# Two-loop electroweak corrections to the running $\alpha_s$ in the $\overline{MS}$ scheme

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# Outline

# Motivation

- Three-loop RGE for all the SM Lagrangian parameters were calculated recently in the $\overline{MS}$ scheme [MSS12, BPV13, CZ13].
- Boundary values at the electroweak (EW) scale are required for a RGE analysis of the model
  - Matching predictions in terms of parameters with "observables" or "pseudo"-observables - in perturbation theory at two loops.
- In a vacuum stability analysis of the SM the uncertainty of the instability scale (or critical values of the SM parameters at the EW scale) is dominated by those of $y_t, \lambda$ and $\alpha_s$ [BKKS12, DDVEM$^+$12]
  - When one determines $\alpha_s(\mu)$ in the SM (from that of $n_f = 5$ flavour QCD) usually only strong interactions are taken into account.
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# The SM RGEs and Vacuum instability

- RGEs allow one to predict the behavior of the higgs effective potential at large values of Higgs field $\phi \gg v$.
- The crucial parameters for the SM stability RGE analysis are the Higgs self-coupling $\lambda$,

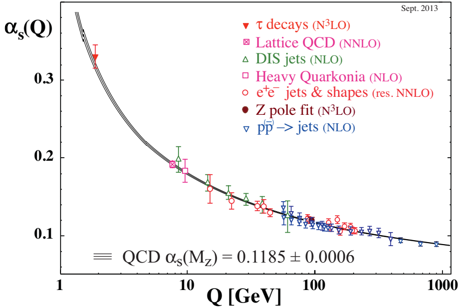
$$V_{eff}(\phi \gg v) \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

top Yukawa coupling $y_t$ and the strong coupling $\alpha_s = g_s^2/(4\pi)$

$$(4\pi)^2 \frac{d\lambda}{dt} = 12\lambda^2 + 6y_t^2\lambda - 3y_t^4 + \ldots$$

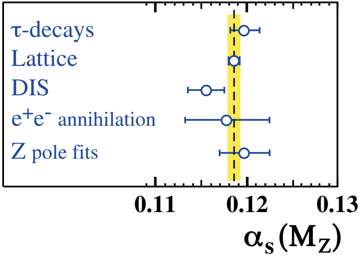
$$(4\pi)^2 \frac{dy_t}{dt} = \frac{9}{4}y_t^3 - 4g_s^2 y_t + \ldots$$

# Observed running of $\alpha_s$




- Observed running of the strong coupling (PDG'14 [O+14])
- World avearge at $\mu = M_Z$ scale in $\overline{MS}$ scheme

# Experimental determination of $\alpha_s$

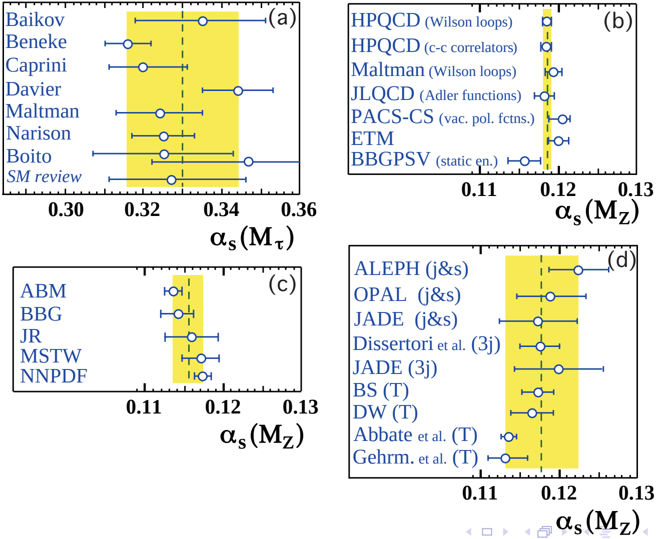



Summary of values of $\alpha_s(M_Z)$ in $n_f = 5$ QCD obtained with "pre-averaging" in certain sub-classes

- $e^+e^-$ annihilation
  - $\alpha_s(M_Z) = 0.1177 \pm 0.0046$
- EW precision fits
  - $\alpha_s(M_Z) = 0.1197 \pm 0.0028$
- DIS
  - $\alpha_s(M_Z) = 0.1154 \pm 0.020$
- $\tau$-lepton
  - $\alpha_s(M_\tau) = 0.330 \pm 0.014 \Rightarrow \alpha_s(M_Z) = 0.1197 \pm 0.016$
- Lattice
  - $\alpha_s(M_Z) = 0.1185 \pm 0.0005$

