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Recent developments in nucleon spin structure with particular focus on pretzelosity $h_{1T}^{\perp}$

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(based on: Phys. Rev. D **77** (2008) 014023 and arXiv:0805.3355)

Overview:

- Introduction: Complicated nucleon spin structure. How to simplify?
- WW-type approximation and $A_{UL}^{\sin 2\phi}$.
- Pretzelosity. What is it?
- Pretzelosity in bag, spectator & constituent models.
- New relation, and consequences
- Prospects: SIDIS & Drell-Yan
- Summary & conclusions

Introduction

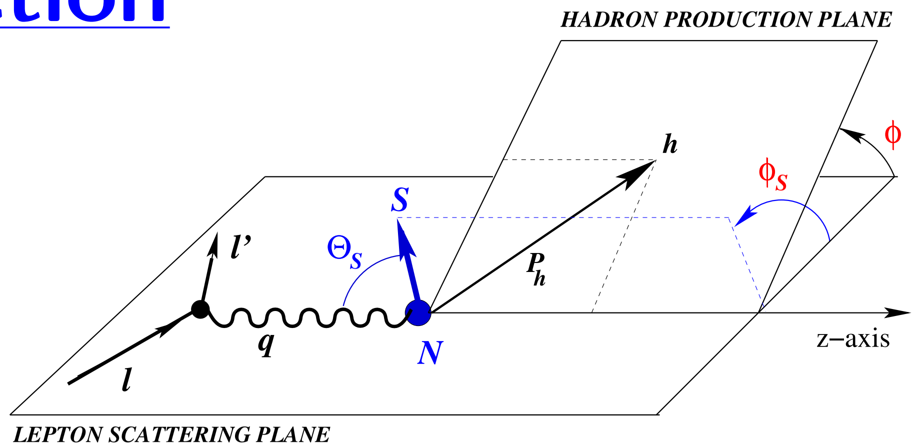
SIDIS $lN \to l'hX$

tree-level: Boer, Mulders, Tangerman 1990s,

factorization: Ji, Ma, Yuan, Collins, Metz 2004,

review: Bacchetta et al., JHEP (2007),

see also Marco Maggiore talk.



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi_h} = F_{UU} + \lambda_e\, S_L F_{LL}$$
$$+ \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + S_L \sin(2\phi) F_{UL}^{\sin(2\phi)} + \lambda_e S_T \cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)}$$
$$+ S_T[\sin(\phi-\phi_S) F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)}]$$

where:

$$\begin{array}{rcl}
F_{UU} \propto \sum_a e_a^2\, f_1^a \otimes D_1^a & , & F_{UU}^{\cos(2\phi)} \propto \sum_a e_a^2\, h_1^{\perp a} \otimes H_1^{\perp a} \\
F_{LL} \propto \sum_a e_a^2\, g_1^a \otimes D_1^a & , & F_{UL}^{\sin(2\phi)} \propto \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a} \\
F_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a & , & F_{UT}^{\sin(\phi+\phi_S)} \propto \sum_a e_a^2\, h_1^a \otimes H_1^{\perp a} \\
F_{UT}^{\sin(\phi-\phi_S)} \propto \sum_a e_a^2\, \underbrace{f_{1T}^{\perp a} \otimes D_1^a}_{\text{chiral-even}} & , & F_{UT}^{\sin(3\phi-\phi_S)} \propto \sum_a e_a^2\, \underbrace{h_{1T}^{\perp a} \otimes H_1^{\perp a}}_{\text{chiral-odd}}
\end{array}$$

$\otimes$ means k_T-convolution.

Hard processes sensitive to parton p_T

light-front correlators ($z^+ = 0$, $p^+ = xP^+$):

$$\phi(x,\vec{p}_T)_{ij} = \int \frac{\mathrm{d}z^-\mathrm{d}^2\vec{z}_T}{(2\pi)^3}\, e^{ipz}\, \langle N(P,S)|\bar{\psi}_j(0)\,\{\text{gauge link}\}\,\psi_i(z)|N(P,S)\rangle$$

parameterized in terms of 8 leading-twist TMDs   8 different aspects!

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\,\phi(x,\vec{p}_T)] = \boldsymbol{f_1} - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N}\,\boldsymbol{f_{1T}^{\perp}}$$

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\gamma_5\,\phi(x,\vec{p}_T)] = S_L\,\boldsymbol{g_1} + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N}\,\boldsymbol{g_{1T}^{\perp}}$$

$$\frac{1}{2}\,\mathrm{tr}[i\sigma^{j+}\gamma_5\,\phi(x,\vec{p}_T)] = S_T^j\,\boldsymbol{h_1} + \frac{\varepsilon^{jk}p_T^k}{M_N}\,\boldsymbol{h_1^{\perp}} + S_L\,\frac{p_T^j}{M_N}\,\boldsymbol{h_{1L}^{\perp}} + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2}\,\boldsymbol{h_{1T}^{\perp}}$$

$\boldsymbol{f_1}/\boldsymbol{g_1}/\boldsymbol{h_1}$ 'collinear' well/known/models, lattice, first data & extractions (Anselmino et all.)

