac{\chi_c+\sigma\chi_{sf}}{(1+\chi_c/2+\sigma\chi_{sf}/2)}=\frac{\chi_c}{1+\chi_c/2}+\sigma\frac{\chi_{sf}}{(1+\chi_c/2)^2}\ ;$$

# $\underline{A_N\text{ - eikonal}}$

$$\Gamma_+^{Eik}(s,b) = (1-\exp[-\chi_{nf}(s,b)])$$



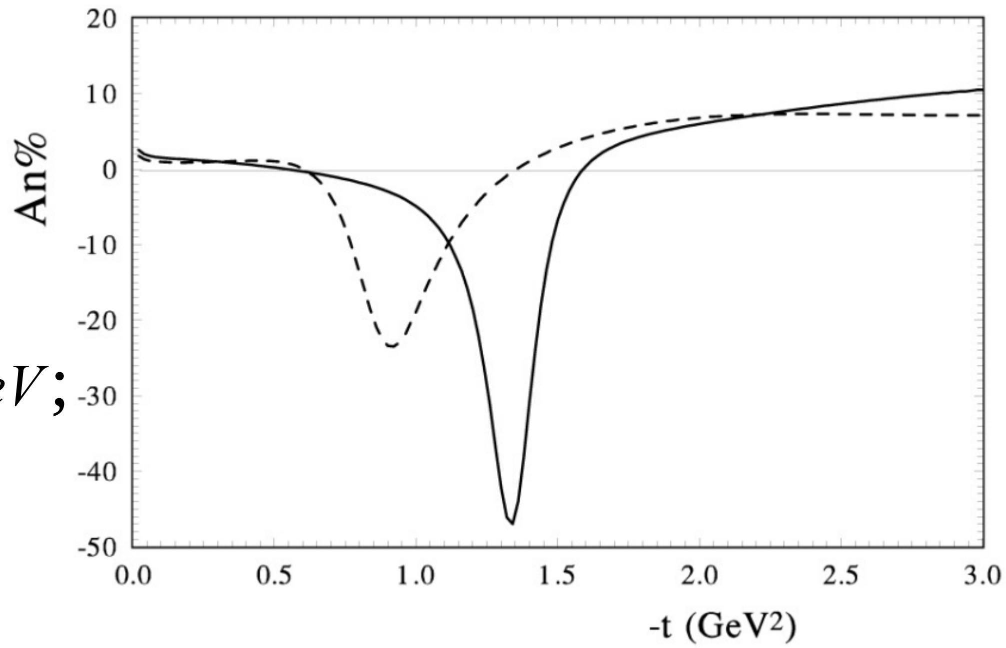

$\sqrt{s} = 50\ GeV;$

$$\Gamma_-^{Eik}(s,b) = \chi_{sf}(s,b)\exp[-\chi_{nf}(s,b)])$$

# $\underline{A_N \text{ - eikonal}}$

$$\Gamma_{+}^{Eik}(s,b) = (1 - \exp[-\chi_{nf}(s,b)])$$

$\sqrt{s} = 50\ GeV;$

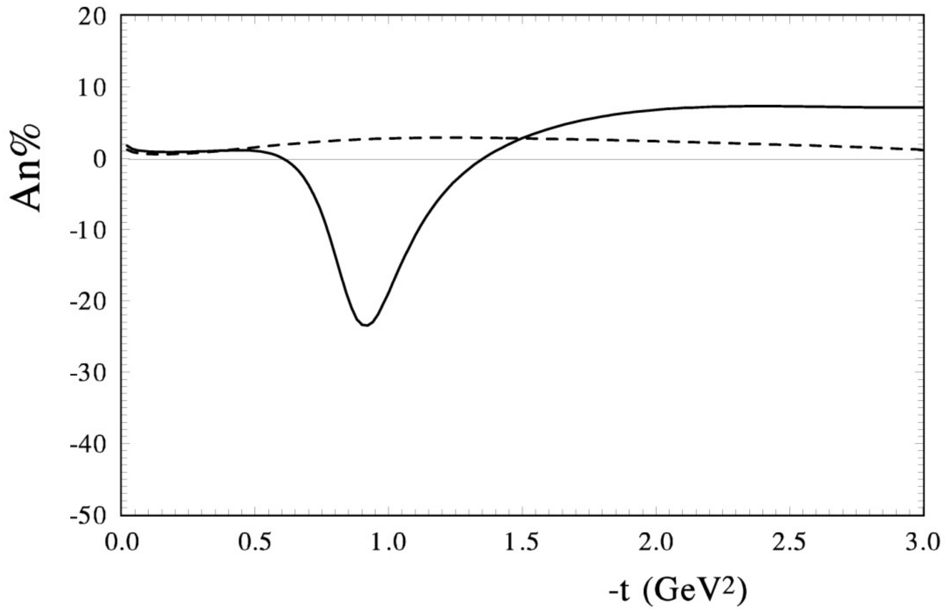



$\sqrt{s} = 500\ GeV;$

$$\Gamma_{-}^{Eik}(s,b) = \chi_{sf}\ \exp[-\chi_{nf}(s,b)])$$

# $A_N$ – Born term

$$\Gamma_{+}^{Born}(s,b) = \chi_{nf}(s,b)$$

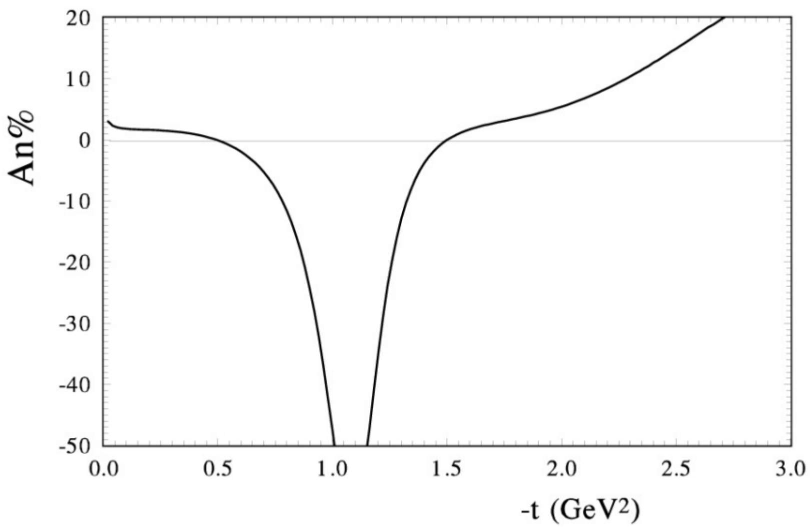



$$\sqrt{s} = 500\ GeV;$$

$$\Gamma_{-}^{Born}(s,b) = \chi_{sf}(s,b)$$

## $\underline{A_N - K\text{-}matrix}$

$$\Gamma_{+}^{K-m}(s,b) = \frac{\chi_{nf}(s,b)}{1+\chi_{nf}(s,b)}$$



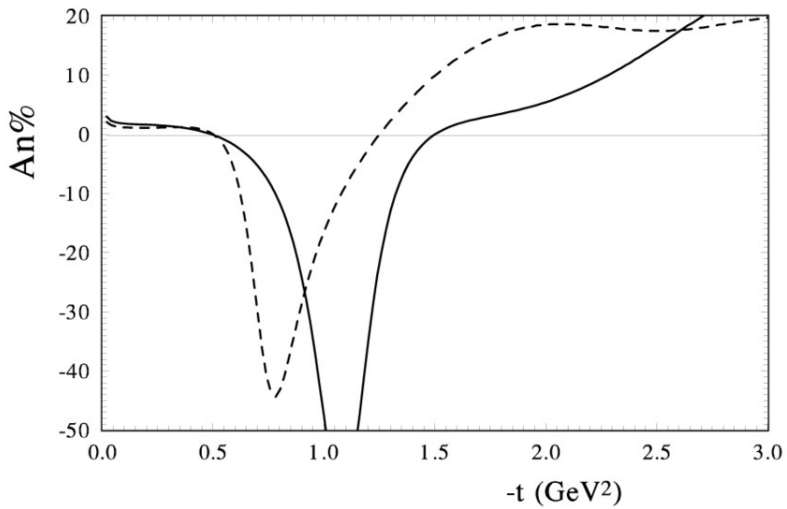

$\sqrt{s} = 50\ GeV;$

$$\Gamma_{-}^{K-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi_{nf}(s,b))^2}$$

## $\underline{A_N - K\text{-}matrix}$

$$\Gamma_{+}^{K-m}(s,b) = \frac{\chi_{nf}(s,b)}{1+\chi_{nf}(s,b)}$$

$\sqrt{s} = 500\ GeV;$

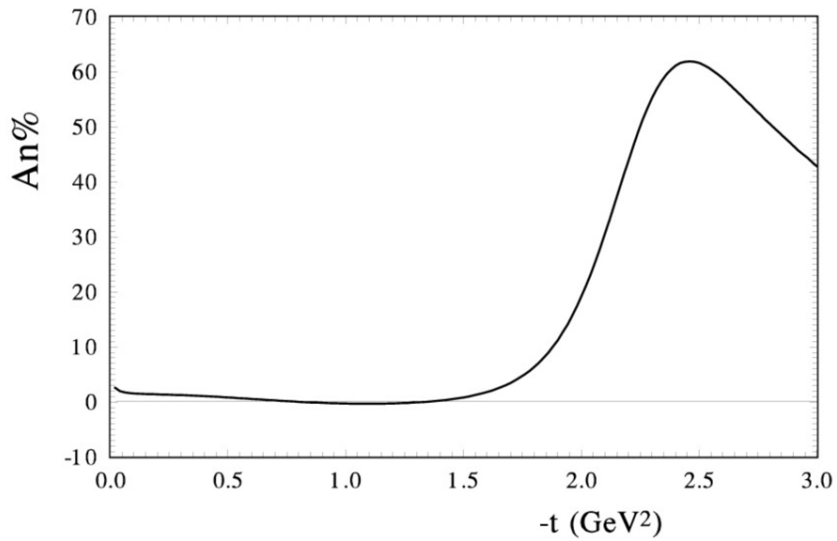



$\sqrt{s} = 50\ GeV;$

$$\Gamma_{-}^{K-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi_{nf}(s,b))^2}$$

