Three-loop beta-functions and anomalous dimensions in the SM

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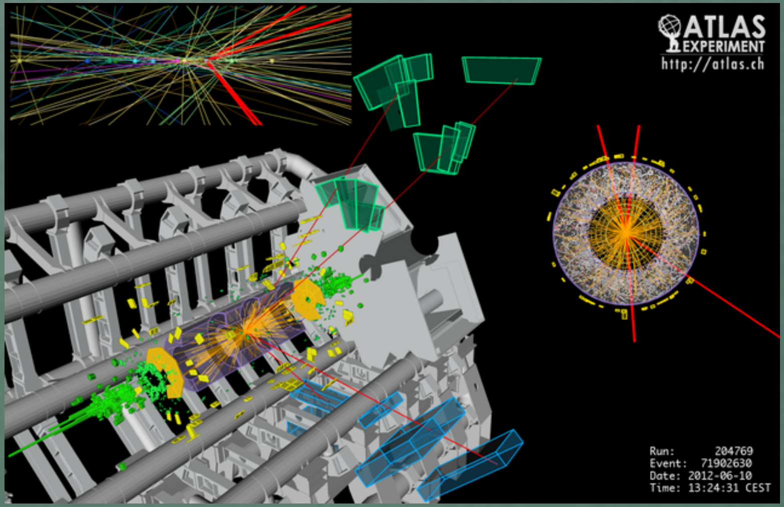
Based on
ArXiv: 1210.6873 (JHEP1301)
ArXiv: 1212.6829 (Phys.Lett.B722)
ArXiv: 1303.4364 (Nucl.Phys.B875)
ArXiv: 1309.1643 (ACAT2013 Proc)
ArXiv: 1310.3806

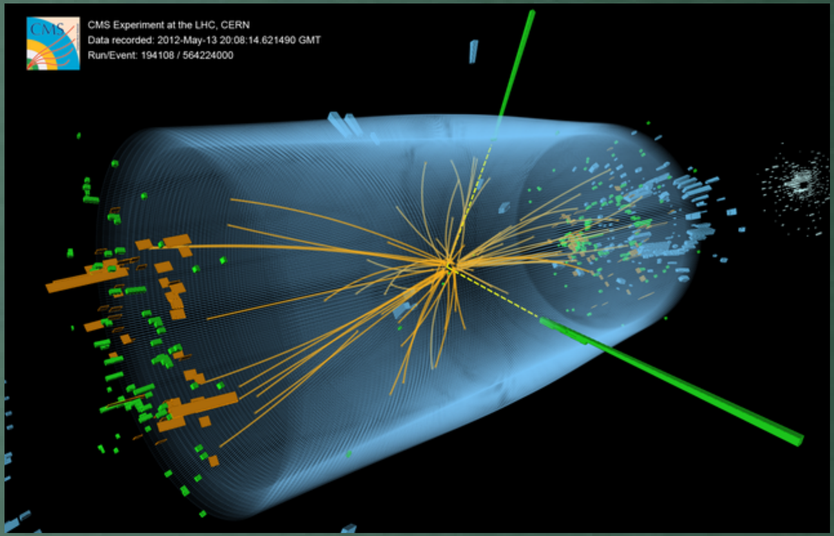
BLTP, 16/10/2013

Some of our motivations

- The discovery of the Higgs boson "finalizes" the SM

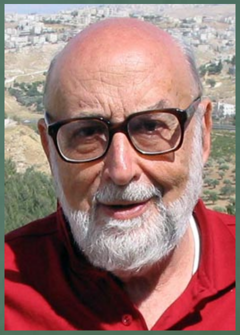


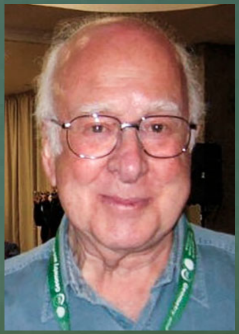
[ATLAS, CMS '11-12]



4 July 2012!

- Nobel Prize'2013...





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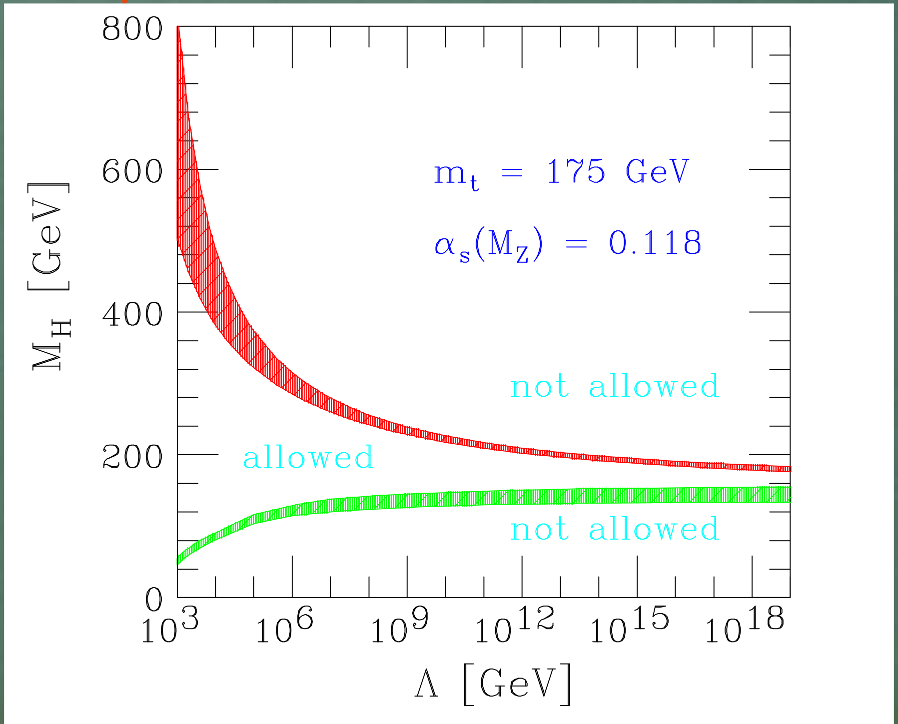
- The discovery of the Higgs boson "finalizes" the SM

[ATLAS, CMS '11-12]

- No clear experimental hints of New Physics

More precise studies of the SM is required



[Hambye,Riesselman'97]

Some of our motivations

- The discovery of the Higgs boson “finalizes” the SM

[ATLAS, CMS '11-12]

- No clear experimental hints of New Physics

More precise studies of the SM is required

- We have all necessary tools

Why not to try? :)

NB: The three-loop Renormalization Group Equations (RGE) in the MSSM are know: [Jack, Jones,Kord,'05]

What TO calculate?

$D = 4 - 2\epsilon$

- Renormalization constants in $\overline{\text{MS}}$ scheme Z_Γ of certain dimensionally regularized 2-,3-, and 4-point Green functions Γ at 1, 2, and 3 loops

$$\Gamma_{\text{Ren}}\left(\frac{Q^2}{\mu^2}, a_i\right) = \lim_{\epsilon\to 0} Z_\Gamma\left(\frac{1}{\epsilon}, a_i\right)\Gamma_{\text{Bare}}\left(Q^2, a_{i,\text{Bare}}, \epsilon\right)$$

$\mathrm{a}_i(\mu)$ - Standard Model parameters in $\overline{MS}$

$$\left(\frac{\partial}{\partial \ln \mu^2} + \beta_{a_i}\frac{\partial}{a_i} + \gamma_\Gamma\right)\Gamma_{\text{ren}}\left(\frac{Q^2}{\mu^2}, a_i\right) = 0$$

What to calculate?

$D = 4 - 2\epsilon$

- Renormalization constants in $\overline{\text{MS}}$ scheme Z_Γ of certain dimensionally regularized 2-,3-, and 4-point Green functions Γ at 1, 2, and 3 loops
- From Z_Γ extract Z_{a_i}- ren. const. For the SM parameters a_i
- Find <u>beta-functions</u> from Z_{a_i} and <u>anomalous dimensions</u> from Z_Γ

SM (running $\overline{\text{MS}}$) parameters

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2}\right)$$

$\hat{\lambda}$ ///

U(1) SU(2) SU(3)

Relation to bare parameters in $\overline{\text{MS}}$ scheme:

$$a_{k,\text{Bare}}\mu^{-2\rho_k\epsilon} = Z_{a_k} a_k(\mu) = a_k + \sum_{n=1}^{\infty} c_k^{(n)} \frac{1}{\epsilon^n},$$

For gauge and Yukawa

$$\rho_k = 1$$

For Higgs self-coupling

$$\beta_i(a_k) = \left.\frac{d a_i(\mu,\epsilon)}{d\ln\mu^2}\right|_{\epsilon=0}, \qquad \beta_i = \beta_i^{(1)} + \beta_i^{(2)} + \beta_i^{(3)} + \ldots$$

Known results

- Gauge couplings:
  - 1 loop:

    [Gross,Wilczeck'73; Politzer'73]

  - 2loop:

    [Jones'74,Caswell'74;Tarasov,Vladimirov'77;**Egorian,Tarasov'79**; Jones'81;Fischler,Hill'82;**Machacek,Vaughn'83**;Jack,Osborn'84]

  - 3 loop:

    [Tarasov,Vladimirov,Zharkov'80,Curtright'80; Jones'80; Steinhauser'98; Pickering,Gracey,Jones'01]

Mihaila,Salomon,Steinhauser'12

(full result for the first time)

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  - 3 loop: [Tarasov,Vladimirov, JINR-E2-80-483]

    [**Tarasov,Vladimirov,Zharkov'80**,Curtright'80; Jones'80; Steinhauser'98; Pickering,Gracey,Jones'01]

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(full result for the first time)

Known results

- Yukawa couplings:
  - 2 loop:

[Fischler,Oliensis'82; **Machacek,Vaughn'83**; Jack,Osborn'84]

  - 3 loop:

Chetyrkin, Zoller'12

(no electroweak couplings, only top Yukawa)

Known results

- Higgs sector:
  - 2 loop:

    [**Machacek, Vaughn'84**; Jack,Osborn'84;

    Ford, JackJones'92; **Luo,Xiao'02**]]
  - 3 loop:

Chetyrkin, Zoller'12
(no electroweak couplings, only top Yukawa)

Chetyrkin, Zoller'13 ← Made their results public a **week** Earlier than our group...
(full result for the first time)

Problems ...

- A lot of Feynman diagrams $\mathcal{O}\left(10^4 - 10^6\right)$

  calculation by hand – impossible...

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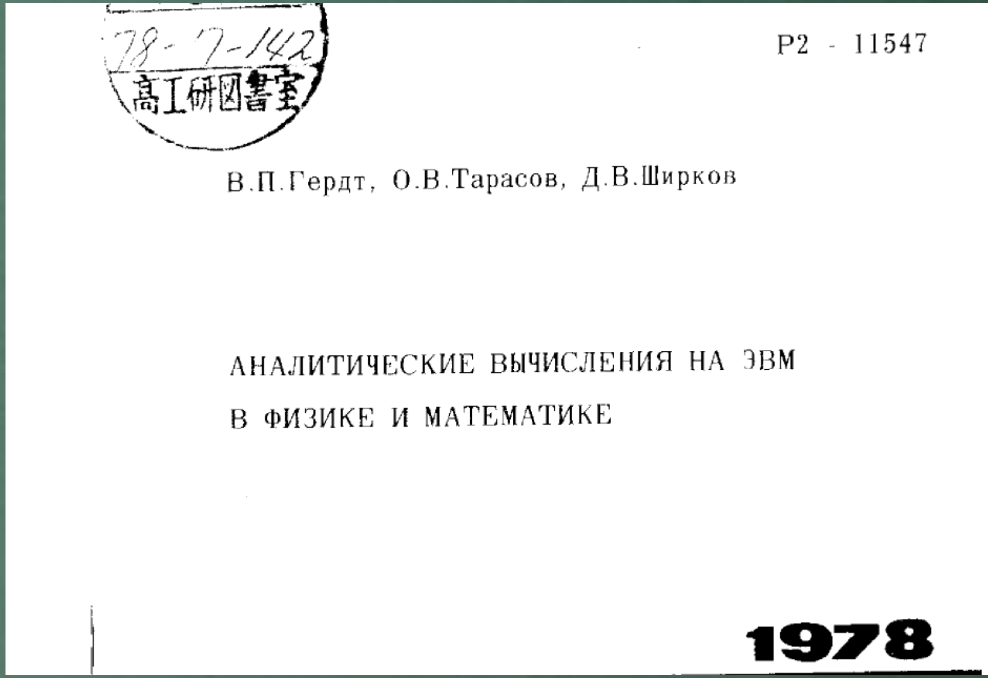

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В.П.Гердт, О.В.Тарасов, Д.В.Ширков

АНАЛИТИЧЕСКИЕ ВЫЧИСЛЕНИЯ НА ЭВМ
В ФИЗИКЕ И МАТЕМАТИКЕ

1978

Problems …

- A lot of Feynman diagrams $\mathcal{O}\left(10^4 - 10^6\right)$

  calculation by hand – impossible...

