Pairing in nuclear and neutron star matter

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Outline

- Part I: Nonadiabtaic theory of nuclear superconductivity
  - Low density neutron matter: Effective potential models
  - Pion spectral functions
  - Off-shell pairing
- Part II: Neutrino production in NS matter
  - Neutrino rates in neutron star matter
  - Effects of superfluidity

The phenomenology

- Low density neutron matter
  - Free-space interactions are well constrained
  - Rotational properties of NS: glitches, precession
  - Cooling: ν radiation and specific heats
  - Stability: damping of oscillations by friction

- Schwinger-Dyson equations

$$G_{\alpha\beta} = G^0_{\alpha\gamma}\left[\delta_{\gamma\delta} + \Sigma_{\gamma\delta}G_{\delta\beta} + \Delta_{\gamma\delta}F^\dagger_{\delta\beta}\right] \tag{1}$$

$$F^\dagger_{\alpha\beta} = {}^{(-)}G^0_{\alpha\gamma}\left[\Delta^\dagger_{\gamma\delta}G_{\delta\beta} +^{(-)} \Sigma_{\gamma\delta}F^\dagger_{\delta\beta}\right] \tag{2}$$

- Matrix structure for finite temperature:

$$\begin{pmatrix} \underline{G}^c_{\alpha\beta}(x,x') & \underline{G}^<_{\alpha\beta}(x,x') \\ \underline{G}^>_{\alpha\beta}(x,x') & \underline{G}^a_{\alpha\beta}(x,x') \end{pmatrix} \tag{3}$$

Wave-function renormalization

- Normal self-energies: $\Sigma^R = \Sigma_S^R + \Sigma_A^R$

$$\xi^*(p) = \xi(p) + \Sigma_S^R(p) \quad (4)$$

- Wave-function renormalization:

$$Z(p) = 1 - \omega^{-1}\Sigma_A^R(\omega). \quad (5)$$

Solution of the Dyson equations

- Normal propagators:

$$G^R(p) = \frac{\omega Z(p) + \xi_p^*}{(\omega + i\eta)^2 Z(p)^2 - \xi_p^{*2} - \Delta^R(p)^2}, \tag{6}$$

- Anomalous propagators:

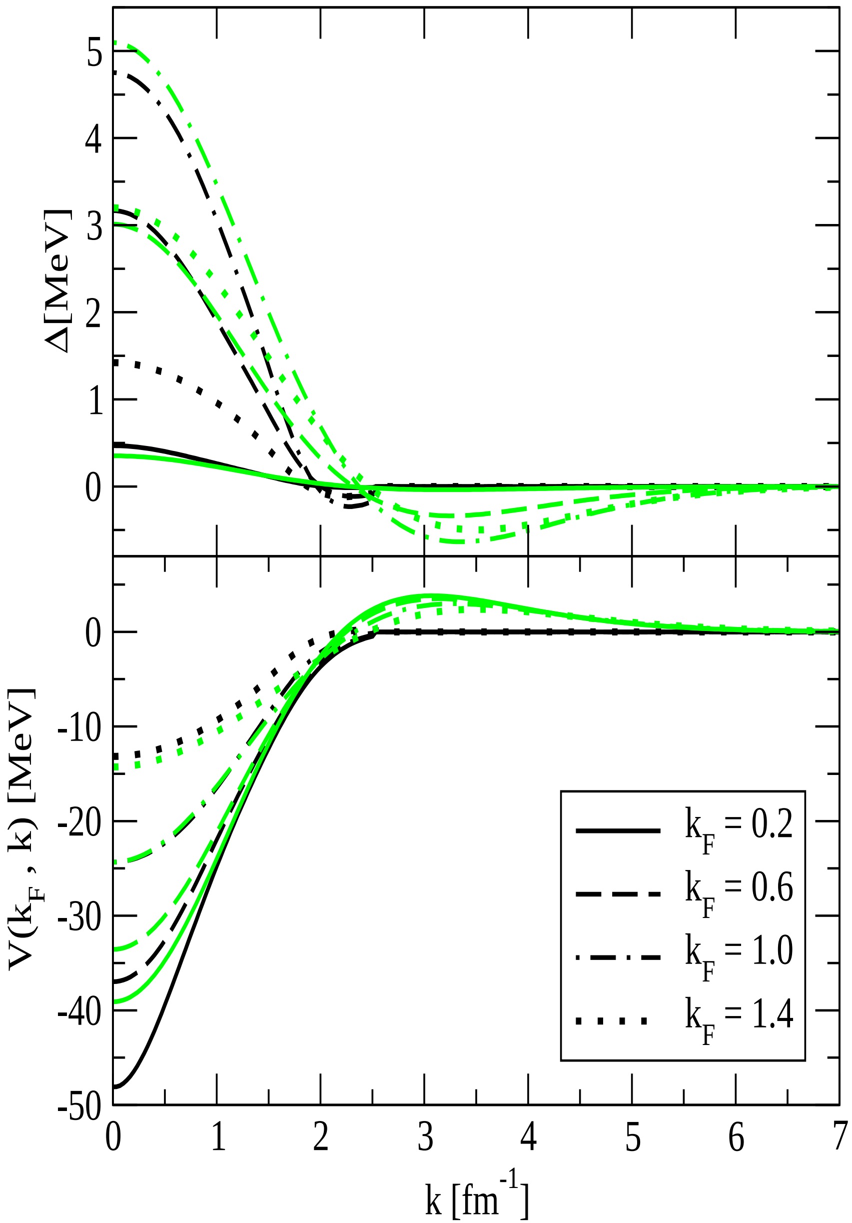
$$F^R(p) = -\frac{\Delta^R(p)}{(\omega + i\eta)^2 Z(p)^2 - \xi_p^{*2} - \Delta^R(p)^2}, \tag{7}$$