# $A_N - U_T$*-matrix*

$$\Gamma_{+}^{U-m}(s,b) = \frac{\chi_{nf}(s,b)}{1+\chi_{nf}(s,b)/2}$$



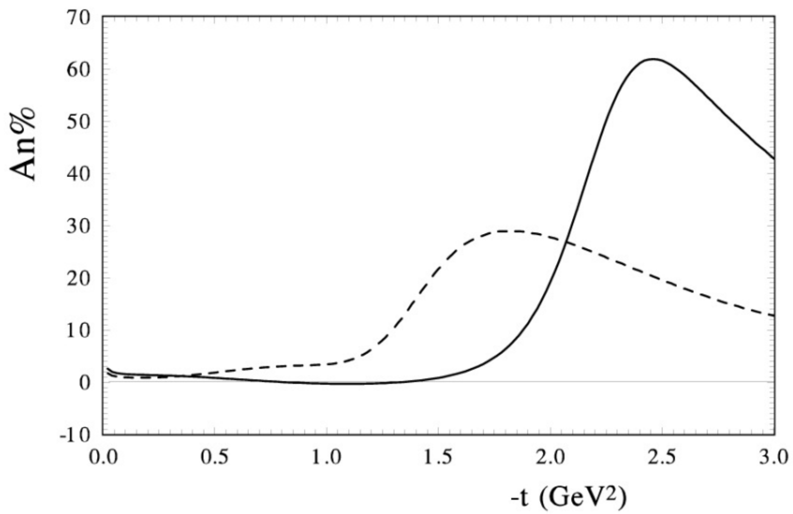

$\sqrt{s} = 50\ GeV;$

$$\Gamma_{-}^{U-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi_{nf}(s,b)/2)^2}$$

## $A_N - U_T$-matrix

$$\Gamma_{+}^{U-m}(s,b) = \frac{\chi_{nf}(s,b)}{1+\chi_{nf}(s,b)/2}$$

$\sqrt{s} = 500\ GeV;$



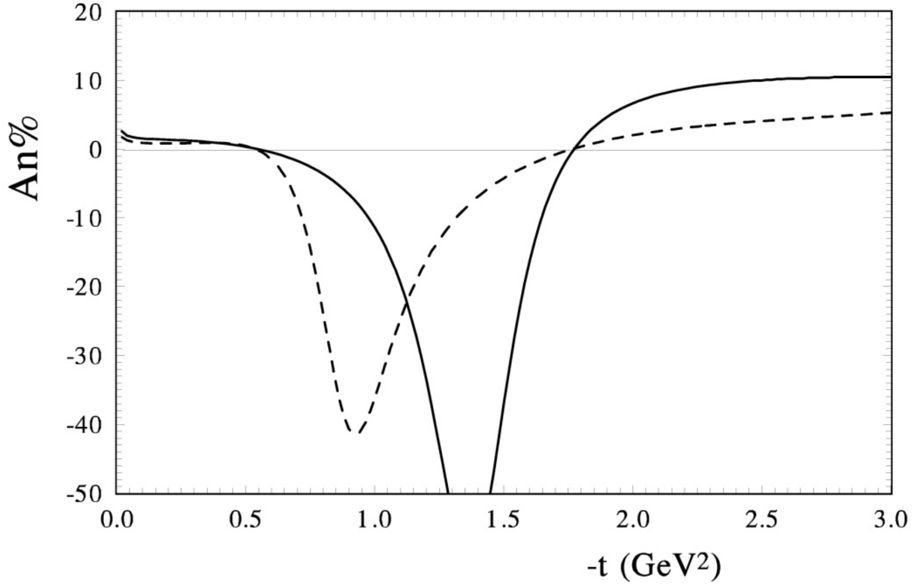

$\sqrt{s} = 50\ GeV;$

$$\Gamma_{-}^{U-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi_{nf}(s,b)/2)^2}$$

$$\chi_{U-m}(s,b) = 2\ tanh\{\frac{1}{2}\chi_{eik}(s,b)\};$$

$$\chi_{U-m}(s,b) = 2\frac{e^{\chi_{eik}(s,b)/2} - e^{-\chi_{eik}(s,b)/2}}{e^{\chi_{eik}(s,b)/2} + e^{-\chi_{eik}(s,b)/2}};$$

$\sqrt{s} = 50\ GeV;$



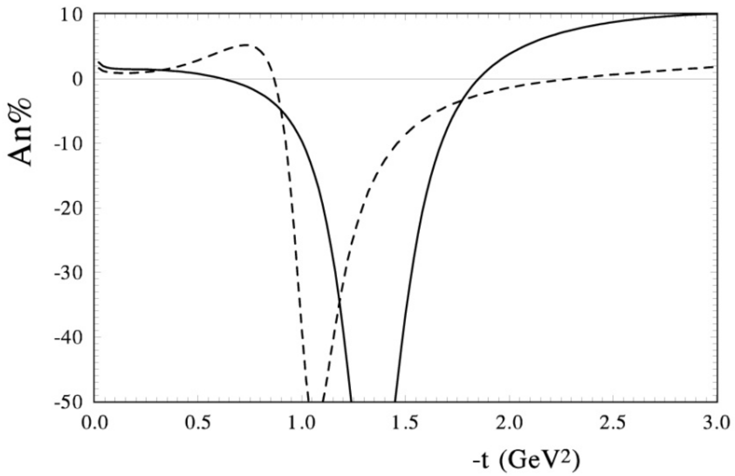

$\sqrt{s} = 500\ GeV;$

## $\underline{A_N - U_T\text{-}matrix \quad (New\ fit\ of\ \chi_{nf}^{fit}(s,b)\ from}$ $\boxed{\frac{d\sigma}{dt}}$

$$\Gamma_{+}^{U-m}(s,b) = \frac{\chi^{fit}{}_{nf}(s,b)}{1+\chi^{fit}{}_{nf}(s,b)/2}$$

$\sqrt{s} = 50\ GeV;$

$\sqrt{s} = 500\ GeV;$




$$\Gamma_{-}^{U-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi^{fit}{}_{nf}(s,b)/2)^2}$$

$\chi_{nf}^{fit}(s,b) \neq \chi_{nf}^{Born}(s,b);$

# Summary

- Unitarization effects is very important for the description spin correlation parameters at RHIC
- Non-linear equations correspond to the different forms of the unitarization schemes. They can have the same asymptotic regime.
- An interpolating form of unitarization can reproduce both the eikonal and the K-matrix (extended U-matrix} unitarization

# Summary

- The true form of the unitarization, which is still to be determined, can help to determine the form of the non-linear equation, which describes the non-perturbative processes at high energies.

# END

- The experiments on proton elastic scattering occupy an important place in the research program at the LHC.

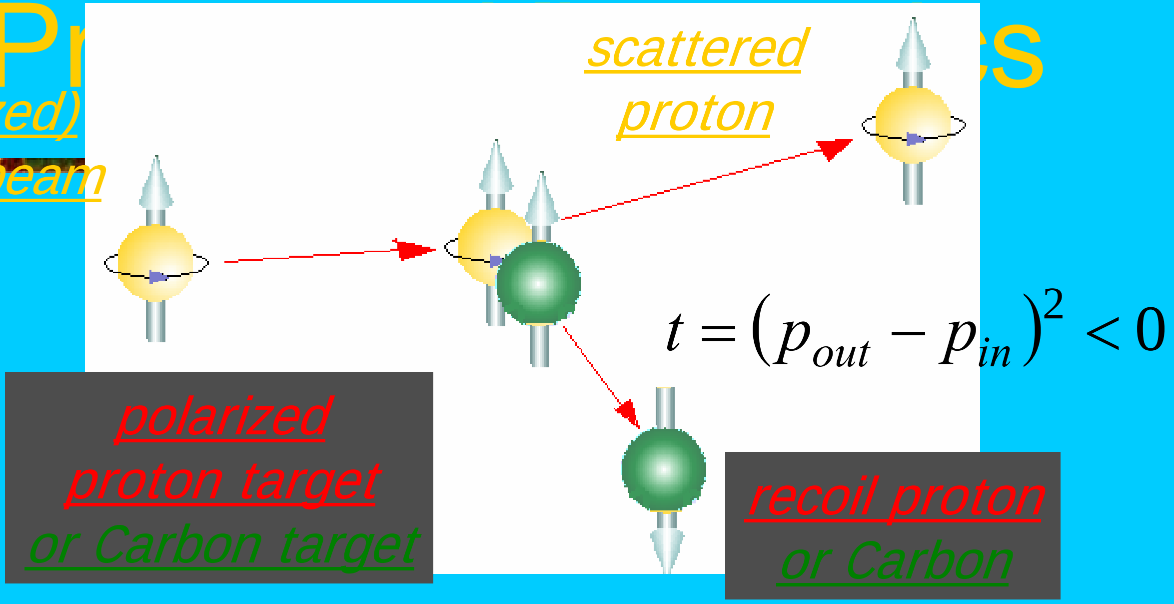
It is very likely that BDL regime will be reached at LHC energies. It will be reflected in the behavior of B(t) and $\rho$(t).

# Double Spin Correlation parameter

$$A_{NN} = \frac{\sigma^{\uparrow\uparrow+\downarrow\downarrow} - \sigma^{\uparrow\downarrow+\downarrow\uparrow}}{\sigma^{\uparrow\uparrow+\downarrow\downarrow} + \sigma^{\uparrow\downarrow+\downarrow\uparrow}}$$

$$A_{NN}(s,t)\frac{d\sigma}{dt} = \frac{4\pi}{s^2}\left\{\ 2|\phi_5|^2 + \mathrm{Re}\left(\phi_1^*\phi_2 - \phi_3^*\phi_4\right)\right\}$$