$\boldsymbol{f_{1T}^{\perp}}/\boldsymbol{h_1^{\perp}}$ 'T-odd' hot!, models, data, extractions (many authors/Drell-Yan)

$\boldsymbol{g_{1T}^{\perp}}/\boldsymbol{h_{1L}^{\perp}}$ certain interest, related to $\boldsymbol{g_1}/\boldsymbol{h_1}$ in Wandzura-Wilczek-type relations (next slides)

$\underbrace{\qquad\quad}_{\otimes D_1}$ $\underbrace{\boldsymbol{h_{1T}^{\perp}}}_{\otimes H_1^{\perp}}$ modest interest, what is that?

all accessible in SIDIS and e^+e^- (Boer, Mulders, Tangerman, Kotzinian 1996-1998)

"Deeper into forest, more firewood!"

Important:

- All 8 leading twist TMDs f_1, g_1, h_1, $f_{1T}^{\perp}$, $h_1^{\perp}$, $g_{1T}^{\perp}$, $h_{1L}^{\perp}$, $h_{1T}^{\perp}$
  but also 16 subleading twist TMDs g_T, h_L, e, $\ldots$ etc. contain
  independent information on the nucleon spin structure.

- There are *no exact relations* among TMDs!

- But having well-motivated "approximations" is valuable!
  At initial stage important (motivations, proposals for experiments).

- For example, Wandzura-Wilczek-type approximations
  (neglect pure-twist-3 & mass terms).

- For example, popular 'non-relativistic limit' prediction: $\boldsymbol{h_1^q(x) = g_1^q(x)}$
  (what do we neglect here???? 'Relativistic effects').

- If confirmed by data (in fact: $g_T(x) - g_T^{\mathrm{WW}}(x) \sim$ very small),
  have to look for a reason (instanton vacuum, lattice).

Wandzura-Wilczek-type approximations

Exact:

$$g_{1T}^{\perp(1)a}(x) = x\, g_T^a(x) + \mathcal{O}(\widetilde{m}) \qquad g_T^a(x) = \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y) + \cancel{\tilde{g}}_T^a(x)^{*}$$

$$\underbrace{-\,2\, h_{1L}^{\perp(1)a}(x) = x\, h_L^a(x) + \mathcal{O}(\widetilde{m})}_{\text{Eq. of motion Mulders, Tangerman 1996}} \qquad \underbrace{h_L^a(x) = 2x \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^a(y) + \cancel{\tilde{h}}_L^a(x)^{**}}_{\text{Wandzura, Wilczek 1977; Jaffe, Ji 1991}}$$

Approximations:

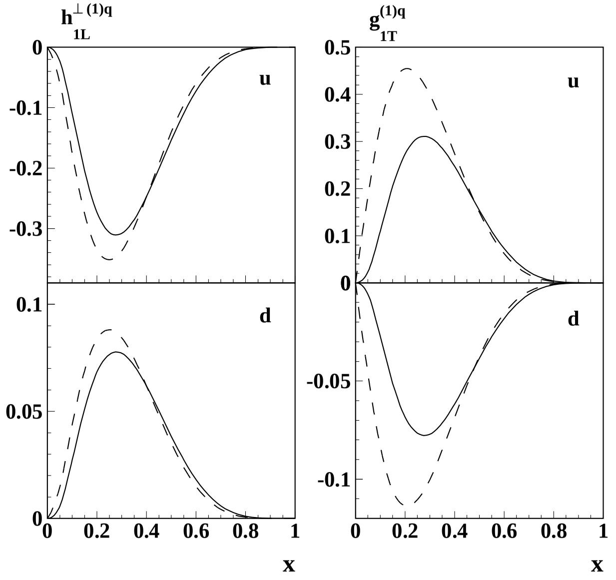
$$g_{1T}^{\perp(1)a}(x) \overset{!?}{\approx} x \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y)$$

$$h_{1L}^{\perp(1)a}(x) \overset{!?}{\approx} -x^2 \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^a(y)$$

Do they work? See in SIDIS:

$$A_{LT}^{\cos(\phi-\phi_S)} \propto \sum_a e_a^2\, g_{1T}^{\perp(1)a}\, D_1^a \; ^{***}$$

$$A_{UL}^{\sin 2\phi} \propto \sum_a e_a^2\, h_{1L}^{\perp(1)a}\, H_1^{\perp a} \; ^{****}$$



Not excluded by HERMES data on $A_{UL}^{\sin 2\phi}$ ($\sim$ zero)

