The G_{Ep}/G_{Mp} Ratio and the 2γ Conundrum*

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*In the sense: a question or problem having only a conjectural answer

Outline

Introduction

Form Factors from Rosenbluth and recoil polarization

The proton Form Factor "discrepancy"

Possible 2γ contribution

Related experiments at JLab

Conclusions

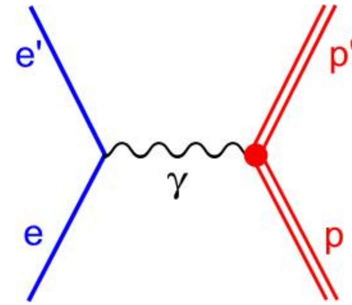
(see also review in "Progress in Particle and Nuclear Physics", C.F.P., V. Punjabi and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 59, 694-764, 2007)

Introduction

- At large Q^2 electromagnetic Form Factors contain structure information on the many-body system of quarks and gluons of the nucleon. At low Q^2 they inform us about the pion cloud.
- When obtained from experiment, the Form Factors are relativistic invariants only to the extent that the probe is a **single** virtual **photon exchanged** between electron and nucleon; higher order contributions destroy this invariance, which one might regain after applying a number of radiative corrections.
- The recent inclusion of 2γ exchange with two hard photons may help reconciliate the discrepancy between Rosenbluth and Recoil Polarization measurements of G_{Ep}/G_{Mp} , but more data are needed.

ep elastic in Born approximation

$j_\mu = \langle e' | \gamma_\mu | e \rangle$



$J_\mu = \langle p' | \Gamma_\mu | p \rangle$

Nucleon vertex: $\Gamma_\mu(p, p') = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2(Q^2)$

Dirac Pauli

F_1 helicity conserving , F_2 helicity non-conserving form factors

Alternately, the Sachs form factors

$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$ $\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$ $\quad \tau = Q^2 / 4M^2$

In the Breit frame, and for $Q^2 \approx 0$, G_E and G_M are Fourier transforms of charge- and current distributions.

Rosenbluth Cross Section

The cross section for single photon exchange (Born term) is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{Mott}}\left(G_{Ep}^2(Q^2) + \frac{\tau}{\varepsilon}G_{Mp}^2(Q^2)\right)/(1+\tau)$$

with:

$$\varepsilon = \left(1 + 2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right)\right)^{-1}$$

The reduced cross section used in Rosenbluth separation :

$$\sigma_{reduced} = \left(\varepsilon(1+\tau)\frac{d\sigma}{d\Omega}\right)\Big/\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \varepsilon G_{Ep}^2 + \tau G_{Mp}^2$$

vs. Recoil Polarization

Polarization transfer in $\vec{e}p \to e\vec{p}$ or spin-target asymmetry $\vec{e}\vec{p} \to ep$ result in either polarization of the recoil proton, or in parallel-transverse asymmetry, respectively.

For recoil polarization, the two polarization components are:

$$hP_eP_t = -hP_e 2\sqrt{\tau(1+\tau)} G_{Ep} G_{Mp} \tan(\frac{\theta_e}{2})/I_0$$

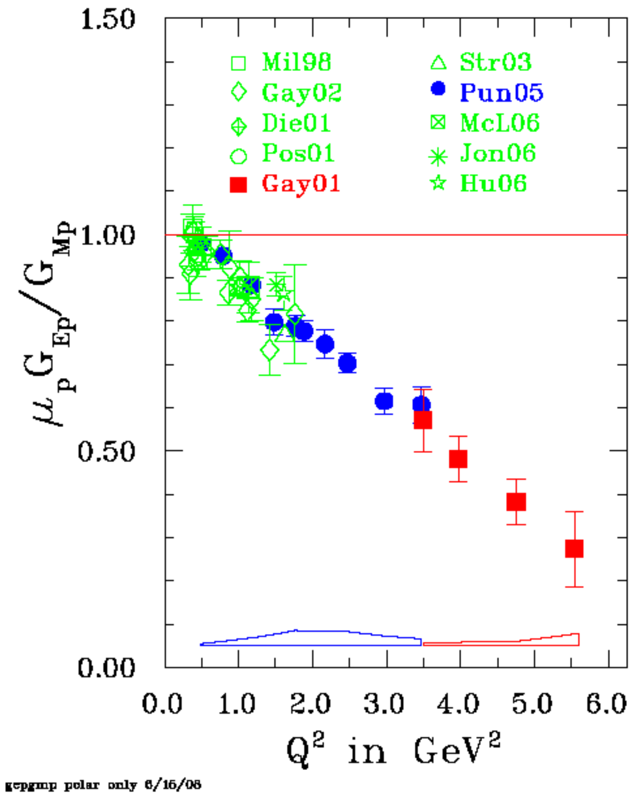
$$hP_eP_\ell = hP_e \frac{(E_e + E_{e'})}{M} G_{Mp}^2 \sqrt{\tau(1+\tau)} \tan^2(\frac{\theta_e}{2})/I_0$$

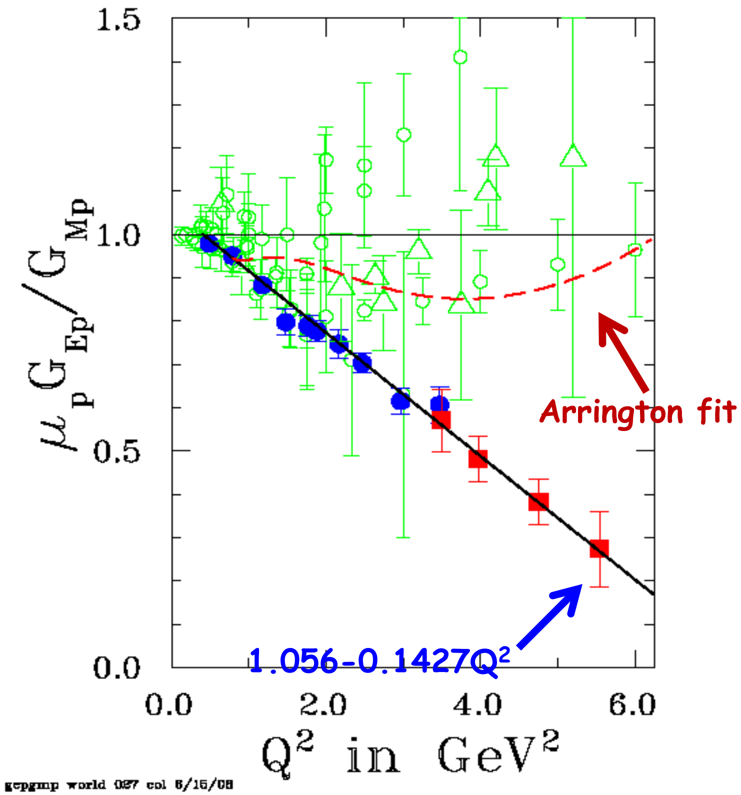
The beauty of the method is that the Form Factor ratio is independent of the electron polarization and of the polarimeter analyzing power:

$$\boxed{\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_\ell} \frac{(E_e + E_{e'})}{2M} \tan\left(\frac{\theta_e}{2}\right) \quad or \quad -\frac{P_t}{P_\ell}\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}}$$

Recoil Polarization Results

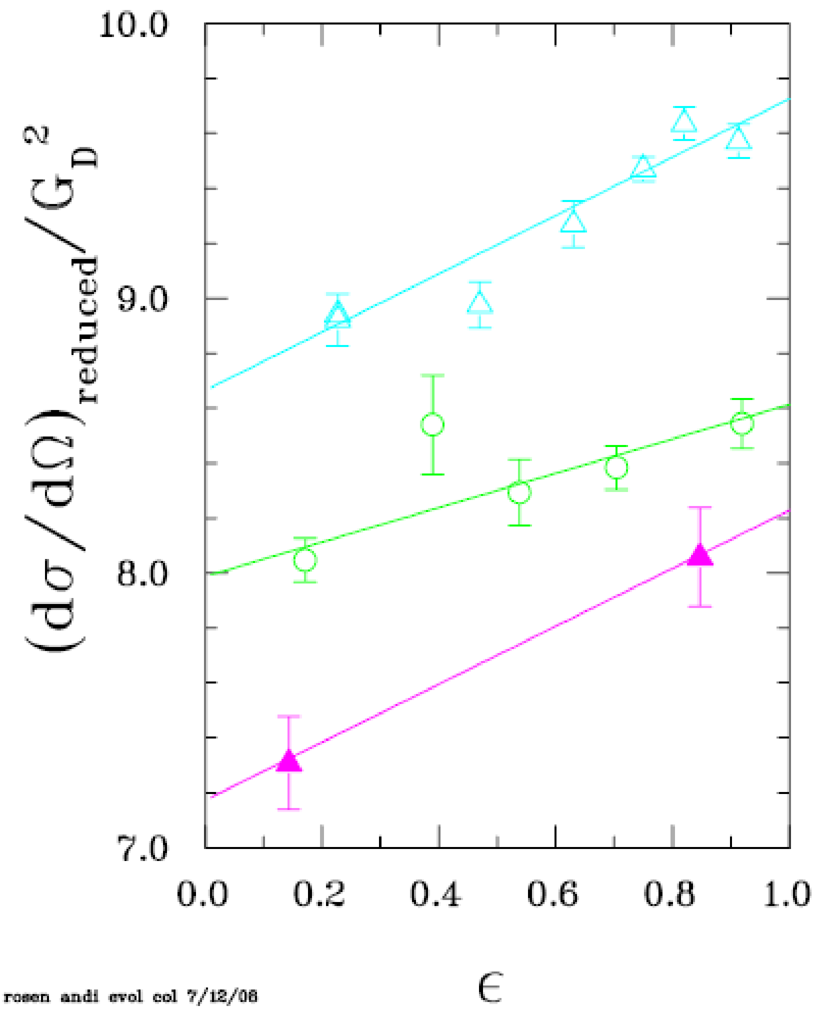
Recoil polarization results are in stark disagreement with Rosenbluth separation data.





