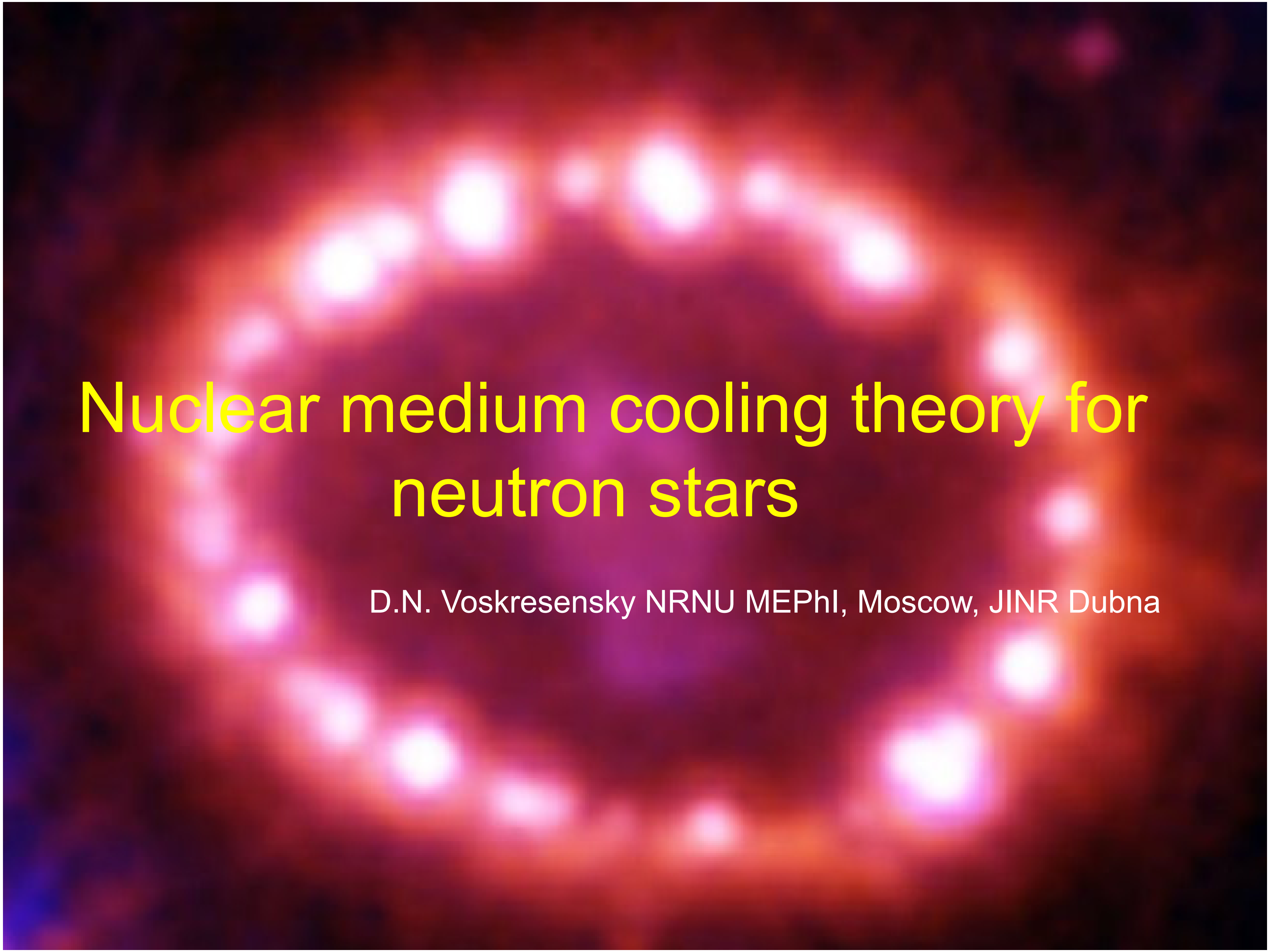


# Nuclear medium cooling theory for neutron stars

D.N. Voskresensky NRNU MEPhI, Moscow, JINR Dubna

# Plan

- Preliminaries: Historical remarks, general information on NS (for pedestrians)

**Cooling of NS**

- "Standard" scenario. "Minimal" cooling. Neutrino reaction rates in normal nuclear matter
- in superfluid nuclear matter
- "Nuclear medium cooling" scenario. Medium effects on reaction rates in normal nuclear matter
- in superfluid nuclear matter

# Preliminaries

**To describe neutron stars and their evolution:**
**we have to construct**
**a bridge between micro and macro worlds**

**20 orders in distance:** from $\sim 10^{-13}$ *cm* to $\sim 10^{3}$ *km*

**15 orders in density:** from mean solar density $\sim g/cm^3$ to $\sim 10^{15}$ $g/cm^3$

**40 orders in time:** from $10^{-23}$ *s* to $\sim 10^{10}$ *years*

**60 orders in mass/energy:** from $m_\pi \sim 10^{-25}$ *g* to $10^{33}$ *g*

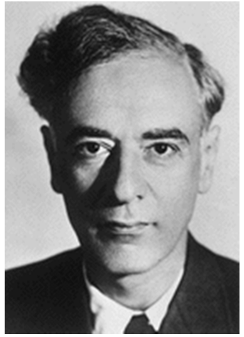
Strong,
electromagnetic,
weak and
gravitational interactions are involved

**Triumph of Universality of Physical Laws!**

# Preliminaries to discovery of NS

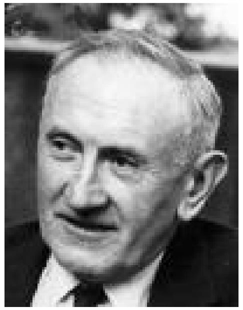
- 1932 Discovery of "$n$" by Chadwick.

From recollections of L. Rosenfeld 1974 (see in Shapiro, Teukolsky book ): next day after discovery of neutron during discussion in NBI seminar Landau suggests possibility of NS



From Yakovlev, Haensel, Baym, Pethick (2013)

**Landau was not in NBI in 1932. In February—March 1931, in Copenhagen, one year before the discovery of the neutron, Landau, Bohr and Rosenfeld discussed a not published paper written by Landau about a possible existence of very dense stars, where atomic nuclei form one giant nucleus. Landau published this paper one year later, in February 1932, the same month when the discovery of the neutron was announced.**
**Landau did not predict neutron stars, only superficially anticipated them (as only a genius could do) !!!**

Baade и Zwicky @ Stanford Meeting, **15-16 Dec. 1933**
**suggest a possibility of existence of NS and their connection to supernovae**

*"…**supernovae** represent the transitions from ordinary stars into **neutron stars**, which in their final stages consist of extremely closely packed neutrons…"*

- 1939 J. Oppenheimer, G. Wolkoff, Phys. Rev., considered ideal $n$ gas in grav. field.

**Why NS is compact?**

(Stability: balance of repulsion and attraction)

$$E_{Fn} \sim \frac{p_{Fn}^2}{2m_n} A, \quad E_G \sim -\frac{GM^2}{R}$$

$$p_{Fn} = \hbar(3\pi^2 \rho_n)^{1/3}, \quad \rho_n = \frac{3A}{4\pi R^3}, \quad M \simeq A m_n$$

$$\rightarrow p_{Fn} \sim \hbar A^{1/3}/R \rightarrow$$

$$R \sim \hbar^2/(GM^{1/3} m_n m_n^{5/3}) \sim 10 km \quad for \quad M \sim M_\odot,$$

Quantum object!

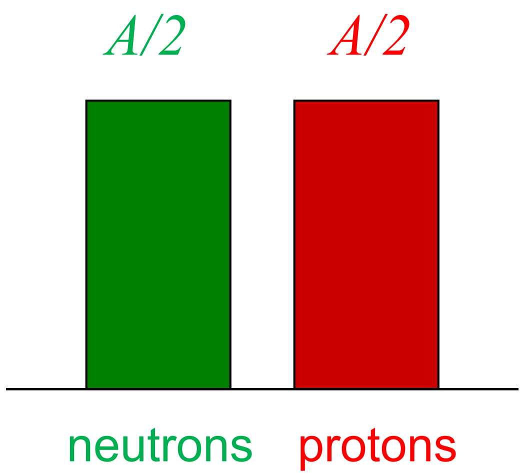
In reality nucleon gas is not ideal. Ignoring electric forces:
nuclear forces are isospin symmetric

➡ Symmetry energy is positive

+ Pauli principle

Distribution of $A$ nucleons


$$E(A, Z = 0) > E(A, Z = A/2)$$




Filling of two Fermi seas is energetically favorable !

## Why NS is of neutrons?

$$E_{Fn} \sim \frac{p_{Fn}^2}{2m_n} A \propto A \quad at \;\; fixed \quad \rho$$

$$E_{sym} \sim (N-Z)^2/A \sim A \quad for \quad N \sim A, \qquad but$$

$$E_{Coul} \sim Z^2 e^2/R \sim Z^2 A^{-1/3} > A \quad if \quad Z \sim A.$$

$\rightarrow$ Profitable to suppress $Z$. How much?

## Composition is determined by the minimum of energy

**Symmetry energy**: $= \bar{a}_{\mathrm{Sym}}\,(n_n - n_p)^2/n \longrightarrow$ *It is favorable to have some protons.*

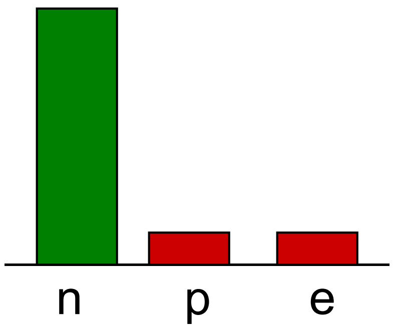
**Electroneutrality**: $n_e = n_p \longrightarrow$ *There will be also some electrons*

**Electron energy** (ultrarel. Fermi gas):

$$\varepsilon_e = 2\int_0^{p_{\mathrm{F}\,e}} \frac{d^3p}{(2\,\pi)^3}\,E_e(p) = \frac{p_{\mathrm{F}\,e}^4}{4\,\pi^2} = \frac{3}{4}\,(3\,\pi^2)^{1/3}\,n_e^{4/3}$$

$\longrightarrow$ *Electron energy increases fast*

Energy minimization: *Mainly neutrons and small admixture of protons and electrons*



- **1930** Z →Z+1 +e +? Energy dis-balance!

N.Bohr suggests energy non-conservation in micro world → problem with relativity.
W.Pauli suggests $\nu$: $n \to p + e + \bar{\nu}$.

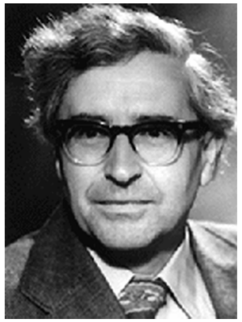
(Originally Pauli called it neutron but it was renamed by Fermi in 1932 as neutrino since Chadwick has used the term ``neutron" for n discovered by him)

- **1934** Fermi, Phenomenological theory of weak interaction

## Period of courageous theorists

- **1942** W. Baade et al, suggested NS in Crab Nebula. (Was observed in 1968.)
- **1959** A.B. Migdal suggests $1S_0$ $nn$ pairing in NS. (Now pulsar glitches are associated with superfluidity, cooling)
- **1960** B. Ambarzumian, G. Saakian suggest hyperonization in NS interiors. $n \leftrightarrow \Lambda$, for $\mu_n \simeq E_{Fn} > m_\Lambda - m_n$. (Now hyperonization at $\rho \gtrsim (2 \div 4)\rho_0$ is discussed including interactions.)
- **1965** S. Tsuruta, A. Cameron, and J. Bahcall, R. Wolf: First scenario for NS cooling.
- **1967** F. Pacini, considered emission of rapidly rotating NS (explains nature of future pulsars)

## Experimental Era



- **08.1967** S.J. Bell discovered radio pulsar "LGM1" or "CP". "Little Green Men" LGM1 or Cambridge Pulsar "CP". Precise cosmic clock! Was unpublished during 6 months: May be signal of out of Earth civilization?
- **02. 1968** A. Hewish, S.J. Bell, et al, Nature.
  Did not know work of Pacini and thought about white dwarf. Electron gas pressure + gravity:

  $$R \sim 10^3 km,$$

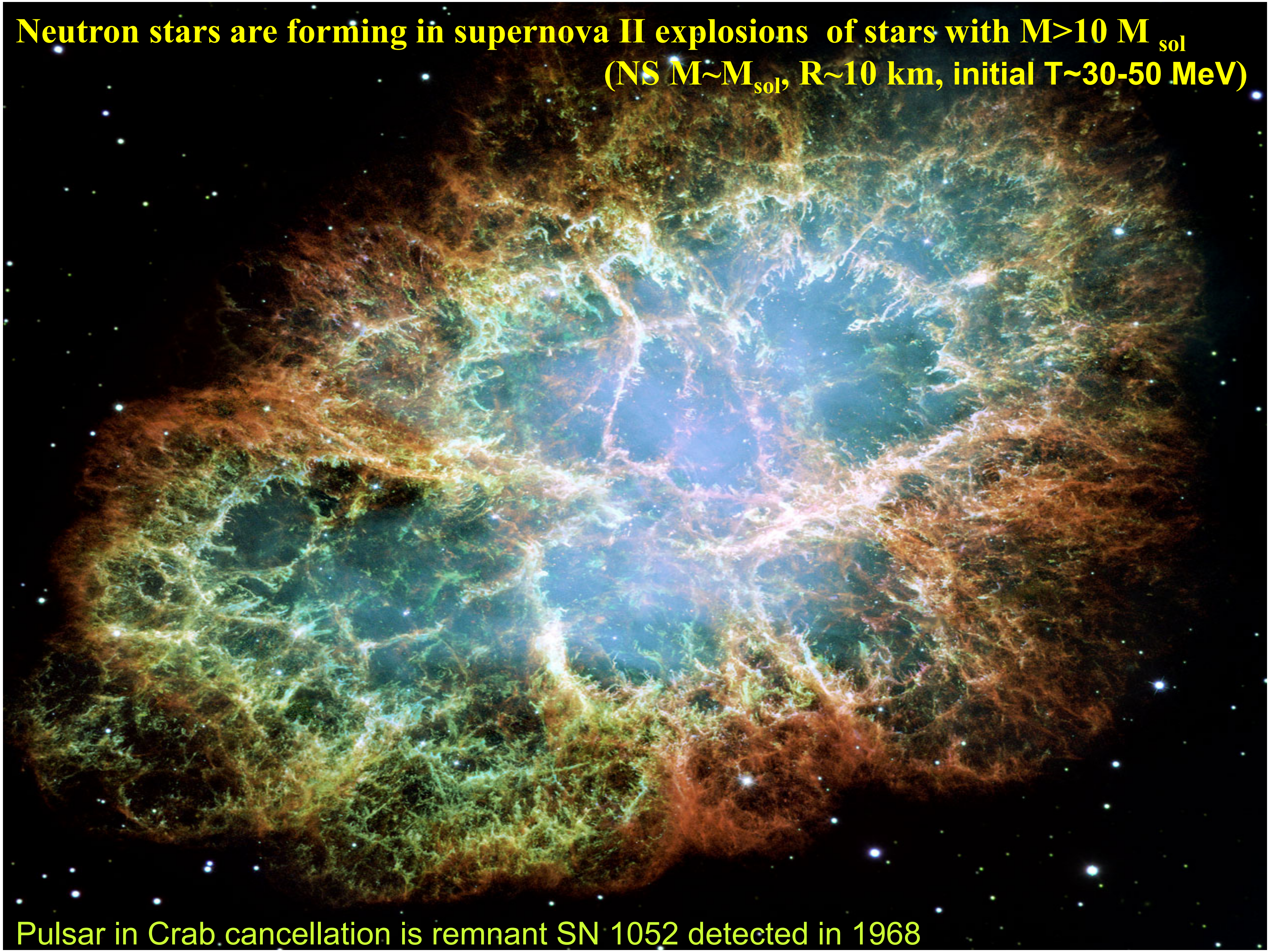
  $P \sim 1s$ can be explained:

  $$\frac{mv^2}{R} < \frac{GMm}{R^2}, \quad v = \frac{2\pi R}{P} \to R < (GM)^{1/3}(\frac{P}{2\pi})^{2/3}.$$

- **06. 1968** T. Gold Nature 218, F. Pacini Nature 219, suggested that pulsars are rapidly rotating NS.
- **end 1968** Discovery of pulsars in Vela ($P = 88ms$) and Crab ($P = 33\,ms$). Clearly NS, connection to SN

## Neutron stars are forming in supernova II explosions of stars with $M>10\ M_{sol}$

**(NS $M \sim M_{sol}$, $R \sim 10$ km, initial $T \sim 30$-$50$ MeV)**



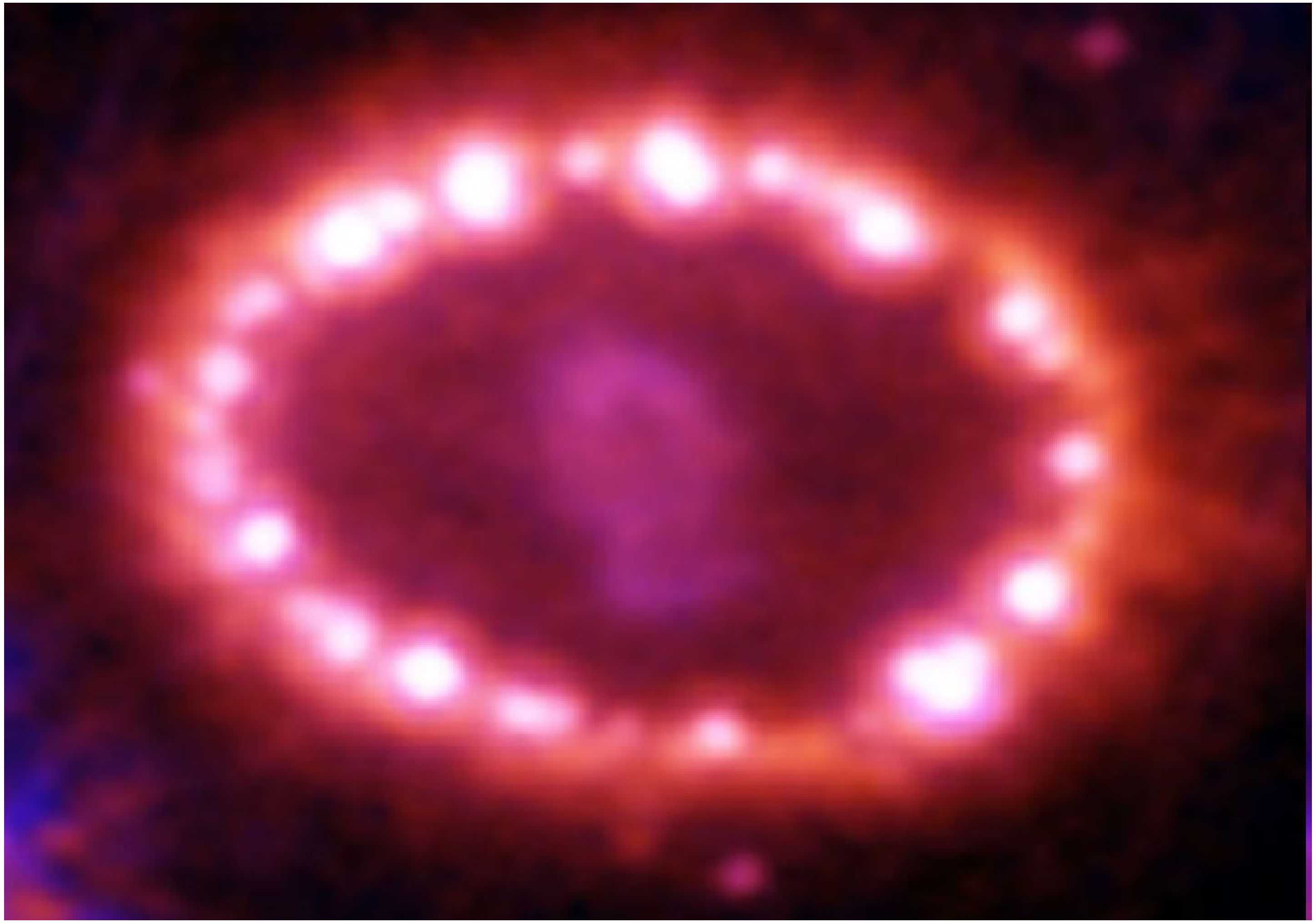
Pulsar in Crab cancellation is remnant SN 1052 detected in 1968

# Observation of Crab born: Japan, China 1054

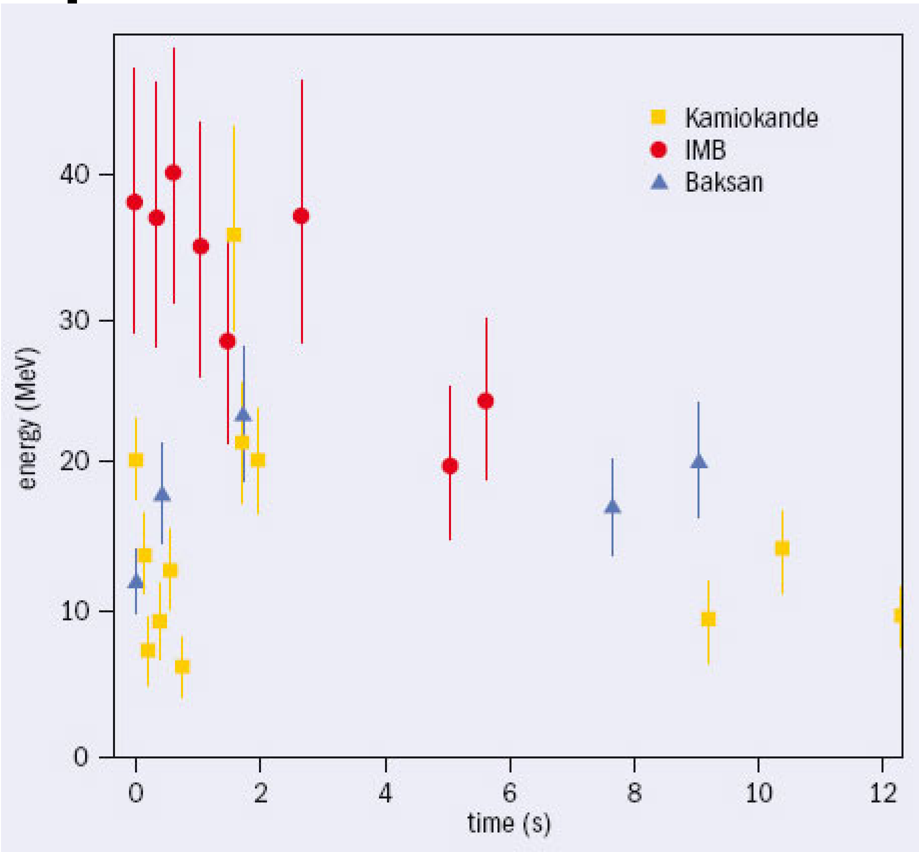
Yang Wei: Exp.: I am humbly prostrating myself. In constellation of Tven Huan (Taurus) I observed the appearance of the guest-star. It was slightly rainbowed. Theory: According to the Emperor order I respectably made a prediction. The quest star will not spoil Aldebaran ($\alpha$-Tauri). This indicates that the country will acquire the great power (finding of I. Shklovskii).

