GIANT RESONANCES

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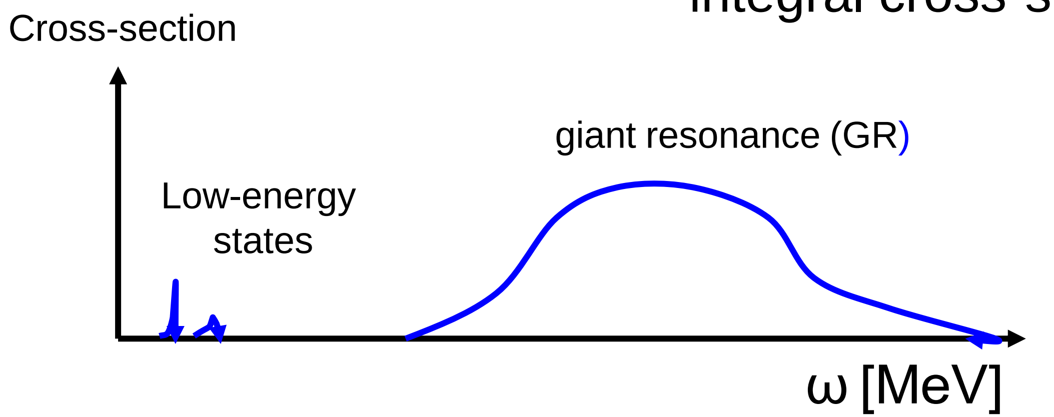
Introduction

GR definition: a collective nuclear excitation exhausting the essential (main) part of the integral cross-section of the reaction.



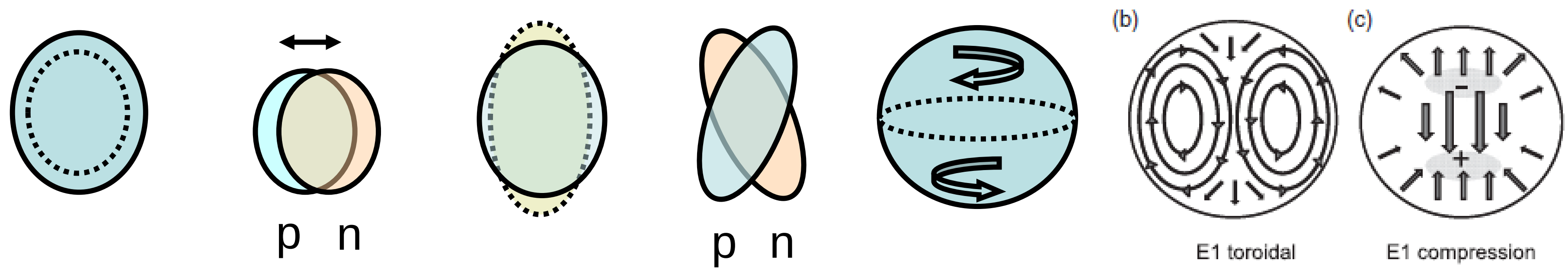
- first GR (giant dipole resonance -GDR) was observed in 1947, other GR were found ~ 30-40 years later.

- now we know a variety of electric and magnetic GR and rather well understand their properties

M. N. Harakeh and A. van der Woude, “Giant Resonances” (Clarendon Press, Oxford, 2001).

Variety of GR

isoscalar (IS,T=0): protons and neutrons oscillate in phase
isovector (IV,T=1): protons and neutrons oscillate in opposite phase



E0(T=0,1)	E1(T=1)	E2(T=0,1)	M1(T=1)	M2(T=0,1)	E1(T=0,1)	E1(T=0,1)
GMR	**GDR**	**GQR**	**scissors**	**twist**	**toroidal**	**compression**

Also spin-flip M1 GR(T=1), …..

- charge-exchange GR: Gamow-Teller GR, … ⟵ omitted in the present talk

As a rule, the origin and main properties of GR are already known.
Then why GR are still actual?

Why GR are actual?

☆ GR are used as a robust test for modern self-consistent mean-field approaches (Skyrme, Gogni, relativistic, …) based on the density functionals (density functional theory).

These approaches

- are **main theoretical tools** to describe GR and other excitations,
- are **self-consistent**: both mean field and residual interaction are obtained from the same initial functional (no additional free parameters)

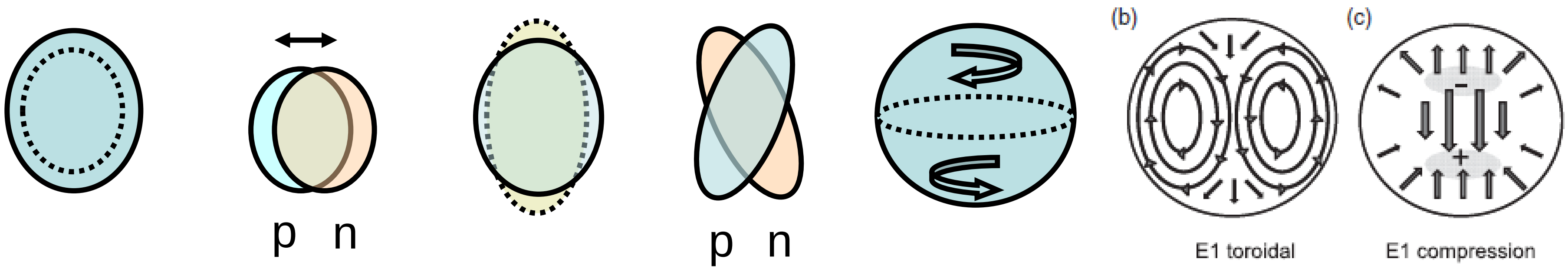
$$E\{J_\alpha(\vec{r},t), b_0,...,b_n\} = \langle \Psi | H | \Psi \rangle$$

$$h(\vec{r},t) = h_0(\vec{r}) + h_{res}(\vec{r},t) = \sum_\alpha \underbrace{[\frac{\delta E}{\delta J_\alpha}]_{J=\bar{J}}}_{\text{mean field}} \hat{J}_\alpha(\vec{r}) + \sum_{\alpha\alpha'} \underbrace{[\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}]_{J=\bar{J}}}_{\text{residual interaction}} \delta J_\alpha(\vec{r},t)\,\hat{J}_\alpha(\vec{r})$$

- pretend to be quite **universal**: description of **astrophysical problems**, **nuclear matter** (symmetric, neutron), and **finite nuclei** through almost all the periodic table,
- pretend to describe both **static nuclear properties** and **nuclear dynamics**
- **parameters** of the initial functional are fitted to describe both finite nuclei (statics, **dynamics**) and nuclear matter

Just GR are used to test functional parameterizations to nuclear dynamics!
Thus GR are extremely important for modern nuclear theory.

GR $\longleftrightarrow$ useful information on features of finite nuclei and nuclear matter



E0(T=0,1)	E1(T=1)	E2(T=0,1)	M1(T=1)	M2(T=0,1)	E1(T=0,1)	E1(T=0,1)
GMR	**GDR**	**GQR**	**scissors**	**twist**	**toroidal**	**compression**
↓	↓	↓	↓	↓	↓	↓
nuclear Incompress.	m^*_1	m^*_0	orbital magnetism	orbital magnetism	vortical motion	nuclear Incompress.

GR still have many open problems:

- troubles to describe simultaneously:
  - GDR(T=1) and GQR(T=0),
  - one-bump and two-bump structures in spin-flip M1 GR
  - GMR in Pb and Sn isotopes
- exotic GR (toroidal) …

GR are still very hot topic!

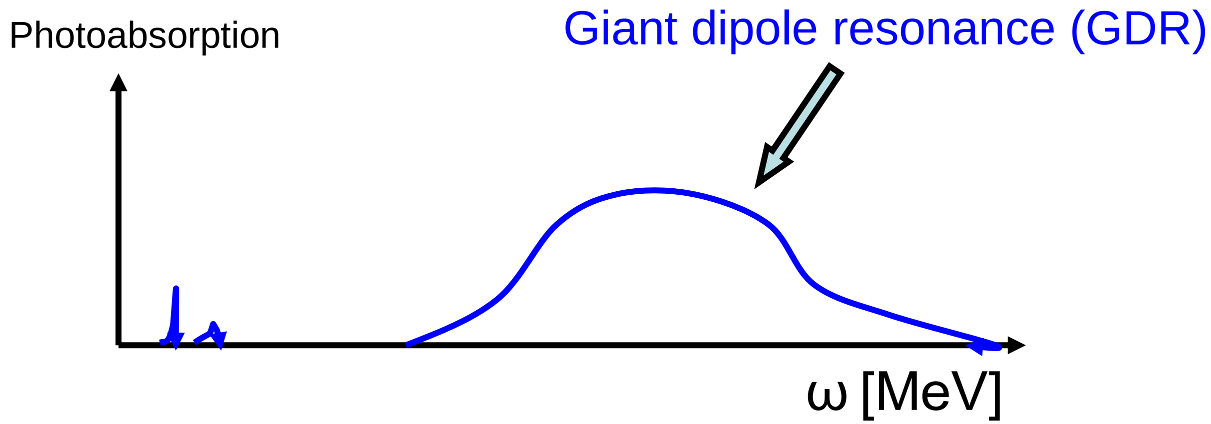
Content:

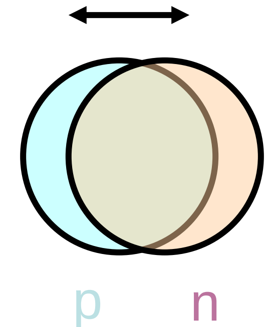
-Variety of giant resonances (GR) in nuclei:
- most important electric GR: E1(T=1), E0(T=0), E2(T=0)
- most important magnetic GR: spin-flip M1, scissors M1
- exotic E1(T=0) GR: toroidal and compression

- Relation to mean field and quantum shells
- Effect of deformation (E0 vs E2, scissors M1)

- Basic theory:
  sum rules, modern self-consistent methods

Giant resonances: definition





E1 GDR (IV)

More accurate definition: giant resonance is a collective multipole (λ^{π}) excitation, exhausting an essential part of the sum rule.

$$m_1^{E\lambda} = \sum_j \omega_j \, | \langle \underbrace{j | F^{E\lambda} | 0}_{\text{response to external field F}} \rangle |^2 \Rightarrow \quad \approx \quad \boxed{\omega_{GDR} \, | \langle GDR | F^{E\lambda} | 0 \rangle |^2}$$

In 1944 (three years before the experimental discovery of GDR), the existence of GDR was predicted by theorist Migdal (USSR) using the sum rule analysis.

Sum rules look complicated but in many cases they are reduced to very simple expressions.