Rosenbluth Separation: Data

A typical example of Rosenbluth separation made prior to the Jlab recoil polarization experiments.



Andivahis et al. PR D50, 5491 (1994)

Selected here are Q^2 values of 2.5,

5 and 7 GeV^2.

So what is the cause for the different results?

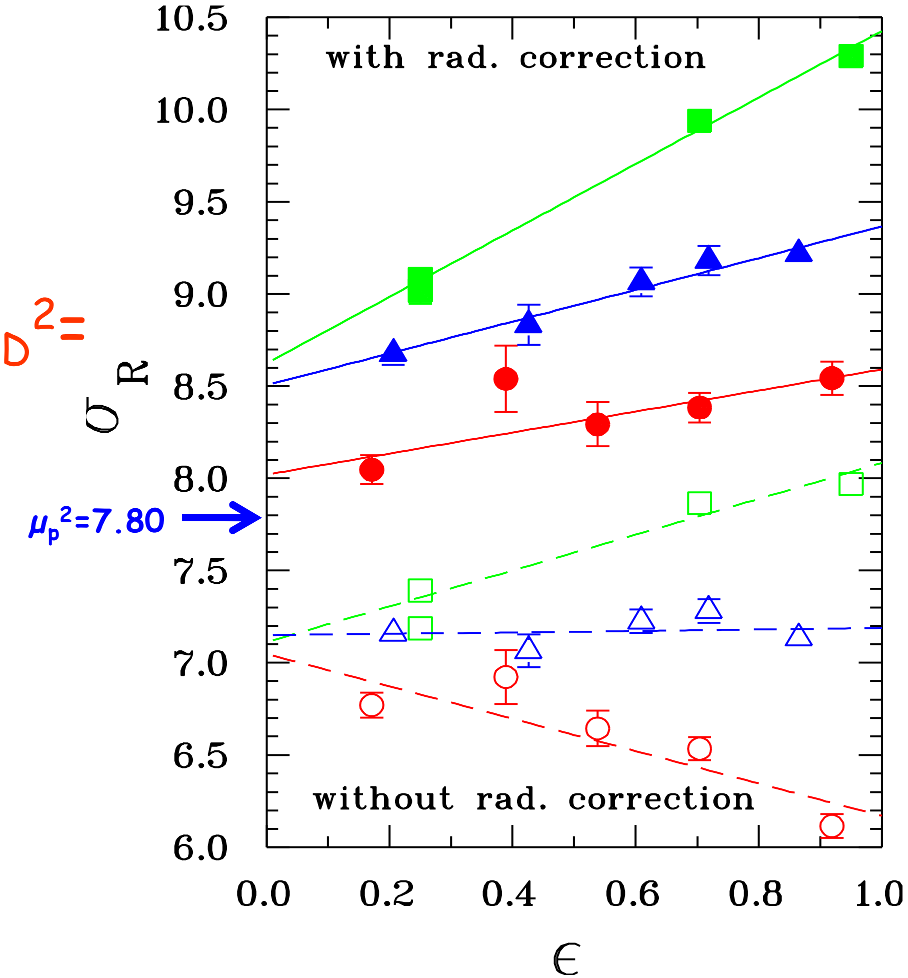
First, radiative corrections at large Q^2 are large and strongly ε-dependent.

$$\sigma_R=[\varepsilon(1+\tau)/\tau][\sigma_{exp}/\sigma_{Mott}]/G_D^2=$$

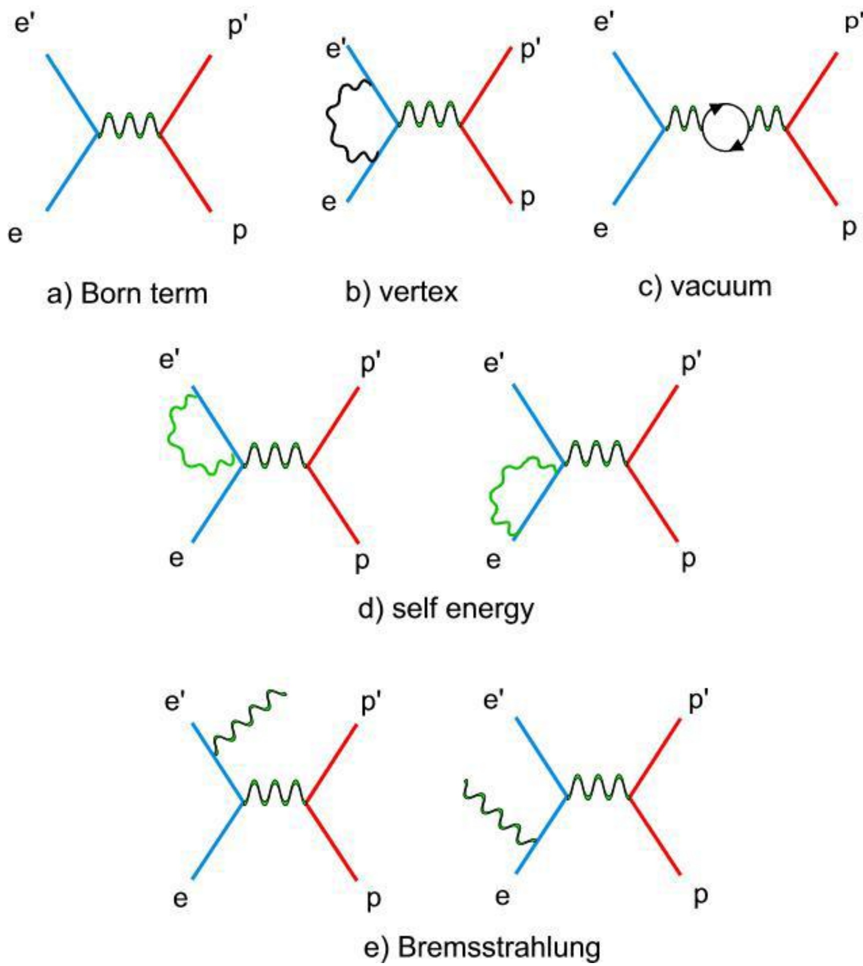
$$=(G^2_{Mp}+\varepsilon\, G^2_{Ep}/\tau)/G_D^2$$

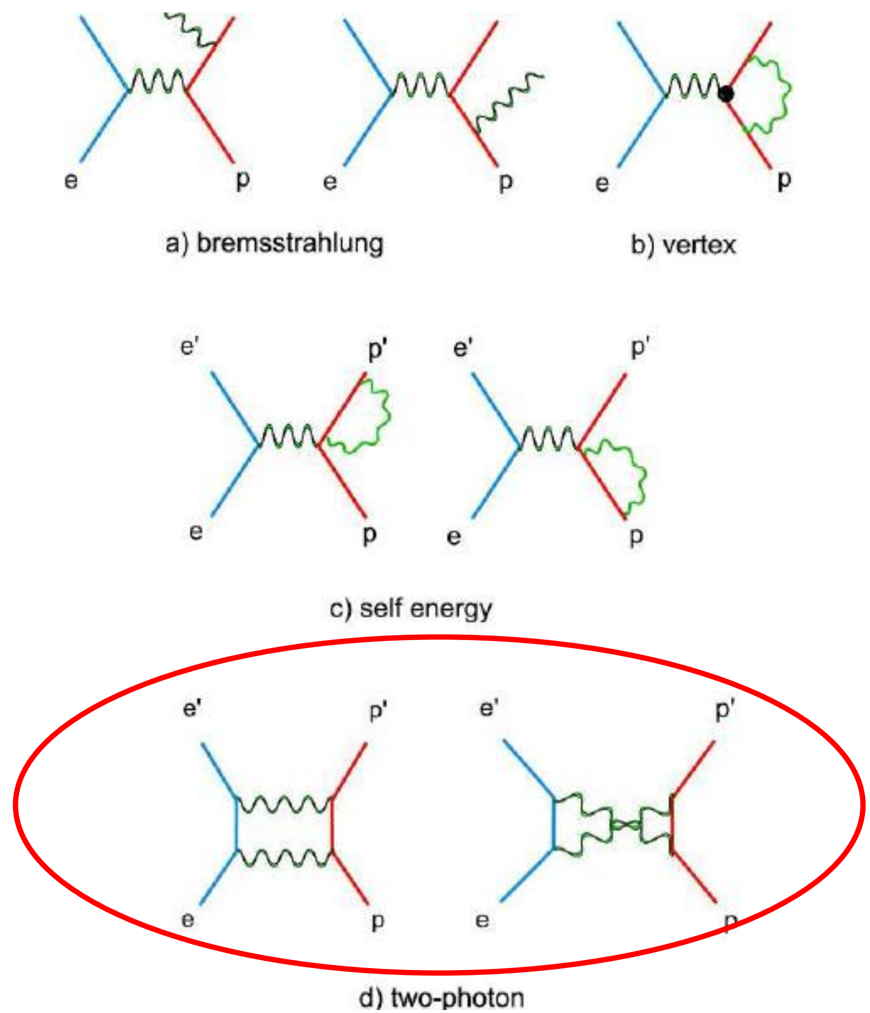
green for 1.75 GeV^2
blue for 3.75 GeV^2
red for 5 GeV^2

Data from Andivahis et al. (1994)



So many corrections!