- γ_5 treatment in dimensional regularization

  Antocommute or not anticommute?

$$\{\gamma_\mu, \gamma_5\} \stackrel{?}{=} 0 \quad \text{vs} \quad \mathrm{tr}\left(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5\right) \stackrel{?}{=} -4i\epsilon^{\mu\nu\rho\sigma}$$

Our solutions (I)

$\overline{MS}$ scheme!

- InfraRed Rearrangement (IRR)
  - We are interesting in UV divergencies only, so it is possible to change IR structure of the diagrams
  - This should be done without introduction of spurious IR divergencies.

[Vladimirov'80]

NB: R*-operation for dealing with both IR and UV

[Chetyrkin, Tkachov,Smirnov'82-'84]

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- Calculation in the <u>unbroken</u> phase of the SM (massless fields)
- Background field-gauge fixing in order to extract gauge beta-functions solely from self-energies

QED-like Ward identities!

see e.g. [Abbot '82], [Denner, Weiglein, Dittmaier,'95]

Our solutions (I)

$\overline{MS}$ scheme!

- InfraRed Rearrangement (IRR)
- Calculation in the unbroken phase of the SM (massless fields)

SU(2) unbroken!

- Background field-gauge fixing in order to extract gauge beta-functions solely from self-energies

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\begin{aligned}
\mathcal{L}_{\rm G} &= -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu}, \\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \\
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^{ijk} W^j_\mu W^k_\nu, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
\end{aligned}$$

Quantum field

Background field

$$G^a_\mu = \tilde{G}^a_\mu + \hat{G}^a_\mu, \qquad a = 1, \ldots, 8$$

$$W^i_\mu = \tilde{W}^i_\mu + \hat{W}^i_\mu, \qquad i = 1, \ldots, 3$$

$$D_\mu = \partial_\mu - i g_s T^a G^a_\mu - i g_2 \tau^i W^i_\mu + i g_1 \frac{Y_W}{2} B_\mu .$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

SU(2) doublets

$$\mathcal{L}_{\rm F} = -Y^u_{ij}(\bar{Q}_i\Phi^c)u_{jR} - Y^d_{ij}(\bar{Q}_i\Phi)d_{jR} - Y^e_{ij}(\bar{L}_i\Phi)E_{jR} + \text{h.c.}$$

SU(2) singlets

$$Q = \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad L = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \qquad i,j = 1,2,3$$

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}, \quad \Phi^c = i\sigma^2\Phi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{pmatrix}$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm H} \quad = \quad (D_\mu \Phi)^\dagger (D_\mu \Phi) + m^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$

The renormalization constant and corresponding beta-function can be extracted from the renormalization of the composite operator

$$\Phi^\dagger \Phi = \left(\frac{h^2 + \chi^2}{2} + \phi^+ \phi^-\right)$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi_G} G_G^a G_G^a - \frac{1}{2\xi_W} G_W^i G_W^i - \frac{1}{2\xi_B} G_B^2,$$

$$\begin{aligned} G_G^a &= \partial_\mu \tilde{G}_\mu^a + g_s f^{abc} \hat{G}_\mu^b \tilde{G}_\mu^c \\ G_W^i &= \partial_\mu \tilde{W}_\mu^i + g_2 \epsilon^{ijk} \hat{W}_\mu^j \tilde{W}_\mu^k \\ G_B &= \partial_\mu \tilde{B}_\mu \end{aligned}$$

Gauge-fixing parameters
(parameter beta-functions
should NOT depend on them)

For non-abelian quantum fields
ordinary derivative is substituted by the
covariant one involving the corresponding
background fields

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm FP} = -\bar{c}_\alpha \frac{\delta G_\alpha}{\delta \theta^\beta} c_\beta \qquad \alpha, \beta = (G, W, B)$$

$$\begin{aligned}
\delta \tilde{G}^a_\mu &= (D_\mu \theta_G)^a = \partial_\mu \theta^a_G + g_s f^{abc} G^b_\mu \theta^c_G \,, \\
\delta \tilde{W}^i_\mu &= (D_\mu \theta_W)^i = \partial_\mu \theta^i_W + g_2 \epsilon^{ijk} W^j_\mu \theta^k_W \,, \\
\delta \tilde{B}_\mu &= \partial_\mu \theta_B \,.
\end{aligned}$$

Infinitesimal quantum gauge transformation

NB: The corresponding Feynman rules
are obtained by means of LanHEP package

(back to) IRR trick

- Variant I: "MINCER"
  - Set all masses to zero
  - Set n external momenta in all relevant diagrams with 2+n legs to zero

<u>single-scale propagator-type integrals</u>

- Pro: Multiplicative renormalizability of Green functions can be used
- Con: naïve application can introduce spurios IR (infrared R^* is needed)

IRR trick

- Variant II: "Bubbles" (BAMBA/MATAD)
  - Expand in external momenta around zero to a sufficient order (or use "exact" decomposition)
  - Introduce an auxilarily mass in each propagator [Misiak,Munz'95, Chetyrkin, Misiak,Munz'97]

$$\frac{1}{(q-p)^2} = \frac{1}{q^2-M^2} + \frac{2qp - p^2 - M^2}{q^2-M^2} \times \frac{1}{(q-p)^2}$$

$q -$ integration momentum, $p -$ external momentum

IRR trick

- Variant II: "Bubbles" (BAMBA/MATAD)
  - Expand in external momenta around zero to a sufficient order (or use "exact" decomposition)
  - Introduce an auxilarily mass in each propagator

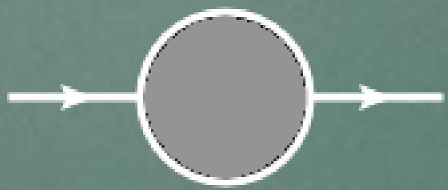
Single-scale vacuum integrals

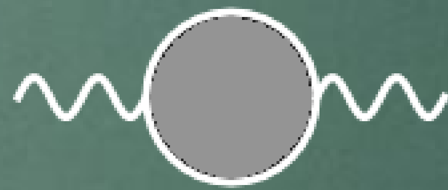
- Pro: No spurious IR divergencies
- Con: Requires explicit introduction of mass counter terms for gauge and scalar fields

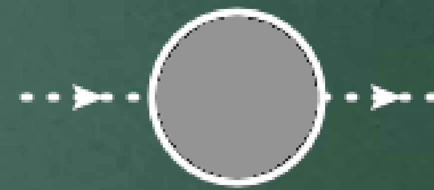
IRR tricks. Which one?

- Variant I:
  - gauge and Yukawa coupling beta-functions
  - Field anomalous dimensions
- Variant II:
  - beta-function for Higgs self-coupling
  - Beta-function for Higgs mass parameter

What WE calculate?







$f = t,\ b,\ \tau$

Z_{f_L} Z_{f_R}

$V = G^a, W^i, B$

$Z_{\hat{V}}$ $Z_{\tilde{V}}$

$h,\ \phi^\pm,\ \chi$

$Z_h = Z_{\phi^\pm} = Z_\chi$

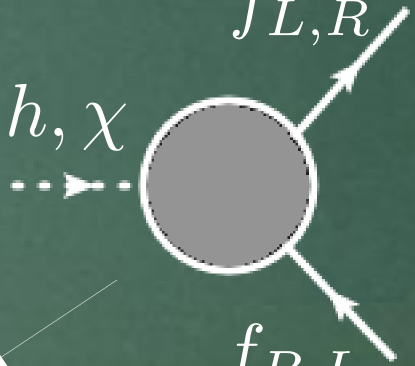
$$Z_{g_1} = Z_{\hat{B}}^{-1/2},\ Z_{g_2} = Z_{\hat{W}}^{-1/2},\ Z_{g_s} = Z_{\hat{G}}^{-1/2}$$

BFG

$$Z_{y_f} = \frac{Z_{ff\phi}}{\sqrt{Z_{f_L} Z_{f_R} Z_\phi}}$$

MINCER

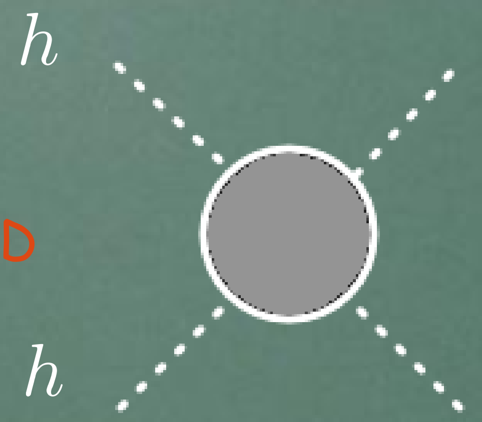
$\phi = h, \chi$

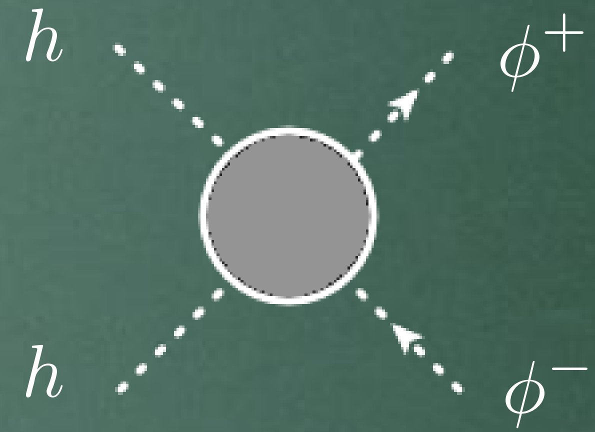


$Z_{ff\phi}$

What WE calculate?