# Issues with $\alpha_s$ determination

Measurements within the sub-classes seems to be marginally compatible with each other within the quoted uncertainties



# QCD embedded in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\rm gauge}_{\rm QCD} + \mathcal{L}^{\rm gauge}_{\rm SU(2)\times U(1)} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm ghosts}$$

- In the QCD embedded in the SM, quark mass terms are generated via Yukawa interactions with the Higgs vacuum expectation value v:

$$m_q = \frac{y_q v}{\sqrt{2}}$$

- Due to spontaneous symmetry breaking (SSB) all other SM masses are also proportional to $v$

$$M_W^2 = \frac{g_2^2 v^2}{4}, \qquad M_Z^2 = \frac{g_1^2 + g_2^2}{4} v^2, \qquad M_h^2 = 2\lambda v^2$$

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- We can express all the SM dimensionless couplings (but $g_s$) via, e.g., SU(2) coupling $g_2$ and different mass ratios:

$$y_q^2 = \frac{g_2^2}{2}\frac{m_q^2}{M_W^2}, \qquad g_1^2 = g_2^2\left(\frac{M_Z^2}{M_W^2} - 1\right), \qquad \lambda = \frac{g_2^2 M_h^2}{8 M_W^2}$$

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- Introducing fine-structure constant $\alpha$ and Weinberg angle $\theta_W$

$$(4\pi)\alpha = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} = g_2^2 \sin^2\theta_W = g_1^2 \cos^2\theta_W$$

- Parametrization used in this work

$$y_q^2 = \frac{4\pi\alpha}{\sin^2\theta_W}\frac{m_q^2}{M_W^2}, \qquad \lambda = \frac{4\pi\alpha}{8\sin^2\theta_W}\frac{M_h^2}{M_W^2}$$

  - All the parameters here are bare (or $\overline{MS}$ renormalizied) ones.
  - NB: In the formal limit $v \to \infty$ the mass ratios are finite.

# Parameter values and the choice of renormlization scheme

- The values of the SM parameter are not predicted by the theory but should be extracted from an experiment via matching procedure.

- However, particlular values are usually scheme- and scale-dependent.
- In the electroweak sector all the couplings can be traded for the measured value of the fine-structure constant $\alpha$ and physical particle masses $M_Z, M_W, M_h$.
  - PROS: Predictions can be expressed in terms of physical quantities.
  - CONS: Predictions can involve potentially large logarithms, e.g., $\ln E/M$ with $E$ being typical energy/momentum transfer of the process and $M$ being some mass.
    - ⋆ Can, in principle, be re-summed by introduction of running couplings in momentum-subtraction MOM scheme.
    - ⋆ NB: In MOM scheme the decoupling theorem holds: corrections due to heavy degrees of freedom are suppressed by their masses.

# Parameter values and the choice of renormlization scheme

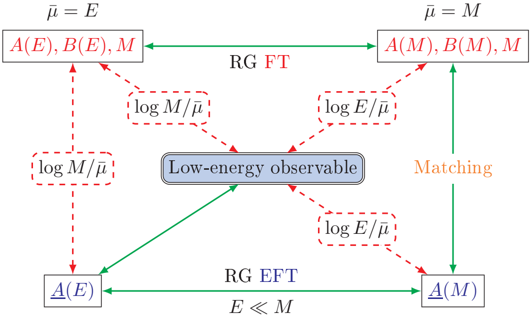
- The values of the SM parameter are not predicted by the theory but should be extracted from an experiment via matching procedure.

- In QCD sector, due to confinement this approach is not convenient, so one usually adopts $\overline{MS}$ scheme to define the running $\alpha_s(\mu)$.
- In order to determine the corresponding value, an observable $\mathcal{O}$ is matched to the corresponding theoretical prediction

$$\mathcal{O} = \alpha_s^k(\mu)\left[c_0(\mu) + c_1(\mu)\alpha_s(\mu) + c_2(\mu)\alpha_s^2(\mu) + \ldots\right],$$

  so that $\alpha_s(\mu_0)$ at some matching $\mu_0$ is extracted.
- To avoid large logarithms the scale $\mu_0$ is usually chosen around the typical scale involved in the measurement of $\mathcal{O}$ (e.g. momentum transfer).
- Nevetheless, in $\overline{MS}$ additional effort is required if the theory involves different mass scales...