On-shell self-energies

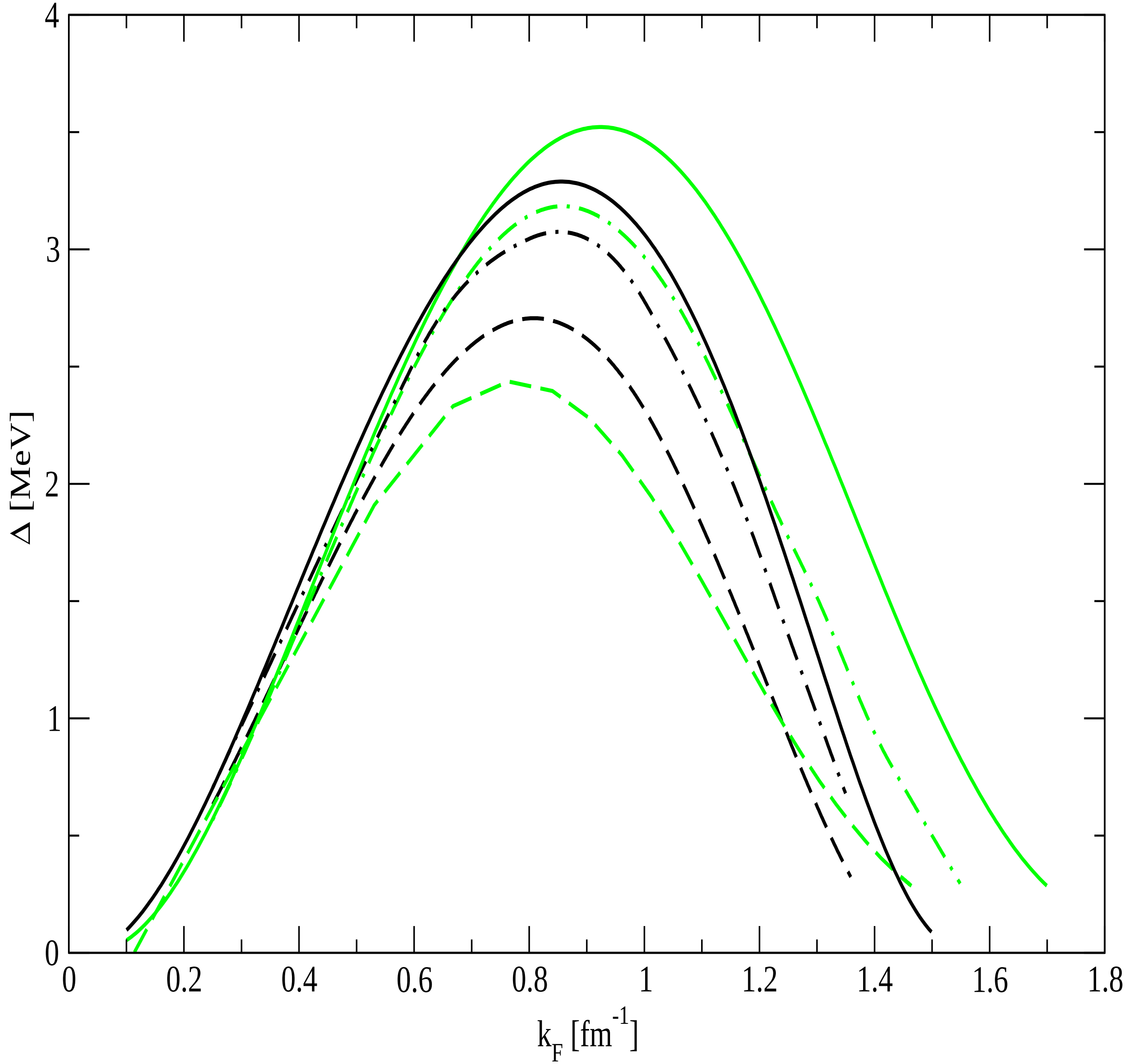
- Reduction to the BCS form

$$\Delta(\vec{p},\omega) = \int \frac{d^3p' d\omega'}{(2\pi)^4} \langle \vec{p},\omega|V|\vec{p}'\omega'\rangle \frac{\Delta(\vec{p}',\omega')}{\sqrt{\varepsilon(p')^2 + \Delta^2(\vec{p}',\omega')}}$$

- Approximate $\langle \vec{p},\omega|V|\vec{p}'\omega'\rangle \simeq \langle \vec{p}|V_{NN}|\vec{p}'\rangle$
  V_{NN} - bare or effective partial-wave interaction
- Compute ε_p (equivalently Σ_p) in the normal state
  $\varepsilon_p \neq \varepsilon_p(\Delta)$.



Δ[MeV]
5
4
3
2
1
0
V(k_F , k) [MeV]
0
-10
-20
-30
-40
-50
0
1
2
3
4
5
6
7
k [fm^-1]
k_F = 0.2
k_F = 0.6
k_F = 1.0
k_F = 1.4



4
3
2
1
0
Δ [MeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
k_F [fm^-1]

Beyond the BCS

- Improving upon the BCS approximation:
  - time-non-local interactions - gaps are then energy dependent
  - keep same approximations for normal and anomalous self-energies (Ward identities)
  - possible in medium modifications of the meson dynamics (e.g. precursor of pion condensation).

Modeling the interactions

At low densities separate the scales with respect to the inverse pion Compton wave-length ~ 1.4 fm.

