Anomaly in QCD And Meson Mixing

A.G.Oganesyan[1], O.V.Teryaev[2] and Y.N.Klopot[2]

[1]Institute of Theoretical and Experimental Physics
Moscow, Russia

[2]Joint Institute for Nuclear Research,
Dubna, Russia

SPIN-Praha-2008

Outline

Consider the matrix element for the transition of the 8th component of octet axial current into two photons with momenta p, p' and polarizations ϵ_α , ϵ'_β:

$$T_{\mu\alpha\beta}(p,p') = \langle p, \epsilon_\alpha; p', \epsilon'_\beta | J^{(8)}_{\mu 5} | 0 \rangle,$$

$$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$$

The general form of $T_{\mu\alpha\beta}(p,p')$ can be expressed as:

$$T_{\mu\alpha\beta}(p,p') = F_1(q^2) q_\mu \epsilon_{\alpha\beta\rho\sigma} p_\rho p'_\sigma + \frac{1}{2} F_2(q^2)(p_\alpha \epsilon_{\mu\beta\rho\sigma} - p'_\beta \epsilon_{\mu\alpha\rho\sigma}) p_\rho p'_\sigma,$$

where $q = p + p'$

Consider the matrix element for the transition of the 8th component of octet axial current into two photons with momenta p, p' and polarizations ϵ_α , ϵ'_β:

$$T_{\mu\alpha\beta}(p,p') = \langle p, \epsilon_\alpha; p', \epsilon'_\beta | J^{(8)}_{\mu 5} | 0 \rangle,$$

$$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}} (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$$

The general form of $T_{\mu\alpha\beta}(p,p')$ can be expressed as:

$$T_{\mu\alpha\beta}(p,p') = F_1(q^2) q_\mu \epsilon_{\alpha\beta\rho\sigma} p_\rho p'_\sigma + \frac{1}{2} F_2(q^2)(p_\alpha \epsilon_{\mu\beta\rho\sigma} - p'_\beta \epsilon_{\mu\alpha\rho\sigma}) p_\rho p'_\sigma,$$

where $q = p + p'$

The functions $F_1(q^2)$ and $F_2(q^2)$ can be represented by nonsubtracted dispersion relations in q^2 and in QCD the anomaly condition gives the sum rule:

$$\int_0^\infty Im\, F_1(q^2)dq^2 = \sqrt{2}\alpha(e_u^2 + e_d^2 - 2e_s^2)N_c = \sqrt{\frac{2}{3}}\alpha,$$

(J.Horejsi, PR 1985; O.Veretin, O.Teryaev, Yad.Fiz. 1995; J.Horejsi, O.Teryaev, Z.Phys. 1995)

- Notice, that in QCD this equation does't have any perturbative corrections , and it is expected that it does not have any nonperturbative corrections too.

Let's saturate the l.h.s. of the above sum rule relation by the η contribution. Use PCAC relation

$$\langle 0 | j^{(8)}_{\mu 5} | \eta \rangle = i f_\eta q_\mu$$

The general form of the η-contribution to $T_{\mu\alpha\beta}(p, p')$ is

$$T_{\mu\alpha\beta}(p, p') = -f_\eta \frac{1}{q^2 - m_\eta^2} \widetilde{A}_\eta q_\mu \epsilon_{\alpha\beta\lambda\sigma} p_\lambda p'_\sigma$$

where $\widetilde{A}_\eta$ is a constant. In the approximation, when only η contribution is accounted in the l.h.s. of the sum rule relation one can find $\widetilde{A}_\eta$

$$\widetilde{A}_\eta = \sqrt{\frac{2}{3}} \frac{\alpha}{\pi} \frac{1}{f_\eta}.$$

Then one can easily calculate the decay width $\eta \to 2\gamma$:

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = \frac{1}{3}\frac{\alpha^2}{32\pi^3}\ \frac{m_\eta^3}{f_\eta^2}$$

- If we put experimental numbers of α, m_η and $f_\eta = 1.2 f_\pi \approx 150 MeV$ we get the numerical value

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = 0.12 keV,$$

which is in a serious disagreement with an experimental value

$$\Gamma_{\eta\to 2\gamma} = 0.510 \pm 0.026 keV$$

- This discrepancy motivates us to consider corrections arising from excited states contributions to the sum rule. Consider the mixing of η and η' mesons.