# Re-summation and effective theories




A well-known example:

- $A(\bar{\mu}) = \alpha_s^{(6)}(\bar{\mu})$,
- $M = M_t$,
- $\bar{A}(\bar{\mu}) = \alpha_s^{(5)}(\bar{\mu})$

Matching 6-flavor QCD with 5-flavor QCD without top quark.

- To combine the simplicity of $\overline{MS}$ and the decoupling feature of the MOM-scheme one employs the notion of effective theories with running couplings $A(\bar{\mu})$ expressible in terms of (running) parameters of "full" theory (FT) - $A(\bar{\mu})$,$B(\bar{\mu})$ and heavy masses $M$.

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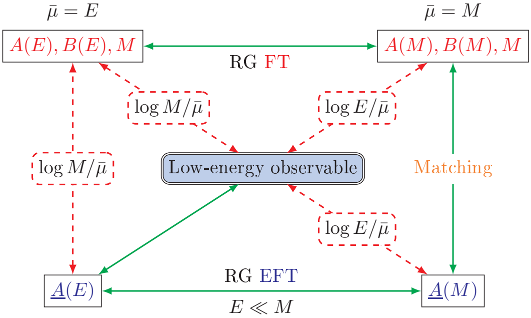


Matching can be used to find $A(\bar{\mu})$ given $\bar{A}(\bar{\mu})$, $B(\bar{\mu})$ and $M$.

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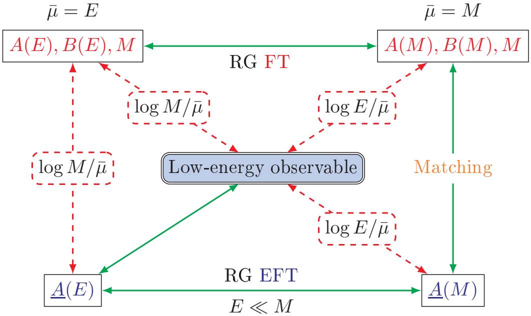


Matching can be used to find $A(\bar{\mu})$ given $\bar{A}(\bar{\mu})$, $B(\bar{\mu})$ and $M$.

This is how $\alpha_s^{(6)}(\bar{\mu})$ is found from the quoted value of $\alpha_s^{(5)}(M_Z)$!

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# An example: QCD with $n_f$ flavours

- Consider $n_f$ flavour QCD with one heavy flavour having large mass $M$.
- At energies $E < M$, one can not produce heavy quarks so one can "integrate them" out, leading to an effective Lagrangian for $n_l$ flavors involving a tower of operators $\mathcal{O}_i$ with dimensions $d_i > 4$ (see [Pic98] for review)

$$\mathcal{L}_{QCD}^{(n_f)} \Leftrightarrow \mathcal{L}_{QCD}^{(n_f-1)} + \sum_{d_i>4} \frac{c_i}{M^{d_i-4}} O_i$$

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$$\mathcal{L}_{QCD}^{(n_f)}\left(\alpha_s^{(n_f)}\right) \Rightarrow \mathcal{L}_{QCD}^{(n_f-1)}\left(\alpha_s^{(n_f-1)}\right)$$

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- At low scales $E \ll M$ one can neglect $O_i$ and consider renormalizable version of ET.
- The two couplings are related through matching condition:

$$\alpha_s^{(n_f-1)}(\mu) = \alpha_s^{(n_f-1)}(\mu) \underbrace{\left[1 + \sum_i \frac{\alpha_s^i(\mu)}{(4\pi)^i} C_i(L)\right]}_{\zeta_{\alpha_s}-\text{decoupling constant}}, \qquad L = \ln\frac{M^2}{\mu^2}$$

- Coefficients $C_i$ are known upto four-loop level, $i = 1, ..., 4$ [CKS00, SS06]

# QED x QCD as an effective low-energy theory

- As a "low-energy" effective theory for the SM we consider a (toy) QCD x QED theory describing strong and electromagnetic interactions of five massless quarks $(u, d, c, s, b)$ and leptons.

$$\mathcal{L}_{SM}\left(\alpha_s^{SM}, g_1, g_2, y_t, \lambda, ...\right) \Rightarrow \mathcal{L}_{QCD\times QED}^{(n_f=5)}\left(\alpha_s^{(5)}, \alpha_{EM}\right)$$

- Similar to the QCD case we "integrate out" top quark, electroweak gauge bosons and Higgs fields. We also neglect Fermi-like non-renormalizable interactions "$G_F\bar{\psi}\psi\bar{\psi}\psi$" with $G_F \propto \frac{1}{M_W^2}$.