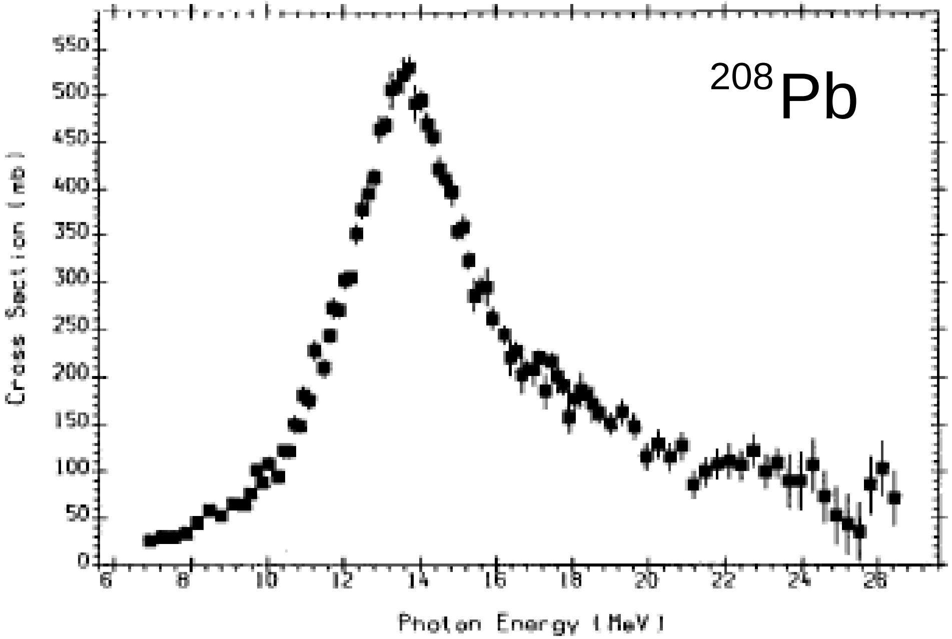
GDR $(\lambda = 1, \pi = -1)$ **:** $m_1^{E1} = \frac{\hbar^2}{2m} \frac{NZ}{A}$ - model independent!

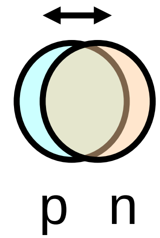
Moreover, GR energies and widths can be estimated in terms of sum rules.

E1(T=1) - GDR

A. V. Varlamov et al, Atlas of Giant Dipole Resonances, INDC(NDS)-394,1999

Experimental photoabsorption cross-section





Goldhaber-Teller model, (1948)

Alternative Steinwedel-Jensen model (1950): out-of-phase oscillations of proton and neutron densities within the sharp and fixed boundary. Gives more realistic A-dependence.

Photoabsorption:

- advantage: mainly excites dipole states,
- shortcoming: experiment is complicated since one should measure contributions of numerous decay channels .

Other reactions for GDR:

- (e,e'), (p,p'), ..one should separate GDR from other modes

Empirical estimation for GDR energy:

$$E_{GDR} = 81A^{-1/3}\ \mathrm{MeV}$$

Probe E1 operator: for $\langle j|F^{E1}|0\rangle$:

$$\hat{F}(E1K) = \frac{N}{A}\sum_{k=1}^{Z}(rY_{1K})_k - \frac{Z}{A}\sum_{k=1}^{N}(rY_{1K})_k$$

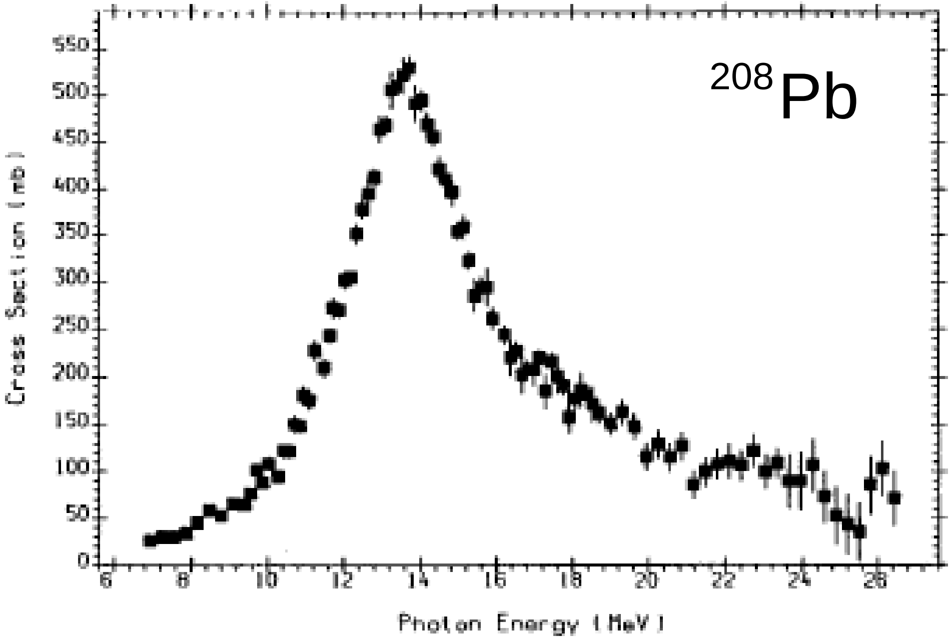
$$rY_{10} \propto z$$

Why not to use the simple operator

$$\hat{M}(E1K) = \sum_{k=1}^{A=Z+N}(rY_{1K})_k \quad ?$$

E1(T=1) - GDR

Experimental photoabsorption cross-section



The operator

$$\hat{\mathrm{M}}(\mathrm{E1K}) = \sum_{k=1}^{A=Z+N} (rY_{1K})_k$$

gives translation of the whole nucleus (center-of-mass motion).
This is not intrinsic excitations. So this is a spurious admixture which must be extracted from description of GDR.
This is obtained by using the proper effective charges in the dipole operator.

Probe E1 operator includes effective charges to remove spurious admixtures:

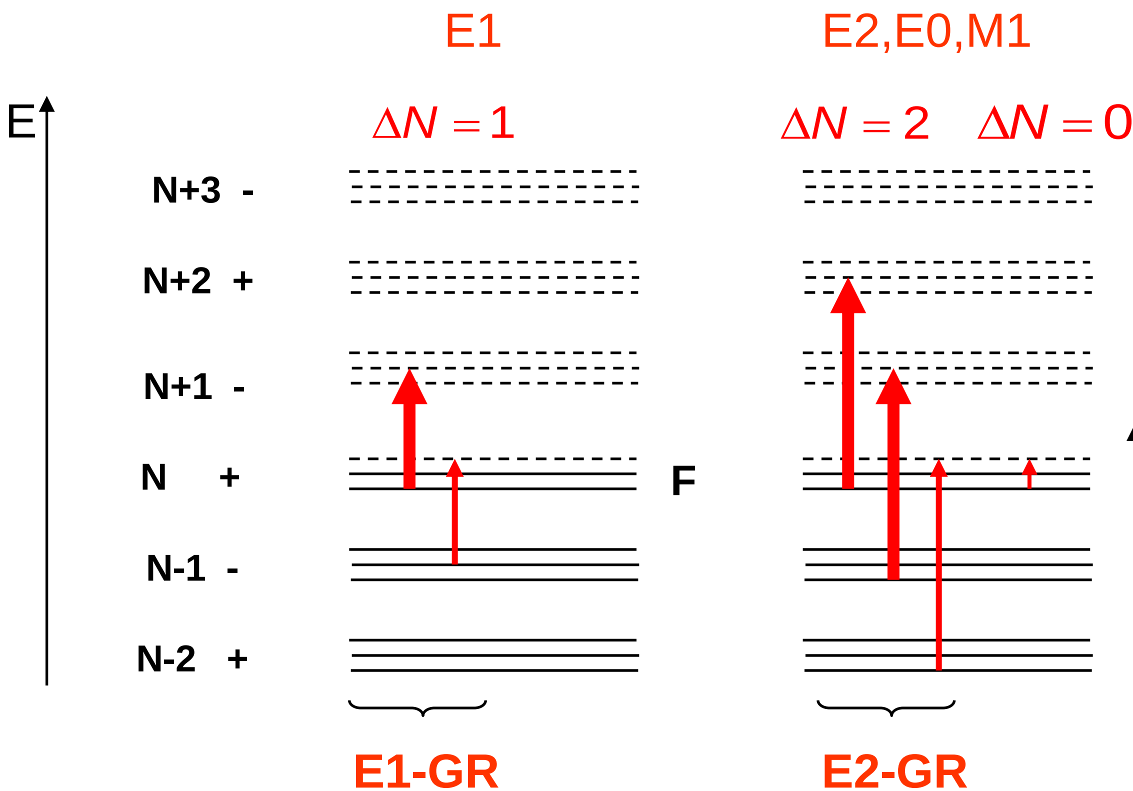
$$\hat{\mathrm{M}}(\mathrm{E1K}) = \frac{N}{A}\sum_{k=1}^{Z}(rY_{1K})_k - \frac{Z}{A}\sum_{k=1}^{N}(rY_{1K})_k$$

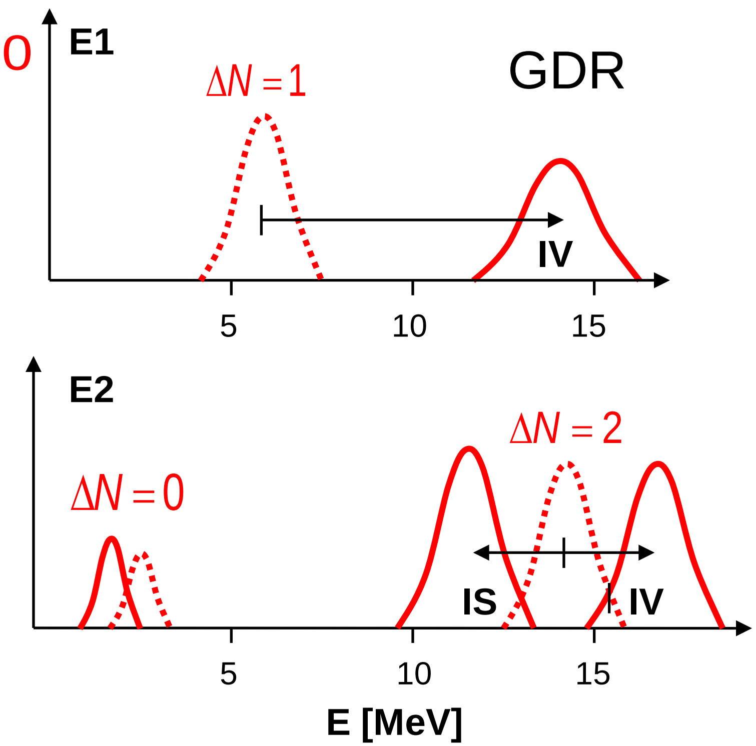
The operator does not “clean” wave functions
but only the response (matrix element of E1 transition): $\langle \mathrm{i}|\hat{\mathrm{M}}(\mathrm{E1})|0\rangle$

