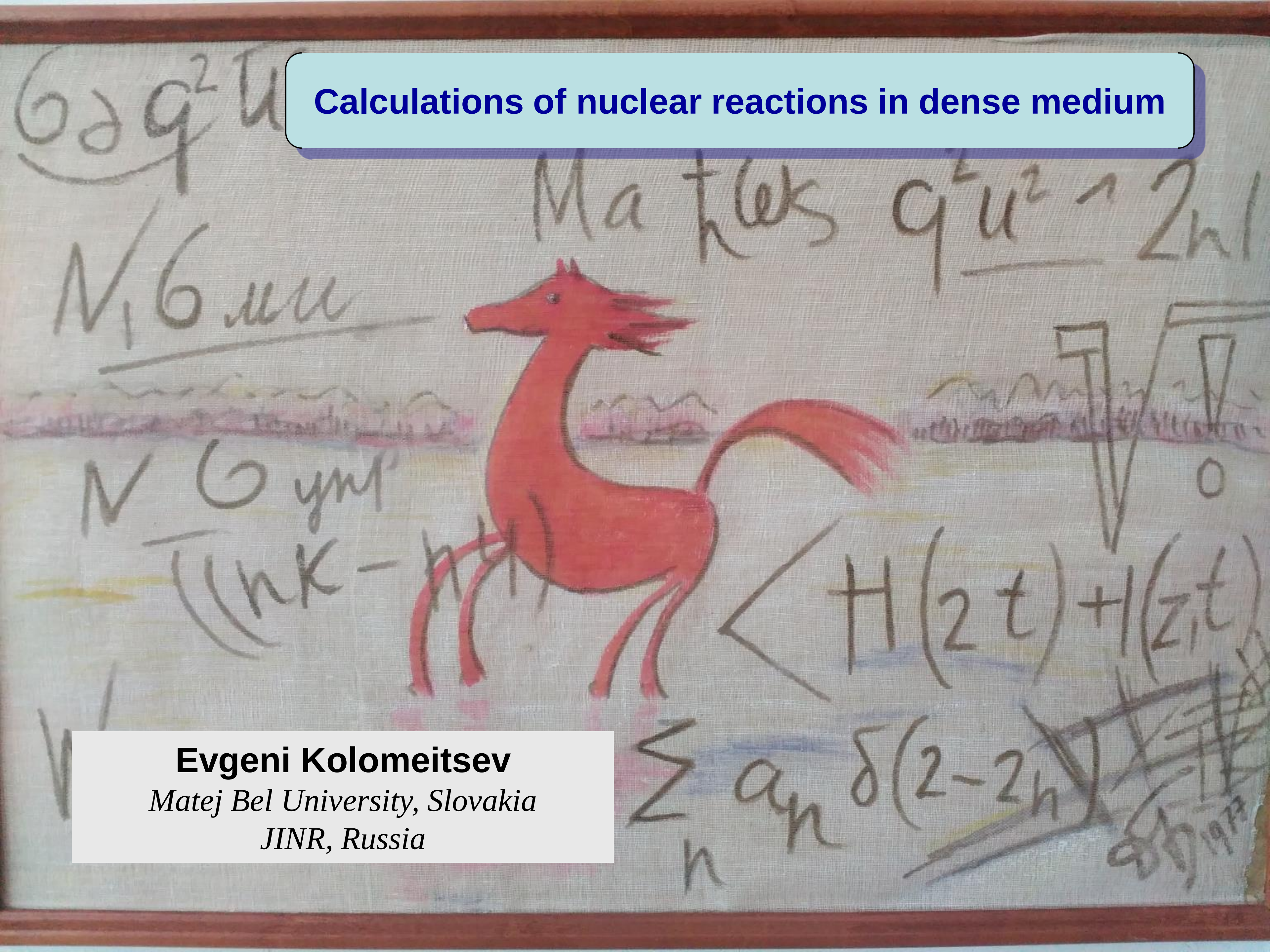
Calculations of nuclear reactions in dense medium



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How to calculate nuclear reactions in dense medium?

1. Green's function method
   equilibrium diagram techniques
   non-equilibrium diagram techniques

2. Double counting problem
   Optical theorem

3. Fermi liquid approach
   quasiparticles
   effective charges

4. Ward identities and current conservation

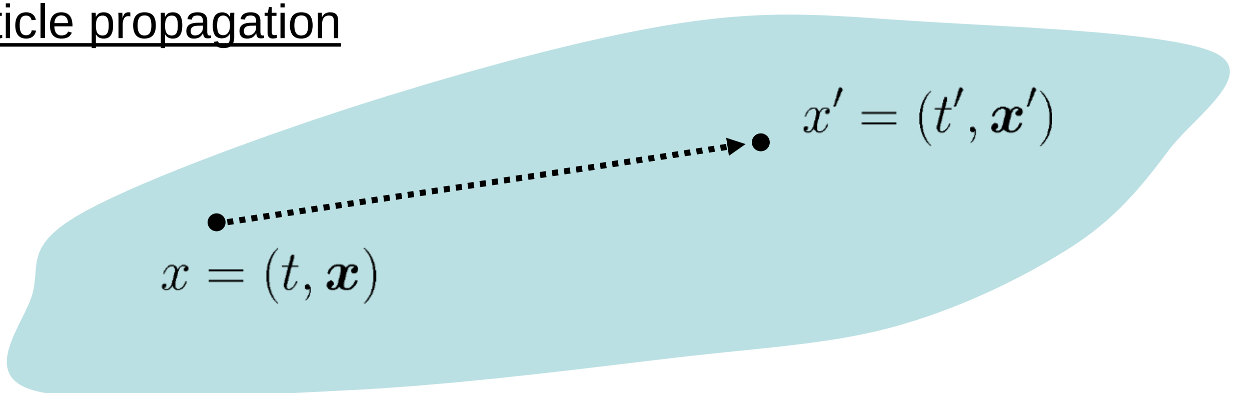
5. Fermi systems with pairing

Green's functions

N-body system: wave function of the whole system $\Psi(x_1,x_2,\ldots,x_N)$ encodes the dynamics of all particles and is very complicated

Introduce the object which describes the dynamics of the reduced number of particles of interest

<u>one-particle propagation</u>



Amplitude of particle transition from a point (x,t) to a point (x',t')

$$\Psi(\boldsymbol{x}', t') = \int d\boldsymbol{x}\, G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t)\Psi(\boldsymbol{x}, t) \qquad t' > t$$

for $t' = t + 0$ $$\Psi(\boldsymbol{x}', t+0) = \int d\boldsymbol{x}\, G^{(+)}(\boldsymbol{x}', t+0; \boldsymbol{x}, t)\Psi(\boldsymbol{x}, t)$$

$$G^{(+)}(\boldsymbol{x}', t+0; \boldsymbol{x}, t) = \delta(\boldsymbol{x}' - \boldsymbol{x})$$