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = \frac{1}{3}\frac{\alpha^2}{32\pi^3}\ \frac{m_\eta^3}{f_\eta^2}$$

- If we put experimental numbers of α, m_η and $f_\eta = 1.2 f_\pi \approx 150MeV$ we get the numerical value

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = 0.12keV,$$

which is in a serious disagreement with an experimental value

$$\Gamma_{\eta\to 2\gamma} = 0.510 \pm 0.026keV$$

- This discrepancy motivates us to consider corrections arising from excited states contributions to the sum rule. Consider the mixing of η and η' mesons.

One Angle $\eta - \eta'$ Mixing Scheme

Let's introduce nonorthogonal states $|P_8\rangle$ and $|P_0\rangle$ and the corresponding fields φ_8, φ_0, coupled to $J^{(8)}_{\mu 5}$ and $J^{(0)}_{\mu 5}$:

$$\langle 0|J^{(k)}_{\mu 5}|P_l\rangle = i\delta_{kl} f_k q_\mu, \quad k = 8, 0.$$

(B.Ioffe, Yad.Fiz. 1979; B.Ioffe, M.Shifman, PL 1980)
Nonorthogonality of the fields φ_3, φ_8 corresponds to the non-diagonal term $\Delta H = m^2_{\eta\pi}\varphi_8\varphi_0$ in the effective interaction Hamiltonian. In the presence of such term the standard PCAC relation is modified in the following way:

$$\partial_\mu J^{(8)}_{\mu 5} = f_\eta (m^2_\eta \varphi_8 + m^2_{\eta\eta'}\varphi_0),$$

The fields φ_8,φ_0 are expressed through the physical fields φ_η, $\varphi_{\eta'}$ as

$$\varphi_8 = \varphi_\eta \cos\theta + \varphi_{\eta'} \sin\theta$$

$$\varphi_0 = -\varphi_\eta \sin\theta + \varphi_{\eta'} \cos\theta,$$

Mixing angle θ can be expressed in terms of masses as:

$$\tan 2\theta = \frac{m^2_{\eta\eta'}}{m^2_{\eta'} - m^2_\eta}$$

Now $ImF_1(q^2)$ is given by the sum of contributions of η and η' mesons. In order to separate the formfactor $F_1(q^2)$, multiply $T_{\mu\alpha\beta}(p,p')$ by q_μ/q^2. Than, taking the imagenary part we get:

$$Im\; q_\mu \frac{1}{q^2}\langle 2\gamma \mid J^{(8)}_{\mu 5} \mid 0\rangle =$$

$$-\frac{f_\eta}{q^2} Im\langle 2\gamma \mid m^2_\eta(\cos\theta\varphi_\eta + \sin\theta\varphi_{\eta'}) + m^2_{\eta\eta'}(-\sin\theta\varphi_\eta + \cos\theta\varphi_{\eta'}) \mid 0\rangle =$$

$$\pi f_\eta[\delta(q^2 - m^2_\eta)A_\eta \cos\theta + \frac{m^2_\eta}{m^2_{\eta'}} \sin\theta\delta(q^2 - m^2_{\eta'})A_{\eta'} -$$

$$\frac{m^2_{\eta\eta'}}{m^2_\eta} \sin\theta\delta(q^2 - m^2_\eta)A_\eta + \frac{m^2_{\eta\eta'}}{m^2_{\eta'}} \cos\theta\delta(q^2 - m^2_{\eta'})A_{\eta'}]$$

where A_η is the amplitude of decay $\eta \to 2\gamma$

If we employ sum rule (...) we'll get:

$$\pi f_\eta [A_\eta \cos\theta + A_{\eta'} \frac{m_\eta^2}{m_{\eta'}^2} \sin\theta - A_\eta \frac{m_{\eta\eta'}^2}{m_\eta^2} \sin\theta + A_{\eta'} \frac{m_{\eta\eta'}^2}{m_{\eta'}^2} \cos\theta] = \sqrt{\frac{2}{3}}\alpha$$