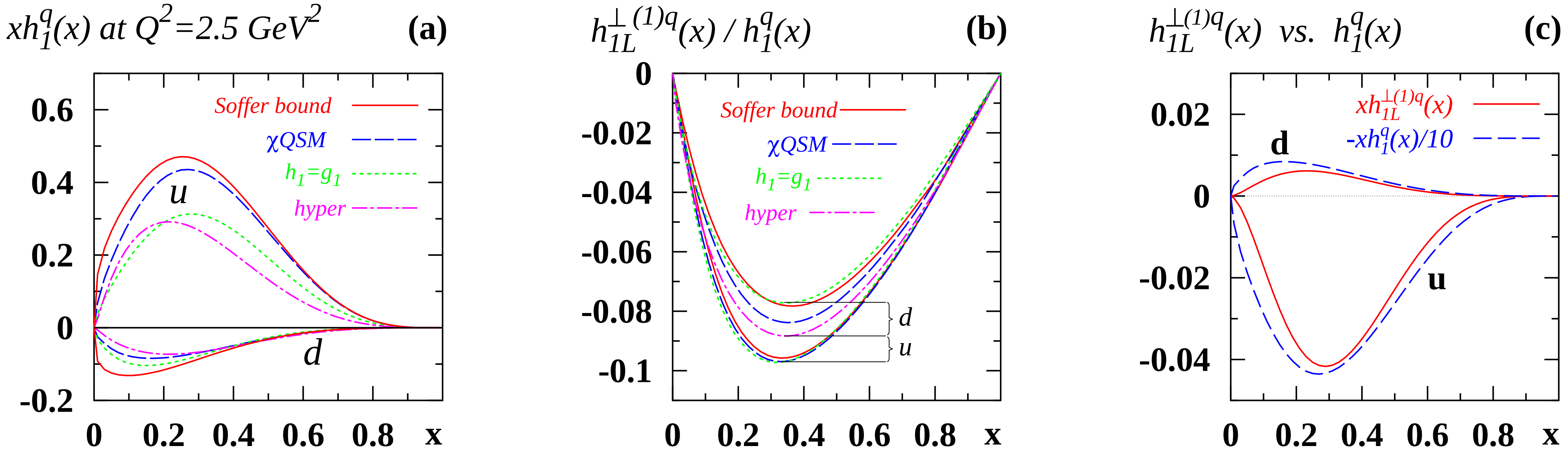
Tests of approximations: COMPASS & CLAS

- * Zheng et al. PRC**70**(2004)065207;
- ** Dressler, Polyakov, PRD**61**(2000)097501
- *** Kotzinian, Parsamyan and Prokudin, PRD 73, 114017 (2006)
- **** Avakian, AE, Goeke, Metz, Schweitzer, Teckentrup, op. cit.

We present a test of $A_{UL}^{\sin 2\phi}$

WW-type approximation for $h_{1L}^{\perp}$

Models for the transversity PDF.



Soffer bound; χQSM, PRD64(01)034013; Nonrelativ.; Hypercentral PRD76(07)051018; Light-cone CQM, Pasquini at al., arXiv:0806.2298[hep-ph]

Ratio $R = h_{1L}^{\perp(1)q}(x)/h_1^q(x)$ little depends on transversity model. A “universal” behaviour:

- $x \to 1$, $R \sim (1-x)$.
- $x \to 0$ behaviour, if $h_1^a(x) \sim x^\alpha$, $R \sim x$ for $\alpha \neq 1$,
- As a common feature $|h_{1L}^{\perp(1)a}(x)/h_1^a(x)| \lesssim 0.1$.

We will use the χQSM and LCQM.

$A_{UL}^{\sin 2\phi}$ in WW-type approximation

Assume Gauss Ansatz for $h_{1L}^{\perp a}(x,\mathbf{p}_T^2)$ and $H_1^{\perp a}(z,\mathbf{K}_T^2)$

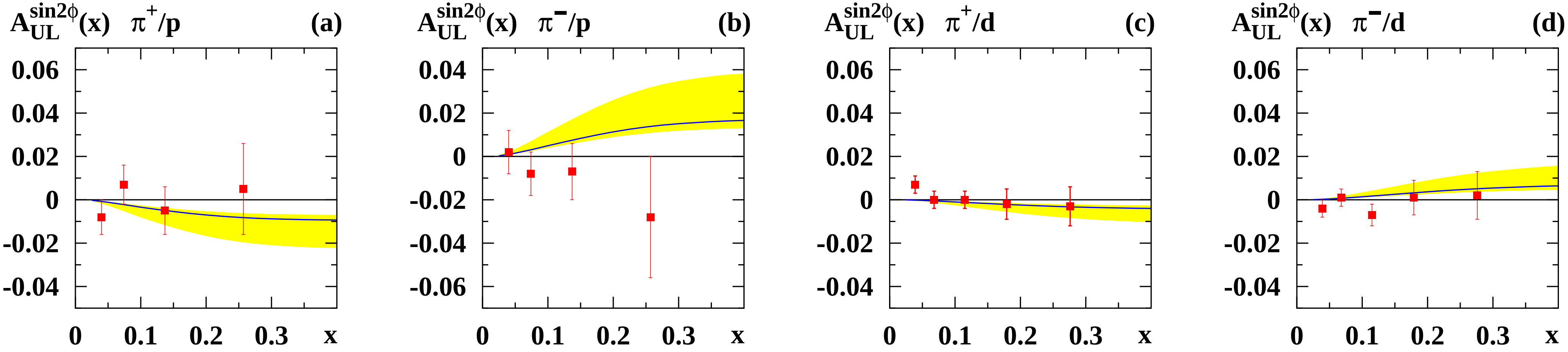
$$A_{UL}^{\sin 2\phi} = \frac{\sum_i \sin(2\phi_i)(N_i^{\leftrightarrows} - N_i^{\rightrightarrows})}{\sum_i \frac{1}{2}(N_i^{\leftrightarrows} + N_i^{\rightrightarrows})} = \frac{\int \mathrm{d}y[\cos\theta_\gamma(1-y)/Q^4]\sum_a e_a^2\, x h_{1L}^{\perp(1)a}(x)\langle 2B_{\text{Gauss}}H_1^{\perp(1/2)a}\rangle}{\int \mathrm{d}y[(1-y+\frac{1}{2}y^2)/Q^4]\sum_a e_a^2 x f_1^a(x)\langle D_1^a\rangle},$$

where $B_{\text{Gauss}} = (1 + z^2\langle\mathbf{p}_{h_1}^2\rangle/\langle\mathbf{K}_{H_1}^2\rangle)^{-1/2}$.