MATAD





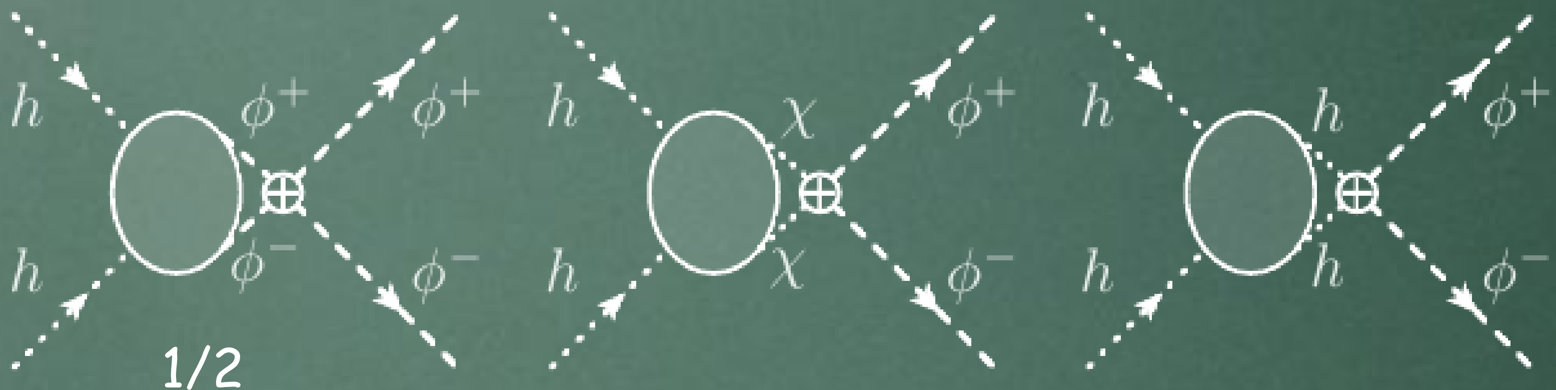
BAMBA

$$Z_{hhhh} = Z_{hh\phi^+\phi^-}$$ SU(2)

$$Z_\lambda = \frac{Z_{hhhh}}{Z_h^2} = \frac{Z_{hh\phi^+\phi^-}}{Z_h Z_\phi}$$

A comment on mass anomalous dimension

Mass parameter m^2 **anomalous dimension** can be found by considering the following diagrams:



i.e., by selecting the diagrams which contribute to $hh\phi^+\phi^-$ Green function and have $\phi^+,\ \phi^-$ external particles connected to a four-vertex

$$Z_{m^2} = \frac{Z_{hh\,[\phi^+\phi^-]}}{Z_h}$$

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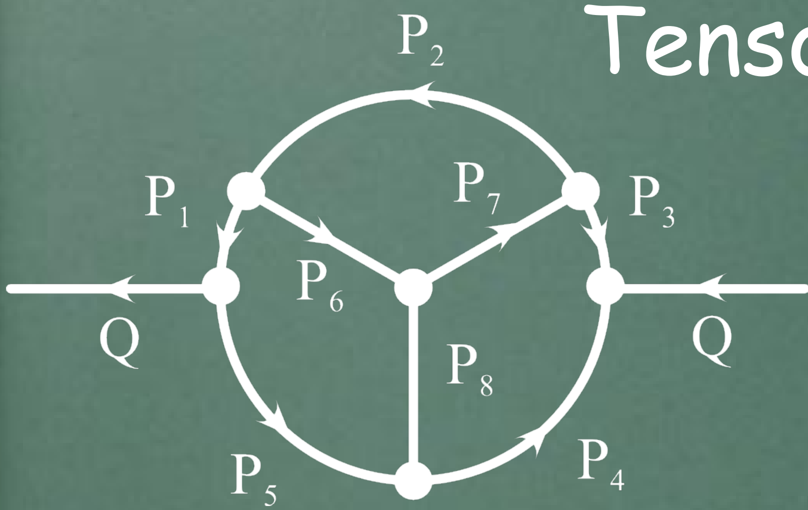
Out computer setup

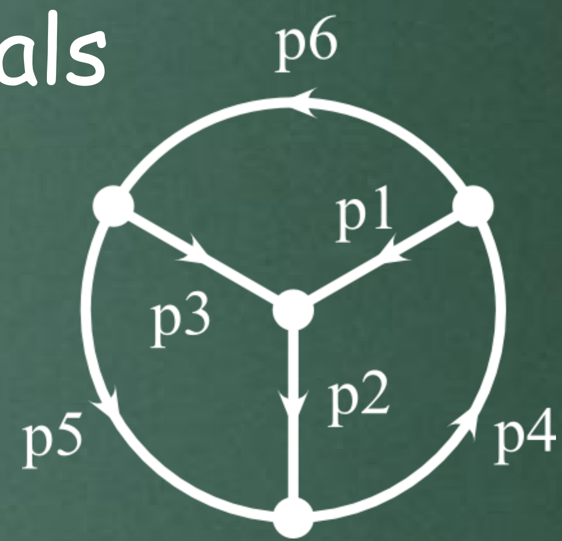
- Public and private computer codes:
  - LanHEP [A.Semenov]
  - FeynArts [Kublbeck,Eck,Mertig,Hahn]
  - Diana (QGRAF) [Fleischer,Tentyuokov] ([Nogueira])
  - <u>MINCER</u> [Gorishii, Larin,Surguladze,Tkachov,Vermaseren]
  - COLOR [van Ritbergen,Schellekens,Vermaseren]
  - <u>BAMBA</u> [Velizhanin]
  - <u>MATAD</u> [Steinhauser]
  - Some C++/awk/python/bash magic

Mincer vs "Bubbles"

BE **D6**



Tensor integrals

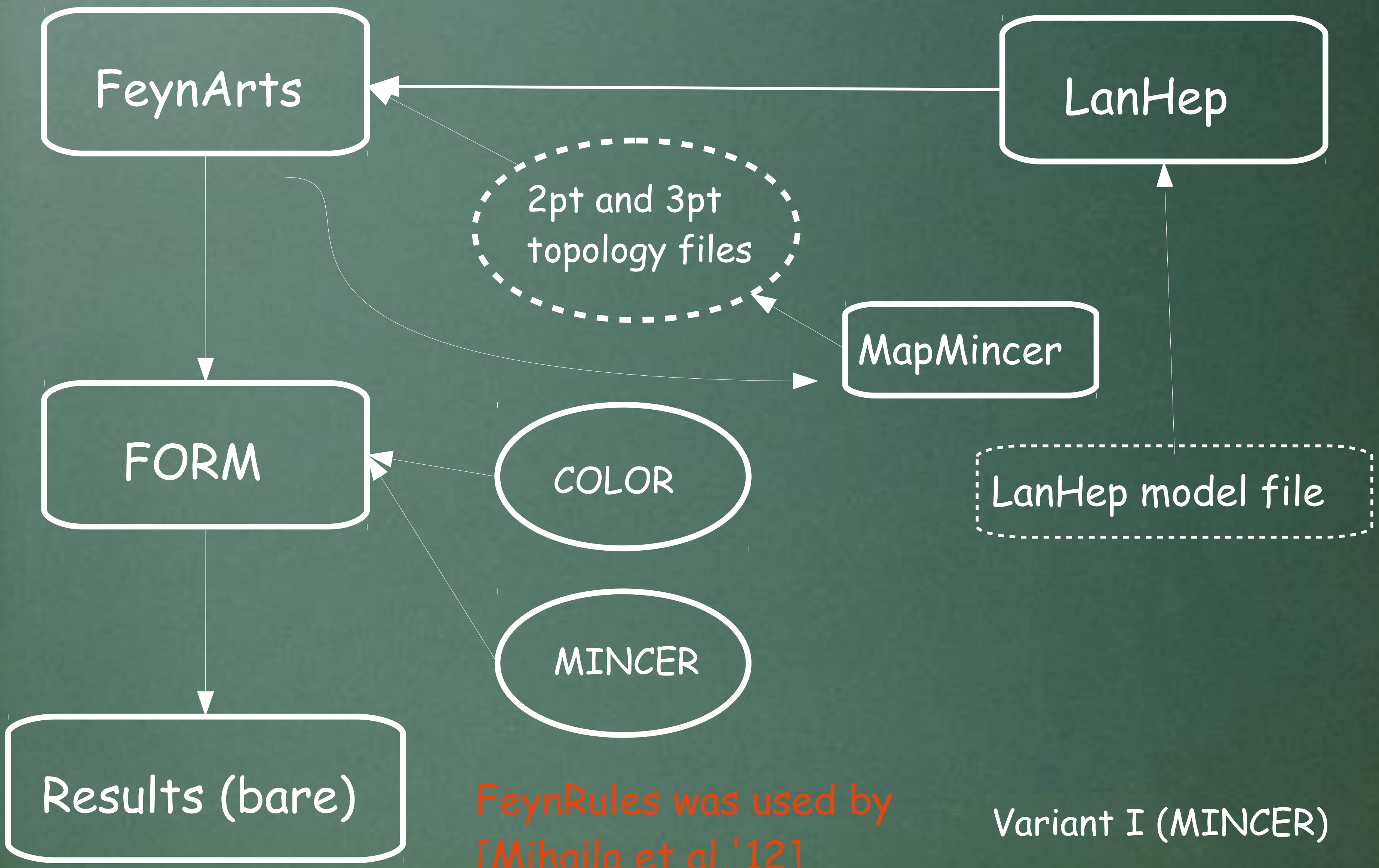


A finite set of **master** integrals

[Tkachov,81,Tkachov,Chetyrkin,81]

$$\int d^D k_i \frac{\partial}{\partial k_i^\mu} \left[q^\mu I(p_1, ..., p_m, k_1, ..., k_l) \right] = 0$$

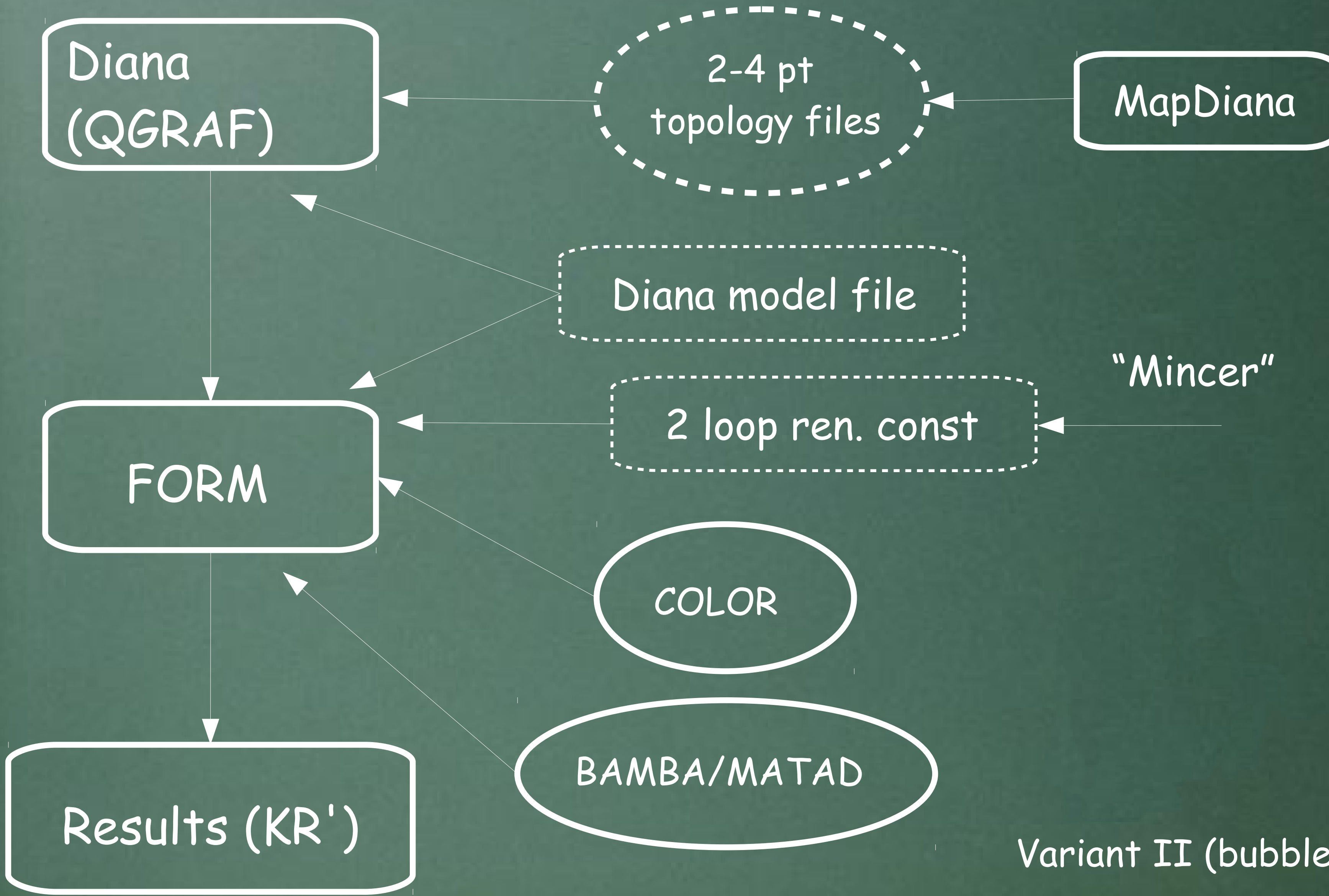
Automatization...



FeynRules was used by [Mihaila et al,'12]

Variant I (MINCER)

Automatization...