# The Elastic Pr... ...cs



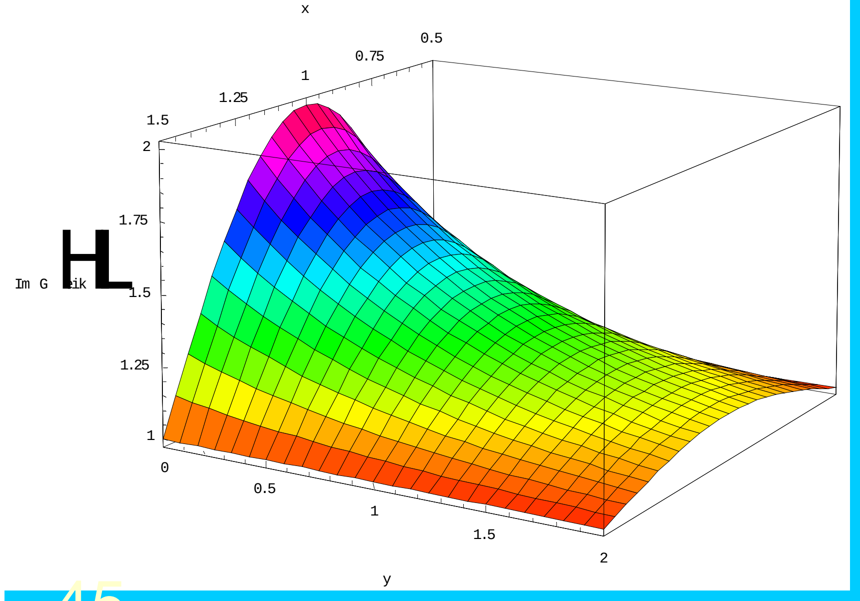

*RHIC beams +*
*internal targets ≡*
*fixed target mode*
*$\sqrt{s} \sim 14$ GeV*

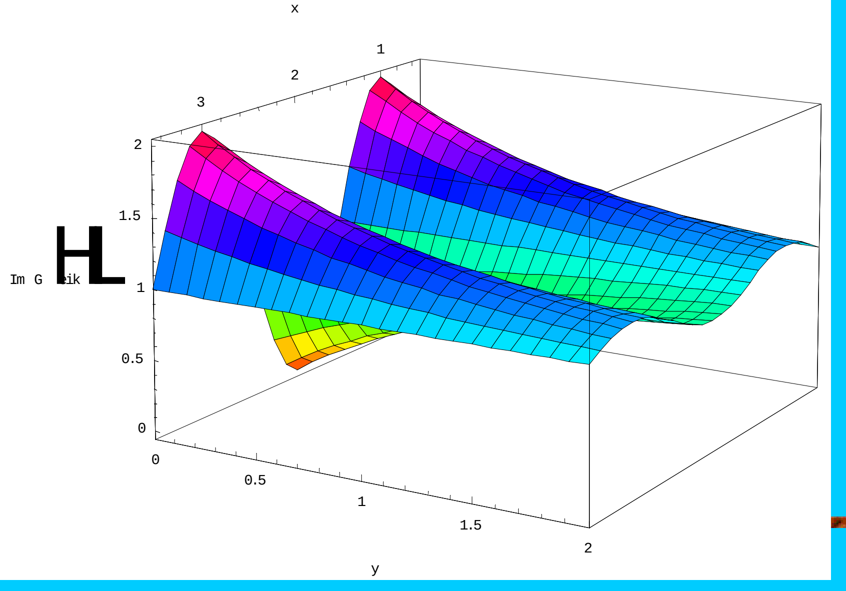
*essentially 1 free parameter:*
*momentum transfer $t = (p_3 - p_1)^2 = (p_4 - p_2)^2 < 0$*
*+ center of mass energy $s = (p_1 + p_2)^2 = (p_3 - p_4)^2$*
*+ azimuthal angle $\varphi$ if polarized !*

*⇒ elastic pp kinematics fully constrained by recoil proton only !*

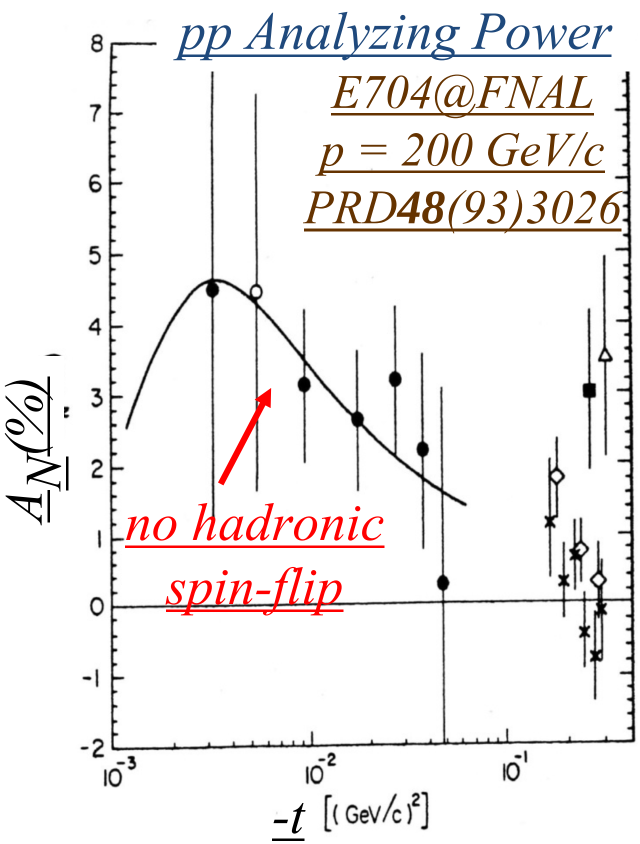
# Complex eikonal and unitarity bound

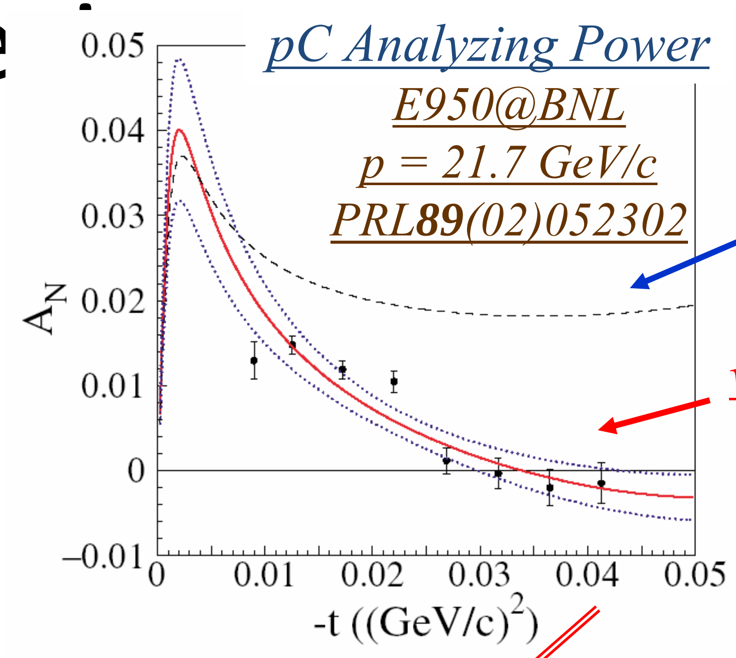
$$G(s,b) = (1 - \exp[-Y(s,b) + i\ X(s,b)])$$



# Some $A_N$ measurements in the CNI re...




*no hadronic spin-flip*

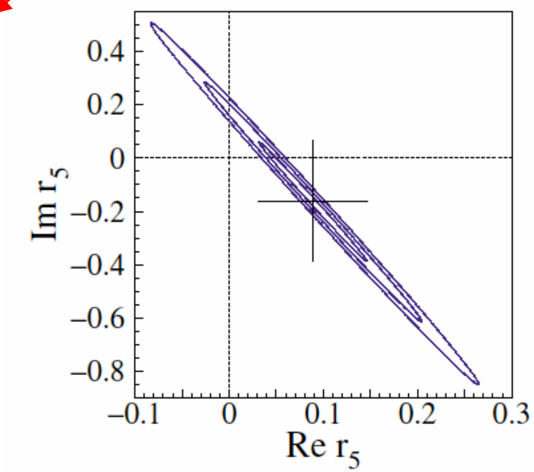
*with hadonic spin-flip*

$r_5^{pC} \propto F_s^{had} / Im\, F_0^{had}$

$Re\, r_5 = \quad 0.088 \pm 0.058$

$Im\, r_5 = \; -0.161 \pm 0.226$

*highly anti-correlated*



# Spin correlation parameter - $A_N$

$$A_N^{beam}(t) = A_N^{target}(t)$$

*for elastic scattering only!*

$$P_{beam} = P_{target} \cdot \varepsilon_B / \varepsilon_T$$

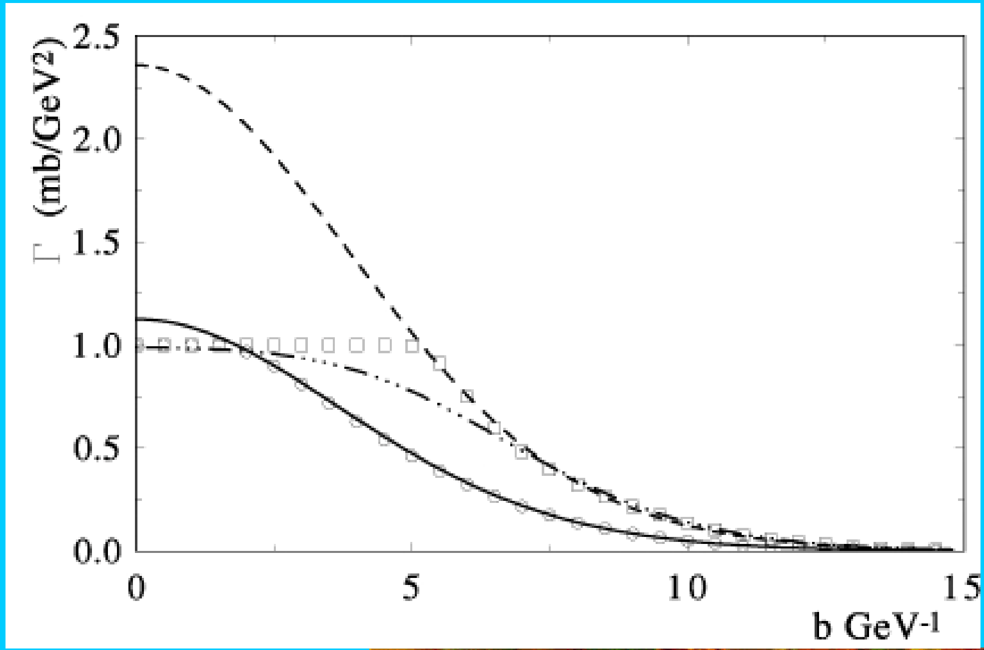
# Soft and hard Pomeron

*Donnachie-Landshoff model;*

*Schuler-Sjostrand model*

$$T(s,t) = [h_1(\frac{s}{s_0})^{\Delta_1} e^{\alpha_1 t \ln(s/s_0)}$$

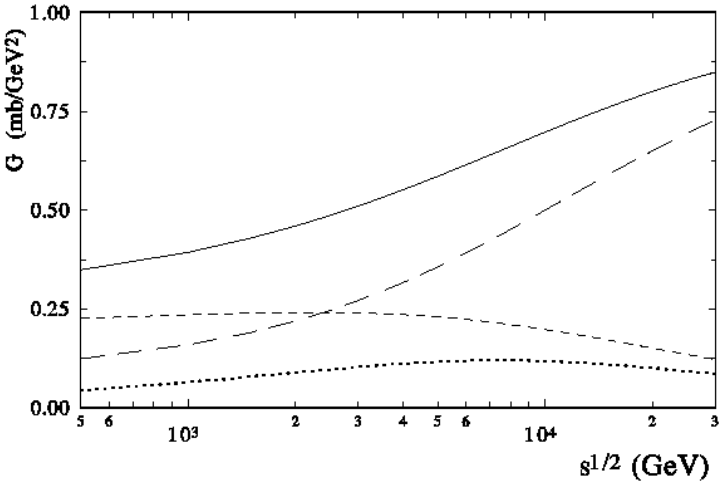
$$+ h_2(\frac{s}{s_0})^{\Delta_2} e^{\alpha_2 t \ln(s/s_0)}]F^2(t)$$

