Helmholtz International Summer School
Nuclear Theory and Astrophysical Applications
July 10-22, 2017

II. Nuclear physics aspects of neutrinoless double beta decay

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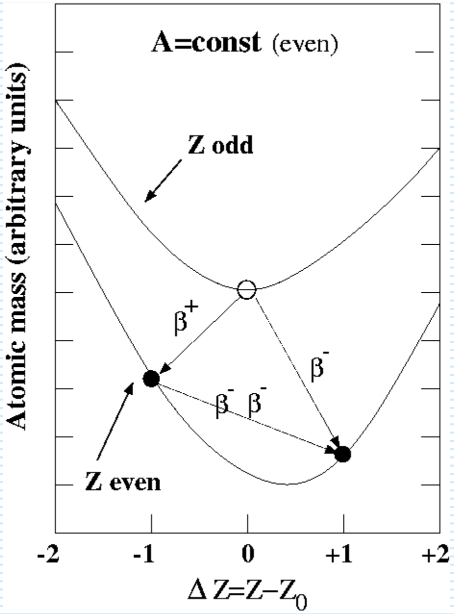
OUTLINE

- ***Introduction***
- **Nuclear matrix elements for the 0νββ-decay**
  different approaches
- **QRPA with isospin restoration**
  Is there a proportionality between 0νββ-decay and 2νββ-decay NMEs?
- **Effect of quenching of g_A on the 0νββ-decay half-life**
  Improved description of the 2νββ-decay => a possible way to fix value of g_A
- **Nuclear structure studies within schematic models**
  violation of SU(4) symmetry, non-linear QRPA
- ***Conclusions***

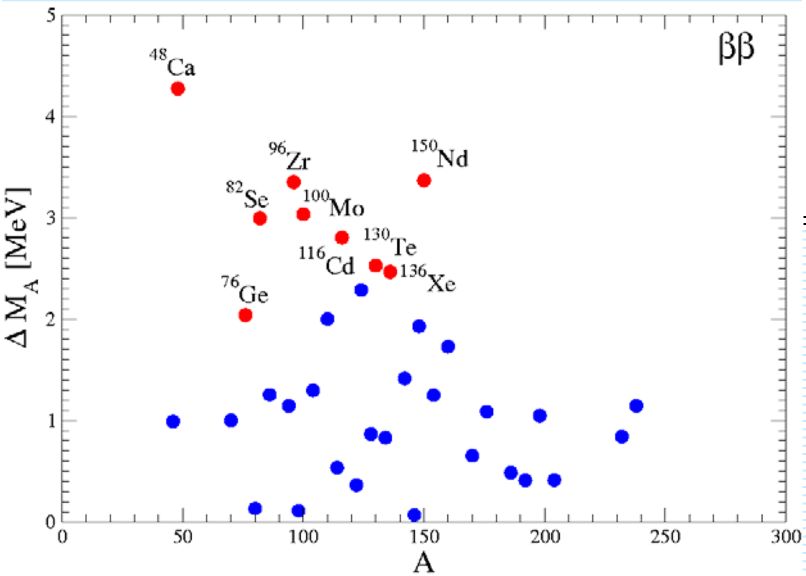
Acknowledgements: **A. Faesler (Tuebingen), P. Vogel (Caltech), J. Terasaki (CTU Prague), M. Krivoruchenko (ITEP Moscow), S. Petcov (SISSA), D. Štefánik (Comenius U.) …**

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

transition	$G^{01}(E_0,Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?



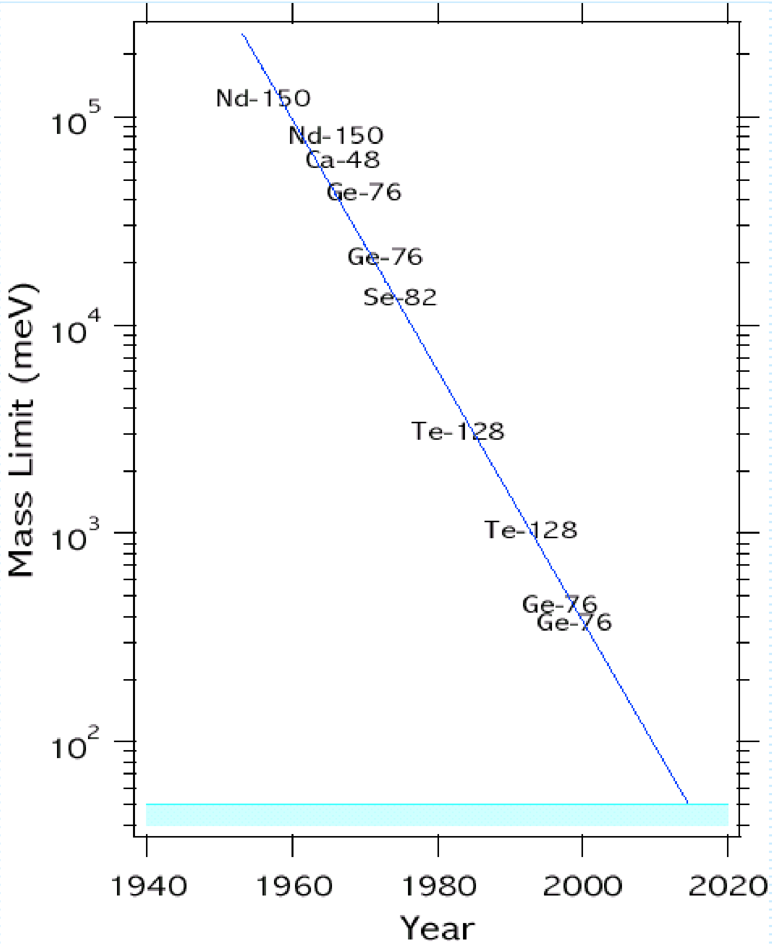
The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved

S.R. Elliott, P. Vogel, Ann.Rev.Nucl.Part.Sci. 52, 115 (2002)

1) *What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).*

2) *How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements.*

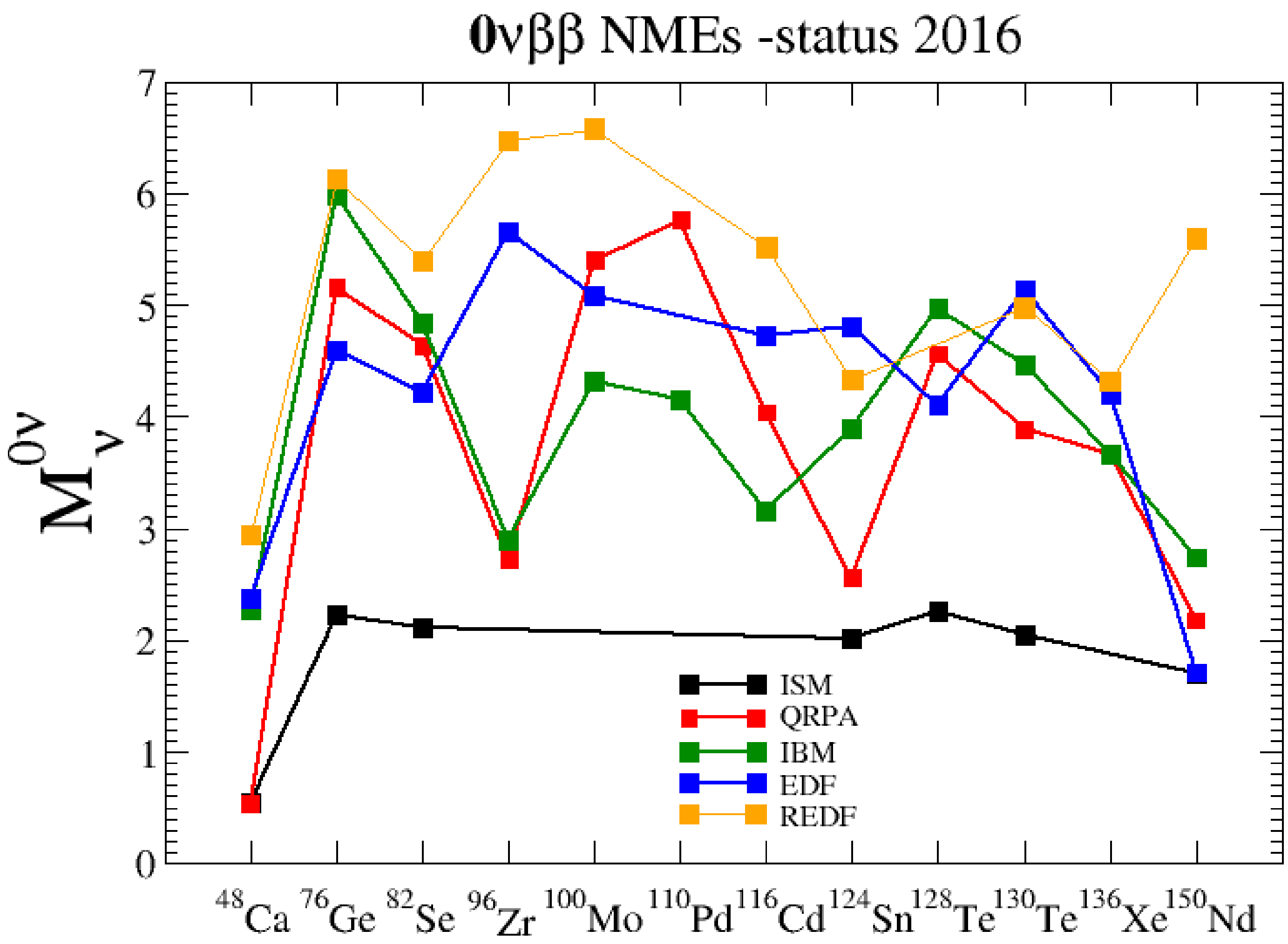


The $0\nu\beta\beta$-decay: A nuclear physics problem

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

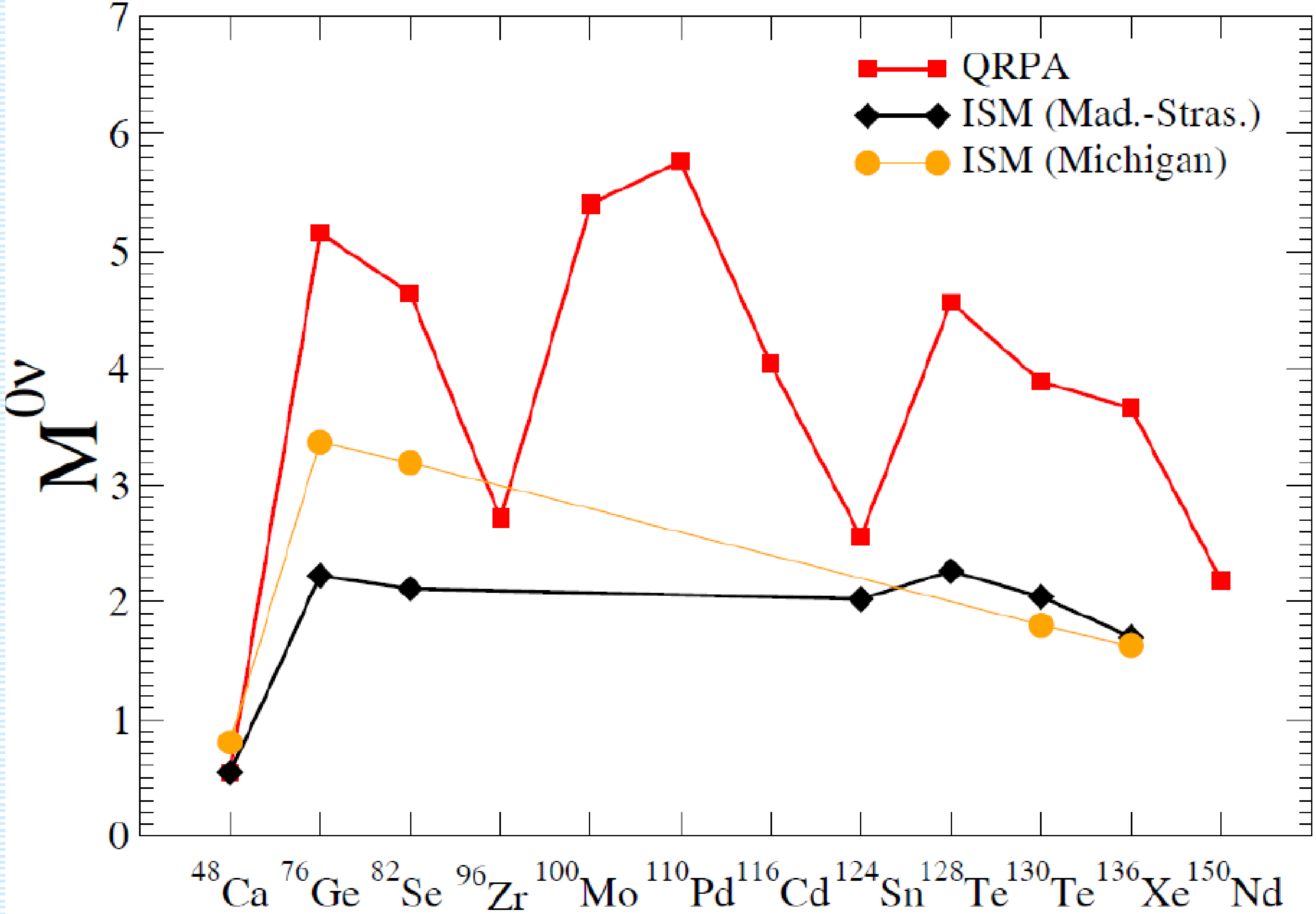
It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.



	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

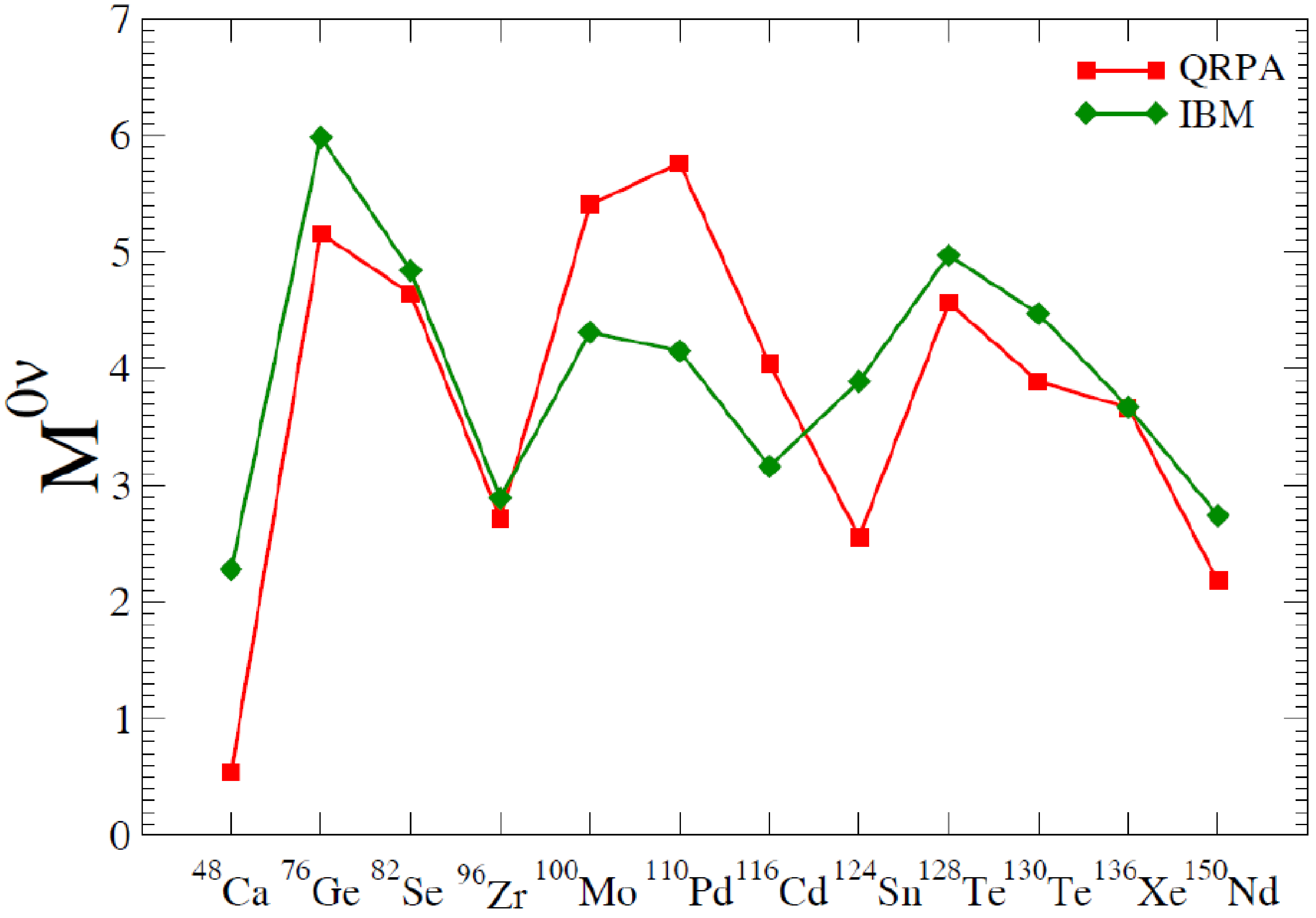
QRPA versus Interacting Shell Model



ISM: Menendez et al. NPA 818 (2009) 139

ISM: Brown, Horoi etc., Phys.Rev.Lett. 113 (2014) 262501 …

QRPA versus IBM



IBM: Barea, Kotila, Iachello, PRC (2013) 014315

Method	g_A	src	$M_\nu^{0\nu}$ ^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd
ISM-StMa	1.25	UCOM	0.85	2.81	2.64			
ISM-CMU	1.27	Argonne	0.80	3.37	3.19			
		CD-Bonn	0.88	3.57	3.39			
IBM	1.27	Argonne	1.75	4.68	3.73	2.83	4.22	4.05
QRPA-TBC	1.27	Argonne	0.54	5.16	4.64	2.72	5.40	5.76
		CD-Bonn	0.59	5.57	5.02	2.96	5.85	6.26
QRPA-Jy	1.26	CD-Bonn		5.26	3.73	3.14	3.90	6.52
dQRPA-NC	1.25	without		5.09				
PHFB	1.25	Argonne				2.84	5.82	7.12
		CD-Bonn				2.98	6.07	7.42
NREDF	1.25	UCOM	2.37	4.60	4.22	5.65	5.08	
REDF	1.25	without	2.94	6.13	5.40	6.47	6.58	
Mean value			1.34	4.55	4.02	3.78	5.57	6.12
variance			0.81	1.20	0.91	2.49	0.58	1.78

Method	g_A	src	$M_\nu^{0\nu}$ ^{116}Cd	^{124}Sn	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
ISM-StMa	1.25	UCOM		2.62		2.65	2.19	
ISM-CMU	1.27	Argonne		2.00		1.79	1.63	
		CD-Bonn		2.15		1.93	1.76	
IBM	1.27	Argonne	3.10	3.19	4.10	3.70	3.05	2.67
QRPA-TBC	1.27	Argonne	4.04	2.56	4.56	3.89	2.18	
		CD-Bonn	4.34	2.91	5.08	4.37	2.46	3.37
QRPA-Jy	1.26	CD-Bonn	4.26	5.30	4.92	4.00	2.91	
dQRPA-NC	1.25	without				1.37	1.55	2.71
PHFB	1.27	Argonne			3.90	3.81		2.58
		CD-Bonn			4.08	3.98		2.68
NREDF	1.25	UCOM	4.72	4.81	4.11	5.13	4.20	1.71
REDF	1.25	without	5.52	4.33		4.98	4.32	5.60
Mean value			4.34	3.07	4.34	3.42	2.59	3.01
variance			0.79	1.01	0.23	1.67	1.10	1.34

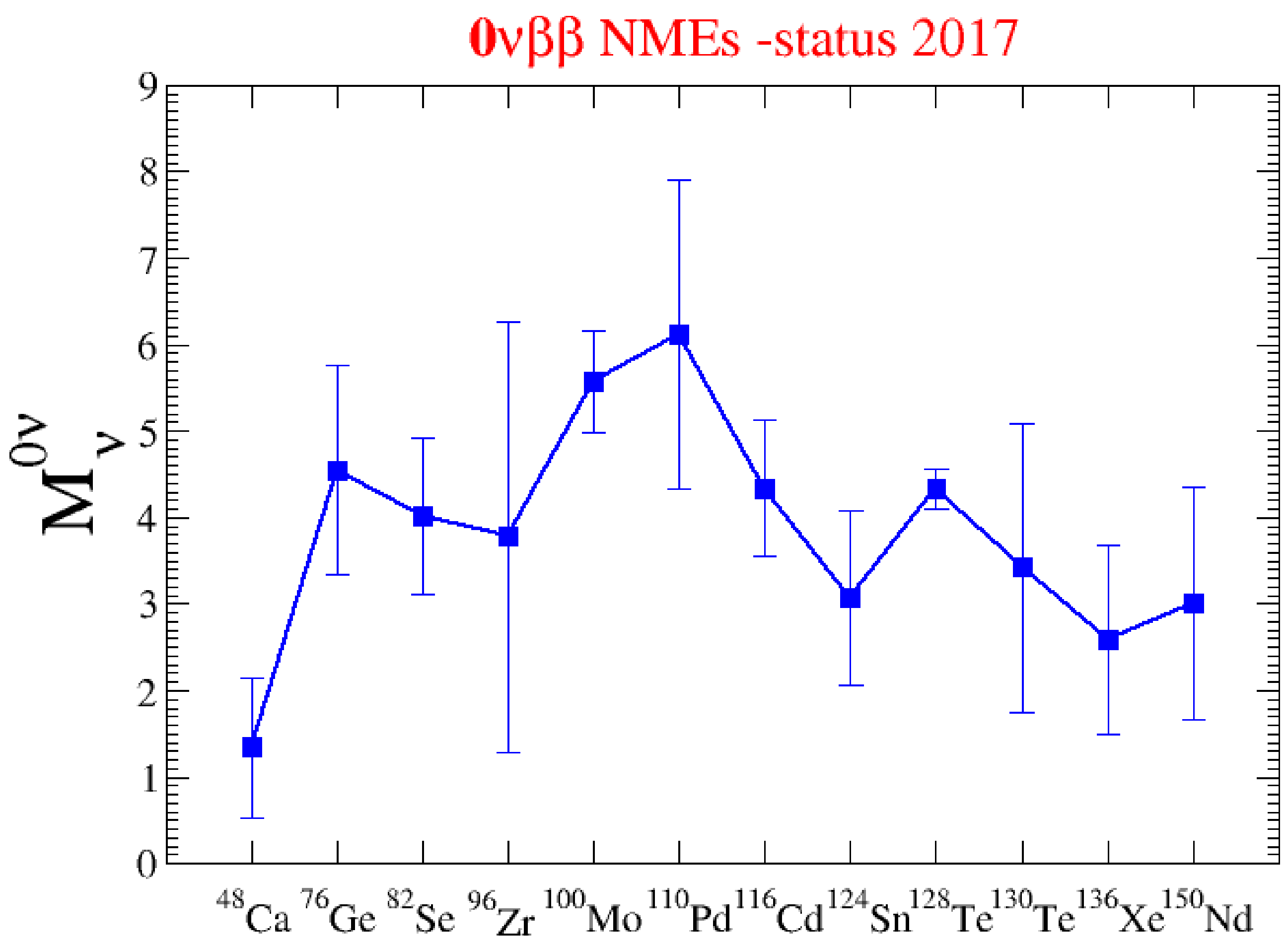
NMEs for unquenched value of g_A

Mean field approaches (PHFB, NREDF, REDF) ⇒ Large NMEs

Interacting Shell Model (ISM-StMa, ISM-CMU) ⇒ small NMEs

Quasiparticle Random Phase Approximation (QRPA-TBC, QRPA-Jy, dQRPQ-NC) ⇒ Intermediate NMEs