- Long range contribution:

$$H_{\pi NN} = -\frac{f_\pi}{m_\pi}(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\boldsymbol{\tau}\cdot\boldsymbol{\phi}),$$

- Short-range contribution:

$$H_{NN,res} = \frac{f_\pi^2}{m_\pi^2}\Big[g'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + h'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, S_{12}(\boldsymbol{n})\Big]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

In-medium pions

- DSE for the full meson propagator:

$$\hat{D}(p) \;=\; \hat{D}_0(p) + \hat{D}_0(p)\hat{\Pi}(p)\hat{D}(p).$$

- Polarization tensor and the vertex:

$$\hat{\Pi}(q) \;=\; -\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\, i\,\hat{\Gamma}_0(q)\hat{G}(p+q)\hat{G}(-p)\hat{\Gamma}(q),$$

$$\hat{\Gamma}(q) \;=\; \hat{\Gamma}_0(q) + \mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\, i\,\hat{\Gamma}_1(q)\hat{G}(p+q)\hat{G}(-p)\hat{\Gamma}(q).$$

Spectral functions

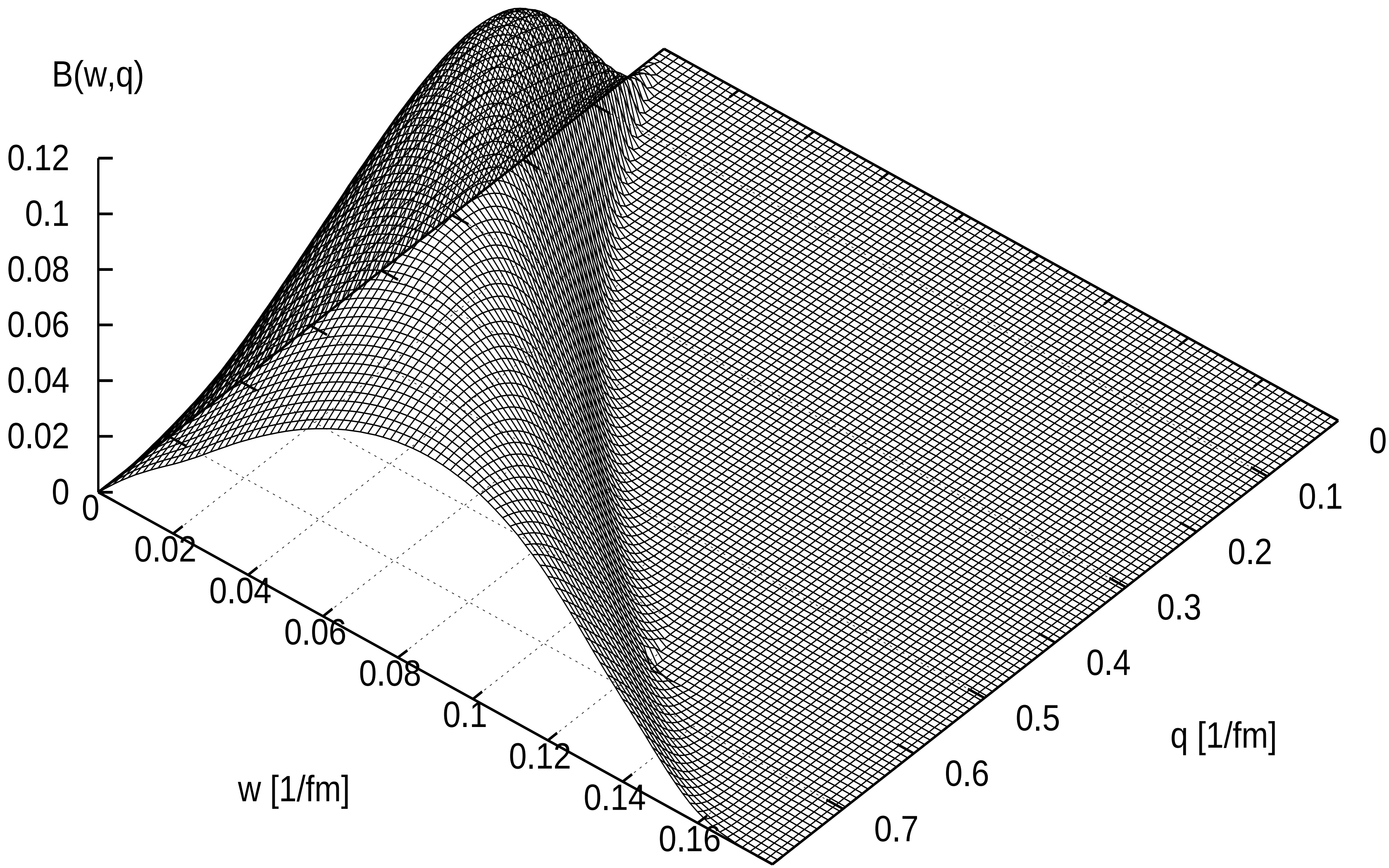
Fixed time correlation functions via the spectral function of pions and Bose occupation probabilities:

$$
\begin{aligned}
D^{<}(q) &= -iB(q)g(\omega) - iB(-q)[1+g(-\omega)], \\
D^{>}(q) &= -iB(q)[1+g(\omega)] - iB(-q)g(-\omega),
\end{aligned}
$$

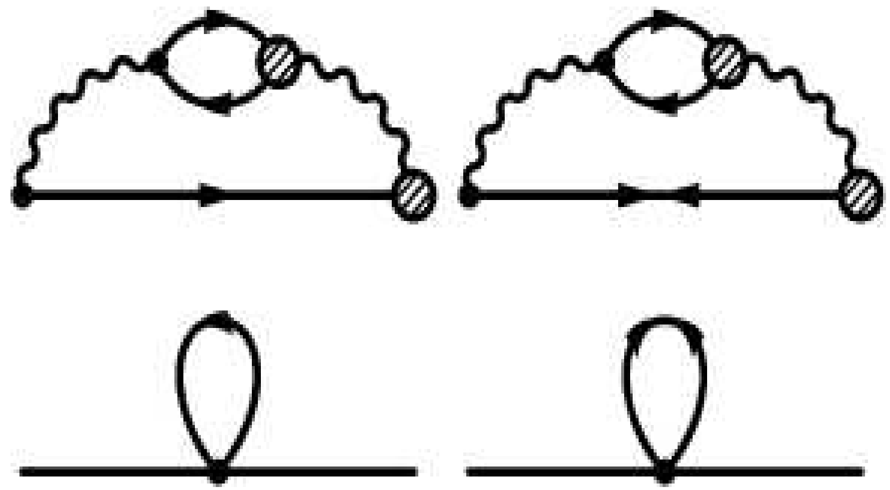
Pion spectral function:

$$
B(q) = \frac{-2\mathrm{Im}\Pi^R(q)}{[\omega^2 - \vec{q}^2 - m_\pi^2 - \mathrm{Re}\Pi^R(q)]^2 + [\mathrm{Im}\Pi^R(q)]^2}.
$$