If ***Ψ(x,t)*** obeys the Schrödinger equation $$[i\partial_t - H(\boldsymbol{x})]\, \Psi(\boldsymbol{x}, t) = 0$$

$$[i\partial_t - H(\boldsymbol{x})]\, G^{(+)}(\boldsymbol{x}, t; \boldsymbol{x}', t') = i\,\delta(t - t')\,\delta(\boldsymbol{x} - \boldsymbol{x}')$$

for homogeneous system : $$G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t) = G^{(+)}((\boldsymbol{x}' - \boldsymbol{x})^2, t' - t > 0)$$

eigenfunctions: $$H\,\varphi_\lambda(\boldsymbol{x}) = \epsilon_\lambda(\boldsymbol{x})\,\varphi(\boldsymbol{x})$$

$$G^{(+)}(\boldsymbol{x}', \boldsymbol{x}, \tau = t' - t) = -\sum_\lambda \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i}\, e^{-i\epsilon\tau} \frac{\varphi_\lambda(\boldsymbol{x}')\,\varphi_\lambda^*(\boldsymbol{x})}{\epsilon - \epsilon_\lambda + i\,0}$$

$$G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t) =< N|\hat{\Psi}(\boldsymbol{x}', t')\,\hat{\Psi}^\dagger(\boldsymbol{x}, t)|N >$$

$$\hat{\Psi}(\boldsymbol{x}, t) = \sum_\lambda \varphi_\lambda(x)\,\hat{a}_\lambda\, e^{-i\,\epsilon_\lambda\, t}$$ $$|N >= a_1^\dagger\, a_2^\dagger\, a_3^\dagger \ldots\, a_N^\dagger\, |0 >$$

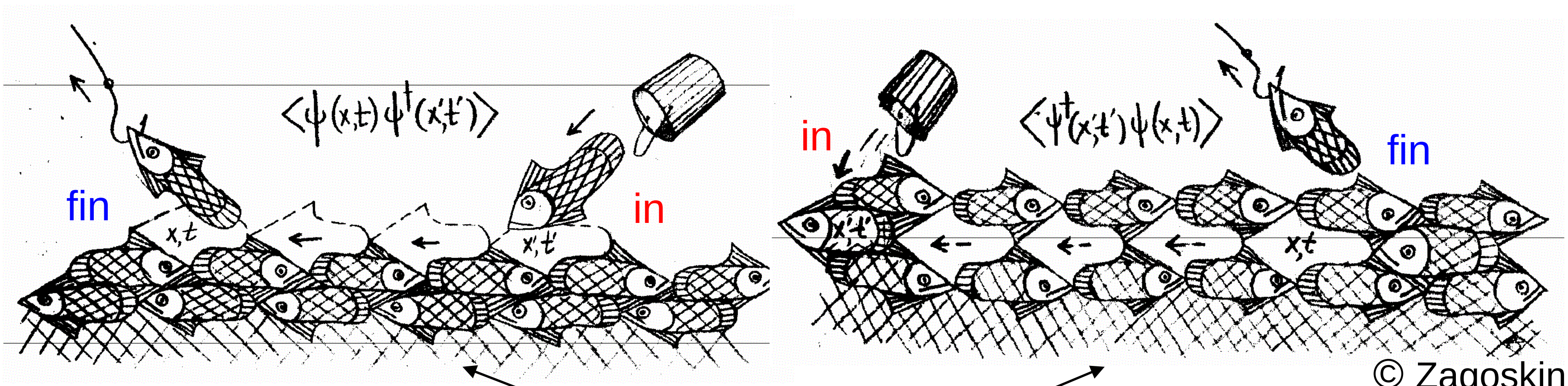
$a_i,\ a_i^\dagger$ annihilation and creation operator

Green's function of non-interacting fermions

$$i\,G(\boldsymbol{x},t;\boldsymbol{x}',t') = < N|T\{\hat\Psi(\boldsymbol{x},t)\,\hat\Psi^\dagger(\boldsymbol{x}',t')\}|N >$$

fin   in

$$= < N|\hat\Psi(\boldsymbol{x},t)\,\hat\Psi^\dagger(\boldsymbol{x}',t')|N > \theta_{t-t'} - < N|\hat\Psi^\dagger(\boldsymbol{x}',t')\,\hat\Psi(\boldsymbol{x},t)|N > \theta_{t'-t}$$



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no interaction

$$G_0(\epsilon,p) = \frac{\mathbf{1-n_p}}{\epsilon-\epsilon_p+i\,0} + \frac{\mathbf{n_p}}{\epsilon+\epsilon_p^h-i\,0}$$

$T=0$

$$n_p = \theta(p_{\rm F}-p)$$

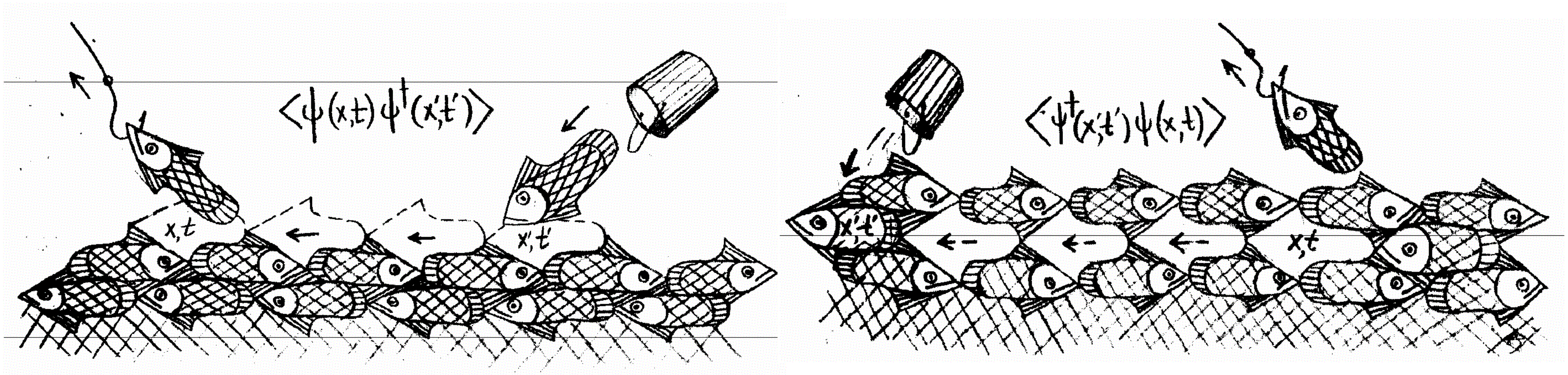
$$\epsilon_p^h = -\epsilon_p$$

$$G_0(\epsilon,\boldsymbol{p}) = \frac{1}{\epsilon-\epsilon_p+i\,\mathrm{sign}(\epsilon)}$$

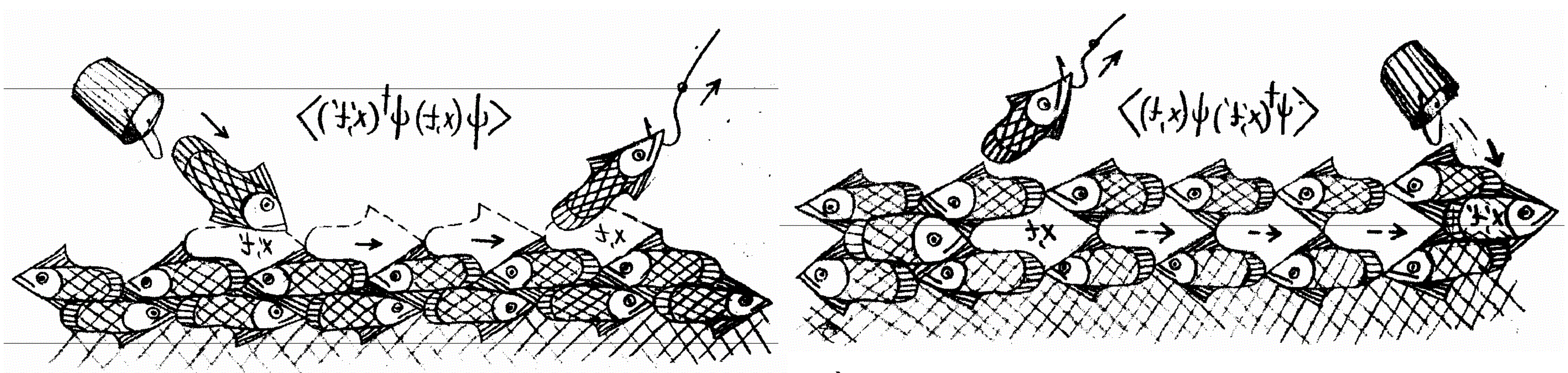
$$\epsilon_p = \frac{p^2-p_{\rm F}^2}{2\,m}$$

particle

G(ε,p)