Radiative Corrections

Following Mo and Tsai (RMP 41, 205 1969)

$$\left.\frac{d\sigma}{d\Omega}\right)_{RC} = e^{\delta} \left.\frac{d\sigma}{d\Omega}\right)_{expt}$$

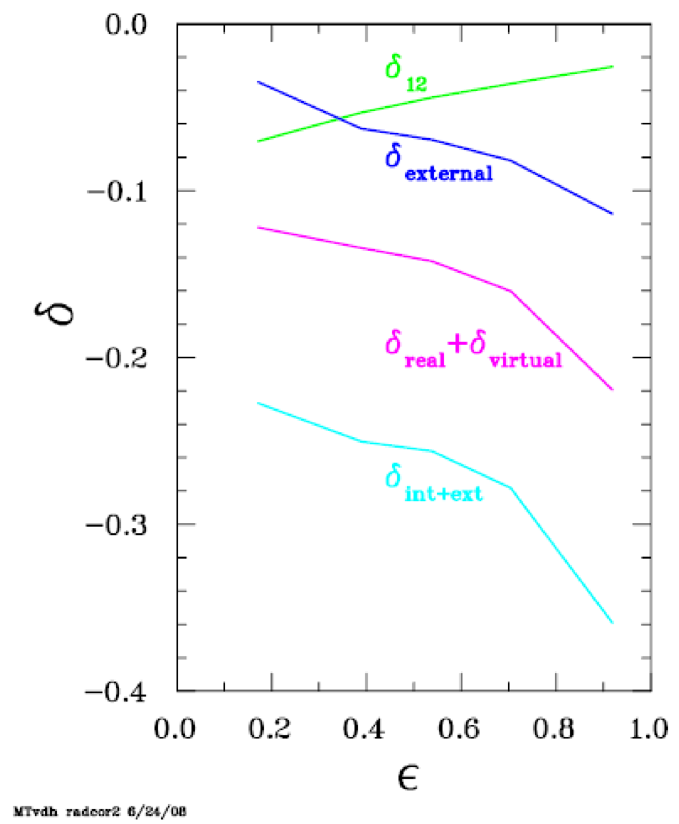
Radiative correction parameter δ comprises several parts:

1) Electron: self-energy, vacuum, bremsstrahlung and vertex

2) Proton: self-energy, vertex, bremsstrahlung ,two photon with one soft

3) Target radiative correction.

Illustration based on Maximon and Tjon, (PR C62:054320, 2000), coded by M. Vanderhaeghen et al.



For 5 GeV^2, Andivahis data

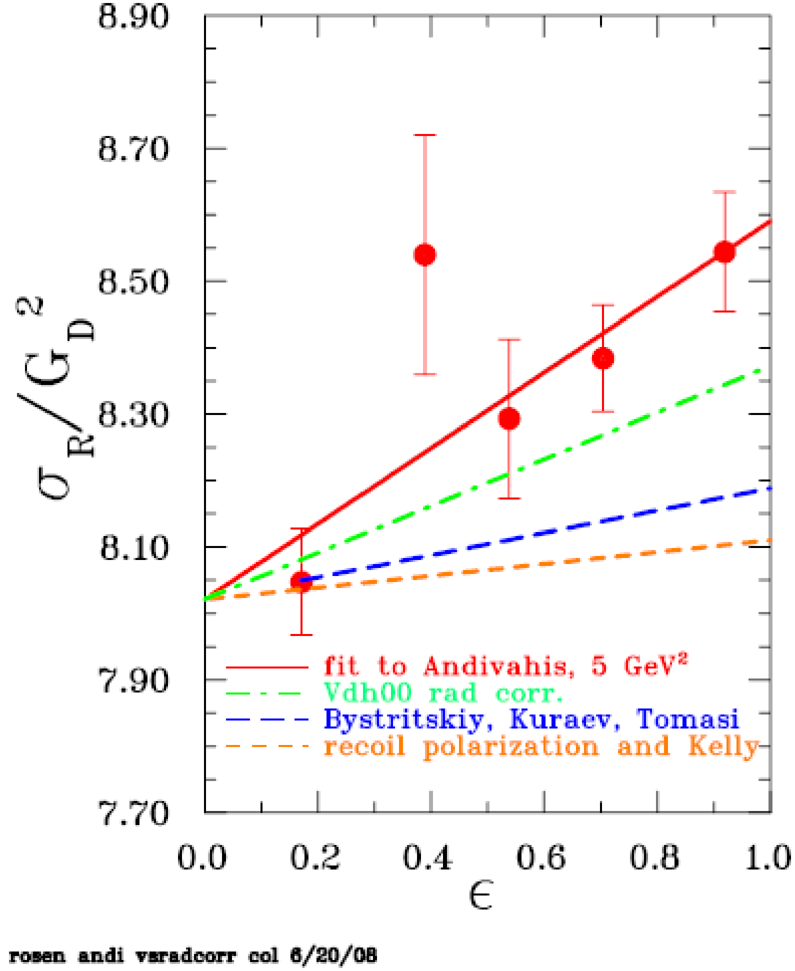
Second, there is a scatter in size of calculated corrections

Andivahis et al: based on Mo and Tsai, (RMP 41, 205 1969) with improvements from Walker et al. (PR D49, 5671 1994)

Vanderhaeghen et al: code based on Maximon and Tjon (PR C62:054320, 2000): exact soft photon, better vertex and exact box diagram calculations.

Bystritskiy, Kuraev, Tomasi-Gustafsson: (PR C75.015207.2007) with structure function (Drell-Yan parton picture). Radiative correction for electron to all orders.

Interpolation from Hall A recoil polarization, G_{Mp} from Kelly fit (PR C 70: 068202 2004).



Is 2γ exchange the Missing Diagram?

- Adding the next-order diagram make the vertex function complex and adds a third FF; there are now 6 amplitudes to measure; requires 6 independent observables. The contribution of the 2γ graph is an interference with the 1γ process.
- Even though smaller by factor e^2 then Born term, effect on Rosenbluth cross section large when $G_{Ep}{}^2$ becomes very small. It was not known 10 years ago that Gep decreases fast with increasing Q2
- But cross sections, unpolarized and polarized, depend upon Real Part only. The hadronic vertex becomes:

$$\Gamma_\mu(p,p') = \gamma_\mu \tilde{G}_{Mp} - \frac{P^\mu}{M}\tilde{F}_2 + \frac{\gamma.KP^\mu}{M^2}\tilde{F}_3$$

with P=p+p', K=k+k', initial and final, proton and electron momenta.

Now for elastic FF, $\tilde{G}_{Mp}$, $\tilde{F}_2$, $\tilde{F}_3$, which are no longer Lorentz invariant.

Evolution of 2γ Hypothesis

Drell and Rudermann (1957), Drell and Fubini (1959) concluded that the correction was at the level of $e^2 \sim 1/137$, and that the Born approximation used by Hofstadter et al was valid up to at least 1 GeV^2.

Greenhut (1969) explicitly refer to 2γ contribution as interference between first and second Born approximation: of order α^3

Validity of the Born approximation was being verified, but no mention of 2 hard photon exchange; the 2γ graph was incorporated in the Mo and Tsai procedure used almost exclusively until the work of L.C. Maximon and J.A. Tjon, but including only 1 hard and 1 soft photons.

The first explicit consideration of exchange of two hard photons was made by P.A.M. Guichon and M. Vanderhaeghen (PRL 91, 142303 2003) , to address the dilemma created by the Jlab recoil polarization data.

Recent review of subject by C. Carlson and M. Vanderhaeghen (Ann.Rev.Nucl.Part.Sci.57:171. 2007)

2γ Continued

- Cross sections and polarizations in Space-Like region are defined by real part of Form Factors.
- Imaginary part defines observables forbidden by parity conservation, induced polarization or single spin asymmetries; and affects Form Factors in Time-Like region.
- To take into account exchange of two hard γ's, must replace Born form factors, which are relativistic invariants, by "effective" form factors, which are not relativistic invariants (and complex), as follows:

$$\boxed{G_{Ep}(Q^2) \Rightarrow \Re e\tilde{G}_{Ep}(Q^2,\varepsilon)} \qquad \boxed{G_{Mp}(Q^2) \Rightarrow \Re e\tilde{G}_{Mp}(Q^2,\varepsilon)}$$

and replace third form factor: $\boxed{\Re e\tilde{F}_3(Q^2,\varepsilon)}$

by:

$$\boxed{Y_{2\gamma} = \sqrt{\tau(1+\tau)}\sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\,\frac{\Re e\tilde{F}_3(Q^2,\varepsilon)}{G_{Mp}}, \quad and \quad \tilde{R} = \frac{\Re e\tilde{G}_{Ep}}{\Re e\tilde{G}_{Mp}}}$$

As discussed by Rekalo and Tomasi-Gustaffson, (NP A742:322,2004) the above is not unique, and may suffer from several defects. We pursue it here because it leads to a solution.