(Efremov et al. PRD**73**(2006)094025; Anselmino et al. PRD**75**(2007)054032)

From transversity at HERMES and Collins PFF at BELLE

$$\langle 2B_{\text{Gauss}}H_1^{\perp(1/2)\text{fav}}\rangle = (3.5 \pm 0.8)\% \text{ and } \langle 2B_{\text{Gauss}}H_1^{\perp(1/2)\text{unf}}\rangle = -(3.8 \pm 0.7)\,.$$



Opposite sign for π^-! Significant higher twist contributions (exclusive ρ and semi-exclusive π^- at large z)? Could be useful however.

Soon to be measured by COMPASS!

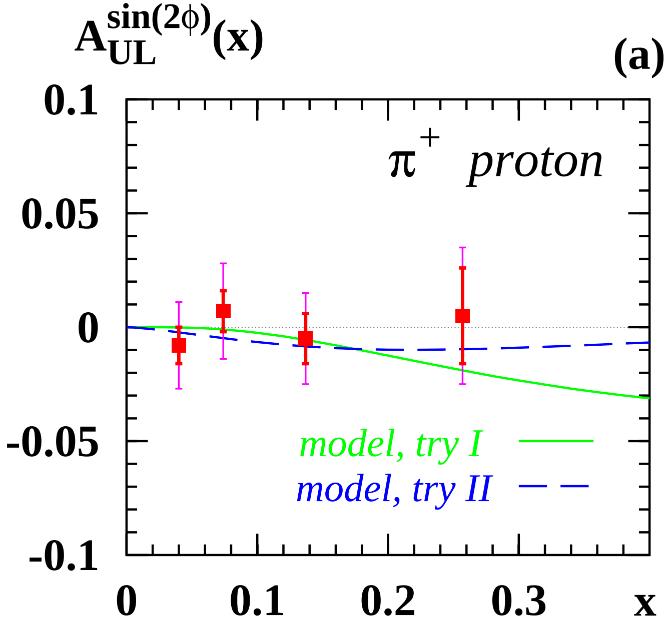
Problem with evolution.

LCQM (and others) gives TMD functions
at low scale μ_0^2.

Evolution equation for $h_{1L}^{(1)\perp}(x,Q^2)$ yet unknown.

Two possibilities:

Model I —— no evolution
(chiral odd, no mixture with gluon)

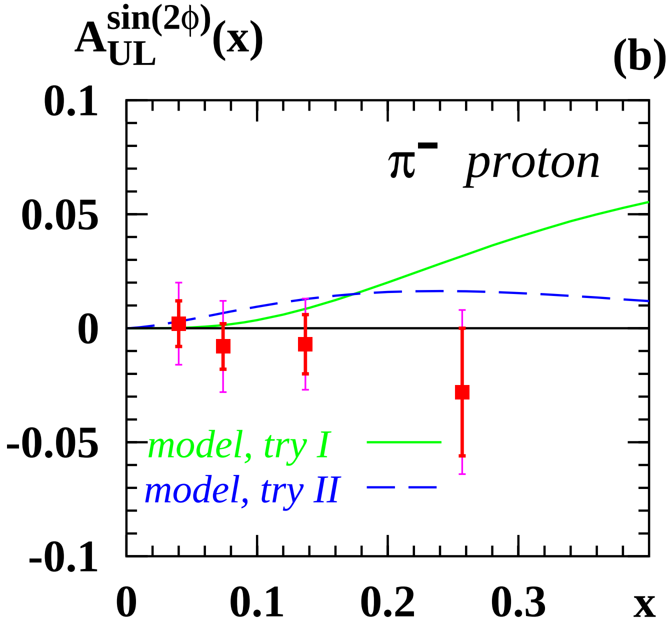


Model II - - - evolution similar to $h_1(x,Q^2)$, i.e.

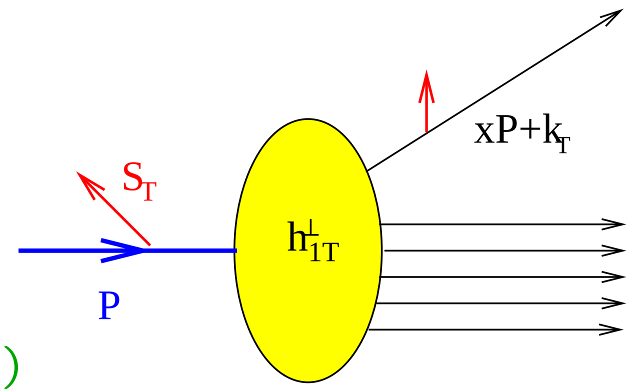
$$h_{1L}^{(1)\perp(x,Q^2)} = h_{1L}^{(1)\perp}(x,\mu_0^2)\frac{h_1(x,Q^2)}{h_1(x,\mu_0^2)}$$

Data HERMES: PRL84(00); NP.Proc.Suppl.79(99).

Seems better agrees with experiment.