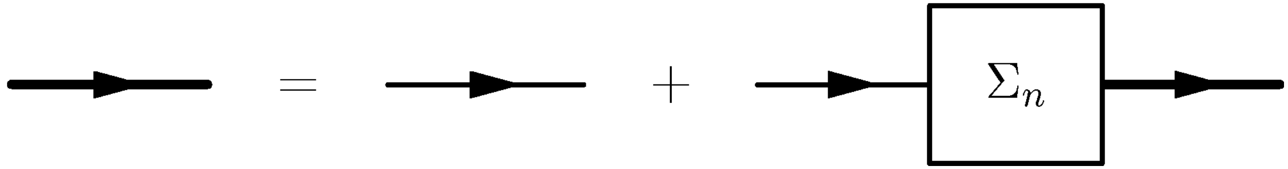
hole



G··(ε,p)

Full Green's function

particle-line

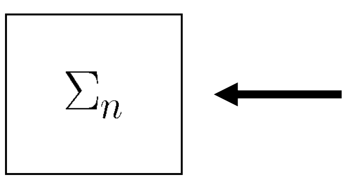


$$\widehat{G}_{\rm n.s.} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}_{\rm n.s.}\,\widehat{G}_{\rm n.s.} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}_{\rm n.s.} \right]^{-1}$$

diagonal in spin-space

$$\hat{G}_0(\epsilon, \boldsymbol{p}\,) = \frac{\hat{1}}{\epsilon - \boldsymbol{p}^{\,2}/2\,m_N + i\,0\,\mathrm{sign}\,(\epsilon - \epsilon_{\rm F})}$$

analogously for the hole-line



particle-particle, particle-hole hole-hole interactions

Diagram technique

Ground state:

$$iG(x,y) = < N|\widehat{T}\{\widehat{\Psi}(x)\widehat{\Psi}^\dagger(y)\}|N > =< N|\widehat{S}^{-1}\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^\dagger(y)\}\widehat{S}|N >$$

in interaction picture: $iG =< N|\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^\dagger(y)\}\widehat{S}|N >< \widehat{S}^{-1} >$

transition from the ground state to the ground state under action of evolution operator

$$\widehat{S} = \widehat{T}\exp\Big\{-i\int_{-\infty}^{\infty}\widehat{V}_I(t)dt\Big\}$$

time ordering

$$\widehat{V}_I(t) = e^{i\widehat{H}_0(\mu)\,t}\widehat{V}\,e^{-i\widehat{H}_0(\mu)t}$$

$$\widehat{H}_0(\mu) = H_0 - \sum_a \mu_a\widehat{N}_a$$

Only one type of Green's functions

Diagram technique "out of non-equilibrium"

For a non-equilibrium state $| N >$

$$iG^{--}(x,y) = < N|\,\widehat{T}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N> \;=\; < N|\,\widehat{S}^{-1}\,\widehat{T}\{\widehat{\Psi}_I(x)\,\widehat{\Psi}^{\dagger}_I(y)\}\,\widehat{S}\,|N>$$

$$\neq\; < N|\widehat{T}\,\{\widehat{\Psi}_I(x)\,\widehat{\Psi}^{\dagger}_I(y)\}\,\widehat{S}\,|N>< \widehat{S}^{-1}>$$

non-equilibrium ground state at -∞ **does not** transit to the same ground state at +∞

due to possible decays **4 Green's functions**

$$iG^{--}(x,y) = < N|\,\widehat{T}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N> \quad iG^{++}(x,y) = < N|\,\widehat{T^{\dagger}}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N>$$

inverse time ordering

Wigner densities (no time ordering operations)

$$iG^{-+}(x,y) = \mp < N|\,\widehat{\Psi}^{\dagger}(y)\,\widehat{\Psi}(x)\,|N> \qquad iG^{+-}(x,y) = < N|\,\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\,|N>$$

Green functions are not independent !

$$G^{--} + G^{++} = G^{-+} + G^{+-}.$$

Diagram technique "out of equilibrium"

Assume suppression of initial correlations ($t \gg t_{cor}$) → Wick theorem

→ averaging of equations of motion for operators we obtain coupled Schwinger-Dyson equations for G^{--}, G^{++}, G^{+-}, G^{-+}

$$G(x,y) = G^0(x,y) + \int G^0(x,z)\Pi(z,z')G(z',y)$$

matrix of full G.F. — matrix of bare G.F. — matrix of self-energies

general structure of the matrix

$$F(x,y) = \begin{pmatrix} F^{--}(x,y) & F^{-+}(x,y) \\ F^{+-}(x,y) & F^{++}(x,y) \end{pmatrix}$$

For Green's functions and self-energies

$$F = \{G, \Pi, \Sigma\}$$

"covariant metric"

$$F_i^j = \sigma_{ik}F^{kj}, \quad \sigma_{ik} = (\sigma_3)_{ik}, \quad \sigma_i^k = \delta_{ik}, \quad i,k = \{-,+\}$$

[Ivanov, Knoll, Voskresensky. NPA 657 (1999); NPA 672 (2000)]

different notations compared with LP (different signs in G^{ij} and Π^{ij}) and KB text books $(>,<)$

Factor $(-i)$ for " $-$ " vertex and $(+i)$ for " $+$ " vertex is used.

- **retarded Green's function** $G^R = G^{--} - G^{-+}$

decouples and defines excitation spectrum

$$G^R_{12} = G^{0,R}_{12} + G^{0,R}_{13} \cdot \Pi^R_{34} \cdot G^R_{42}$$

- No Wick rotation
- Same diagrams as for ground-state system

Thermal equilibrium

In equilibrium only one Green's function (self-energy) (G^R,Σ^R)is required :

$$F(p) \;=\; \begin{pmatrix} F^R \pm i f(E)\mathcal{A} & \pm i\, f(E)\mathcal{A} \\ -i(1 \mp f(E))\mathcal{A} & -F^A \mp i\, f(E)\mathcal{A} \end{pmatrix}$$

for Green's functions $\mathcal{A} = A$

for self-energies $\mathcal{A} = \Gamma$

$$F = \{G, \Pi, \Sigma\}$$

particle occupation factors: $f(E) = \dfrac{1}{e^{(E/T)} \pm 1}$ Wigner densities:

spectral function: $A(p) = -2\,\mathrm{Im}\, G^R = \dfrac{\Gamma}{M^2 + \Gamma^2/4}$ **width** $\Gamma = -2\,\mathrm{Im}\,\Pi^R$

no relation between energy and momentum

mass operator $M = -Q(p) + \mathrm{Re}\,\Pi$ with $Q = \epsilon - \dfrac{\boldsymbol{p}^2}{2m}$ non-relativistic particles