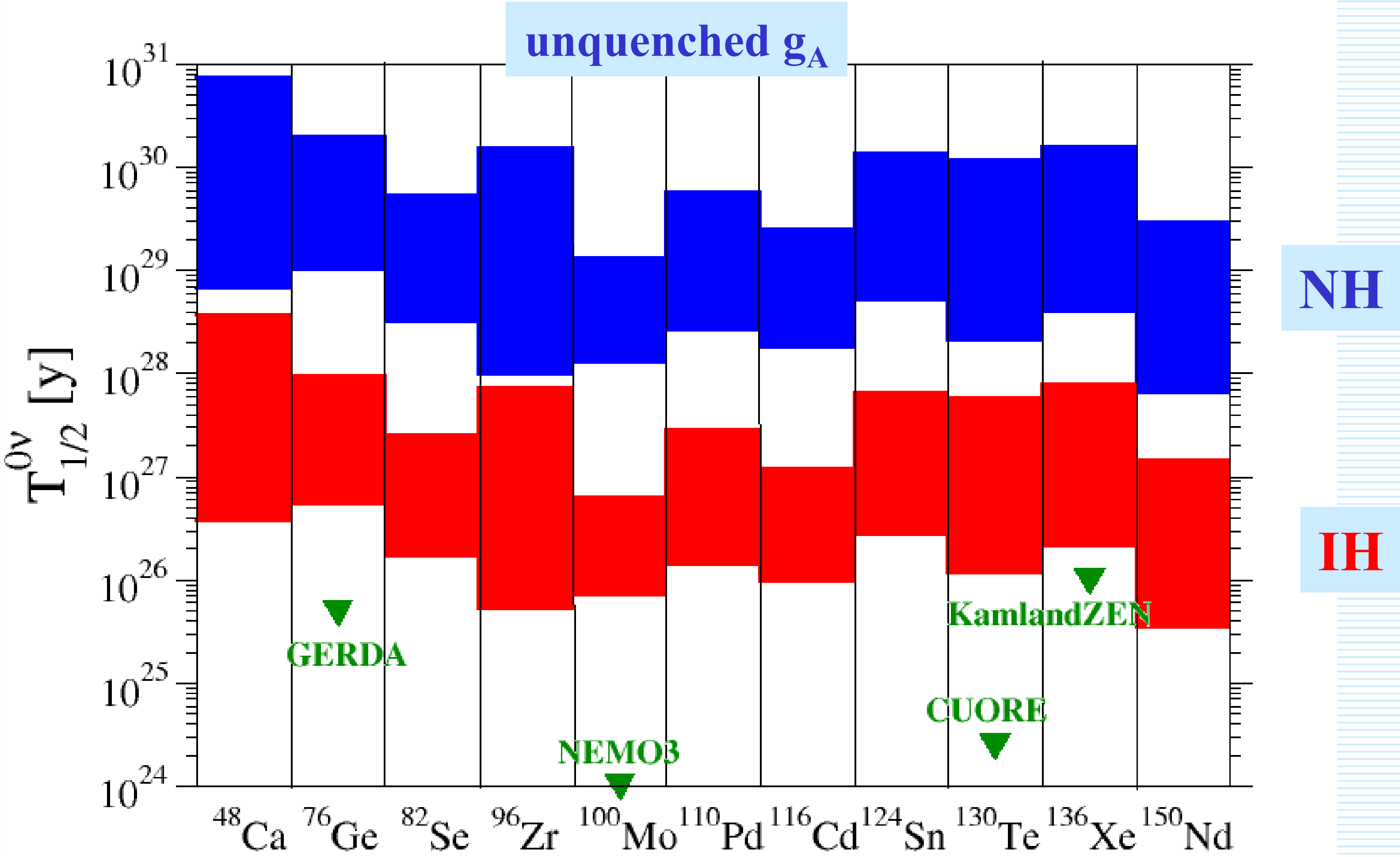
Interacting Boson Model (IBM) ⇒ Close to QRPA results



J.D.Vergados, H. Ejiri, , F.Š., Int. J. Mod. Phys. E25, 1630007(2016)

	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted

0νββ –half lives for NH and IH with included undertainties in NMEe



NH: $m_1 \ll m_2 \ll m_3$ $\quad m_3 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}. \quad m_2 \simeq \sqrt{\delta m^2}$

$1.4 \text{ meV} \leq m_{\beta\beta} \leq 3.6 \text{ meV}$

IH: $m_3 \ll m_1 < m_2$ $\quad m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

$m_3 \ll \sqrt{\Delta m^2}$

Lightest ν-mass equal to zero

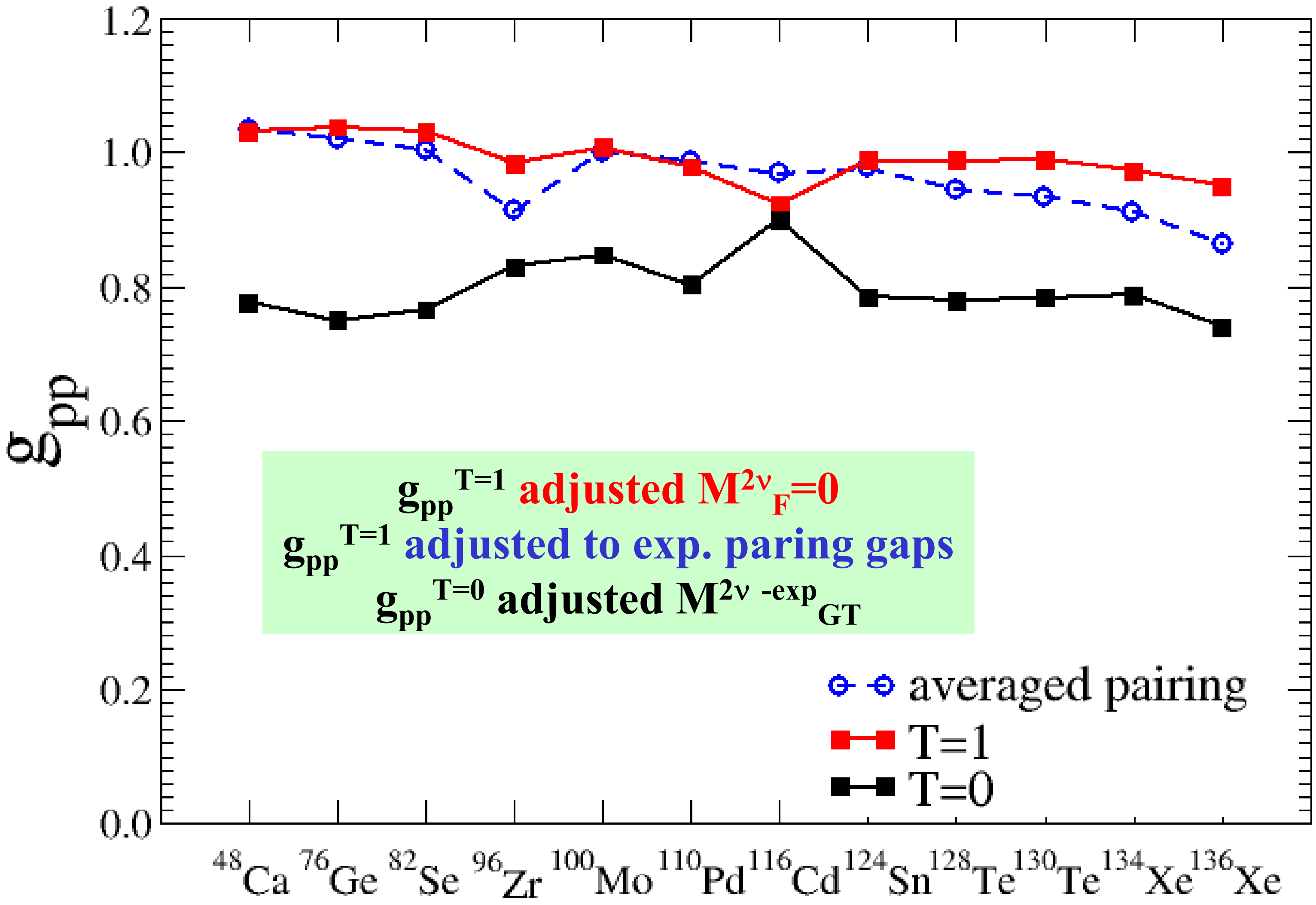
$20 \text{ meV} \leq m_{\beta\beta} \leq 49 \text{ meV}$

QRPA and isospin symmetry restoration

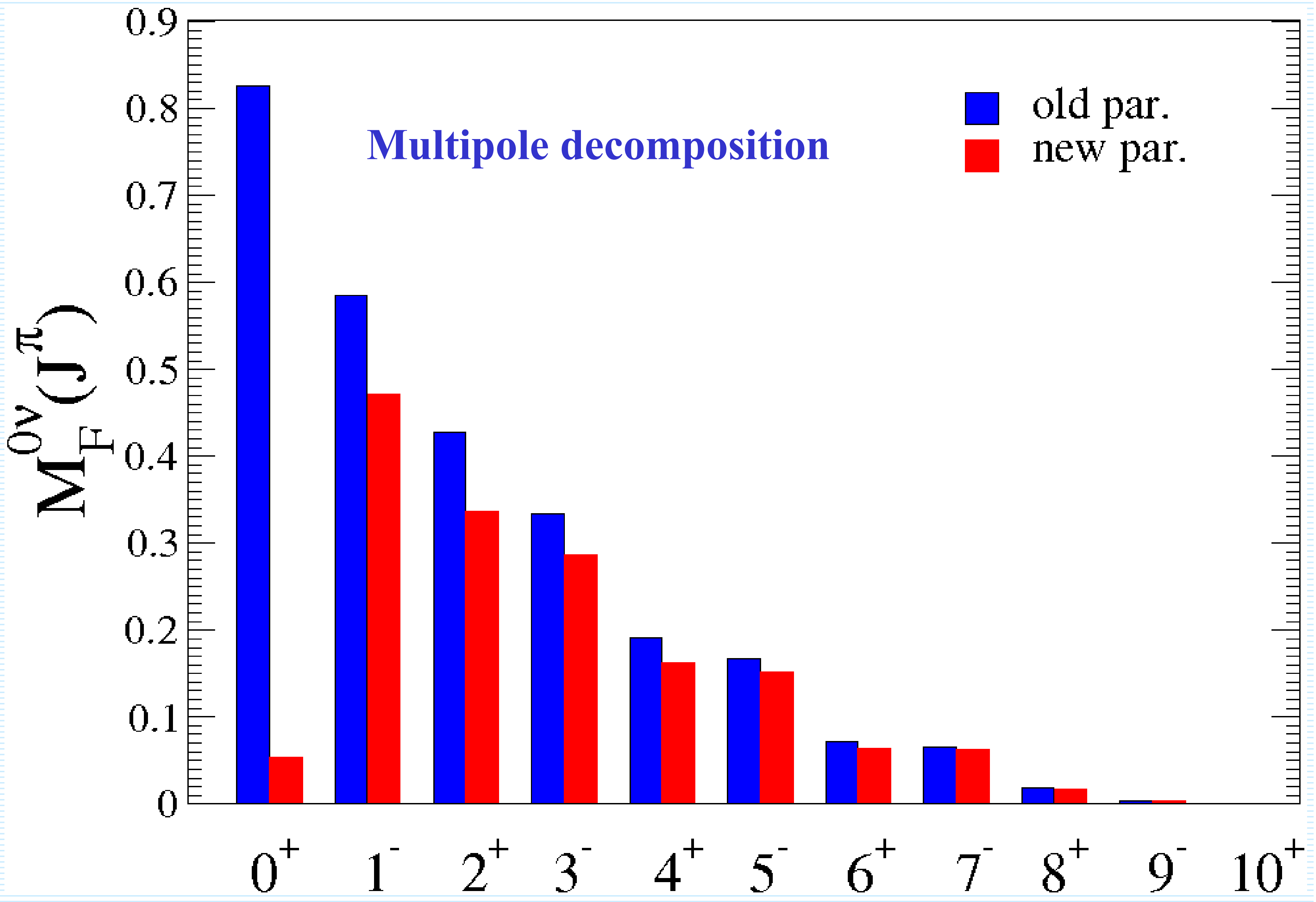
F.Š., V. Rodin, A. Faessler, and P. Vogel

PRC 87, 045501 (2013)

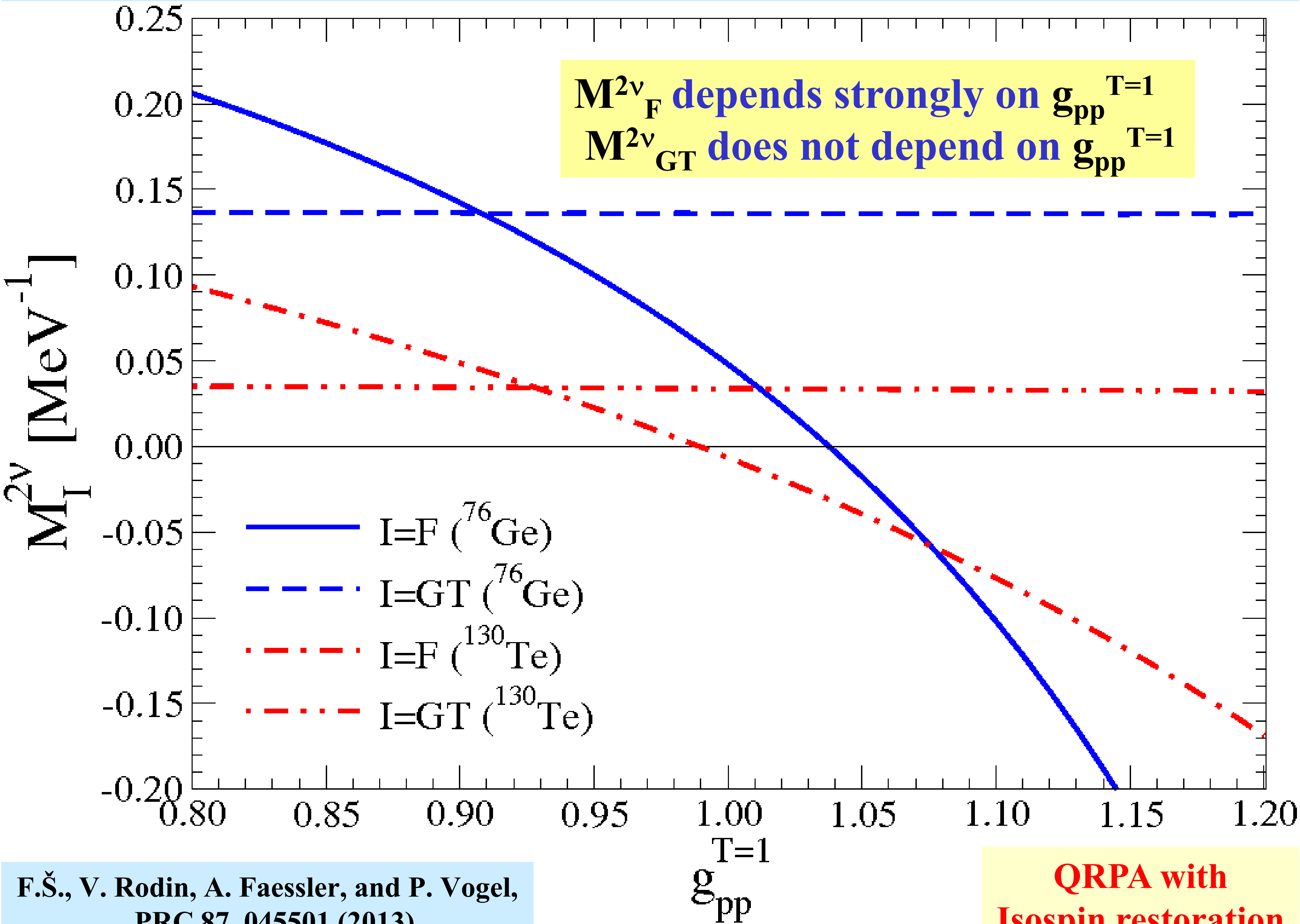
Close values ■ and ■ => no new parameter



Separation of g_{pp} into $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$

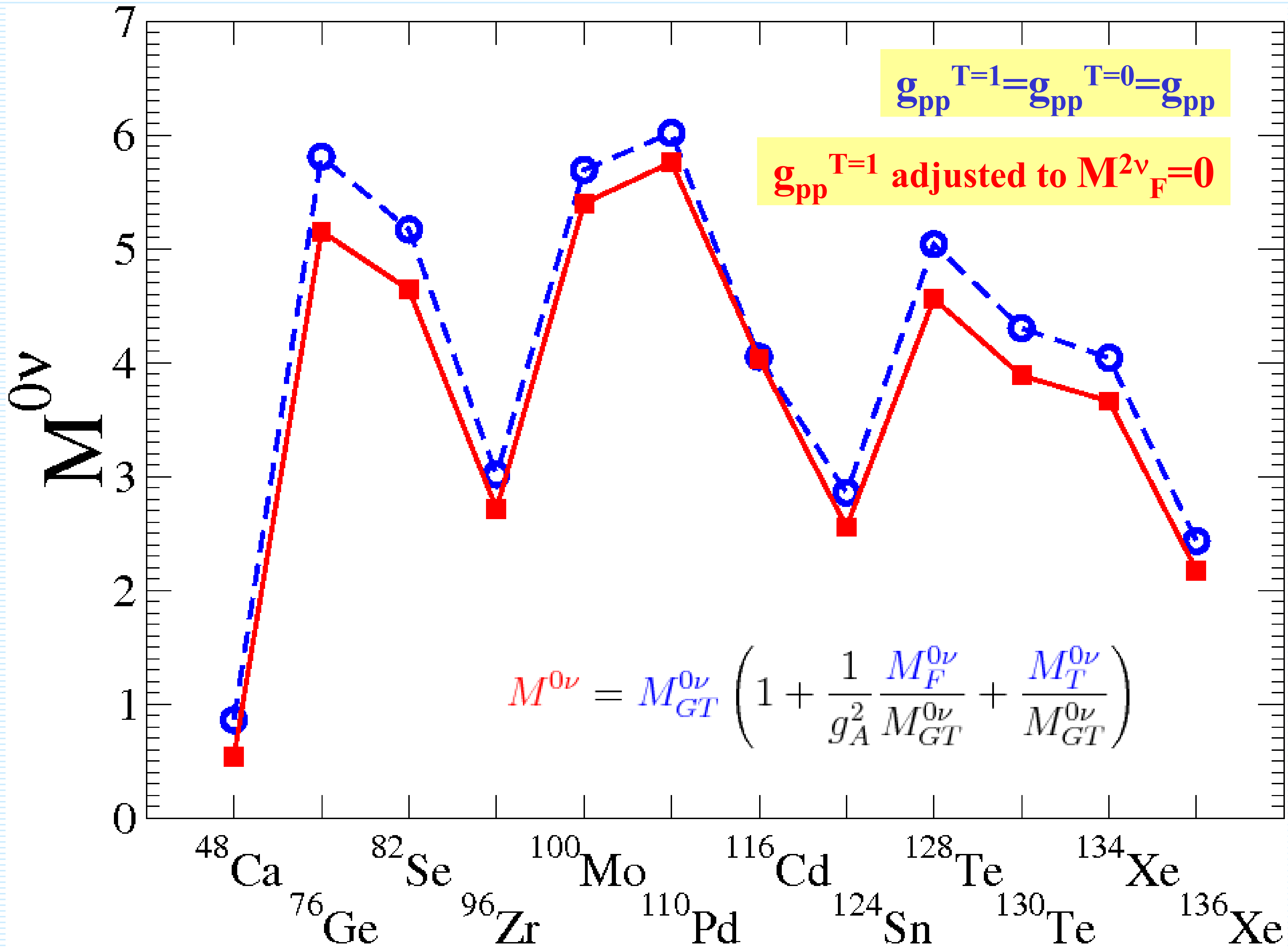


Multipole decomposition
old par.
new par.
$M_F^{0\nu}(J^\pi)$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0^+ 1^- 2^+ 3^- 4^+ 5^- 6^+ 7^- 8^+ 9^- 10^+