Pion spectral function



Neutron (baryon) self-energies



$$
\begin{aligned}
H &= -\frac{f_\pi}{m_\pi}(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\boldsymbol{\tau}\cdot\boldsymbol{\phi}), \\
&+ \frac{f_\pi^2}{m_\pi^2}\Big[g'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + h'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, S_{12}(\boldsymbol{n})\Big]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,
\end{aligned}
$$

Fock (retarded) self-energies

$$\Sigma^R(\omega,\vec{p}) \;=\; \mathrm{Tr}\int\frac{d^3q d\varepsilon}{(2\pi)^4}\Gamma_0(\vec{q})A_G(\varepsilon,\vec{p}-\vec{q})C(\omega,\varepsilon,\vec{q})\Gamma(\vec{q}),$$

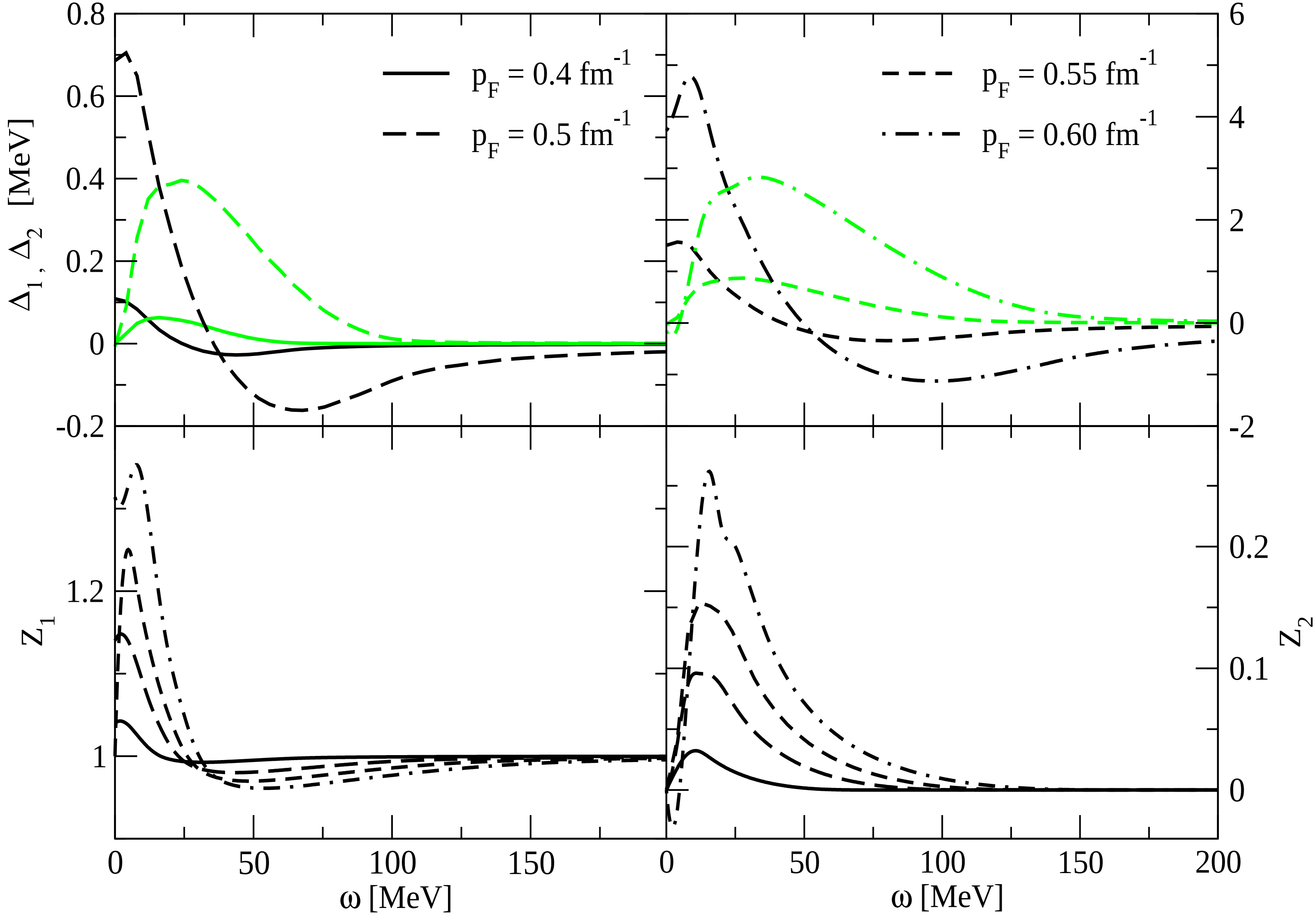
$$\Delta^R(\omega,\vec{p}) \;=\; \mathrm{Tr}\int\frac{d^3q d\varepsilon}{(2\pi)^4}\Gamma_0(\vec{q})A_F(\varepsilon,\vec{p}-\vec{q})C(\omega,\varepsilon,\vec{q})\Gamma(\vec{q}),$$