Properties of pretzelosity $h_{1T}^{\perp u}$



- Chiral-odd, no gluons
  (Mulders, Rodrigues '01; Meissner, Metz, Goeke '07)

- Suppressed at small and large x vs. f_1^q
  (Avakian, Brodsky, Deur, Yuan; Brodsky, Yuan; Burkardt)

- $h_{1T}^{\perp u} = -h_{1T}^{\perp d}$ modulo $1/N_c$ corrections (Pobylitsa 2003)

- Model-dependent relation to chirally odd GPDs (lattice)
  (Burkardt; Meissner, Metz, Goeke 2007)

- Presence of wave-function components with two units orbital momentum difference needed; e.g. quadratic in p-wave component (Burkardt 2007)

- Measures in nucleon deviation from spherical shape. Is it a "pretzel", "bagel", "peanut"? (Miller, nucl-th/0708.2297; Burkardt, hep-th/0709.2966) hence **pretzelosity**

Pretzelosity in inequalities

- Positivity condition (Bacchetta, Boglione, Henneman and Mulders, 2000)

$$\left| h_{1T}^{\perp(1)a}(x, \vec{p}_T^{\,2}) \right| \leq \frac{1}{2} \Big(f_1^a(x, \vec{p}_T^{\,2}) - g_1^a(x, \vec{p}_T^{\,2}) \Big)$$

useful since f_1^a and g_1^a known, with $\boxed{h_{1T}^{\perp(1)a}(x, \vec{p}_T^{\,2}) = \frac{\vec{p}_T^{\,2}}{2M_N^2} h_{1T}^{\perp a}(x, \vec{p}_T^{\,2})}$

(Notice p_T-dependence!)

- Combine with $|h_1^a(x)| \leq \frac{1}{2}(f_1^a + g_1^a)(x)$ (Soffer 1994)

$$|h_{1T}^{\perp(1)a}(x)| + |h_1^a(x)| \leq f_1^a(x)$$

less useful, since $h_1^a(x)$ less known but interesting:

transversity and pretzelosity cannot be both large.

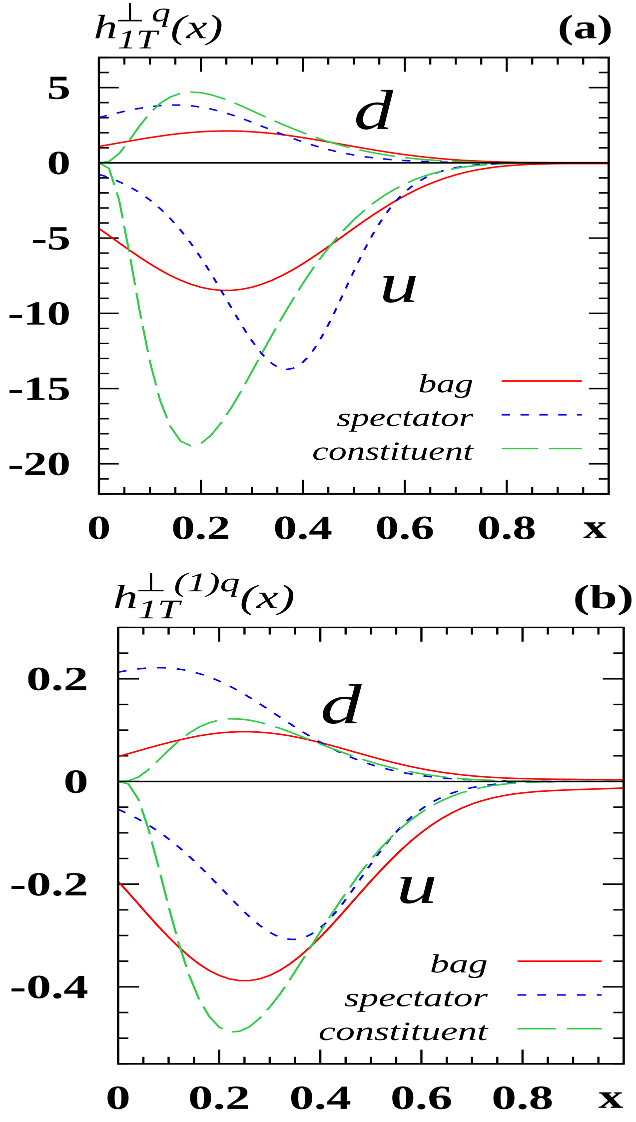
Pretzelosity in models

$$\boxed{h_{1T}^{\perp a}(x) = \int d^2\vec{p}_T\, h_{1T}^{\perp a}(x, \vec{p}_T^{\,2})}$$

- It is large!
- Larger than $f_1^q(x)$, $h_1^q(x)$.
- Not constrained by positivity.
- Signs opposite to transversity.
- Qualitative agreement bag, spectator & constituent models.

$$\boxed{h_{1T}^{\perp(1)a}(x) = \int d^2\vec{p}_T\, \frac{\vec{p}_T^{\,2}}{2M_N^2}\, h_{1T}^{\perp a}(x, \vec{p}_T^{\,2})}$$

- Models consistent with large N_c (but $N_c = 3$ and corrections visible)
- $|h_{1T}^{\perp(1)a}(x)/h_1^a(x)| \sim \mathcal{O}(20\%)$, see below
- Positivity satisfied, and not small e.g. $|h_{1T}^{\perp(1)q}(x)| + |h_1^q(x)| \leq f_1^q(x)$, u-flavour explores $\frac{2}{3}$ of bound (d-flavour $\frac{1}{3}$).
- Exist relations among TMDs.