F.Š., V. Rodin, A. Faessler, and P. Vogel, PRC 87, 045501 (2013)

QRPA with Isospin restoration



$g_{pp}^{T=1}=g_{pp}^{T=0}=g_{pp}$
$g_{pp}^{T=1}$ adjusted to $M^{2\nu}_{F}=0$
$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_{F}}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_{T}}{M^{0\nu}_{GT}}\right)$
$M^{0\nu}$
0
1
2
3
4
5
6
7
^{48}Ca
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{110}Pd
^{116}Cd
^{124}Sn
^{128}Te
^{130}Te
^{134}Xe
^{136}Xe

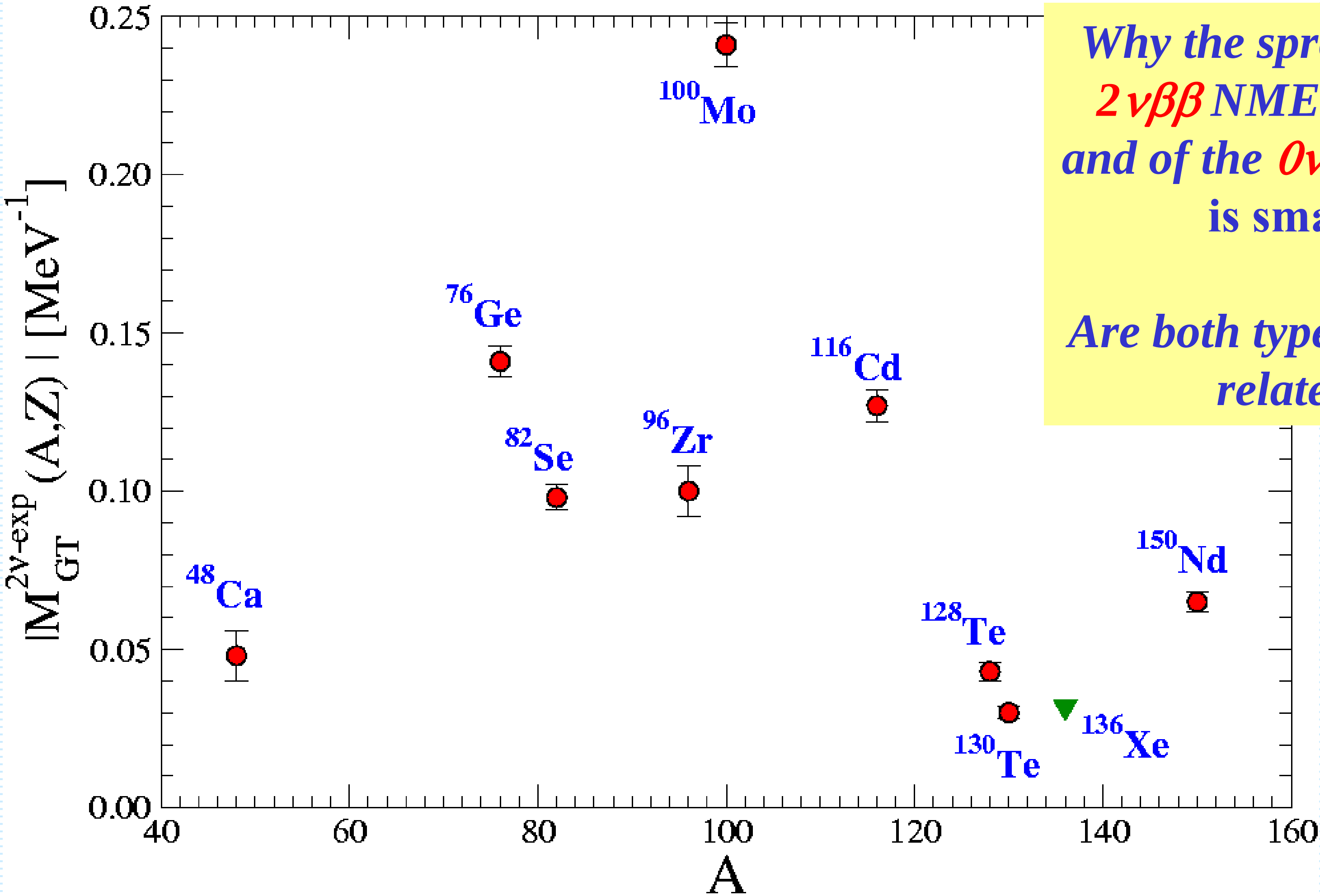
On the relation between $0\nu\beta\beta$-decay and $2\nu\beta\beta$-decay (GT) NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

2νββ-decay NMEs

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0,Z)\; g_A^4\; |M_{GT}^{2\nu}|^2$$

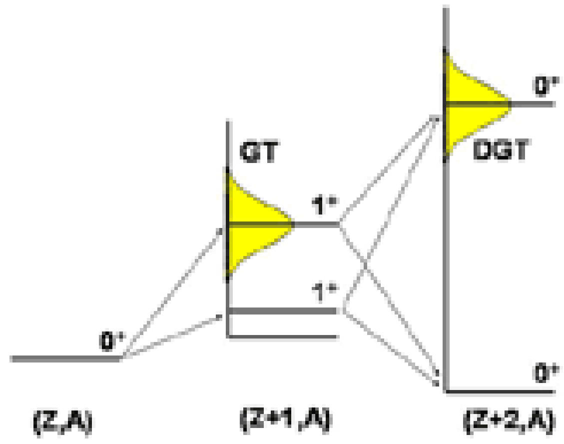


***Why the spread of the 2νββ NMEs is large and of the 0νββ* NMEs is small?**

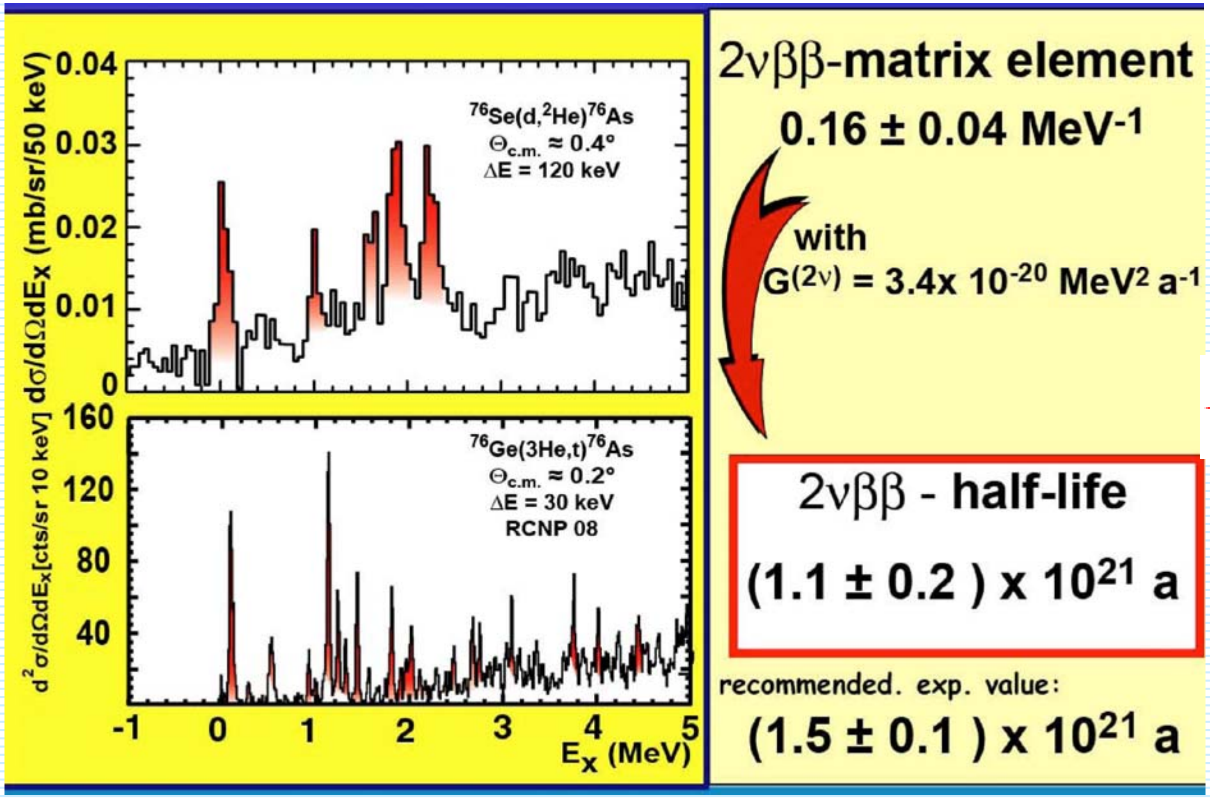
Are both type of NMEs related?

Differencies among 2νββ-decay NMEs: up to factor 10

***The cross sections of* (t,^{3}He) *and* (d,^{2}He) *reactions give* B(GT$^\pm$) *for* β^+ *and* β^-*, product of the amplitudes* (B(GT)$^{1/2}$) *entering the numerator of* $M^{2\nu}_{GT}$**



$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\; M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$



Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\; M^{(-)}_{GT}(m)$$

SSD hypothesis

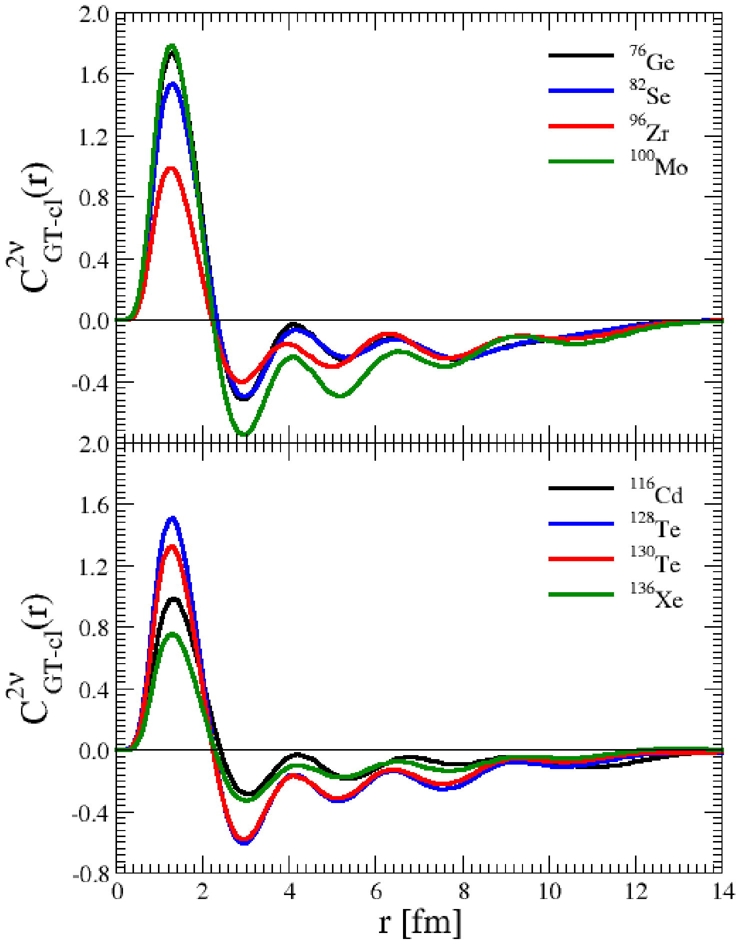
$$g_A^2\; M^{2\nu}_{GT-cl} = \frac{3\; D}{\sqrt{ft_{EC}\, ft_{\beta^-}}}$$

Grewe, …Frekers at al, PRC 78, 044301 (2008)

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

***r*- relative distance of two nucleons**



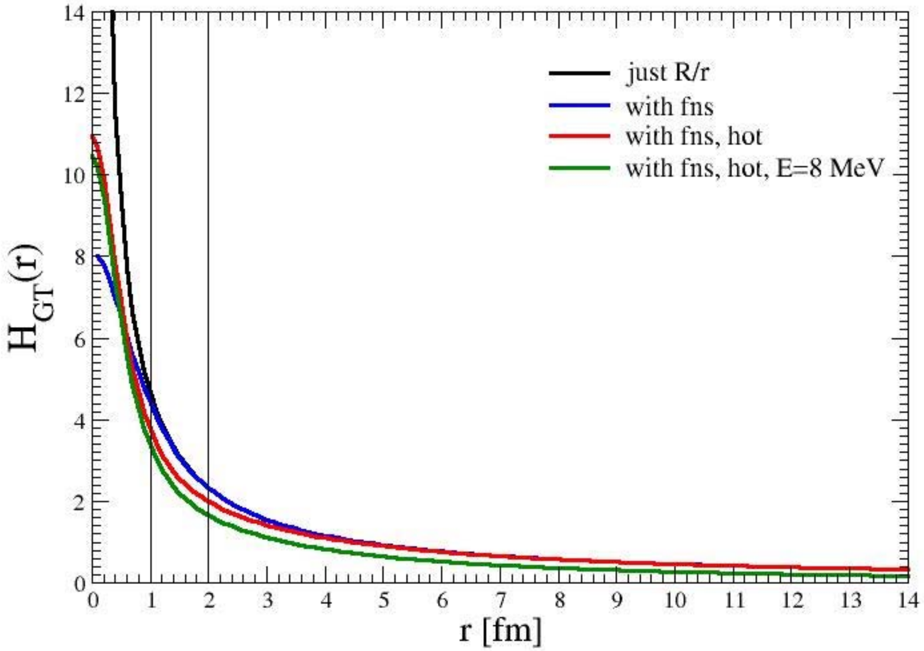
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f^2_{FNS}(q^2) g_{HOT}(q^2) dr$$



Neutrino potential prefer short distances

Closure 2νββ GT NME

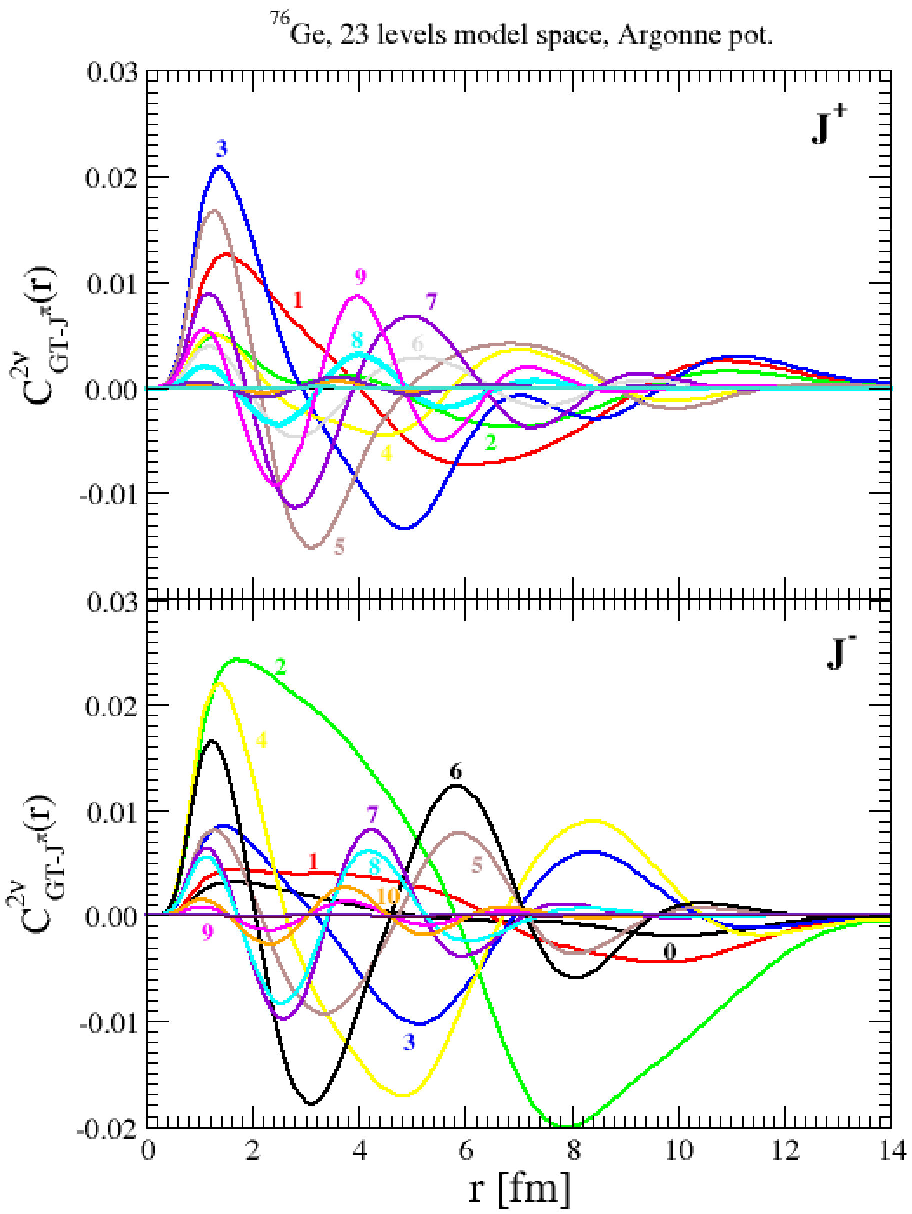
The only non-zero contribution from $J^{\pi}=1^{+}$

$$M^{2\nu}_{GT-cl} =$$

$$\sum_{J^{\pi},m} < 0_f^+|\tau^+\vec{\sigma}|J^{\pi},m > \cdot < J^{\pi},m|\tau^+\vec{\sigma}|0_i^+ >$$

$$= \sum_{m} < 0_f^+|\tau^+\vec{\sigma}|1^+,m > \cdot < 1^+,m|\tau^+\vec{\sigma}|0_i^+ >$$

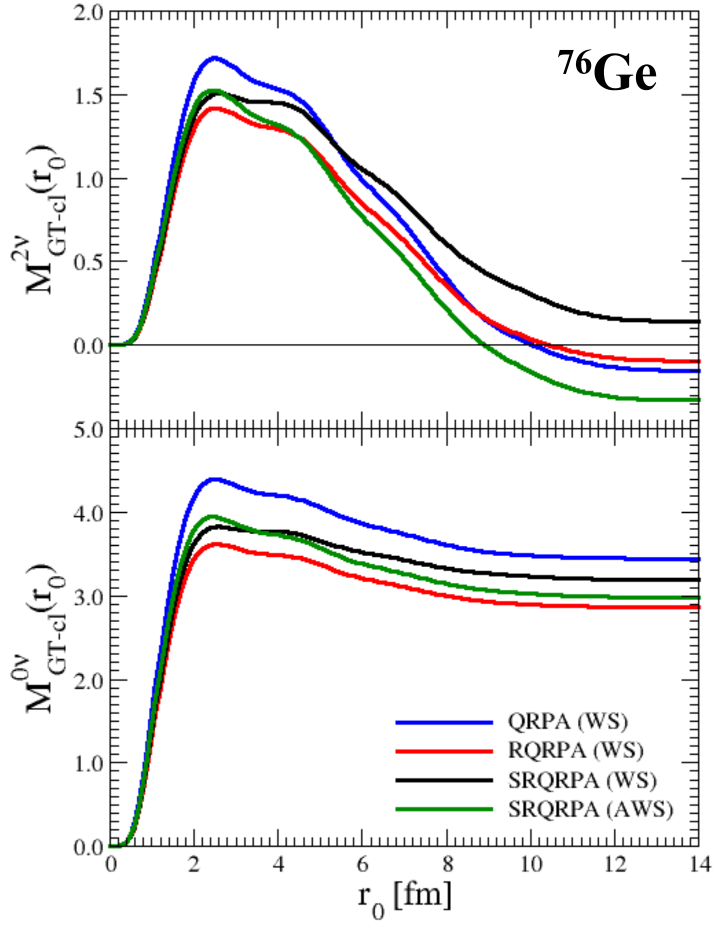
$$M^{2\nu}_{GT-cl} = \sum_{J^{\pi}} \int_0^{\infty} C^{2\nu}_{GT-J^{\pi}}(r)dr$$



$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

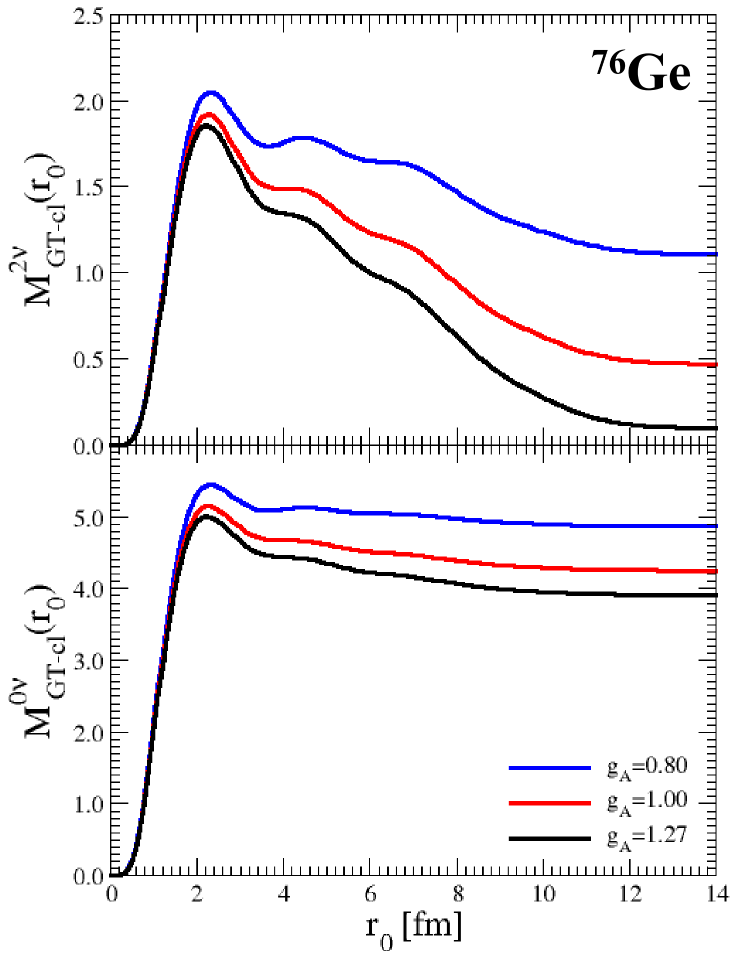
$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$