where

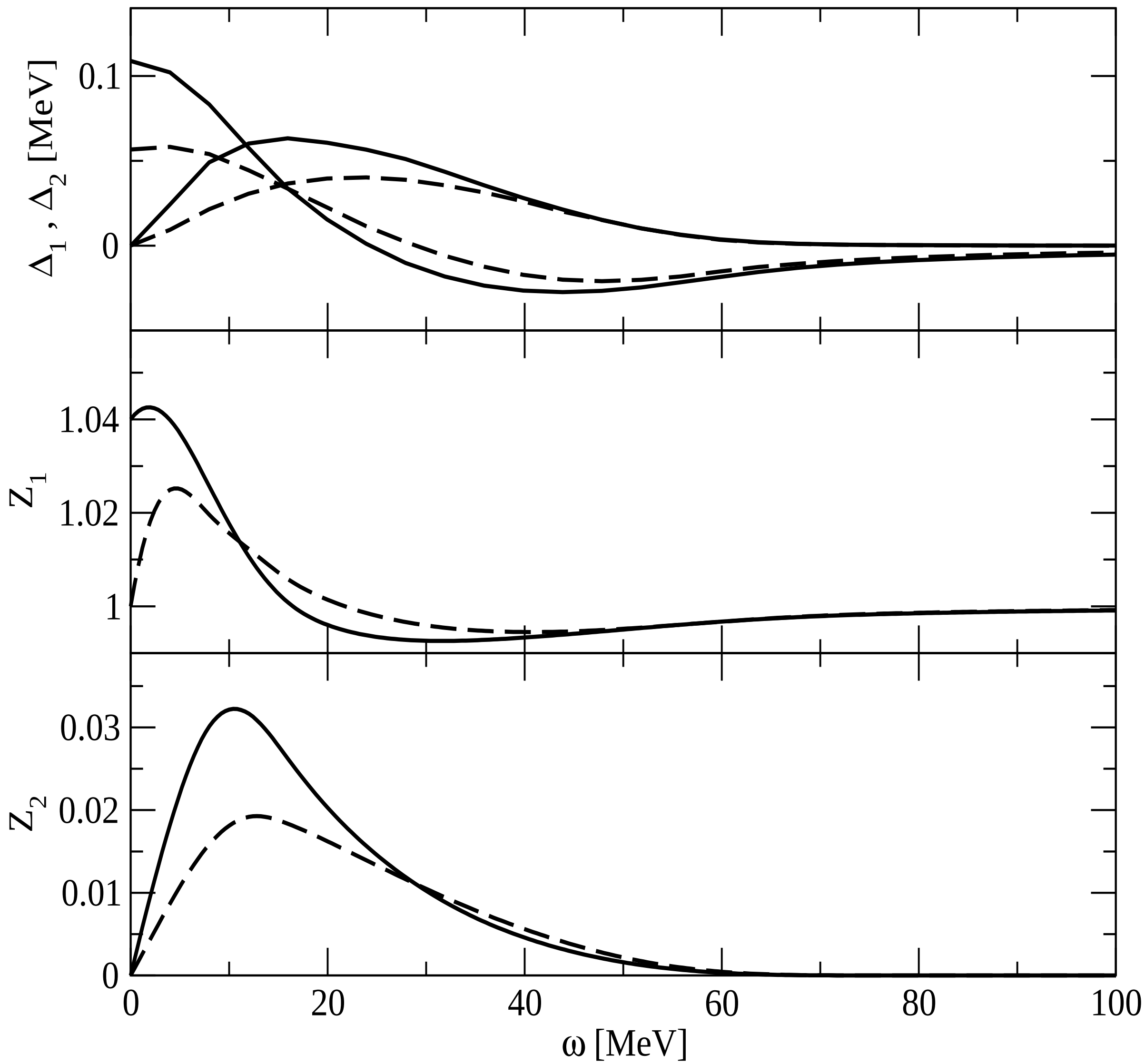
$$C=\int_0^\infty\frac{d\omega'}{2\pi}B(\omega',\vec{q})\left[\frac{f(\varepsilon)+g(\omega')}{\varepsilon-\omega'-\omega-i\eta}+\frac{1-f(\varepsilon)+g(\omega')}{\varepsilon+\omega'-\omega-i\eta}\right].$$

SOLVING NUMERICALLY

- Need to solve self-consistently 4 integral equations (= 2 complex equation for the normal and anomalous self-energies).
- The kernels of these integral equations are singular either at the boundary or within the integration range.
- A 2d mesh in the energy range 0-400 MeV with 200 $\times$ 200 mesh points
- Iterative procedure; good convergence after 15-20 iterations.



0.8
0.6
0.4
0.2
0
-0.2
Δ_1, Δ_2 [MeV]
p_F = 0.4 fm^{-1}
p_F = 0.5 fm^{-1}
p_F = 0.55 fm^{-1}
p_F = 0.60 fm^{-1}
6
4
2
0
-2
Z_1
1.2
1
0.2
0.1
0
Z_2
0
50
100
150
0
50
100
150
200
ω [MeV]
ω [MeV]



0.1
0
Δ_1, Δ_2 [MeV]
1.04
1.02
1
Z_1
0.03
0.02
0.01
0
Z_2
0
20
40
60
80
100
ω [MeV]

Summary: Part I

- BCS would predict constant real gap in the entire off-shell energy range.
- Nonadiabatic superconductivity model:
  - The complex gap is a complicated function of frequency; the pair correlations are damped above several MeV and the damping exceeds the eigen-frequency at $\omega \geq 10$ MeV
  - The wave-function renormalization (the so-called E-mass) is in enhanced (≥ 1) off the mass-shell and is damped.

Classifying reactions...