GR and mean field

$\tau = (-1)^{\lambda}$ **- natural parity** $E\lambda$

$\tau = (-1)^{\lambda+1}$ **- unnatural parity** $M\lambda$





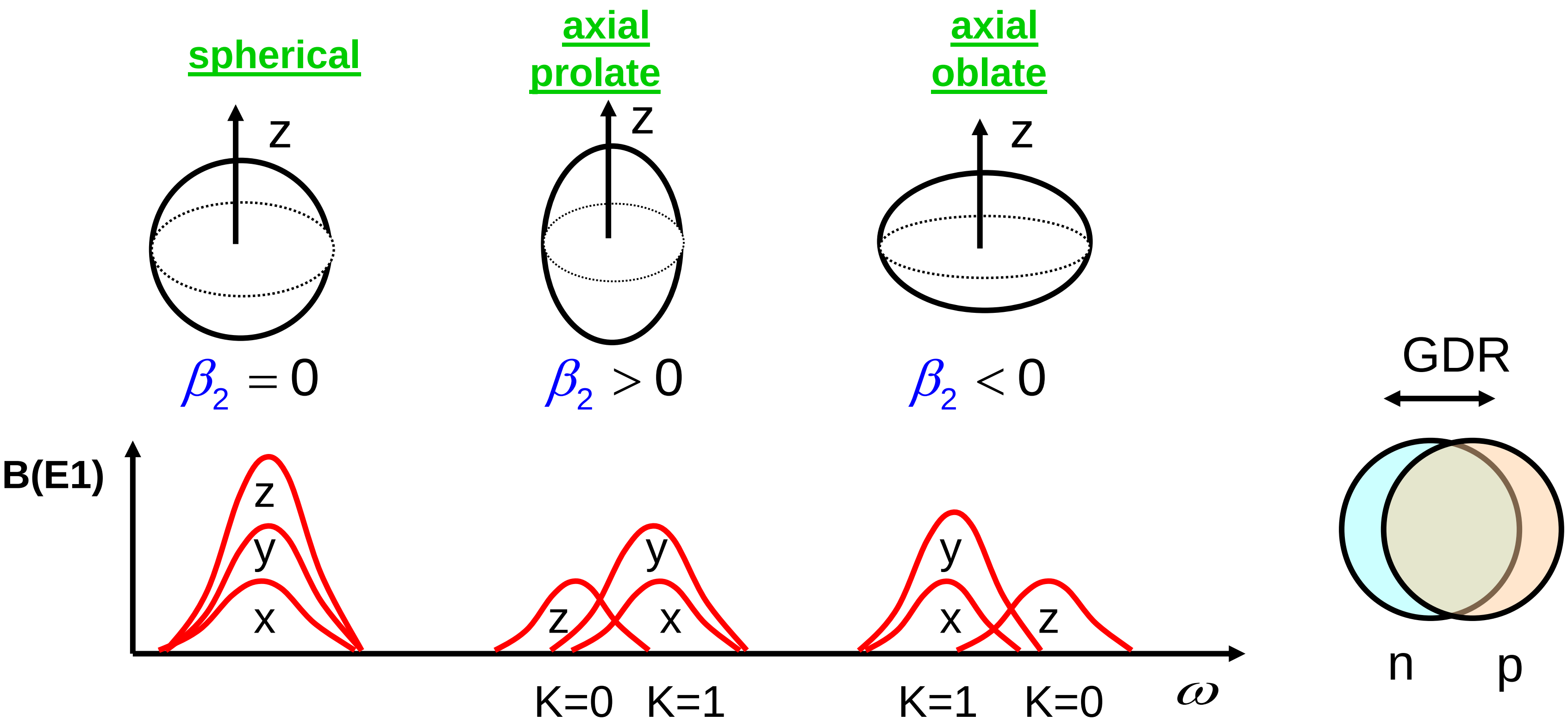
$E(\Delta N = 1) \approx 41 A^{-1/3}$ MeV (~ 7 MeV for ^{208}Pb)

$$E(E1-GR) \approx 7\ MeV$$

$$E(E2-GR) \approx 14\ MeV$$

Effect of axial deformation on GDR

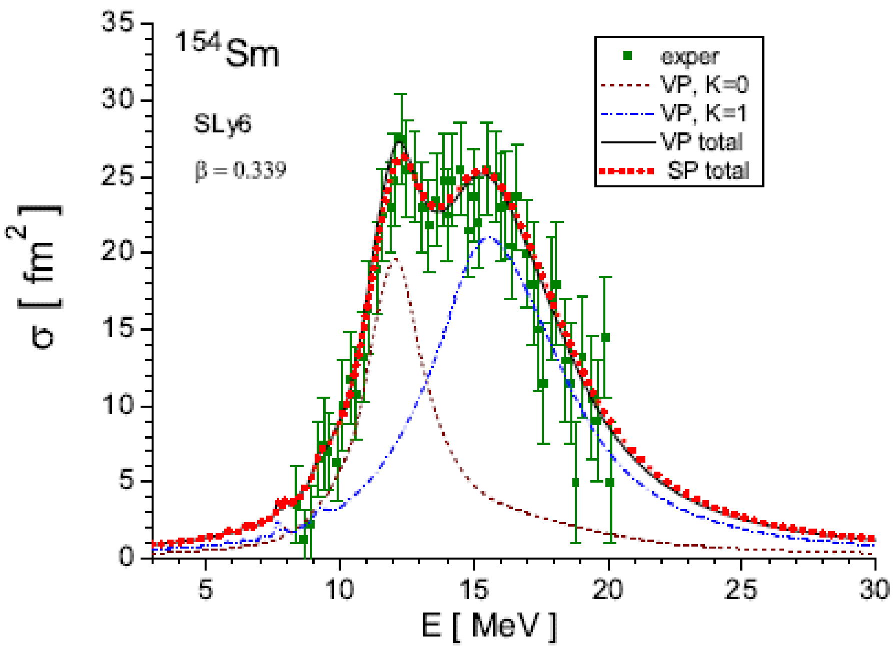
Nuclei with fully occupied valence shell are spherical (magic and semi-magic nuclei).
Nuclei with partly occupied valence shell can be deformed.
The nuclear shape with a minimum of the system energy is actually realized.



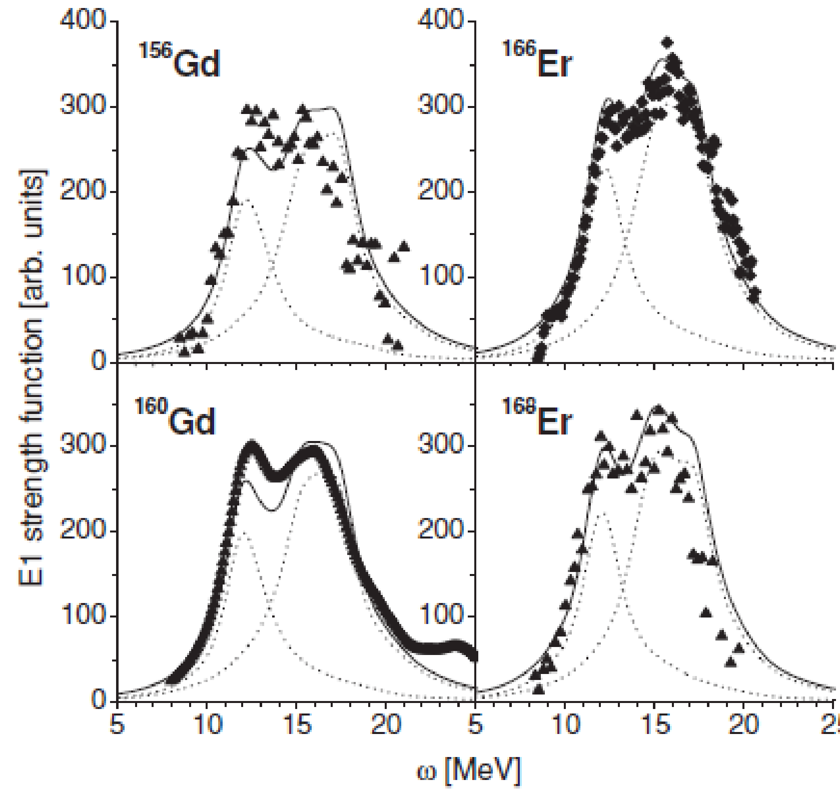
$$\omega_i \propto \frac{1}{R_i}$$

Axial quadrupole deformation causes splitting of GDR into K=0 and K=1 branches

Examples of deformation-induced GDR splitting



A Repko, J. Kvasil, V.O.N.
arXiv:1705.05436v1[nucl-th]



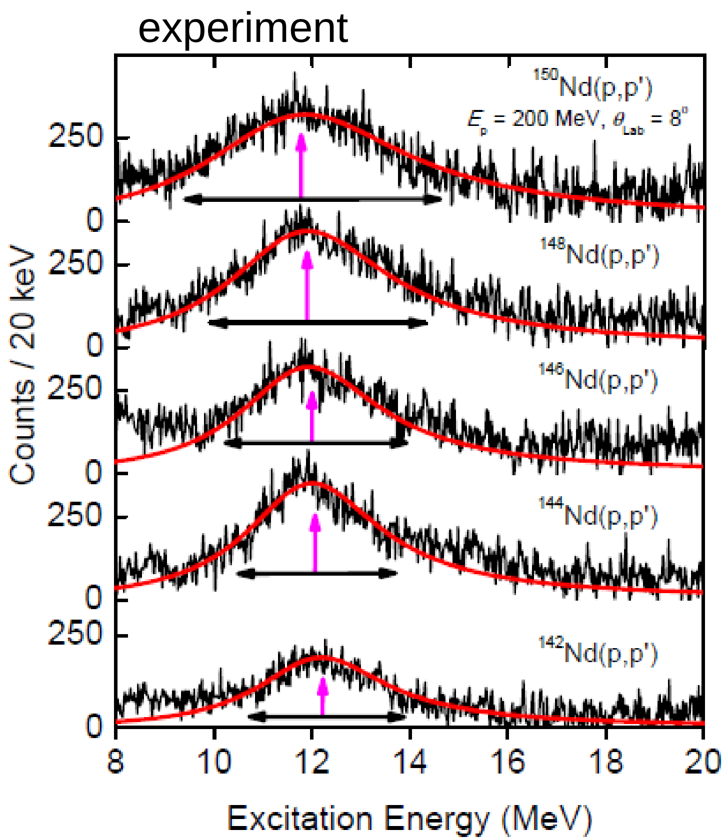
W. Kleinig , V.O.N., J. Kvasil, P.-G. Reinhard,
P. Vesely, PRC78, 044313 (2008)

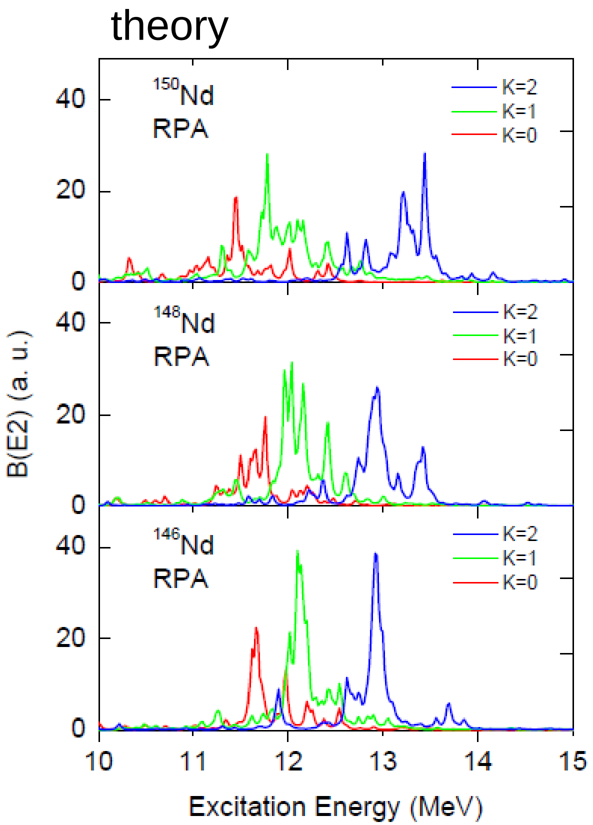
Particular GR in more detail:

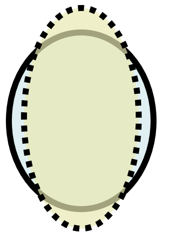
- E2(T=0),
- E0(T=0),
- spin-flip M1,
- scissors M1

Giant quadrupole resonance E2(T=0) - GQR

C.O. Kureba et al, PLB'2017







$$E = 64\,A^{-1/3}\ \text{MeV}$$

Open problems:

- dependence on the isoscalar effective mass m_0^*,
- problem of simultaneous description of E1(T=1) and E2(T=0) with Skyrme forces
- wavelet analysis of GQR fine structure (deformation splitting)

Giant monopole resonance E0(T=0) - GMR

GMR is the main source of information on nuclear incompressibility K_∞ and m_0^*

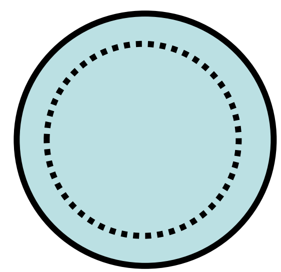
Blaizot:

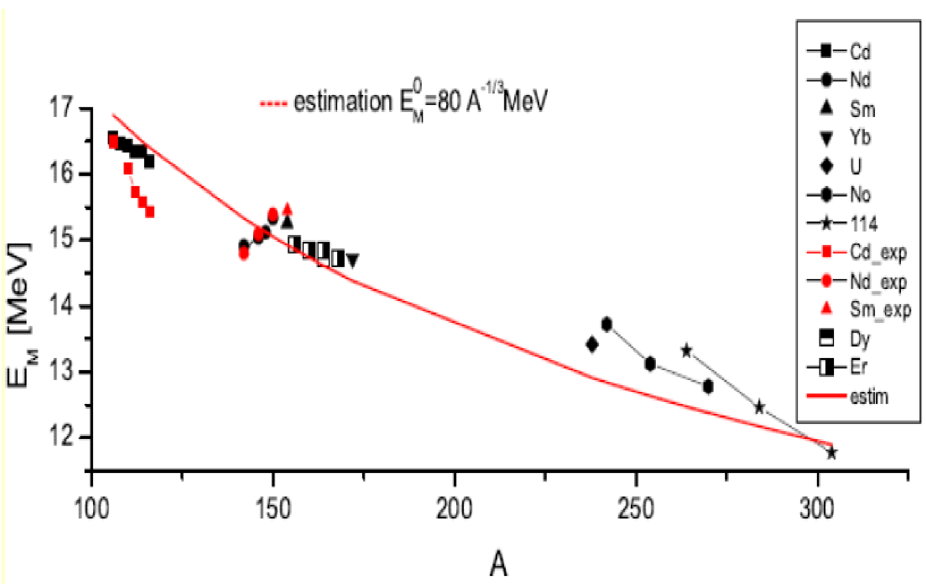
$$E_M = \sqrt{\frac{\hbar^2 K_A}{m_0^* \langle r^2 \rangle_0}} \qquad K_A = K_v + K_s A^{-1/3} + (K_{\tau v} + K_{\tau s} A^{-1/3}) \frac{(N-Z)^2}{A^2} + \ldots$$

$$K_\infty = 9\rho^2 \frac{\partial^2}{\partial \rho^2}\left[\frac{E(\rho)}{A}\right]_{\rho=\rho_0} \qquad K_A = 0.64 K_\infty - 3.5\,MeV$$

$$K_\infty = 225 - 240\,MeV$$

GMR





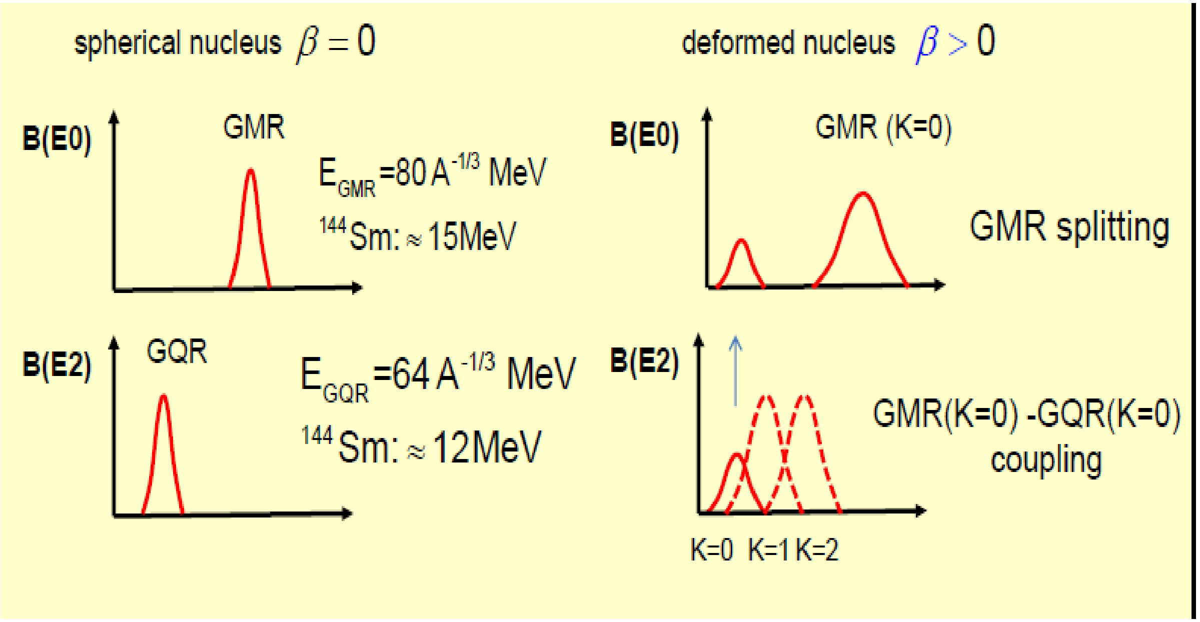
$$E_{GMR} = 80\,A^{-1/3}\ \text{MeV}$$

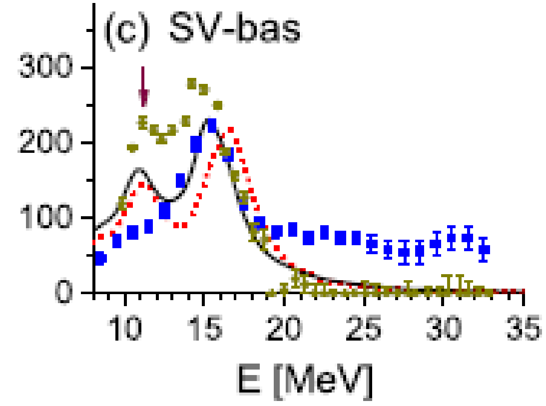
Deviations from the trend for deformed nuclei

Giant monopole resonance E0(T=0) - GMR

J. Kvasil , V.O.N., A. Repko, W. Kleinig, and P.-G. Reinhard, PRC,94, 064302 (2016)

Till now mainly GMR in spherical nuclei was used to get K_∞
But most of nuclei are deformed! Is it possible to use deformed nuclei?



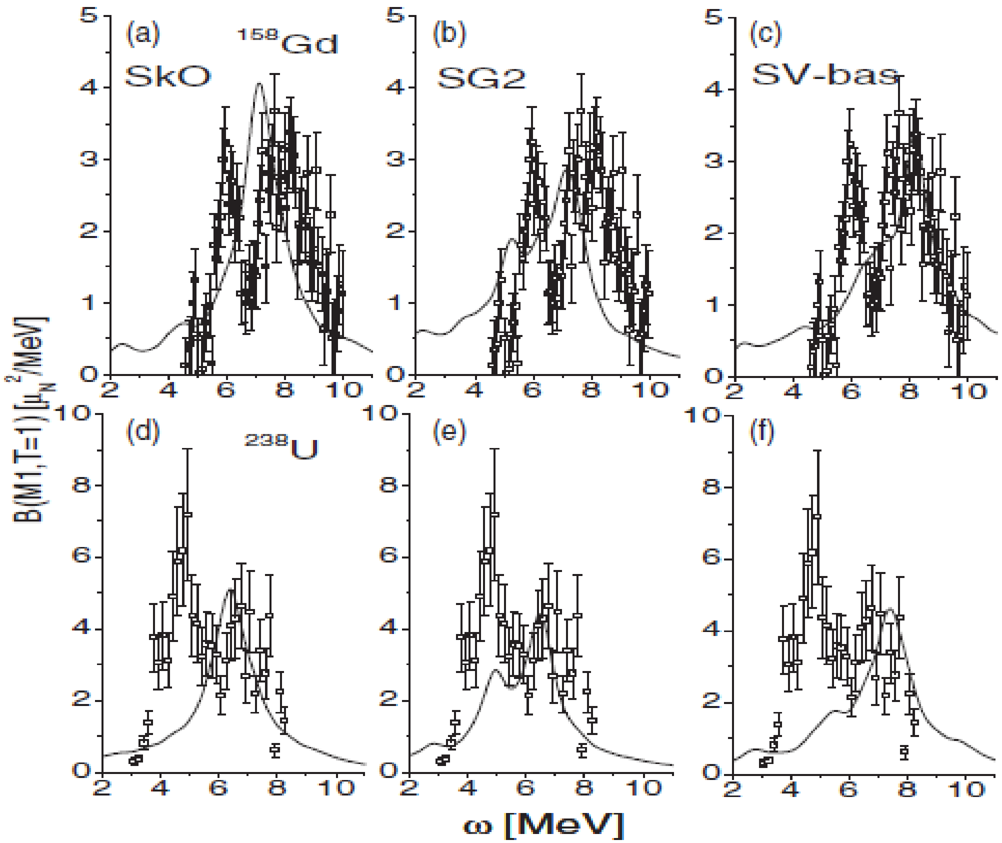


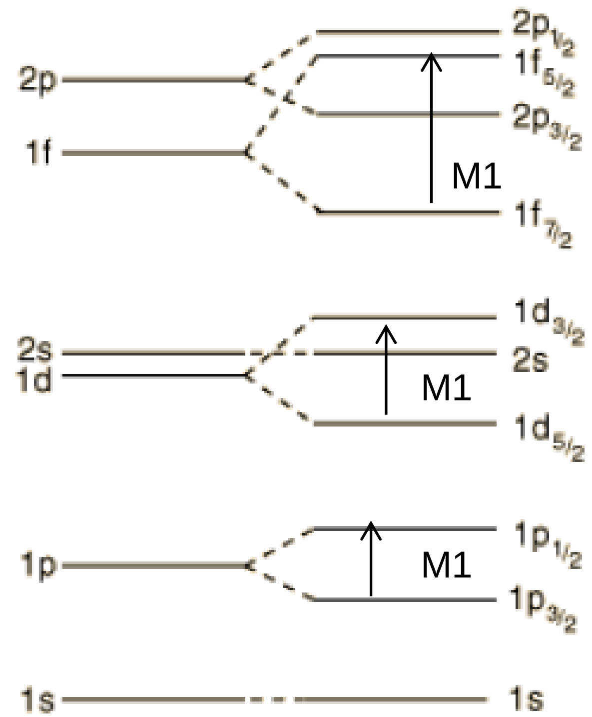
GMR and QMR are independent in spherical nuclei but coupled in axial deformed nuclei.
This leads to double-peak structure of GMR.