Now let's express amplitudes in terms of decay widths, employ the relation for mixing parameter $m_{\eta\eta'}^2$ and finally get the equation for mixing angle:

$$\boxed{\cos\theta + \beta\frac{m_\eta^2}{m_{\eta'}^2}\sin\theta - \frac{1}{2}(\frac{m_{\eta\eta'}^2}{m_\eta^2} - 1)\tan 2\theta \sin\theta + \frac{\beta}{2}(1 - \frac{m_{\eta\eta'}^2}{m_{\eta'}^2})\tan 2\theta \cos\theta = \xi,}$$

where

$$\beta = \frac{A_\eta}{A_{\eta'}} = \sqrt{\frac{\Gamma_{\eta'\to 2\gamma}}{\Gamma_{\eta\to 2\gamma}} \frac{m_\eta^3}{m_\eta'^3}},$$

$$\xi = \sqrt{\frac{\alpha^2 m_\eta^3}{96\pi^3 \Gamma_{\eta\to 2\gamma}} \frac{1}{f_\eta^2}}$$

As an input we'll use experimental data (PDG Review 2006):

$m_\eta = 547.51 \pm 0.18$ MeV
$m'_\eta = 957.78 \pm 0.14$ MeV
$\Gamma_{\eta \to 2\gamma} = 0.510 \pm 0.026$ keV
$\Gamma_{\eta' \to 2\gamma} = 4.30 \pm 0.15$ keV
$f_\pi = 130 \pm 0.5$ MeV

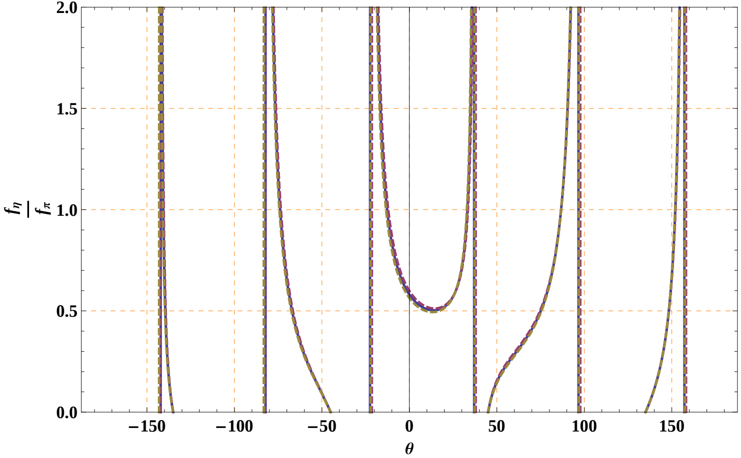


Figure: Mixing angle θ as a function of decay constant f_η in the one angle mixing scheme

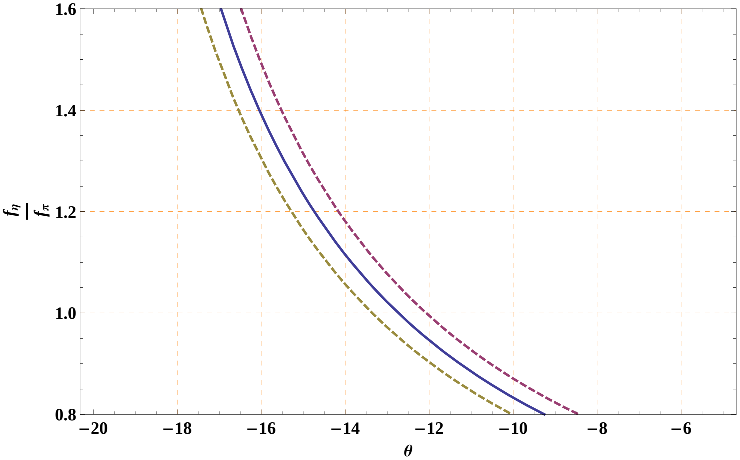


Figure: Mixing angle θ as a function of decay constant f_η in the one angle mixing scheme

For $f_\eta = 1.25$ we get the mixing angle

$$\theta = -15.2 \pm 0.6$$

Two Angle $\eta - \eta'$ Mixing Scheme

Let's introduce the fields φ_8,φ_0 which are expressed through the physical fields φ_η, $\varphi_{\eta'}$ as