Processes on fermions

- Neutral current processes (Z_0 exchange)

$$\begin{cases} f_1 \longrightarrow f_2 + \nu_f + \bar{\nu}_f & \text{(brems)} \\ f_1 + f_1' \longrightarrow f_2 + f_2' + \nu_f + \bar{\nu}_f \end{cases} \tag{8}$$

- Charged current processes ($W^\pm$ exchange)

$$\begin{cases} f_1 \longrightarrow f_2 + e + \bar{\nu}_e & \text{(Urca)} \\ f_1 + f_1' \longrightarrow f_2 + f_2' + e + \bar{\nu}_e \end{cases} \tag{9}$$

Transport equations

- ν and $\bar{\nu}$ - Boltzmann equations

$$\left[\partial_t + \vec{\partial}_q\,\omega_\nu(\vec{q})\vec{\partial}_x\right] f_\nu(\vec{q},x)$$
$$= \int_0^\infty \frac{dq_0}{2\pi}\mathrm{Tr}\left[\Omega^<(q,x)S_0^>(q,x) - \Omega^>(q,x)S_0^<(q,x)\right],$$

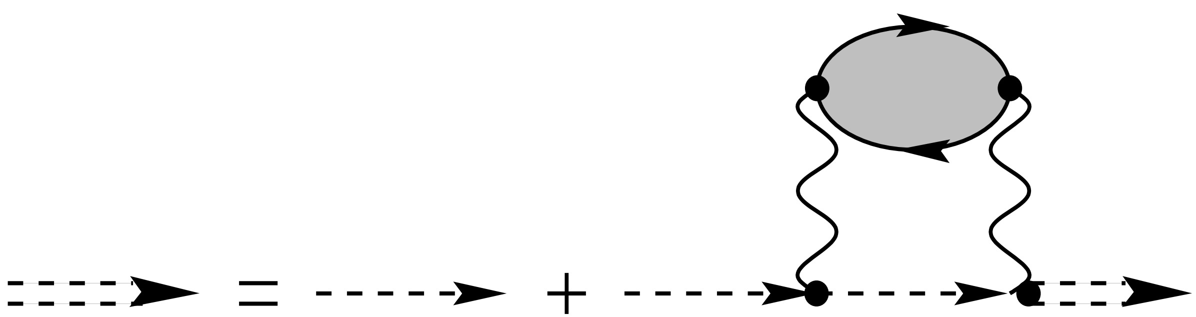
- ν-quasiparticle propagators:

$$S_0^<(q,x) = \frac{i\pi\,\not{q}}{\omega_\nu(\vec{q})}\Big[\delta\left(q_0-\omega_\nu(\vec{q})\right) f_\nu(q,x) - \delta\left(q_0+\omega_\nu(\vec{q})\right)\left(1-f_{\bar{\nu}}(-q,x)\right)\Big]. \quad (10)$$

Self-energies

- ν and $\bar{\nu}$-self-energies (second order in weak force)

$$-i\Omega^{>,<}(q_1,x) = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1-q_2-q)$$
$$i\Gamma^{\mu}_{L\,q}\, iS_0^{<}(q_2,x) i\Gamma^{\dagger\,\lambda}_{L\,q} i\Pi^{>,<}_{\mu\lambda}(q,x), \quad (11)$$

- 

Bremsstrahlung emissivity

- energy loss per unit time and volume

$$\epsilon_{\nu\bar{\nu}} = \frac{d}{dt}\int \frac{d^3q}{(2\pi)^3}\left[f_\nu(\vec{q}) + f_{\bar{\nu}}(\vec{q})\right]\omega_\nu(\vec{q}) \qquad (12)$$

- expressed through the collision integrals

$$\epsilon_{\nu\bar{\nu}} = -2\left(\frac{G}{2\sqrt{2}}\right)^2 \sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\vec{q}_2)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(\vec{q}_1)} \int \frac{d^4q}{(2\pi)^4}$$
$$(2\pi)^4\delta^3(\vec{q}_1 + \vec{q}_2 - \vec{q})\delta(\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2) - q_0)\ \left[\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2)\right]$$
$$g_B(q_0)\left[1 - f_\nu(\omega_\nu(\vec{q}_1))\right]\left[1 - f_{\bar{\nu}}(\omega_\nu(\vec{q}_2))\right]\Lambda^{\mu\lambda}(q_1, q_2)\Im\mathrm{m}\,\Pi^R_{\mu\lambda}(q).$$

Urca emissivity

- energy loss per unit time and volume

$$\epsilon_{Urca} = \frac{d}{dt}\int \frac{d^3q}{(2\pi)^3}\left[f_{\bar\nu}(\vec{q})\right]\omega_\nu(\vec{q}) \tag{13}$$

- expressed through the collision integrals

$$\begin{aligned}\epsilon_{\bar\nu} = & -2\left(\frac{\tilde G}{\sqrt{2}}\right)^2 \int \frac{d^3q_1}{(2\pi)^3 2\omega_e(\boldsymbol{q}_1)} \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\boldsymbol{q}_2)} \\ & \int d^4q\,\delta(\boldsymbol{q}_1+\boldsymbol{q}_2-\boldsymbol{q})\delta(\omega_e+\omega_\nu-q_0)\,\omega_\nu(\boldsymbol{q}_2) \\ & g_B(q_0)\left[1-f_e(\omega_e)\right]\Lambda^{\mu\zeta}(q_1,q_2)\Im\mathrm{m}\,\Pi^R_{\mu\zeta}(q),\end{aligned} \tag{14}$$