Open problems:
- essential discrepancy in **TAMU** and **RCNP** experimental data
- simultaneous description of GMR in Pb - Sn

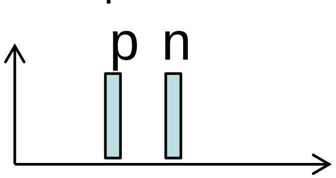
V.O.N., J. Kvasil, P. Vesely, W. Kleinig, P.-G. Reinhard,
V.Yu. Pomomarevet al, J. Phys. G.37, 064034 (2010).

Spin-flip M1 GR:
transitions between spin-orbit partners





Spin-flip M1 usually have two peaks (in both spherical and deformed nuclei)
The peaks are caused by neutron and proton spin-flip transitions).



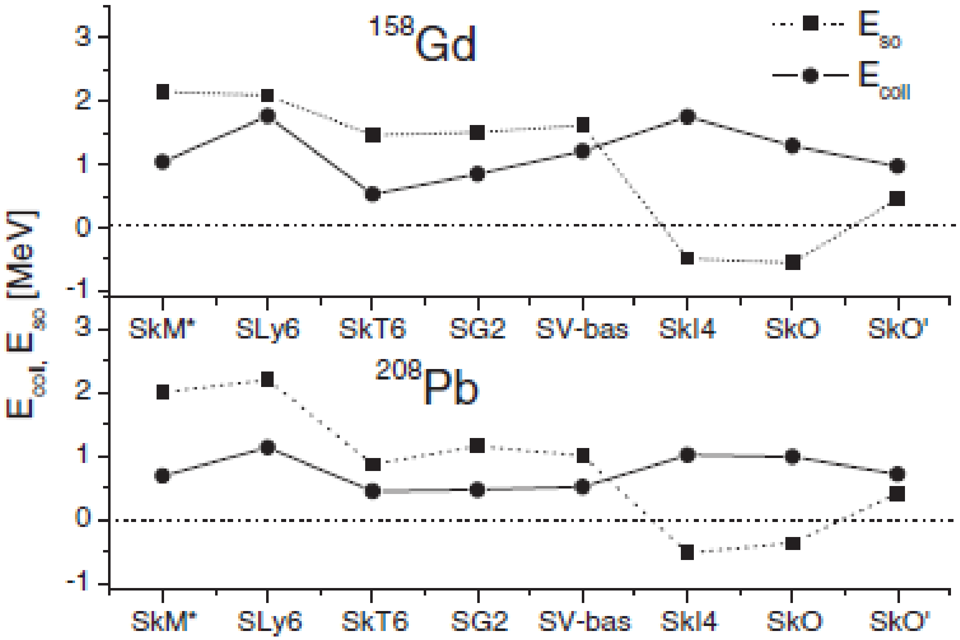
So spin-flip M1 GR is a good test for nuclear spin-orbit interaction

In some nuclei (208Pb) spin-flip M1 has one peak. Why?

Spin-flip M1 GR:

V.O.N. et al, J. Phys. G.37, 064034(11), (2010);.

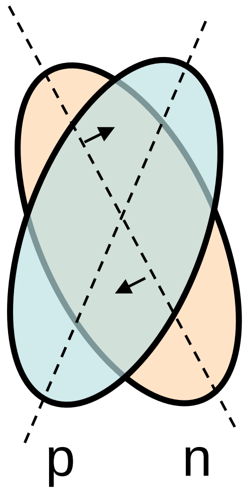
The residual interaction can mix two M1 peaks into one.
So the result depend on the competition between spin-orbit splitting and residual interaction (collective shift).



Open problems:

- For the moment no one Skyrme parameterization can simultaneously describe one- and two-bump spin-flip GR. Each parameterization gives always two-peak or always one-peak.
- Spin-flip M1 is the counterpart of Gamow-Teller GR in the neutral channel. If we poorly describe spin-flip M1, then we badly describe Gamow-Teller GR!

Scissors (orbital) M1 GR



- general property of two-component deformed systems

- well know in nuclei and other quantum systems (atomic clusters, trapped atomic Bose-Einstein condensate, quantum dots, ..)

- low excitation energy and strong M1(ΔK=1) transitions to gs

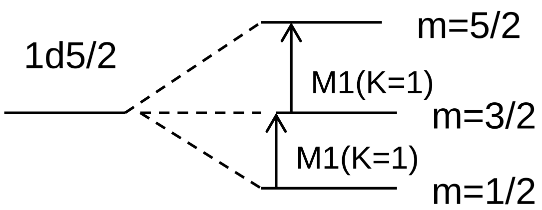
- exists only in deformed systems

$$\omega \propto N^{-1/3}\beta_2\,, \quad B(M1) \propto N^{4/3}\beta_2$$

$$\hat{M}(M1) = \mu_B \sqrt{\frac{3}{8\pi}} \sum_{q=n,p} [g_q^s \hat{s} + g_q^l \hat{l}\,]$$ -operator of M1-transition

spin-flip orbital

So the scissors mode is the test for orbital M1 transitions!



Scissors mode is formed by M1(K=1) transitions between neighbor levels produced by the deformation splitting

Open problems:

- Spin- scissors?

Exotic dipole resonances

Pygmy, toroidal and compression
E1 resonances

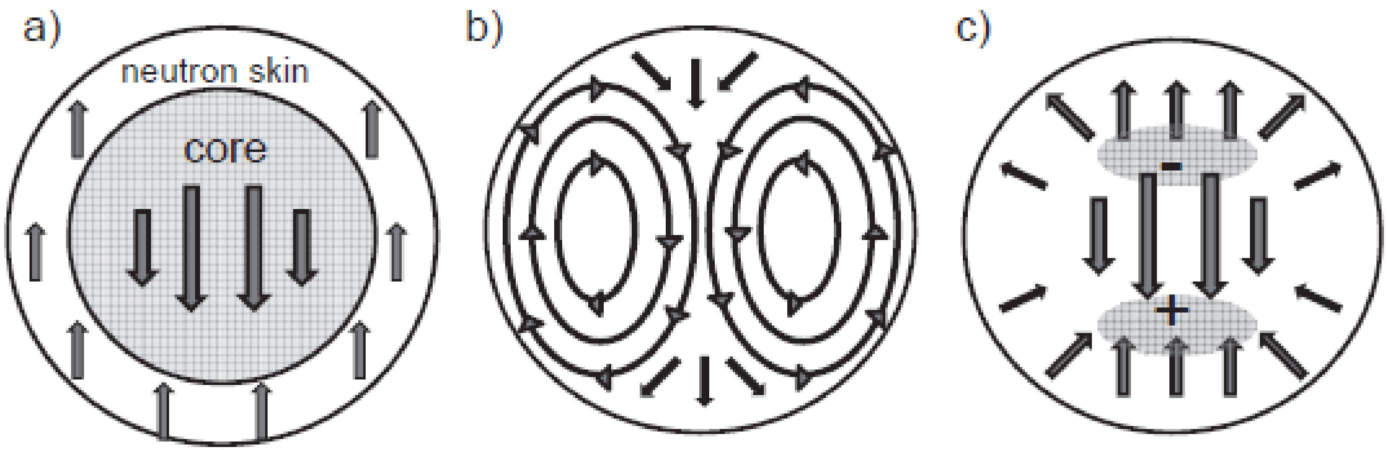
Isoscalar giant dipole resonance

Exotic dipole resonances

R. Mohan et al (1971),

V.M. Dubovik (1975)
S.F. Semenko (1981)

M.N. Harakeh (1977)
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

E1 pygmy E1 toroidal E1 compression

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational vortical irrotational

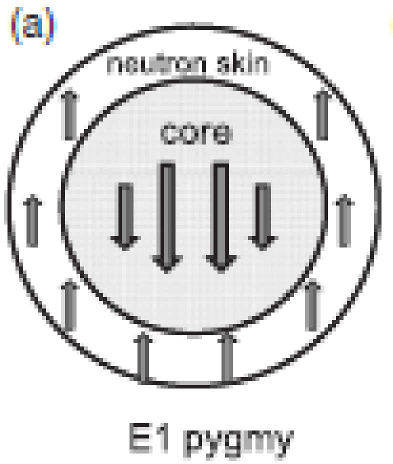
$$E = 50 \div 60\ A^{-1/3} MeV \qquad E = 50 \div 70\ A^{-1/3} MeV \qquad E = 132\ A^{-1/3} MeV$$

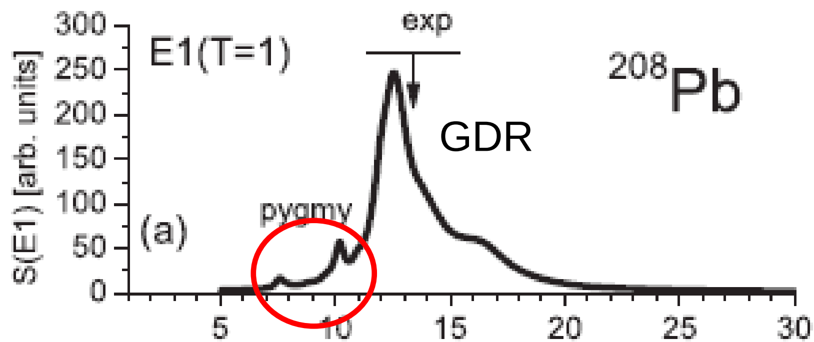
Reviews:
N. Paar et al, Rep. Prog. Phys. 70 691 (2007);
D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)
V.O.N. et al, Phys. Atom. Nucl. 79, 842 (2016).

- Different kinds of dipole oscillations with fixed c.m.
- TR: elastic, at fixed boundaries
- TR: the only known electric vortical mode

Pygmy E1 resonance

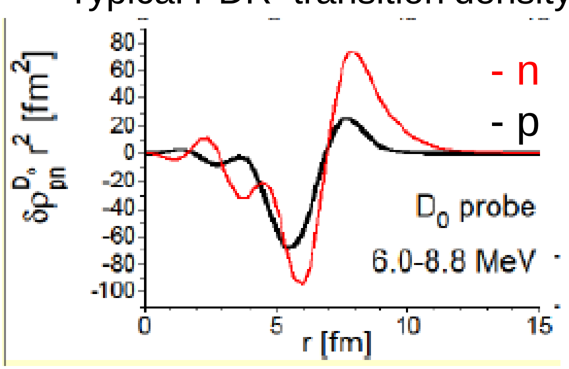
Reviews:
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007);
D. Savran, T. Aumann, and A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013)





Only few per cent from dipole EWSR!