Nucleon *Nuclear physics*



F.Š.,

Nucleon *Nuclear physics*



Different QRPA-like approches

Fedor Sim

Dependence on axial-vector coupling

Phenomenological estimation of $M^{0\nu}_{GT}$

Nucleus	$T_{1/2}^{2\nu-exp}$ [y] [years]	$\|M_{GT}^{2\nu-exp}\|$ $[MeV^{-}1]$	SSD $\|M_{GT-cl}^{2\nu}\|$	SSD $\|M^{0\nu-ph}\|$	ChER $\|M_{GT}^{2\nu}\|$ $[MeV^{-}1]$	ChER $\|M_{GT-cl}^{2\nu}\|$	ChER $\|M^{0\nu-ph}\|$
^{48}Ca	4.4×10^{19}	0.046	-	-	0.083	0.220	1.98
^{76}Ge	1.5×10^{21}	0.0.136	-	-	0.159	0.522	5.46
^{96}Zr	2.3×10^{19}	0.090	-	-	-	0.222	3.45
^{100}Mo	7.1×10^{18}	0.231	0.350	4.02	-	-	-
^{116}Cd	2.8×10^{19}	0.126	0.349	4.21	0.064	0.305	3.67
^{128}Te	1.9×10^{24}	0.126	0.033	0.41	-	-	-

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f_{FNS}^2(q^2)g_{HOT}(q^2)dr$$

$$\begin{aligned} M_{GT}^{0\nu} &= H_{GT}(r=0)\ M_{GT-cl}^{2\nu} \\ &\quad -\int_0^\infty \mathcal{F}(r)C_{GT-cl}^{2\nu}(r)dr \\ &= M_{GT}^{0\nu-ph} - M_{GT}^{0\nu-rest} \end{aligned}$$

with Taylor expansion

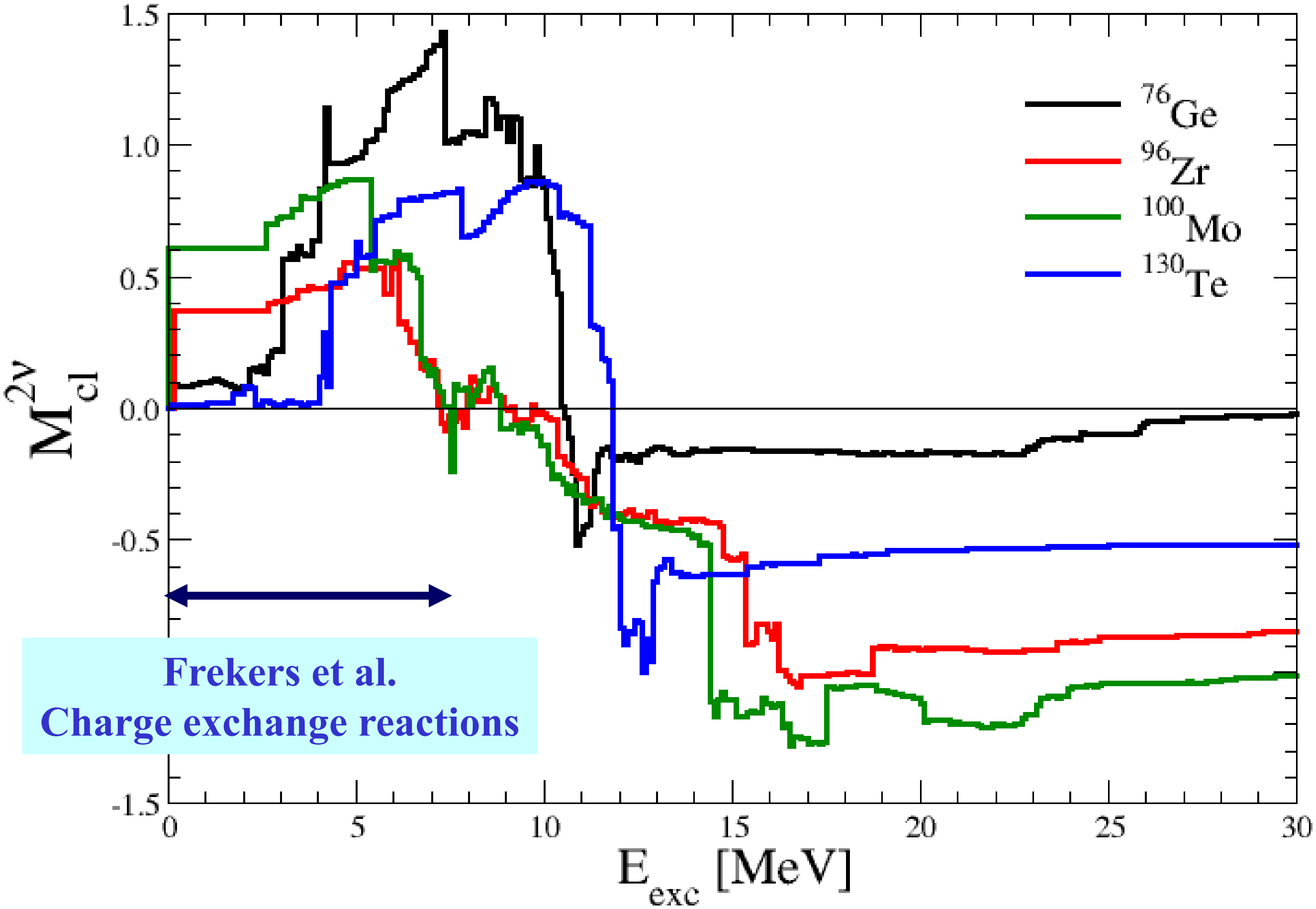
$$\begin{aligned} j_0(qr) &= 1 - \frac{1}{6}(qr)^2 + \frac{1}{120}(qr)^4 - \cdots \\ &= 1 - \mathcal{F}(r) \end{aligned}$$

A: Phenomen. prediction:
Too large
(~ factor 2)

B: Need to be calculated
Not negligable

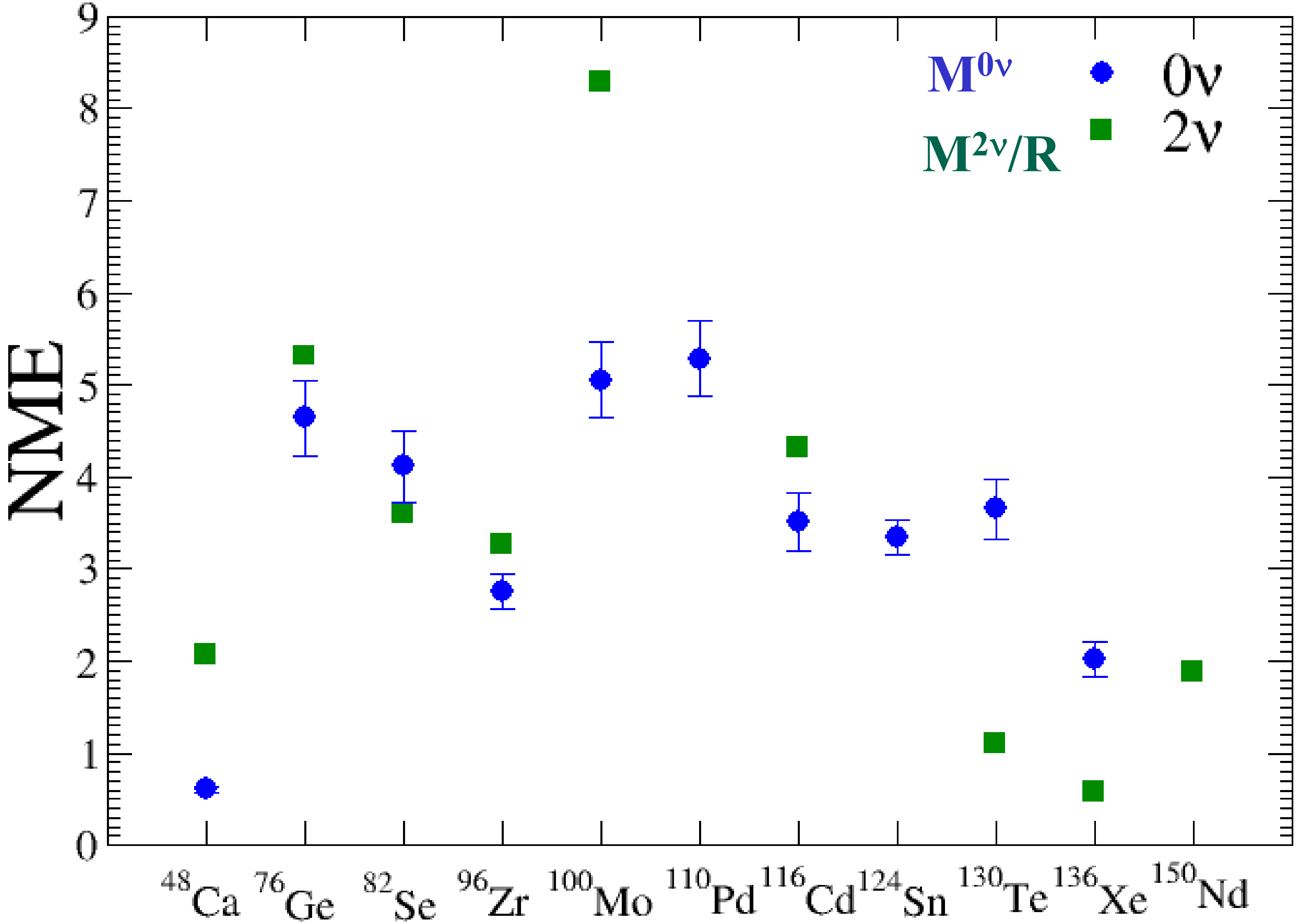
There is no proportionality between $M^{0\nu}_{GT}$ and $M^{2\nu}_{GT}$

There is no proportionality between 0νββ-decay and 2νββ-decay NME!!!



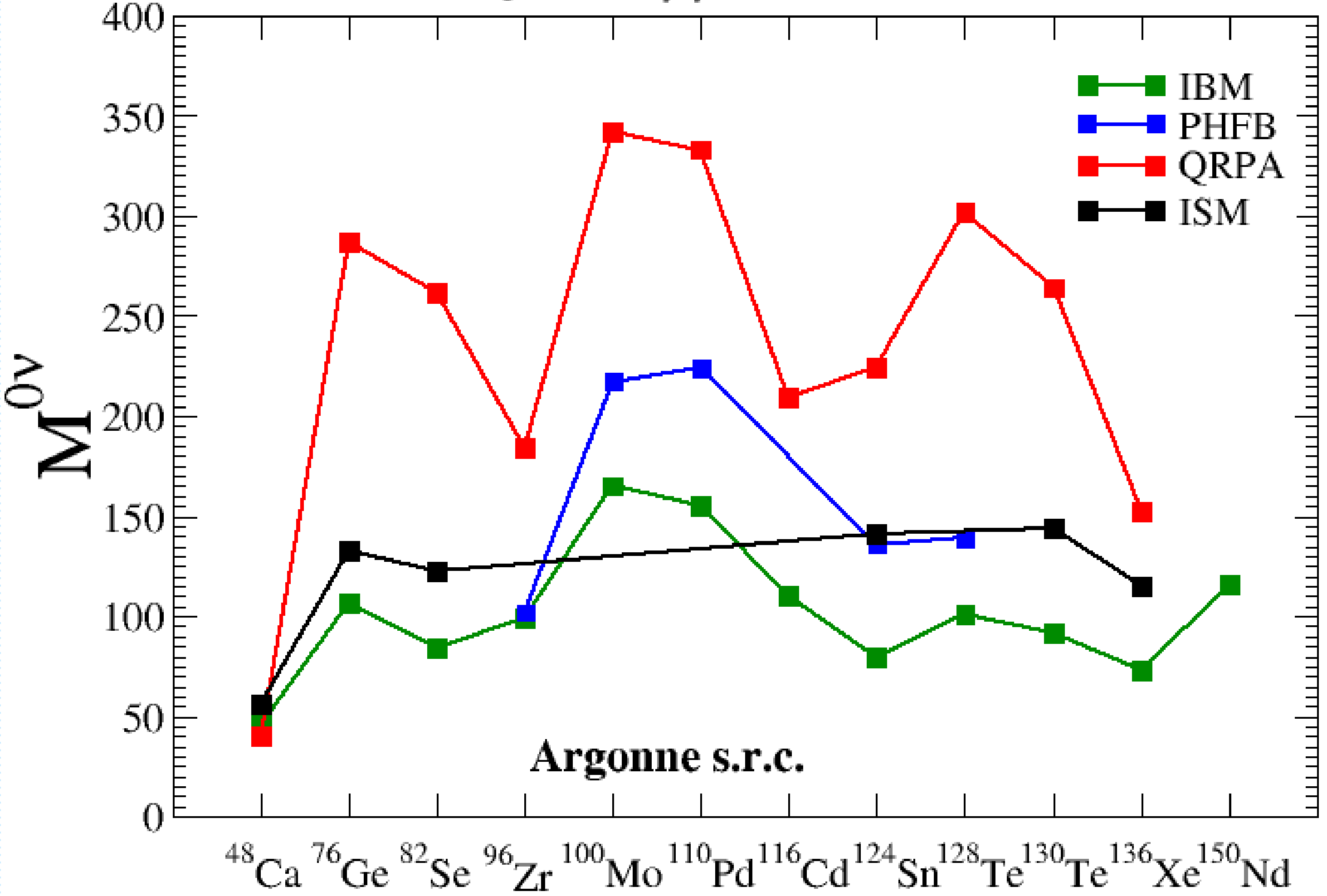
Frekers et al.
Charge exchange reactions

Is there a connection between 0νββ- and 2νββ-decay NME?



F. Š., Nucl. Phys. B, Proc. Suppl. (2014) – Proceeding of NOW14 conference

Heavy ν: 0νββ NMEs -status 2014



PHFB: K. Rath et al., PRC 85 (2012) 014308
IBM: Barea, Kotila, Iachello, PRC (2013) 014315

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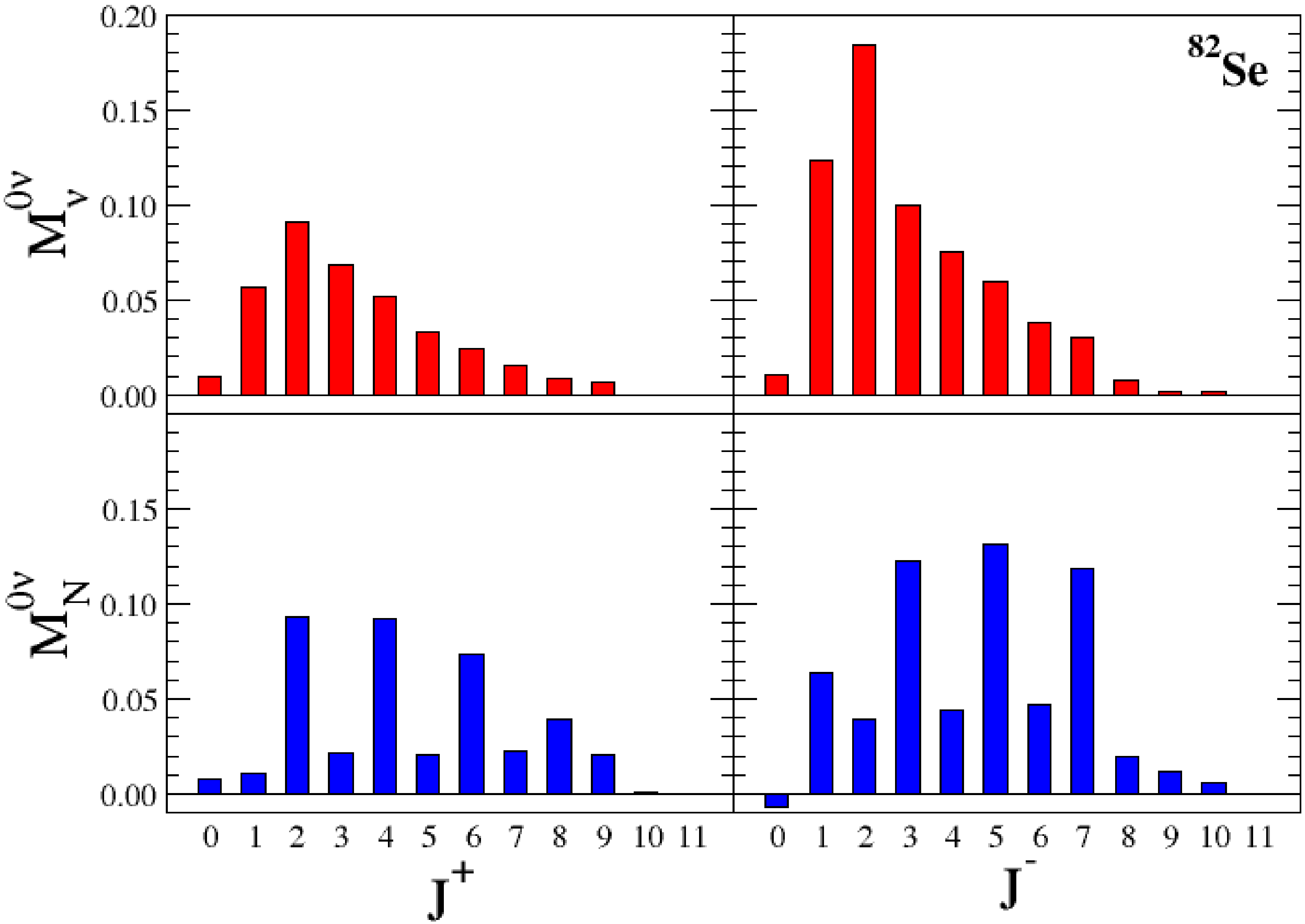
QRPA: Faessler, Gonzales, , F. Š., Kovalenko, PRD 90 (2014) 096010 ``Vergados, Ejiri, F. Š., RPP 75 (2012) 106301
ISM: Menendez, privite communications

Multipole decomposition of NMEs normalized to unity



$g_A{}^4 = (1.269)^4 = 2.6$

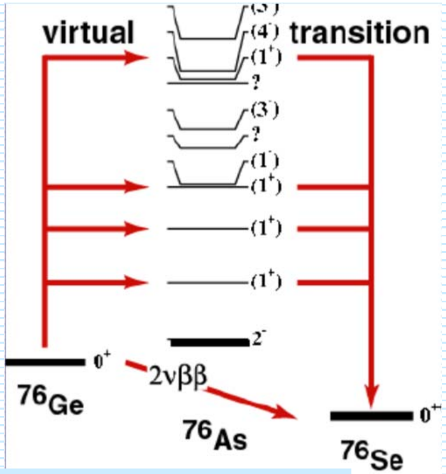
Quenching of g_A (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)

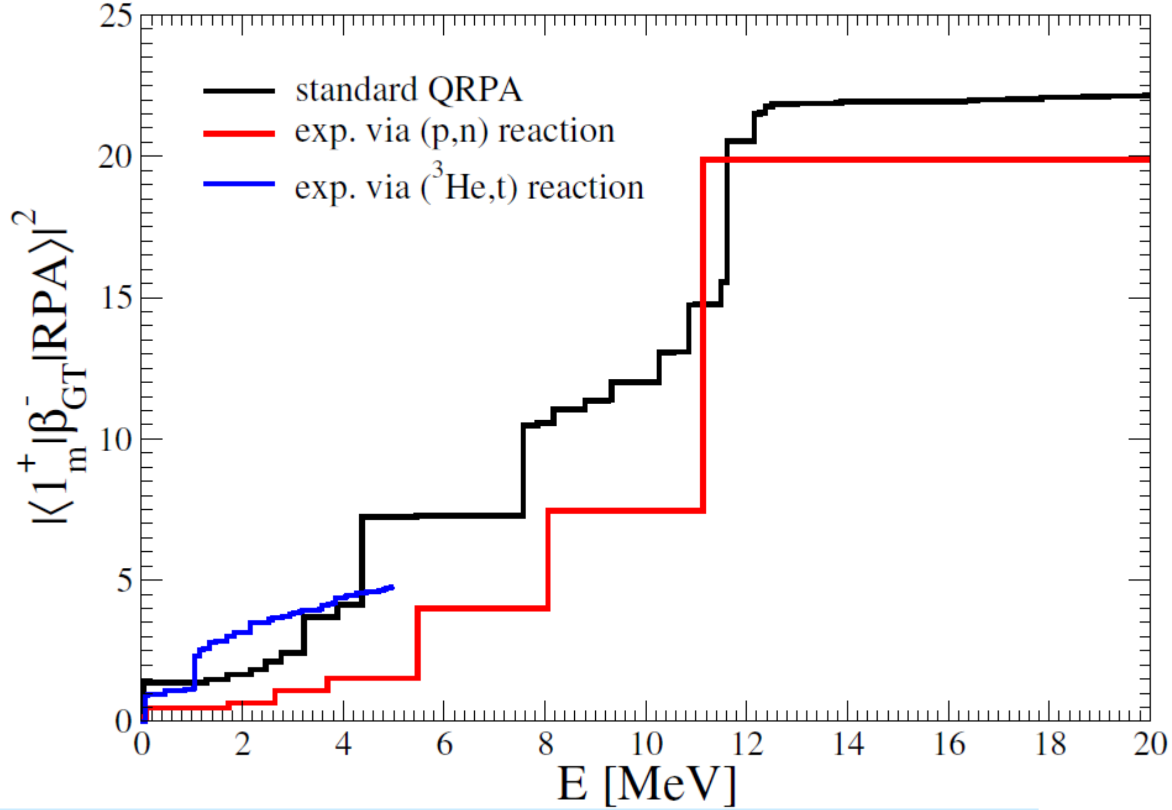
$(g^{eff}_A)^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment

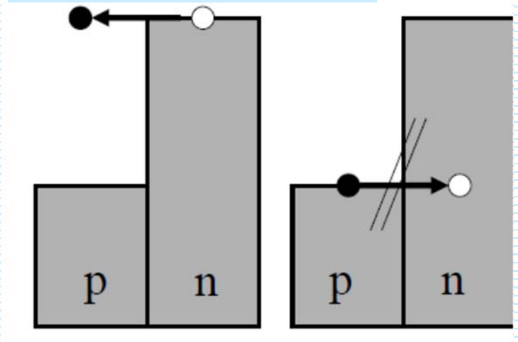
$^{76}_{32}Ge_{44} \Rightarrow$

$S_{\beta^-} - S_{\beta^+} = 3(N\text{-}Z) = 36$





Pauli blocking



Cross-section for charge exchange reaction:

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \,|\, \sigma\tau \,|\, i \rangle|^2$$

$q = 0!!$

largest at 100 - 200 MeV/A

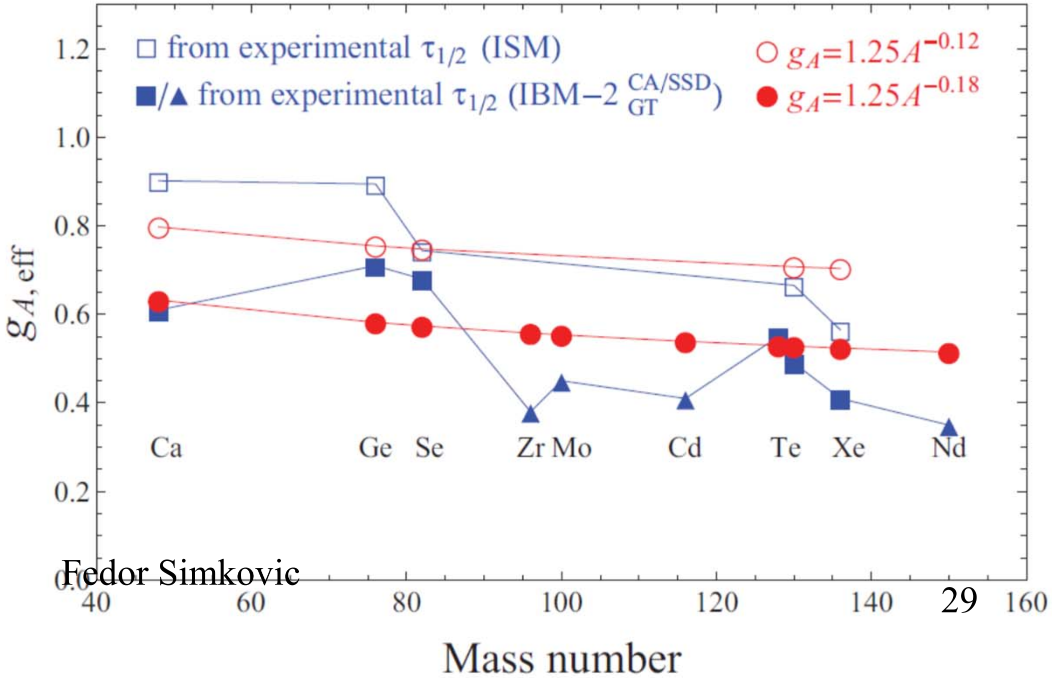
Quenching of g_A *(from theory: $T_{1/2}^{0\nu}$ up 50 x larger)*

$(g^{eff}_A)^4 \simeq 0.66$ (^{48}Ca), 0.66 (^{76}Ge), 0.30 (^{76}Se), 0.20 (^{130}Te) and 0.11 (^{136}Xe)
The Interacting Shell Model (ISM), which describes qualitatively well energy spectra, does reproduce experimental values of $M^{2\nu}$ only by consideration of significant quenching of the Gamow-Teller operator, typically by 0.45 to 70%.

$(g^{eff}_A)^4 \simeq (1.269\, A^{-0.18})4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

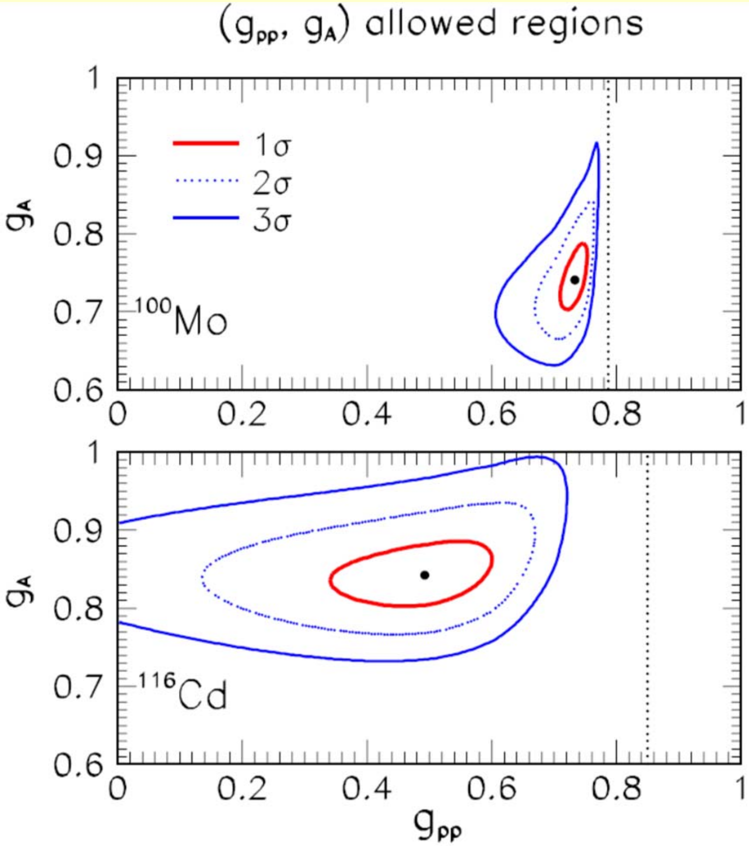
J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).

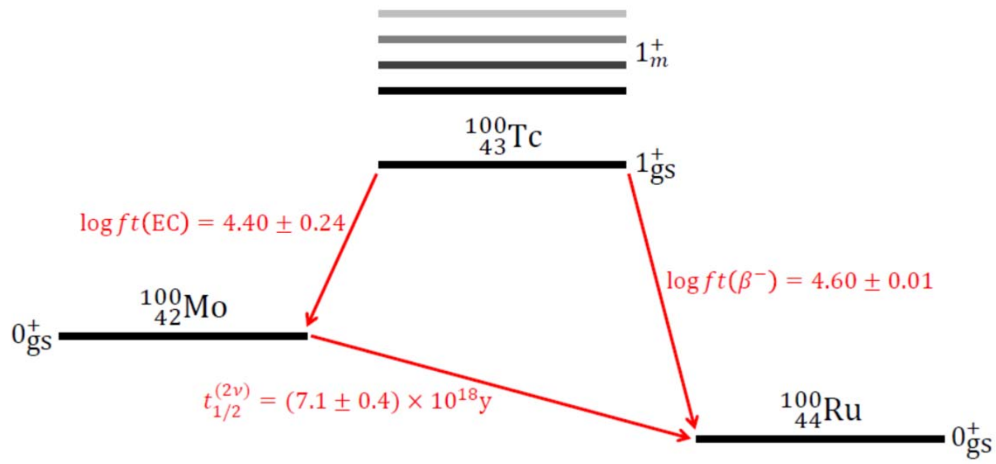
It has been determined by theoretical prediction for the 2νββ-decay half-lives, which were based on within closure approximation calculated corresponding NMEs, with the measured half-lives.



Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$(g^{eff}_A)^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd, respectively (The QRPA prediction).
g^{eff}_A was treated as a completely free parameter alongside g_{pp} (used to renormalize particl-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g^{eff}_A and g_{pp}, where possible, to the β-decay rate and β+/EC rate of the $J = 1^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the 2νββ rates of the initial nuclei, leads to an effective g^{eff}_A of about 0.7 or 0.8.





Extended calculation also for neighbour isotopes performed by

F.F. Depisch and J. Suhonen, arXiv:1606.02908[nucl-th]

r Simkovic

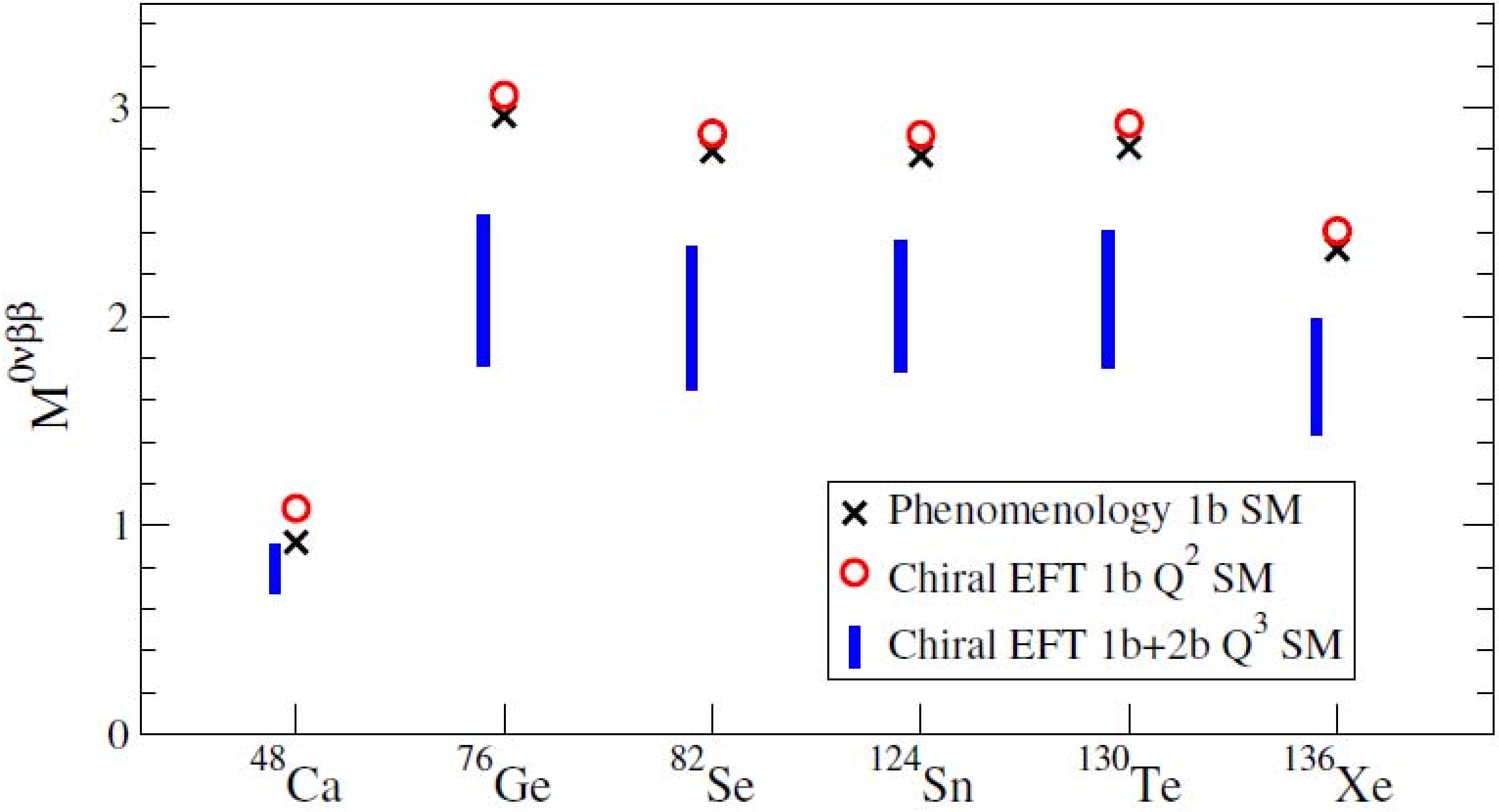
Dependence of g^{eff}_A on A was not established.

Quenching of g_A and two-body currents

Menendez, Gazit, Schwenk, PRL 107 (2011) 062501; MEDEX13 contribution

$$\mathbf{J}^{\rm eff}_{i,2b} = -g_A \boldsymbol{\sigma}_i \tau_i^- \frac{\rho}{F_\pi^2}\left[\frac{2}{3}c_3\frac{p^2}{4m_\pi^2+p^2} + I(\rho,P)\left(\frac{1}{3}(2c_4-c_3)+\frac{1}{6m}\right)\right] = -g_A\delta(p)\boldsymbol{\sigma}_i\tau_i^-$$

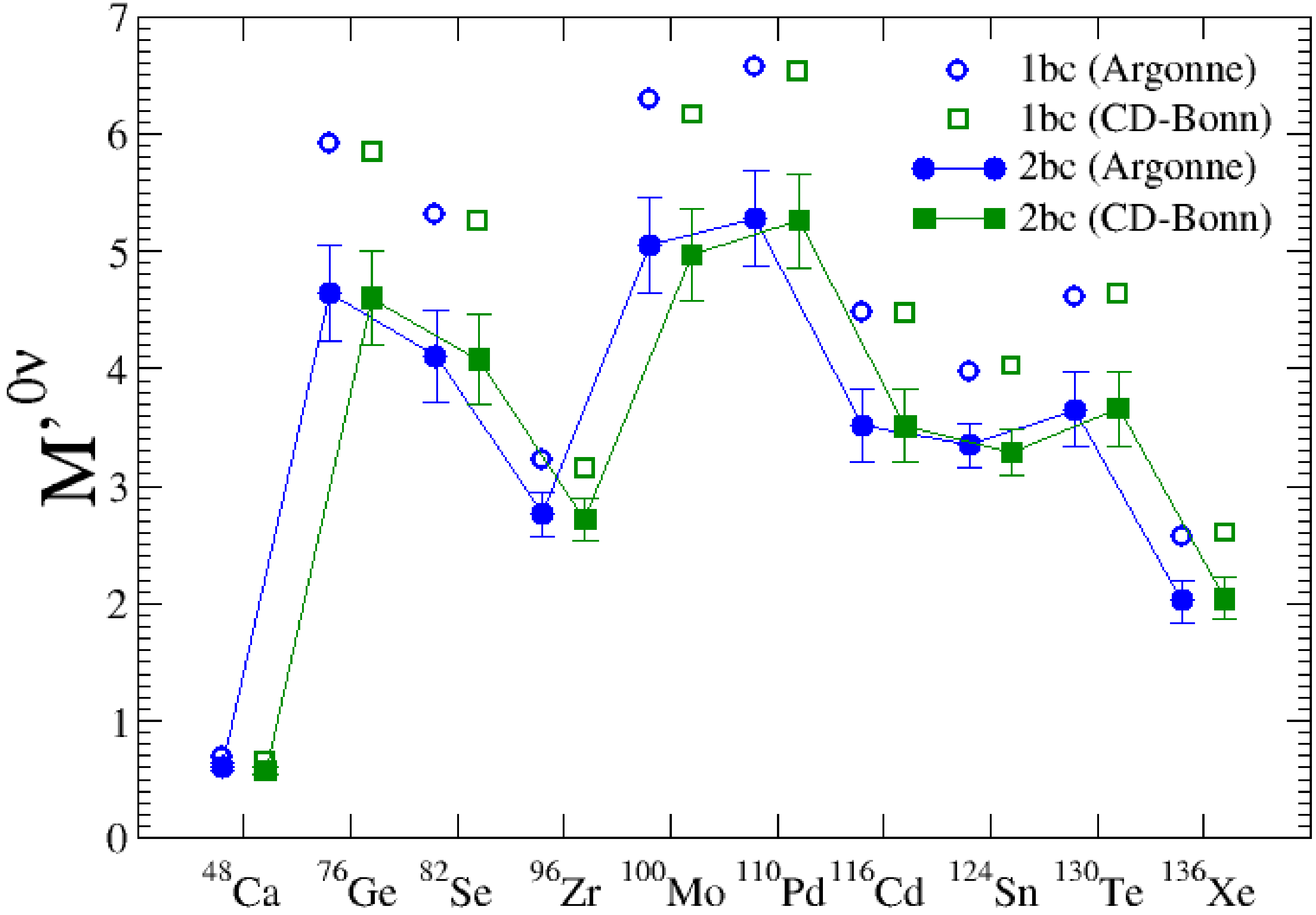
The 0νββ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related to the quenching of Gamow-Teller transitions found in nuclear structure calc.



Quenching of g_A, two-body currents and QRPA

(Suppression of the 0νββ-decay NME of about 20%)

Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308



But, a strong suppression of 2νββ-decay half-life, ($g_A^{eff} = g_A \delta(p=0) = 0.7\text{-}1.0$)

Understanding of the $2\nu\beta\beta$-decay NMEs is of crucial importance for correct evaluation of the $2\nu\beta\beta$-decay NMEs

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states. Both change two neutrons into two protons.

Explaining $2\nu\beta\beta$-decay is necessary but not sufficient

There is no reliable calculation of the *$2\nu\beta\beta$-decay NMEs*

Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.)
ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF

Improved description of the 2$\nu\beta\beta$-decay rate

F.Š., R. Dvornický, D. Štefánik and A. Faessler, to be submitted

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 \left(g_A^{\text{eff}}\right)^4 I^{2\nu}$$

Half-life without factorization of NMEs and phase space

$$\begin{aligned} I^{2\nu} &= \frac{1}{m_e^{11}} \int_{m_e}^{E_i-E_f-m_e} F_0(Z_f, E_{e_1}) p_{e_1} E_{e_1} dE_{e_1} \\ &\times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f, E_{e_2}) p_{e_2} E_{e_2} dE_{e_2} \\ &\times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1} \end{aligned}$$

$$\mathcal{A}^{2\nu} = \left[\frac{1}{4}|M_{GT}^K + M_{GT}^L|^2 + \frac{1}{12}|M_{GT}^K - M_{GT}^L|^2\right]$$

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i+E_f)/2}{[E_n - (E_i+E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$M_n = \langle 0_f^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 1_n^+ \rangle \langle 1_n^+ \parallel \sum_m \tau_m^- \sigma_m \parallel 0_i^+ \rangle$$

$$\begin{aligned} \epsilon_K &= \left(E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}\right)/2 \\ \epsilon_L &= \left(E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}\right)/2 \end{aligned}$$

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

Standard approximation which allows factorization of NME and phase space

$$M_{GT}^{K,L} \simeq M_{GT}^{2\nu} = m_e \sum_n \frac{M_n}{E_n - (E_i + E_f)/2}$$

Let perform Taylor expansion

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

$$E_n - \frac{E_i + E_f}{2} = \frac{Q}{2} + m_e + (E_n - E_i) > |\epsilon_{K,L}|$$

Improved description of the 0νββ–decay rate

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \equiv \frac{\Gamma^{2\nu}}{\ln(2)} \simeq \frac{\Gamma_0^{2\nu}+\Gamma_2^{2\nu}+\Gamma_4^{2\nu}}{\ln(2)}$$

$$\frac{\Gamma_0^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \mathcal{M}_0 G_0^{2\nu}$$

$$\frac{\Gamma_2^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \mathcal{M}_2 G_2^{2\nu}$$

$$\frac{\Gamma_4^{2\nu}}{\ln(2)} = \left(g_A^{\rm eff}\right)^4 \left(\mathcal{M}_4 G_4^{2\nu}+\mathcal{M}_{22} G_{22}^{2\nu}\right)$$

Taylor expansion up to ε⁴

$$G_J^{2\nu} = \frac{c_{2\nu}}{m_e^{11}}\int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1})p_{e_1}E_{e_1}dE_{e_1} \times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2})p_{e_2}E_{e_2}dE_{e_2} \times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}_J^{2\nu} dE_{\nu_1}, \quad \text{(J=0, 2, 4, 22)}$$

$$\mathcal{A}_0^{2\nu} = 1 \quad \mathcal{A}_2^{2\nu} = \frac{\varepsilon_K^2+\varepsilon_L^2}{(2m_e)^2},$$

$$\mathcal{A}_{22}^{2\nu} = \frac{\varepsilon_K^2\varepsilon_L^2}{(2m_e)^4} \quad \mathcal{A}_2^{2\nu} = \frac{\varepsilon_K^4+\varepsilon_L^4}{(2m_e)^4}$$

Phase space factors

	$2\nu\beta\beta$-decay			
nucl.	$G_0^{2\nu}$ [yr^{-1}]	$G_2^{2\nu}$ [yr^{-1}]	$G_4^{2\nu}$ [yr^{-1}]	$G_{22}^{2\nu}$ [yr^{-1}]
^{76}Ge	$4.816\ 10^{-20}$	$1.015\ 10^{-20}$	$1.332\ 10^{-21}$	$6.284\ 10^{-22}$
^{82}Se	$1.591\ 10^{-18}$	$7.037\ 10^{-19}$	$1.952\ 10^{-19}$	$8.931\ 10^{-20}$
^{100}Mo	$3.303\ 10^{-18}$	$1.509\ 10^{-18}$	$4.320\ 10^{-19}$	$1.986\ 10^{-19}$
^{130}Te	$1.530\ 10^{-18}$	$4.953\ 10^{-19}$	$9.985\ 10^{-20}$	$4.707\ 10^{-20}$
^{136}Xe	$1.433\ 10^{-18}$	$4.404\ 10^{-19}$	$8.417\ 10^{-20}$	$3.986\ 10^{-20}$

$$\mathcal{M}_0 = \left|M^{2\nu}_{GT-1}\right|^2$$

$$\mathcal{M}_2 = \Re\{M^{2\nu}_{GT-1} M^{2\nu}_{GT-3}\}$$

$$\mathcal{M}_{22} = \frac{1}{3}\left|M^{2\nu}_{GT-3}\right|^2$$

$$\mathcal{M}_4 = \frac{1}{3}\left|M^{2\nu}_{GT-3}\right|^2 + \Re\{M^{2\nu}_{GT-1} M^{2\nu}_{GT-5}\}$$