Now $>$ 3000 supernovas are observed. NS are born in SNII type explosions of stars with $M \gtrsim 10M_{\odot}$ with a frequency several times per century in the Galaxy.

- 23.02.1987 First registration of $\nu$ from SN by Kamiokande (Japan), IBM(USA), Baksan (USSR):



# Explosion of SN1987A




Registration of 24 neutrinos $E_\nu$~ 10-40 MэВ

If Supernova will be exploded in our Galaxy (frequency 1/(30-100 yr)),

Superkamiokande will register ~$10^3$ neutrinos!

*certainly, if not too close to our Sun, otherwise it will be last exp. result ☺ !!!*

## Yet exotics

- **1971** A.B.Migdal, $\pi$ condensate for $\rho \sim \rho_0$, possibility of abnormal superdense nuclei glued by $\pi$ condensate (depends on unknown $\rho$ dependence of Landau-Migdal parameter $g'(\rho)$; **1976** A.B.Migdal, **1976** , A.B.Migdal,G.Sorokin, O.Markin, I.Mishustin: $\pi$ condensate superdense nuclei $N \sim Z$, $A \lesssim 10^2$, neutron nuclei $A \gtrsim 10^3$, **1977** D.V., G.Sorokin, A.Chernoutsan: superdence nuclei-stars of arbitrary size.
- **1971** A.R. Bodmer: $u, d, s$ quark very small size systems.
- **1984** E.Witten: $u, d, s$ strange nuclei-stars (depends on uncertainty in $B$, $m_s$).
- **1974** T.D.Lee, G.Wick: $\sigma$ superdense nuclei.
- **1978** A.B.Migdal, A.Chernoutsan, I.Mishustin, Dynamics of I order phase transition to superdence $\pi$ cond. state.

## Yet exotics

- **1979** V.Berezovoy, I.Krive, E.Chudnovsky: idea of $P$ wave kaon condensation, **1995** E.Kolomeitsev, D.V., B.Kampfer: $P$ wave kaon condensation in NS.
- **1986** D.Kaplan, A.Nelson: $S$ wave kaon condensation.
- **1992** N.Glendenning: mixed phases: $npe \leftrightarrow (\pi_c, K_c, q)$ constructed by Gibbs conditions, **1993** H.Heiselberg, C.Pethick, E.Staub: Coulomb+surface effects, **2002** T.Tatsumi, D.V., M.Yasuhira: shrinking of mixed phases due to screening effects.
- **1978** S.Frautchi, **1984** D.Balin, A.Love, suggested color superconductivity $\Delta_q \lesssim 1$MeV,
- **1995** M.Dyakonov, H. Forkel, M.Lutz, **1998** M.Alford, K.Rajagopal, F. Wilczek; and R.Rapp, T. Schaefer, E. Shuryak, M. Velkovsky: $\Delta_q \lesssim 100$MeV, variety of phases.
- **1997** D.V., idea of charged rho-condensation in NS

# Structure of NS

- **atmosphere**, $\sim mm \div 10\ cm$, $\rho \lesssim 10^6\ g/cm^3$, plasma: determines photon radiation, $T_{surf}, B_{surf}$ affect EOS.
- **outer crust**, $\sim 10^2\ m$, $\rho < \rho_d = 4 \cdot 10^{11}\ g/cm^3$, solid of heavy nuclei + rel. electrons;
- **inner crust**, $\sim km$, $\rho \lesssim 0.5 \div 0.7\rho_0$, neutronized nuclei + neutron gas + rel. electrons.

  $0.3\rho_0 \lesssim \rho \lesssim 0.5 \div 0.7\rho_0$, nuclear pasta = mixed phase: nuclear drops, rods, slabs, etc.

# Structure of NS

- **outer core**, $\rho \lesssim 2 \div 4\rho_0$, $\gtrsim$ several km., m.b. up to center, superfluid (at $T \lesssim MeV$) of $nn$, $pp$ + normal electrons.
- **inner core**, up to center (larger for massive NS),

  Kingdom of Exotics: possible mixed phases between $npe$, $\pi_c$, $K_c$, $H$, $q$-CSC? and pure phases $\pi_c$, $K_c$, $H$, $q$-CSC?

**Pion, kaon, charged rho-condensates, and quark deconfinement phase transitions result in similar consequences for NS**

# Cooling of neutron stars

After passing a minute, during $10^5$ years a neutron star cools down by neutrino emission, then by photon emission from the surface

$$\lambda_\nu \gg R \simeq 10\mathrm{km}$$

(except first minutes after NS formation during which NS cools down up to few MeV)

White-body radiation problem (at low $T<T_{opac}$ ~1-few MeV):

direct reactions: **Similar to di-lepton radiation problem in HIC**

bring information straight from the dense interior

$T_{surf}$ (t) is related to $T_{in}$ (t); problem to compute $T_{in}$ (t)

# Measuring pulsar age

- **pulsar spin-down** rotation frequency $\Omega = 2\pi/P$ period

for non-accreting systems, period increases with time

power-law spin-down $\dot{\Omega} = \frac{d\Omega}{dt} = -k\,\Omega^n$ braking index $n = \ddot{\Omega}\,\Omega/\dot{\Omega}^2$

for magnetic dipole spin-down ***n=3***

$$t = \frac{P}{(n-1)\,\dot{P}}\left[1 - \left(\frac{P_0}{P}\right)^{n-1}\right]$$

"spin-down age"

- **kinematic age**

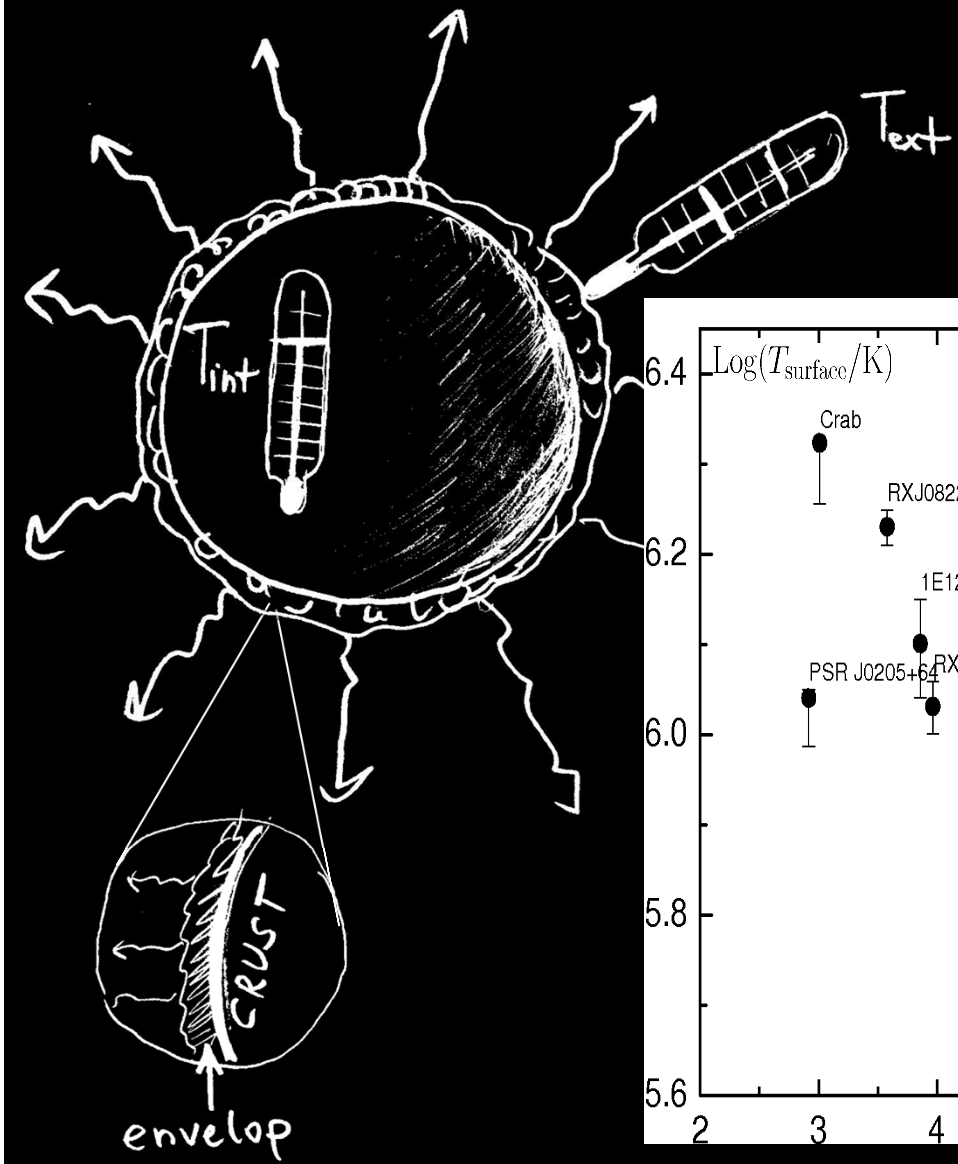
1) age of the associated SNR

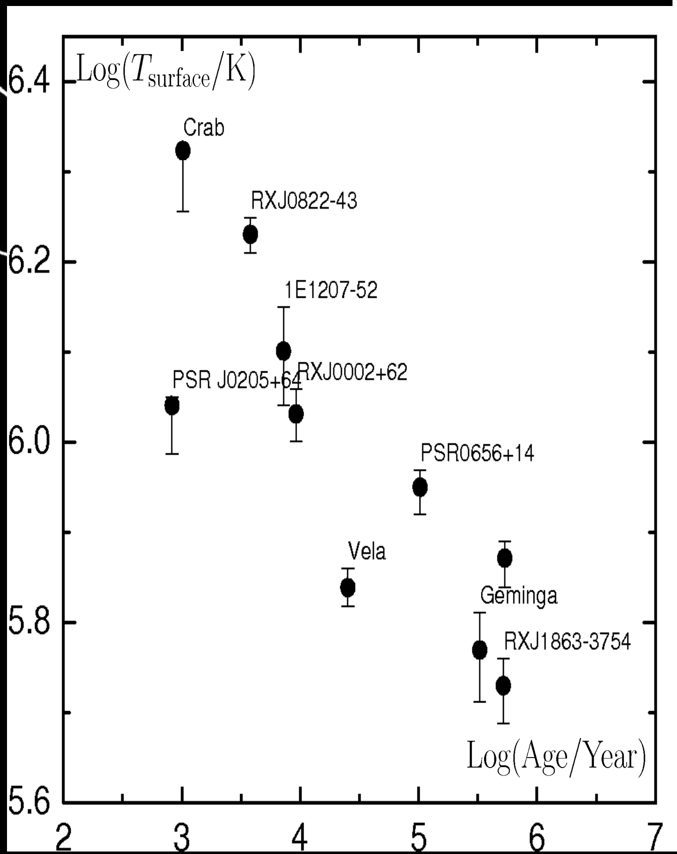
3) historical events

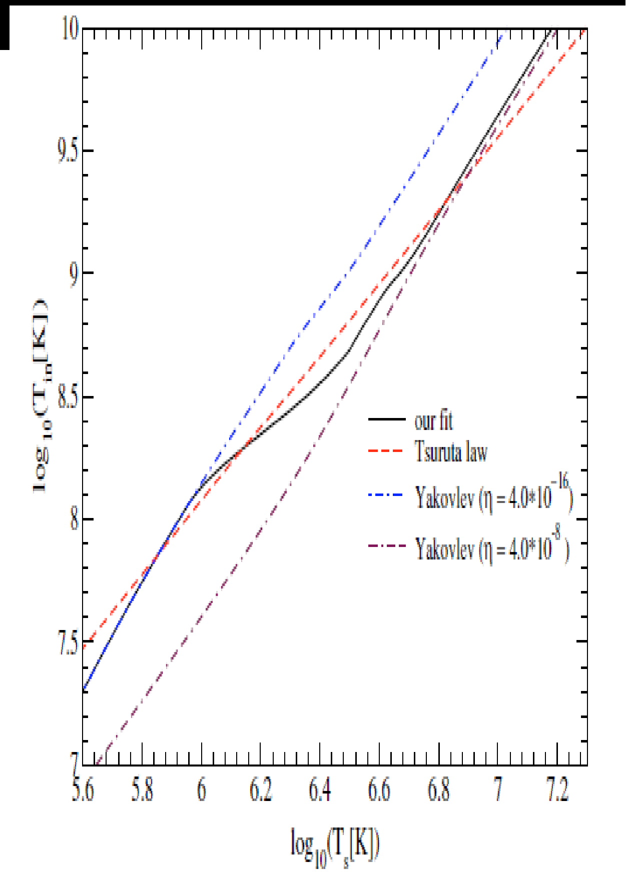
Crab : 1054 AD
Cassiopeia A: 1680 AD
Tycho's SN: 1572 AD

# Measuring pulsar temperature

- internal vs. external temperature



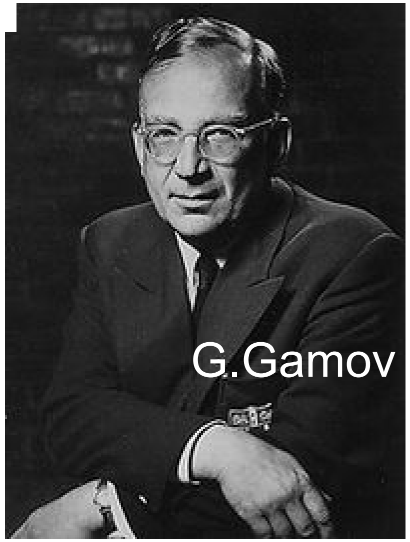




Yakovlev et al

# Direct reactions in standard scenario

- **1965** S. Tsuruta, A. Cameron, and J. Bahcall, R. Wolf:
  First scenario for NS cooling.




**Cooling**: crust is light and interior is massive

→ most important are reactions in dense interior

(where baryon density $n \gtrsim n_0$ $n_0$ is the nuclear saturation density)

**Phase-space separation**

**one-nucleon reactions:** $n \to p + e + \bar{\nu}$ direct Urca (DU)

URCA "Unrecordable Cooling Agent" (by Gamov 1941)

Casino da Urca in Brazil-waist of money;
pilferer, thief in Odessa

**two-nucleon reactions:** $n + n \to n + p + e + \bar{\nu}$ modified Urca (MU)

$n + n \to n + n + \nu + \bar{\nu}$ nucleon bremsstrahlung (NB)
(less important)

# Neutrino emission reactions

For T> min., T<$T_{opac}$~ MeV neutron star is transparent for neutrino

$$\frac{\partial}{\partial t}(Te^{\phi}) = -\frac{\epsilon_\nu}{c_V}e^{2\phi} + \frac{e^{\lambda}}{c_V r^2}\frac{\partial}{\partial r}\left(\kappa\, r^2 e^{\phi} + \lambda \frac{\partial}{\partial r}\left(Te^{\phi}\right)\right)$$

*$c_V$ - specific heat density,*
*$\varepsilon_\nu$ – neutrino emissivity,*
*Φ, λ – metric coefficients*

*Ķ- heat conductivity*

for t>300-500 yr isothermal stage

$$C_V \frac{dT}{dt} = -L$$

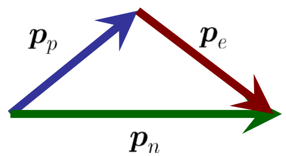
$C_V$ – specific heat,
L-luminosity

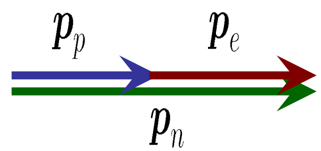
Strategy: Emissivity $\epsilon(T_{in})$, specific heat $C_V$, thermal cond. $\kappa$ from calcul., $\rightarrow T_{in}(t)$ from transport calcul., $T_s = f(T_{in})$ from calcul., $\rightarrow T_s(t) \rightarrow$ compare with $T_s(t)$ known from observations.

# NS cooling

- **1965** S. Tsuruta, A. Cameron, and J. Bahcall, R. Wolf: First scenario for NS cooling.

DU process $n \longrightarrow p + e + \bar{\nu}$




momentum conservation: $p_{Fn} \geq 2p_{Fp}$.

$$n_p = n_e\,, \quad p_{F,p} = q_{F,e}$$

For the gas of free quasiparticles

$$\mu_n = E_{F,n} \simeq \frac{p_{F,n}^2}{2m_N^*}\,, \quad \mu_p = E_{F,p} \simeq \frac{p_{F,p}^2}{2m_N^*}\,, \quad \mu_e = E_{F,e} \simeq q_{F,e}$$

$n_{c,DU} \approx 30\, n_0$ → **DU process is forbidden in NS**

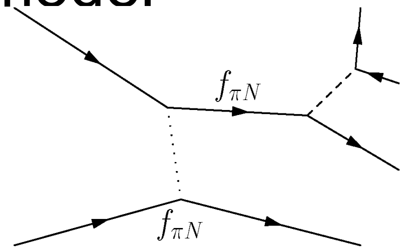
# Standard scenario

Tsuruta, S. 1979, Phys. Rep., 56

Shapiro, S., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects (New York: Wiley ), Chap. 11

**Main permitted process is MU:** $n + n \to n + p + e + \bar{\nu}$

**1979** Friman and Maxwell computed MU in FOPE model



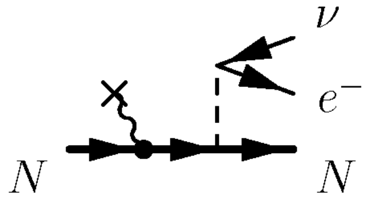

+ simple $T_s$ –$T_{in}$ relation (Tsuruta law $T_s^{\rm Tsur} = (10\ T_{\rm in})^{2/3}$)

➡ only slow cooling

# Standard + exotics scenario

**1977** Maxwell, O., Brown, G. E., Campbell, D. K., Dashen, R. F., Manassah, J. T. 1977, ApJ, 216

**added process on pion condensate**

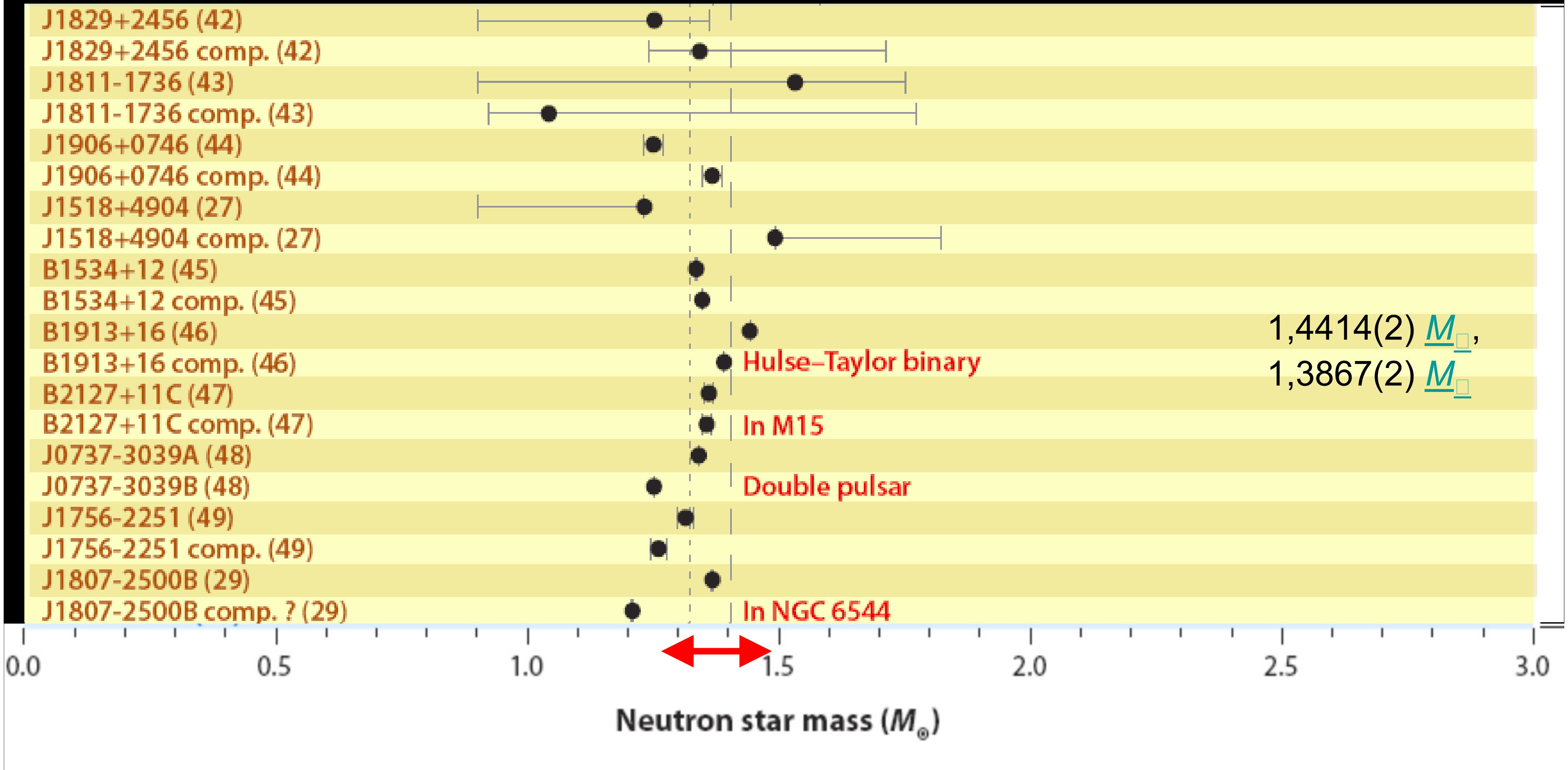



➡ only rapid cooling

that time most of researches believed that all NS have the very same masses ≈1.4 $M_{sol}$

so, only slow coolers either rapid ones could be explained

# Masses of NS in double neutron star binaries

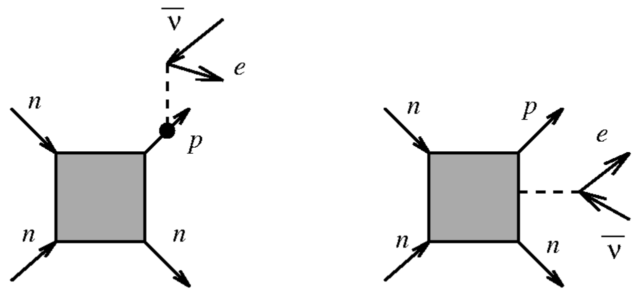



majority of measured NS masses are focused near the value 1.4 $M_{sol}$

# Nuclear medium cooling

- D.V., A. V. Senatorov JETP Lett.1984, JETP 1986 found strong density dependence for MU emissivity