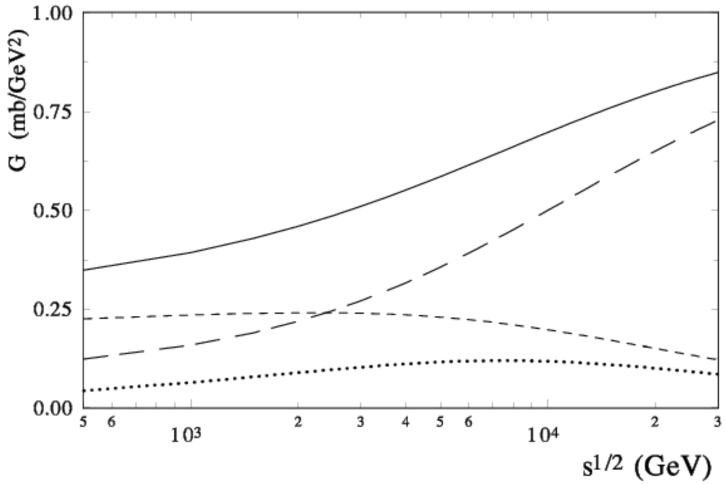


# Such form of U-matrix does not have the BDL regime

## *Predictions at LHC (14 TEV)*

$$\sigma_{tot}(s) = 230mb; \qquad \frac{\sigma_{el}(s)}{\sigma_{tot}(s)} = 0.67;$$



# Corresponding phases

$$\chi_{eik}(s,b) = log\{1 + \chi_{U-m}(s,b)\};$$

$$\chi_{U-m}(s,b) = e^{\chi(s,b)}\{1 - e^{-\chi_{eik}(s,b)}\};$$

# Radius of the saturation

$$R^2(s) \sim 4[R_0\, Ln(\frac{h}{2B}) + [R_0 + \frac{\alpha^{'}}{\Delta} Ln(\frac{h}{2(d + \Delta Ln(s))})]\Delta Ln(s) + \Delta\,\alpha^{/} Ln^2(s)]$$

<u>*Low s*</u> $\qquad R^2(s) \sim Ln(s);$

$s \to \infty \qquad R^2(s) \sim Ln^2(s).$

$$soft \quad IPomeron \quad \alpha^{/}\,\Delta = 0.1 * 0.3 = 0.03$$

$$hard \quad IPomeron \quad \alpha^{/}\,\Delta = 0.4 * 0.1 = 0.04$$

$$2\,\mathrm{Im}\,\Gamma(s,b) - |\,\mathrm{Im}\,\Gamma(s,b)\,|^2 + |\,\mathrm{Re}\,\Gamma(s,b)\,|^2 = g_{in} > 0$$

$$g_{in} < 1$$

$$\Gamma(s,b) = i\,[1-(1-g_{in})\,exp(i\Phi)]$$

$$\Gamma(s,b) = i\,[1-exp(-\Omega+i\Phi)]$$

$$\Gamma(s,b) = 1-\exp[i\,\chi(s,b)]$$

$$= 1-\exp[-\mathrm{Re}\ \Omega(s,b)][cos(\Omega)-i\,sin(\Omega)]$$

*momentum transfer* $t = (p_3 - p_1)^2 = (p_4 - p_2)^2 < 0$

*+ center of mass energy* $s = (p_1 + p_2)^2 = (p_3 - p_4)^2$

*+ azimuthal angle* $\varphi$ *if polarized !*

*⇒ elastic pp kinematics fully constrained by recoil proton only*

# Soft and hard Pomeron

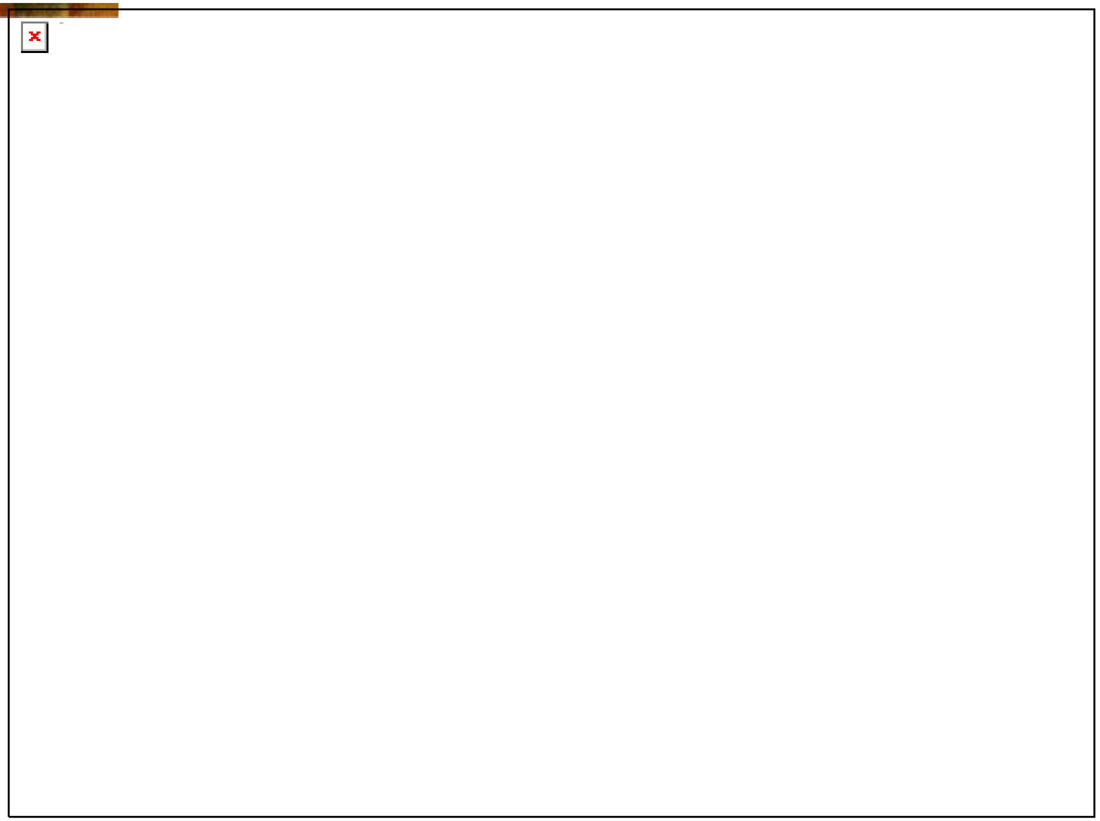
Donnachie-Landshoff model;

Schuler-Sjostrand model

Cudell-Lengyel-Martynov-Selyugin

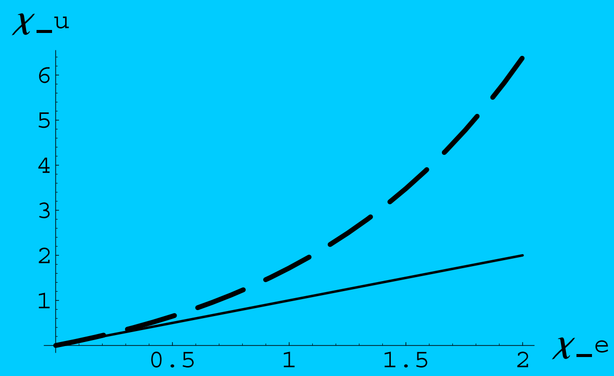
$$T(s,t) = [h_1\left(\frac{s}{s_0}\right)^{\Delta_1} e^{\alpha_1 t \ln(s/s_0)}$$

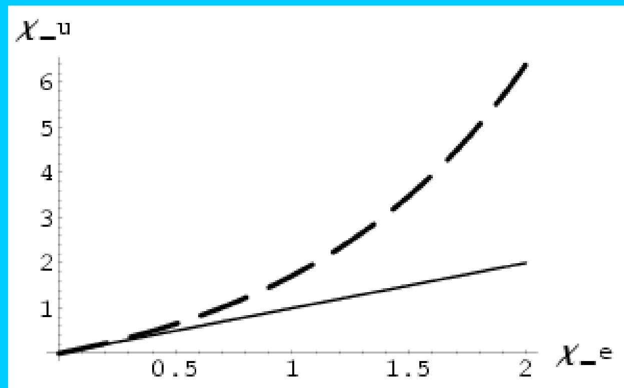
$$+ h_2\left(\frac{s}{s_0}\right)^{\Delta_2} e^{\alpha_2 t \ln(s/s_0)}]F^2(t)$$