3 different NMEs

$$M^{2\nu}_{GT-1} \equiv M^{2\nu}_{GT}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

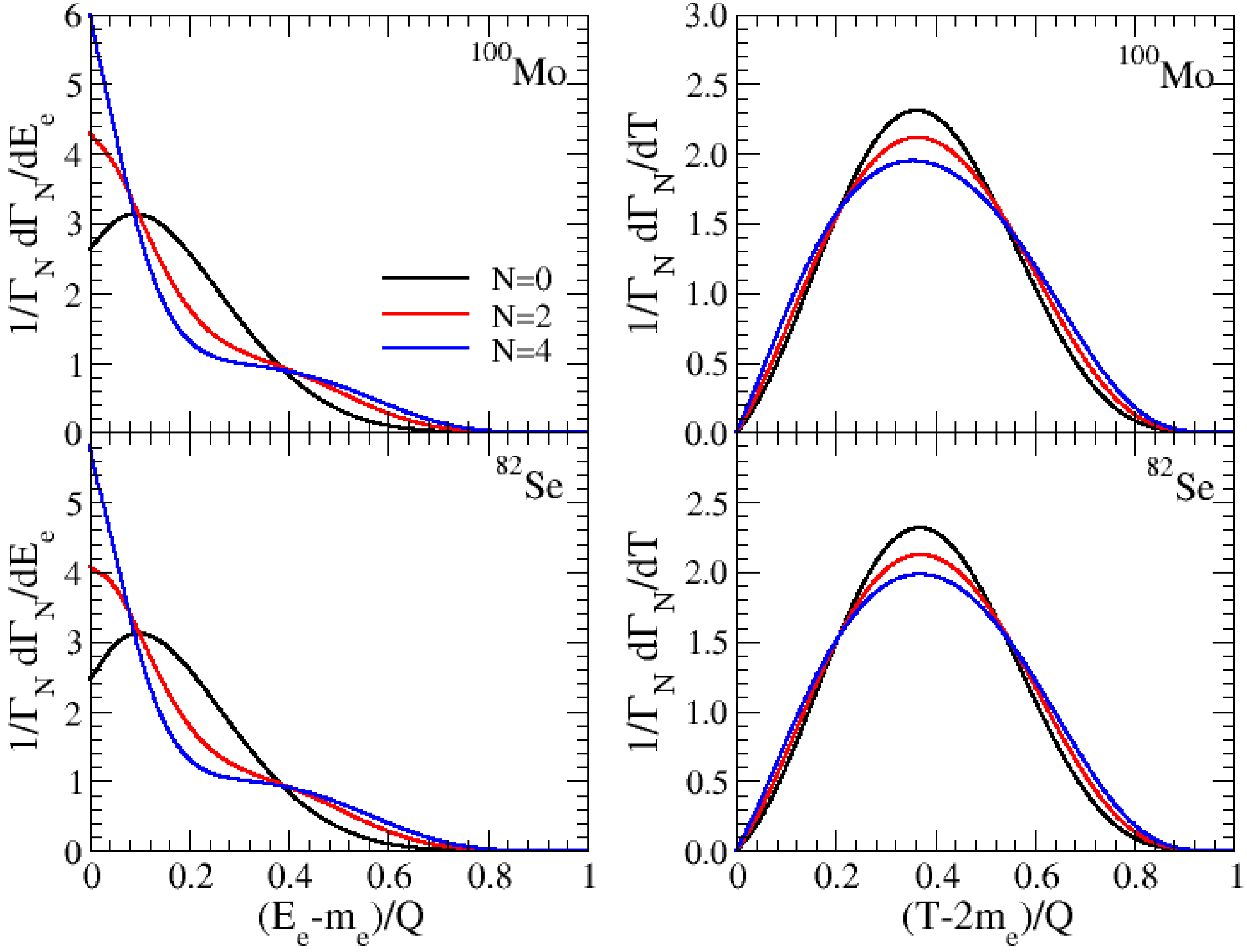
$$M^{2\nu}_{GT-5} = \sum_n M_n \frac{16\ m_e^5}{(E_n - (E_i + E_f)/2)^5}$$

QRPA

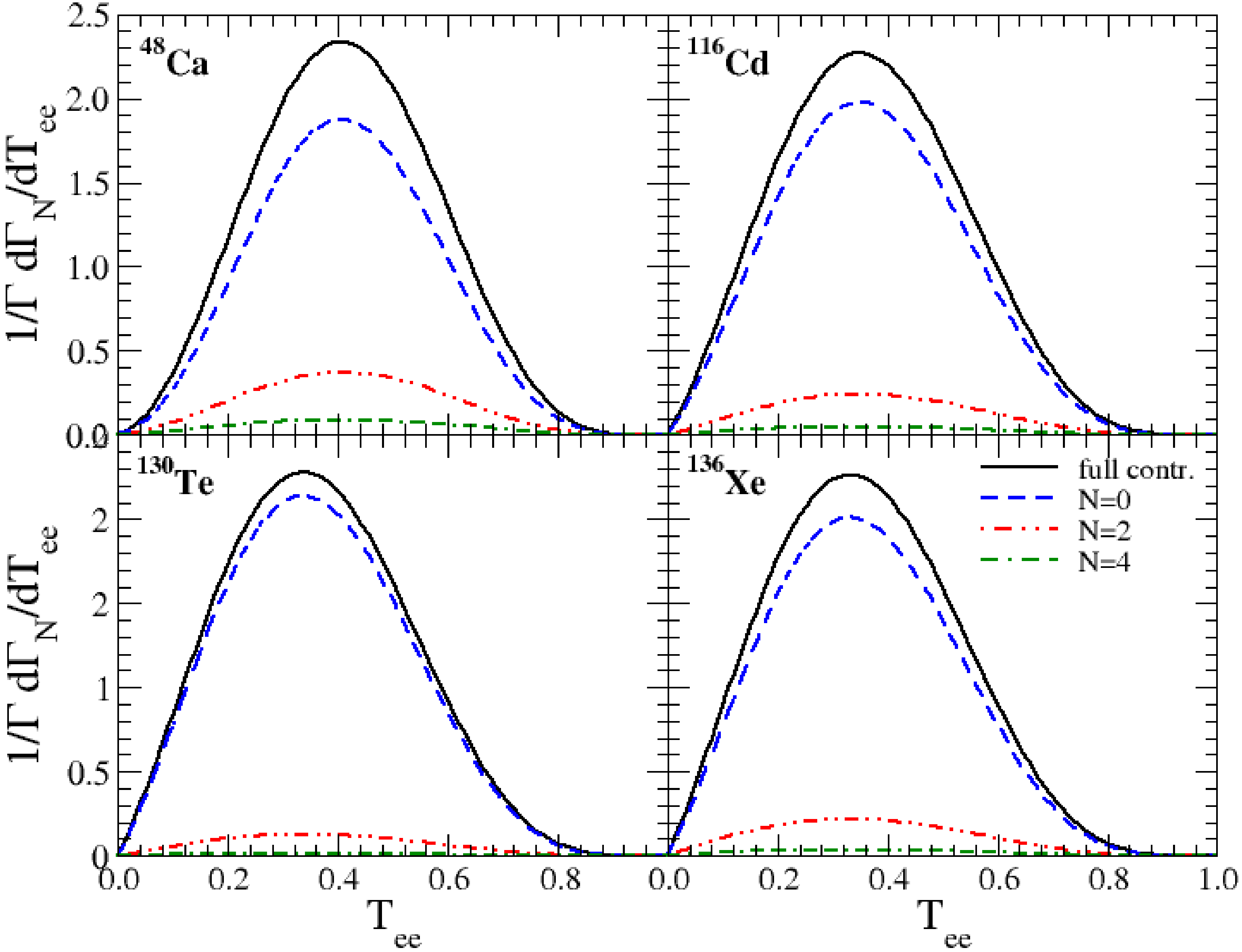
$2\nu\beta\beta$-decay NMEs and their ratios

nucl.	$g_A^{\rm eff}$	$M^{2\nu}_{GT-1}$	$M^{2\nu}_{GT-3}$	$M^{2\nu}_{GT-5}$	$\xi^{2\nu}_{13}$	$\xi^{2\nu}_{15}$	$P^{2\nu}_0$	$P^{2\nu}_2$	$P^{2\nu}_4$	$T^{2\nu-exp}_{1/2}$ [yr]
^{76}Ge	0.800	0.175	0.0214	0.00445	0.1220	0.0254	0.9741	0.0250	0.0009	$1.65\ 10^{21}$
	1.000	0.111	0.0133	0.00263	0.1204	0.0237	0.9745	0.0247	0.0008	
	1.269	0.689	0.00716	0.00716	0.1040	0.0170	0.9780	0.0214	0.0006	
^{82}Se	0.800	0.124	0.0216	0.00645	0.1745	0.0521	0.9213	0.0711	0.0076	$0.92\ 10^{20}$
	1.000	0.0795	0.0129	0.00355	0.1620	0.0446	0.9271	0.0664	0.0065	
	1.269	0.0498	0.00643	0.00136	0.1290	0.0272	0.9421	0.0538	0.0041	
^{100}Mo	0.800	0.292	0.123	0.0453	0.4230	0.1553	0.8163	0.1578	0.0259	$7.1\ 10^{18}$
	1.000	0.184	0.0876	0.0322	0.4752	0.1745	0.7972	0.1731	0.0297	
	1.269	0.112	0.0633	0.0233	0.5646	0.2075	0.7661	0.1976	0.0363	
^{130}Te	0.800	0.0466	0.00873	0.00239	0.1873	0.0512	0.9389	0.0569	0.0042	$6.9\ 10^{20}$
	1.000	0.0298	0.00577	0.00144	0.1937	0.0482	0.9371	0.0588	0.0041	
	1.269	0.0185	0.00373	0.00078	0.2015	0.0420	0.9352	0.0610	0.0038	
^{136}Xe	0.800	0.0268	0.00706	0.00232	0.2637	0.0866	0.9190	0.0745	0.0065	$2.19\ 10^{21}$
	1.000	0.0170	0.00526	0.00169	0.3098	0.0995	0.9059	0.0863	0.0078	
	1.269	0.0104	0.00403	0.00126	0.3867	0.1207	0.8848	0.1051	0.0101	

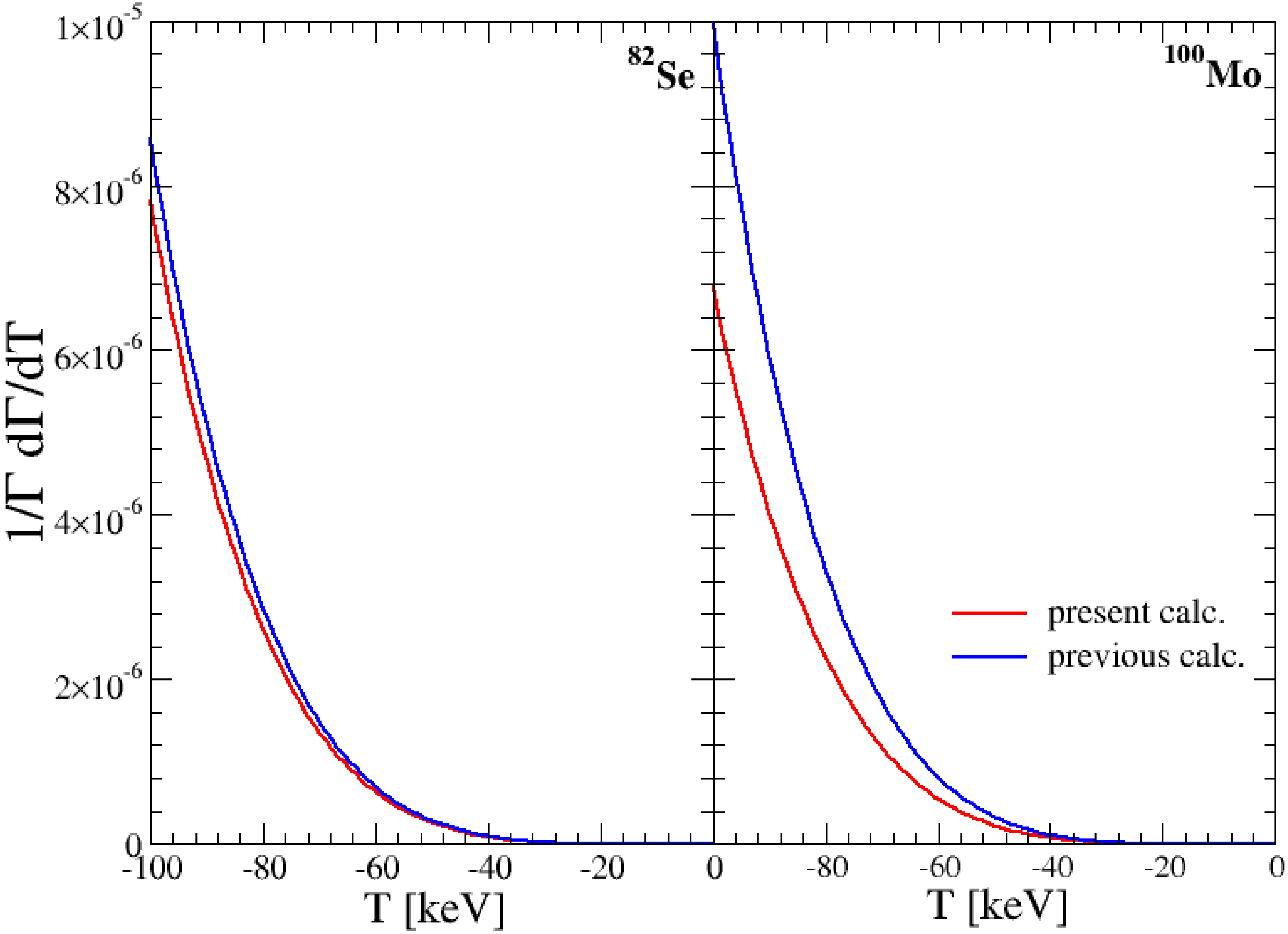
Normalized to unity different partial energy distributions



The sum electron energy distribution



The endpoint of the spectrum of differential decay rate vs. the sum of kinetic energy of emitted electrons



The half-life and ratios of NMEs

$$\left[T^{2\nu\beta\beta}_{1/2}\right]^{-1} = \left(g_A^{\rm eff}\right)^4 \left|M^{2\nu}_{GT-1}\right|^2 \left(G^{2\nu}_0 + \Re\{\xi^{2\nu}_{13}\} G^{2\nu}_2 + \frac{1}{3}\left|\xi^{2\nu}_{13}\right|^2 G^{2\nu}_{22} + \left(\frac{1}{3}\left|\xi^{2\nu}_{13}\right|^2 + \Re\{\xi^{2\nu}_{15}\}\right) G^{2\nu}_4\right)$$

$$\xi^{2\nu}_{13} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}} \qquad \xi^{2\nu}_{15} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$

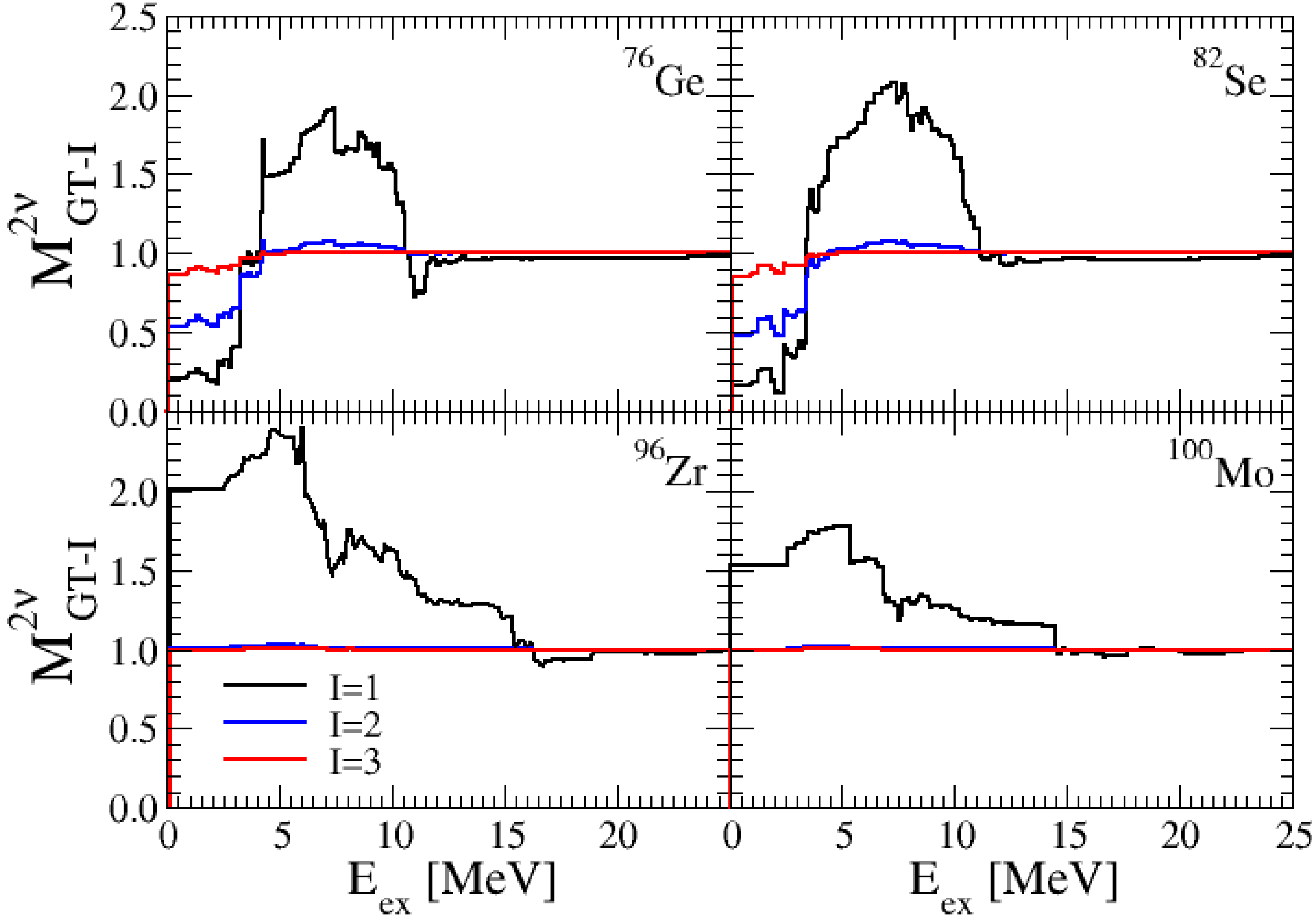
The half-life expressed with only one ratio of NMEs

$$\left[T^{2\nu\beta\beta}_{1/2}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M^{2\nu}_{GT-3}\right|^2 \frac{1}{\left|\xi^{2\nu}_{13}\right|^2} \left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)$$

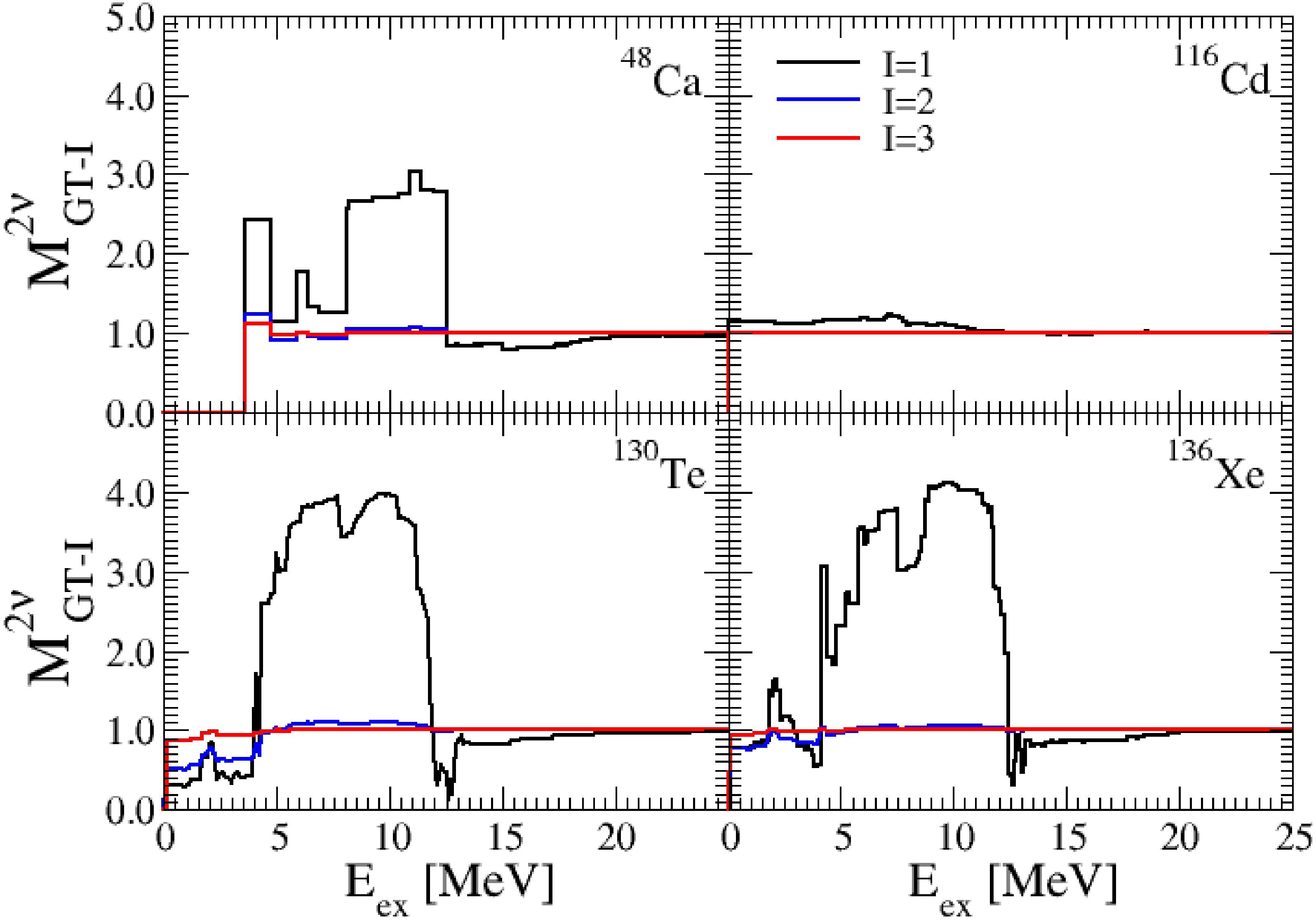
The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)