(now called Medium MU process) $n + n \to n + p + e + \bar{\nu}$

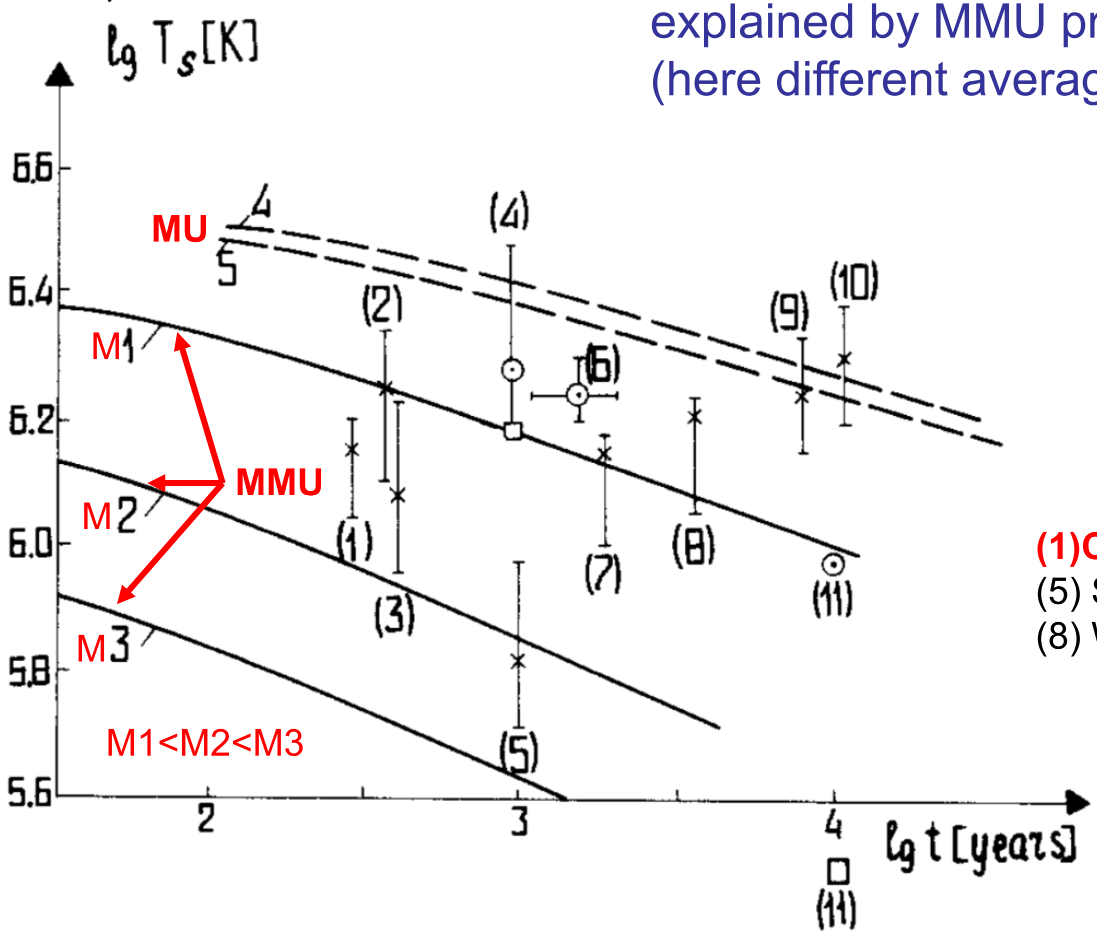



and suggested that NS (might be seen in soft X rays)

**should have essentially different masses.**

**Heavier NS cool down substantially faster!**

D.V., Senatorov JETP **1986:** all the data (upper limits to $T_s$ known to that time) were explained by MMU process assuming different masses (here different average densities) of NS




circles: observed sources,
crosses: upper limits,
squares: $T_s$ of Crab and Vela, adjusted from an analysis of their frequency glitches

**(1)Cas A**, (2) Kepler, (3) Tycho, (4) Crab, (5) SN 1006, (6) RCW 103, (7) RCW 86, (8) W28, (9) G350, 018, (10) G22, 7-02, (11) Vela

If in the future central sources are discovered in supernova remnants with low values of $T_s$ (see Fig. 6), then they could be associated with neutron stars having a denser internal region than other neutron stars with higher $T_s$

# New data: masses are essentially different

## Pulsar J1614-2230

$$M = (1.97 \pm 0.04)\ M_{sol}$$

P.Demorest et al., Nature 467 (2010)

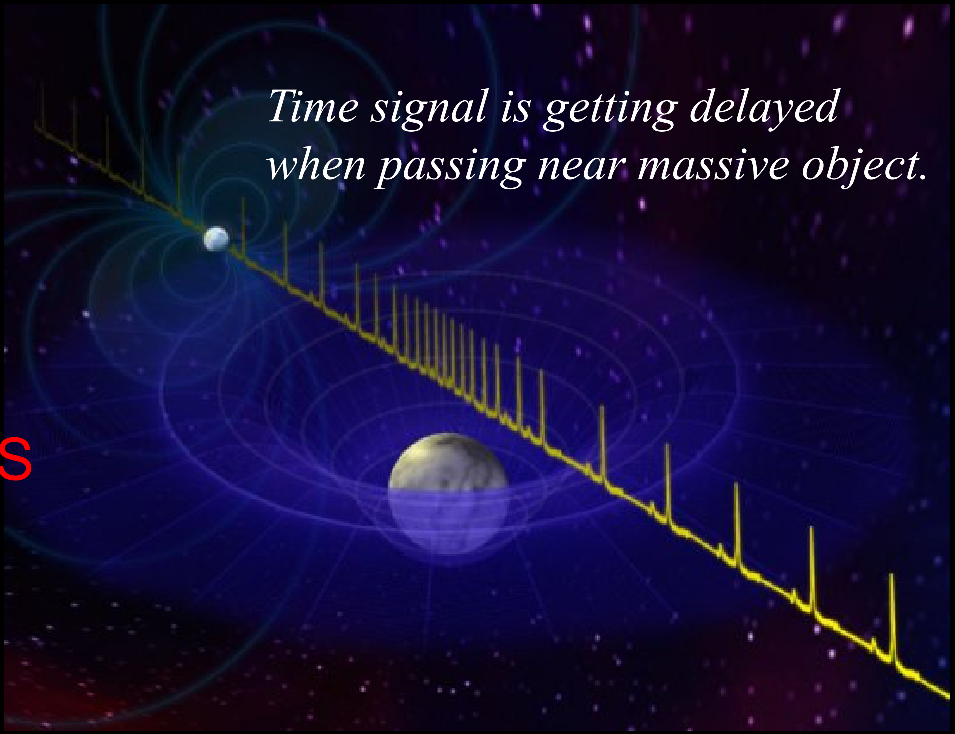
## **Pulsar J0348-04232**

$M = (2.01 \pm 0.04\ )\ M_{sol}$

J. Antoniadis et al., Science (2013)

Highest well-known masses of NS

Measured Shapiro delay with high precision

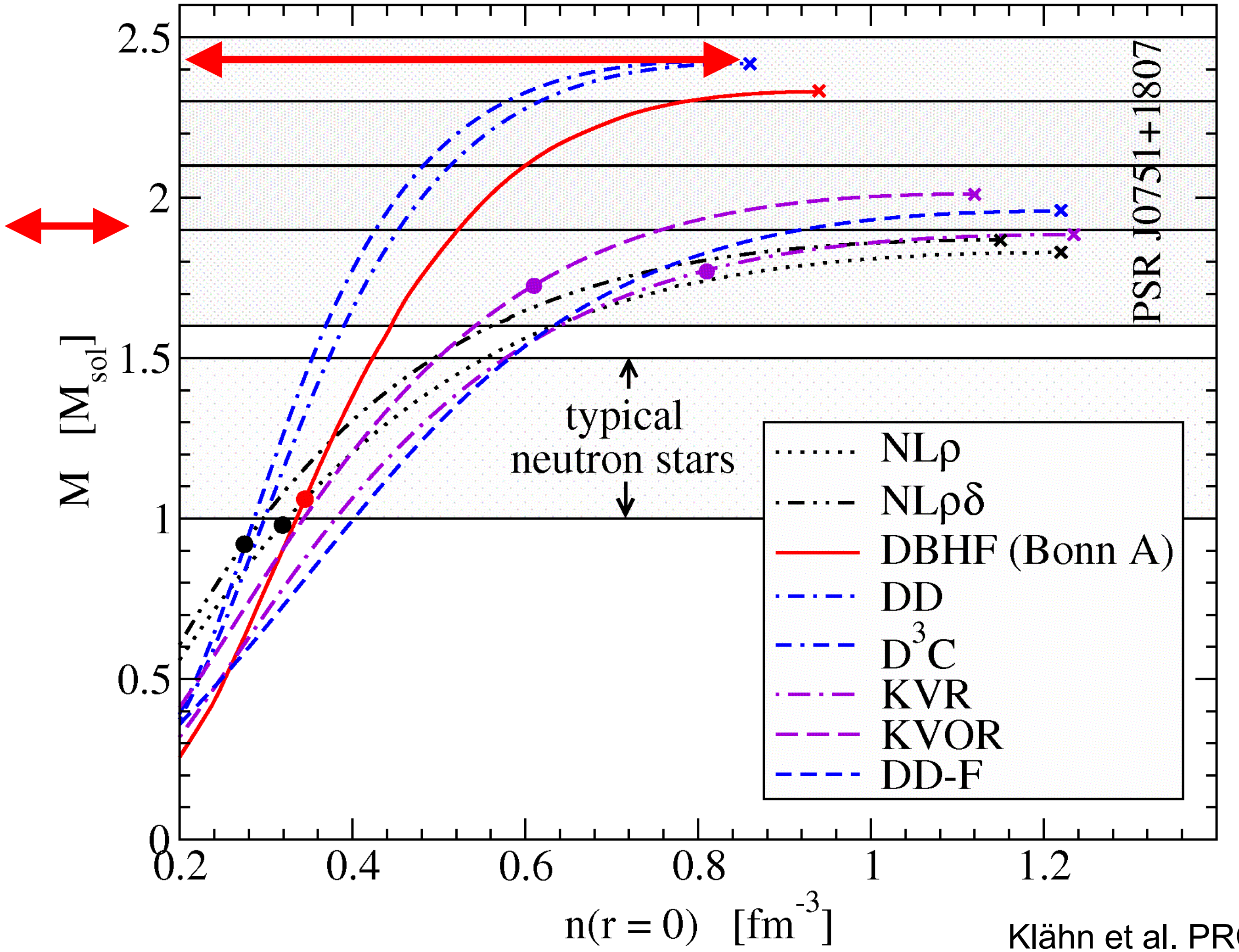



there are heavier, but far less precisely measured candidates)

$2.44^{+0.27}_{-0.27}\ M_\odot$ for 4U 1700-377, $2.39^{+0.36}_{-0.29}\ M_\odot$ for PSR B1957+20 both in X-ray binaries, and $2.74 \pm 0.21\ M_\odot$ for J1748-2021B in neutron star-white dwarf binaries

Lightest NS    PSR J1807-2500B: $M=1.2064+-0.0020\ M_{sol}$

# NS mass-central density diagram for different EoS



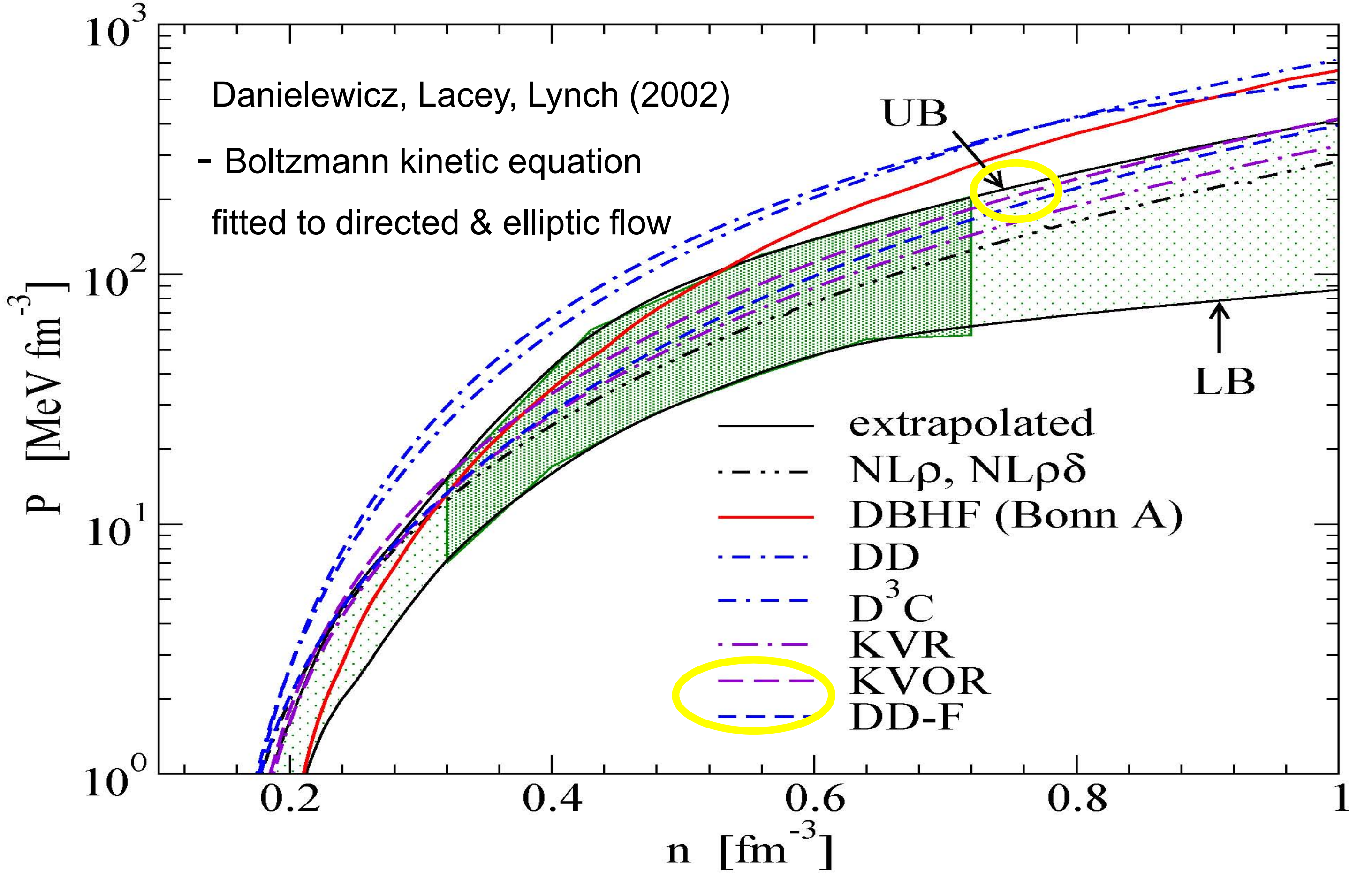

Klähn et al. PRC 74, 035802 (2006)

**If M>2.4 $M_{sol}$ ( ↔ ) were observed, all these EoS would be invalid!**

**Central densities in various NS are different! ➡ Studying various NS we may test density dependences of EoS and NN interaction**

# HEAVY ION COLLISIONS MEET NEUTRON STARS

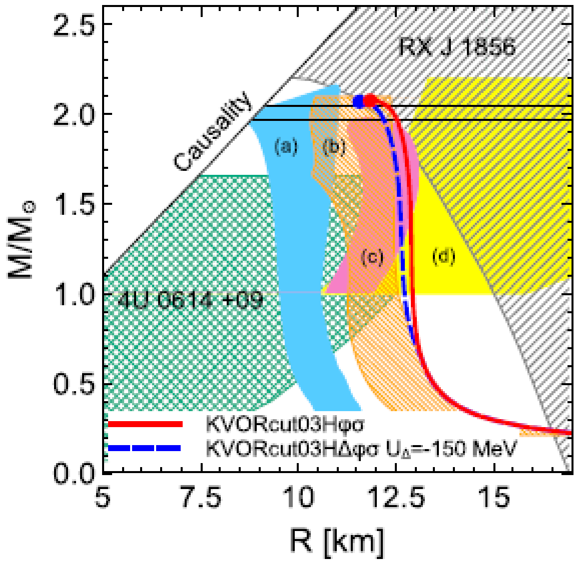
(common constraints)

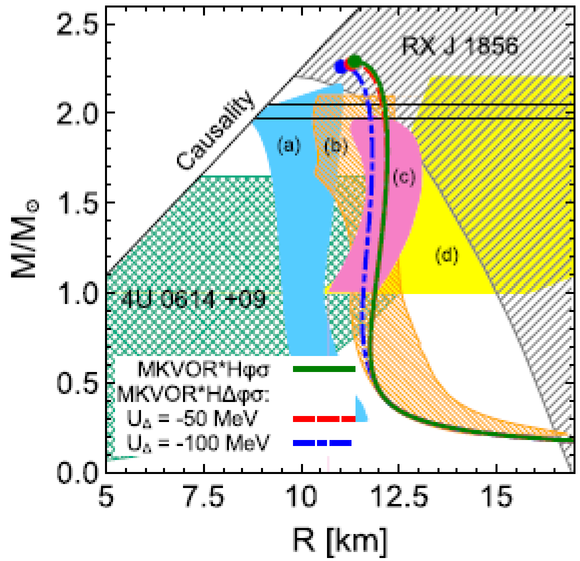



**Stiffest EoS do not satisfy the HIC constraint.**

**Only EoS near the upper boundary of the box satisfy both the HIC and NS mass constraints yielding $M_{max}$>1.97 $M_{sol}$**

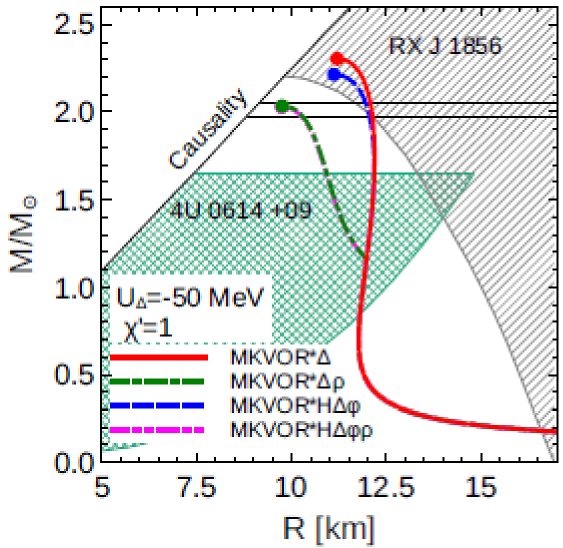
# With included hyperons and Delta-isobars




Kolomeitsev E E, Maslov K A and Voskresensky D N 2017 *Nucl. Phys.* A **961**

With included charged ρ condensate



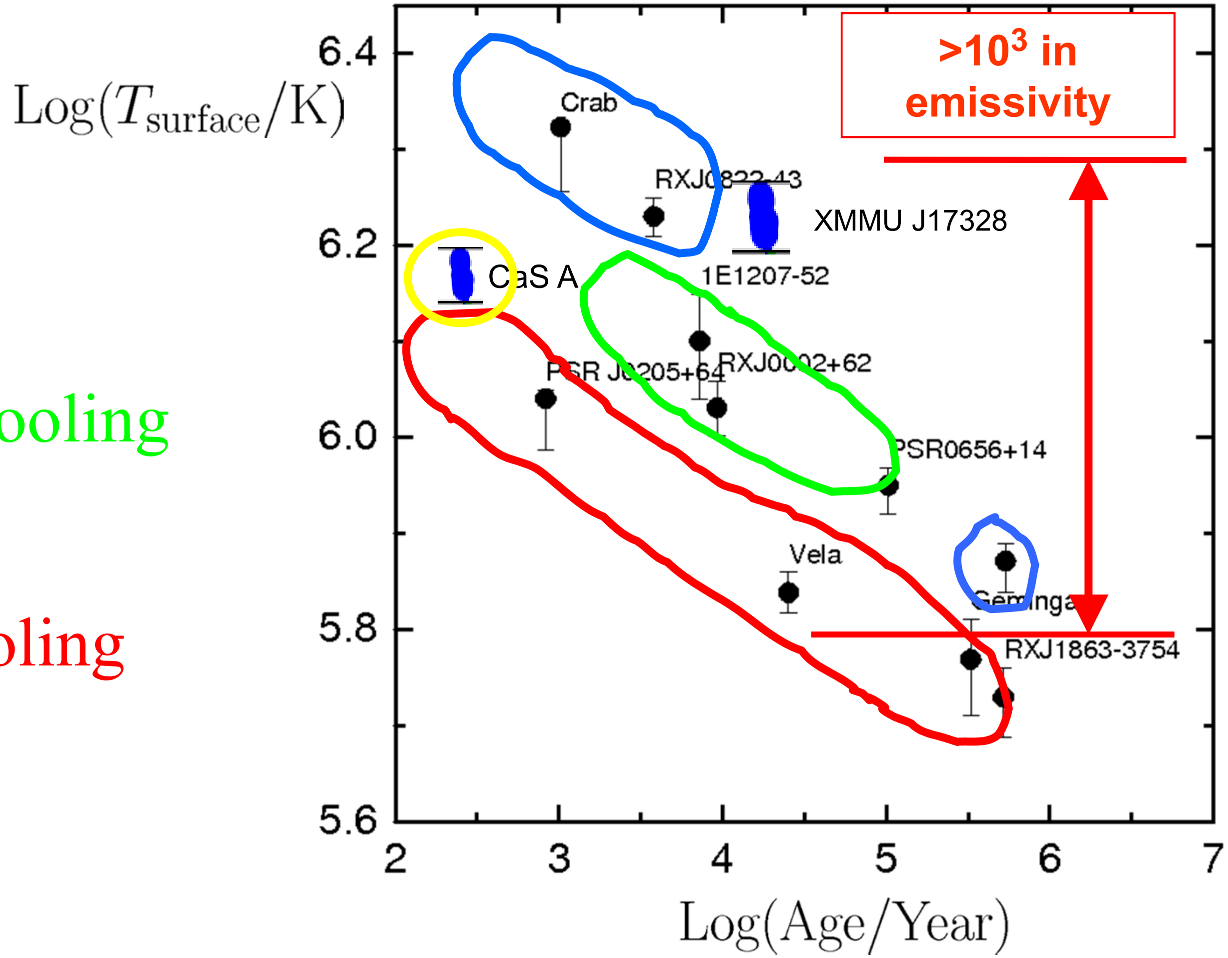
# NS cooling data

3 groups+Cas A:

slow cooling

intermediate cooling

rapid cooling




How to describe all groups within one cooling scenario?