- Formally, we consider the limit $v \to \infty$, which is different from that $y_t, g_2, \lambda \to \infty, v = \textit{fixed}$ usually implied in discussions of "non-decoupling" feature of the models with SSB (see [Pic98] for discussion).

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- From the phenomelogical point of view we miss a lot of electroweak physics, goverened at low energies by the Fermi constant $G_F$!

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- Nevertheless, our task is to study the running of $\alpha_s^{SM}(\mu)$ in $\overline{MS}$ extracted from $\alpha_s^{(5)}(\mu)$ at some matching scale $\mu_0 \simeq 100 - 200$ GeV
- Due to the *chosen $\overline{MS}$ scheme*, the result is also valid in the effective QED×QCD×Fermi theory!

# How to find matching relation?

- In order to match the SM and our effective theory, one, in principle, needs to consider some low-energy observables predicted in both models.
- Asymptotic expansion in large mass $M$ (LME) of the SM result should reproduce the effective theory prediction in each order of $\frac{1}{M^2}$.
- The rules of LME tells us that the expansion (in terms of Feynman diagrams) consists of
  - the "hard part" [all internal momenta $q_i \sim M$
  - the "soft part" [all internal momenta $q_i \ll M$]
  - a mixture of hard and soft lines, some internal lines have $q_i \simeq M$ and some have $q_k \ll M$

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# How to find matching relation?

It turns out that only the "hard part" contributes to the matching relation between the couplings of the theories at the given loop level.

- Due to this, it is tempting to consider only the "hard part" which corresponds to the Taylor expansion of the integrand in small external momentum and masses.
- An obvious subtlety: such an expansion can generate (spurious) infra-red (IR) divergencies upon integration, which should be "subtracted" in a proper way.
- A convenient way to deal with this problem is to use dimensional regularization and perform matching at the bare level, e.g.,

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0} \times \alpha_{s,0}, \qquad \alpha_{s,0} \equiv \alpha_{s,0}^{SM}$$

# Matching bare parameters

$$\alpha_{s,0}^{(5)} = \zeta_{\alpha_s,0}[\alpha_{s,0}, \alpha_0, M_0] \times \alpha_{s,0}$$

Due to SU(3) gauge invariance, the bare decoupling constant $\xi_{\alpha_{s,0}}$ can be found in a number of ways:

$$\zeta_{\alpha_s,0} = \zeta^2_{cGc,0}\zeta^{-2}_{c,0}\zeta^{-1}_{G,0} = \zeta^2_{qGq,0}\zeta^{-2}_{q,0}\zeta^{-1}_{G,0} = \ldots$$

in which different $\zeta s$ are found by considering three- and two-point 1PI green functions in the SM so that

- $\zeta_{cGc,0}$and $\zeta_{qGq,0}$ correspond to the leading terms in Taylor expansion of the integrand of the ghost-gluon and (light)-quark-gluon vertices, respectively.
- $\zeta_{c,0}, \zeta_{G,0}, \zeta_{q,0}$ involve only $\ln M/\mu$ terms coming from ghost, gluon and quark propagators.

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Taylor expansion can produce spurious IR-divergent $\frac{1}{(q^2)^2}$ terms, which, upon integration, lead to additional IR poles in $\epsilon = (4-d)/2$ in bare $\zeta s$.

# Matching bare parameters

$$\alpha_s^{(5)}(\mu) = \frac{Z_{\alpha_s}[\alpha_s, \alpha, M]}{Z_{\alpha_{s(5)}}\left[\alpha_s^{(5)}\right]} \zeta_{\alpha_s,0}\left[Z_{\alpha_s}\alpha_s, Z_\alpha \alpha, Z_M M\right] \times \alpha_s(\mu)$$

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But the spurious IR poles are canceled in the matching relation for the running couplings *after* renormalization.