$Q = p^2 - m^2$ relativistic bosons

➡ $G^{-+} \propto f(E)$! ➡ specific role of G^{-+}

Thermal equilibrium

- **Quasiparticle limit** $\Gamma \to 0$ $A \to 2\pi\,\delta(M)$

  that fixes in-medium mass-shell $M = -Q(p) + \mathrm{Re}\,\Pi = 0$

  in quasiparticle limit $T \ll E_{\rm F}$ each extra G^{-+} line brings a factor $\propto \dfrac{T^2}{E_{\rm F}^2}$

  "perturbation series" in number of (- +) Green functions (T-counting)

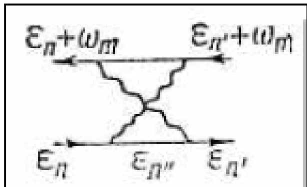
- **gas limit** particles are on mass-shell and $\mathrm{Re}\,\Pi^R \to 0$

$$G_0^{-+} = +2\pi\,i\,f(E)\,\delta(E+\mu-E_p) \quad G_0^{+-} = -2\pi\,i\,(1-f(E))\,\delta(E+\mu-E_p)$$

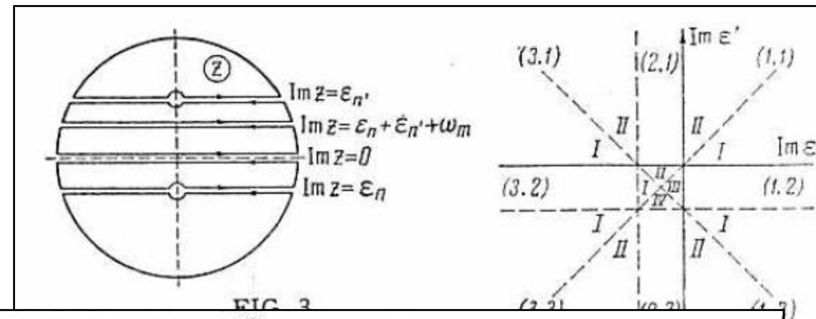
In equilibrium, the usage of “+-” notations is a matter of taste

TRANSPORT EQUATION FOR A DEGENERATE SYSTEM OF FERMI PARTICLES

G. M. ÉLIASHBERG



$$\Gamma_1(\varepsilon_n, \varepsilon_{n'}; \omega_m) = T \sum_{n''} G(\varepsilon_{n''}) G(\varepsilon_n + \varepsilon_{n'} + \omega_m - \varepsilon_{n''}) \times D(\varepsilon_n - \varepsilon_{n''}) D(\varepsilon_{n''} - \varepsilon_{n'}).$$



sponding to the interaction. It is most convenient to study the analytic properties of this diagram by substituting for the summation over n″ an integration:

$$\Gamma_1(\varepsilon_n, \varepsilon_{n'}; \omega_m) = \frac{1}{4\pi i} \int_C dz \,\text{th}\,\frac{z}{2T} G(z) G(\varepsilon_n + \varepsilon_{n'} + \omega_m - z) D(\varepsilon_n - z) D(z - \varepsilon_{n'}) + T[G(\varepsilon_n) G(\varepsilon_{n'} + \omega_m) D(0) D(\varepsilon_n - \varepsilon_{n'}) + G(\varepsilon_{n'}) G(\varepsilon_n + \omega_m) \times D(\varepsilon_n - \varepsilon_{n'}) D(0)]. \quad (7)$$

$$\Gamma_1(\varepsilon_n, \varepsilon_{n'}; \omega_m) = \frac{1}{4\pi i} \int_{-\infty}^{\infty} d\varepsilon'' \Big\{ \text{th}\,\frac{\varepsilon''}{2T} [G^R(\varepsilon'') - G^A(\varepsilon'')] G(\varepsilon_n + \varepsilon_{n'} + \omega_m - \varepsilon'') \times D(\varepsilon_n - \varepsilon'') D(\varepsilon'' - \varepsilon_{n'}) + \text{th}\,\frac{\varepsilon''}{2T} [G^A(-\varepsilon'') - G^R(-\varepsilon'')] G(\varepsilon'' + \varepsilon_n + \varepsilon_{n'} + \omega_m) D(-\varepsilon'' - \varepsilon_{n'} - \omega_m) \times D(\varepsilon'' + \varepsilon_n + \omega_m) + \text{cth}\,\frac{\varepsilon''}{2T} [D^A(-\varepsilon'') - D^R(-\varepsilon'')] G(\varepsilon'' + \varepsilon_n) G(\varepsilon_{n'} + \omega_m - \varepsilon'') D(\varepsilon'' + \varepsilon_n - \varepsilon_{n'}) + \text{cth}\,\frac{\varepsilon''}{2T} [D^R(\varepsilon'') - D^A(\varepsilon'')] G(\varepsilon'' + \varepsilon_{n'}) \times G(\varepsilon_n + \omega_m - \varepsilon'') D(\varepsilon_n - \varepsilon_{n'} - \varepsilon'') \Big\}. \quad (8)$$

Kinetic equations

$f(t,\boldsymbol{r},\boldsymbol{p})$ distribution of particles in the phase space

- **Boltzmann KE** $$\underbrace{\frac{\partial f}{\partial t} + \boldsymbol{v}\nabla_{\boldsymbol{x}} f + \boldsymbol{F}_{\rm ext}\nabla_{\boldsymbol{p}} f}_{\text{drift term}} = I[f]$$

Between collisions “particles” move along *characteristic* determined by an external force $F_{\rm ext}$

collision term (binary collisions):

$$I[f] = \int \mathrm{d}\boldsymbol{p}'\mathrm{d}\boldsymbol{p}_1\mathrm{d}\boldsymbol{p}'_1 W(\boldsymbol{p},\boldsymbol{p}_1|\boldsymbol{p}',\boldsymbol{p}'_1)\left[f(t,\boldsymbol{r},\boldsymbol{p}')\,f(t,\boldsymbol{r},\boldsymbol{p}'_1) - f(t,\boldsymbol{r},\boldsymbol{p})\,f(t,\boldsymbol{r},\boldsymbol{p}_1)\right]$$

- **Quantum KE** $$\widehat{D}\,F(X,p) - \{\Gamma_{\rm in}(X,p), \Re G^R(X,p)\} = C(X,p)$$

$$(X,p) = (t,\boldsymbol{q},E,\boldsymbol{p})$$

drift operator: $$\widehat{D}(\dots) = \underbrace{\left[v_\mu - \frac{\partial\Re\Sigma^R(X,p)}{\partial p^\mu}\right]}_{Z^{-1}_\mu(X,p)}\frac{\partial}{\partial X_\mu} + \frac{\partial\Re\Sigma^R(X,p)}{\partial X^\mu}\frac{\partial}{\partial p_\mu}$$

$$v^\mu = (1,\boldsymbol{p}/m)$$

for non-relat. part.

collision term: $$C(X,p) = \Gamma^{\rm in}(X,p)\,\widetilde{F}(X,p) - \Gamma^{\rm out}(X,p)\,F(X,p)$$

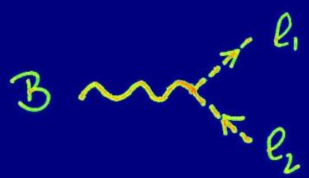
Gain term (production rate): $$\Gamma^{\rm in}(X,p) = \mp i\Sigma^{-+}(X,p)$$