L. Pentchev's method

Now, we can find values for 3 observables ($Y_{2\gamma}$, $\tilde{G}_M^2$ and A_y, the analyzing) from the 3 quantities ($d\sigma$, Pt and $A_y \times P_\ell$) we have measured at 3 values of ε, by inverting their relation as follows:

$$P_t = \sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\,\frac{\tilde{G}_M^2}{d\sigma_{reduced}}\left(\tilde{R} + Y_{2\gamma}\right)$$

$$P_\ell = \sqrt{(1+\varepsilon)(1-\varepsilon)}\,\frac{\tilde{G}_M^2}{d\sigma_{reduced}}\left(1 + \frac{2}{1+\varepsilon}\varepsilon Y_{2\gamma}\right)$$

$$\frac{d\sigma_{reduced}}{\tilde{G}_M^2} = 1 + \frac{\varepsilon}{\tau}\tilde{R}^2 + 2\left(1 + \frac{R}{\tau}\right)\varepsilon Y_{2\gamma}$$

For example from P_ℓ and P_t

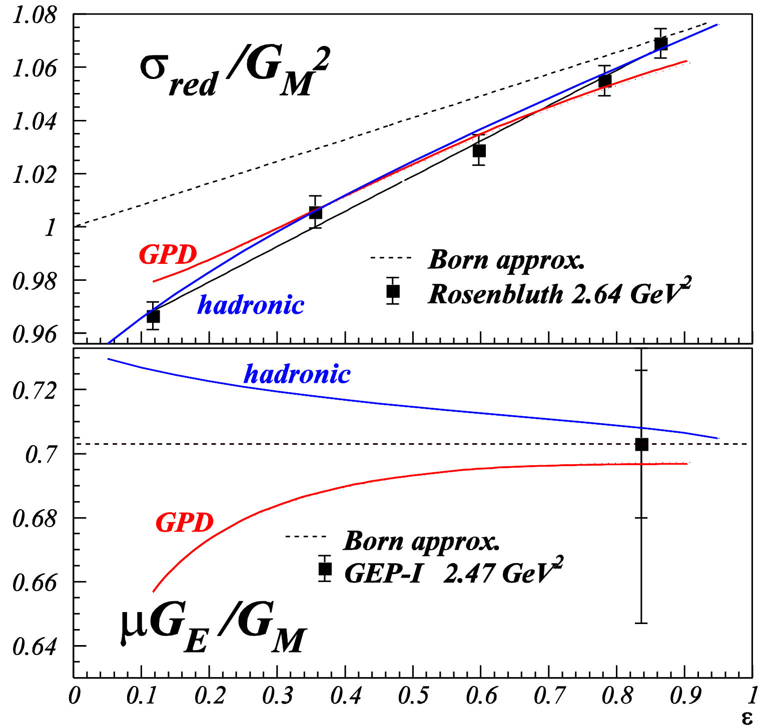
$$Y_{2\gamma} = \frac{\tilde{R} - \sqrt{\tau} F r}{\sqrt{\tau} r / F - 1}$$

where

$$\tilde{R} = \frac{\tilde{G}_{Ep}}{\tilde{G}_{Mp}}, \quad r = \frac{P_t}{P_\ell} \quad and \quad F = \sqrt{\frac{1+\varepsilon}{2\varepsilon}}$$

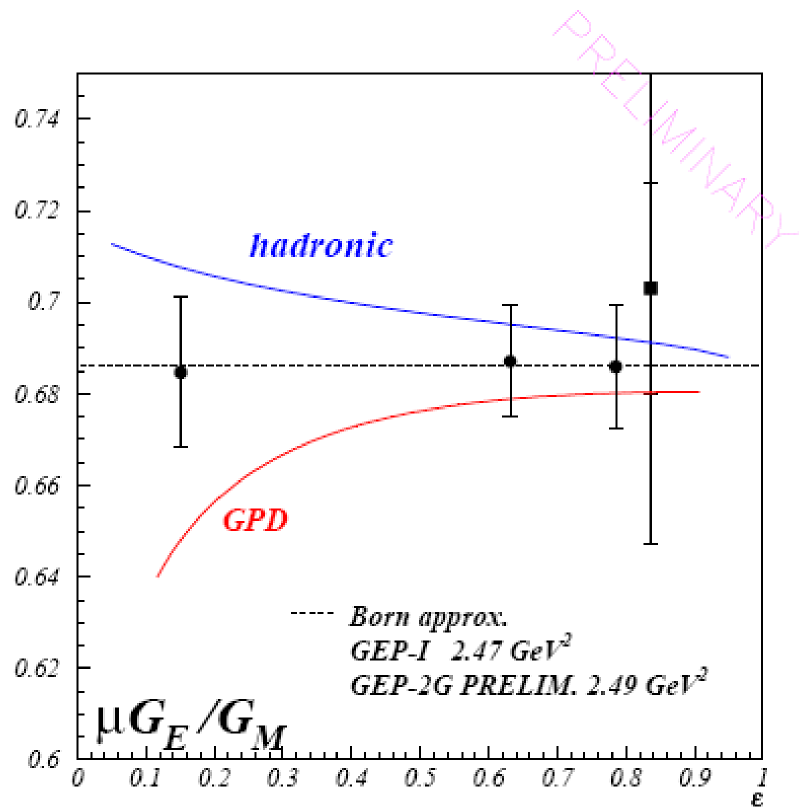
r is measured, so above gives $Y_{2\gamma}$ versus $\tilde{R}$; likewise for $\tilde{G}_M^2$ and A_y, the analyzing power.

2γ-Gamma Model Prediction



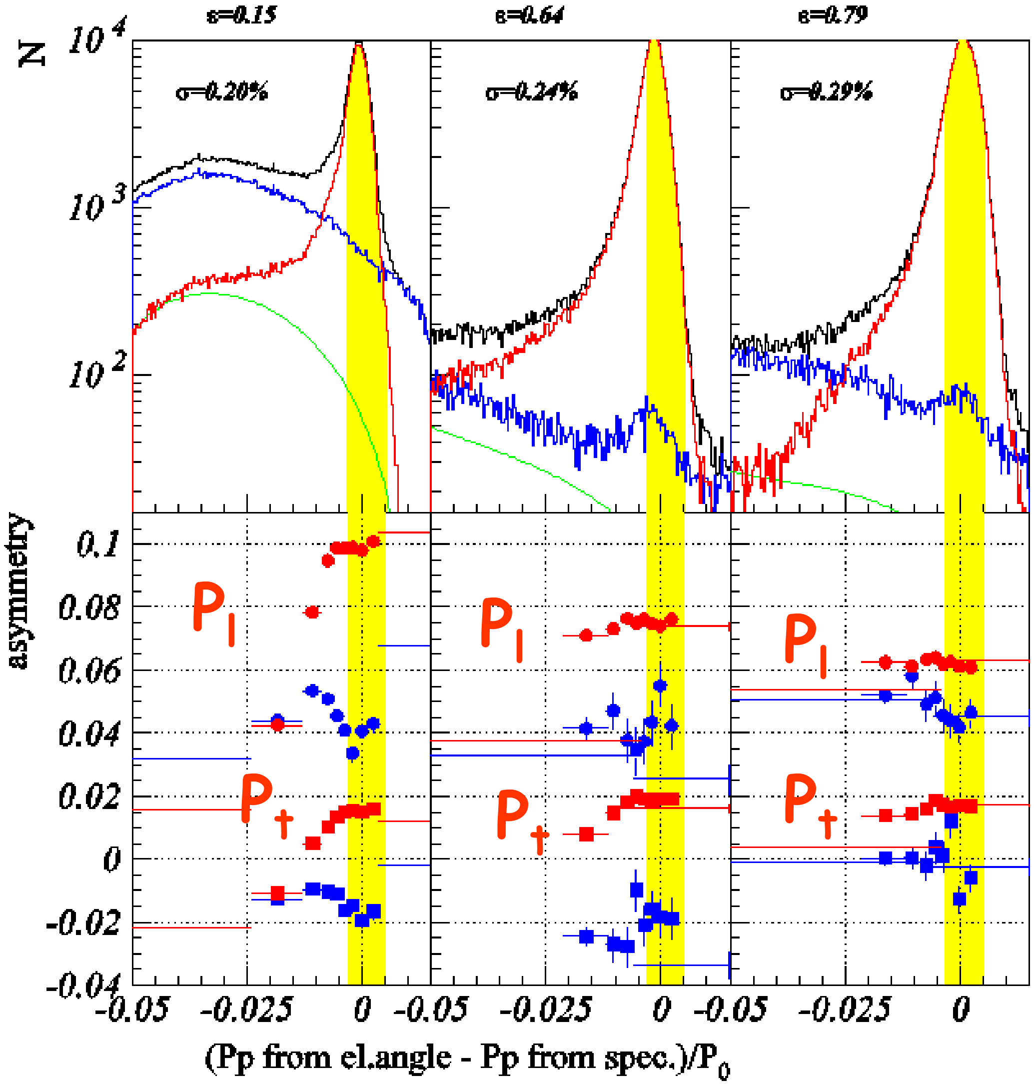
The inclusion of hard 2γ exchange calculated by Chen et al (with GPDs) and Blunden et al (hadronic model), creates a very slight non-linearity in Rosenbluth plot (top), but distinct behaviors for $\mu_p G_E/G_M$ from polarization measurements (bottom).

The Preliminary Results



The preliminary results from experiment 04-019 at JLab, for Q^2=2.49 GeV^2 and three values of ε. Error bars not final. No ε -dependence of $\mu_p G_{Ep}/G_{Mp}$ at the 0.01 level.

Separated P_t and P_l



1) Distribution of relative difference between P from θ_p and P from spectrometer
2) Physical background
3) Background subtracted
4) Residual background

Elastic amplitude reconstruction different from full Born/non-Born separation: requires e+/e- and triple polarization observables (M.P.Rekalo and E. Tomasi-Gustafsson Nucl.Phys.A740:271-286,2004)

Here one can constrain the contribution from the third non-Born amplitude $Y_{2\gamma}$.