# A comment on Gauge independence and tadpole diagrams

The calculation was carried out in a general $R_\xi$ gauge, parametrized by four gauge-fixing parameters $(\xi_G, \xi_W, \xi_Z, \xi_\gamma)$

$$\begin{aligned}\mathcal{L}_{\text{g.f,}} &= -\frac{1}{2\xi_G}(\partial_\mu G_\mu)^2 - \frac{1}{2}(\partial_\mu A_\mu)^2 \\ &\quad -\frac{1}{\xi_W}\left|\partial_\mu W_\mu^+ - i\xi_W M_W \phi^+\right|^2 - \frac{1}{2\xi_Z}(\partial_\mu Z_\mu - \xi_Z M_Z \chi)^2\end{aligned}$$

- The result presented above are expressed in terms of pole masses and is free from gauge-fixing parameters.
- However, the bare result looks gauge-dependent (e.g., due to the top quark self-energy) if tadpoles are not properly accounted for (see [FJ81]).

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In a model with SSB one has to be sure that the true minimum is used.
NB: The v.e.v of the higgs field is a gauge-dependent quantity!
Tadpoles (i.e., green functions with one external leg) should be zero.

# A comment on Gauge independence and tadpole diagrams

- Equivalently, loop-generated tadpoles are canceled (already at the bare level) by a tree-level tadpole, since bare vev $v_0$ minimizes the effective potential

$$i \cdot t_0 \quad - \quad i \cdot T \quad = 0$$

- It is convenient to cast the bare vev into the following form with non-minimal $Z_{v0}$. The latter is determined in PT by canceling tadpoles order by order (we follow [ACOV03])

$$v_0 = Z_{v_0}^{\frac{1}{2}} \cdot v_{tree,0}, \qquad v_{tree,0}^2 \equiv \frac{m_0^2}{\lambda_0} \Rightarrow \frac{M_{h,0}^2}{2\lambda_0}$$

$$t_0 = \left[\frac{M_h^2 M_W \sin\theta_w}{e}\right]_0 (Z_{v_0} - 1) Z_{v_0}^{\frac{1}{2}}$$

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$$v_0 = Z_{v_0}^{\frac{1}{2}} \cdot v_{tree,0}, \qquad v_{tree,0}^2 \equiv \frac{m_0^2}{\lambda_0}$$

- The "tree-level" bare $v_{tree,0}$ is gauge-invariant by construction, since it is defined in terms of the Lagrangian parameters.
- This allows one to define gauge-invariant bare and $\overline{MS}$ renormalized particle masses, e.g., for the Higgs mass

$$\begin{aligned}
\left[3\lambda_0 v_0^2 - m_0^2\right] \quad &\rightarrow \quad M_{h,0}^2 + \frac{3}{2} M_{h,0}^2 (Z_{v_0} - 1) \\
M_{h,0}^2 \quad &\equiv \quad 2\lambda_0 v_{tree,0}^2 = 2 m_0^2 \\
M_{h,0}^2 \quad &= \quad Z_{M_h^2}(\mu) m_h^2(\mu), \qquad Z_{M_h^2} = Z_\lambda Z_v = Z_{m_0^2}
\end{aligned}$$

with minimal renormalization constants $Z_{M_{h^2}}, Z_\lambda, Z_{m^2}$, and $Z_v$ .

- The same is true for other masses (in particular, $M_t$)!

# A comment on Gauge independence and tadpole diagrams

- This approach allows us to obtain bare $\zeta_{\alpha_s,0}$ free from gauge-fixing parameters and , as a consequence, an explicit gauge-independent expression for

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left( 1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s \alpha} + \dots \right),$$

in which $\delta\zeta$s are given in terms of $\overline{MS}$ parameters and involve $\ln \frac{m_t^2(\mu)}{\mu^2}$ instead of $\ln \frac{M_t^2}{\mu^2}$.