The direct Urca process

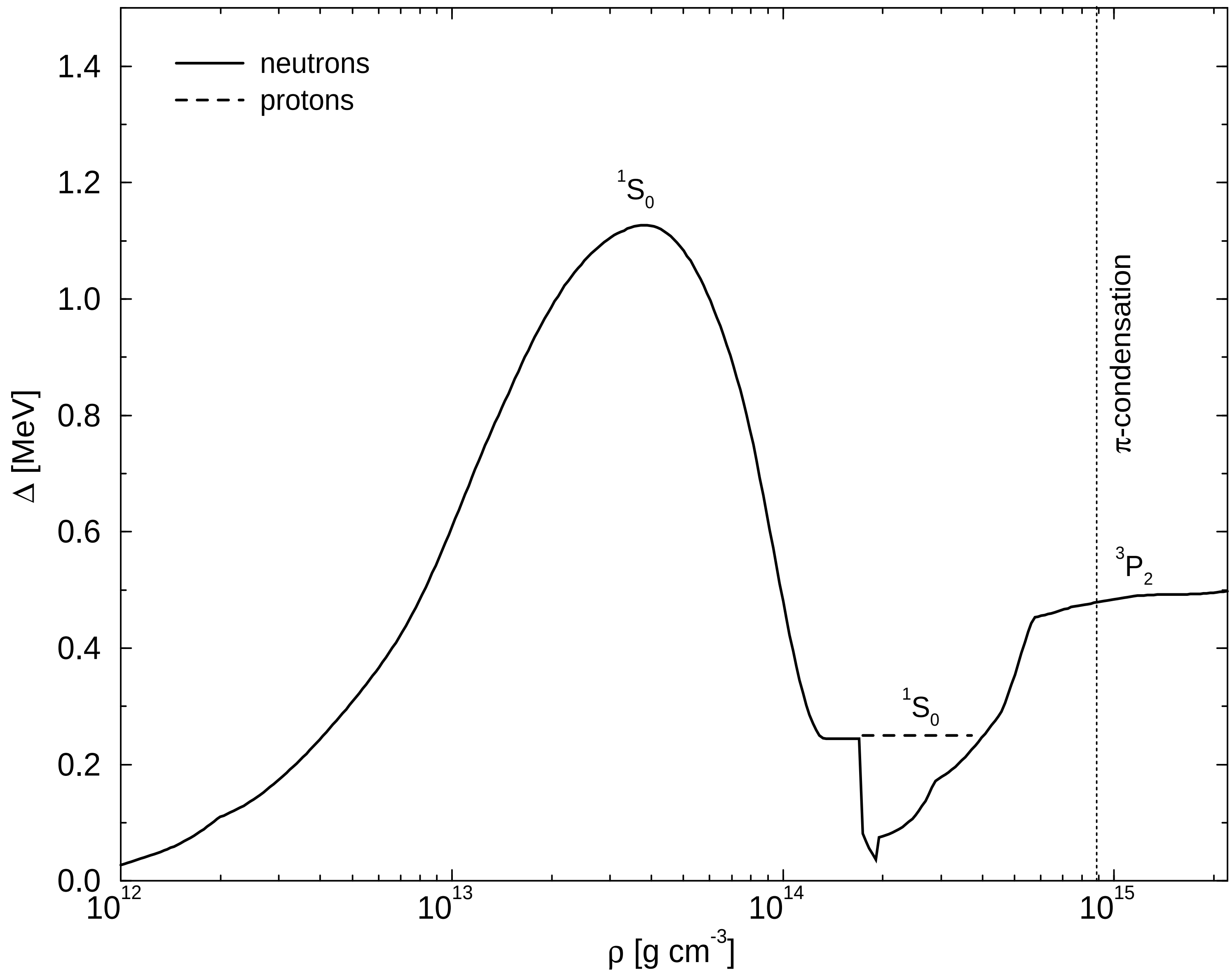
Simplest charge-current process is the β-decay

$$n \to p + e^- + \bar{\nu} \qquad\qquad e^- + p \to n + \nu \tag{15}$$

Direct Urca is forbidden by kinematics if the matter is strongly asymmetric, for proton fractions $x_p \geq 11 - 13\%$

$$\epsilon_{\bar{\nu}} = (1 + 3g_A^2)\frac{3\tilde{G}^2 m_n^* m_p^* p_{Fe}}{2\pi^5\beta^6}\int dy\; g_B(y)\,\ln\frac{1+e^{-x_{\min}}}{1+e^{-(x_{\min}+y)}}$$
$$\times \int dzz^3 f_e(z-y) \simeq 10^{26} \times T^6 \text{ erg cm}^{-3}\text{ s}^{-1}, \tag{16}$$