The L. Pentchev plot

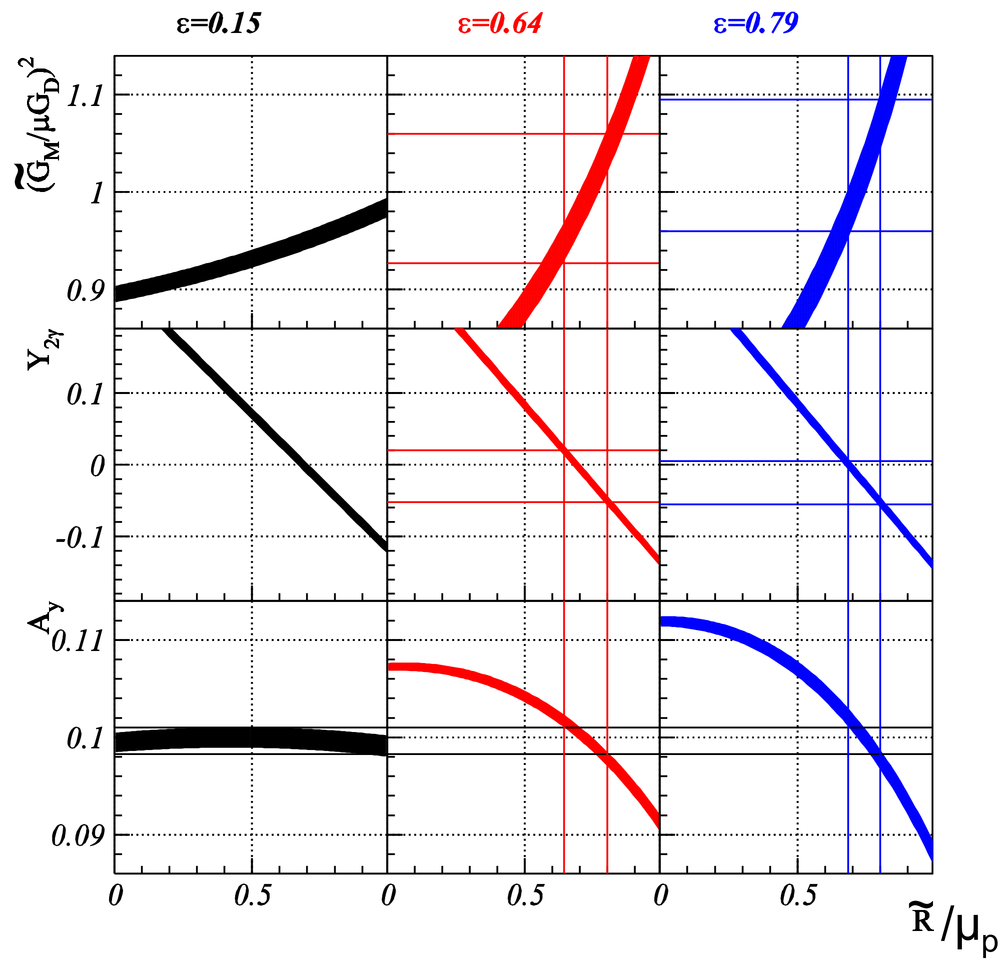
Measured P_t/P_l, $A_y{*}P_\ell$, $d\sigma$

Plot $\Re e(\tilde{G}_{Mp})$, $Y_{2\gamma}$ and A_y, calculated by inverting P_t, P_ℓ and $d\sigma$ relations, versus unknown ratio $\tilde{R}$

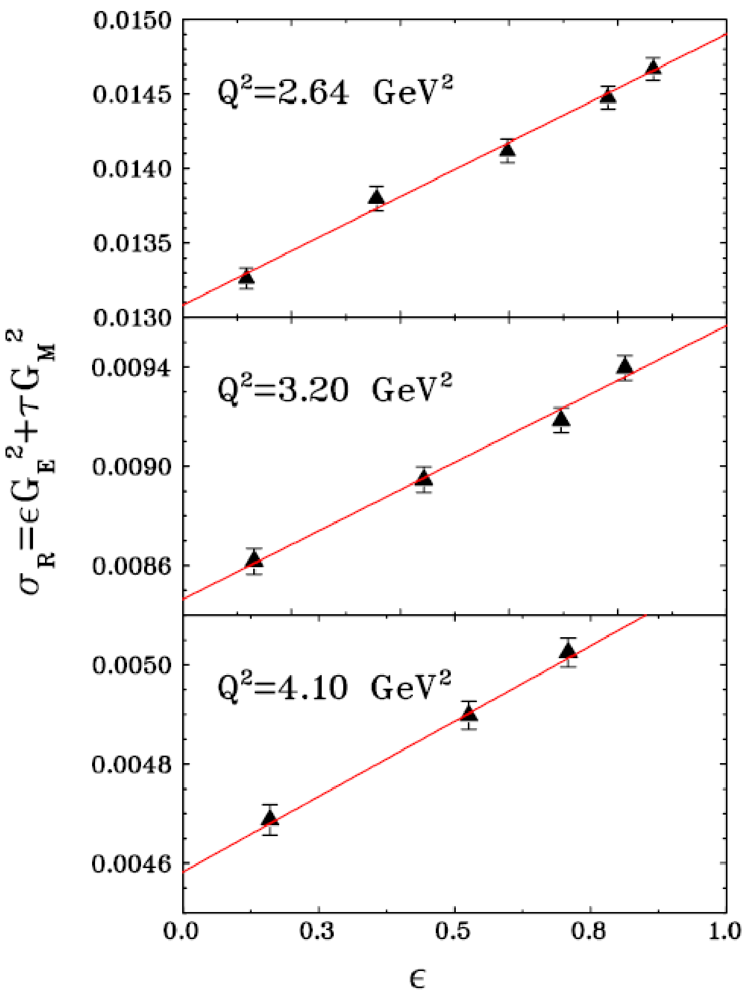
A_y same for all three ε's because same Q^2: horizontal lines.

Defines $\tilde{R}$ at 2 other ε's: ~0.7

Intersections of vertical lines with colored bands define $Y_{2\gamma}$ and $\tilde{G}_{Mp}$.



The Rosenbluth data shows no non-linearity!



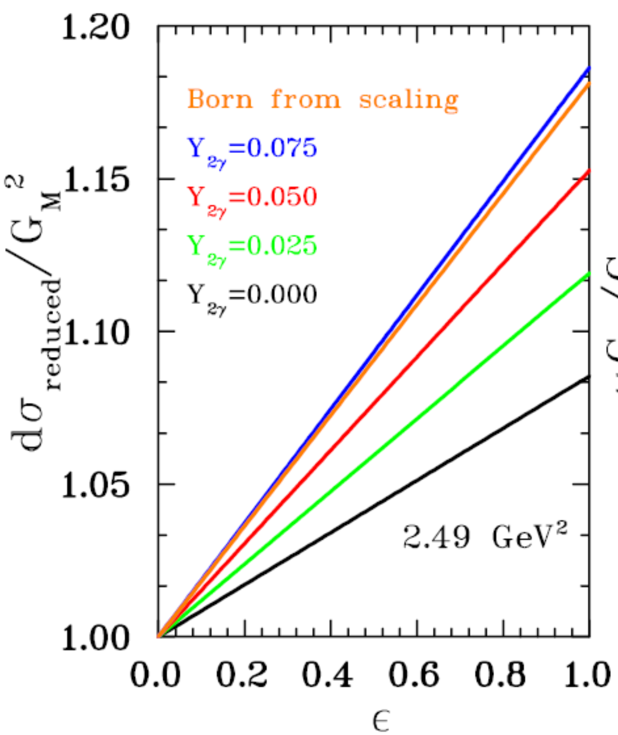
Even if $\tilde{F}_3(Q^2,\varepsilon)$ was ε-independent, the 2γ-term would imply a strong non-linear ε-dependence of the Rosenbluth cross section, because of the $(1+\varepsilon)/(1-\varepsilon)$ term in the definition of $Y_{2\gamma}$.

Most recent data from the (super-) Rosenbluth experiment suggest no obvious non-linearity.

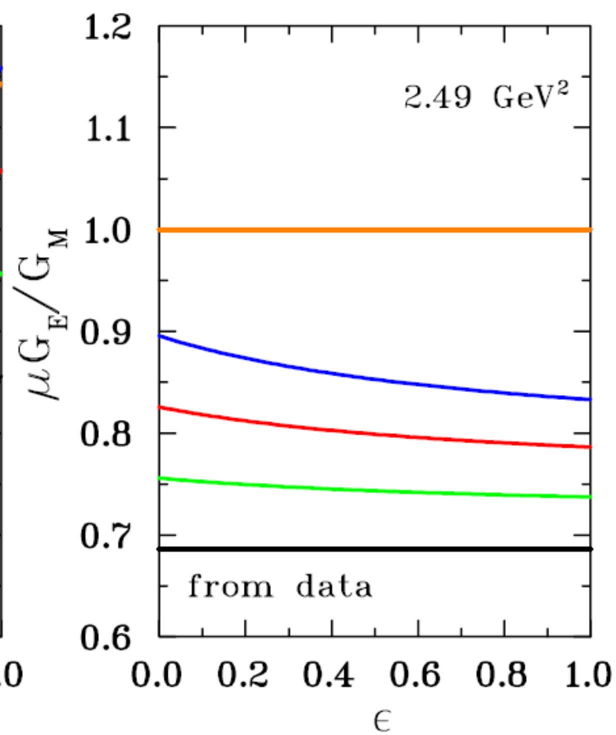
I.A. Qattan et al, Phys. Rev. Lett. 94, 142301 (2005)

A Toy Model for the 2γ "Correction"

Rosenbluth



Recoil Polarization



twophoton Y2gammaX 07/2/2008

A bad example of application of Occam's razor?