# Calculation of processes. Suppressed medium effects.

$$L^{\rm int} = \frac{G}{\sqrt{2}} j_\mu l^\mu \qquad G = 1.16\cdot 10^{-5}\ {\rm GeV}^{-2}$$ the weak interaction constant

lepton current $\quad l_\mu = \bar{u}(q_1)\,\gamma_\mu(1-\gamma_5)\,u(q_2) \qquad \sum_{spin} u(q)\,\bar{u}(q) = \gamma_\mu\, q^\mu$

nucleon current $\quad < N|j_\mu|N > \;=\; V_\mu^{NN} - A_\mu^{NN} = \bar{g}_V\,(\bar{N}\,\gamma_\mu N) - \bar{g}_A(\bar{N}\,\gamma_\mu\,\gamma_5 N)$

$$V_\mu^{np} \approx g_V\,\chi_p^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_n(p)$$

$$V_\mu^{nn} \approx -\frac{g_V}{2}\,\chi_n^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_n(p)$$

$$V_\mu^{pp} \approx +\frac{g_V}{2}\,\mathbf{c_V}\chi_p^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_p(p)$$

$$A_\mu^{np} = -2\,A_\mu^{pp} = -2\,A_\mu^{nn} \approx g_A\,\chi_p^\dagger(p')\big(\boldsymbol{\sigma}\cdot\boldsymbol{v},\boldsymbol{\sigma}\big)\chi_n(p)$$

$$g_A \simeq 1.26$$

$$g_V = 1 \qquad \boldsymbol{v} = \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_N}$$

**~v (Fermi velocity) corrections are important**

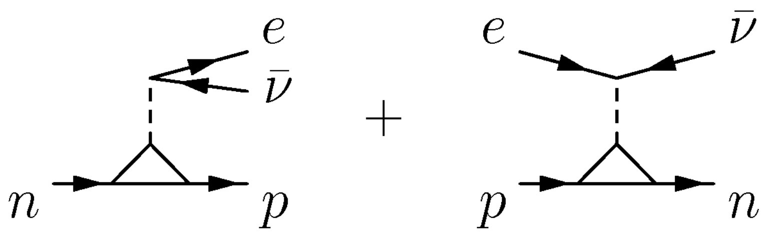
$$\mathbf{c_V} = 1 - 4\sin^2\theta_W \simeq \mathbf{0.08}$$

Note 1/2 in neutral channel,
since *Z* boson is neutral and *W* is charged!

# One-nucleon processes (DU). No medium effects

For $n > n_c^{\rm DU}$ $(M > M_c^{\rm DU})$

**emissivity (Fermi golden rule):**




**bare vertices !**

$$\epsilon_\nu^{\rm DU} = 2\int \frac{d^3p_n}{(2\pi)^3} f_n \int \frac{d^3p_p}{(2\pi)^3}(1-f_p) \int \frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e) \int \frac{d^3q_{\bar\nu}\,\omega_{\bar\nu}}{2\omega_{\bar\nu}(2\pi)^3}(2\pi)^4\delta^{(4)}(P_f-P_i)\sum_{\rm spins}|M|^2$$

**Counting powers of *T*:**

each external nucleon and electron line ~$T$ $\quad\Rightarrow\quad \epsilon_\nu^{\rm DU} \sim T^6$

neutrino phase space × neutrino energy $n_{\rm bos}\,\omega_{\bar\nu} \times \delta(\omega_{\bar\nu} - \dots)\,\omega_{\bar\nu}^2\, d\omega_{\bar\nu} \sim T^3$

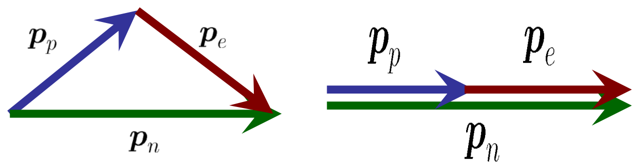
$$\epsilon_\nu^{\rm DU} \simeq 4\cdot 10^{27}\,(n_e/n_0)^{1/3}\,T_9^6\,\Theta(2p_{{\rm F},p} - p_{{\rm F},n})\,\frac{\rm erg}{{\rm cm}^3\cdot{\rm s}}$$

$T_9 = T/10^9\,{\rm K}$

$n_0 \simeq 0.17\ {\rm fm}^{-3}$

- one-nucleon phase-space volume (» $10^{27}$-$10^{28}$ factor)
- $T^6$ dependence
- threshold behavior ($n > n_c^{DU}$ , $n_c^{DU}$ depends on EoS)
- very moderate density dependence

$|\boldsymbol{p}_p| = |\boldsymbol{p}_e|$

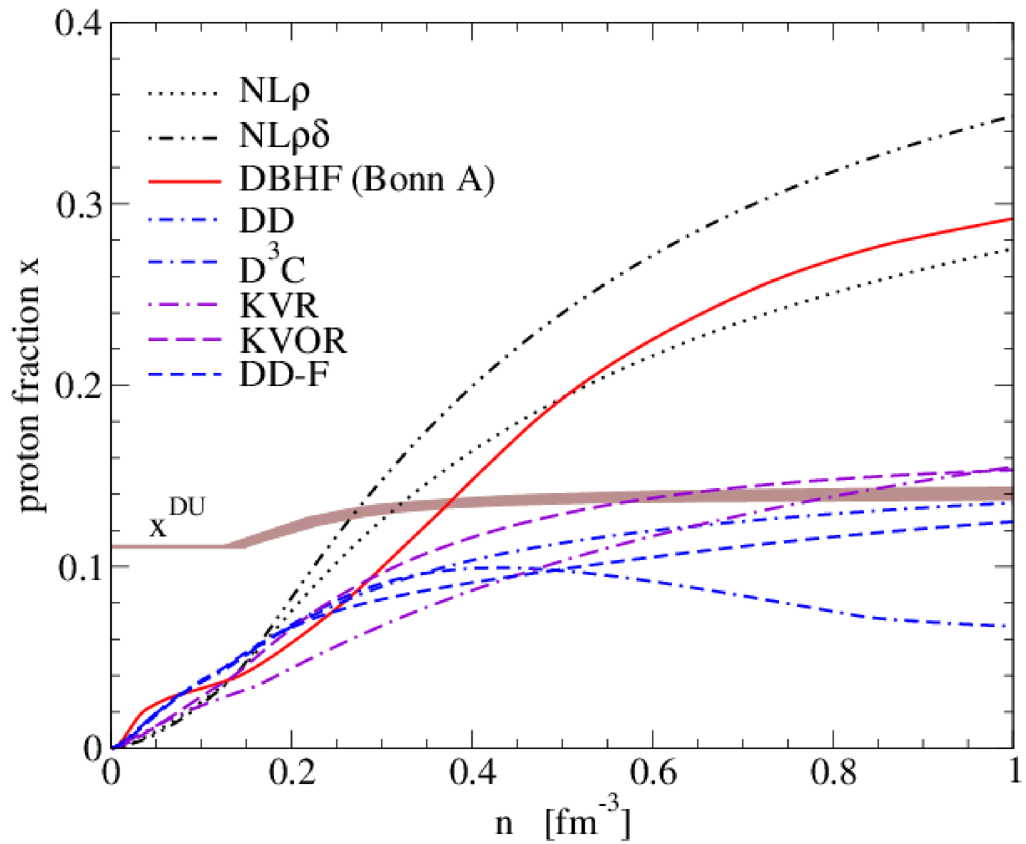


For realistic EoS DU threshold m.b. decreased!

Suggested by Boguta, Bodmer NPA 1977 in RMF model, new life of DU: Lattimer, Prakash, Pethick, Haensel, PRL 1991

$$\mu_i = \frac{\partial E}{\partial n_i}$$

**Not as for ideal Fermi gas !**

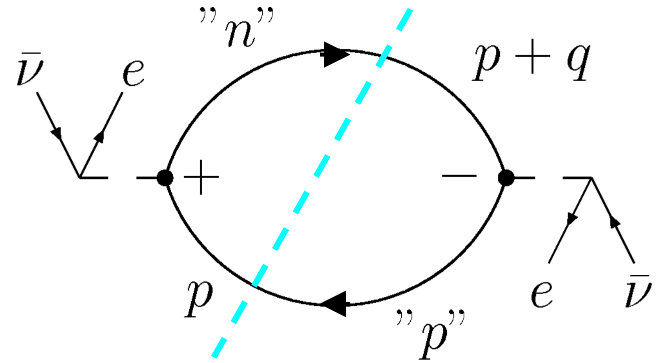


# Optical theorem in non-equilibrium diagram technique

D.V., Senatorov Yad.Fiz.(1987)

self-energy with free non-equilibrium Green's functions

$$-i\,\Pi_0^{-+} =$$




$$-i\Pi_0^{-+} = \frac{G^2}{2}\mathrm{Tr}\{l_1^\mu l_2^\nu\}\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\{(-iJ_\mu)\,iG_n^{-+}(p+q)\,(+iJ_\nu)\,iG_p^{+-}(p)(-1)\}$$

$$\epsilon_\nu^{\mathrm{DU}} = 2\int\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar{\nu}}}{2\,\omega_{\bar{\nu}}\,(2\pi)^3}\omega_{\bar{\nu}}\,[-i\Pi_0^{-+}(q_e+q_{\bar{\nu}})]$$

$$-i\,\Pi_0^{-+} = 2\,n_{\mathrm{bos}}(\omega)\,\mathrm{Im}\,\Pi^R(\omega),$$

$$G_0^{-+} = \pm 2\,\pi\,i\,f(E)\,\delta(E+\mu-E_p) \quad G_0^{+-} = -2\,\pi\,i\,(1\mp f(E))\,\delta(E+\mu-E_p)$$

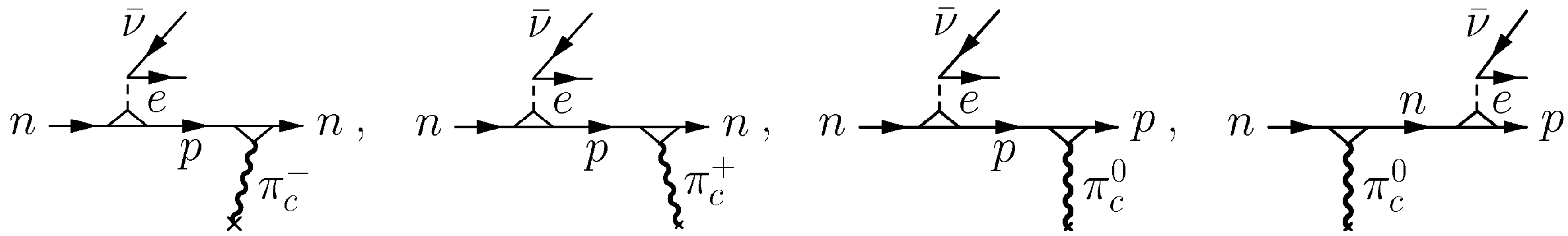
**Cut of the diagram means removing of $dE$ integration due to $\delta$-function**

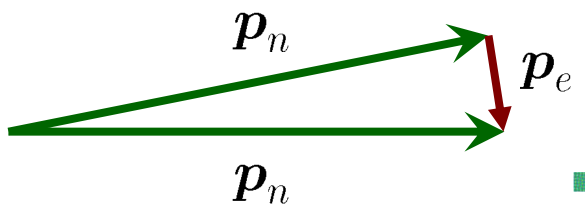
$$\epsilon_\nu^{\mathrm{DU}} = 2\int\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar{\nu}}}{2\,\omega_{\bar{\nu}}\,(2\pi)^3}\omega_{\bar{\nu}}\,[-i\Pi_0^{-+}(q_e+q_{\bar{\nu}})]$$

# Pion Urca processes

**PU is also one-nucleon process** (if the model permits pion condensation)

For $n > n_c^{\rm PU}$ $(M > M_c^{\rm PU})$ pion Urca (PU) processes:



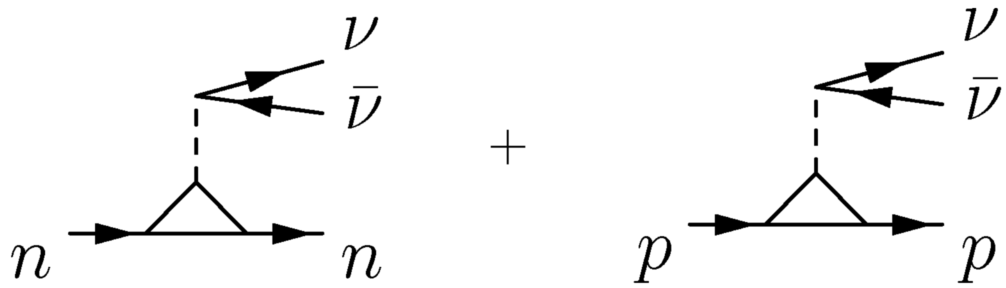

neutrons in both initial and final states

→ energy-momentum conservation is easily fulfilled

with bare vertices: $\epsilon_\nu \sim 10^{26}\, T_9^6 (n/n_0)^{1/3} \; \dfrac{\rm erg}{\rm cm^3 \; sec}$

All “exotic” one-nucleon processes start only when the density exceeds some <u>critical density</u>

# One-nucleon processes on neutral currents




**energy-momentum conservation**

$$\delta(\boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{q}_1 - \boldsymbol{q}_2)\,\delta(E_1 - E_2 - \omega_1 - \omega_2)$$

➡

$$E_1 = E(p_1) = \frac{p_1^2}{2m_N} = \frac{(\boldsymbol{p}_1 - \boldsymbol{q}_1 - \boldsymbol{q}_2)^2}{2m_N} + \omega_1 + \omega_2$$

$$\approx \frac{p_1^2}{2m_N} - v_F\,|\boldsymbol{q}_1 + \boldsymbol{q}_2|\,\cos\theta + |\boldsymbol{q}_1| + |\boldsymbol{q}_2|$$

➡ requires $v_{\rm F} \geq 1$

**In absence of pairing processes on neutral currents are forbidden!**

# Two-nucleon process (MU)

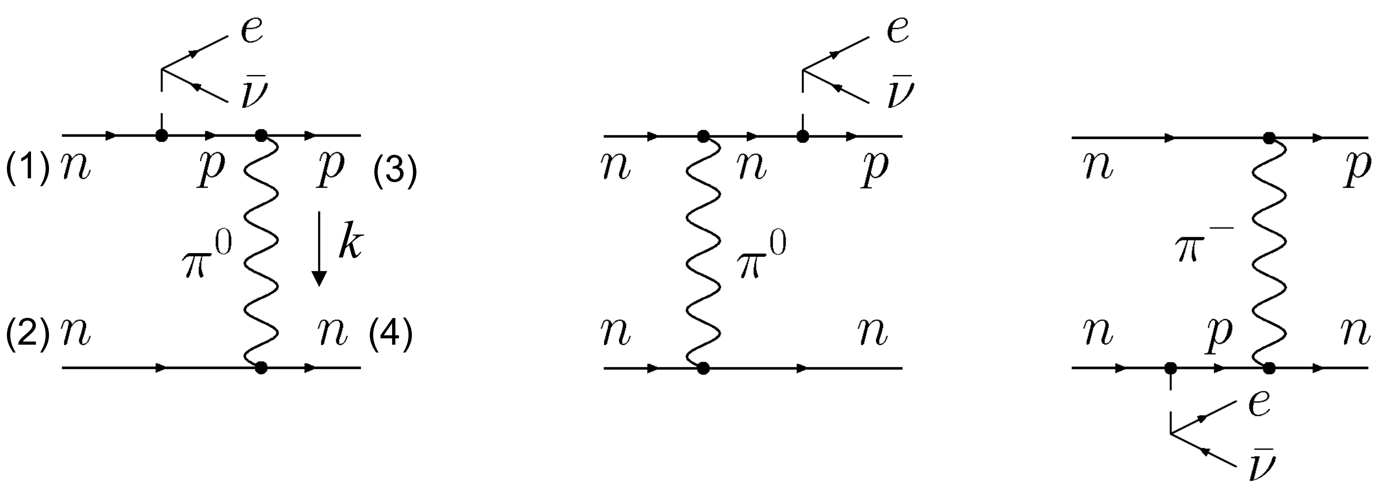
$$n + p \longrightarrow p + p + e + \bar{\nu} \qquad\qquad n + n \longrightarrow n + p + e + \bar{\nu}$$

- ✔ no critical density
- ✔ 5 fermions ➡ suppressed phase-space volume (compared to one-nucleon processes)
- ✔ $T^8$ dependence of the emissivity (5 fermions ➡ $\sim T^5$, $\omega_{\bar{\nu}}\,\delta(\omega_{\bar{\nu}} + \ldots)\,\omega_{\bar{\nu}}^2\,d\omega_{\bar{\nu}}$ ➡ $T^3$)

# Two-nucleon process (Modified Urca)

Friman & Maxwell AJ (1979) $n + n \rightarrow n + p + e + \bar{\nu}$

**FOPE model of NN interaction (no medium effects)**




Additionally one should take into account exchange reactions (identical nucleons)

**FOPE model continues to be used by different groups,**
e.g. by Page et. All, Yakovlev et al.

# Two-nucleon process (Modified Urca)

**Emissivity:**

$$\epsilon_\nu^{\rm MU} = \prod_{i=1}^{4} \int \left[ \frac{d^3 p_i}{(2\pi)^3} \right] f_1 \, f_2 \, (1 - f_3) \, (1 - f_4) \, \frac{d^3 q_e \, (1 - f_e)}{2 \, \omega_e \, (2\pi)^3}$$
$$\times \, \frac{d^3 q_{\bar\nu}}{2 \, \omega_{\bar\nu} \, (2\pi)^3} \, \omega_{\bar\nu} \, (2 \, \pi)^4 \, \delta^{(4)}(P_f - P_i) \, \frac{1}{s} \sum_{spins} |M|^2,$$

$s=2$ is symmetry factor. Reactions with the electron in an initial state yield extra factor 2.

Finally

$$\epsilon_\nu^{\rm MU} = \frac{11513}{60480 \, \pi} \, G^2 \, g_A^2 \, f_{\pi NN}^4 \, m_n^3 \, m_p \, p_{{\rm F},e} \, T^8 \, 1.3 \simeq 8 \cdot 10^{21} \, (n_p/n_0)^{1/3} \, T_9^8 \, \times \, \frac{\rm erg}{{\rm cm}^3 \cdot {\rm s}}$$

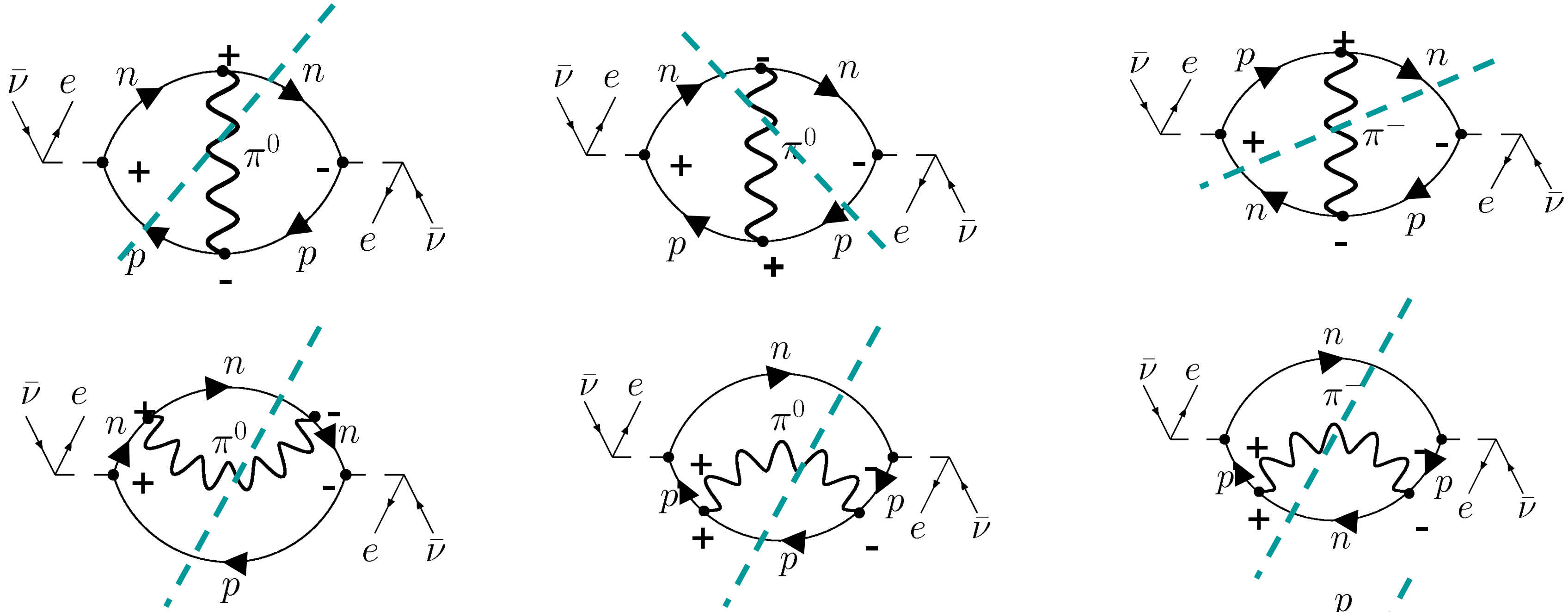
↑ due to exchange reactions

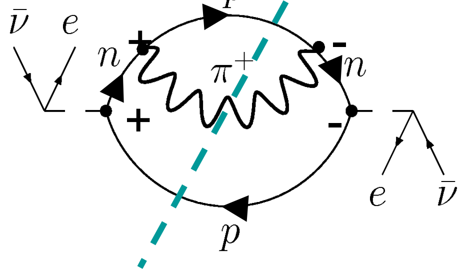
**Coherence:** only axial-vector term contributes (!)