Variant II (bubbles)

...and (our) "solutions" (II)

$$\gamma_5?$$

In D dimensions

See review [Jegerlehner,'00]

$$\{\gamma_\mu, \gamma_5\} = 0 \quad \Longrightarrow \quad \mathrm{tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 0$$

Naive DREG

How about?

$$\mathrm{tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) \stackrel{?}{=} -4i\epsilon^{\mu\nu\rho\sigma}$$

4D object!

Semi-naive Gamma5

- Semi-naive treatment of Gamma5:
  - Use $\{\gamma_5, \gamma^\mu\} = 0, \quad \gamma_5^2 = 1$

    to put all the gamma5's to the rightmost position in a fermion chain*
  - "Even" traces (no γ_5 left) pose no problem
  - In "Odd" traces (one γ_5 left) we use

$$\gamma_5 = -\frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$$

  - Contract Eps-tensors as in 4D!

$\mathcal{O}(\epsilon)$

Difference!

Semi-naive Gamma5

Do we have a contibution ("EPS-contibution") from this?

$$\mathrm{tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma_5\right) = -4i\epsilon^{\mu\nu\rho\sigma}$$

Vector indices should be contracted either with external momenta or with each other

→ Two closed fermion loops are required

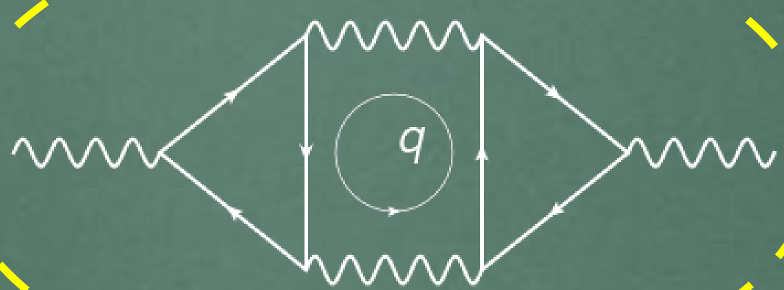
What kind of fermion loops appear in our calculations?

Semi-naive Gamma5

What kind of fermion loops appear in our calculations?

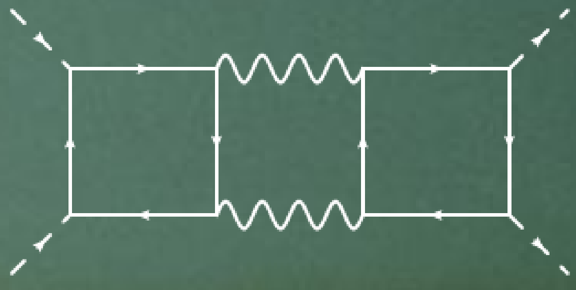
Two internal fermion loops

Only bosonic external fields
at three loops



Gauge-Anomaly cancelations: No EPS-contibution upon summation over the SM (anti)fermions

$$\mathrm{Tr}(\{T^a, T^b\}T^c) = 0$$



[Gross,Jackiw'69]

"Dangerous" diagrams

$\frac{\mathcal{O}(\epsilon)}{\epsilon^2}$ error

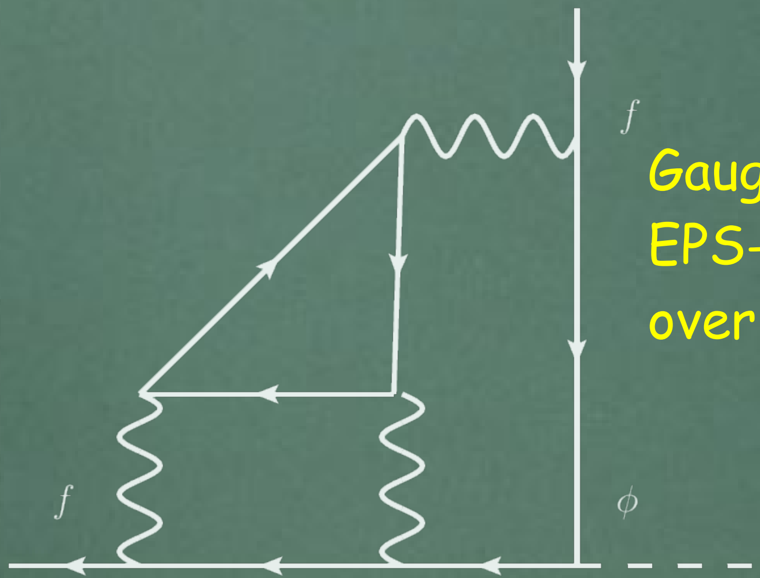
Semi-naive Gamma5

What kind of fermion loops appear in our calculations?

One internal loop and one external fermion chain

Relevant for our calculation of
Yukawa coupling Z's

Become a loop in Dirac space
upon contraction with a
projector



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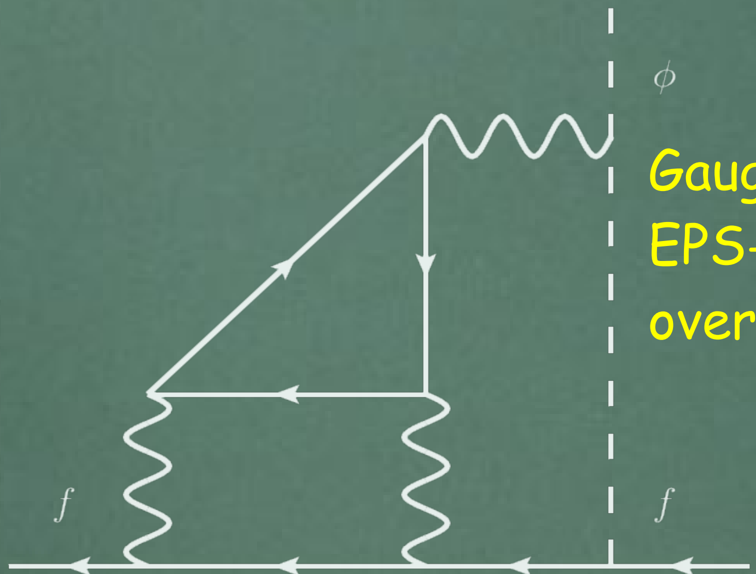

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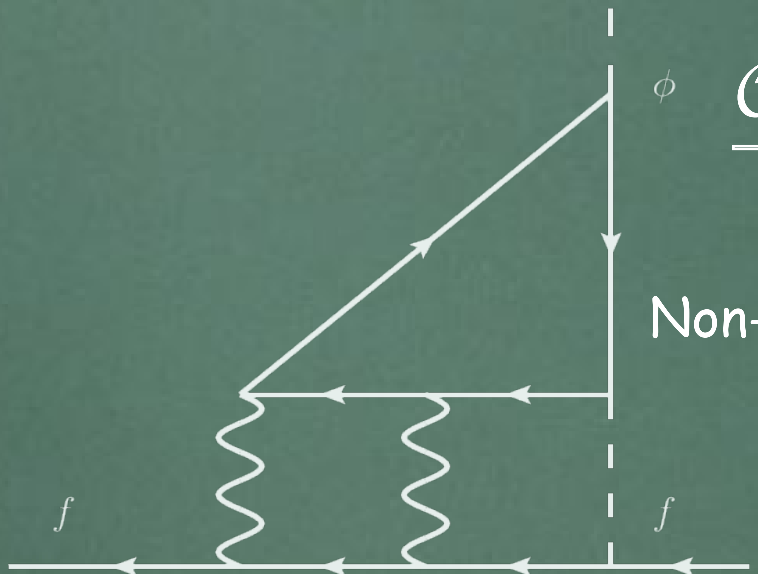

$$\frac{\mathcal{O}(\epsilon)}{\epsilon}$$ error → Not important

Non-trivial contribution!

[Chetyrkin,Zoller'12]

Semi-naive Gamma5

A comment about non-cyclicity of Trace operation...

The price of simultaneous application of the above-mentioned rules is the fact that there is an ambiguity in positioning of gamma5 in a trace, e..g.

$$g^{\mu_1\mu_6}\left[\mathrm{Tr}\left(\gamma_5\gamma_{\mu_1}\ldots\gamma_{\mu_6}\right)-\mathrm{Tr}\left(\gamma_{\mu_6}\gamma_5\gamma_{\mu_1}\ldots\gamma_{\mu_5}\right)\right]\propto(D-4)\epsilon_{\mu_1\ldots\mu_5}$$

[Kreimer'94]

Can spoil gauge anomaly cancelations if treated non-consistently...

We are lucky! **FeynArts** and **DIANA** uniquely define "cut" points of closed fermion chains for all the diagrams with the same "Generic" propotype

Towards the results...

The boundary values should be obtained by matching, i.,e comparing the predictions with (psedo)-observables.

Threshold corrections at the weak scale

	LO	NLO	NNLO	NNNLO
	0 loop	1 loop	2 loop	3 loop
g_2	$2M_W/V$	full	Work in progress	—
g_Y	$2\sqrt{M_Z^2 - M_W^2}/V$	full	Work in progress	—
y_t	$\sqrt{2}M_t/V$	$\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha_s^2, \alpha_s\alpha_{1,2})$ full	$\mathcal{O}(\alpha_s^3)$
λ	$M_h^2/2V^2$	full	for $g_{1,2} = 0$ full	—
m^2	M_h^2	full	full	—

Here we have defined $V \equiv (\sqrt{2}G_\mu)^{-1/2}$ and $g_1 = \sqrt{5/3}g_Y$.

From [Buttazzo et al'13]

Towards the results...

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Towards the results...

The boundary values should be obtained by matching, i.,e comparing the predictions with (psedo)-observables.

[PDG'13]

$$\alpha^{-1} = 137.035999$$
$$\alpha_s(M_Z) = 0.1184(7)$$
$$\mathrm{G}_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$$
$$\mathrm{M}_t = 173.5(1.0)\ \mathrm{GeV}$$
$$\mathrm{M}_h = 125.5(1.5)\ \mathrm{GeV}$$
$$\mathrm{M}_Z = 91.1876(21)\ \mathrm{GeV}$$
$$\mathrm{M}_W = 80.385(15)\ \mathrm{GeV}$$
$$\mathrm{m}_b(m_b) = 4.18(3)\ \mathrm{GeV}$$
$$\mathrm{M}_\tau = 1.77682(16)\ \mathrm{GeV}$$

Towards the results...

The boundary values should be obtained by matching, i.,e comparing the predictions with (psedo)-observables.