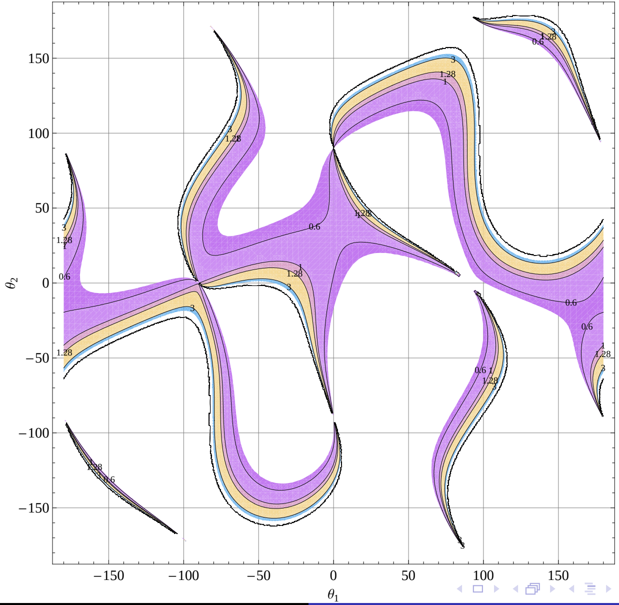
$$\varphi_8 = \varphi_\eta \cos\theta_2 + \varphi_{\eta'} \sin\theta_1$$
$$\varphi_0 = -\varphi_\eta \sin\theta_2 + \varphi_{\eta'} \cos\theta_1,$$

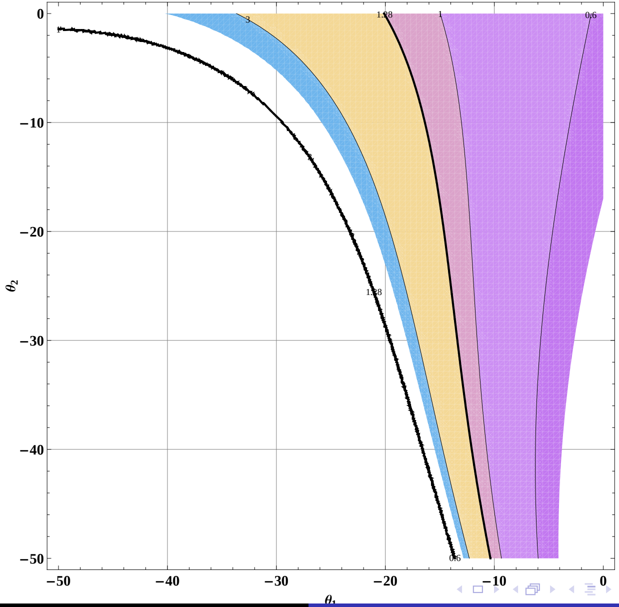
And the master-equation for two angle mixing scheme will be than:

$$\cos\theta_2 + \beta\frac{m_\eta^2}{m_{\eta'}^2}\sin\theta_1 - \frac{m_{\eta\eta'}^2}{m_\eta^2}\sin\theta_2 + \beta\frac{m_{\eta\eta'}^2}{m_{\eta'}^2}\cos\theta_1 = \xi,$$

where

$$m_{\eta\eta'}^2 = \frac{1}{2}\frac{m_\eta^{2\,\prime}\sin 2\theta_1 - m_\eta^2 \sin 2\theta_2}{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2},$$
$$\beta = \frac{A_\eta}{A_{\eta'}} = \sqrt{\frac{\Gamma_{\eta'\to 2\gamma}}{\Gamma_{\eta\to 2\gamma}}\frac{m_\eta^3}{m_\eta'^3}},$$
$$\xi = \sqrt{\frac{\alpha^2 m_\eta^3}{96\pi^3\Gamma_{\eta\to 2\gamma}}\frac{1}{f_\eta^2}}$$


θ_2
θ_1
150
100
50
0
−50
−100
−150
0
50
100
150


0
−10
−20
−30
−40
−50
−50
−40
−30
−20
−10
0
θ2
θ1
3
1.28
1
0.6

Summary

- Anomaly condition in it's dispersive approach allowed us to get a precise value for a mixing angle in a one-angle mixing scheme.
- For a two-angle mixing scheme the limits for both mixing angles was gotten.