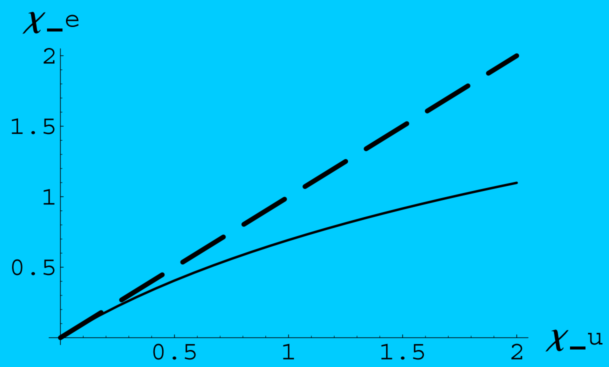


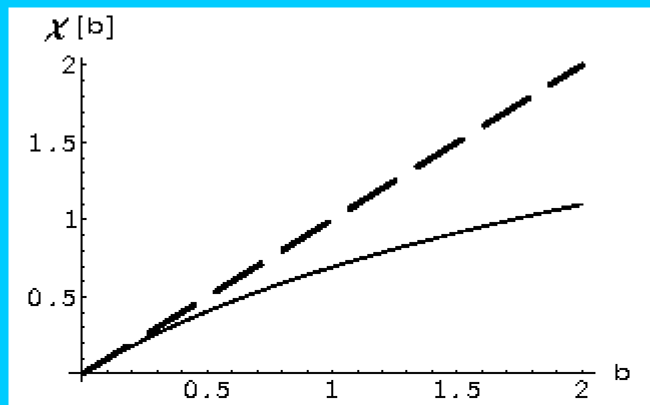
$$\frac{dN}{dy} = \chi \; Exp[-\chi];$$

# Corresponding phases 2



# Extended forms of the unitarization

*Ter-Martirosyan-Kaidalov*

$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\,db \frac{1}{2C} (1 - \exp[-2C\,\delta(s,b)]) \qquad C \approx 1.2;$$

*Giffon-Martynov-Predazzi*

$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\,db \frac{1}{2C} (1 - \exp[-2C\,\delta(s,b)]) \qquad C \geq \frac{1}{2};$$

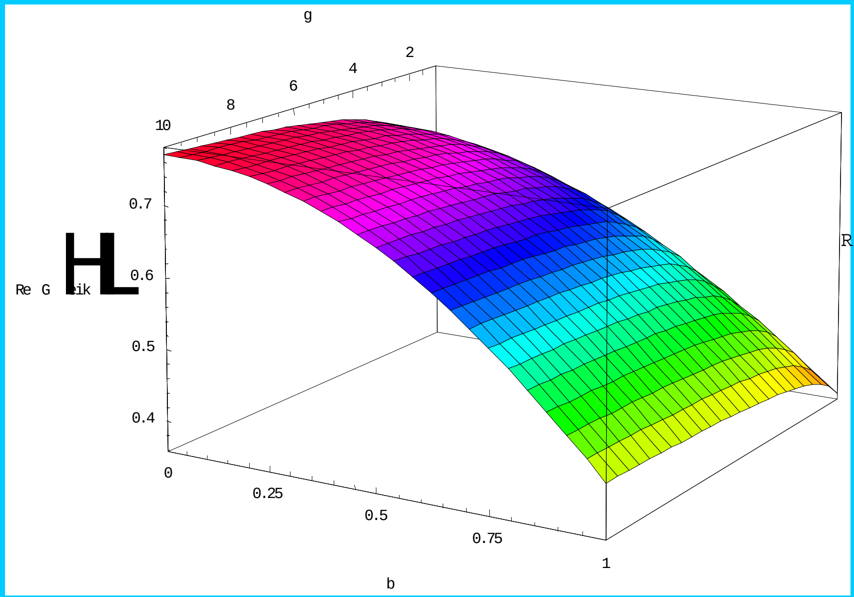
$$\sigma_{tot}(s) = 8\pi \int_0^\infty b\,db \frac{\delta(s,b)}{1 + 2C\,\delta(s,b)};$$

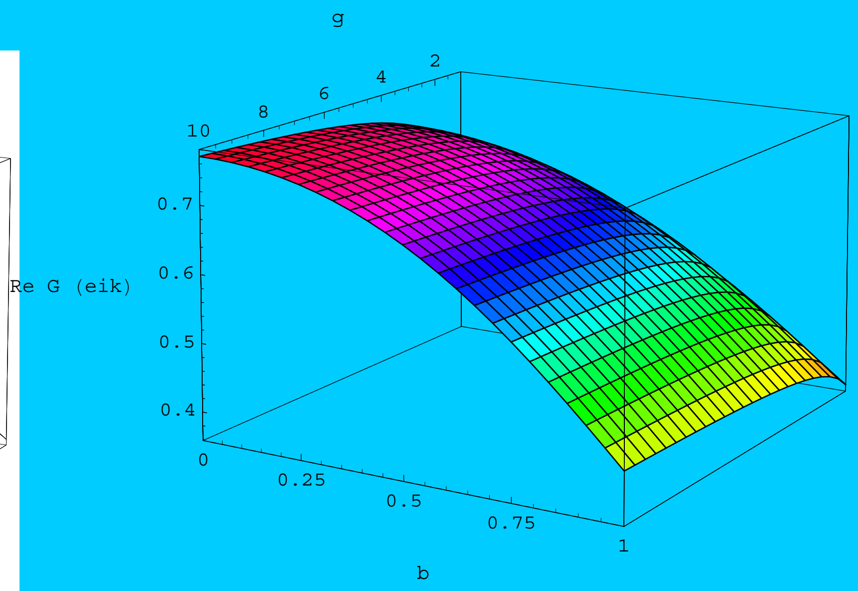
$$\Gamma(s,b) = \frac{1}{2i} \sum_{n=1}^{\infty} \frac{G(n)}{n!} [2i\;\delta(s,b)]^n$$

$G(n) = C^{n-1}$ *eikonal*

$G(n) = n!\,C^{n-1}$ *U-matrix*

# Interpolating form of unitarization



# Elastic scattering amplitude

$pp \to pp$ $\qquad\qquad$ $p\bar{p} \to p\bar{p}$

$$\frac{d\sigma}{dt} = 2\pi[|\Phi|_1^2 + |\Phi|_2^2 + |\Phi|_3^2 + |\Phi|_4^2 + 4|\Phi|_5^2]$$

$$\Phi_i(s,t) = \Phi_i^h(s,t) + \Phi_i^e(t)\, e^{i\alpha\varphi}$$

$$\varphi(s,t) = \mp [\gamma + \ln(\mathrm{B}(s,t)\,|t|/2) + \nu_1 + \nu_2]$$

$\gamma = 0{,}577...$ ( *the Euler constant* )

$\nu_1$ *and* $\nu_2$ *are small correction terms*

## Impact parameter representation and unitarization schemes

$$T(s,t) = is\int_0^\infty b\,db\,J_0(bq)\;\Gamma(b,s)$$

$$T(s,t) = is\int_0^\infty b\,db\,J_0(bq)\;(1-\exp[i\,\chi(s,b)])$$

$$T(s,t) = is\int_0^\infty b\,db\,J_0(bq)\;\frac{U(s,b)}{1-iU(s,b)};$$

# General form of Unitarization

$$\Gamma(s,b) = \frac{1}{n}[1 - \exp(-n\,\chi(s,b))];$$

*Ter-Martirosyan, Kaidalov*

*Desgrolard, Martynov*

$$\Gamma_n(s,b) = \frac{1}{n}[1 - \frac{1}{(1+\chi(s,b))^n}]$$

$$\Gamma_\lambda(s,b) = 1 - [1 + \frac{\chi(s,b)}{\lambda}]^{-\lambda}$$

*Miettinen, Thomas*

$\lambda \to \infty$ *eikonal*

$\lambda \to 1$ *K-matrix*

$$\Phi^{B}_{\lambda_3\lambda_4\lambda_1\lambda_2}(s,b) = U_{\lambda_3\lambda_4\lambda_1\lambda_2}(s,b) + i\rho(s)\sum_{\mu,\nu} g^{i}_{\lambda_3\lambda_1} g^{i}_{\lambda_4\lambda_2}[\sqrt{-t}\,]^{|\lambda_3-\lambda_1|+|\lambda_4-\lambda_2|}(\frac{s}{s_0})^{\alpha_i}(1\pm\exp[-i\pi\alpha_i])$$

$$N[y] = \frac{i\,\chi(s,b)}{1+\chi(s,b)};$$

$$N = \frac{0.1\,s^{0.4}\;Exp[(b/3)^2]}{1+0.1\,s^{0.4}Exp[(b/3)^2]}$$

$$N = 1 - Exp[-0.1 s^{\Delta} Exp[(b/3)^2]]$$

$$\int_0^z x^{-1.3}(1-x)^{-1.3}\,Exp[x^{-1.3}(1-x)^{-1.3}]\,dx$$

$$F(s,t) = \Phi_1(s,t) + \Phi_3(t)$$
$$= F_N(s,t) + F_C(t)\, e^{i\alpha\varphi}$$

$$F_C(t) = \mp 2\alpha\, G^2(t) / |t|$$

$$B(s,t) = \frac{d}{dt}\left(\ln \frac{d\sigma}{dt}(s,t)\right),$$

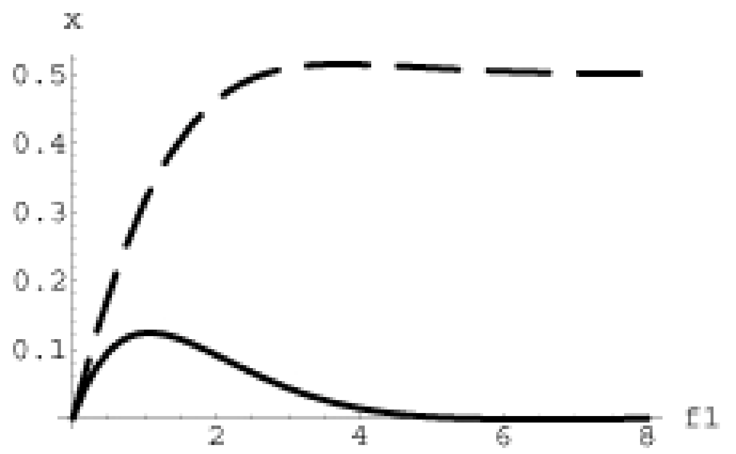
$$\sigma_{tot}(s) = 4\pi\, \mathrm{Im}\, F(s,t)$$

$$\rho(s,t) = \frac{\mathrm{Re}\, F_N(s,t)}{\mathrm{Im}\, F_N(s,t)};$$

# Eikonal

$$\Gamma(s,b) = i/2\{1 - Exp(-b\,Exp[-\Delta i\pi/2])\}$$

$$= \frac{i}{2}\{1 - Exp[-b\ cos[\Delta\pi/2]]*[\cos(b\sin(\Delta\pi/2)) + i\sin(b\sin(\Delta\pi/2)]$$