[PDG'13]

Initial values:

$$g_1 = 0.3576$$
$$g_2 = 0.6514$$
$$g_s = 1.2063$$
$$y_t = 0.9665$$
$$y_b = 0.016$$
$$y_\tau = 0.01$$
$$\lambda = 0.13$$
$$\mu = 100\ \mathrm{GeV}$$

←

$$\alpha^{-1} = 137.035999$$
$$\alpha_s(M_Z) = 0.1184(7)$$
$$\mathrm{G}_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$$
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F.Bezrukov: http://www.inr.ac.ru/~fedor/SM

Results. U(1) gauge coupling

Three-loop contribution to the three-loop beta-function:)

Initial values:

$$a_1 = 0.00134996$$

$$a_2 = 0.00268702$$

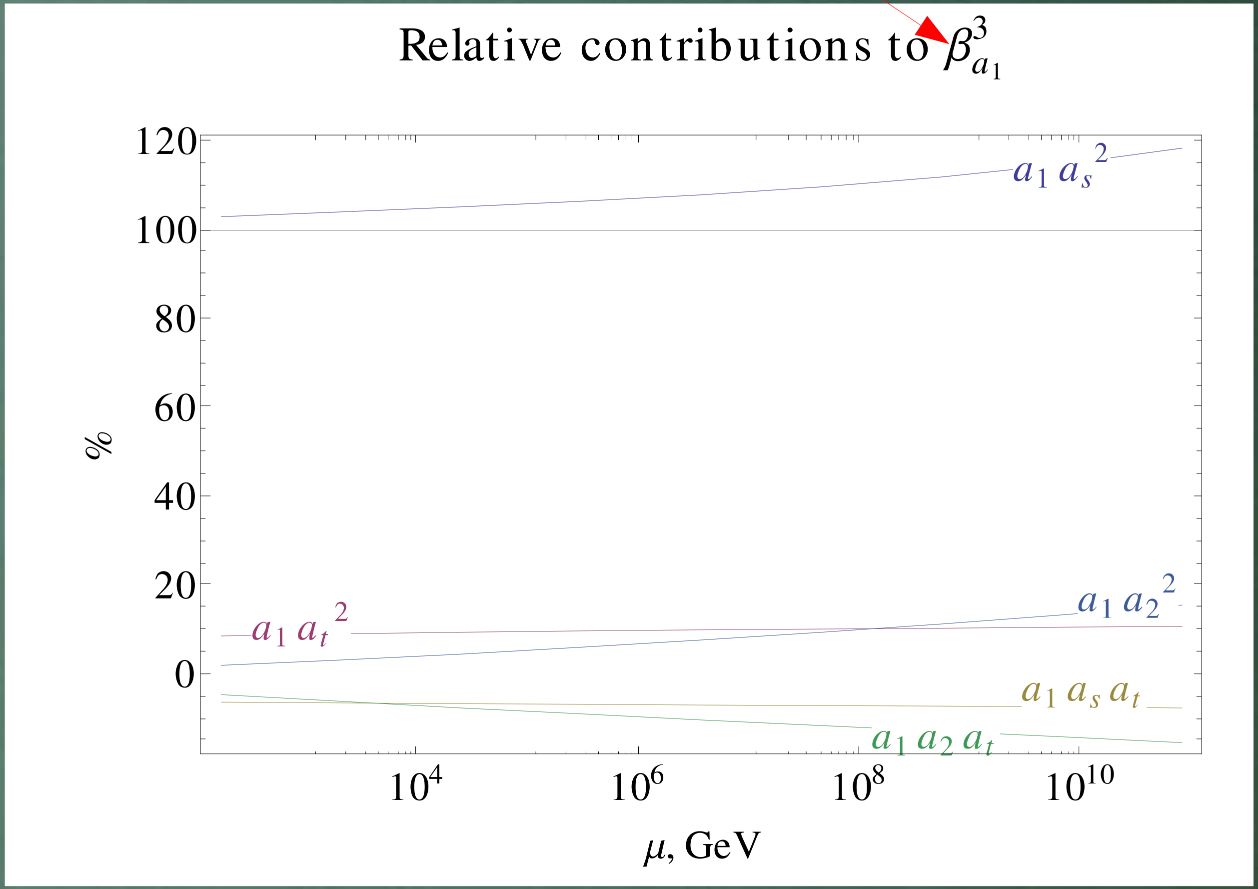
$$a_s = 0.009215$$

$$a_t = 0.00592$$

$$a_b = 1.62 \cdot 10^{-6}$$

$$a_\tau = 6.33 \cdot 10^{-7}$$

$$\hat{\lambda} = 0.00088$$



F.Bezrukov: http://www.inr.ac.ru/~fedor/SM

http://arxiv.org/src/1210.6873/anc

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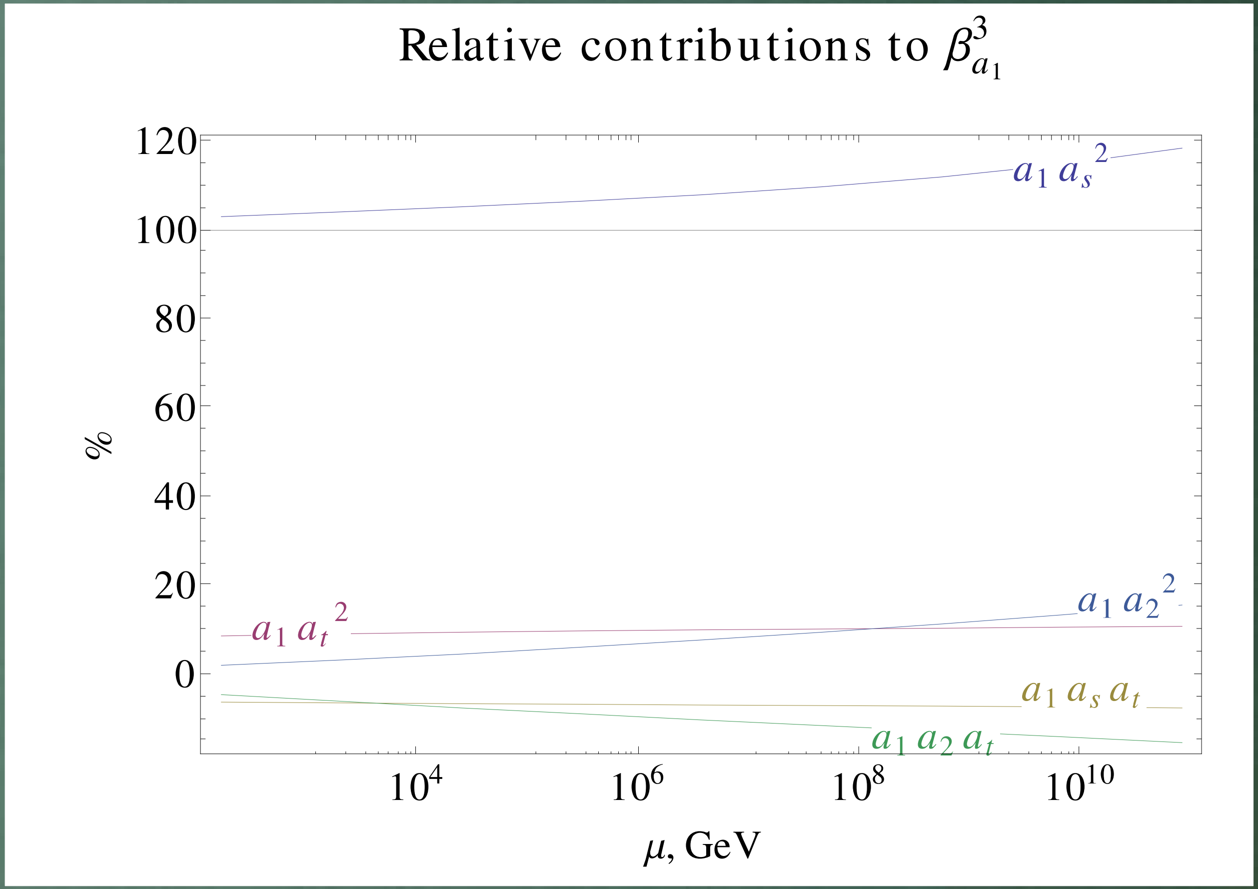


http://arxiv.org/src/1210.6873/anc

Results. SU(2) gauge coupling

Initial values:

$a_1 = 0.00134996$

$a_2 = 0.00268702$

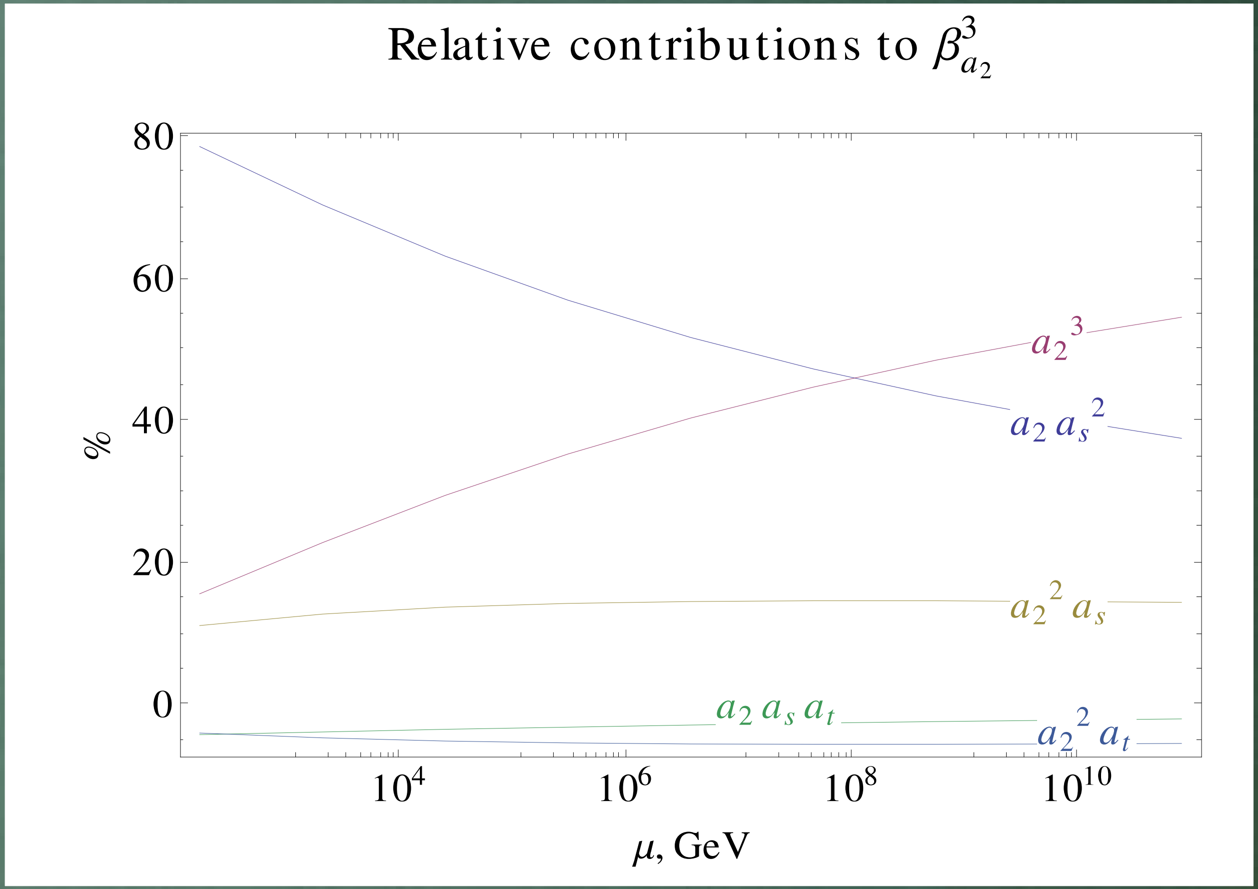
$a_s = 0.009215$

$a_t = 0.00592$

$a_b = 1.62 \cdot 10^{-6}$

$a_\tau = 6.33 \cdot 10^{-7}$

$\hat{\lambda} = 0.00088$

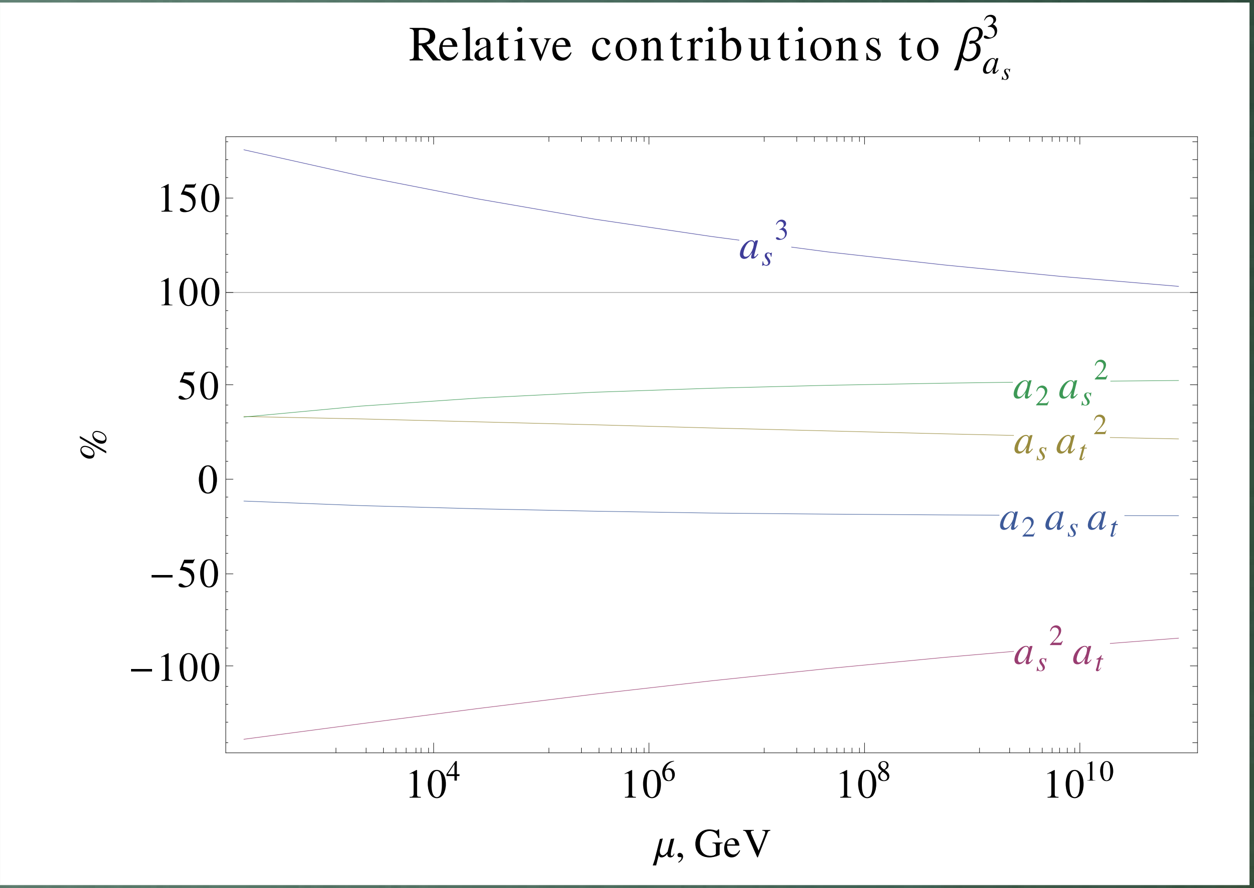


http://arxiv.org/src/1210.6873/anc

Results. SU(3) gauge coupling

Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$

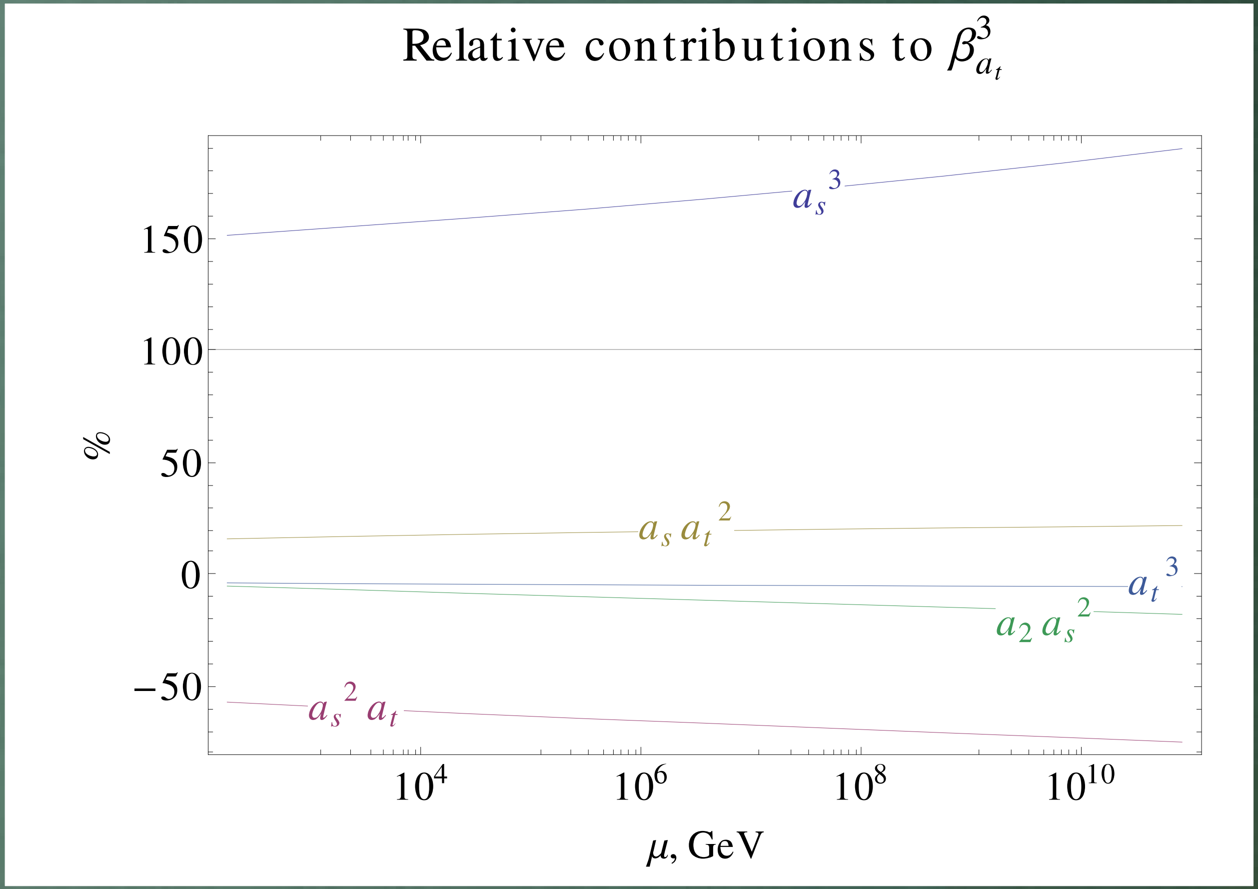


http://arxiv.org/src/1210.6873/anc

Results. Top Yukawa

Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$

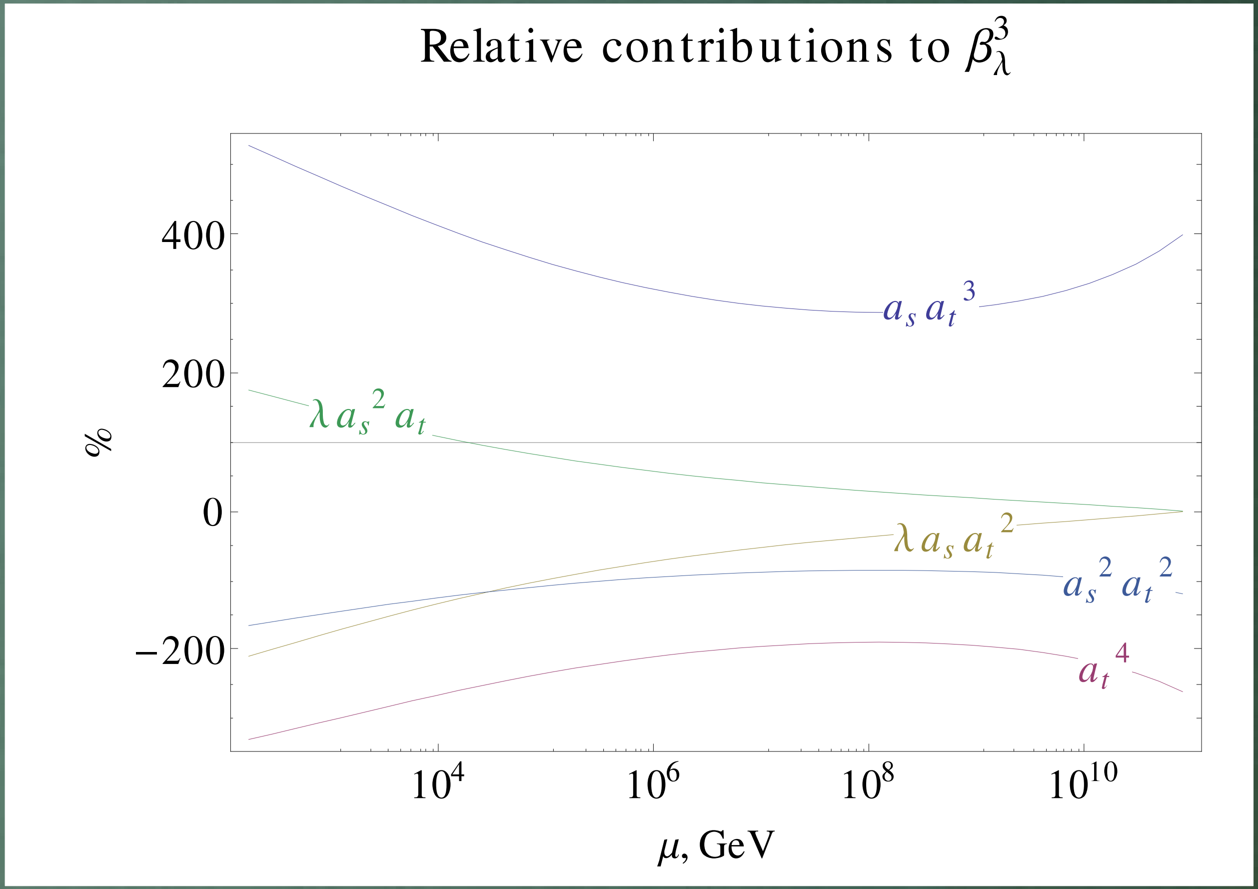


http://arxiv.org/src/1212.6829/anc

Results. Higgs self-coupling

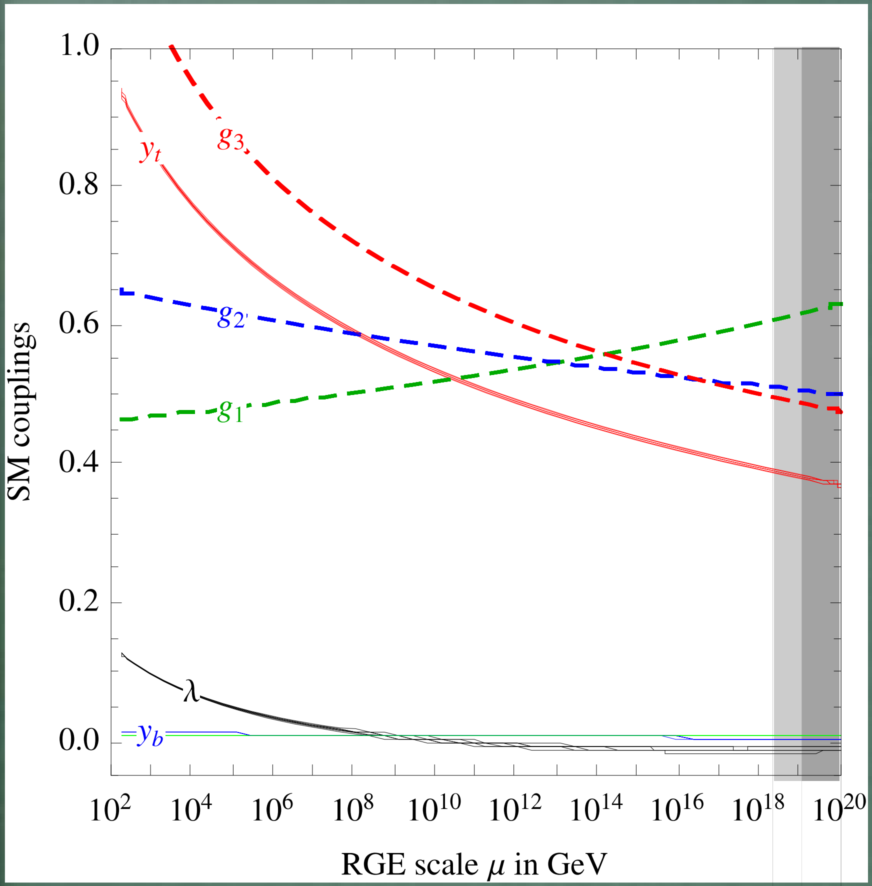
Initial values:

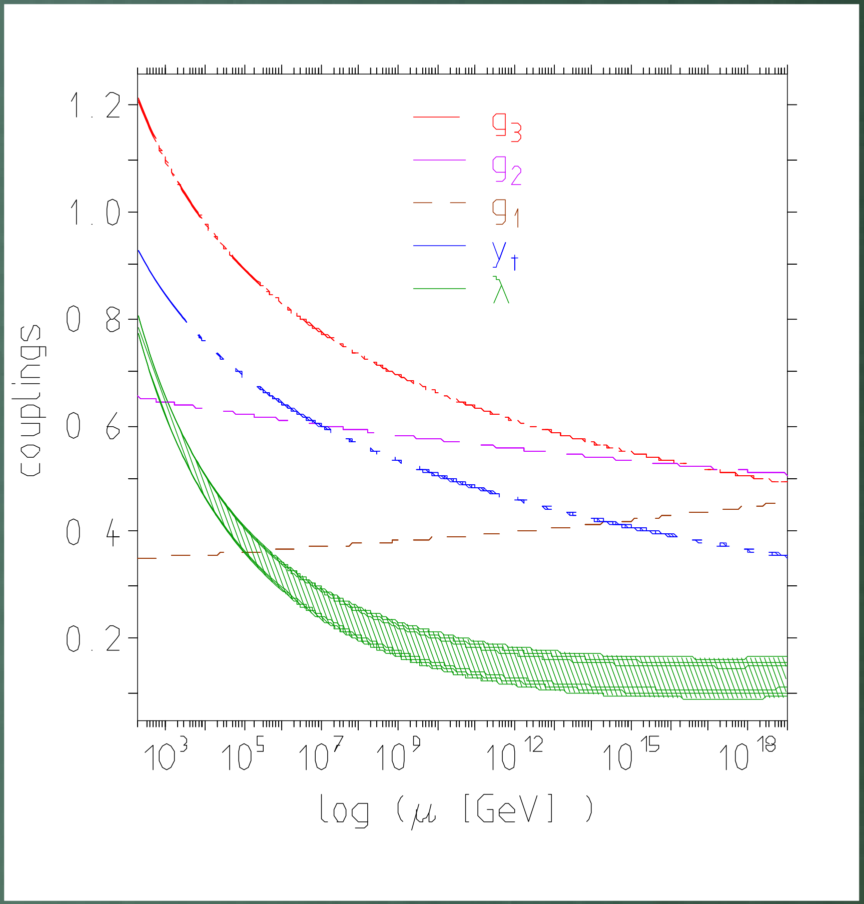
$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$



http://arxiv.org/src/1303.4364/anc

SM Running





[Degrassi et al, 12] vs [Jegerlehner, 13]

BLTP, 16/10/2013

Application – Vacuum Stability

The SM Vacuum can be studied by considering the full effective potential for background higgs field.

Given the existence of EW vacuum with VEV v
The effecitve potential for classical fields **h>>v**
can be approximated via

[Weinberg,Coleman'73]
[Ford,Jack,Jones'92]

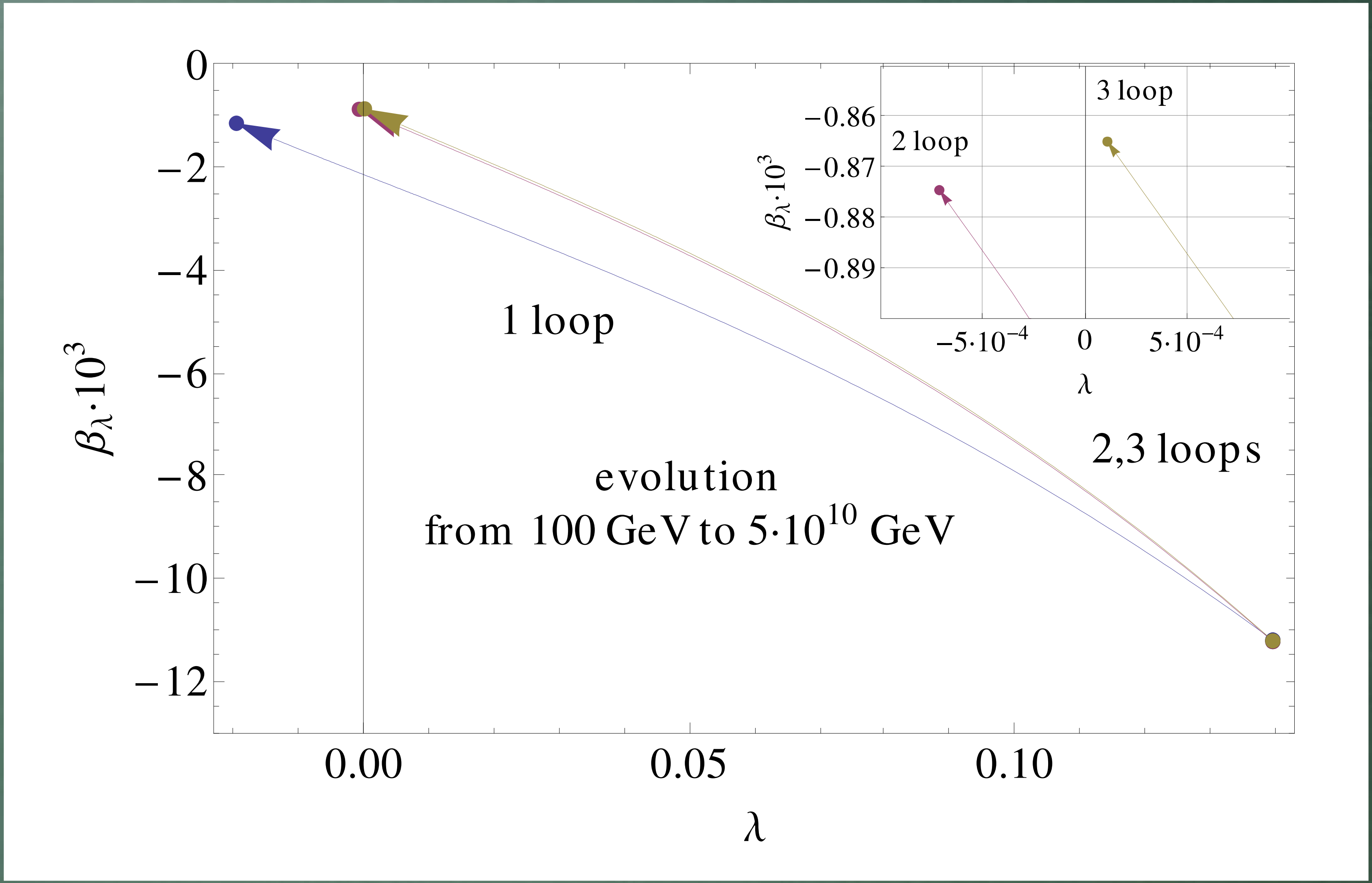
$$V_{eff}(h) \simeq \lambda(h) h^4/4$$

With $\lambda(\mu)$ being the running Higgs self-coupling

The self-coupling is a crucial parameter for the analysis of the SM vacuum stabitliy....

http://arxiv.org/src/1303.4364/anc

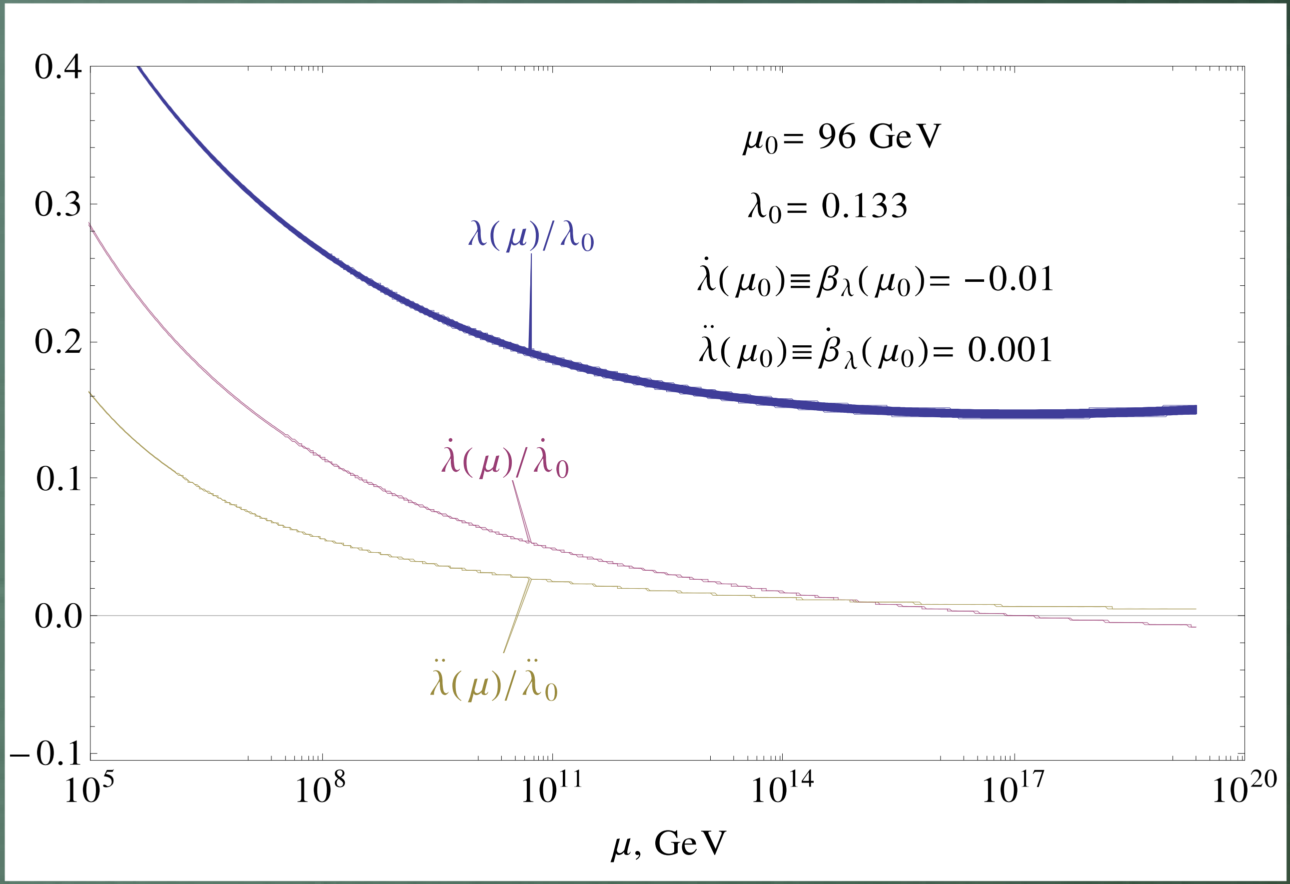
Results. Higgs self-coupling



http://arxiv.org/src/1303.4364/anc

But, again,...

Different implementation of matching procedure leads to different results...



Initial $\overline{MS}$ parameters from [Jegerlehner'13]

The issue should be clarified..

Threshold correction to Yukawa top!

Conclusions

- 3-loop beta-functions for all the fundamental parameters of the SM are obtained and a full agreement is found with

[Mihaila, Salomon, Steinhauser,'12]
[Chetyrkin, Zoller,'12-'13]

(3-loop Yukawa beta-functions - new result :)

2:1

Gauge and Higgs couplings

Yukawa couplings

NB: two different groups from Karlsruhe

Conclusions

- 3-loop beta-functions for all the fundamental parameters of the SM are obtained and a full agreement is found with

  [Mihaila, Salomon, Steinhauser,'12]
  [Chetyrkin, Zoller,'12-'13]

  (3-loop Yukawa beta-functions - new result :)

- A framework is established for calculation of three-loop RGEs within "arbitrary" QFT model
  (with the help of LanHEP/FeynRules)

Conclusions

- All the results can be found online as ancillary files of the arXiv versions of the corresponding papers...

- But: do not forget about another big problem: two-loop “matching”. There is a discrepancy in two different apporaches..