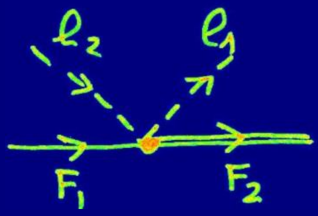
Loss term (absorption rate): $$\Gamma^{\rm out}(X,p) = i\Sigma^{+-}(X,p)$$

DOUBLE COUNTING PROBLEM
(diplopia)

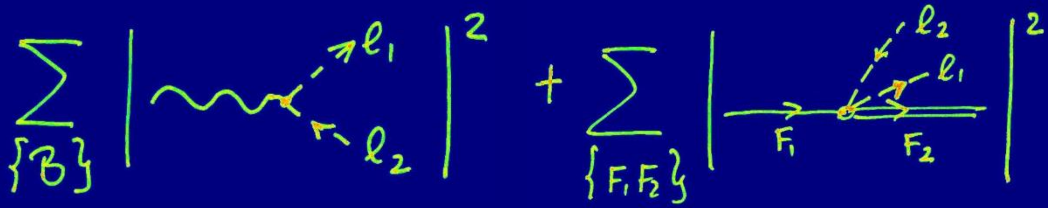
How to calculate reaction rates in medium?

Let a lepton pair (λ_1, λ_2) be coupled to a boson (B) or to a fermion pair (Φ_1, Φ_2)





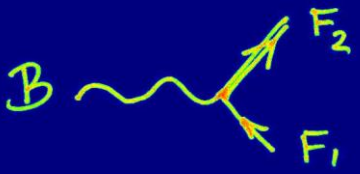
Lepton production rate in medium consisting of the bosons and fermions is given by



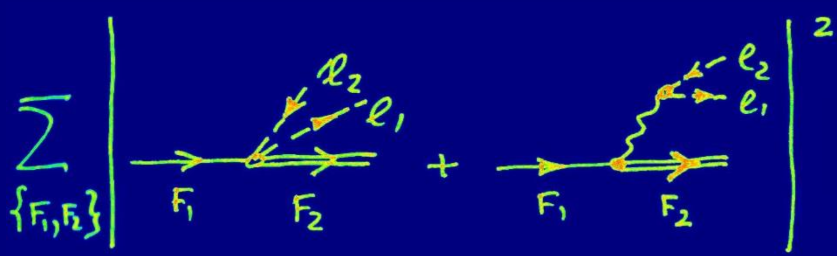
Feynman diagrams

summation over the phase space of initial particle (occupation factors)

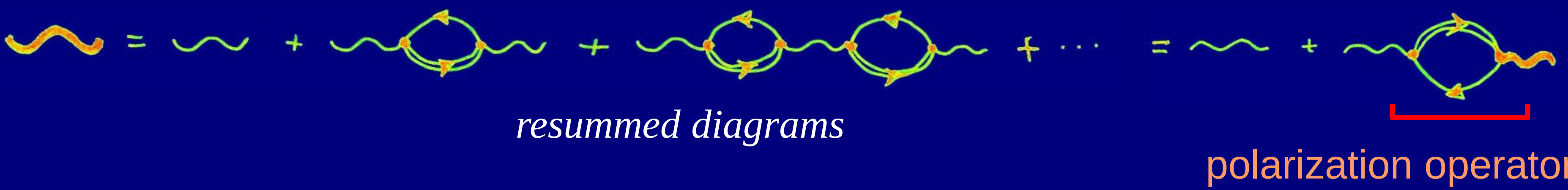
Introduce a coupling among boson and fermions:



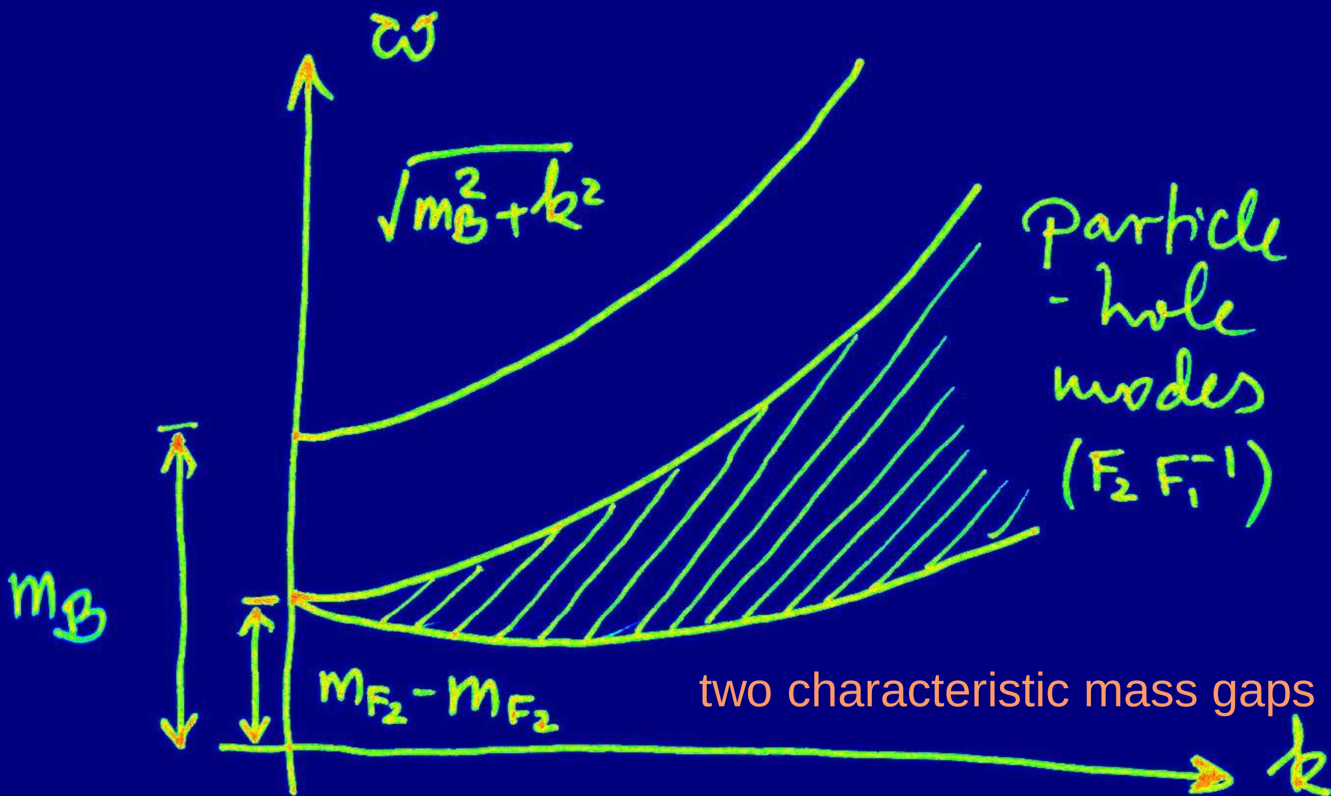
in vacuum



in medium consisting of fermions Φ_1 the boson propagator is

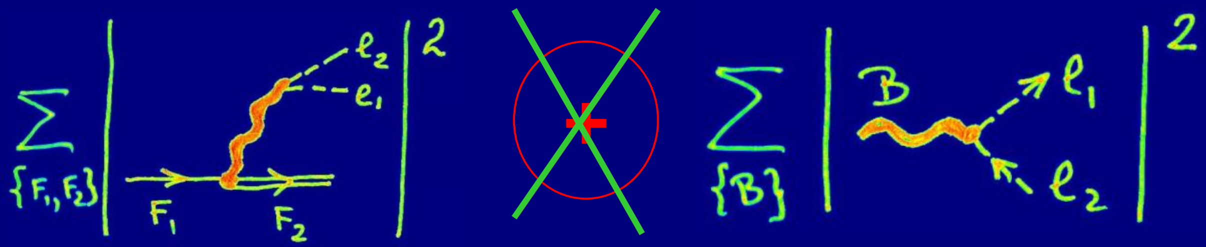


Spectrum of excitations with the quantum numbers of bosons B



To calculate the lepton production rates we cannot use Feynman diagrams with in-medium (dressed propagators).