# Impact parameters dependence

$$T(s,t) = is\int_0^\infty b\,db\,J_0(bq)\,(1-\exp[i\,\chi(s,b)])$$

The Froissart bound

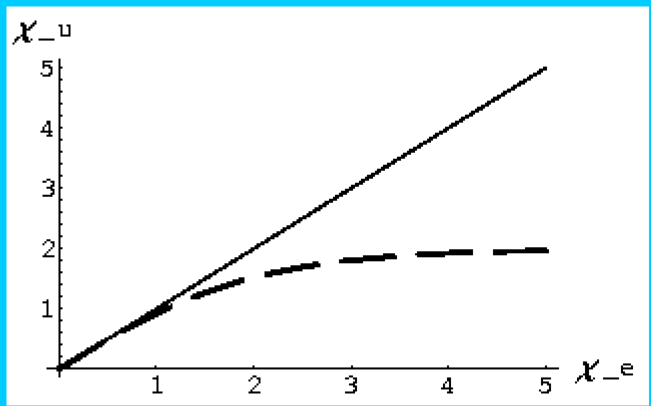
$$\sigma_{tot}(s) \le a\,\log^2(s)$$

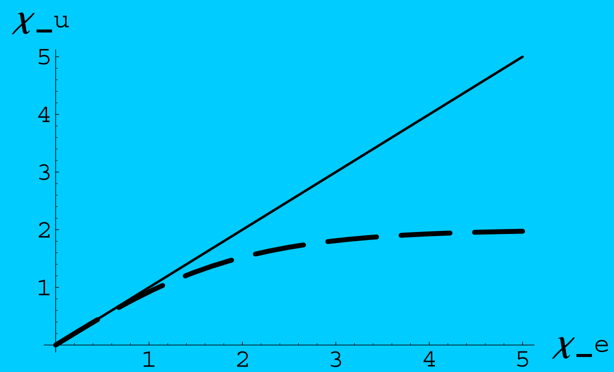
Factorization

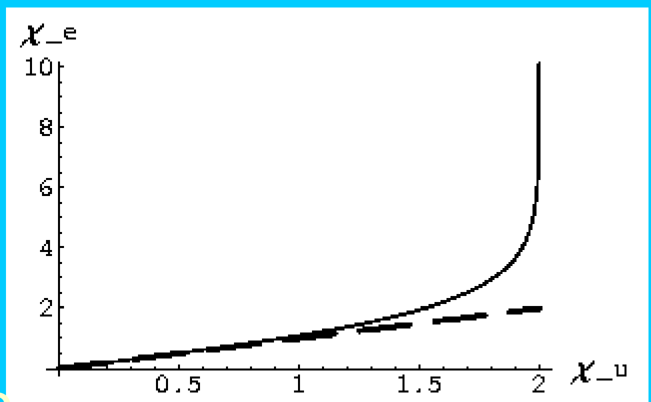
$$\chi(s,b) = h(s)\,f(b); \qquad h(s) \sim s^{\Delta}$$

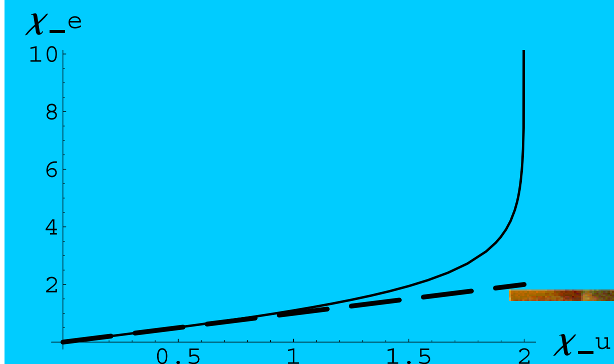
# Corresponding phases

## ***Bound on the region of validity of the U-matrix phase***



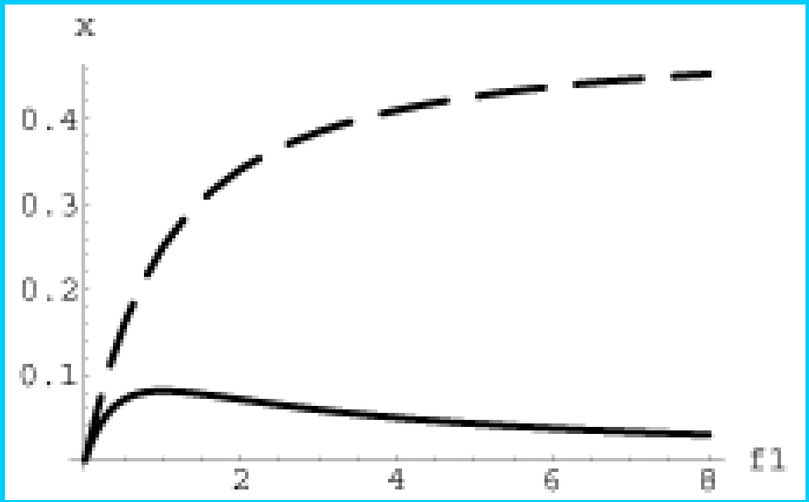



# U-matrix

$$\Gamma(s,b) = \frac{i}{2} b\ \exp[-\Delta\pi i/2)]/(1+b\ \exp[-\Delta\pi i/2)] =$$

$$= \frac{i}{2}\{1 - \frac{1}{1+b^2+2b\,cos[\Delta\pi/2]}(1+b\ cos[\Delta\pi/2]+i\,b\sin(\Delta\pi/2)]\}$$

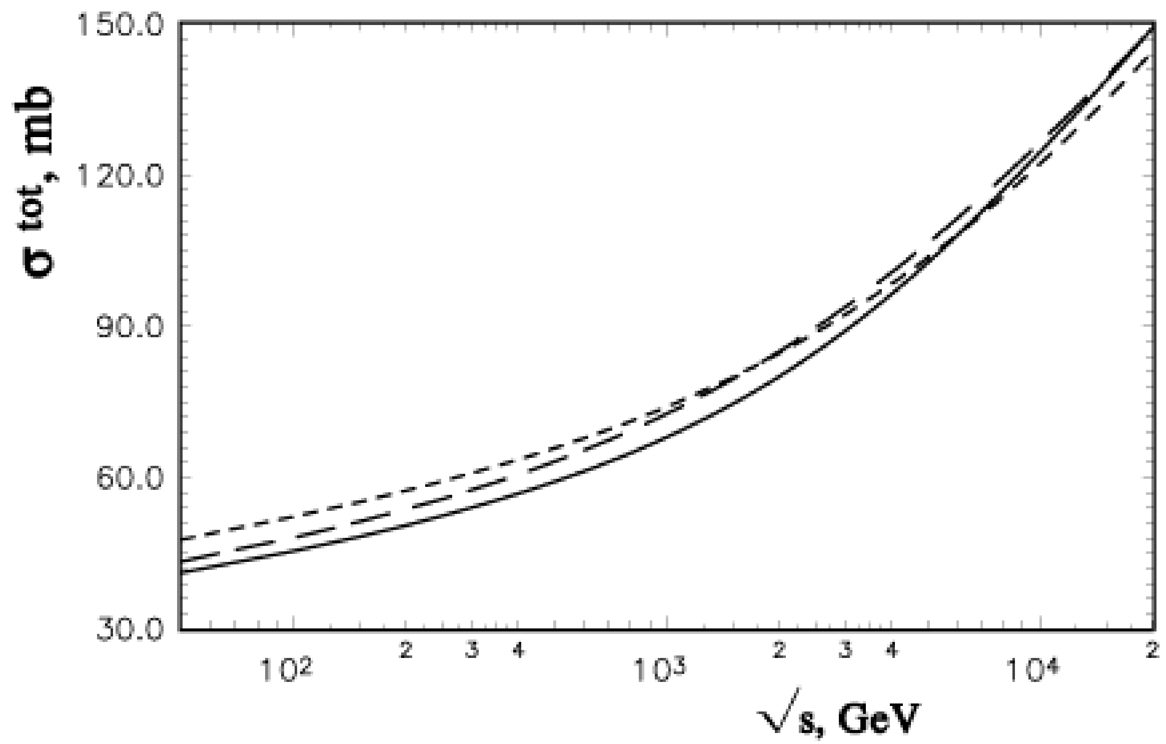


# TANGH

$$\frac{dN}{dy} = N^2 (1 - N^2);$$

$$\Gamma(s,b) = Tanh[\chi(s,b)]$$

# Total cross sections at the LHC



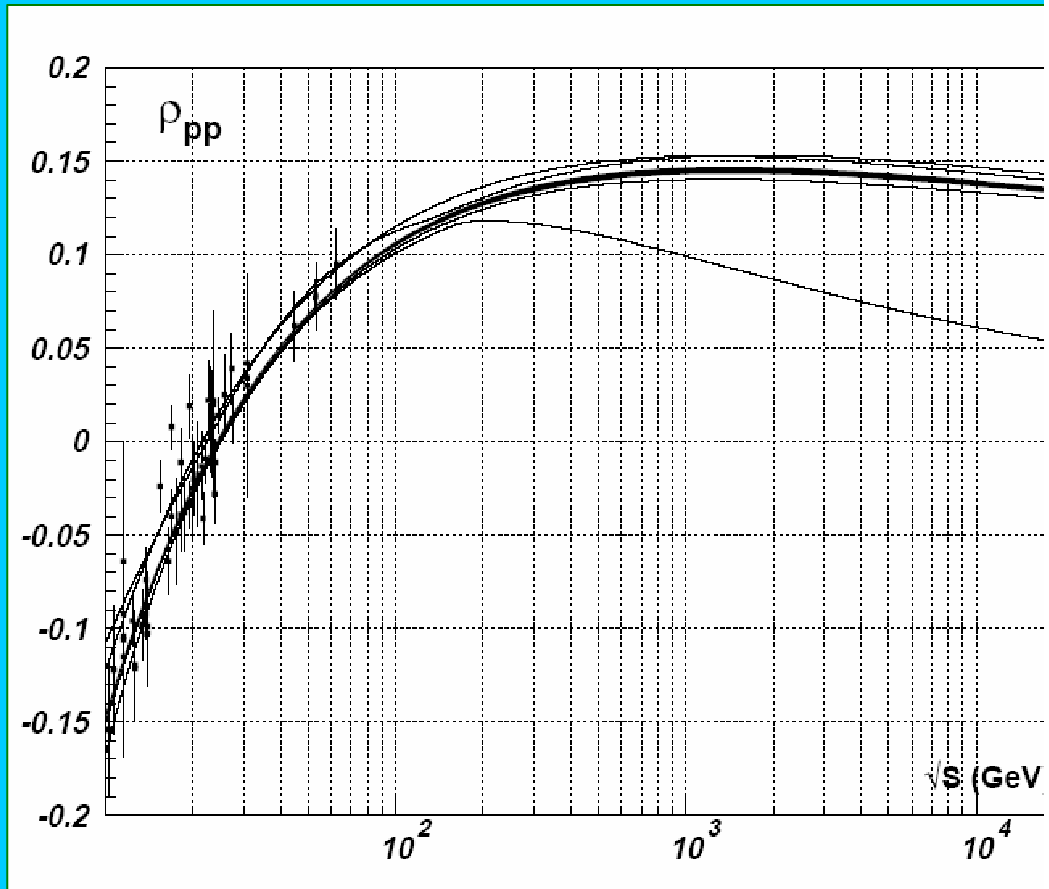
## The $\rho$ parameter

linked to $\sigma_{tot}$ via dispersion relations

sensitive to $\sigma_{tot}$ beyond the energy

at which is measured

- predictions of $\sigma_{tot}$ beyond LHC energies
- Or, are dispersion relations still valid at LHC energies?