Typical PDR transition density:



- Treatment: oscillations of the neutron excess against the core with N=Z
- of a high interest nowadays:
  - important for some astrophysical problems (EOS, neutron stars, …)
  - related to neutron skin (neutron-rich nuclei)
  - relating for building isospin part of equation of state (EOS), namely, the symmetry energy .

$$E = -a_V A + a_S A^{2/3} + a_0 \frac{Z^2}{A^{1/3}} + a_C \frac{(N-Z)^2}{A} + \ldots$$

$$EOS: E(\rho_n, \rho_p) = E_0(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \ldots$$

V.M. Dubovik and A.A. Cheshkov,
Sov. J. Part. Nucl. v.5, 318 (1975).

Toroidal moment:

- appears in multipole decomposition of nuclear current density

Following theorems of Helmholtz and Chandrasekhar/Moffat,
the current distribution can be decomposed as

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times[\vec{r}\,\psi(\vec{r})] + \vec{\nabla}\times\vec{\nabla}\times[\vec{r}\,\chi(\vec{r})]$$

electric moments | magnetic moments | electric + **toroidal** moments

transversal

Multipole electric operator (external field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\;\; j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

Toroidal operator appears as the second order term in long-wave expansion of the electric operator

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^{\lambda}Y_{\lambda\mu} \longleftarrow$$

standard electric operator
In long wave approximation

Toroidal E1 operator:

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\, [r^3 + \frac{5}{3} r < r^2 >_0]\, \vec{Y}_{11\mu}(\hat{\vec{r}}) \cdot [\underbrace{\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{vortical flow}}]$$

- second-order part of the electric operator

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\, [r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu}\, [\underbrace{\vec{\nabla} \cdot \hat{\vec{j}}_{nuc}(\vec{r})}_{\textbf{irrotational flow}}]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\, \hat{\rho}(\vec{r})\, [r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu}$$

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

$$\hat{M}_{tor} = \frac{-i}{2\sqrt{3}c}\int d\vec{r}\, \hat{\vec{j}}_{nuc}(\vec{r}) \cdot \vec{\nabla} \times (\vec{r} \times \vec{\nabla}) \underbrace{[r^3 - \frac{5}{3} r < r^2 >_0]}_{\text{compression}}$$

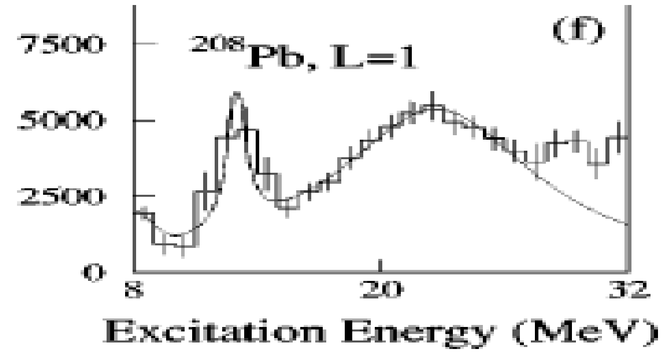
toroidal and compression modes are coupled

TDR and CDR constitute low- and high-energy ISGDR branches (?)

Familiar treatment → LE (toroidal) HE (compression)

Experiment: (α, α')

^{208}Pb
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004
M.Uchida et al, PRC 69, 051301(R) (2004)



There are also exp ISGDR data in

^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

TDR

CDR

G. Colo et al, PLB 485, 362 (2000)
D. Vretenar et al, PRC, 65, 021301(R) (2002)
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007);

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

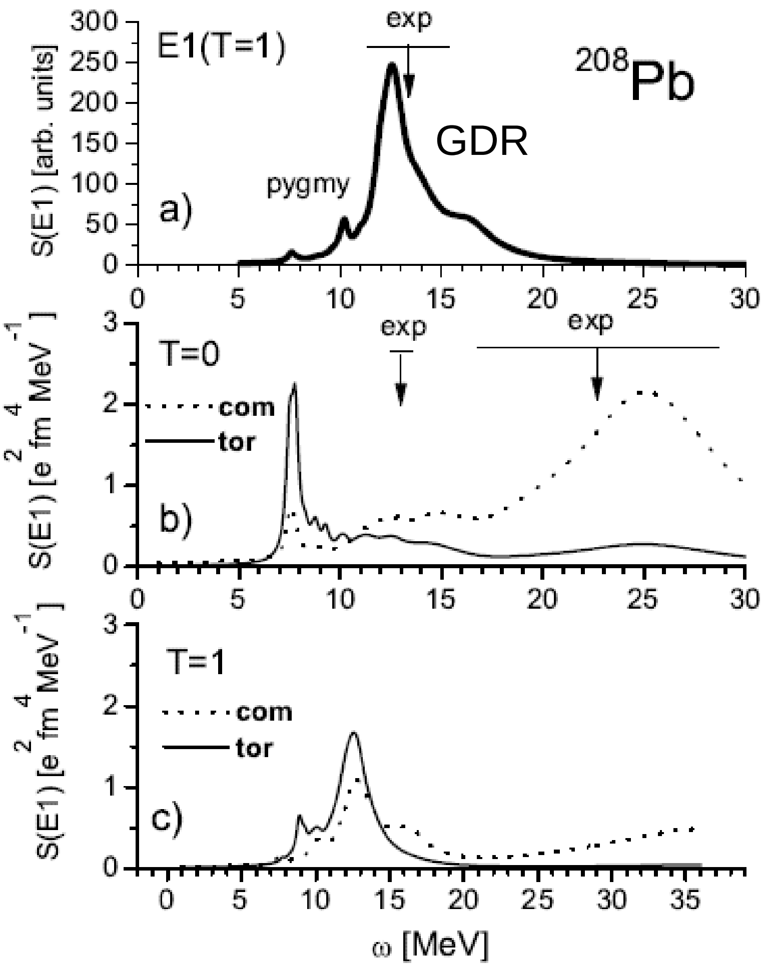
Perhaps Uchida observed at 10-17 MeV not TDR but mixed CDR/TDR low-energy bump. The main peaked TDR must at the lower energy ~ 7-9 MeV.

Relation of E1 toroidal and pygmy resonances

Is PDR a local part of TDR?

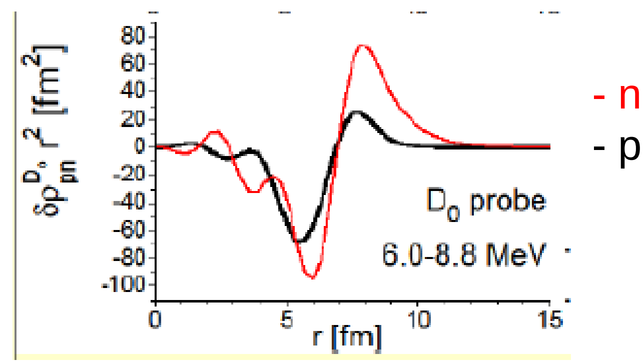
Strength functions SLy6

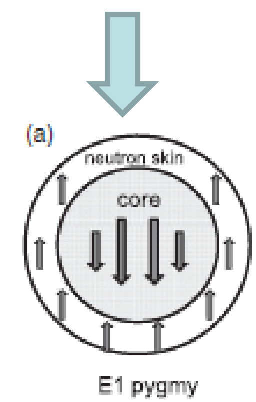
A. Repko, P.G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013)



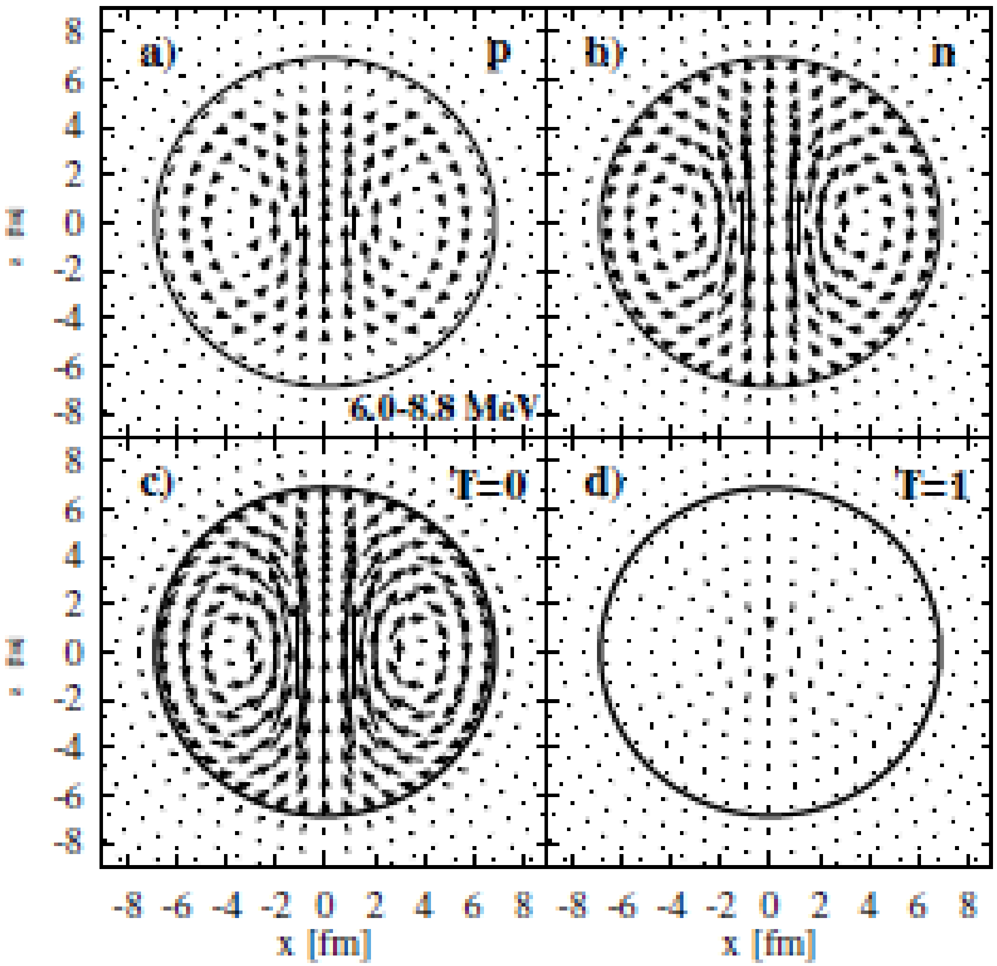
PDR region hosts TDR and CDR!