It can lead to double counting!!



.... additional complications due to vertex corrections

OPTICAL THEOREM

Optical theorem. Closed diagrams

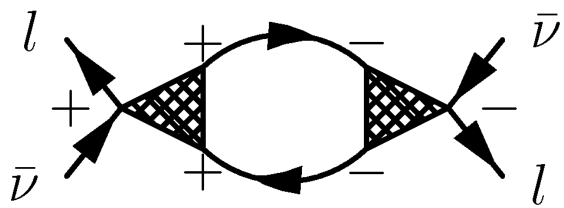
Perturbative diagrams are irrelevant for calculation of in-medium processes.

In general case one should deal with

closed diagrams in terms of dressed Green's functions

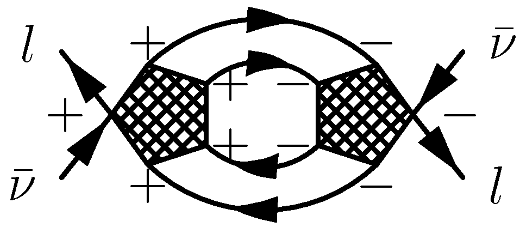
[Voskresensky, Senatorov, Sov. Nucl. Phys. 45 (1987); Knoll, Voskresensky, Ann. Phys. 249 (1996)]

$$\frac{d\mathcal{W}^{\rm tot}_{\bar\nu l}}{dt} = \frac{(1-n_l)d^3q_{\bar\nu}dq_l^3}{(2\pi)^6\,4\,\omega_{\bar\nu}\,\omega_l}\Bigg($$



one-nucleon reactions

$+$



two-nucleon reactions

$$+ \quad \ldots \Bigg)$$

Series in number of full $\{-,+\}$ Green's functions

for $T \ll \epsilon_{\rm F}$ each G^{-+} brings a factor $(T/\epsilon_{\rm F})^2$

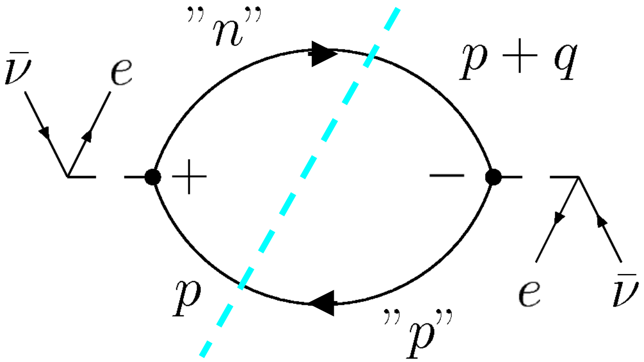
for superfluid systems: [Kolomeitsev, Voskresensky, Phys. Atom. Nucl. 74 ,1316 (2011)]

Optical theorem in non-equilibrium diagram technique

Let us calculate self-energy

with free Green's functions

$$-i\,\Pi_0^{-+} =$$



$$-i\Pi_0^{-+} = \frac{G^2}{2}\mathrm{Tr}\{l_1^\mu l_2^\nu\}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\{(-iJ_\mu)\,iG_n^{-+}(p+q)\,(+iJ_\nu)\,iG_p^{+-}(p)(-1)\}$$

$$= -i\mathcal{L}_0^{-+}\sum_{\mathrm{spin}}|M|^2$$

$iG_n^{-+}(p+q) \to 2\,\pi\,i\,f(E+\omega)\,\delta(E+\omega+\mu-E^{(n)}_{\boldsymbol{p}+\boldsymbol{q}})$

$iG_p^{+-}(p) \to -2\,\pi\,i\,(1-f(E))\,\delta(E+\mu-E_p)$

the loop function

$$\int dE$$

$$-i\mathcal{L}_0^{-+} = \int\frac{d^3p}{(2\pi)^3}\,f_n(\boldsymbol{p}+\boldsymbol{q})\,[1-f_p(\boldsymbol{p})]\,2\,\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(\boldsymbol{p})-\mu_n+\mu_p]$$

Cutting the diagram means removing of *dE* integration due to δ-function

Comparing with standard expression for emissivity

$$\epsilon_\nu^{\rm DU} = 2\int \frac{d^3p_n}{(2\pi)^3} f_n \frac{d^3p_p}{(2\pi)^3}(1-f_p)\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar\nu}\,\omega_{\bar\nu}}{2\omega_{\bar\nu}(2\pi)^3}(2\pi)^4\delta^{(4)}(P_f-P_i)\sum_{\rm spins}|M|^2$$

$$\epsilon_\nu^{\rm DU} = 2\int \frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\pi)^3}\omega_{\bar\nu}\left[-i\Pi_0^{-+}(q_e+q_{\bar\nu})\right]$$

to calculate Direct Urca emissivity

we need only (no medium effects) simple free W boson "- +" loop

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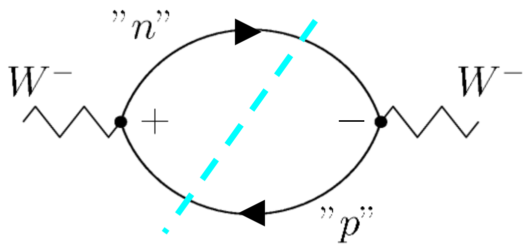
Using relation $i\,\Pi^{-+} = -\frac{2\,{\rm Im}\Pi^R}{e^{\omega/T}-1}$ we may calculate cross-sections as an integral of $|M|^2$ over the phase space **OR** as an imaginary part of W^- boson self-energy



perturbative expansion: second-order term in weak coupling and **zeroth-order** term in strong coupling

Terms of higher order in strong couplings must be included! $\Pi_0^{-+} \longrightarrow \Pi^{-+}$

[Voskresensky, Senatorov, Sov, J. Nucl. Phys. 45 (1987)]

- **Bose occupation number out of fermion loop**

$$f_{\rm F}(E_1)\,[1-f_{\rm F}(E_2)] = [f_{\rm F}(E_2)-f_{\rm F}(E_1)]\,f_{\rm B}(E_1-E_2)$$

$$\begin{aligned}
-i\mathcal{L}_0^{-+} &= \int \frac{d^3p}{(2\pi)^3}\, f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})\,[1-f_{{\rm F}p}(\boldsymbol{p})]\,2\,\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p] \\
&= f_B(\omega)\int \frac{d^3p}{(2\pi)^3}[f_{{\rm F}p}(\boldsymbol{p})-f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})]\,2\,\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p] \\
&= -2\,f_B(\omega)\,{\rm Im}\int \frac{d^3p}{(2\pi)^3}\,\frac{f_{{\rm F}p}(\boldsymbol{p})-f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})}{E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p-i0} \\
&= -f_B(\omega)\,2\,{\rm Im}\,\mathcal{L}_{np}^R
\end{aligned}$$

$$\mathcal{L}_{ab}^R = \int \frac{d^3p}{(2\pi)^3}\frac{f_{{\rm F}a}(\boldsymbol{p}+\boldsymbol{q})-f_{{\rm F}b}(\boldsymbol{p})}{\omega+E^b(p)-E^a(\boldsymbol{p}+\boldsymbol{q})+\mu_a-\mu_b+i0}$$