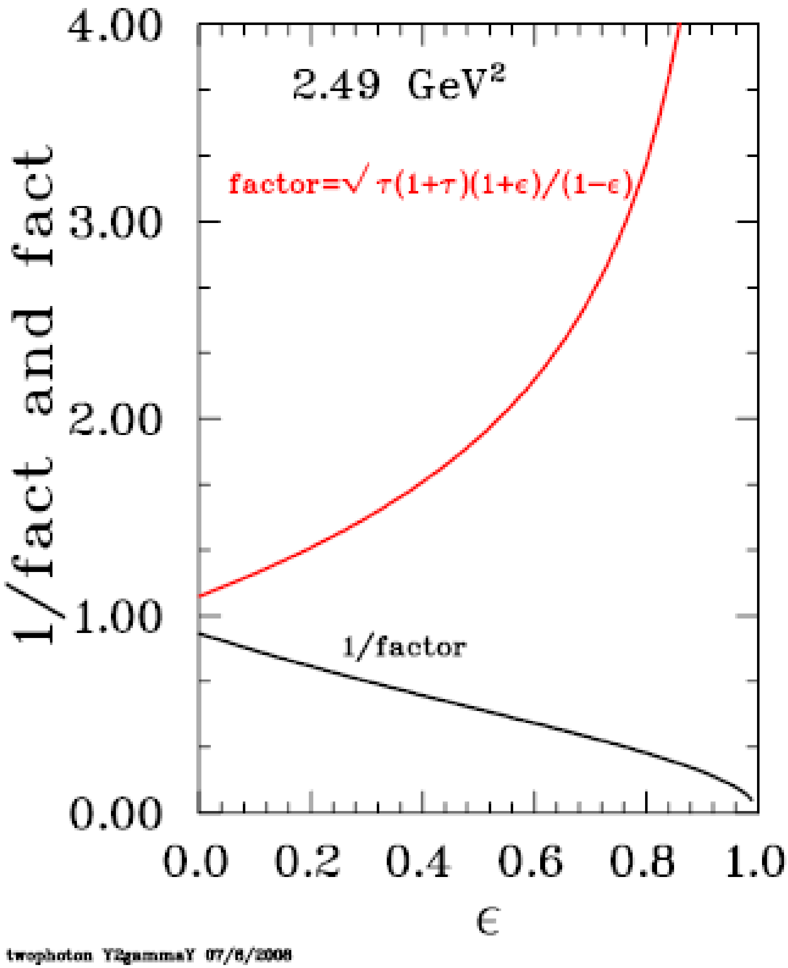
The proton FF data before 1998 did not require anything beyond the standard set of Radiative Corrections and implied scaling: $\mu_p G_{Ep}/G_{Mp} \sim 1$

Pluralitas non est ponenda sine neccesitate (14th century)

Assumes an ε-independent $Y_{2\gamma}$

Now we have a plurality of calculations of the 2γ "correction"

What if ?

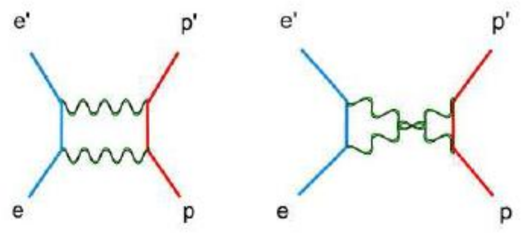


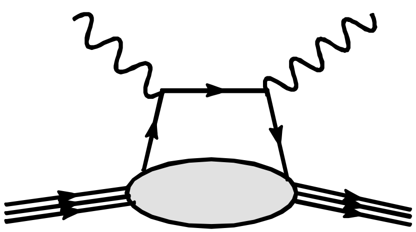
Rosenbluth plots are ~ linear in ε, at least so far; future deviations are bound to be small (see Qattan plots). The relation between $Y_{2\gamma}$ and $Re\tilde{F}_3$ is intrinsically non-linear.

So of three things one; either:
The $Re\tilde{F}_3$ FF, and therefore $Y_{2\gamma}$, is very small, and cannot be detected.

Then discrepancy between Rosenbluth and Recoil Polarization must come from incomplete or not accurate enough "Radiative Correction",
Or:
$Re\tilde{F}_3$ decreases with ε almost linearly, in which case $Y_{2\gamma}$ is nearly constant, and undetectable in a Rosenbluth experiment.
Or: neither one is true.

How to calculate contribution from exchange of two hard photon?



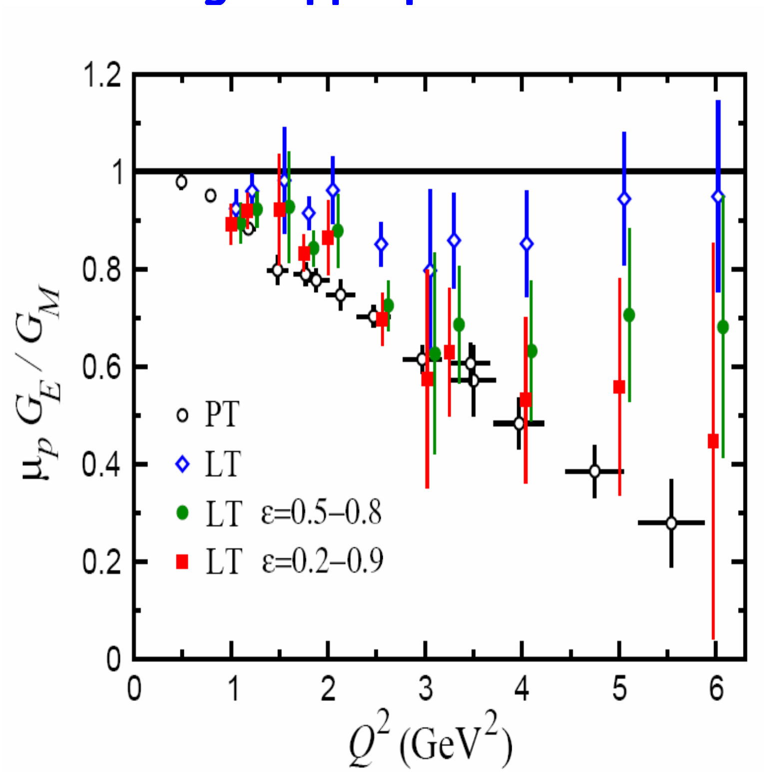


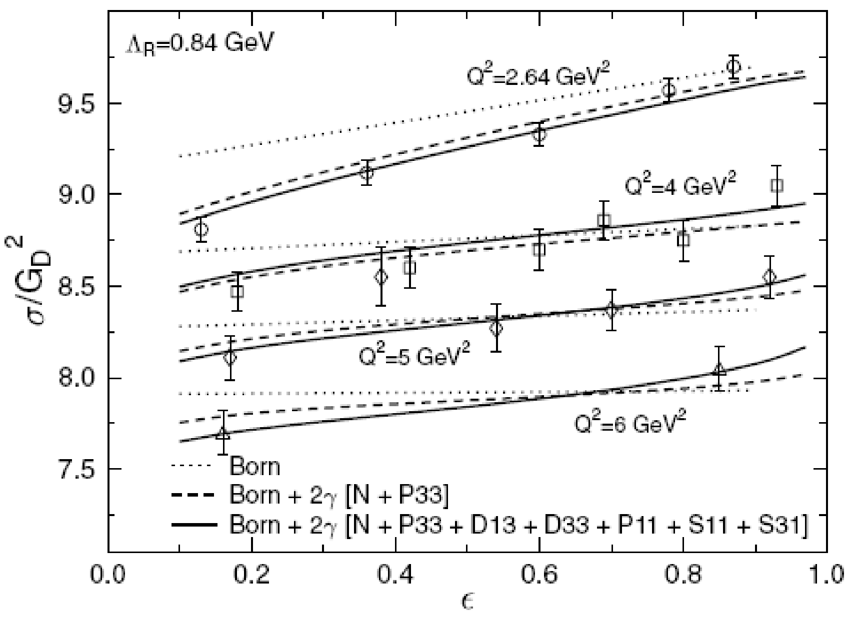
The proton in the intermediate state is virtual, in its ground state, or any baryonic state compatible with spin and parity. So far three main approaches to calculate box diagram:

- Generalized parton distribution or GPD, where the photon interacts with a single quark (Afanasev, Brodsky, Carlson, Chen and Vanderhaeghen, PR 72, 013008, 2005)
- Hadronic models (Kondratyuk and Blunden, PR 75, 038201, 2007).
- Include box diagram in K-matrix Drell Young structure function (Bistritskiy, Kuraev, Tomasi-Gustafsson. PR C75.015207.2007)