Effects of pairing on direct Urca process

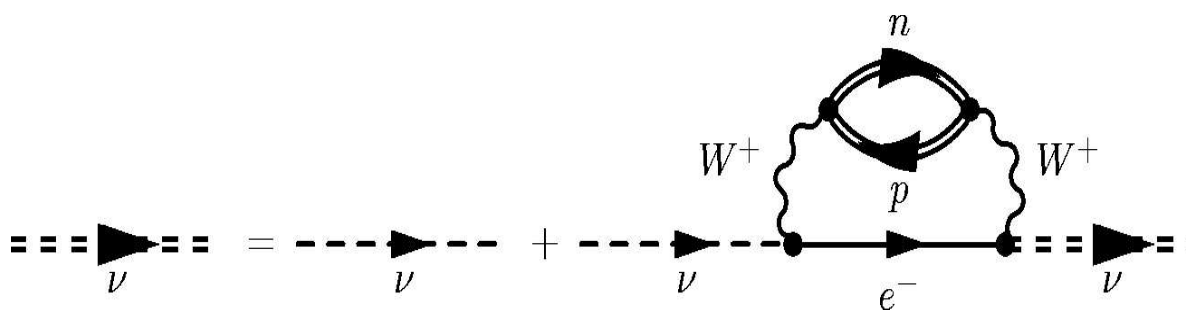


Effects of pairing on direct Urca process

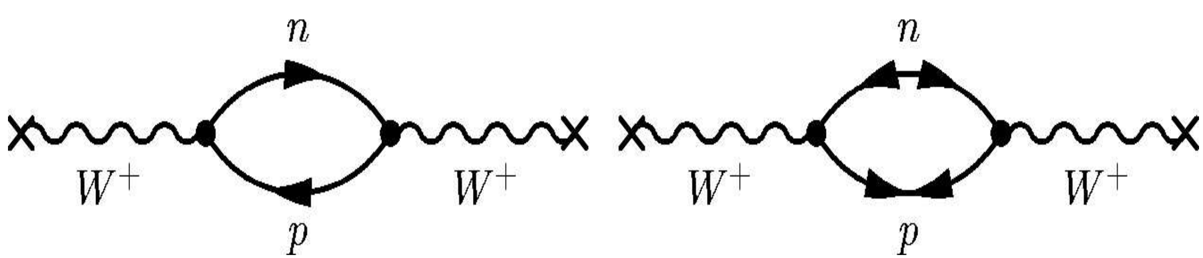
Naive picture prescribes a suppression of the Urca process by the pairing gap [$\Delta_{\max} = \Delta_n,\ \Delta_p$]

$$\epsilon_{\bar{\nu}} \rightarrow \epsilon_{\bar{\nu}} \times \exp\left(-\frac{\Delta_{\max}}{T}\right). \tag{17}$$

A more systematic way ...



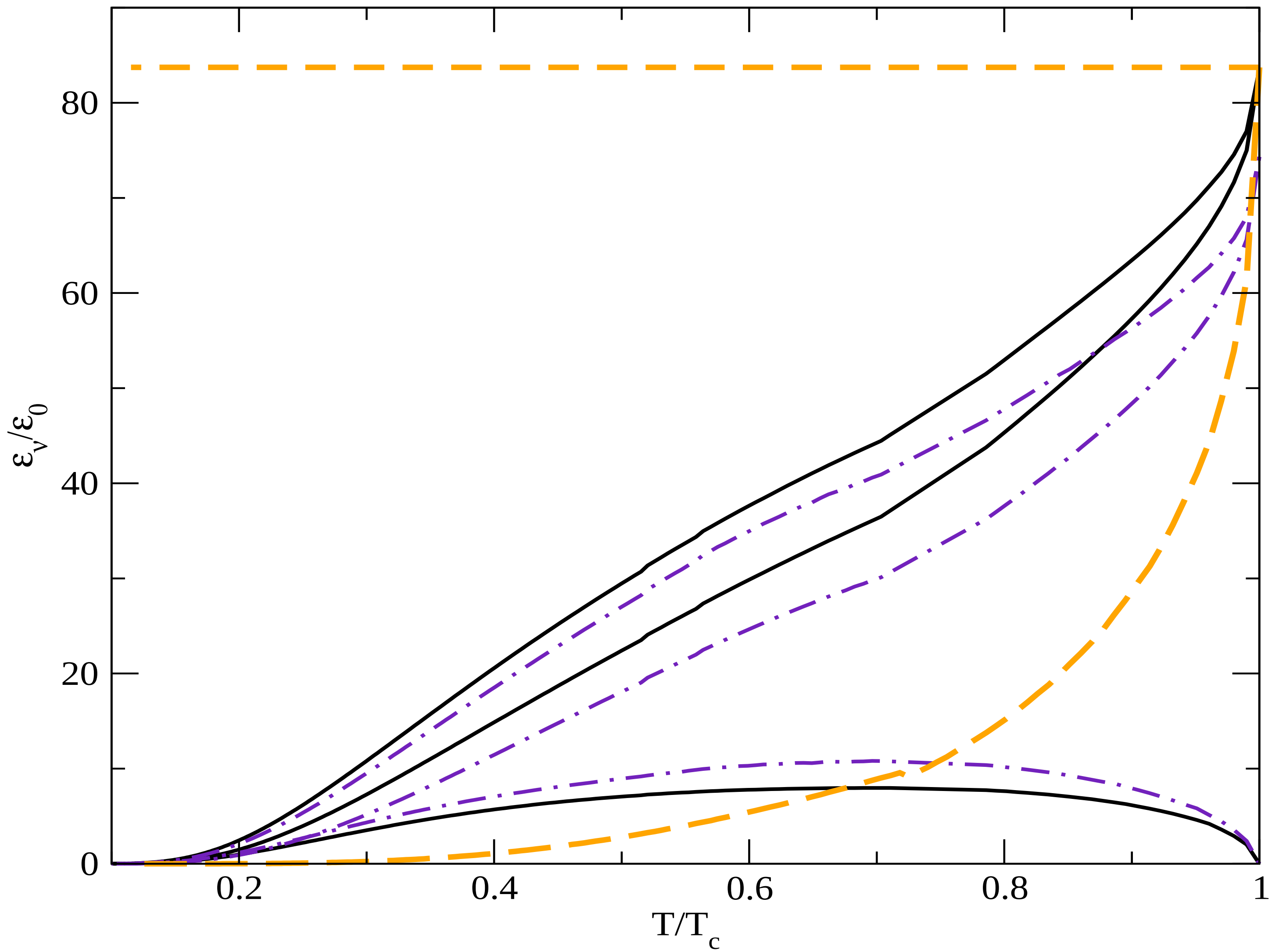
Polarization tensor at one loop



$$\Pi^R_{V/A}(\boldsymbol{q},\omega) = \sum_{\sigma,\vec{p}} \left\{ \left(\frac{u_p^2 u_k^2}{\omega + \varepsilon_p - \varepsilon_k + i\delta} - \frac{v_p^2 v_k^2}{\omega - \varepsilon_p + \varepsilon_k + i\delta} \right) [f(\varepsilon_p) - f(\varepsilon_k)] \right.$$

$$\left. + \left(\frac{u_p^2 v_k^2}{\omega - \varepsilon_p - \varepsilon_k + i\delta} \right) [1 - f(\varepsilon_p) - f(\varepsilon_k)] \right\}, \quad (18)$$

One-loop vs naive suppression



Pair-breaking contribution