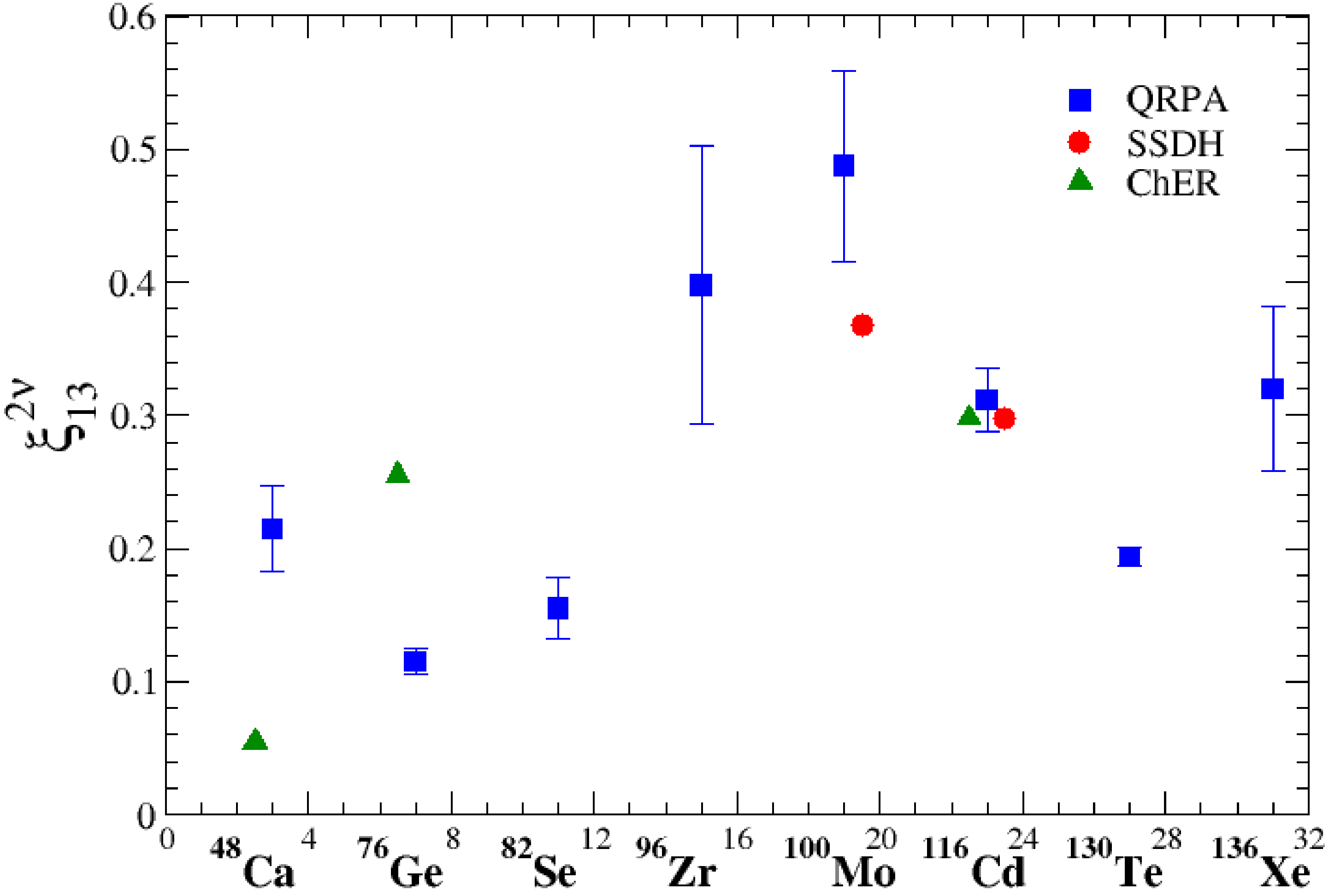
$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|} \frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)}}$$

The running sum of the 2νββ–decay NMEs



The running sum of the 2νββ–decay NMEs





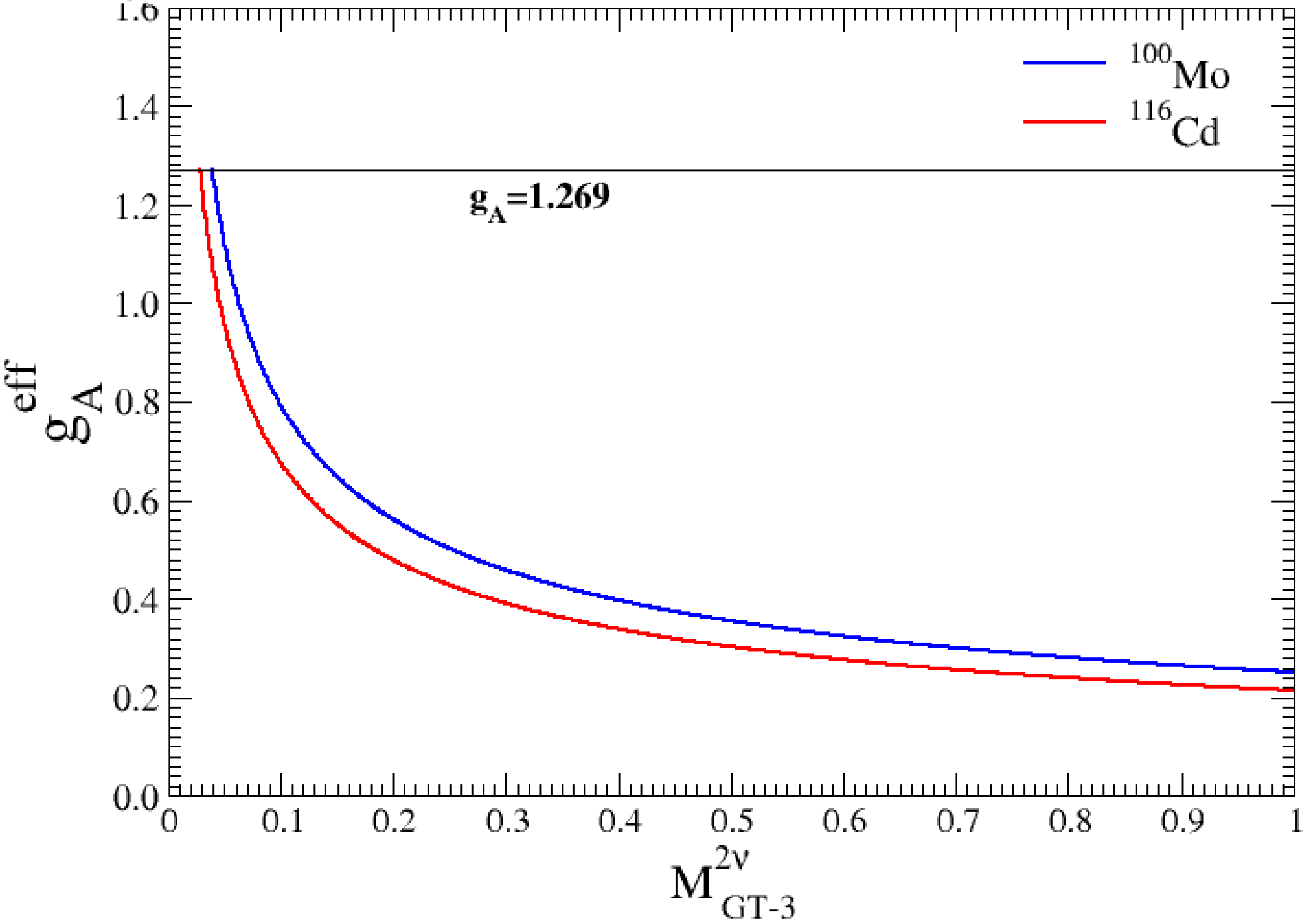
$\xi_{13}^{2\nu}$
0.6
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
16
20
24
28
32
^{48}Ca
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{116}Cd
^{130}Te
^{136}Xe
QRPA
SSDH
ChER

Solution: NEMO3/Supernemo measurement of ξ and calculation of M_{GT-3}

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

$$g_A^{\rm eff}(^{100}{\rm Mo}) = \frac{0.251}{\sqrt{M_{GT-3}^{2\nu}}}$$

$$g_A^{\rm eff}(^{100}{\rm Cd}) = \frac{0.214}{\sqrt{M_{GT-3}^{2\nu}}}$$



The DBD Nuclear Matrix Elements and the SU(4) symmetry

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

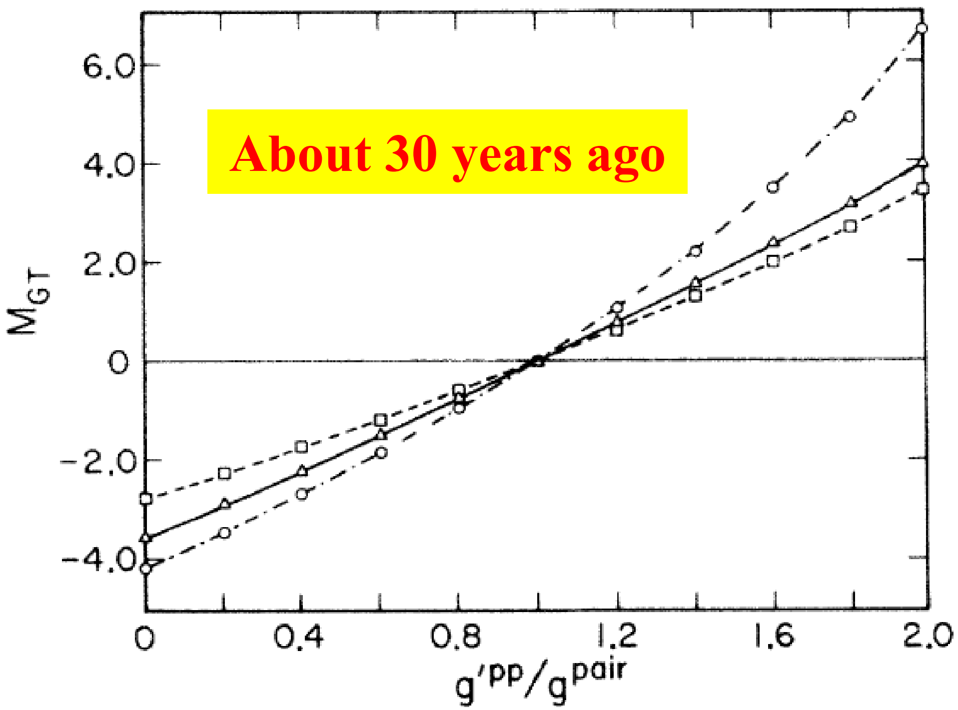
Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11

E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53

…



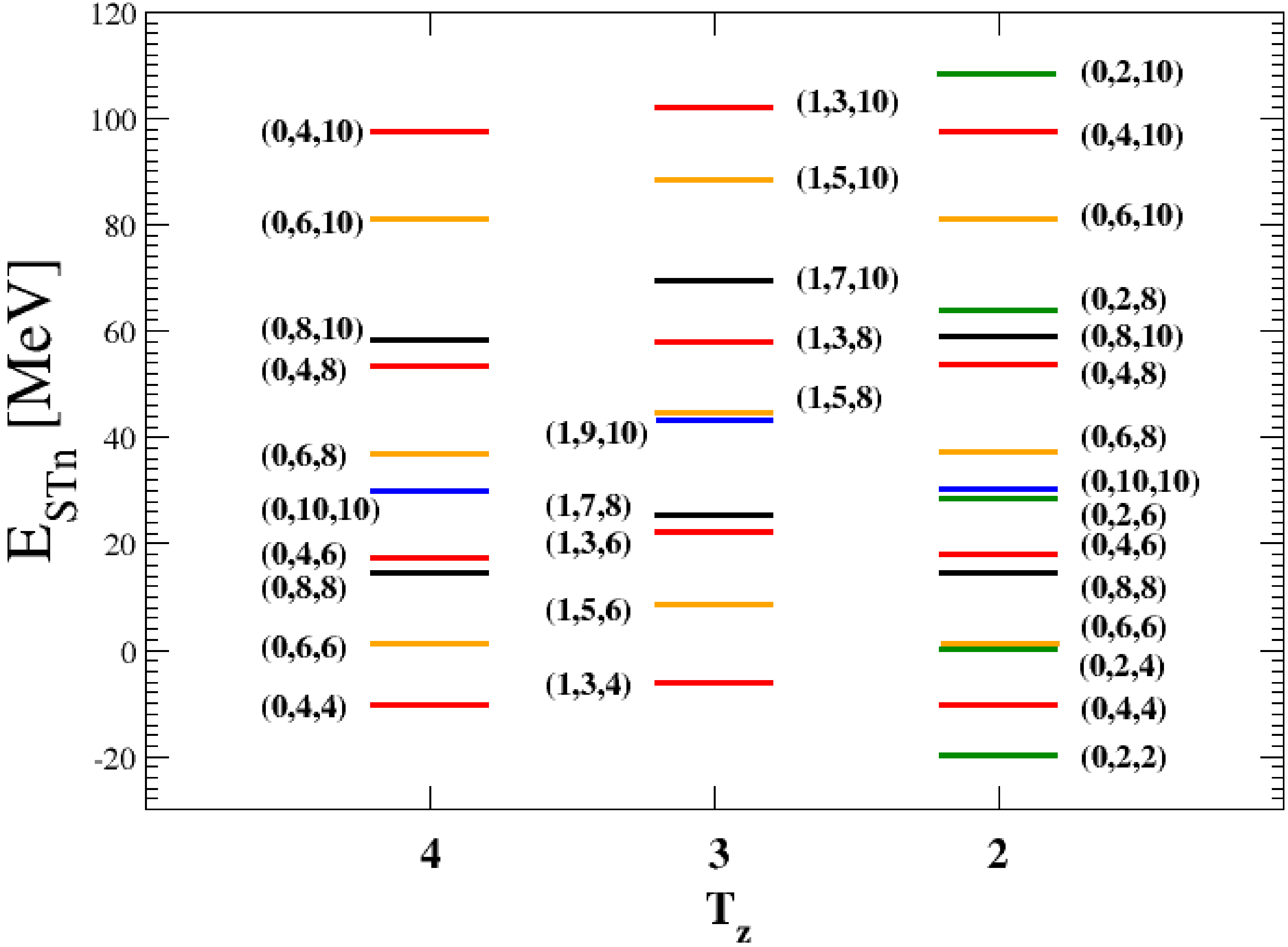
The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior? Is it an artifact of the QRPA?

Energies of excited states for the case of conserved SU(4) symmetry

M_F=0, M_{GT}=0 (see SU(4) multiplets)



s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1})A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

H_I violates SU(4) symmetry

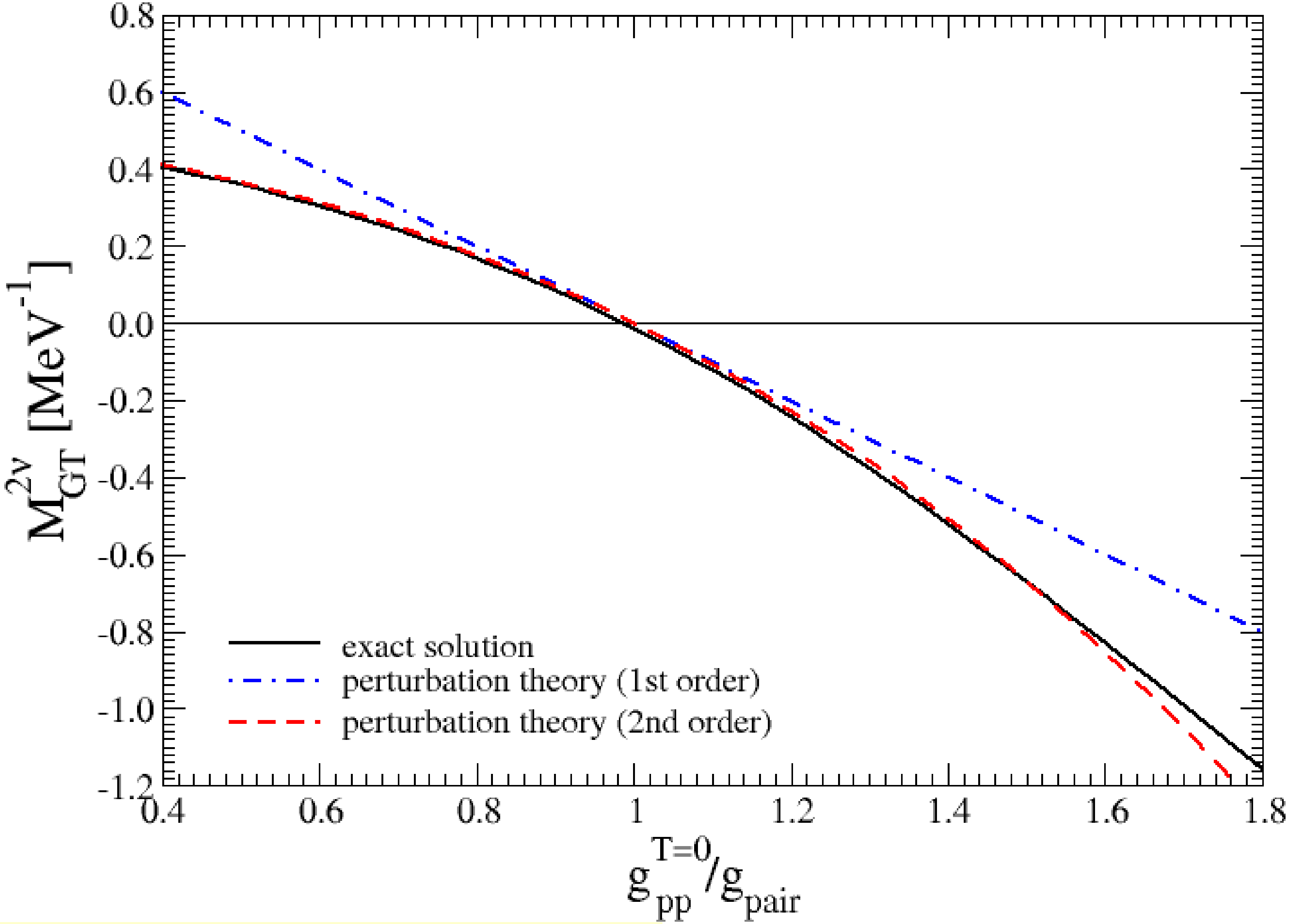
g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, $M_T=\pm1$)
$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, $M_T=0$)
$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)
g_{ph}- strength of particle-hole force

M_F and M_{GT} do not depend on the mean-field part of **H** and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of **H**

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\,(g_{pair} - g_{pp}^{T=1})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

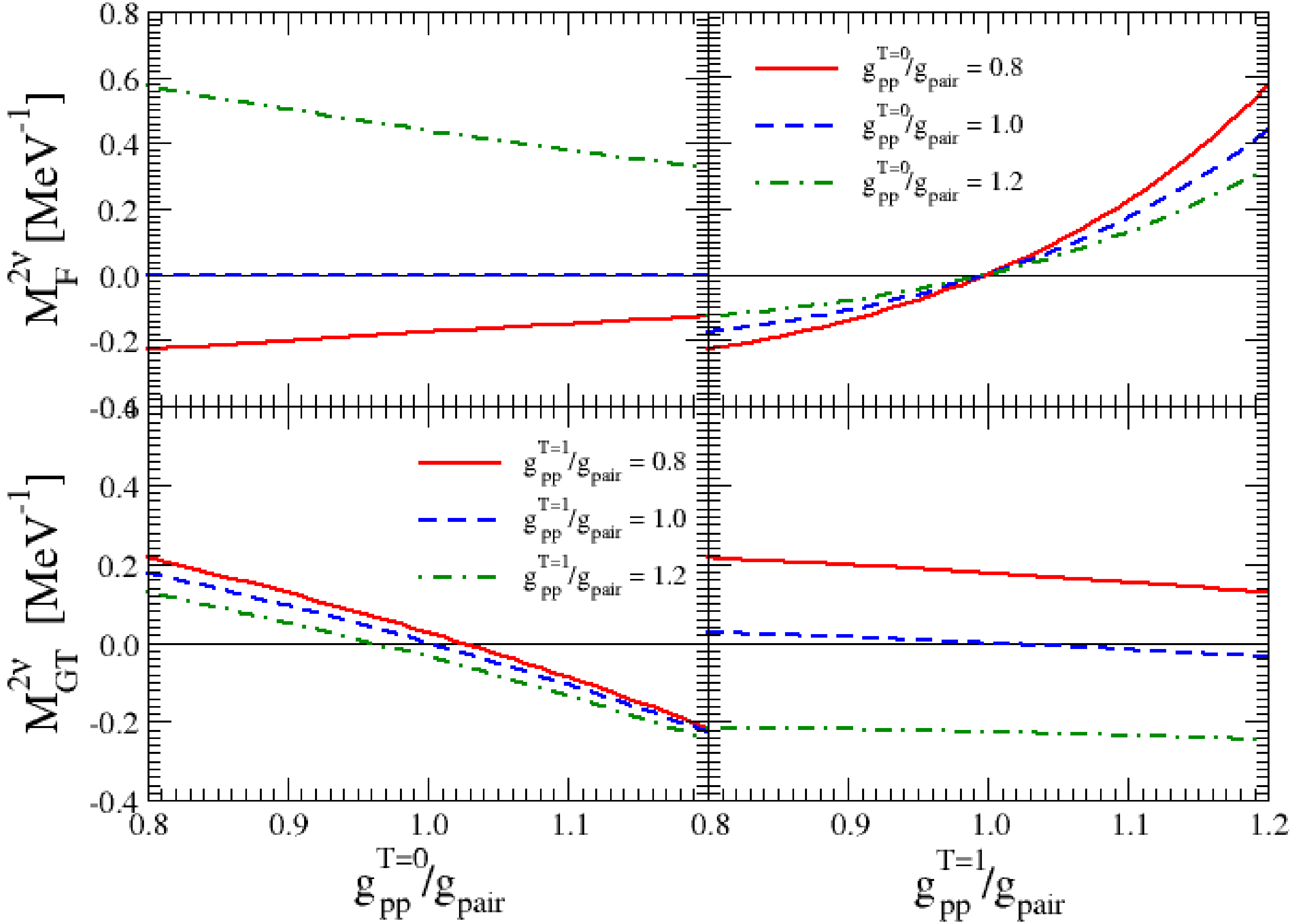
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}}\left\{\frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}\right\}$$