# Matching running parameters

(One of) our final expression (s):

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left( 1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2}\ \delta\zeta^{(2)}_{\alpha_s\alpha} + \dots \right),$$

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In terms of the top pole mass $M_t$ (all $\mu$-dependence of $X$s is explicit)

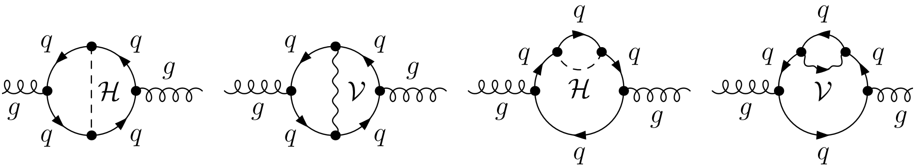
$$\begin{aligned}
\delta\zeta^{(1)}_{\alpha_s} &= X^{(1)}_{\alpha_s} \ln\frac{M_t^2}{\mu^2}, \qquad X^{(1)}_{\alpha_s} = \frac{4}{3}T_f = \frac{2}{3} \\
\delta\zeta^{(2)}_{\alpha_s} &= X^{(0)}_{\alpha_s^2} + X^{(1)}_{\alpha_s^2}\ln\frac{M_t^2}{\mu^2} + X^{(2)}_{\alpha_s^2}\ln^2\frac{M_t^2}{\mu^2}, \\
X^{(0)}_{\alpha_s^2} &= \left(\frac{32}{9}C_A - 15C_F\right)T_f = -\frac{14}{3} \\
X^{(2)}_{\alpha_s^2} &= \frac{16}{9}T_f^2 = \frac{4}{9}, \qquad X^{(1)}_{\alpha_s^2} = \left(\frac{20}{3}C_A + 4C_F\right)T_f = \frac{38}{3}
\end{aligned}$$

# Matching running parameters

(One of) our final expression (s):

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left( 1 + \frac{\alpha_s}{4\pi} \delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\, \delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s \alpha}{(4\pi)^2}\, \delta\zeta^{(2)}_{\alpha_s\alpha} + \dots \right),$$

Diagrams contributing to $\delta\zeta^{(2)}_{\alpha_s\alpha}$ ($\mathcal{H} = h_0, \phi^\pm, \chi$ - higgs and would be goldstone bosons, $\mathcal{V} = W^\pm, Z$, $q$ - different quarks)




The corresponding integrands are expanded in external momentum $Q$ and masses of light quarks (all but $t$). For consistency, Yukawa interactions of light quarks are also neglected.

# Matching running parameters

(One of) our final expression (s):

$$\alpha_s{}^{(5)} = \alpha_s \zeta_{\alpha_s} = \alpha_s \left(1 + \frac{\alpha_s}{4\pi}\delta\zeta^{(1)}_{\alpha_s} + \frac{\alpha_s^2}{(4\pi)^2}\,\delta\zeta^{(2)}_{\alpha_s} + \frac{\alpha_s\alpha}{(4\pi)^2}\,\delta\zeta^{(2)}_{\alpha_s\alpha} + \ldots\right),$$

In terms of PDG'14 particle pole masses (all $\mu$-dependence of $X$s is explicit) new result is given by ($x_{ij} \equiv M_i/M_j$)

$$\delta\zeta^{(2)}_{\alpha_s\alpha} = \frac{M_t^2}{M_W^2 s_W^2}\left(X^{(1)}_{\alpha_s\alpha}\ln\frac{M_t^2}{\mu^2} + X^{(0)}_{\alpha_s\alpha}\right), \qquad \frac{M_t^2}{M_W^2 s_W^2} = 20.8(2)$$

$$X^{(1)}_{\alpha_s\alpha} = -1 + x_{wt}^2\left(\frac{2}{9} + \frac{22}{9}x_{wz}^2\right) + \frac{11}{6}x_{zt}^2 = -0.034(15)$$

$$X^{(0)}_{\alpha_s\alpha} = -1.17(2) \quad \text{to be compared with } X^{(0)}_{\alpha_s^2} = -\frac{14}{3}$$

See arXiv:1410.7603 [Bed14] for analytic result in terms of $x_{ij}$

Enhancement factor due to the top Yukawa coupling $y_t$ : $\alpha_s\alpha\frac{M_t^2}{M_W^2 s_W^2} \sim \alpha_s^2$