*2 last combination don't satisfied the cgarge invariants and change signe of time*

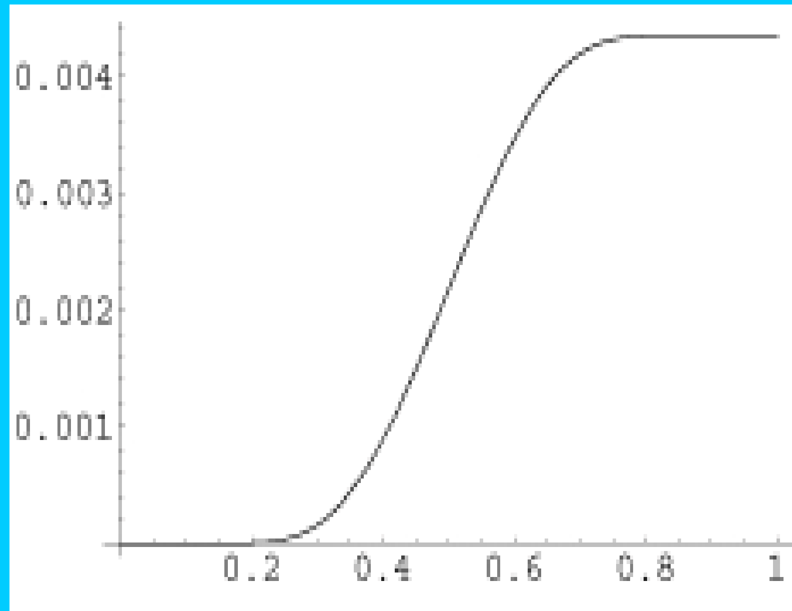
*If all particles are the same the amplitude do not changes with Change з1бз2 on л1бл2 and P->K K->P*