(Avakian, AE, Schweitzer, Yuan, op.cit.)
(Jakob, Mulders, Rodrigues 1997)
(Pasquini, et al. hep-ph:0806.2298)

Very specific relations in Valence Quark Models

(1) $f_1^q(x,\vec{p}_T^{\,2}) = N_q\, f_1(x,\vec{p}_T^{\,2})$ with $N_u = 2,\ N_d = 1$

(2) $g_1^q(x,\vec{p}_T^{\,2}) = \ P_q\, g_1(x,\vec{p}_T^{\,2})$ $P_u = \frac{4}{3},\ P_d = -\frac{1}{3}$ from SU(6)

(h_1, $h_{1T}^{\perp}$ analog)

"Bare" distributions satisfy:

(1) $f_1(x,\vec{p}_T^{\,2}) + g_1(x,\vec{p}_T^{\,2}) = 2h_1(x,\vec{p}_T^{\,2})$

(2) $h_1(x,k_\perp) - h_{1T}^{\perp(1)}(x,k_\perp) = f_1(x,k_\perp)$

(3) $h_{1L}^{\perp(1)}(x,k_\perp) = -g_{1T}(x,\vec{p}_T^{\,2})$

(1), (2) and (3) hold in LCQM (Pasquini et al. PRD72(2005), hep-ph:0806.2298)

(1) and (2) hold in Bag and also in "Zavada Model"

(Avakian at al. hep-ph/0805.3355; P.Zavada talk)

(3) holds in spectator model, (1) and (2) are recovered
only if $M^{qq}_{axial} \rightarrow M^{qq}_{scalar}$ (Jakob at al. NPA626(1997))

More general and exciting relation:

In all mentioned models:

$$g_1^q(x) - h_1^q(x) = h_{1T}^{\perp(1)q}(x)$$

Popular statement: helicity − transversity = 'measure' of relativistic effects.

More precise statement: **helicity − transversity = pretzelosity!**

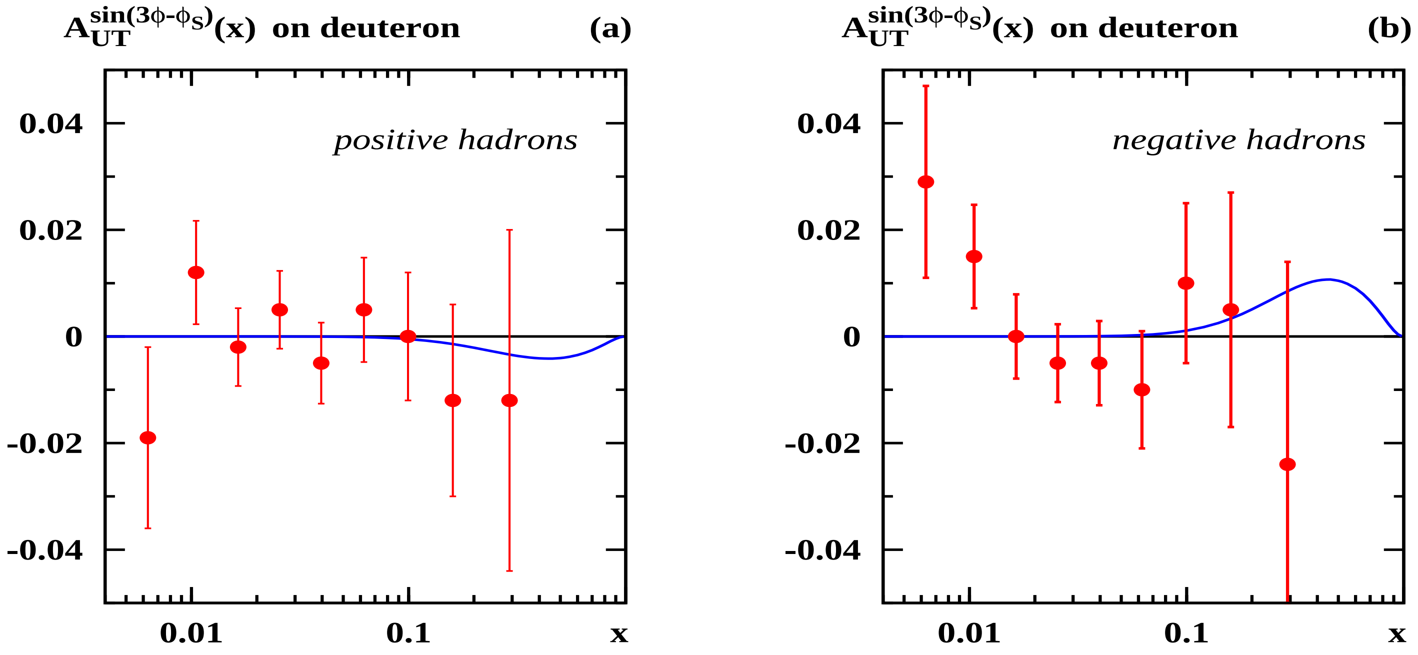
Statement valid at low scale in large class of relativistic models,
not valid in models with gluons (Meissner, Metz, Goeke 2007),
not valid in QCD (all TMDs independent, not preserved by evolution).

Corollary: pretzelosity → 0 in non-relativistic limit
define pretzelosity = 'measure' of relativistic effects (it is not "peanut"!)