# Optical theorem for modified URCA reactions

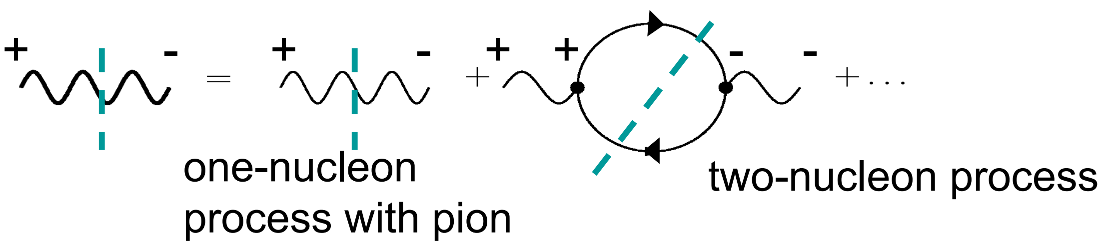
$$\epsilon_\nu^{\mathrm{MU}} = \int \frac{d^3 q_e\,(1-f_e)}{2\,\omega_e\,(2\,\pi)^3}\,\frac{d^3 q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\,\pi)^3}\,\omega_{\bar\nu}\,[-i\Pi^{-+}_{\mathrm{MU}}(q_e+q_{\bar\nu})]$$

To get correct 2-order $\Pi^{-+}$ one should add diagrams with $\pi^-$ corresponding to $np \to ppe\bar\nu$ reaction. They should be added coherently.

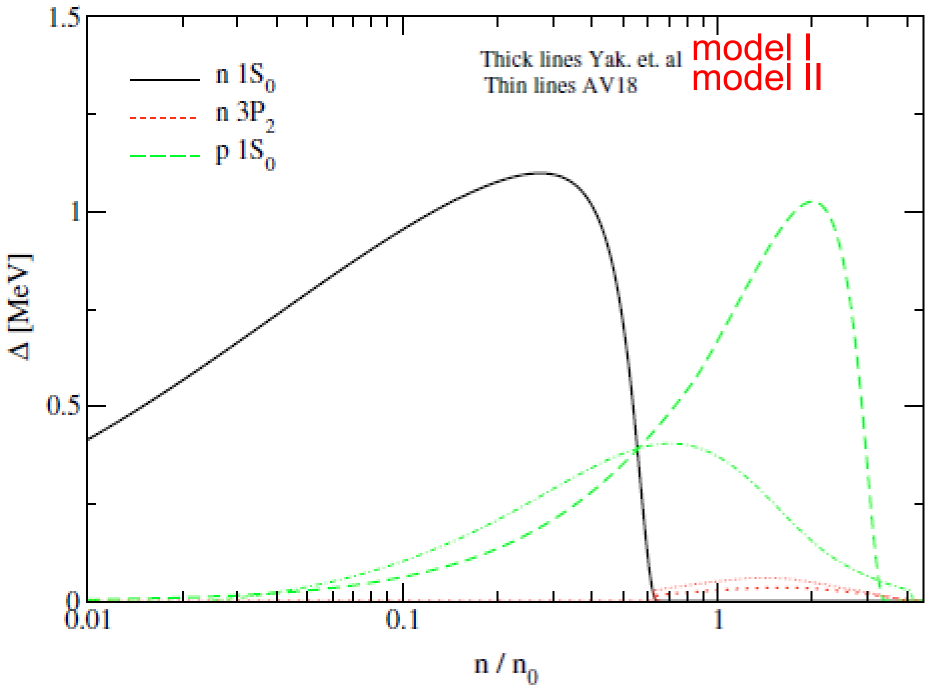



thick pion line (here up to 2nd order):

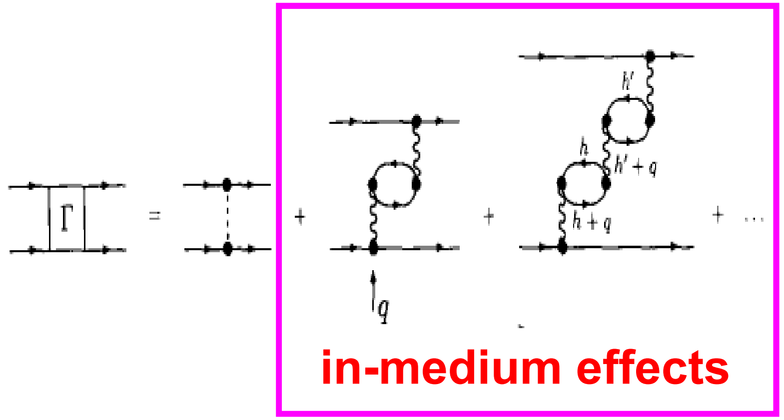


# Pairing in NS matter




A.B.Migdal (1959)

$$\Delta \simeq 2E_F \exp(-1/(NV)), \quad N = m^* p_F/\pi^2,$$



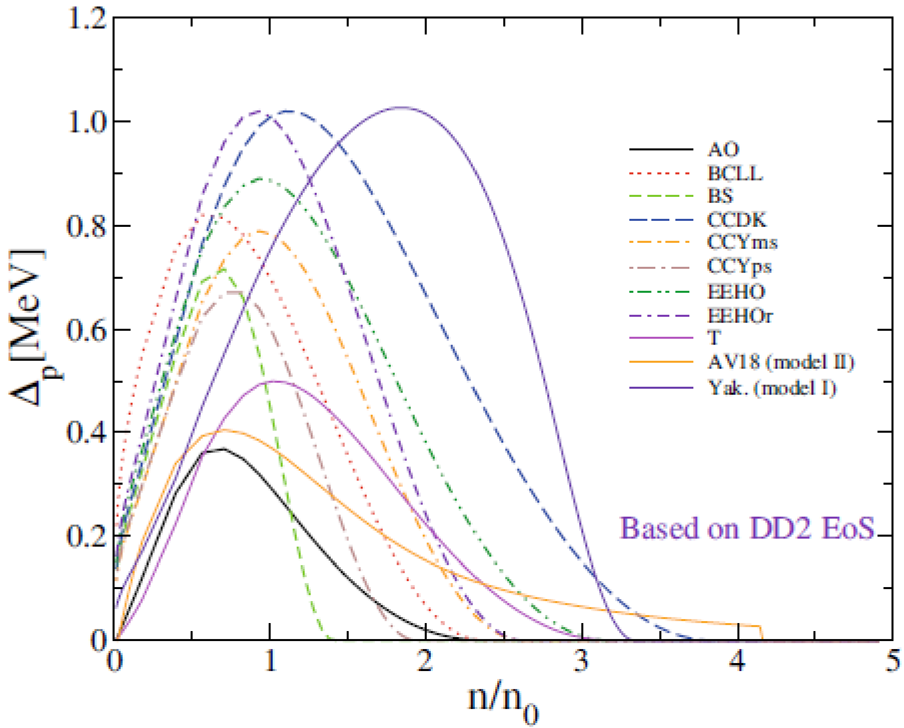

U. Lombardo and H.-J. Schulze (2000)

- in all models $1S_0$ gaps drop above ~$4n_0$
- Cooling is most sensitive to pairing in dense matter (to $3P_2$ neutron gap and $1S_0$ proton gaps)
- Gaps are very sensitive to inclusion of **in-medium effects**

Schwenk, Friman, PRL (2004) triplet paring is supperessed by medium-induced spin-orbit interaction, $3P_2$ gap <10 keV, *we exploit this result*,

Others use BCS-based estimates $\Delta(3P_2)$ ~0.1 MeV

# $1S_0$ proton pairing gap models




**Without (rather strong) proton-proton pairing it is impossible to explain slow cooling objects!**

# Pairing in nuclear matter

excitation spectrum

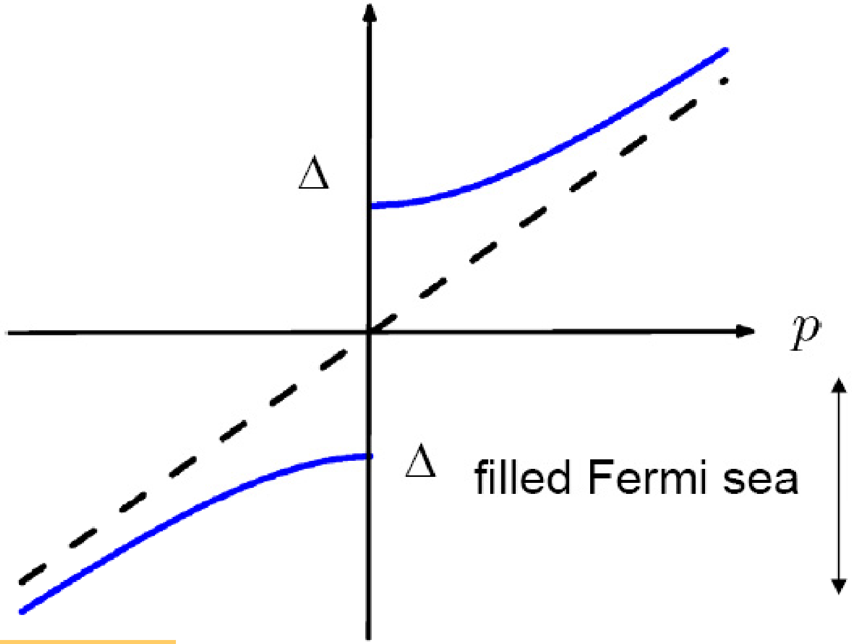
- **without pairing**

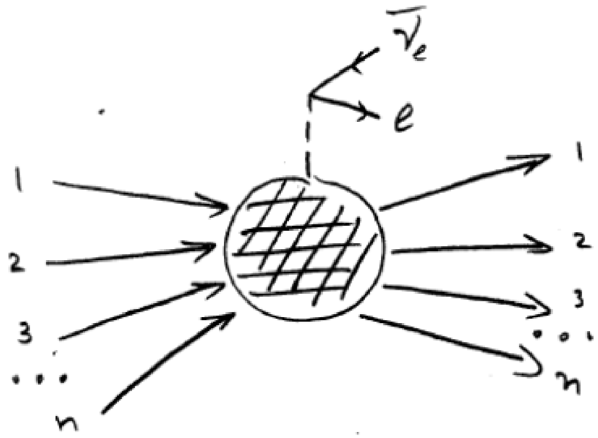
$$E = \epsilon_p - \epsilon_F \simeq v_{\rm F}\,(p - p_{\rm F})$$

- **with pairing**

$$E = \pm\sqrt{v_F^2\,(p - p_{\rm F})^2 + \Delta^2}$$

pairing gap



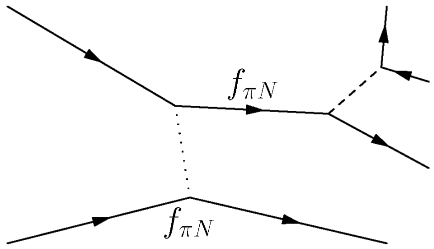




suppressed by factor $\sim e^{-n\,\Delta/T}$

# Standard scenario + exotics

standard $\underline{T < T_{\rm opac} \sim 10^{-1} \div 10^{0}\ \text{MeV}}$

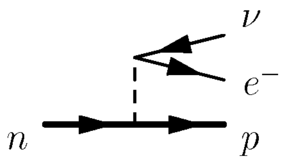
**MU:**




$$10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \times e^{-2\Delta/T}$$

exotics

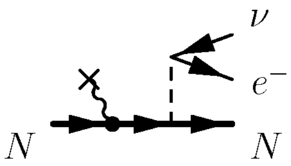
**DU:**




$$10^{27} \times T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \times e^{-\Delta/T}$$

allowed if $n > n_c^{\rm DU}$

**PU:**




$$10^{27} \times T_9^6 \frac{|\varphi_c|^2}{m_\pi^2} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \times e^{-\Delta/T}$$

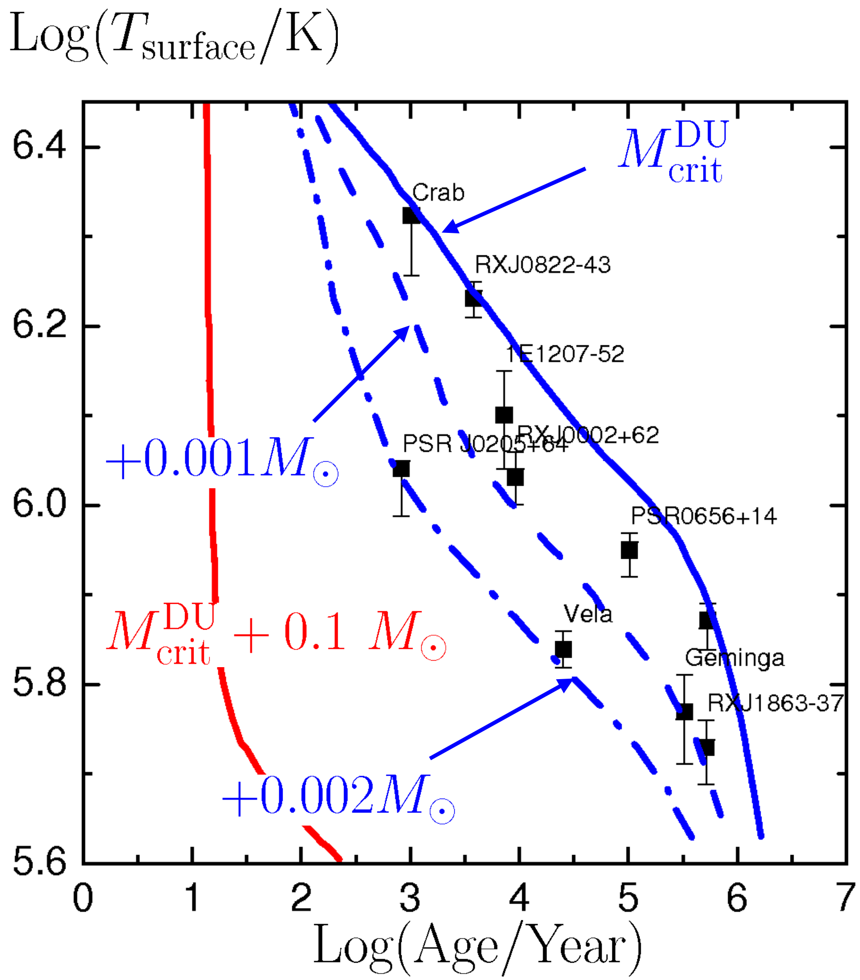
allowed if $n > n_c^{\rm PU}$

# Neutron Star Cooling Scenario

standard scenario (MU+pairing)

only "slow" cooling can be described

Neutron stars with $M > M_{\rm crit}^{\rm DU}$ will be too cold




→ **DU constraint:**

**DU process schould be „exotics"**

*(if DU starts it is difficult to stop it)*

since in reality masses of NS are not close to each other →

**Either EoS with low DU threshold should be rejected or pp- gap should be very large**

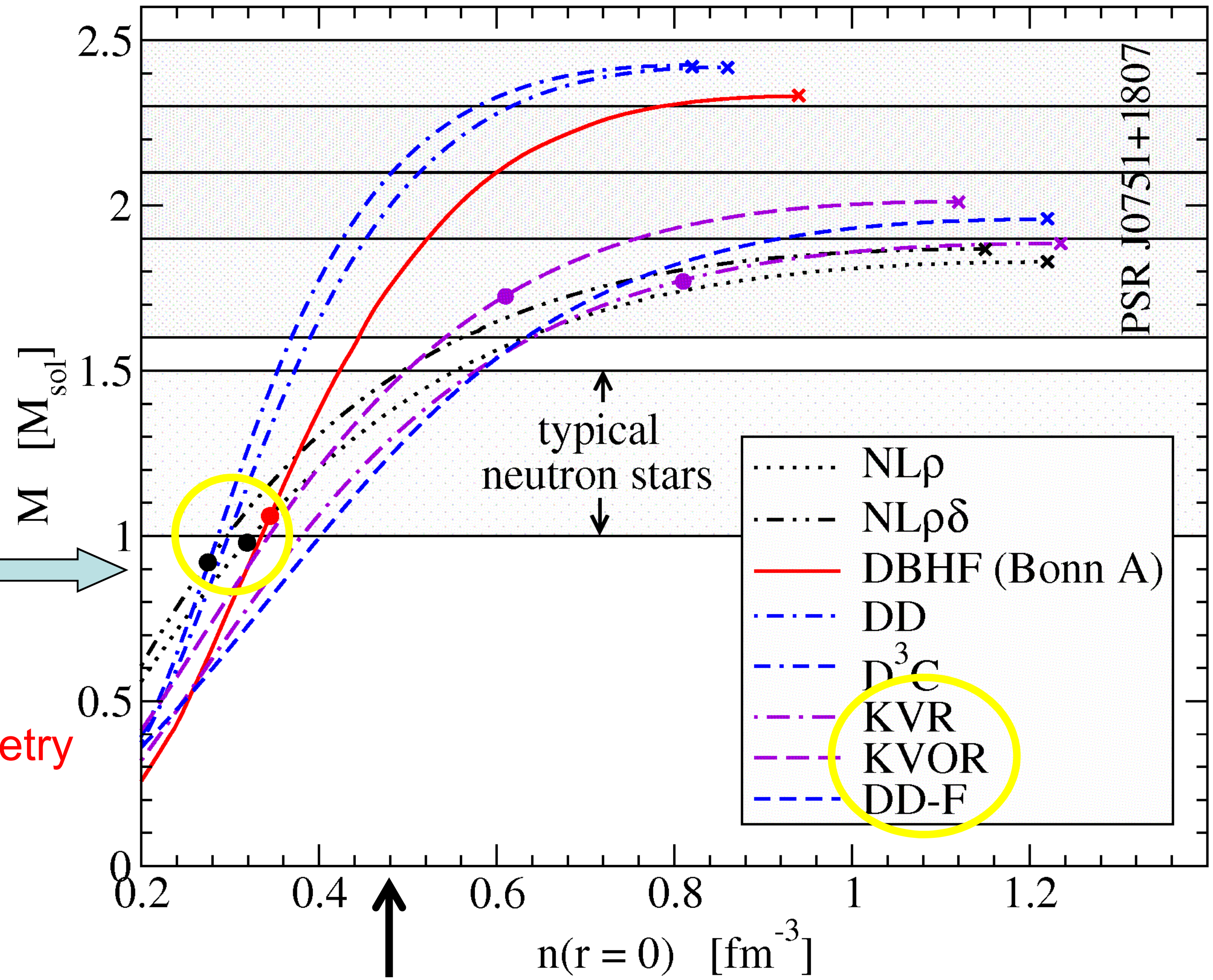
$M^{DU} > 1.35\text{-}1.5\ M_{sol}$ [Kolomeitsev, D.V. (2005), Klahn et al. (2006)]

**If were not so, why objects seen in soft X rays have low masses whereas most measured NS masses are in range $M \sim 1.3\text{-}1.5\ M_{sol}$ ?**

# DU -- information about nuclear EoS

should be modified

Information about density dependence of the symmetry energy

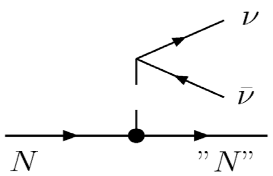


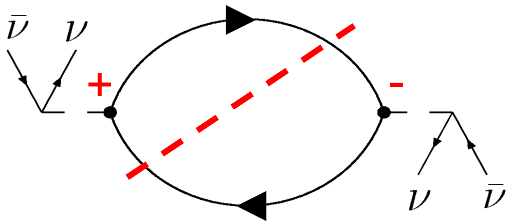

3 $n_0$

Klähn et al. PRC 74, 035802 (2006)

# Breaking and Formation of Cooper pairs (PBF)

normal matter




are forbidden

with free vertices and Green's functions

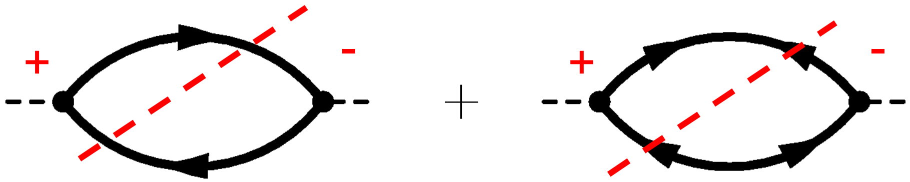
superfluid matter

**new "quasi"-one-nucleon-like** processes (one-nucleon phase space volume) become permitted

[Flowers, Ruderman, Sutherland, AJ 205 (1976),
D.V.& Senatorov, Sov. J. Nucl. Phys. 45 (1987) ]

Diagrams with normal and anomalous Green func.