Typical PDR transition density:





QRPA : nuclear current for E1-excitations at 6.0-8.8 MeV (PDR region)

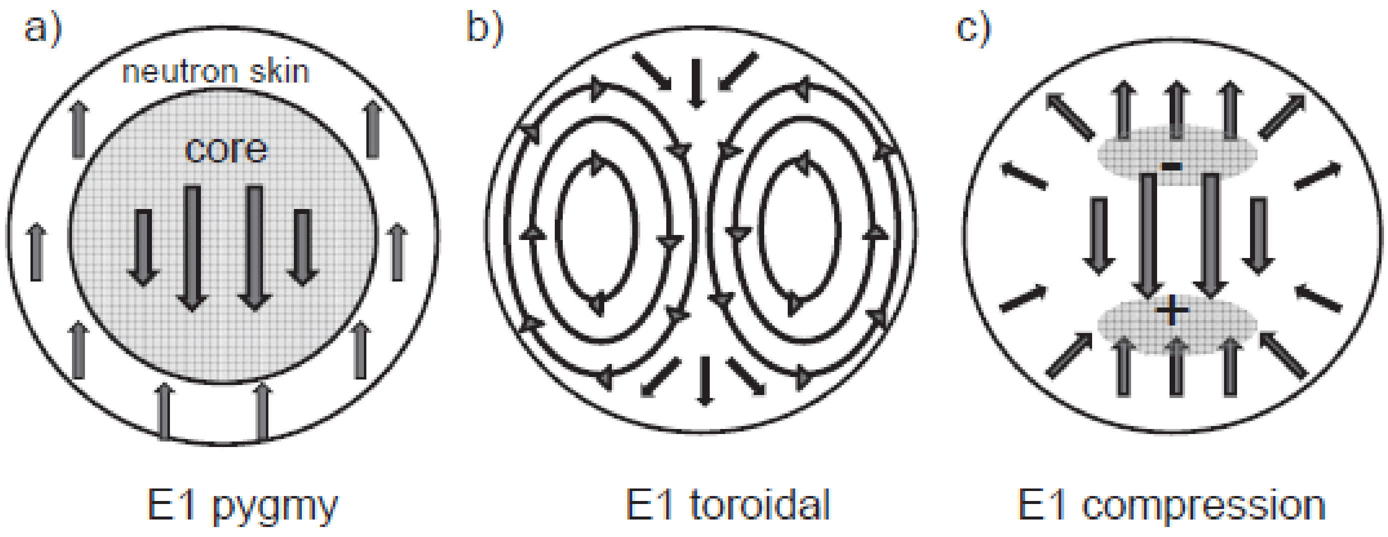


^{208}Pb

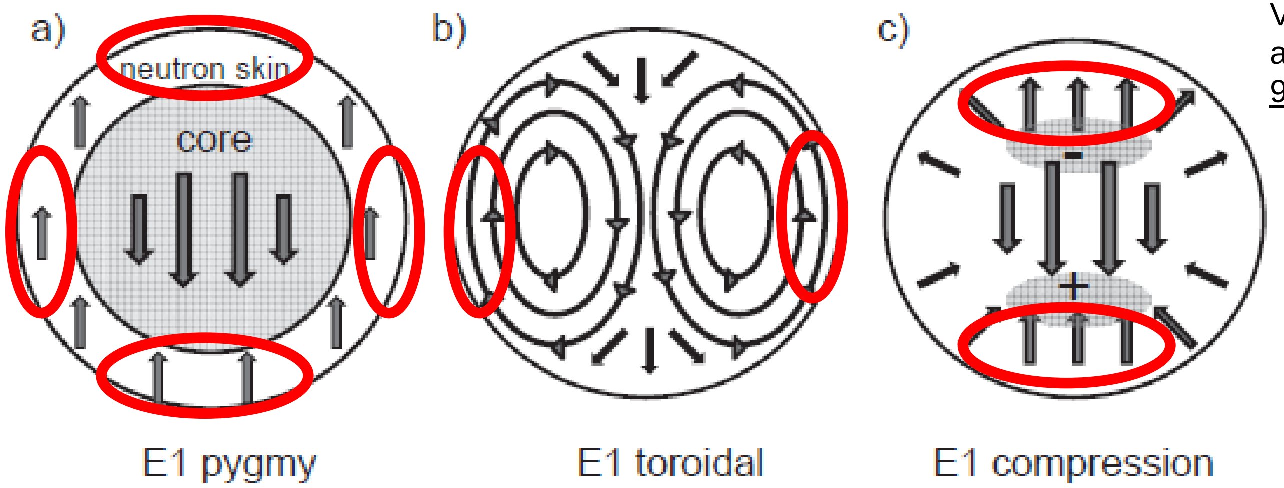
- mainly isoscalar toroidal flow in PDR energy region!

--so PDR is actually the toroidal motion?

Does the toroidal flow contradicts the familiar PRD picture?



?



V.O. N., A. Repko, P.-G. Reinhard, and J. Kvasil, EPJ Web of Conf., 93, 01020 (2015)

PDR can be viewed as a local peripheral part of TDR and CDR!

Theoretical models for GR:

- sum rules,
- self-consistent mean-field methods

Sum rules (1):

$$m_1^{E\lambda} = \sum_j \omega_j \,|\langle j|F^{E\lambda}|0\rangle|^2 = \langle 0|[F^{E\lambda},[F^{E\lambda},H]]|0\rangle \qquad F^{E\lambda\mu}(\vec{r}) = \sum_{i=1}^{Z} e r_i^{\lambda} Y_{\lambda\mu}(\Omega_i)$$

$$= \langle 0|[F^{E\lambda},[F^{E\lambda},T]]|0\rangle$$

If V and F depend only on coordinates, then
[F,H] = [F,T] + [F,V] = [F,T]

$$= \frac{eh^2}{8\pi m}\lambda(\lambda+1) Z \langle 0|\underbrace{r^{2\lambda-2}}_{=1\ \text{для}\ \lambda=1}|0\rangle \longleftarrow$$

model independent !

$$m_1 = \tfrac{1}{2}\langle 0|[F,[F,H]]|0\rangle$$
$$= \tfrac{1}{2}\langle 0|(F[F,H]-[F,H]F)|0\rangle$$
$$= \tfrac{1}{2}\langle 0|(2FHF - FFH - HFF)|0\rangle$$

$$I = \sum_i |i\rangle\langle i|$$
$$\langle i|F|j\rangle = \langle j|F^{\dagger}|i\rangle^*$$
$$\langle 0|F|i\rangle\langle 0|F|i\rangle^* = |\langle 0|F|i\rangle|^2$$

$$m_1 = \tfrac{1}{2}\sum_i \left(2\langle 0|FH|i\rangle\langle i|F|0\rangle - \langle 0|F|i\rangle\langle i|FH|0\rangle - \langle 0|HF|i\rangle\langle i|F|0\rangle\right)$$

$$= \tfrac{1}{2}\sum_i \left(2E_i\,|\langle 0|F|i\rangle|^2 - 2E_0\,|\langle 0|F|i\rangle|^2\right) \qquad \omega_i = E_i - E_0$$

$$= \sum_i \omega_i\,|\langle 0|F|i\rangle|^2$$

Sum rules (2):

☆ **Sum rules are related to basic quantum chacteristics:**

Sum rules are moments

$$m_p = \int \omega^p S(\omega) d\omega = \sum_i \omega_i^p \, |\langle i | F | 0 \rangle|^2$$

p = … -1, 0,1, 2, 3,…

of the strength function

$$S(\omega) = \sum_i \delta(\omega - \omega_i) \, |\langle i | F | 0 \rangle|^2$$

Sum rule determine limits of the polarizability:

$$\alpha(F,\omega)|_{\omega\to\infty} = 2[\frac{1}{\omega^2} m_1 + \frac{1}{\omega^4} m_3 + \ldots] \qquad \alpha(F,\omega)|_{\omega\to 0} = -2[m_{-1} + \omega^2 m_{-3} + \ldots]$$

Sum rules (3):

☆ **SR allow to estimate the GR energy and width**

$$m_p = \sum_i \omega_i^p \, |\langle i \,|\, F \,|\, 0 \rangle|^2 \quad \Rightarrow \quad \omega_{GR}^p \, |\langle GR \,|\, F \,|\, 0 \rangle|^2$$

$$\omega_p = \sqrt{\frac{m_p}{m_{p-2}}} = \sqrt{\frac{\omega_{GR}^p \langle GR \,|\, F \,|\, 0 \rangle^2}{\omega_{GR}^{p-2} \langle GR \,|\, F \,|\, 0 \rangle^2}} = \sqrt{\frac{\omega_{GR}^p}{\omega_{GR}^{p-2}}}$$

$$\omega_1 \le \bar{\omega} \le \omega_3, \quad \sigma^2 \le \tfrac{1}{4}(\omega_3^2 - \omega_1^2)$$

☆ **SR with p=1 and 3 are most used**

$$m_1 = \tfrac{1}{2}\langle 0 \,|\, [F,[F,H]] \,|\, 0 \rangle,$$

$$m_3 = \tfrac{1}{2}\langle 0 \,|\, [[F,H],[H,[H,F]]] \,|\, 0 \rangle.$$

They appear in oscillator Hamiltonian $\quad H = \frac{1}{m_1}\ddot{x} + m_3 x^2$

as

m_3	**- spring parameter (restoring force)**
$m_1 \propto N$	**- inertia or mass parameter**

Sum rules (4):

SR are generally expressed through commutators and anticommutators:

$$m_0 = \tfrac{1}{2}\langle 0 | \{F,F\} | 0 \rangle - \langle 0 | F | 0 \rangle^2,$$

$$m_1 = \tfrac{1}{2}\langle 0 | [F,[F,H]] | 0 \rangle,$$ ⇐ **only commutators**

$$m_2 = \tfrac{1}{2}\langle 0 | \{[F,H],[H,F]\} | 0 \rangle,$$

$$m_3 = \tfrac{1}{2}\langle 0 | [[F,H],[H,[H,F]]] | 0 \rangle, \quad \ldots$$ ⇐ **only commutators**

SR with odd p=2n+1 **expressed via commutators and most useful.**

Thouless:

$$m_1^{RPA} = \sum_i \omega_i \langle i|F|RPA\rangle^2 = \frac{1}{2}\langle RPA|[F,[H,F]]|RPA\rangle$$

$$\approx \frac{1}{2}\langle HF|[F,[H,F]]|HF\rangle$$ ⇐ **Insensitivity of commutators to g.s. correlations ! The same for m_3 .**

$$= \sum_{ph} \varepsilon_{ph} \langle ph|F|HF\rangle^2$$ ⇐ **Can be used as a test for completeness of the single-particle basis.**

Strength functions

Usually GR are calculated in terms of strength functions:

$$S(\omega) = \sum_i \delta(\omega - \omega_i)\, \omega_i \, |\langle i \,|\, \hat{F} \,|\, 0 \rangle|^2$$

- has δ -function spikes, is <u>not convenient</u> for comparison with smooth experimental data

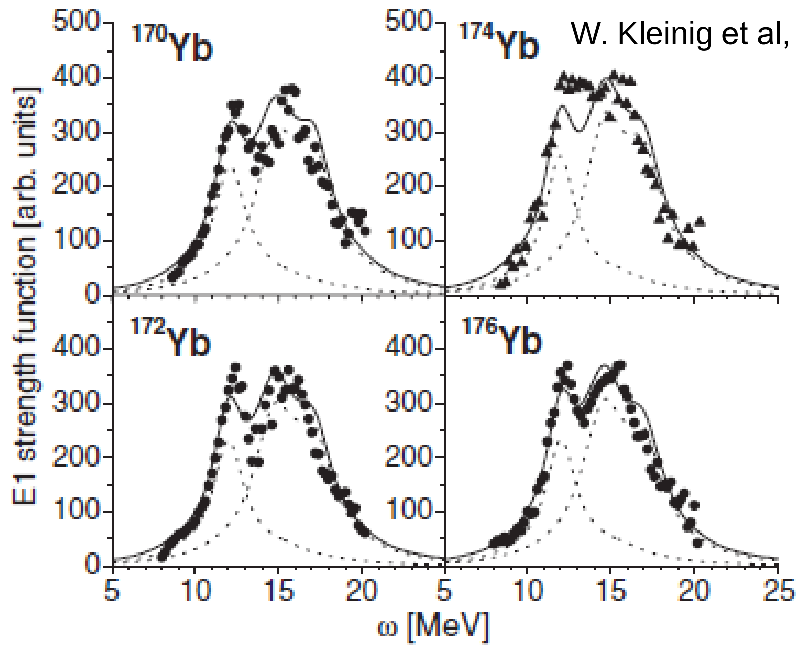
It is more convenient to use strength function

$$S(F;\omega) = \sum_{i \neq 0} \varsigma(\omega - \omega_i)\, \omega_i \, |< i \,|\, \hat{F} \,|\, 0 >|^2$$

with the Lorentz averaging weight where the averaging parameter is Δ =1-2 MeV

$$\varsigma(\omega - \omega_\nu) = \frac{1}{2\pi} \frac{\Delta}{[(\omega - \omega_\nu)^2 + \frac{\Delta^2}{4}]}$$