Pretzelosity in SIDIS: first insights

Preliminary COMPASS deuteron data on $A_{UT}^{\sin(3\phi_h-\phi_S)} \propto \sum_a e_a^2\, h_{1T}^{\perp(1)a} \otimes H_1^{\perp a}$
A.Kotzinian [on behalf of COMPASS collaboration], arXiv:0705.2402 [hep-ex]

Use $H_1^\perp$ from HERMES & Belle (V & Y; AE, Goeke, Schweitzer; Anselmino et al.)
Use Light-cone CQM (Boffi, AE, Pasquini, Schweitzer, in preparation)



COMPASS data hint at suppression at small x, or opposite signs for u and d (as expected)
and do not exclude large pretzelosity at intermediate and large x!

Pretzelosity in SIDIS: prospects

There will be data from COMPASS proton target (small x)
There will be data from HERMES (so far only 'is small')
There will be more insights!

Most preferable conditions:

- intermediate $x \sim (0.1 - 0.4)$
- high luminosity

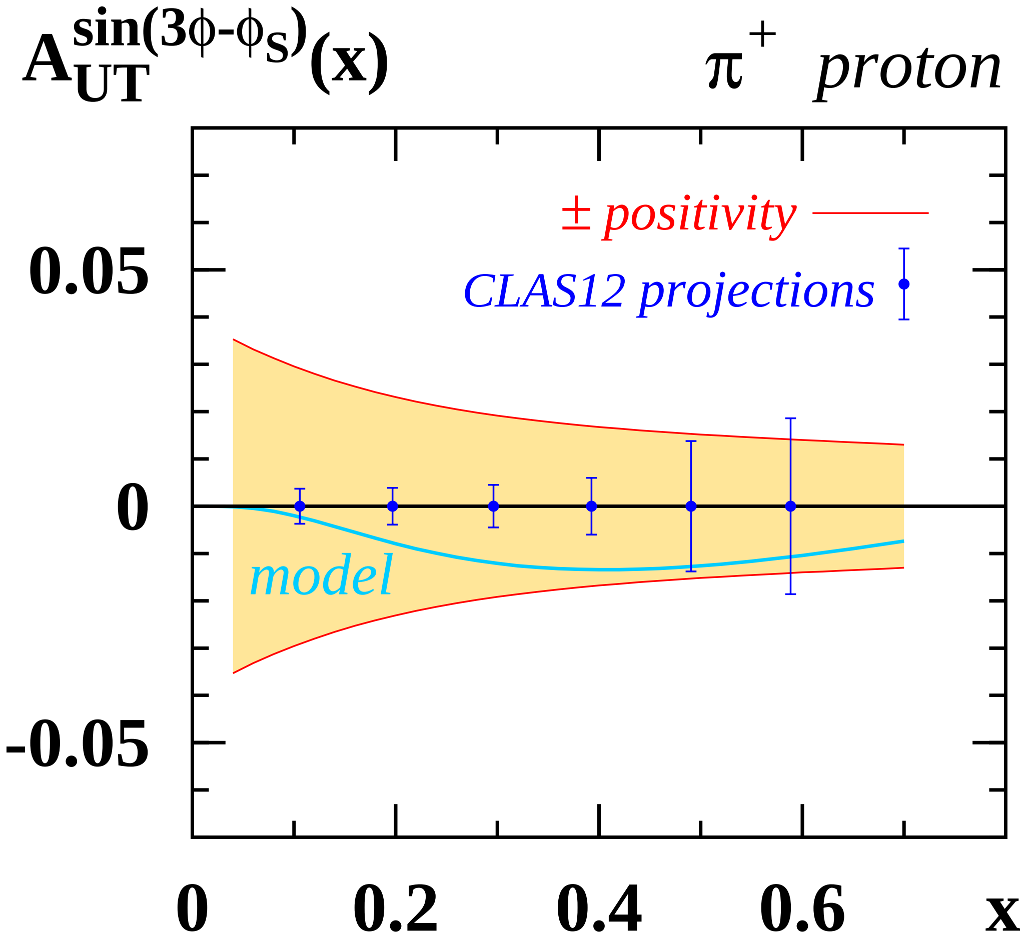
  → JLab

CLAS with 12 GeV (H.Avakian et al,
LOI 12-06-108)

—- $|h_{1T}^{(1)\perp q}| < f_1^q - g_1^q$
—- Light-Cone CQM

Error projections for 2000
hours run time at CLAS12

(Boffi, AE, Pasquini, Schweitzer, in preparation)