Naive generalization:



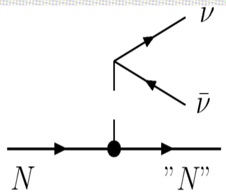

are allowed

$$(G^{-+})^a_b = 2\,\pi\,i\,f(\boldsymbol{p})\,[u_p^2\,\delta(E-E(p)) + v_p^2\,\delta(E+E(p))]\,\delta^a_b$$

$$(F^{-+})_{ab} = -2\,\pi\,i\,f(\boldsymbol{p})\,u_p\,v_p\,[\delta(E-E(p)) - \delta(E+E(p))]\,g_{ab}$$

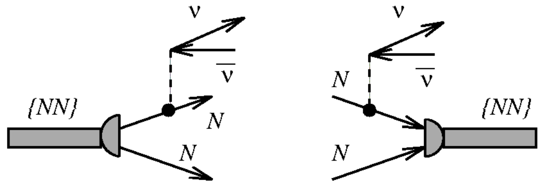
Next step! **Breaking and Formation of Cooper pairs (PBF)**

In normal matter one-nucleon processes




are forbidden

In superfluid ($T<T_c<0.1$-$1$ MeV)




are allowed

$$\epsilon_\nu \sim 10^{20} T_9^7 \, \xi_{nn}^2$$

$\Delta_{nn}$ is neutron gap and $\xi_{nn}=\exp(-\Delta_{nn}/T)$

Flowers, Ruderman, Sutherland, APJ (1976)

computed without inclusion of medium effects

emissivity of the process on "p" is $10^2$ times suppressed if one uses free p-vertices

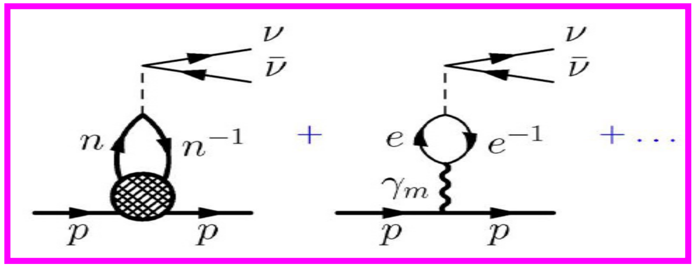
Then D.V., Senatorov Sov J. Nucl. Phys.(**1987**)

$$10^{28} \times \left(\frac{\Delta}{\mathrm{MeV}}\right)^7 \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \ \frac{\mathrm{erg}}{\mathrm{cm^3\,s}}$$

pre-factor $\Delta^7_{nn}$ rather than $T^7$ !

Already with a naive inclusion of in-medium effects

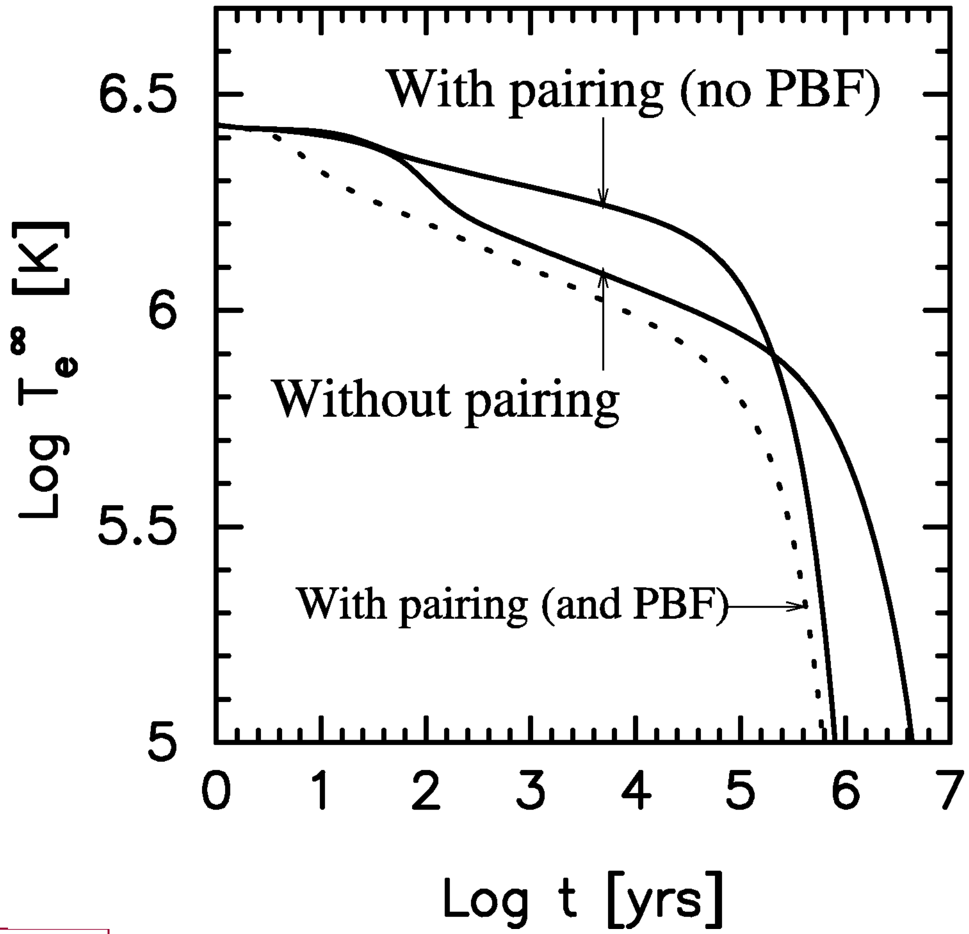
process on p is efficient: **purely medium effect**



# Effects of pairing on the neutron star cooling

✔ enhanced cooling

strongly depending on Δ(n)




**pair breaking and formation (PBF) processes are important for cooling!**

D.V.& Senatorov, (1987)

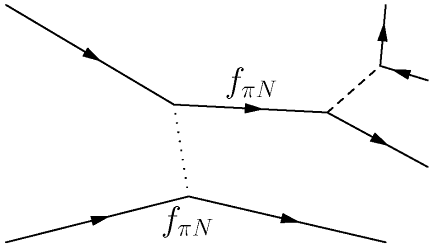
Schaab et al. (1996)

[Page, Geppert, Weber , NPA 777, 497 (2006)]

# Minimal cooling paradigm D.Page et al., D.G. Yakovlev et al.

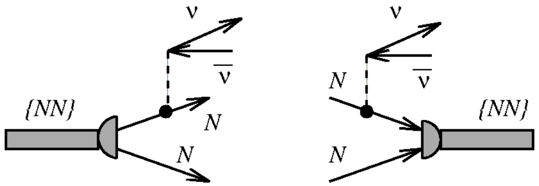
## Reactions in presence of pairing

**MU:**




$$10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}} \times e^{-2\Delta/T}$$

**PBF:**




$$10^{28} \times \left(\frac{\Delta}{\mathrm{MeV}}\right)^7 \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}$$

dominates for $T \ll \Delta$ (at least)

and for Δ> 0.1 MeV

**attempts to fit cooling data by fitting Δ (n) dependencies and using different $T_s$-$T_{in}$ for different NS**

They state that ***Info on internal neutrino emission is disguised by unknown composition of heat blanket***

- **Minimal cooling paradigm does not allow to explain all available data (problems or with slow coolers or with rapid coolers).**

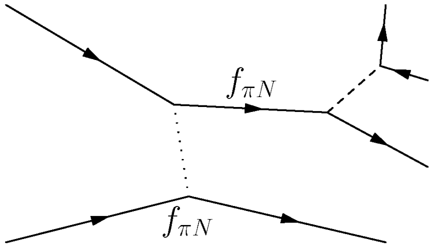
# Minimal (+exotics) scenario

## Neutrino emission reactions

for $T<T_c$

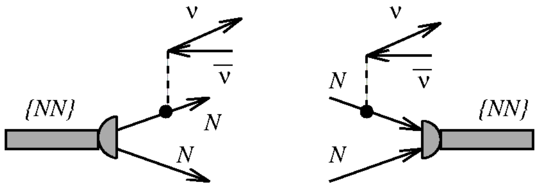
**minimal**

**MU:**




$$10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \quad \times e^{-2\Delta/T}$$

**PBF:**

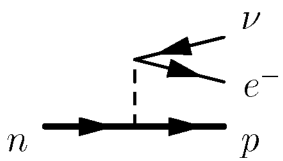



$$10^{28} \times \left(\frac{\Delta}{\text{MeV}}\right)^7 \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \quad \frac{\text{erg}}{\text{cm}^3\,\text{s}}$$

dominates for $T \ll \Delta$ (at least)

**exotics**

**DU:**

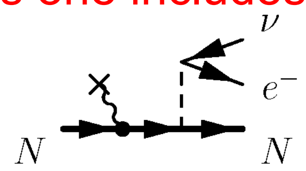



$$10^{27} \times T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \quad \times e^{-\Delta/T}$$

allowed if $n > n_c^{\text{DU}}$

Sometimes one includes PU

**PU:**




$$10^{27} \times T_9^6 \frac{|\varphi_c|^2}{m_\pi^2} \frac{\text{erg}}{\text{cm}^3\,\text{s}} \quad \times e^{-\Delta/T}$$

allowed if $n > n_c^{\text{PU}}$

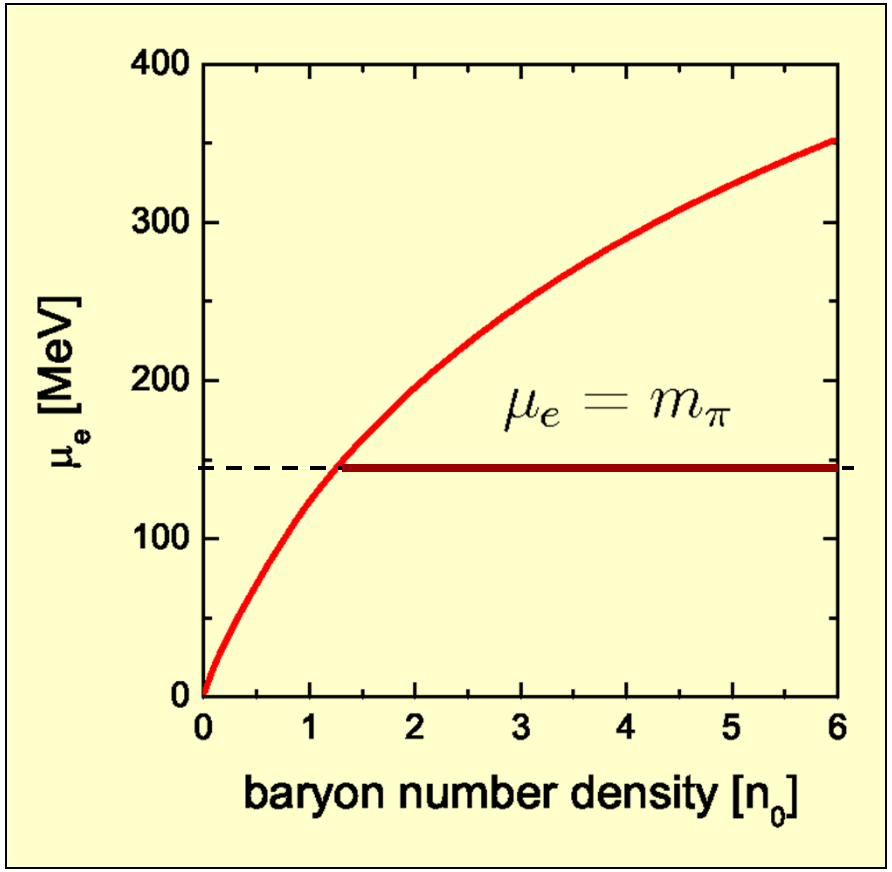
# P-wave pion condensate or s-wave condensate?

But P-wave pion condensation is purely in-medium effect

## Pionization (Bose-Einstein cond.)

Weak reactions start

$$e^- \longrightarrow \pi^- + \nu_e$$

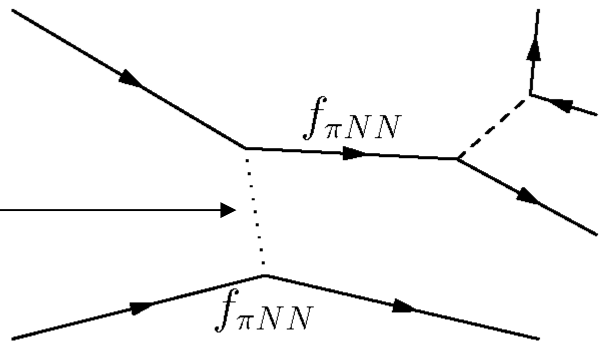



**In Minimal Cooling Scenario one silently ignores pionization! But within their concept it must be included! If included, pionization results in a very rapid cooling for all NS.**

# Inconsistencies of FOPE model

The only diagram in FOPE model which contributes to the MU and NB is

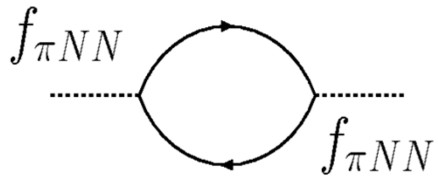
Free one-pion exchange →




**For consistency one needs to calculate corrections of the second-order in $f_{\pi NN}$ in other values. Otherwise -- problems with unitarity.**

**Pion polarization operator in dispersion relation at order $f_{\pi NN}^2$ :**

$$D^{-1}(\omega, k) = \omega^2 - m_\pi^2 - k^2 - \Pi_0^R(\omega, k, n) = 0$$




$$-D^{-1}(0, k_m \simeq p_{\rm F}) =: \omega^{*\,2}(k_m) < 0$$

measure of pion softening

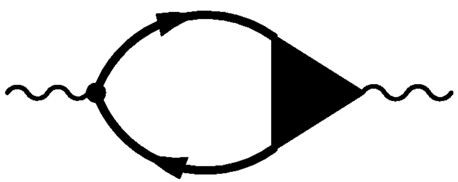
**Pion condensation already at $n > 0.3\ n_0$**

But there is no pion condensation in atomic nuclei

# Solution of the puzzle

One should replace FOPE by the full NN interaction,
essential part of which is due to MOPE
with vertices corrected by NN correlations.

*$NN^{-1}$ part of the pion polarization operator is*



$$\approx \Pi_0^R(\omega, k, n)\,\gamma(g', \omega, k, n)$$

suppressed by the factor $\gamma(g', \omega = 0, k \simeq p_F, n \simeq n_0) \simeq 0.35 \div 0.45.$

In isospin-symmetric matter no pion condensation at $n \lesssim n_0$

**Another inconsistencies of standard scenario:**

- it uses FOPE but allows for P-wave pion condensate processes for $n > n_c^{PU} > n_0$:
  **P-wave pion condensation arises only due to pion softening!**
- It does not include pionization processes (S-wave pion condensation) which are allowed already for low density, if πN interactions are ignored

# Ideas of Fermi liquid theory

**We need $NN$ interaction amplitude for $\rho \gtrsim \rho_0$** to describe excitations and their contribution to reactions

Couplings are strong $\rightarrow$ perturbation theory does not work.

**1956** L.Landau, Sov. JETP

Application to nuclei, see A.B.Migdal, Theory of finite Fermi systems, Willey,N.Y. 1967.

Idea: Separation of short and long scales.

Short scale relates to $r_\Lambda \simeq 0.2\, fm$, also to $\sigma$, $\rho$, $\omega$, $N$, $r_\sigma$, $r_\rho$, $r_\omega$, $r_N$.

$\longrightarrow$ local quantities = constants in momentum space.

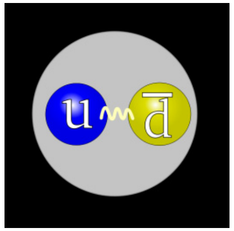
Long scale relates to low-lying excitations: $NN^{-1}$, $\pi$, $\Delta N^{-1}$ ($\mathrm{m}_\Delta - m_N \simeq 2m_\pi$).

All processes for $\omega, k \lesssim (2\div 3)m_\pi$ are treated explicitly.

Particular role of pion: $\rho_0 = 0.5$ in units $m_\pi = 1$, i.e. of order of one, $f_{\pi NN} = 1.01$ (strong coupling) $\longrightarrow$

strong medium effects for $\rho \gtrsim \rho_0$

Pion in vac.

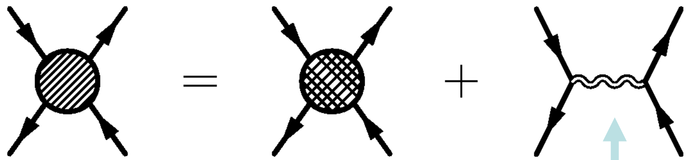



Pions in medium

pion (flower): in honor of Pean –doctor of Olimpic Gods.

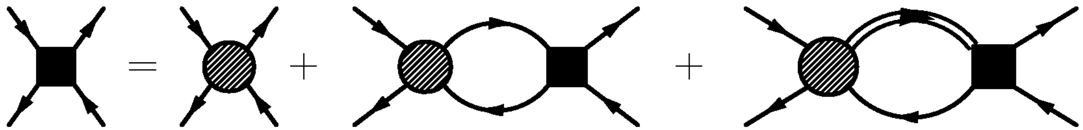
# Fermi liquid approach

- **explicit pionic degrees of freedom**

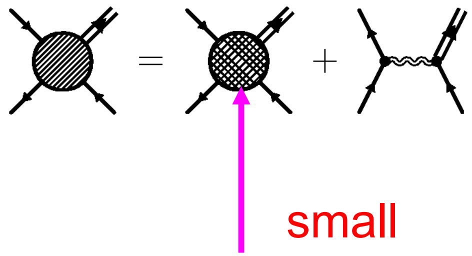


pion with residual (irreducible in $NN^{-1}$ and $\Delta N^{-1}$) s-wave $\pi$ N interaction and $\pi\pi$ scattering``

- **explicit $\Delta$ degrees of freedom** explicit nucleon-nucleon hole and Delta-nucleon hole degrees of freedom



Part of the interaction involving $\Delta$ isobar is analogously constructed:



small

- Reduction of the more local interaction to the point-like interaction

$$= C_0 \left( f_{12} + g_{12} \boldsymbol{\sigma_1 \sigma_2} \right),$$

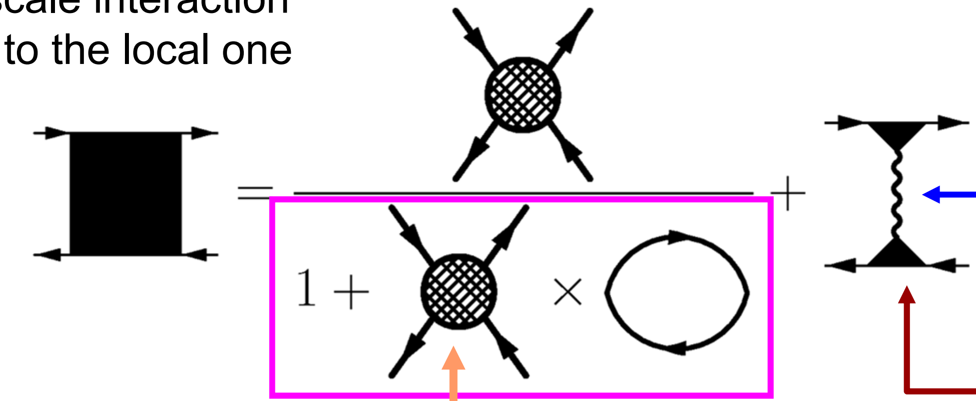
Assumption that the Landau-Migdal parameters, $f_{12}$, $g_{12}$, are constants Is a rough approximation.