See., e.g., [Bezrukov, Kalmykov, Kniehl, Shaposhnikov,'12] and [Degrassi,Di Vita,Elias-Miro,Espinosa,Giudice,Isidori,Strumia,'12]

New results are on the way

We recalculated the RGEs with general complex Yukawa matrices which encompass general flavour structure of the SM

Higgs potential parameters – [1310.3806]
Yukawa matrices – to be published….

Plans*...

- Extract the RGEs for CKM matrix elements from the obtained results for Yukawa matrices
- Calculate and study leading four-loop contribution to the Higgs self-coupling beta-function
- Carry out a careful study of 2-loop matching conditions in the SM (the above-mentioned discrepancy)

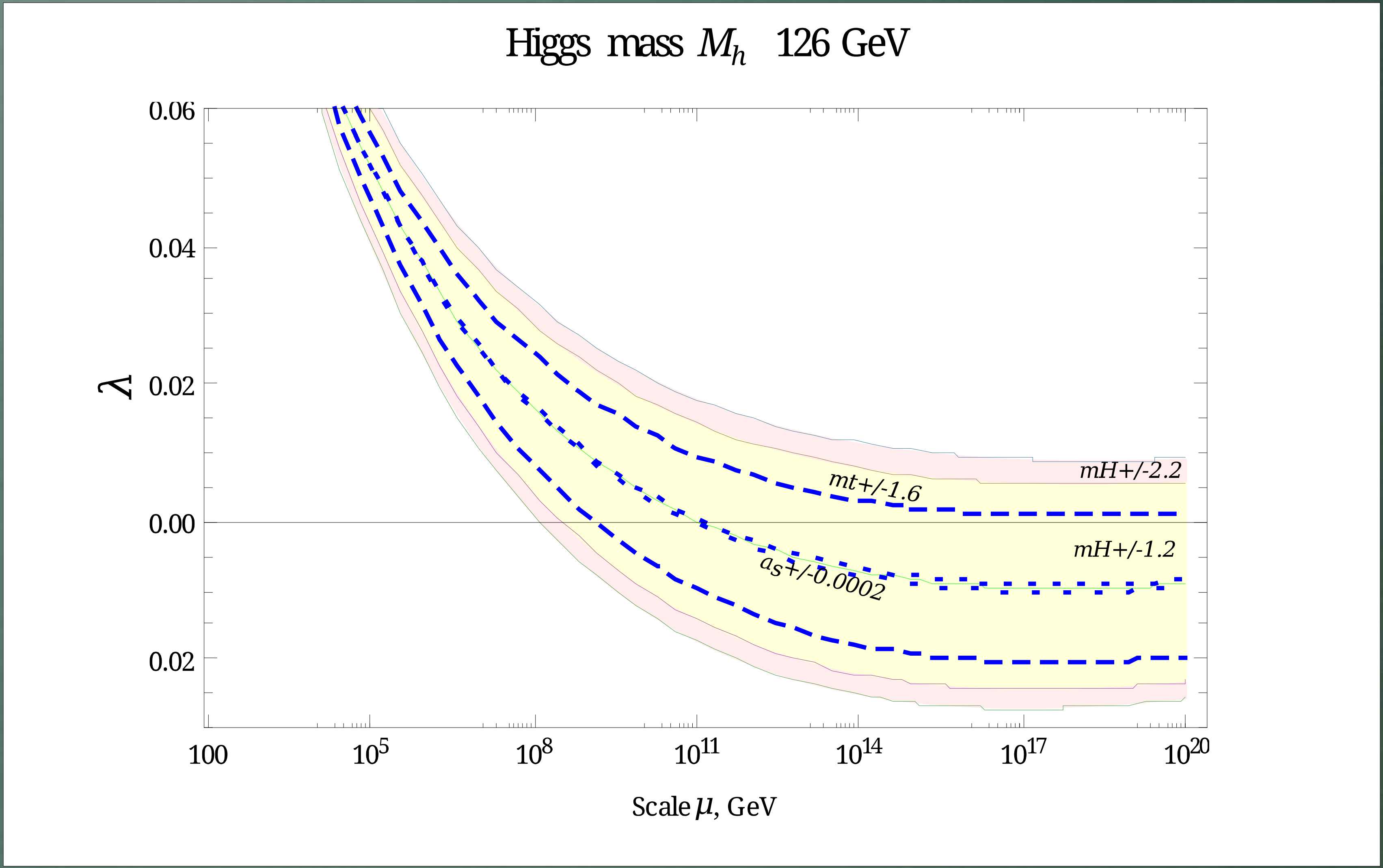
* either with or without the President's grant

Plans...

- Find four-loop gauge-coupling beta-function in non-supersymmetic QCD with with quarks, “gluinos” and “squarks” (a request from S. Mikhailov and A. Kataev)
- Evaluate four-loop SM gauge beta-functions to verify the predictions based on Weyl consistency conditions [Antipin et al,13]
- Calculate three-loop running of the parameters of the 2HDM and the corresponding two-loop matching...

Thank you for your attention!

Dependence on Mt



Results. Bottom Yukawa

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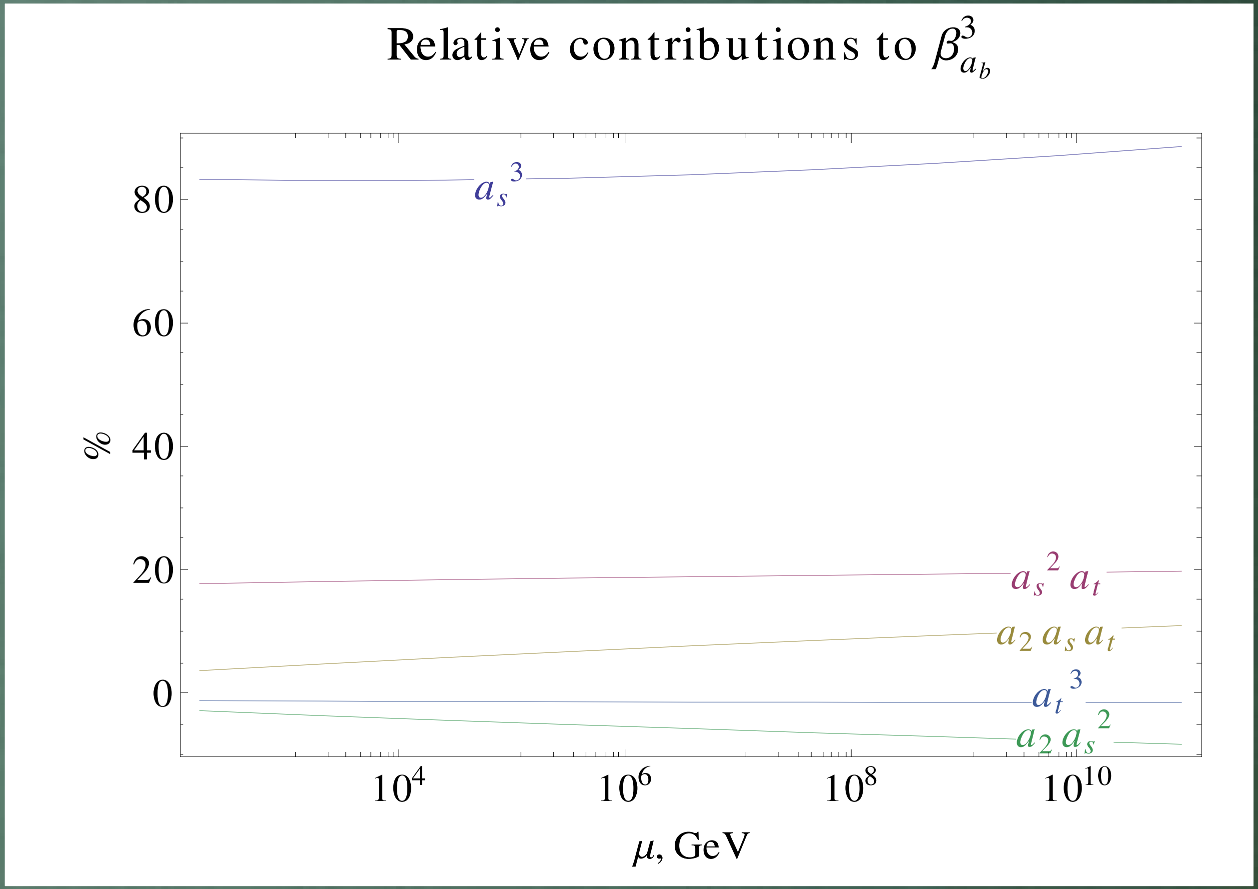

http://arxiv.org/src/1212.6829/anc

Results. Tau Yukawa

Initial values:

$$a_1 = 0.00134996$$
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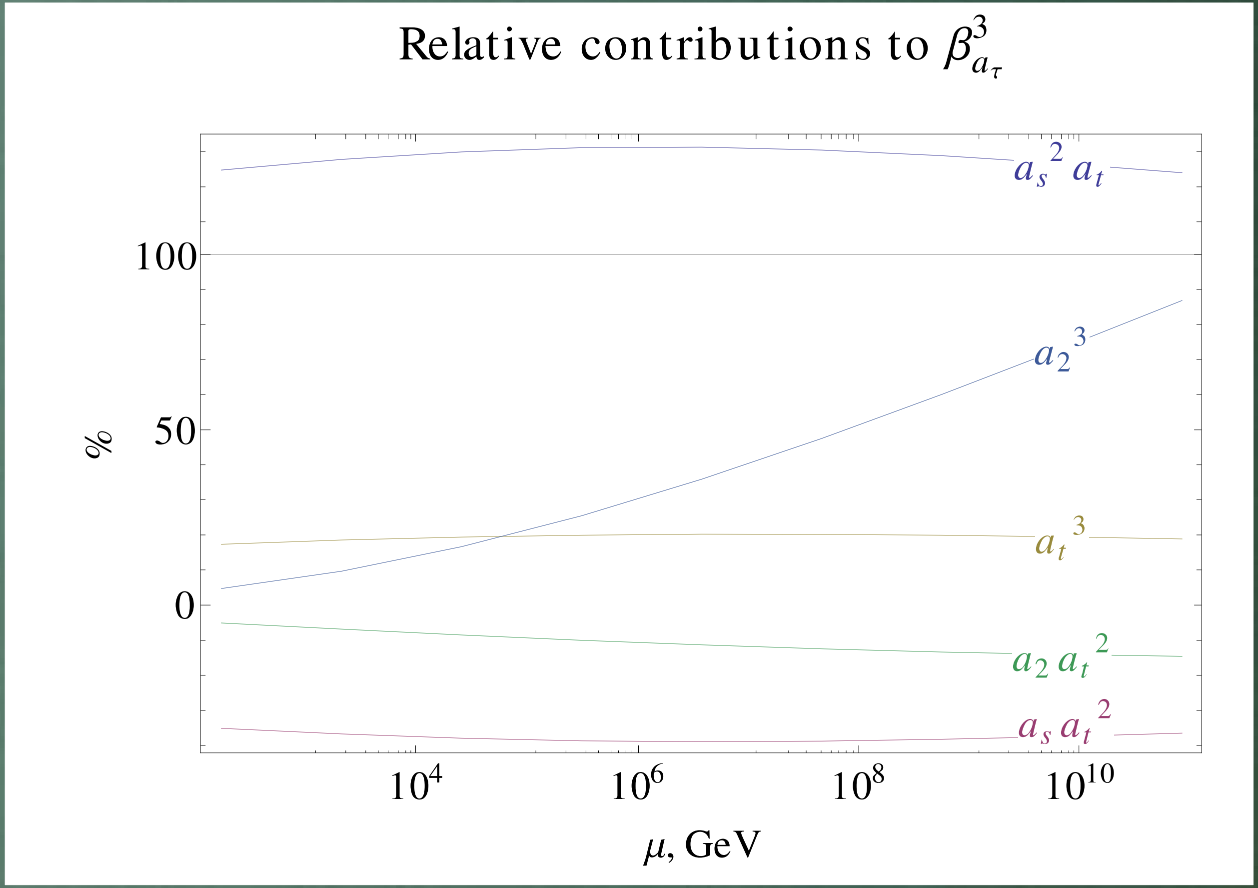

http://arxiv.org/src/1212.6829/anc