Hadronic Evaluations of 2γ

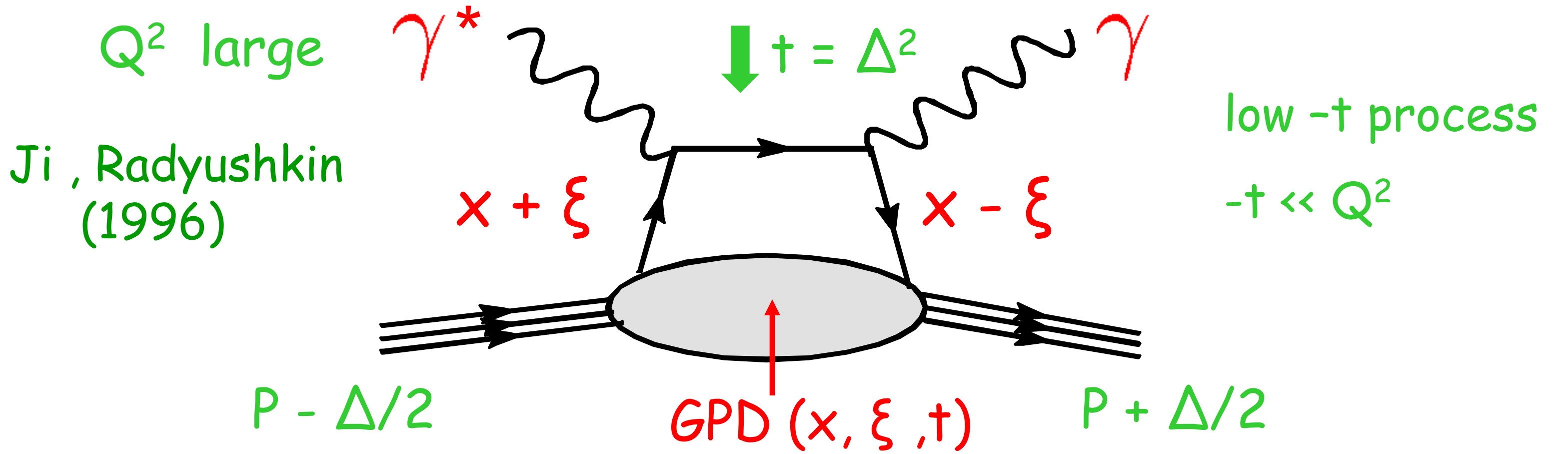
Blunden and Melnitchouk, Blunden, Melnitchouk and Tjon (PR 72, 034612, 2005). Proton contribution only first (2005). Later added Δ contribution. Finite size of "Nucleon" included through appropriate FF.





Latest: Kondratyuk and Blunden added 5 low lying resonances to previous calculation with nucleon and Δ, and use polarization data as "Born" FF: higher resonances ~ cancel each other.

Generalized parton distributions



$(x + \xi)$ and $(x - \xi)$: longitudinal momentum fractions of quarks

at large Q^2 : QCD factorization theorem ➡ hard exclusive process can be described by 4 transitions (GPDs) :

Vector : $H\ (x, \xi, t)$ **Axial-Vector** : $\tilde{H}\ (x, \xi, t)$

Tensor : $E\ (x, \xi, t)$ **Pseudoscalar** : $\tilde{E}\ (x, \xi, t)$

Relations of GPDs to observables

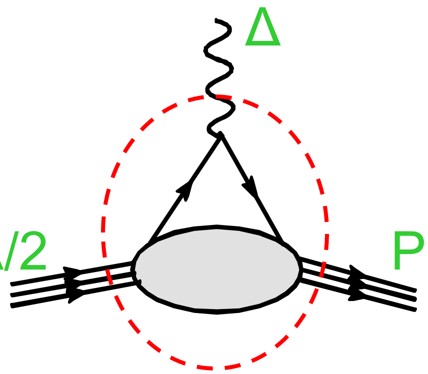
- forward limit : ordinary parton distributions

$$H^q(x, \xi = 0, t = 0) = q(x)$$ unpolarized quark distribution

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$ polarized quark distribution

E^q , $\tilde{E}^q$: do NOT appear in DIS ➡ new information

- first moments : nucleon electroweak form factors



ξ independence :
Lorentz invariance

$$\int_{-1}^{1} dx\, H^q(x, \xi, t) = F_1^q(t)$$ Dirac

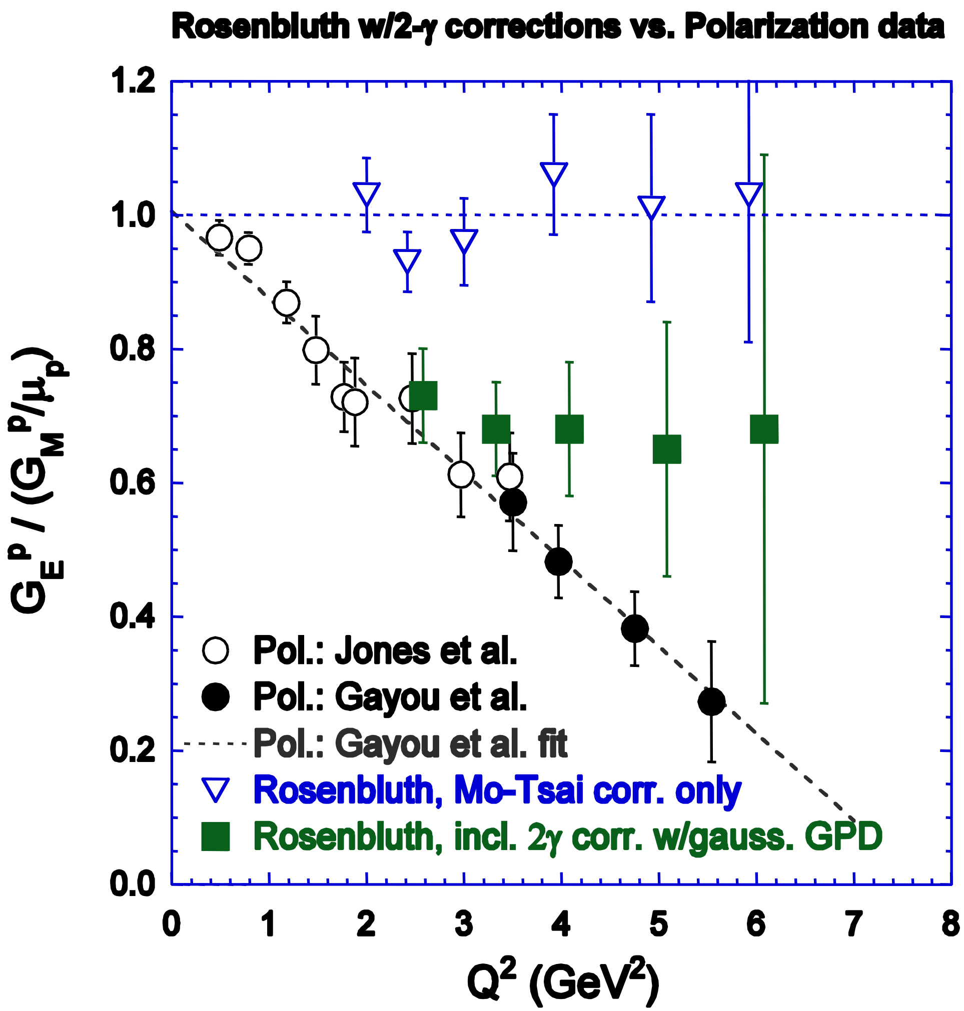
$$\int_{-1}^{1} dx\, E^q(x, \xi, t) = F_2^q(t)$$ Pauli

$$\int_{-1}^{1} dx\, \tilde{H}^q(x, \xi, t) = G_A^q(t)$$ axial

$$\int_{-1}^{1} dx\, \tilde{E}^q(x, \xi, t) = G_P^q(t)$$ pseudo-scalar

Evaluation of 2γ from GPDs

Afanasev, Brodsky, Carlson, Y.C. Chen, Vanderhaeghen, Phys. Rev. D 72 (2005) 013008; using GPDs fitted to FF data, Guidal et al. (2004)