# Low energy excitations in nuclear Fermi liquid (Landau-Migdal approach)

**Re-summed NN interaction**

based on a separation of long and short scales

provided short-scale interaction can be reduced to the local one




full pion propagator: enhancement of the amplitude

dressed vertex: suppression

Similar to Debye screening in plasma

Landau-Migdal parameters of short-range interaction are extracted from atomic nuclei

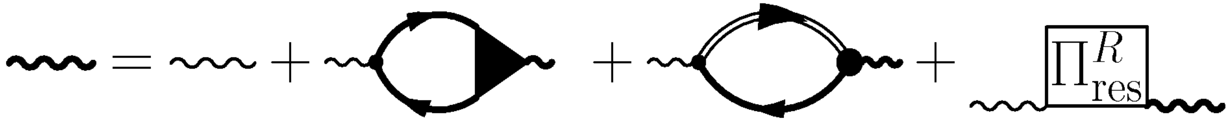
**Poles yield zero-sound modes in scalar and spin channels**

known phenomena in Fermi liquid

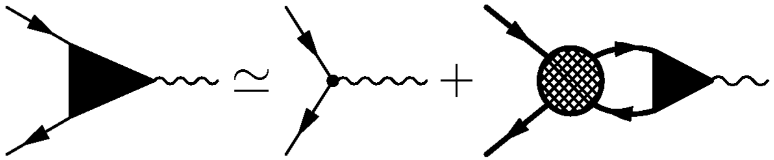
see Migdal et al., Phys.Rep. (1990)

# Virtual pion mode

Dyson equation for **the full retarded pion Green function**:




The $\pi N\Delta$ full-dot-vertex includes a phenomenological background correction due to presence of higher-lying resonances.
**The full $\pi NN$ vertex** takes into account $NN$ correlations:



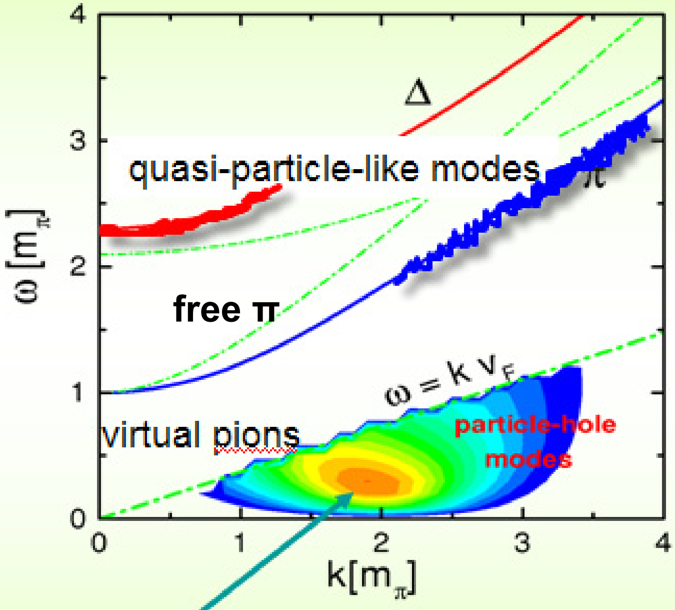
The value of the $NN$ interaction, e.g. in the neutral pion channel, is determined by the full pion propagator at small $\omega$ and $k \simeq p_{\rm Fn}$:

$$(\omega^*)^2(k) = -(D^R_\pi)^{-1}(\omega = 0, k, \mu_\pi).$$

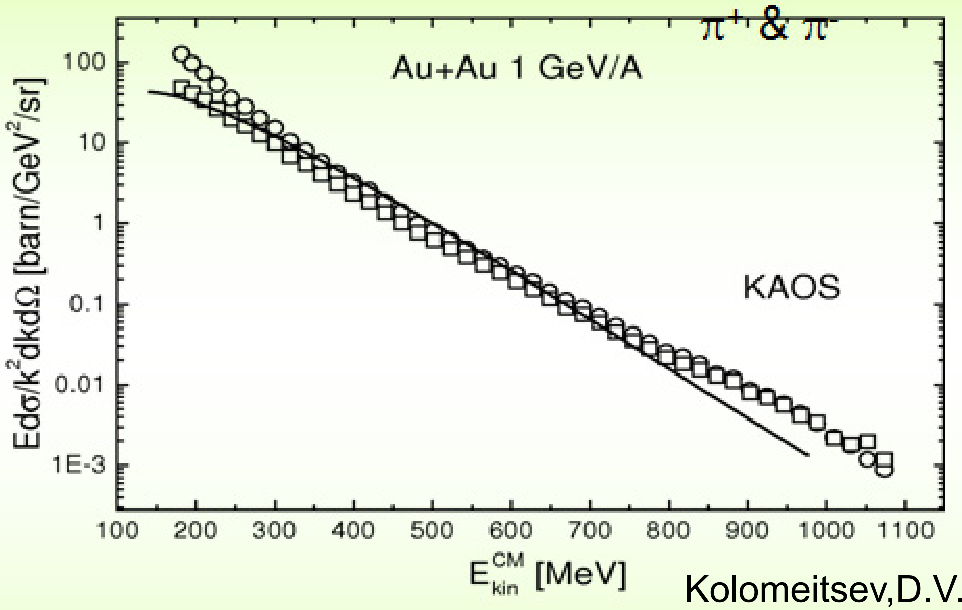
For $n > n_{c1}$ ($n_{c1} < n_0$) the quantity $\omega^*(k)$ has minimum for $k = k_m \simeq (0.9 \div 1)p_{\rm Fn}$.
The quantity $\omega^*(k_m)$ (hereafter $\omega^*$) has the meaning of the *effective pion gap*.

# Pion spectra in nuclear matter

Pion spectrum in nuclear matter at saturation.
similar for $\pi^0$ in neutron matter




Pion production in Au+Au collision with energy 1 GeV per nucleon




Kolomeitsev,D.V. (1996)

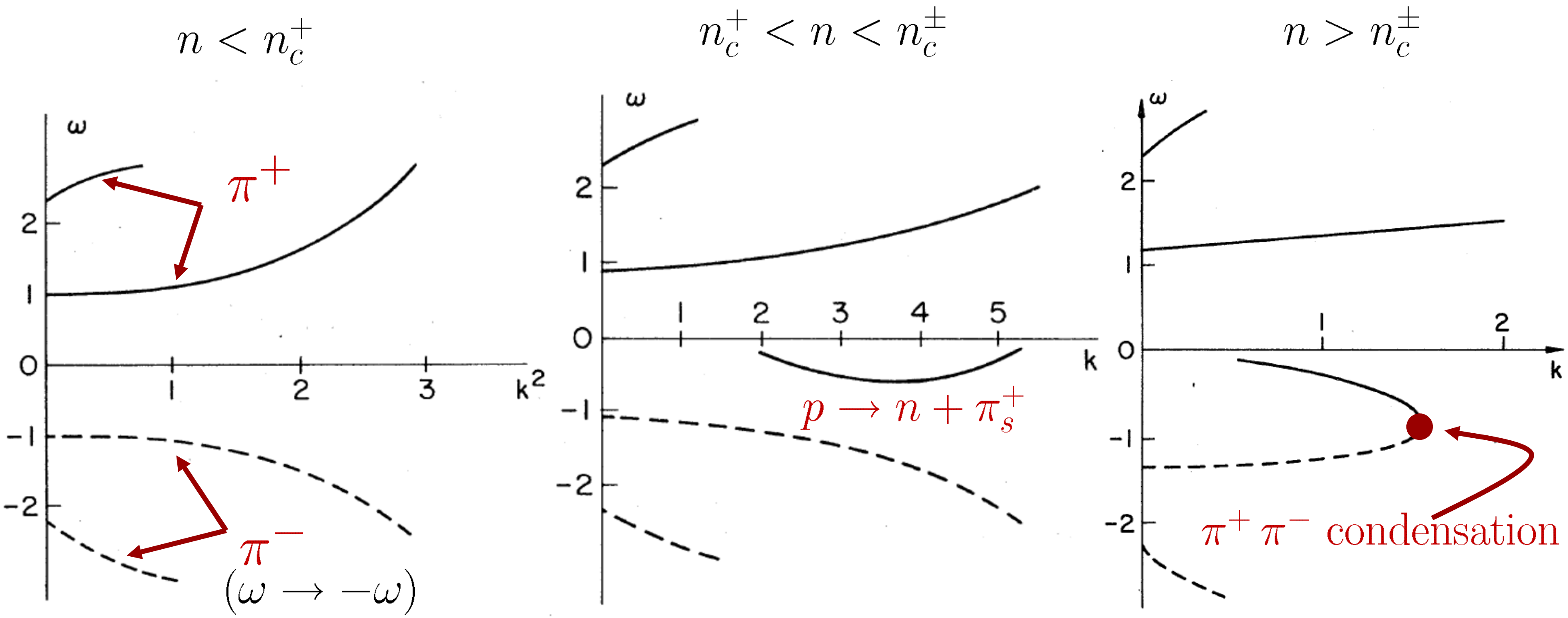
**Pions have short mean-free path and are radiated at freeze-out**

the smaller collision energy, the larger is in-medium effect

Possibility of the P-wave pion condensation in dense NS interiors : $\omega^2<0$ for $n>n_{cr}$

A.B. Migdal ZhETF (1971)

# Charged pion spectra in neutron matter and pion condensation




[Migdal,Markin,Mishustin, JETP (1974)]

$$n_c^+ \lesssim n_0\,, \quad n_c^\pm \sim (1-3)\,n_0\,, \quad n_c^{\pi^0} \sim (1-3)\,n_0$$

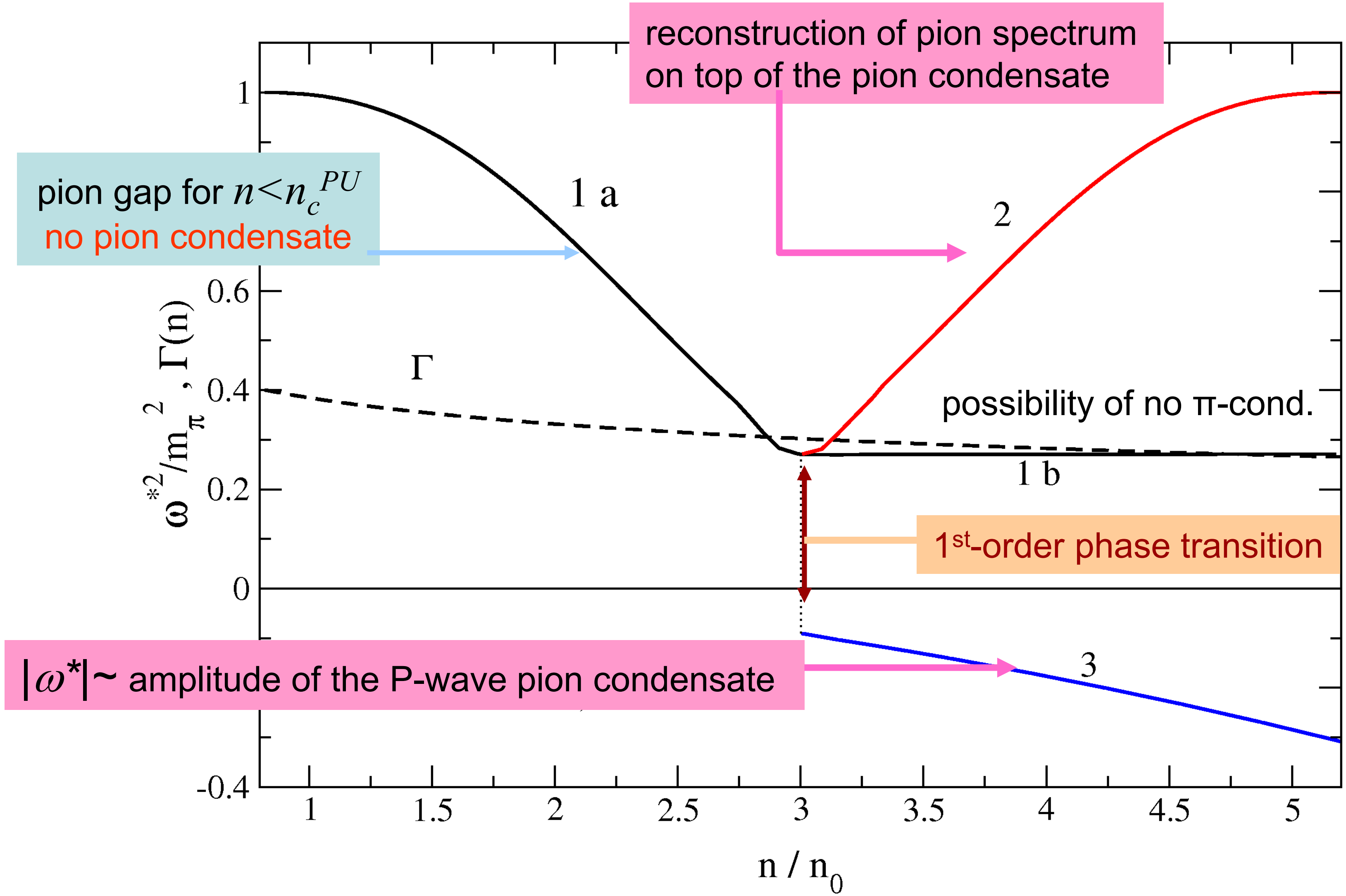
variational calculations [Akmal, Pandharipande, Ravenhall, PRC58 (1998)]:

pion condensate: $n_c \simeq 2\,n_0 \quad N = Z$

neutral pion condensate: $n_c \simeq 2\,n_0 \quad N = Z\,, \qquad n_c = 1.3\,n_0 \quad N >> Z$

# Pion softening with increase of the density

$$\omega^{*2}(k) = -[D_\pi^R(\omega = 0, k, \mu_\pi)]^{-1}$$




Γ –vertex suppression factor

From the cooling fit $n_c$ >1.5-2 $n_0$ for stif EoS

# Repulsive $\pi^-$ N interaction in S-wave

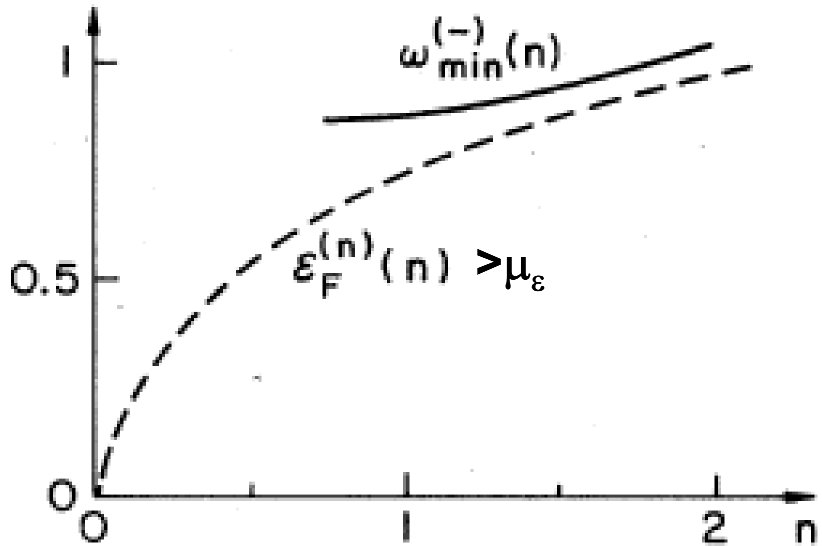
$$\Pi_S(\omega) = -T^{(-)}(\omega)\,(\rho_p - \rho_n) - T^{(+)}(\omega)\,(\rho_p + \rho_n)$$

repulsive in neutron reach matter

repulsive for $\omega > m_\pi$

$$T^{(-)}(\omega) = \frac{\omega}{2\,f_\pi^2}\left(1 + C\,\frac{\omega^2}{8\,\pi^2\,f_\pi^2}\right) \qquad T^{(+)}(\omega) = \frac{\sigma_{\pi N}}{f_\pi^2}\left(1 - \frac{\omega^2}{m_\pi^2}\right)$$

$$C > 0$$



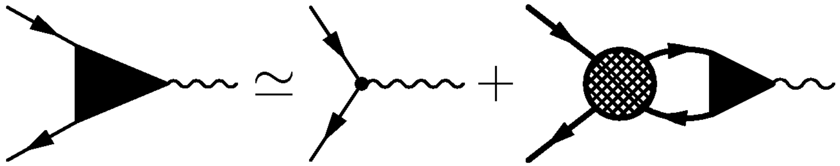

**No S-wave pion condensation (Migdal 1973)**
**Only P-wave pion condensation is allowed!**

# MEDIUM EFFECTS IN NEUTRINO PRODUCTION

In the medium many reaction channels are opened up

# Re-summed weak interaction

The weak coupling vertex is renormalized in medium:



wavy line corresponds to weak current

For the $\beta$-decay:

$$V_\beta = \frac{G}{\sqrt{2}} \left[ \widetilde{\gamma}(f')\, l_0 - g_A\, \widetilde{\gamma}(g')\, \boldsymbol{l\sigma} \right]$$

For processes on the neutral currents $N_1 N_2 \to N_1 N_2 \nu \bar{\nu}$

$$V_{nn} = -\frac{G}{2\sqrt{2}} \left[ \gamma(f_{nn})\, l_0 - g_A\, \gamma(g_{nn})\, \boldsymbol{l\sigma} \right]$$

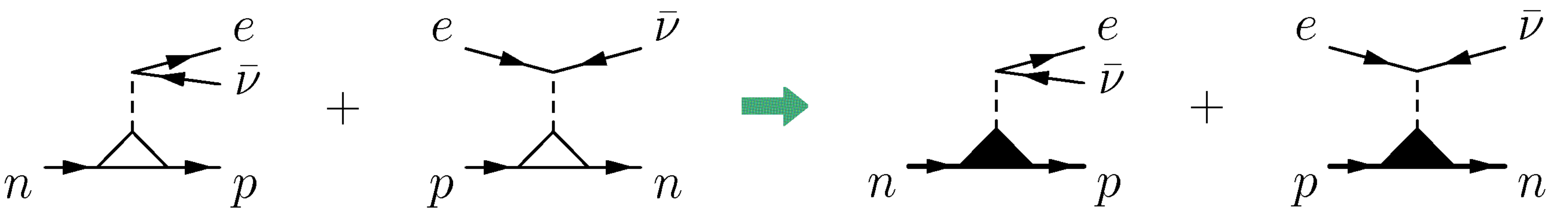
$$V_{pp}^N = \frac{G}{2\sqrt{2}} \left[ \kappa_{pp}\, l_0 - g_A\, \gamma_{pp}\, \boldsymbol{l\sigma} \right]$$

with the correlation functions

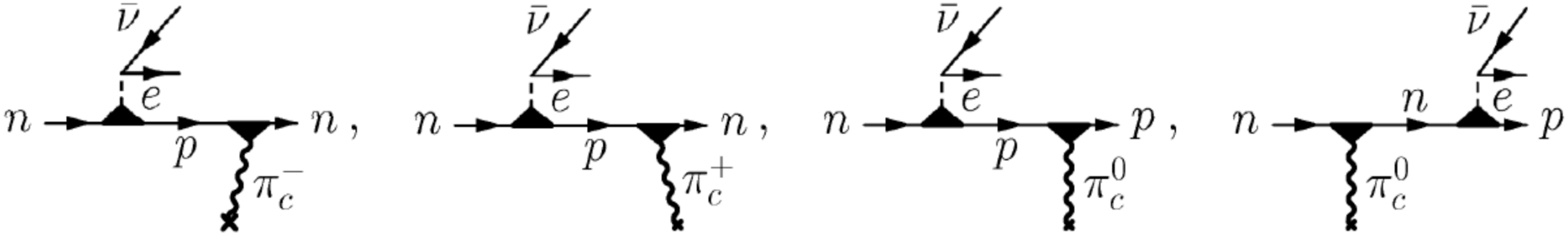
$$\kappa_{pp} = c_V - 2 f_{np}\, \gamma(f_{nn})\, C_0\, L_{nn}, \;\; \gamma_{pp} = (1 - 4\, g\, C_0\, L_{nn})\; \gamma(g_{nn}),$$

[D.V., Senatorov, Sov. J. Nucl. Phys. 45 (1987)]

# Proper DU processes




Due to full vertices ➡ a factor $\Gamma^2_{\mathrm{w-s}}$ in emissivity.
(rather minor modification, since $\omega \simeq p_{\mathrm{F},e} \gg q \sim T$).

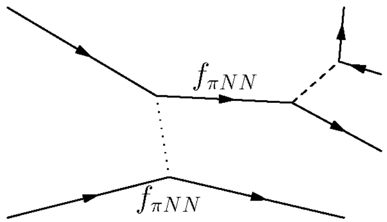


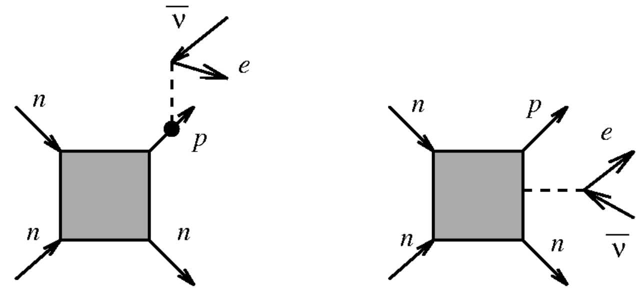

with full vertices: $\epsilon_\nu \sim 10^{26} \Gamma_s^2 \Gamma^2_{\mathrm{w-s}} \, T_9^6 (n/n_0)^{1/3} \, \frac{\mathrm{erg}}{\mathrm{cm^3 \ sec}}$

➡ $\Gamma_s^2 \Gamma^2_{\mathrm{w-s}} \sim 10^{-1} - 10^{-2}$

# Medium effects in two-nucleon processes

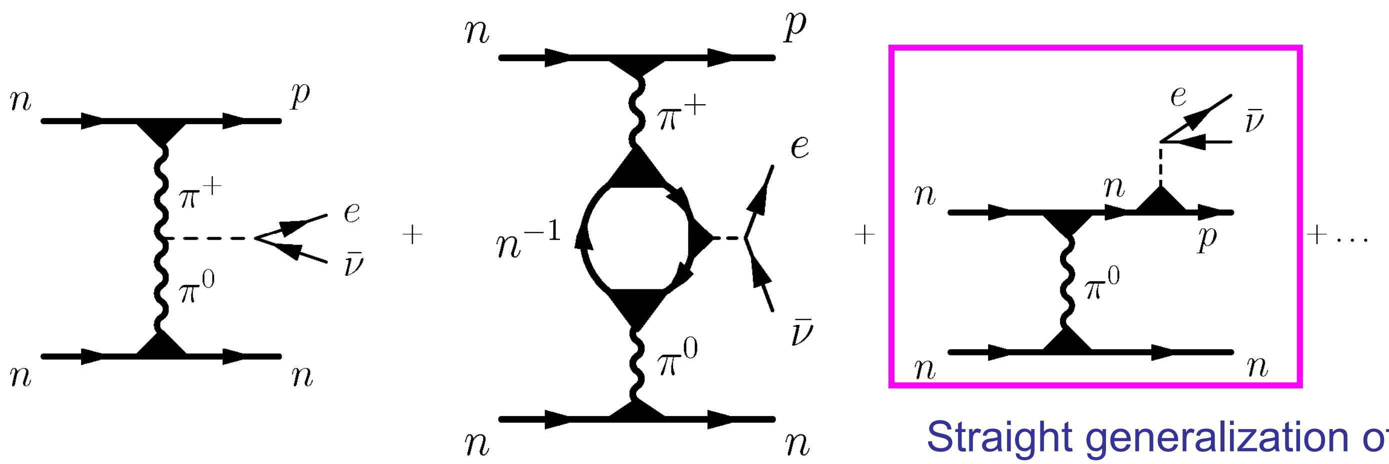
MU(FOPE) :




---

MOPE




Straight generalization of MU

**emissivity:** larger → smaller

Very important in our scenario!

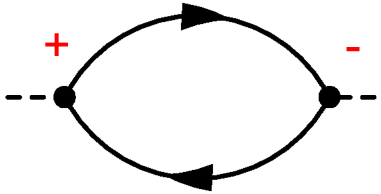
$$\frac{\epsilon_\nu[\mathrm{MMU}]}{\epsilon_\nu[\mathrm{MU}]} \sim 3 \left(\frac{n}{n_0}\right)^{10/3} \frac{[\Gamma(n)/\Gamma(n_0)]^6}{[\omega^*(n)/m_\pi]^8}$$

**Very strong density dependence**

enhancement factor $\sim 10^3$ -- $10^5$ for $n \sim (1.5\text{-}4)\, n_0$

# Vector current conservation

normal matter



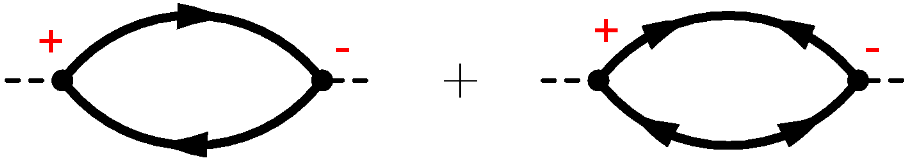

free vertices and Green's functions

The Ward identity is fulfilled and the current is conserved

superfluid matter

Diagrams with normal and anomalous Green func.