The Lorentz weight simulates smoothing effects beyond the calculations



W. Kleinig et al, PRC'08

Operator of the external field (probe):

$$\hat{F}(E1,T=1) = \frac{N}{A} \sum_{k=1}^{Z} (rY_{1K})_k - \frac{Z}{A} \sum_{k=1}^{N} (rY_{1K})_k$$

$$\hat{F}(E2,T=0) = \sum_{k=1}^{A} (r^2 Y_{2K})_k$$

Modern self-consistent mean field methods:

- relativistic (lecture of R. Jolos)
- Skyrme contact interaction
- Gogny finite range forces

These approaches

- are main theoretical tools to describe GR and other excitations,
- are self-consistent: both mean field and residual interaction are obtained from the same energy functional (no additional free parameters)

$$E\{J_\alpha(\vec{r},t), b_0,...,b_n\} = \langle \Psi | H | \Psi \rangle$$

$$h(\vec{r},t) = h_0(\vec{r}) + h_{res}(\vec{r},t) = \sum_{\alpha} [\frac{\delta E}{\delta J_\alpha}]_{J=\bar{J}} \hat{J}_\alpha(\vec{r}) + \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}]_{J=\bar{J}} \delta J_\alpha(\vec{r},t) \hat{J}_\alpha(\vec{r})$$

- parameters of the initial functional are fitted to describe both finite nuclei (almost all the periodic table) and nuclear matter (symmetric, neutron, …)
- many various parameterizations, a universal parameterization is absent

Skyrme forces in nuclear physics

$$\hat{H}(\vec{r}_1,\ldots,\vec{r}_A,t)=\sum_i \frac{\hbar^2}{2m}\nabla_j^2+\sum_{i>j}V_{ij}+\sum_{i>j>k}V_{ijk}+\ldots$$

$$V_{12}=t\delta(\vec{r}_1-\vec{r}_2),\quad V_{123}=t_3\delta(\vec{r}_1-\vec{r}_2)\delta(\vec{r}_2-\vec{r}_3)$$

$$\left.\begin{aligned}V_{12}&=t_0(1+x_0P_\sigma)\delta(\vec{r}\,')\\&+\frac{1}{2}t_1(1+x_1P_\sigma)[\vec{k}\,'^2\,\delta(\vec{r}\,')+\delta(\vec{r}\,')\vec{k}^2]\end{aligned}\right\}$$ - central term

$$+t_2(1+x_2P_\sigma)\vec{k}\,'\cdot\delta(\vec{r}\,')\vec{k}$$ - non-local term

$$+\frac{1}{6}t_3(1+x_3P_\sigma)[\rho(\vec{r}\,)]^\alpha\,\delta(\vec{r}\,')$$ - density-dependent term

$$+iW\vec{\sigma}\cdot[\vec{k}\,'\times\delta(\vec{r}\,')\vec{k}]$$ - spin-orbit term

$$\vec{r}=(\vec{r}_1+\vec{r}_2)/2,\quad \vec{r}\,'=\vec{r}_1-\vec{r}_2 \qquad \vec{\sigma}=\vec{\sigma}_1+\vec{\sigma}_2,\quad P_\sigma=(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)/2$$

$$\vec{k}=\vec{\nabla}_1-\vec{\nabla}_2)/2i,\quad \vec{k}\,'=-(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)/2i$$

Skyrme energy density:

V.O.Nesterenko, et al, PRC 66, 044307 (2002)
P. Vesely et al, PRC, 80, 031302(R) (2009).

$$E=\int \Psi^* H\Psi d\vec{r}, \qquad \Psi^* H\Psi = T + H_{Sk} + H_{Coul}$$

$$\Psi=\frac{1}{\sqrt{A!}}\det\begin{pmatrix}\phi_{k_1}(\vec{r}_1) & \dots & \phi_{k_1}(\vec{r}_A)\\ \dots & \dots & \dots \\ \phi_{k_A}(\vec{r}_1) & \dots & \phi_{k_A}(\vec{r}_A)\end{pmatrix}$$

$$\mathcal{H}_{Sk}(\vec{r}) = \frac{b_0}{2}\rho^2 - \frac{b_0'}{2}\sum_{\tau=n,p}\rho_\tau^2 - \frac{b_2}{2}\rho(\Delta\rho) + \frac{b_2'}{2}\sum_{\tau=n,p}\rho_\tau(\Delta\rho_\tau)$$

$$+b_1(\rho\tau - \vec{j}^{\,2}) - b_1'\sum_{\tau=n,p}(\rho_\tau\tau_\tau - \vec{j}_\tau^{\,2})$$

$$+\frac{b_3}{3}\rho^{\alpha+2} - \frac{b_3'}{3}\rho^\alpha\sum_{\tau=n,p}\rho_\tau^2$$

$$-b_4[\rho(\vec{\nabla}\vec{\Im}) + \vec{s}\quad(\vec{\nabla}\times\vec{j})]$$

$$-b_4'\sum_{\tau=n,p}[\rho_\tau(\vec{\nabla}\vec{\Im}_\tau) + \vec{s}_\tau\quad(\vec{\nabla}\times\vec{j}_\tau)]$$

conventional terms for electric modes

$$+\tilde{b}_1[\vec{s}\vec{T} - \vec{\Im}^2] + \tilde{b}_1'\sum_{\tau=n,p}[\vec{s}_\tau\vec{T}_\tau - \vec{\Im}_\tau^2]$$

$$+\frac{\tilde{b}_0}{2}\vec{s}^{\,2} - \frac{\tilde{b}_0'}{2}\sum_{\tau=n,p}\vec{s}_\tau^{\,2} + \frac{\tilde{b}_3}{2}\rho^\alpha\vec{s}^{\,2} - \frac{\tilde{b}_3'}{2}\rho^\alpha\sum_{\tau=n,p}\vec{s}_\tau^{\,2}$$

$$-\frac{\tilde{b}_2}{2}\vec{s}(\Delta\vec{s}) + \frac{\tilde{b}_2'}{2}\sum_{\tau=n,p}\vec{s}_\tau(\Delta\vec{s}_\tau)$$

spin-isospin terms

(+) nucleon $\rho(\vec{r}) = \sum_j |\phi_j|^2$

(+) kin. en. $\tau(\vec{r}) = \sum_j |\vec{\nabla}\phi_j|^2$

(+) spin-orb. $\Im(\vec{r}) = -i\sum_j \phi_j^+\nabla\times\vec{\sigma}\phi_j$

(-) current $\vec{j}(\vec{r}) = \frac{-i}{2}\sum_j(\phi_j^+\vec{\nabla}\phi_j - \phi_j\vec{\nabla}\phi_j^+)$

(-) spin $\vec{s}(\vec{r}) = -i\sum_j \phi_j^+\vec{\sigma}\phi_j$

(-) spin-kin. en. $\vec{T}(\vec{r}) = \sum_j \vec{\nabla}\phi_j\vec{\sigma}\cdot\vec{\nabla}\phi_j$

Justification of Skyrme densities

Formal arguments:

Single-particle density matrix:

$$\rho(\vec{r}\sigma,\vec{r}\,'\sigma') = \sum_i \phi_i(\vec{r},\sigma)\phi_i^*(\vec{r}\,',\sigma')$$

$$= \frac{1}{2}[\rho(\vec{r},\vec{r}\,')\delta_{\sigma\sigma'} + \sum_\nu \langle\sigma|\hat{\sigma}_\nu|\sigma'\rangle s_\nu(\vec{r},\vec{r}\,')]$$

$$\rho(\vec{r},\vec{r}\,') = \sum_\sigma \rho(\vec{r}\sigma,\vec{r}\,'\sigma),$$

$$s_\nu(\vec{r},\vec{r}\,') = \sum_{\sigma\sigma'} \rho(\vec{r}\sigma,\vec{r}\,'\sigma')\langle\sigma'|\hat{\sigma}_\nu|\sigma\rangle$$

Other densities are first and second derivatives of basic densities : $\rho, \vec{s}$

$$\rho(\vec{r}) = \rho(\vec{r},\vec{r}) \qquad \vec{s}(\vec{r}) = \vec{s}(\vec{r},\vec{r})$$ basic densities

$$\vec{j}(\vec{r}) = \frac{1}{2i}[(\vec{\nabla}-\vec{\nabla}')\rho(\vec{r},\vec{r}\,')]_{\vec{r}=\vec{r}\,'} \qquad \vec{\Im}_{\mu\nu}(\vec{r}) = \frac{1}{2i}[(\vec{\nabla}_\mu-\vec{\nabla}_\mu')s_\nu(\vec{r},\vec{r}\,')]_{\vec{r}=\vec{r}\,'}$$ their momenta (first derivatives)

$$\tau(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\rho(\vec{r},\vec{r}\,')]_{\vec{r}=\vec{r}\,'} \qquad \vec{T}(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\vec{r},\vec{r}\,')]_{\vec{r}=\vec{r}\,'}$$ their kin. energies (second derivatives)

Functional involves all possible bilinear combinations of the basic densities $\rho, \vec{s}$ and their first and second derivatives.

- Some kind of gradient expansion (important for non-uniform systems)
- Combinations of densities in the functional must:
  - a) be time-even,
  - b) fulfill local gauge (Galilean) invariance

Conclusions

Features of different GR have been studied for many years
and now, as a rule, are basically known.

However investigation of GR is still very important because:

- GR are used for fitting modern self-consistent models to nuclear dynamics
- provide valuable information on nuclear matter characteristics
- still have many open problems
  - simultaneous description of:
    - E1(T=1) and E2(T=0)
    - E0(T=0) in Pb and Sn isotopes
    - one- and two-bump structure of spin-flip M1 in different nuclei
    - toroidal E1(T=0): still should be found experimentally
    - fine structure of GR and wavelet analysis,
  - …..

Still very hot topic!

References:

Sum rules:

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3) G.F. Bertsch and R.A. Broglia, "Oscillations in Finite Quantum Systems", Cambridge univ. press, 1994, ISBN 0 521 41148

4) P. Ring, P. Schuck, "Nuclear Many-Body Problem", Springer-Verlag, NY, 1980.

Giant resonances

1) M.N. Harakeh and A. van der Woude, "Giant Resonances" (Oxford: Clarendon Press) 2001.

2) "Electric and magnetic giant resonances in nuclei", ed. J. Speth (Singapore: World Scientific) 1991.

Thank you for your attention!