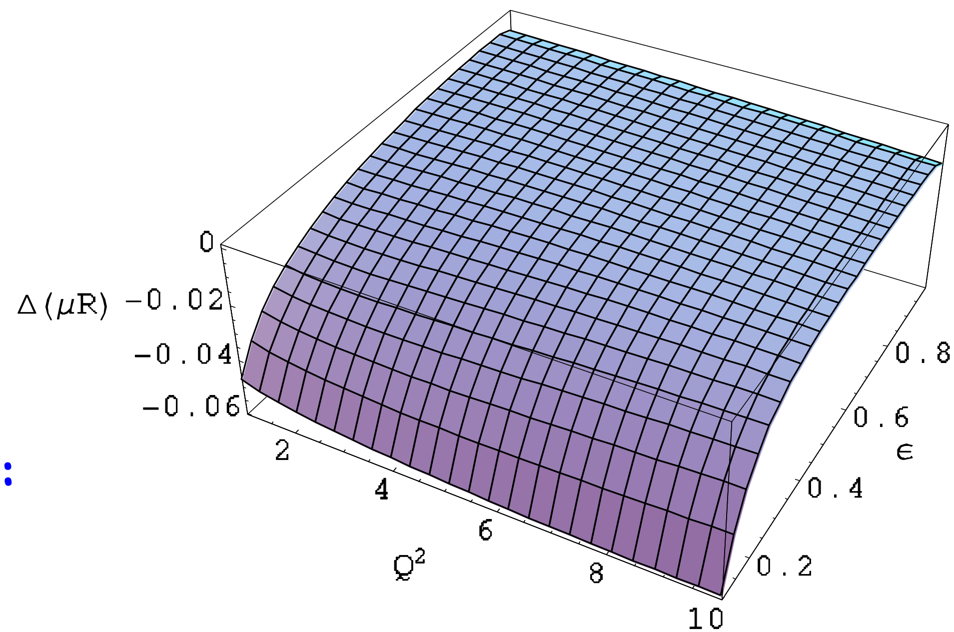


GPD calculation of 2γ Correction

Absolute correction to FF ratio $\mu G_E/G_M$ from recoil polarization

Note slow Q^2 variation, but strong ε variation at small ε

Domain of validity of calculation: high Q^2 or large ε



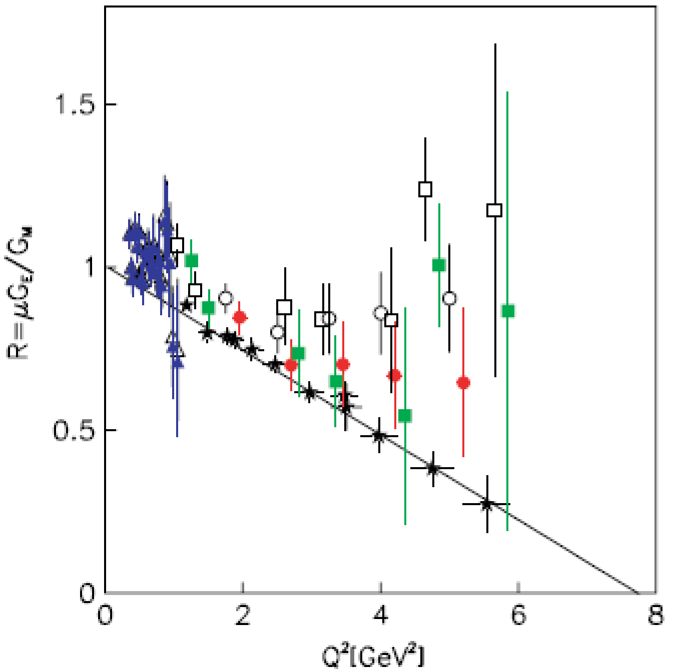
More calculations

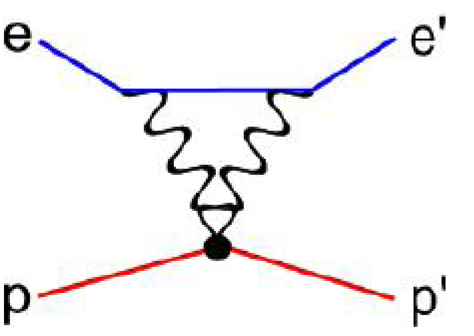
- Bystricky, Kuraev, Tomasi-Gustafsson: structure function (Drell-Yan) method: include 2γ contribution; negligible effect1 on CS to 5 GeV^2 !

- Criticized by A. Afanasev: use of fixed energy cutoff (0.97) above is cause of agreement with data (arXiv:0711.3065)

- Dubnicka and coworkers: possible effect of BS by incident electron. Polarization experiments are exclusive, with very small MM acceptance (see V. Punjabi's talk).

- Kuhn & Weigel add one-loop contribution to box diagram in chiral soliton model; contribution from 2 pion diagrams; found small 2γ effect.





Whether two-photon exchange is entirely responsible for the FF "crisis" or not at all, is to be determined experimentally

For example: Real part of $Y_{2\gamma}$

1) ε-independence of G_{Ep}/G_{Mp} in recoil polarization ⟶ Hall C 04-019, completed
2) cross section difference in e^+ and e^- proton scattering ⟶ Hall B 04-116; also Olympus/Doris with refurbished BLAST detector
3) non-linearity of Rosenbluth plot ⟶ Hall C 05-017; being analyzed

Also imaginary part

4) from induced out-of-plane polarization ⟶ by-product of 04-019/04-108?
5) single-spin target asymmetry ⟶ Hall A 05-015 (^{3}He↑)

Positron/Electron Difference

The e^+/e^- cross section data, J. Mar et al. (1969). Beam energy 4 to 10 GeV, Q^2 from 0.2 to 5 GeV^2; no systematics in ε-value.

In principle best way to determine 2γ contribution, because sign of 1γ-2γ interference changes with sign of electron charge. Experiments with positrons will determine the Born values of the FF:

$$\sigma^{(+)} - \sigma^{(-)} = 2\sigma_0\left[\varepsilon G_{Ep}{}^2(Q^2) + \tau G_{Mp}{}^2(Q^2)\right]$$

$$\frac{1}{2} I_0 (P_t{}^{(+)} + P_t{}^{(-)}) = P_e \sqrt{2\varepsilon(1-\varepsilon)\tau}\, G_{Ep}(Q^2) G_{Mp}(Q^2)$$

$$\frac{1}{2} I_0 (P_\ell{}^{(+)} + P_\ell{}^{(-)}) = P_e \tau \sqrt{2\varepsilon(1-\varepsilon)}\, G_{Mp}{}^2(Q^2)$$

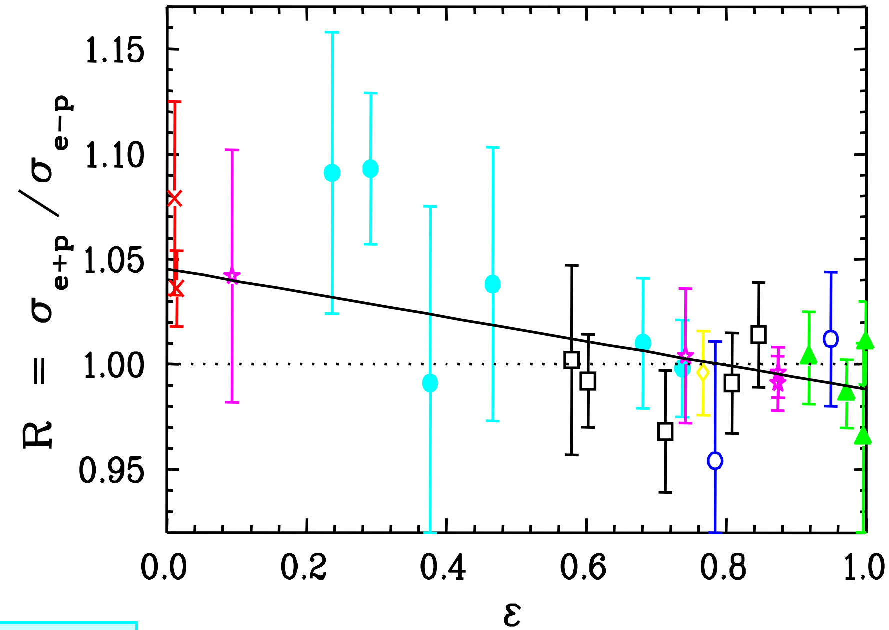
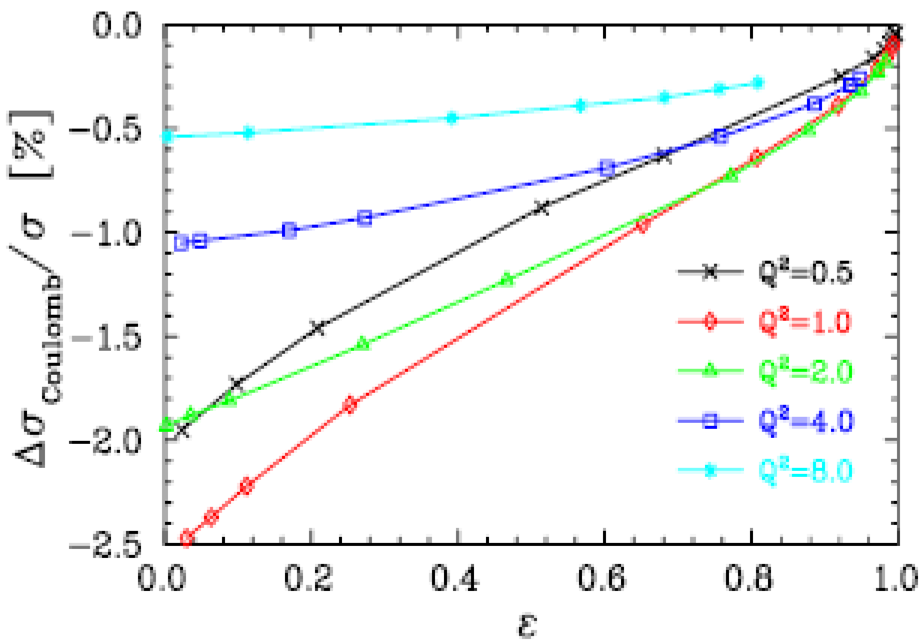


Figure from J. Arrington (2003)

Other Corrections?

Correction for Coulomb distortion of the in- and outgoing electron waves is another effect which has been neglected in the past.

Distorsion due to diagrams with 1 hard photon, and 1 or more soft photons.



Arrington and Sick (P.R. C70:028203 (04)) have calculated correction to *ep* cross section: effect strongest at 1 GeV^2, then decreases with Q^2.

Does not begin to explain Rosenbluth/Polarization discrepancy.

Conclusion, Perspective

Both experimental characterization and phenomenological understanding of the structure of the proton, have changed drastically since 1998, year of the first recoil polarization experiment in Hall A.

Rapid decrease of G_{Ep} with Q^2 not a surprise: predicted in at least 3 papers: Iachello Jackson and Lande (73) with VMD, Frank, Jennings and Miller (96) withCQM, and Holzwarth (96) with chiral soliton.

Currently under development are efforts to get a full understanding of two-photon effects, and revision of standard radiative correction calculation codes.

In my view clear experimental evidence for two-photon exchange as the explanation for the discrepancy between Rosenbluth and recoil polarization, is not in.

The End

Experiments are the only means of knowledge
at our disposal. The rest is poetry, imagination.

Max Planck