M_{GT} up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H



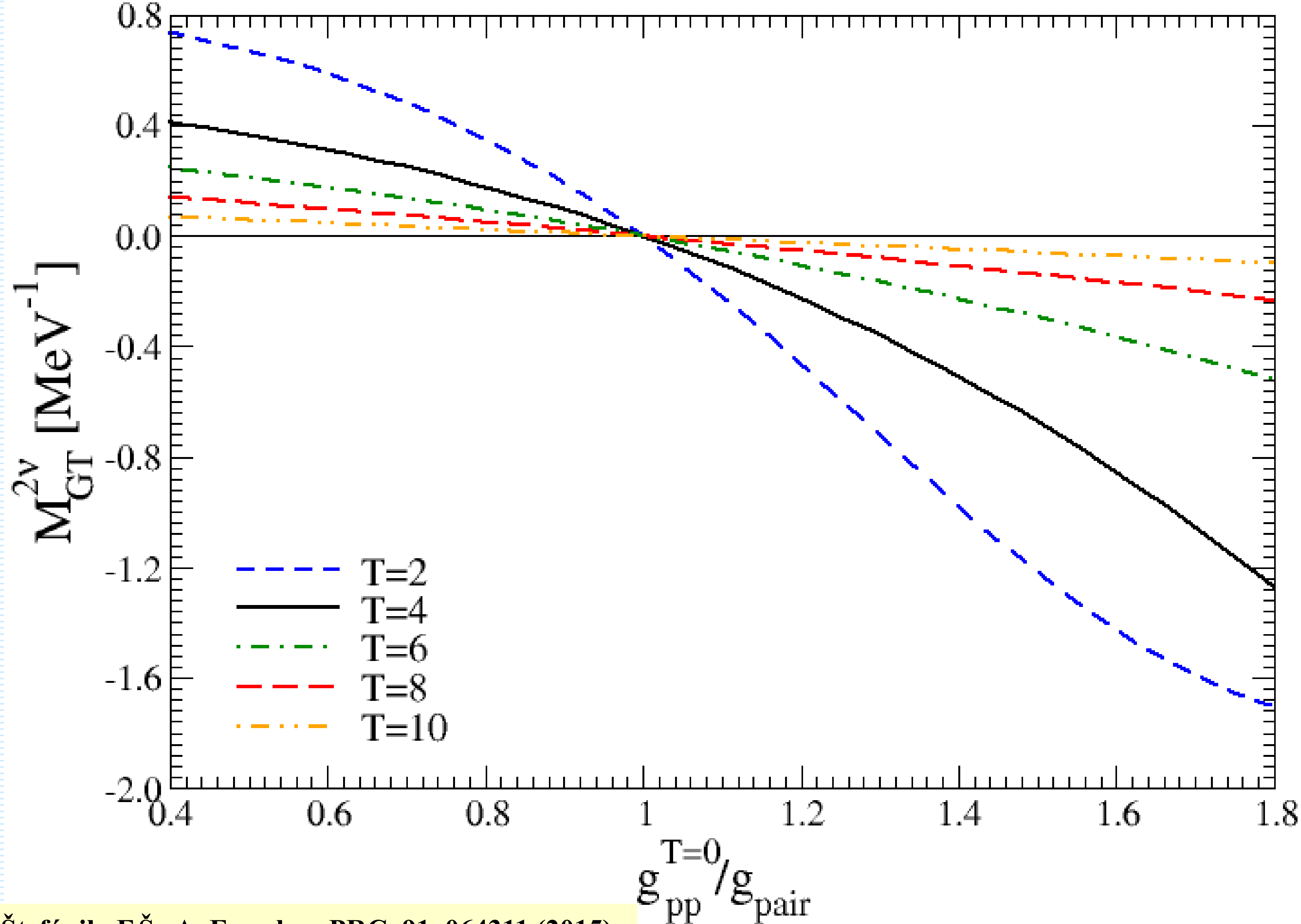
D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

Results confirm dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA



D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

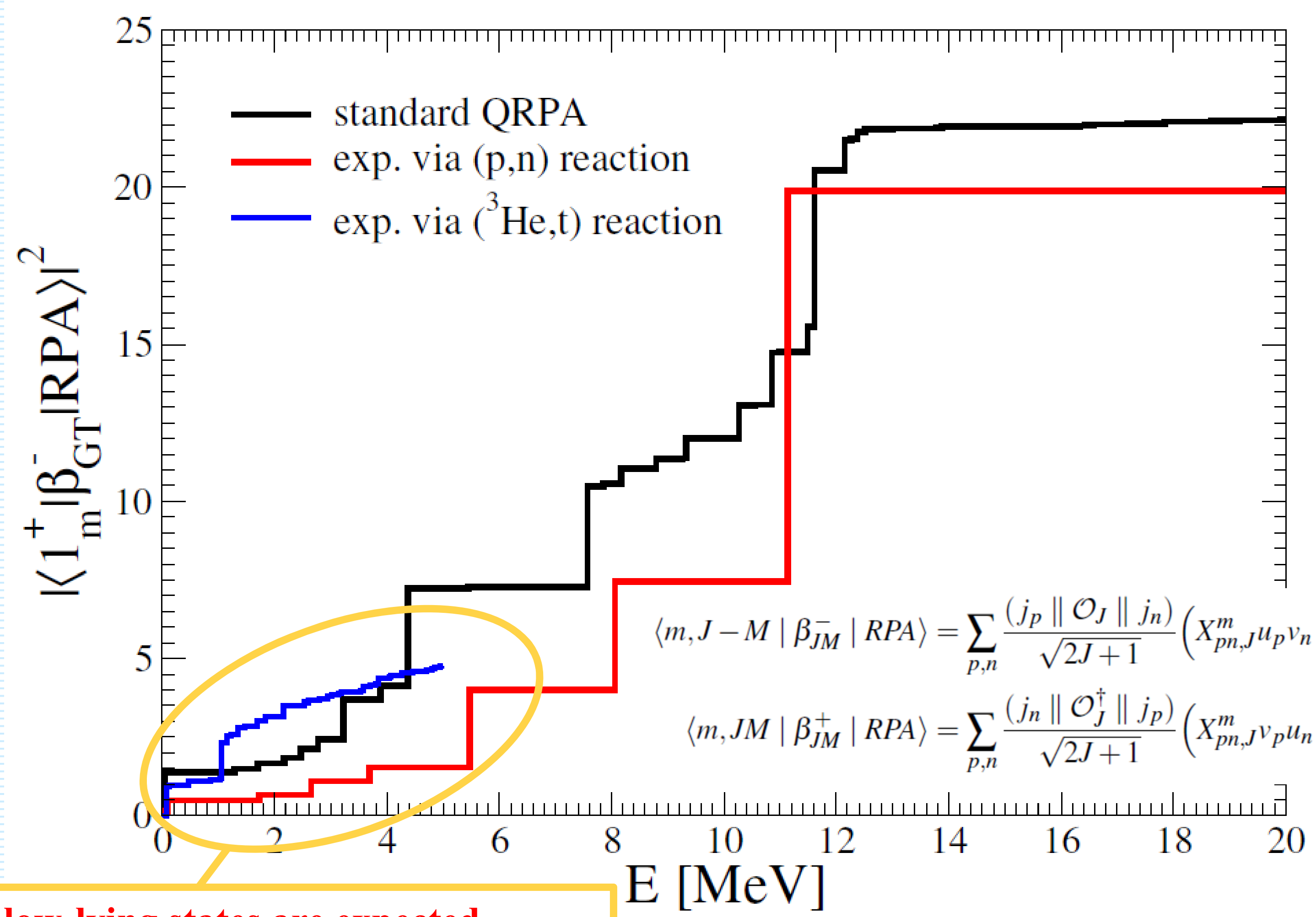
By assuming a fixed violation of the SU(4) symmetry by particle-particle int.
M_{GT} decreases by increase of isospin of the ground state



D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

β^- transitions in the standard QRPA

Calculate what can be confronted with experiment.



$$\langle m, J-M \mid \beta^-_{JM} \mid RPA \rangle = \sum_{p,n} \frac{(j_p \parallel \mathcal{O}_J \parallel j_n)}{\sqrt{2J+1}} \left(X^m_{pn,J} u_p v_n - Y^m_{pn,J} v_p u_n \right),$$

$$\langle m, JM \mid \beta^+_{JM} \mid RPA \rangle = \sum_{p,n} \frac{(j_n \parallel \mathcal{O}^\dagger_J \parallel j_p)}{\sqrt{2J+1}} \left(X^m_{pn,J} v_p u_n - Y^m_{pn,J} u_p v_n \right).$$

- **low-lying states are expected to be important for 2νββ decay**
- **we need improvement in this region**

Limitations of the standard QRPA

We want to fix the following limitations of the standard QRPA:

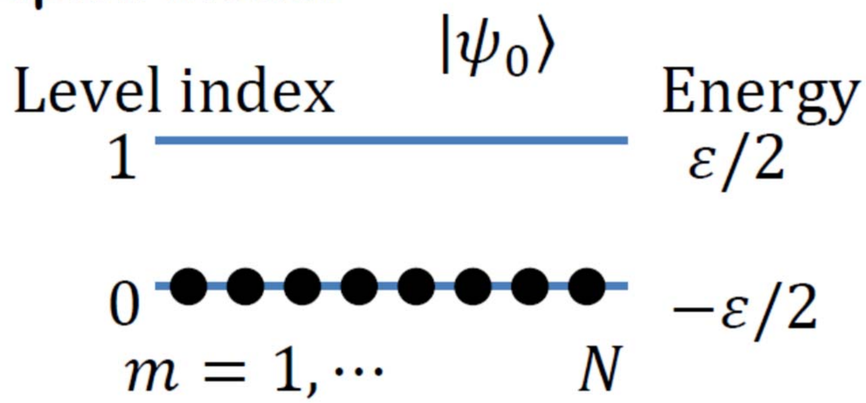
1. **Due to the QBA Pauli principle is broken and the QRPA collapses for the higher values of coupling parameters, which might be of physical interest.**
2. **Excited states of multi-phonon structure are neglected. Only the linear terms in phonon operator are considered.**

Reproduction of exact solutions of Lipkin model by nonlinear higher random-phase approximation

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

Useful for test of theory often used.
H.J. Lipkin et al., N.P. **62**, 188 (1965)

Lipkin model



Hamiltonian

$$H = \varepsilon J_z + \frac{V}{2}\left(J_+^2 + J_-^2\right)$$

The nonlinear phonon operator

$$Q_k^{o\dagger} = \sum_{l=1}^{n} (X_{2l-1}^k \mathcal{J}_+^{2l-1} + Y_{2l-1}^k \mathcal{J}_-^{2l-1}),$$

(odd-order subspace)

$$Q_k^{e\dagger} = c_k + \sum_{l=1}^{n} (X_{2l}^k \mathcal{J}_+^{2l} + Y_{2l}^k \mathcal{J}_-^{2l}),$$

(even-order subspace)

Algebra

$$[J_z, J_+] = J_+$$
$$[J_z, J_-] = -J_-$$
$$[J_+, J_-] = 2J_z$$

RPA ground state

$$Q_k|\Psi_0\rangle = 0$$

Eigen states, wave functions, total energies, excitation energies and phonon-creation operators obtained for N=2 by the nonlinear higher RPA.

Eigenstate	Wave function	Total energy
Ground	$\lvert\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}-\varepsilon)}}\left(1 - \frac{E^o_{10}-\varepsilon}{2V}J^2_+\right)\lvert\psi_0\rangle$	$-E^o_{10}$
Odd-order excited	$Q^{o\dagger}_1\lvert\Psi_0\rangle = \frac{1}{\sqrt{2}}J_+\lvert\psi_0\rangle$	0
Even-order excited	$Q^{e\dagger}_1\lvert\Psi_0\rangle = \frac{V}{\sqrt{2E^o_{10}(E^o_{10}+\varepsilon)}}\left(1 + \frac{E^o_{10}+\varepsilon}{2V}J^2_+\right)\lvert\psi_0\rangle$	E^o_{10}

Eigenstate	Excitation energy	Phonon-creation operator
Ground	0	
Odd-order excited	$E^o_{10} = \sqrt{\varepsilon^2 + V^2}$	$Q^{o\dagger}_1 = \frac{\sqrt{E^o_{10}}}{2\varepsilon}\left(\frac{V}{\lvert V\rvert}\sqrt{E^o_{10}+\varepsilon}J_+ + \sqrt{E^o_{10}-\varepsilon}J_-\right)$
Even-order excited	$E^e_{10} = 2E^o_{10}$	$Q^{e\dagger}_1 = \frac{V}{\lvert V\rvert}\left(\frac{V}{2\varepsilon} + \frac{E^o_{10}+\varepsilon}{4\varepsilon}J^2_+ + \frac{E^o_{10}-\varepsilon}{4\varepsilon}J^2_-\right)$

J. Terasaki, A. Smetana, F. Š., M.I. Krivoruchenko, arXiv:1701.08368 [nucl-th]

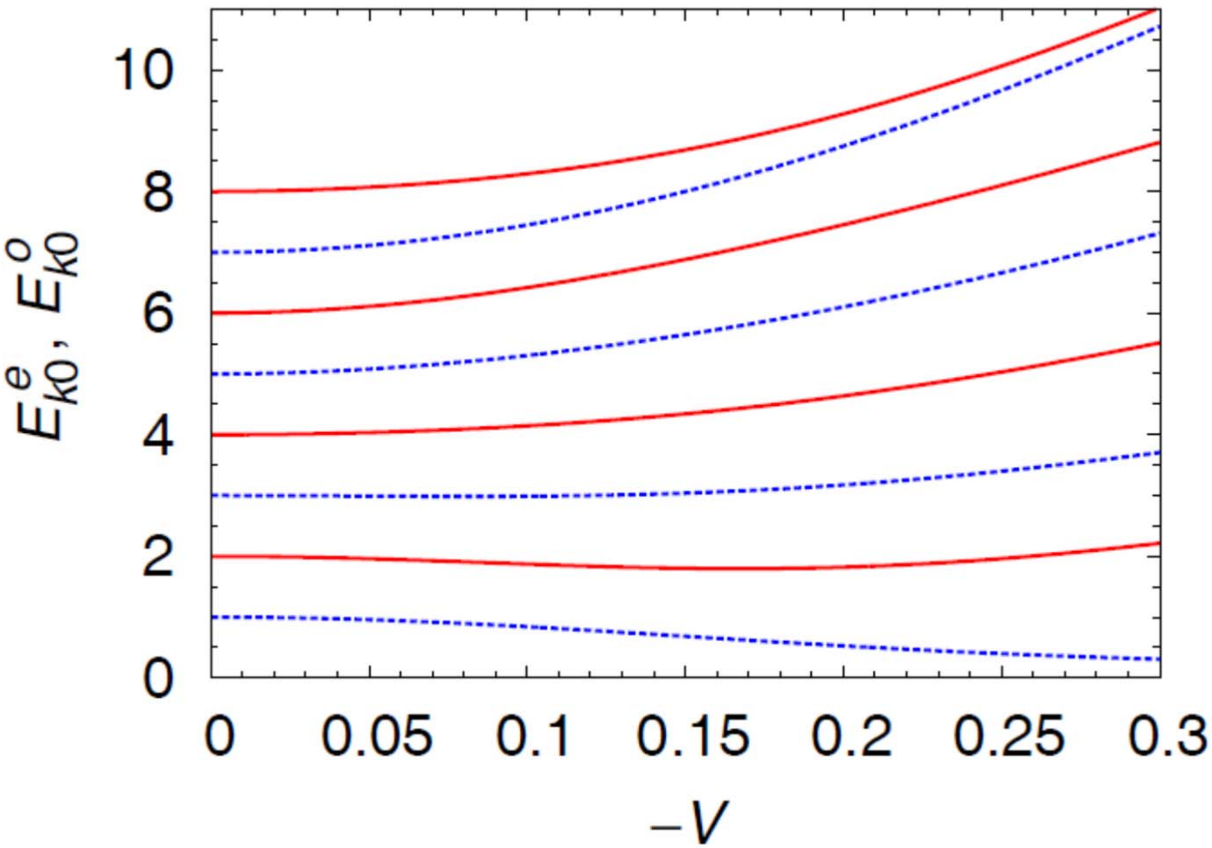
RPA equation

$$\begin{pmatrix} A^o & B^o \\ B^o & A^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix} = E_{k0}^o \begin{pmatrix} U^o & O \\ O & -U^o \end{pmatrix} \begin{pmatrix} X_k^o \\ Y_k^o \end{pmatrix}$$

$$\begin{aligned} A_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle \\ B_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, H, \mathcal{J}_-^{2j-1}] | \Psi_0 \rangle \\ U_{ij}^o &= \langle \Psi_0 | [\mathcal{J}_-^{2i-1}, \mathcal{J}_+^{2j-1}] | \Psi_0 \rangle \end{aligned}$$

$$[A, B, C] = (1/2)[[A, B], C] + (1/2)[A, [B, C]]$$

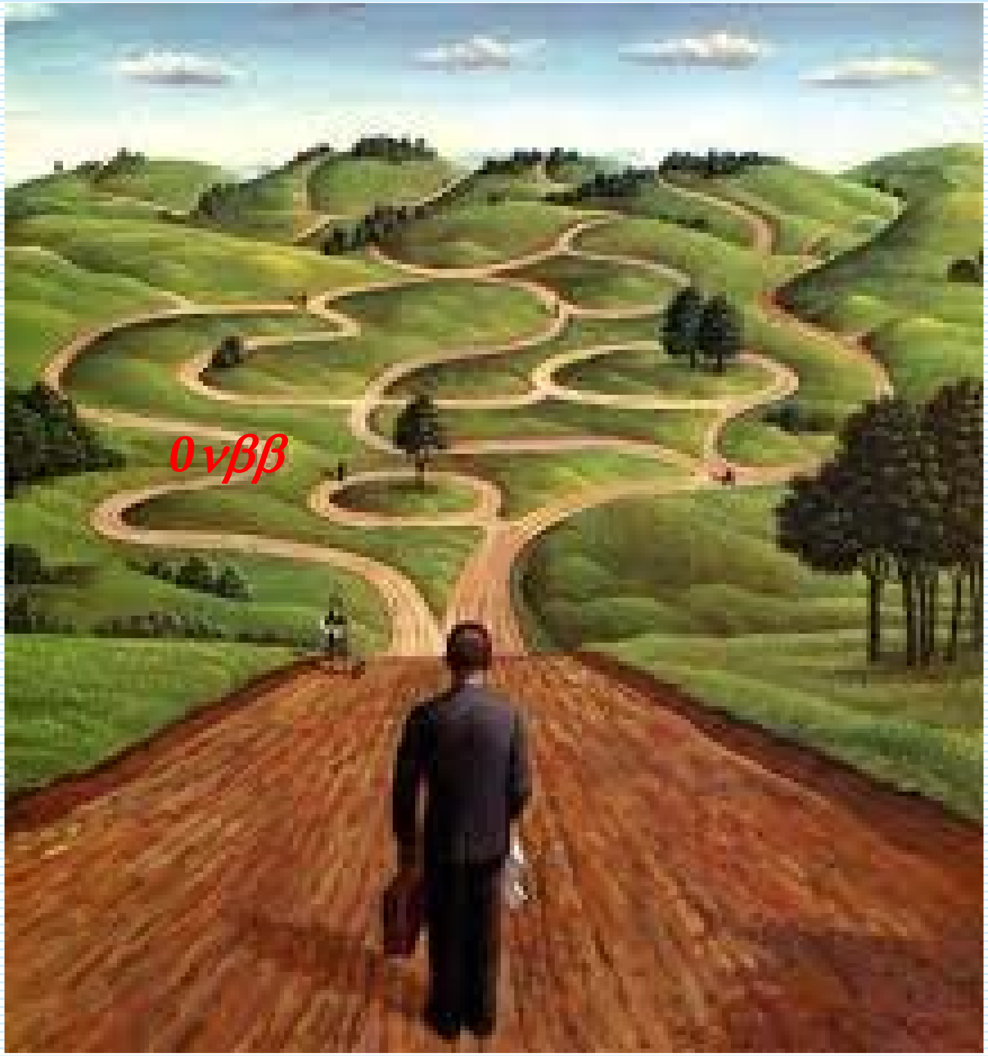
N=8, ε=1
Breaking point of RPA is V=-0.143



Exact agreement of RPA results with those obtained by diagonalization of H

Instead of Conclusions

Progress in nuclear structure calculations is highly required



Progress in nuclear structure calculations is highly required