*So the 6 cobination also disappear.*

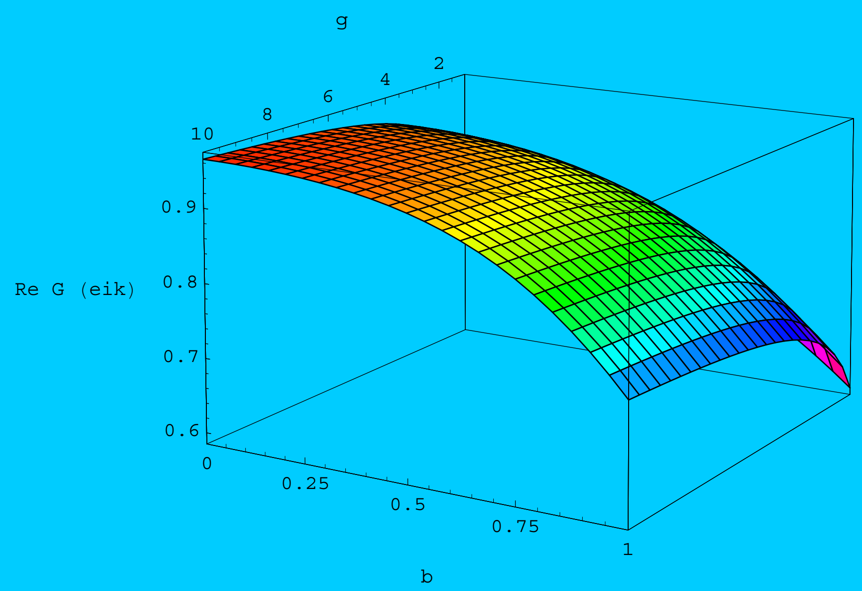
# Saturation of the Gluon dencity

$$xG(x) = x^{-\Delta}(1-x)^{-\Delta}$$

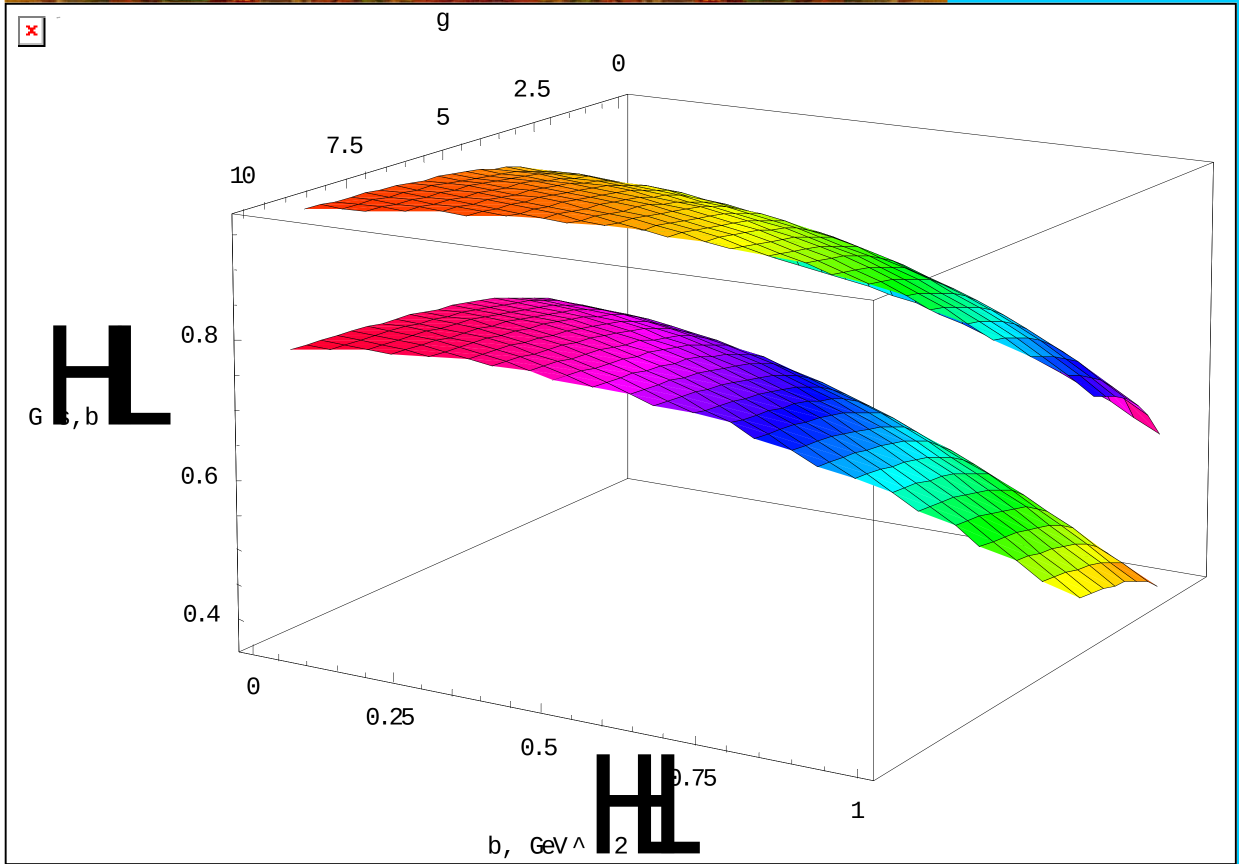
$$\frac{dN}{dx} = \chi\ Exp[-\chi];$$



# Interpolating form of unitarization

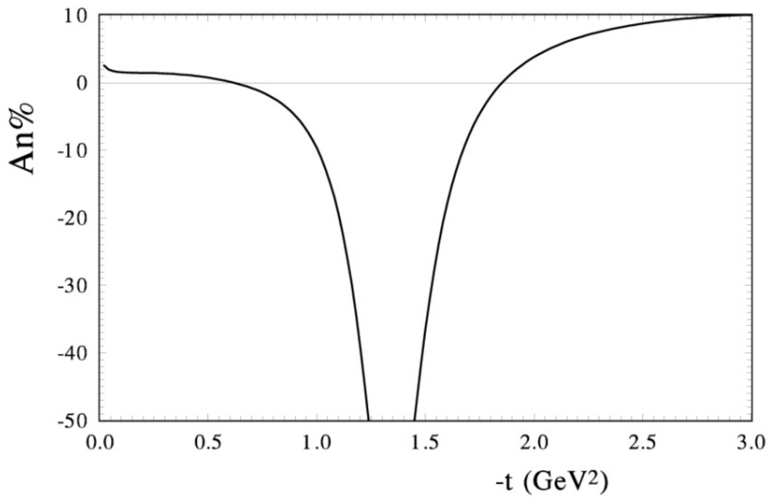


# Interpolating form of unitarization

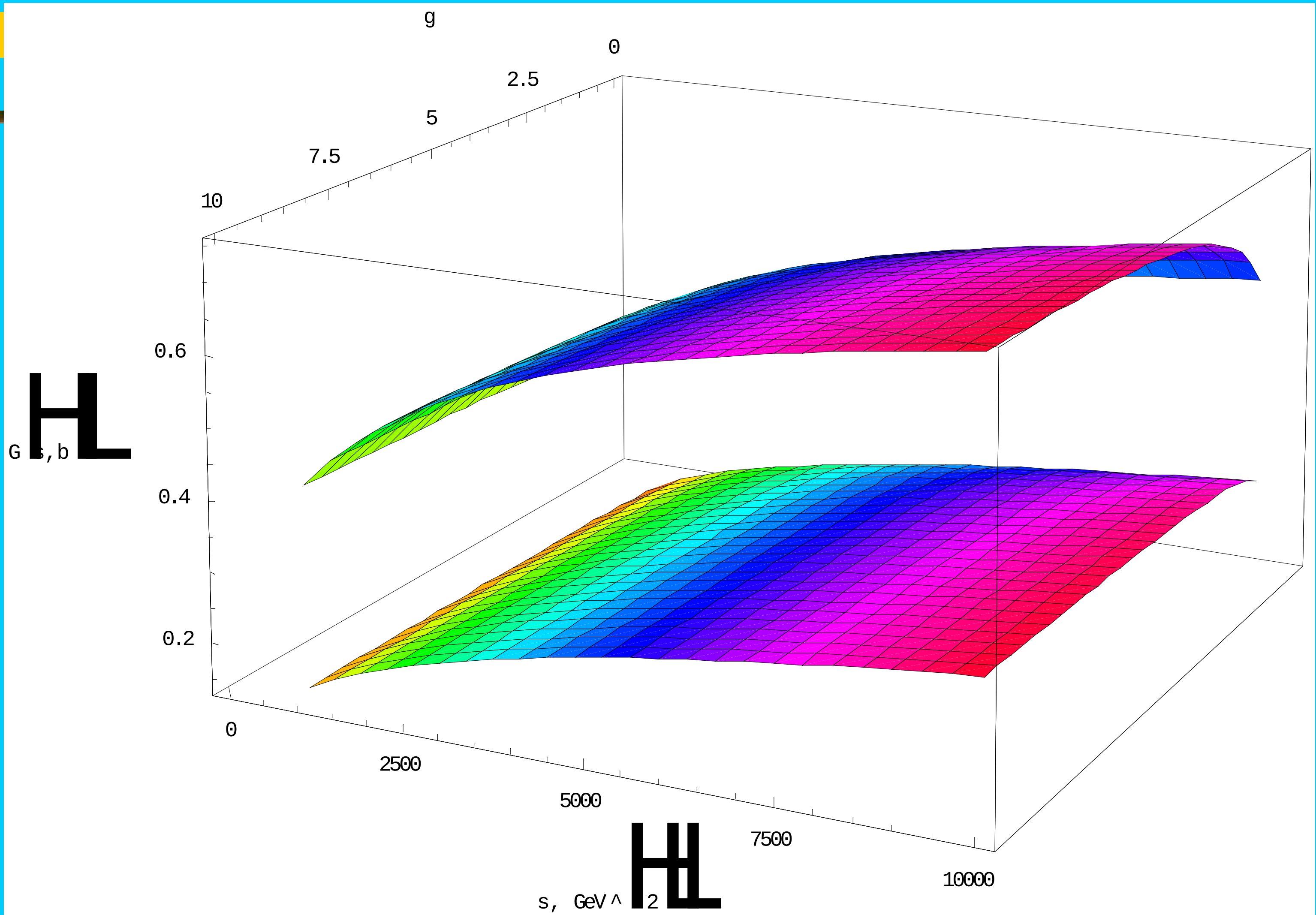


# $A_N - U_T$-matrix

$$\Gamma_{+}^{U-m}(s,b) = \frac{\chi^{fit}{}_{nf}(s,b)}{1+\chi^{fit}{}_{nf}(s,b)/2}$$




$$\Gamma_{-}^{U-m}(s,b) = \frac{\chi_{sf}(s,b)}{(1+\chi^{fit}{}_{nf}(s,b)/2)^2}$$



g
0
2.5
5
7.5
10
0.6
0.4
0.2
G s,b
0
2500
5000
7500
10000
s, GeV^2

# The nuclear slope parameter b

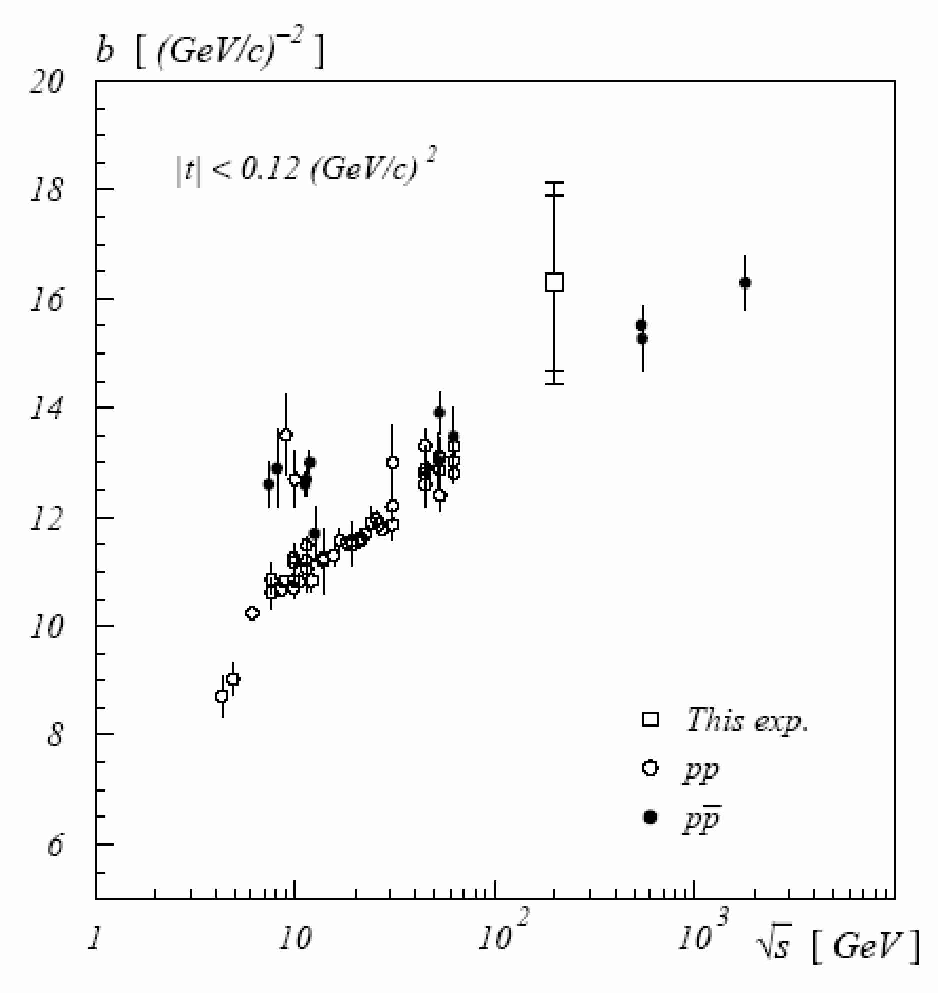
- t-region of $10^{-2} \div 10^{-1}$ GeV$^2$

- The b parameter is sensitive to the exchange process
  - Its measurement will allow to understand the QCD based models of hadronic interactions

- “Old” language : shrinkage of the forward peak
  - $b(s) \propto 2\,\alpha' \log s$ ; where $\alpha'$ is the slope of the Pomeron trajectory $\approx 0.25$ GeV$^2$

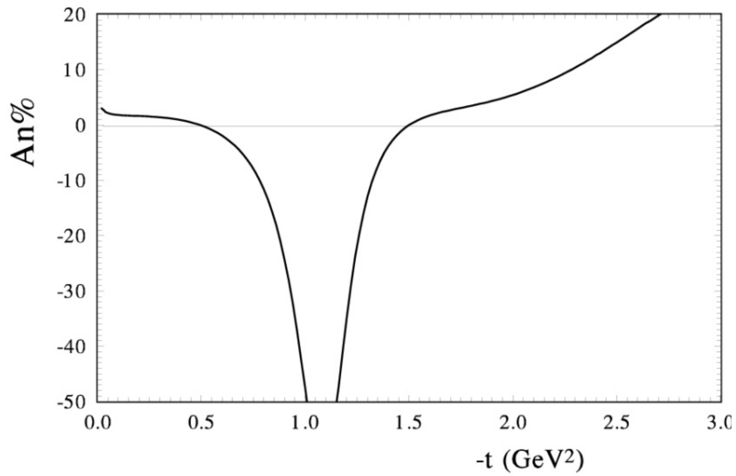
- Not simple exponential - t-dependence of local slope

- Structure of small oscillations?



# <u>*An-UfT*</u>

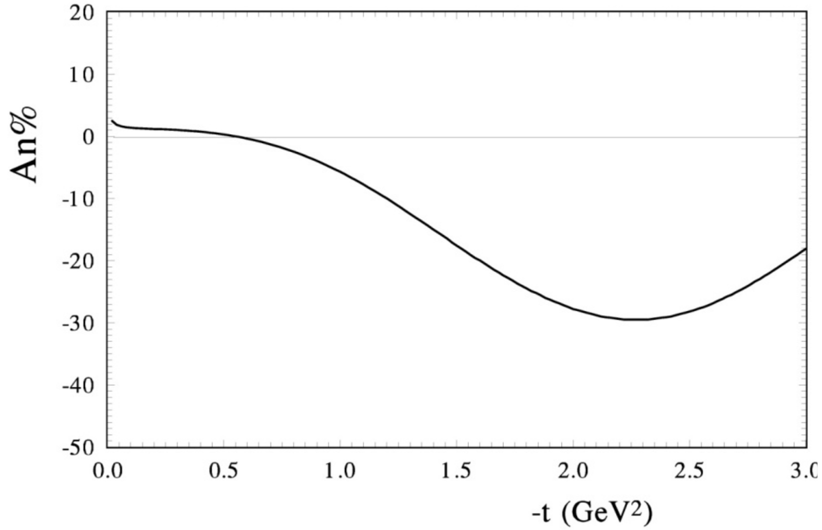
$$\Gamma_+^{U_{eik}}(s,b) = \frac{\chi_{fit}(s,b)}{1-\chi_{fit}(s,b)}$$




$$\Gamma_-^{U_{eik}}(s,b) = \frac{\chi^s(s,b)}{(1+\chi_{fit}(s,b))^2}$$

# <u>*AN UmTei*</u>

$$\Gamma_+^{U_{eik}}(s,b) = \frac{\chi_{fit}(s,b)}{1-\chi_{fit}(s,b)/2}$$




$$\Gamma_-^{U_{eik}}(s,b) = \frac{\chi^s(s,b)}{(1+\chi_{fit}(s,b)/2)^2}$$

# Non-linear equations

$$N(y) = \Gamma(s, b_{fix})$$

## eikonal

$$\frac{dN}{dy} = (-\log[1-N])\ (1-N);$$

$$N = 1 - exp[-0.1 s^{\Delta}]$$

## U-matrix

$$\frac{dN}{dy} = N(1-N);$$

$$N = \frac{0.1\ s^{\Delta}}{1 + 0.1\ s^{\Delta}}$$