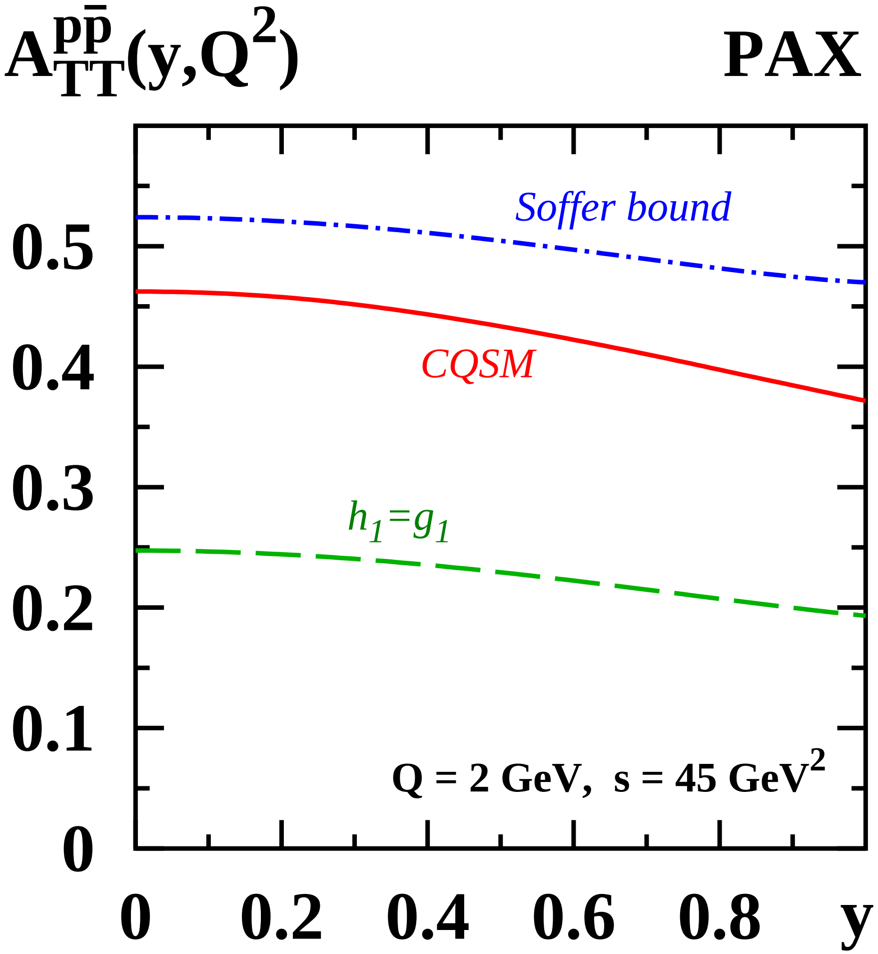


Consequences of pretzelosity

Can be accessed in Drell-Yan in certain azimuthal spin asymmetries.
But most immediate consequence:

- $g_1^u(x) - h_1^u(x) = \underbrace{h_{1T}^{\perp(1)u}(x)}_{\text{negative}}$
- $|h_{1T}^{\perp(1)u}| \sim 20\% |h_1^u|$

$\longrightarrow h_1^u$ is $\sim 20\%$ larger
than g_1^u at low scale



PAX

$A_{TT}^{p\bar{p}} \propto \sum_a e_a^2 h_1^a\, h_1^a \approx \frac{4}{9}\, h_1^u\, h_1^u$

$A_{TT}^{p\bar{p}} \approx (1.2)^2 A_{TT}^{p\bar{p}}|_{g_1=h_1}$

Promising!
(of course, corrections)

Conclusions

- Data do not exclude the possibility that WW-type approximations for $A_{UL}^{\sin 2\phi}$ work.

- For more definite statements precise measurements of these SSAs are necessary, preferably in the region around $x \sim 0.3$.

- CLAS upcoming run will certainly improve our current understanding of this and other SSAs and shed light on spin-orbit correlations.

- Experimental confirmation of the utility of the WW-type approximation would mean the possibility to extract information on transversity from a longitudinally polarized target.

- Pretzelosity is a new interesting PDF!

- Measures 'relativistic effects' and obeys $g_1^a(x) - h_1^a(x) = h_{1T}^{\perp(1)a}(x)$ at low scale in large class of relativistic quark models

- Gives rise to $A_{UT}^{\sin(3\phi-\phi_S)}$ in SIDIS. first insights from COMPASS (**bravo!**), soon also HERMES, Hall A JLab
  (Please: extract! Not only cross check for $\sin(\phi \pm \phi_S)$ fits.)

- Preferable condition: intermediate $x \sim (0.1 - 0.5)$, high luminosity at Jefferson Lab CLAS 12

- Consequences for Drell-Yan: $A_{TT}^{p\bar{p}} \sim 1.5\, A_{TT}^{p\bar{p}}|_{g_1=h_1}$ looking forward to PAX!

- Pretzelosity $\in$ 'beyond' Sivers and Collins $\neq$ peanuts but fascinating!!!

Thank you!

Appendix

- many (not all) models predict $h_1^u > g_1^u$
  including bag & spectator model, where $g_1^u(x) - h_1^u(x) = h_{1T}^{\perp(1)u}(x) < 0$
- Torino-Cagliari group: $h_1^u \approx g_1^u$ within statistical error bars
  hard work done (compliments!), I would not know how to do it better, but:
- SIDIS $= h_1\ H_1^\perp = (\lambda h_1)\ (\frac{1}{\lambda} H_1^\perp)$ does not tell us the normalization of h_1
- Normalization fixed by Belle e^+e^- data $= H_1^\perp\ H_1^\perp$
- $H_1^\perp(z, k_T, \mu^2 =$ 110 GeV^2| Belle) $\overset{?}{\leftrightarrow}$ $H_1^\perp(z, k_T, \mu^2 =$ 2.5 GeV^2| HERMES)
  certain reasonable *assumptions* used $\rightarrow h_1$ has *systematic* uncertainty
- In principle, we know AE-Teyaev-Collins-Soper-Sterman formalism.
  In practice: hard work needed to implement into SIDIS and fitting codes.
- If in the end it turns out: *assumptions* were justified within 20%, then:
  Torino-Cagliari group will be happy. Many models will be happy.
  Pretzelosity will have room. We will see!