Naive generalization:



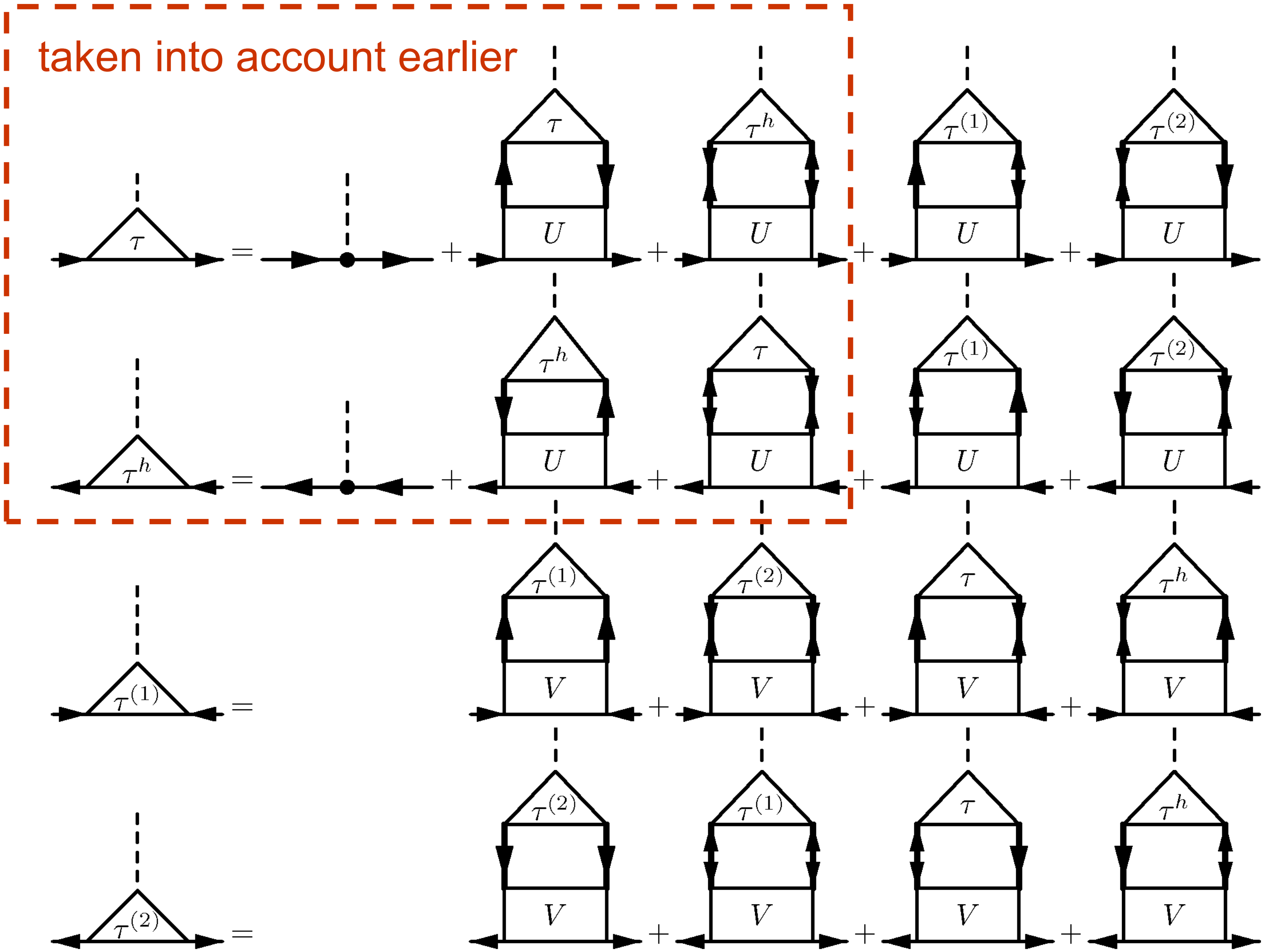

Gap appears due to a non-trivial self-energy

Vertex must be modified accordingly. Otherwise the vector current is not conserved

free vertices and non-interacting quasi-particles with a gapped spectrum $E_p^2 = \epsilon_p^2 + \Delta^2$

Problem!

# Larkin-Migdal equations




Cannot be written in matrix form in Nambu-Gor'kov space since U ≠ V

# Emissivity in pair formation breaking reactions

$$\epsilon^{(n)}_{\nu\nu,A} \simeq \left(1 + \frac{11}{21} - \frac{2}{3}\right) v^2_{\mathrm{F},n}\, \epsilon^{(0n)}_{\nu\nu,A}$$

moderate suppression

Kolomeitsev,D.V. (2008)

$$\epsilon^{(n)}_{\nu\nu,V} \simeq \frac{4}{81}\, v^4_{\mathrm{F},n}\, \epsilon^{(0n)}_{\nu\nu,V}$$

strong suppression

Leinson,Perez (2006),Kolomeitsev,D.V. (2008)

with free vertices

$$\epsilon^{(0n)}_{\nu\nu} = \frac{4\rho_n\, G^2 \Delta^7_n}{15\,\pi^3} I\left(\frac{\Delta_n}{T}\right) \qquad I(z) = \int_1^\infty \frac{\mathrm{d}y\, y^5}{\sqrt{y^2-1}} e^{-2zy} \,,$$

$$R(\mathrm{nPFB}) = \frac{\epsilon^{\mathrm{nPBF}}_{\nu\nu}}{\epsilon^{(0n)}_{\nu\nu}} \simeq \frac{\epsilon^{\mathrm{nPBF}}_{\nu\nu,A}}{\epsilon^{(0n)}_{\nu\nu}} \simeq \frac{6}{7}\, g^{*\,2}_A\, v^2_{\mathrm{F},n} = F_n\, v^2_{\mathrm{F},n} \,.$$

$$\epsilon^{p\mathrm{PBF}}_{\nu\nu,A} \simeq \epsilon^{(0p)}_{\nu\nu}\, \tfrac{6}{7} g^{*\,2}_A v^2_{\mathrm{F},p}.$$

Main contribution is due to the axial current.

Suppression is of the order ~0.1

# Thermal conductivity

with taking into account of medium effects

Blaschke, Grigorian, D.V. 2013

lepton term
with inclusion of
Landau damping

$$\kappa_e = 8.5 \cdot 10^{21} \left( \frac{p_{\mathrm{F},e}}{\mathrm{fm}^{-1}} \right)^2 f_e \,\mathrm{ergs\, s^{-1} cm^{-1} K^{-1}}, \quad (3)$$

$$f_e \simeq \frac{2.7}{e^{1.3T/T_{cp}} - 1},$$

yields suppression of previous Baiko result

for $T < T_{cp}$ and $f_e = 1$ for $T > T_{cp}$. For simplicity a contribution of muons is neglected.

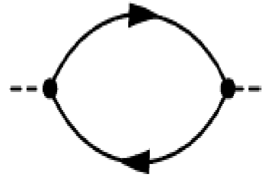
P. S. Shternin and D. G. Yakovlev, Phys. Rev. D **75**, 103004 (2007).

nn- term with inclusion
of pion softening

$$\kappa_b = \kappa_b^{\mathrm{SY}} \left(\omega^*(n)/m_\pi\right)^3 \left(\Gamma(n_0)/\Gamma(n)\right)^4 n_0/n$$

# NS Cooling after 2004. New bits and pieces

1. **Calculation of thermal conductivity** by Shternin and Yakovlev 2007
With inclusion of Debye screening in the photon propagator (medium effect)
decrease in comparison with the previous results by Baiko et al (2001)



**In our "nuclear medium cooling scenario" we include loops and dressed vertices for all species !**

[Blaschke, Grigorian, D.V. 2013]

2. **New measurement of NS masses**: PSR J1614-2230: $M=1.97+-0.04\ M_{sol}$
[P.Demorest et al., Nature 467 (2010)]
PSR J0348-04232: $M=2.01+-0.04\ M_{sol}$
[J. Antoniadis et al., Science (2013) ]

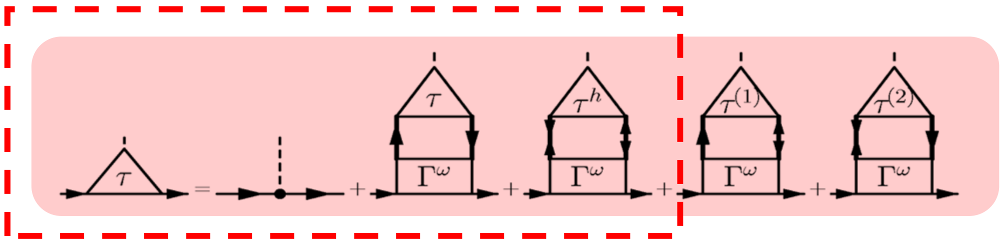
Lightest NS PSR J1807-2500B: M=1.2064+-0.0020

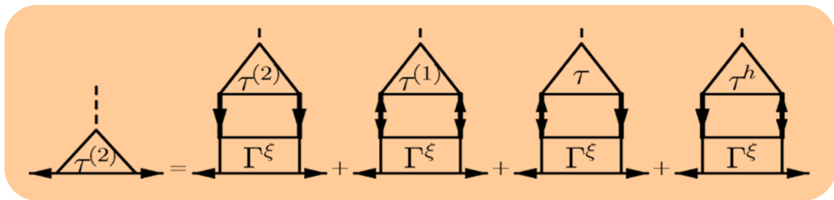
**NS masses can be very different. Need for a stiffer EoS**

3. **Improved calculations PFB reaction**: exact conservation of baryon current requires self consistent inclusion of vertex corrections. For $1S_0$ pairing:

vector term: strong suppression $\sim v_{F,n}^4 \sim 0.01$
Leinson,Perez PLB (2006),
Kolomeitsev,D.V. PRC (2008)

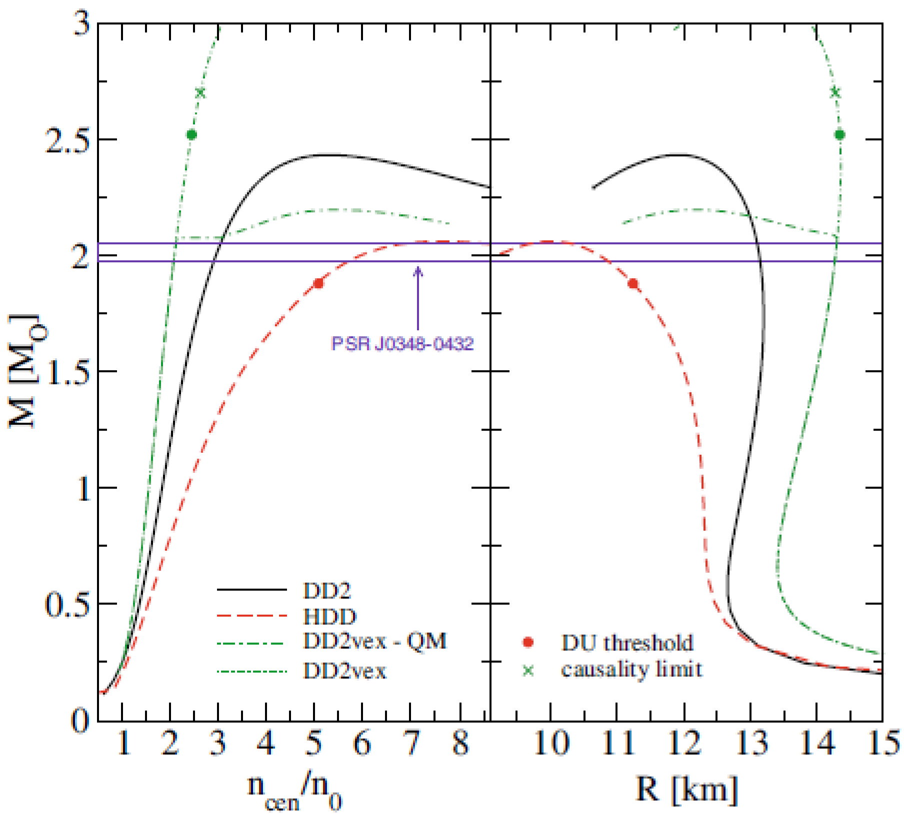
axial-vector term: moderate suppression $\sim v_{F,n}^2 \sim 0.1$
Kolomeitsev,D.V. PRC (2008,2010)

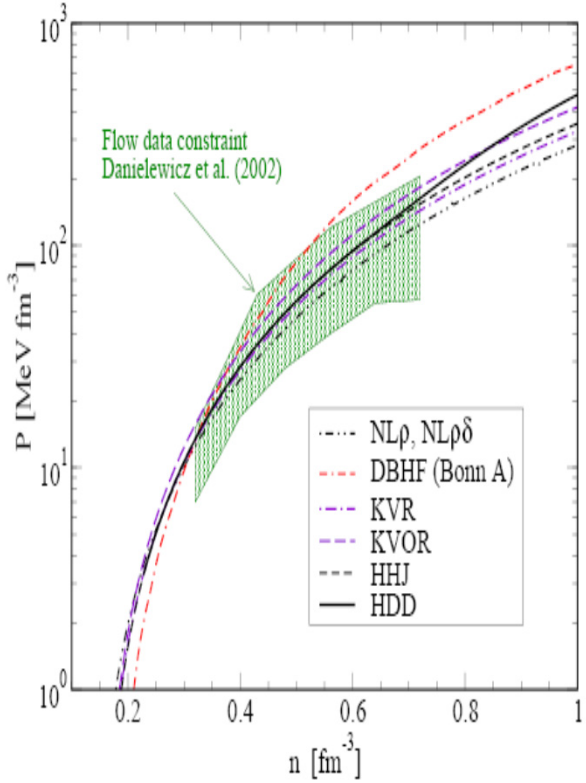




# NS Mass-central density plot for EoSs that we use

Blaschke, Grigorian, D.V. 2013




We incorporated excluded volume effect: HDD EoS is very close to KVOR, APR EoS for n<4 $n_0$ (thus we satisfy the HIC-flow constraint) but EoS stiffens for n>4$n_0$ increasing $M_{max}$. DD2 does not fulfil the flow constraint.

# Nuclear medium cooling scenario

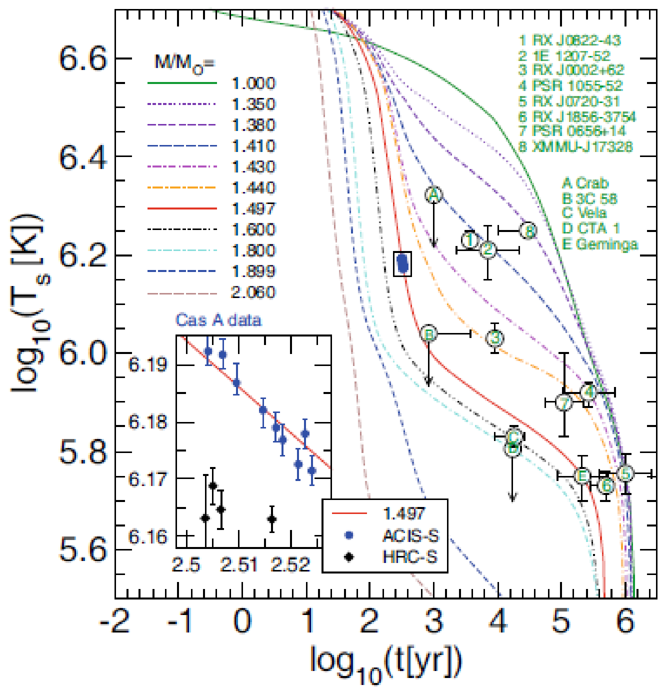
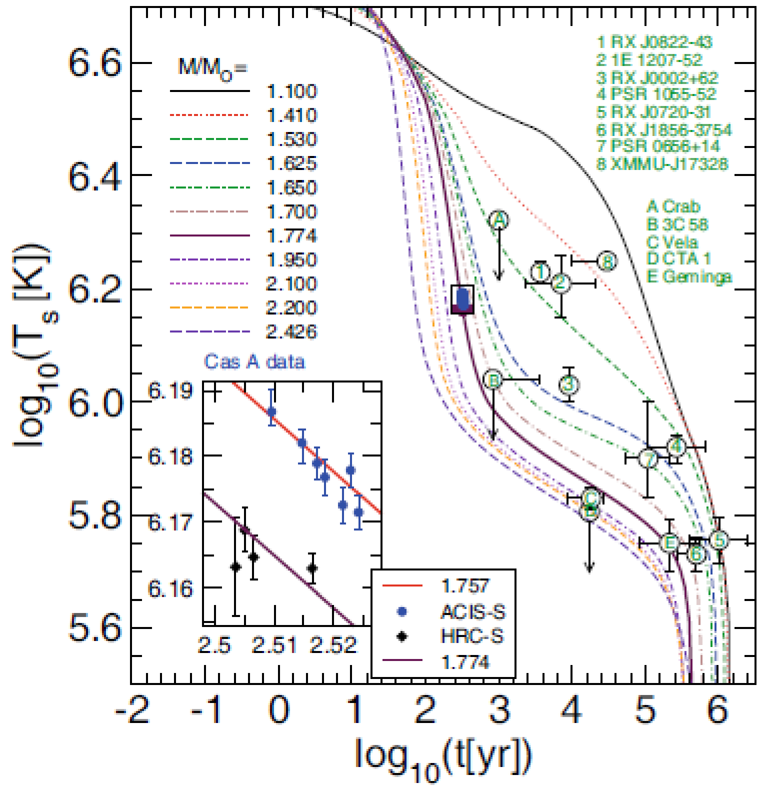



**Fig. 4.** Cooling curves for a NS sequence according to the hadronic HDD EoS; $T_s$ is the redshifted surface temperature, $t$ is the NS age. The effective pion gap is given by the solid curve 1a+1b in fig. 2, $n_c^{\pi} = 3n_0$. The $1S_0$ *pp* pairing gap corresponds to model I. The mass range is shown in the legend. Comparison with Cas A ACIS-S and HRC-S data is shown in the inset. Cooling ACIS-S data for Cas A are explained with a NS mass of $M = 1.497M_\odot$.




An example for DD2 EoS

# Deficiencies of Minimal cooling scenario avoided in Nuclear Medium Cooling Scenario

- **Calcul. In Minimal Cooling Scenario silently ignore pionization! But within their concept it must be included! In Nucl. Med. Cool. Scenario pionization does not occur only owing to interactions.**
- **FOPE model and P-wave pion condensation are incompatible. MediumOPE model is compatible with pion cond.**
- **Calcul. in Minimal Cooling Scenario includes medium effect on electron heat conductivity and on vector current in PBF process but ignore many other important medium effects. Most important in-medium effects are included only within Nucl. Med. Cool. Scenario.**

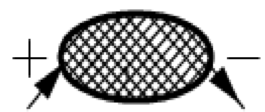
## **General consideration:** Knoll, D.V. Ann. Phys. 249 (1996)

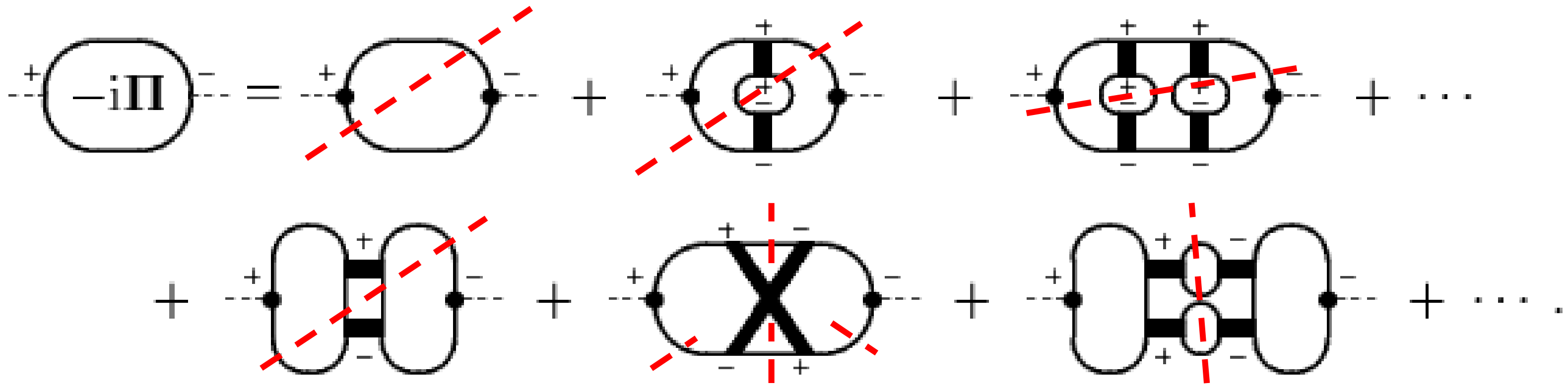
# white body radiation problem

**Direct reactions from piece of matter ($\nu$ in NS, e+e-, γ, $K^+$ in HIC)**

expansion in full non-equilibrium $G^{-+}$

$$\frac{dW}{d^3q/[(2\pi)^3 2\omega_q]} = -i\Pi^{-+} =$$





Only for low $T \ll \varepsilon_F$, quasiparticle approximation is valid

(each $G^{-+}$ yields $T^2$, allows to cut diagrams over $G^{-+}$ )

For soft radiation: quasiclassics (all graphs in first line are of the same order):LPM effect

Белка песенки поет, да орешки все грызет
А орешки не простые, в них скорлупки – золотые,
Ядра – чистый изумруд,
*но, быть может, люди врут*
*А. С. Пушкин*

Squirel sings amazing song,
Nut is puzzle, must be solved!
Bulk is pion condensate,
Shell from pasta phase is made,
**Kernel is pure emerald**,
Radiates neutrino light,
And who knows if I am right.

*from A.S. Pushkin (in my frivolous translation ☺)*

a lot of work still remains