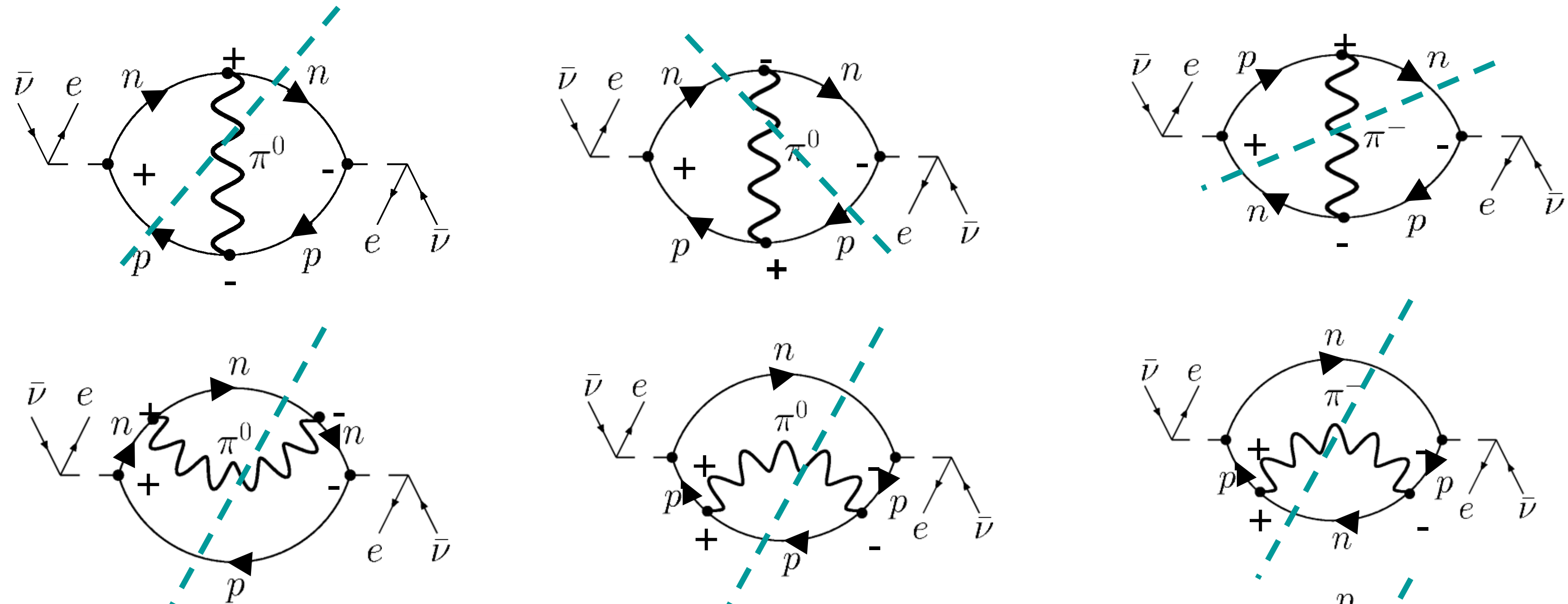
Lindhard function

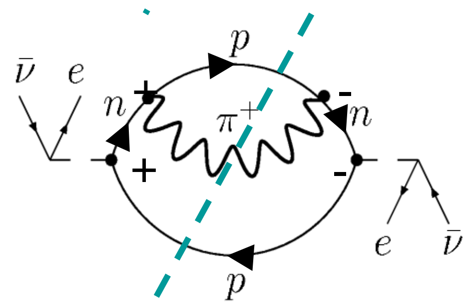
very sharp function of ω and $\boldsymbol{\kappa}$

Optical theorem for modified URCA reactions

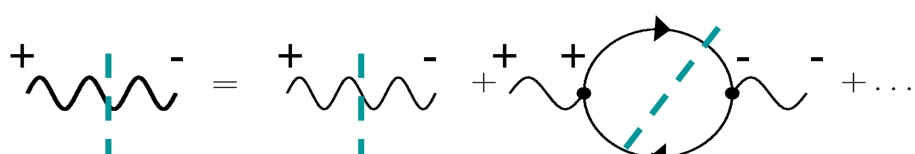
$$\epsilon_\nu^{\rm MU} = \int \frac{d^3 q_e\,(1-f_e)}{2\,\omega_e\,(2\,\pi)^3}\,\frac{d^3 q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\,\pi)^3}\,\omega_{\bar\nu}\,[-i\Pi^{-+}_{\rm MU}(q_e+q_{\bar\nu})]$$

To get correct 2-order Π^{-+} one should add diagrams with π^- corresponding to $np \to ppe\bar\nu$ reaction. They should be added coherently.





thick pion line (here up to 2[nd] order):



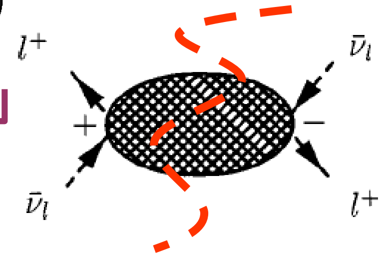
one-nucleon process with pion

two-nucleon process

Neutrino emissivity

$$\varepsilon_{l_1 l_2} = \frac{G^2}{2} \int \frac{d^3 q_1 \,(1 - f_1)}{(2\pi)^3\, 2\,\omega_1} \frac{d^3 q_2\,(1 - f_2)}{(2\pi)^3\, 2\,\omega_2}\, \omega\, n_{\rm bos}(\omega)\, 2\,\Im \sum_{\text{lept. spin}} \chi(q)$$

$$q = (\omega, \boldsymbol{q}\,) = q_1 + q_2$$



weak current susceptibility

vector (V) and axial (A) currents

$$\chi(q) = \int d^4x e^{-i\,q\cdot(x-y)} \left\langle \big(V_\mu(x) l^\mu(x) - A_\mu(x) l^\mu(x)\big) \big(V^\nu(y) l^\dagger_\nu(y) - A^\nu(y) l^\dagger_\nu(y)\big)\right\rangle$$

$$\chi(q) = \chi_V(q) + \chi_A(q)$$

$$\chi_V(q) \propto \langle V_\mu(x) l^\mu(x) V^\nu(y) l^\dagger_\nu(y)\rangle \longrightarrow \big\langle V_0(x) l_0(x) V_0(y) l^\dagger_0(y)\big\rangle$$

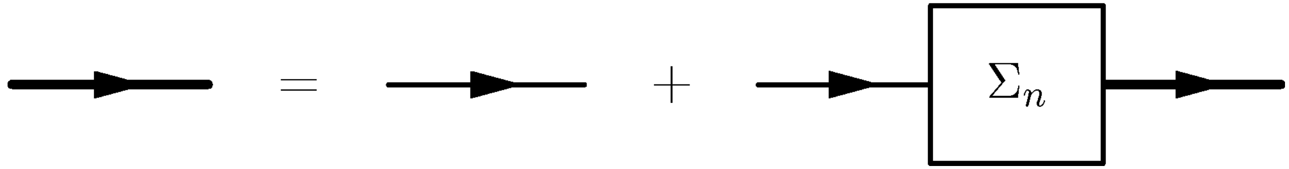
non-relativistic limit

$$\chi_A(q) \propto \langle A_\mu(x) l^\mu(x) A^\nu(y) l^\dagger_\nu(y)\rangle \longrightarrow \Big\langle \big(\boldsymbol{A}\boldsymbol{l}(x)\big)\,\big(\boldsymbol{A}\,\boldsymbol{l}^\dagger(y)\big)\Big\rangle$$

relativistic corrections can be large !