# Extraction of $\alpha_s^{SM}$ from $\alpha_s^{(5)}$

- By construction, given the parameters of the SM one can find the value of the effective coupling $\alpha_s^{(5)}$.
- However, it is $\alpha_s^{(5)}(\mu)$ which is fitted to observables the QCD.
- Due to this, one is interested in the inverse relation (obtained in PT):

$$\alpha_s = {\alpha_s}^{(5)} \left( 1 + \frac{{\alpha_s}^{(5)}}{4\pi} \delta\zeta^{(1)}_{\alpha_s'} + \frac{\left({\alpha_s}^{(5)}\right)^2}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'} + \frac{{\alpha_s}^{(5)}\alpha}{(4\pi)^2} \delta\zeta^{(2)}_{\alpha_s'\alpha} \right)$$

$$\begin{aligned} \delta\zeta^{(1)}_{\alpha_s'} &= \delta\zeta^{(1)}_{{\alpha_s}^{(5)}} = -\delta\zeta^{(1)}_{\alpha_s} \\ \delta\zeta^{(2)}_{\alpha_s'} &= -\left( \delta\zeta^{(2)}_{\alpha_s} - 2(\delta\zeta^{(1)}_{\alpha_s})^2 \right) \\ \delta\zeta^{(2)}_{\alpha_s'\alpha} &= -\delta\zeta^{(2)}_{\alpha_s\alpha} \end{aligned}$$

# Numerical analysis of the $\mathcal{O}(\alpha_s \alpha)$ correction

- In order to analyze the calculated correction we take the matching scale is $\mu = M_Z$ and use PDG'14 values of the pole masses.
- The quoted world averages $\alpha_s^{(5)}(M_Z) = 0.1185$, $\alpha^{-1} = 127.04$ is assumed to be fitted within the effective theory.
- At $Z$ - boson mass scale (three-loop contribution $\mathcal{O}(\alpha_s^3)$ is also shown):

$$\alpha_s(M_Z) = 0.1185 \cdot \left[ 1 - \underbrace{0.008067}_{\alpha_s} - \underbrace{0.000965}_{\alpha_s^2} + \underbrace{0.000143}_{\alpha_s \alpha} + \underbrace{0.000018}_{\alpha_s^3} \right],$$

- In principle, final result for the running $\alpha_s^{SM}(\mu \gg M_Z)$ should not depend on the matching scale. However, due to truncation of the series, there is a residual dependence on $\mu$
- As a consequence, the matching scale is usually chosen of the order of electroweak scale so that no large logs appear in the relation (effectively re-sum logarithms $\ln M_Z/\mu$).

# Scale dependence of the decoupling corrections

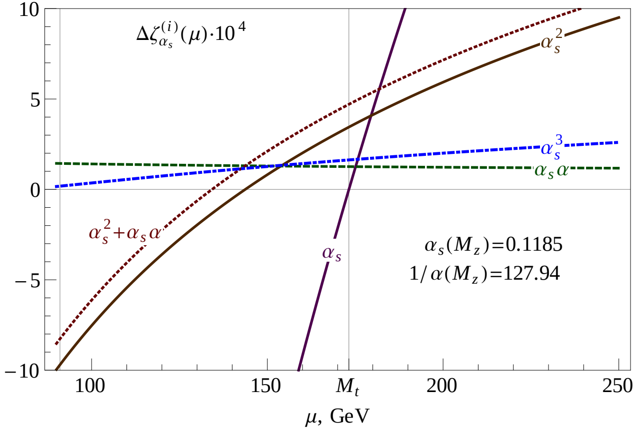
The scale dependence of different matching corrections:

$\alpha_s$ in terms of $\alpha_s^{(5)}$

$$\Delta\zeta_{\alpha_s}^{(\alpha_s)} \equiv \frac{\alpha_s^{(5)}}{(4\pi)}\delta\zeta_{\alpha_s^{(5)}}^{(1)},$$

etc

Four-loop running up to the matching scale via RunDec [CKS00] package.



# Conclusions

- Electroweak corrections to the matching relation between $\alpha_s$ of the SM and effective $\alpha_s{}^{(5)}$ are found and expressed either in terms of particle pole masses or $\overline{MS}$ running masses in an explicit gauge-invariant way.
- The corrections,when evaluated at the electroweak scale, are found to be comparable with pure three-loop QCD contribution usually taken into account in RGE analysis of the SM.
- However, the relative value of $\mathcal{O}(\alpha_s\alpha)$ correction is typically around $10^{-4}$, which currently below the uncertainty in determination of $\alpha_s{}^{(5)}$.
- Nevetherless, we hope that the result presented here is a necessary step towards future precise analysis of the SM.

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