INCLUSION OF STRONG INTERACTIONS

"Strong" Self-Energy

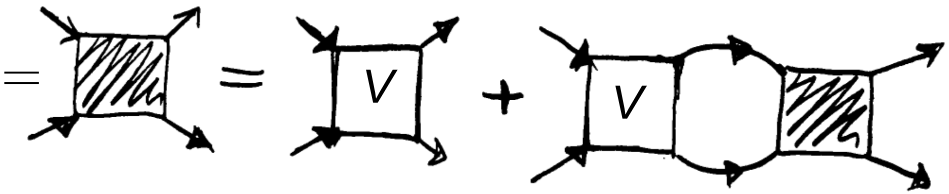


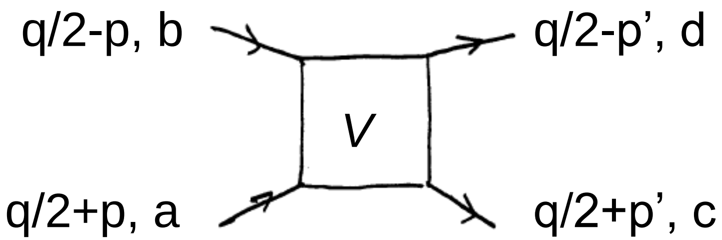
particle-particle, particle-hole hole-hole interactions

$$\widehat{G}_{\text{n.s.}} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}_{\text{n.s.}}\,\widehat{G}_{\text{n.s.}} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}_{\text{n.s.}}\right]^{-1}$$

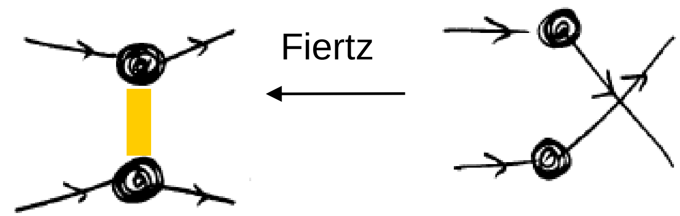
Particle-particle interaction

$$-i\,T_{pp}(p,p';q) =$$

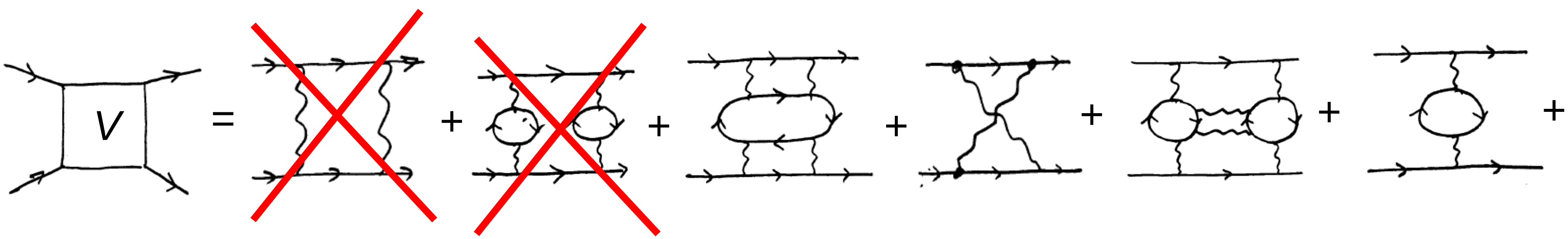




$$\left[\widehat{V}(p,p',q)\right]_{cd,ab} = V_0(p,p',q)(i\sigma_2)_{dc}(i\sigma_2)_{ab}$$
$$+ V_1(p,p',q)(\boldsymbol{\sigma} i\sigma_2)_{dc}(i\sigma_2\boldsymbol{\sigma})_{ab}$$



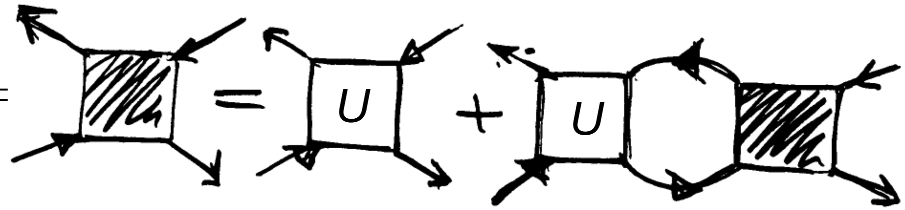
two-particle irreducible interaction

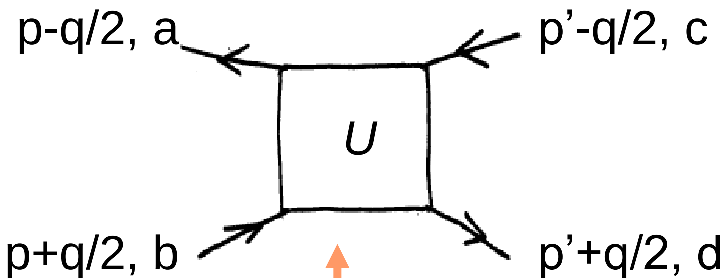


$$\widehat{T}_{\rm pp}(p,p',q) = \widehat{V}(p,p',q) + \int \frac{\mathrm{d}^4 p''}{(2\pi)^4\, i} \widehat{V}(p,p'',q)\, \widehat{G}(q/2+p'')\, \widehat{G}(q/2-p'')\, \widehat{T}_{\rm pp}(p'',p',q)$$

Particle-hole interaction

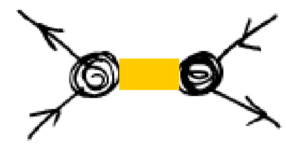
$$-i\,T_{\rm ph}(p,p';q) =$$

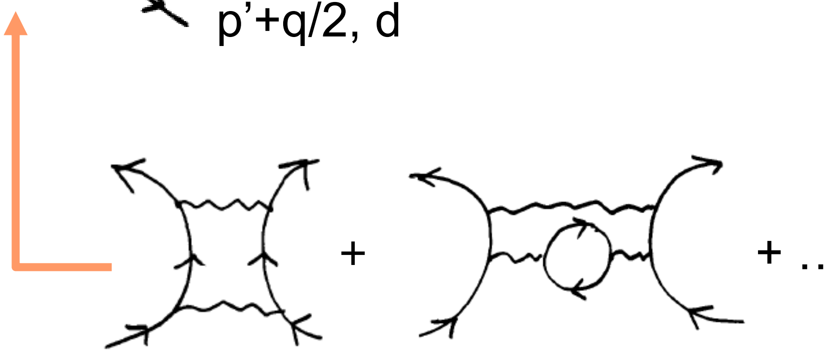




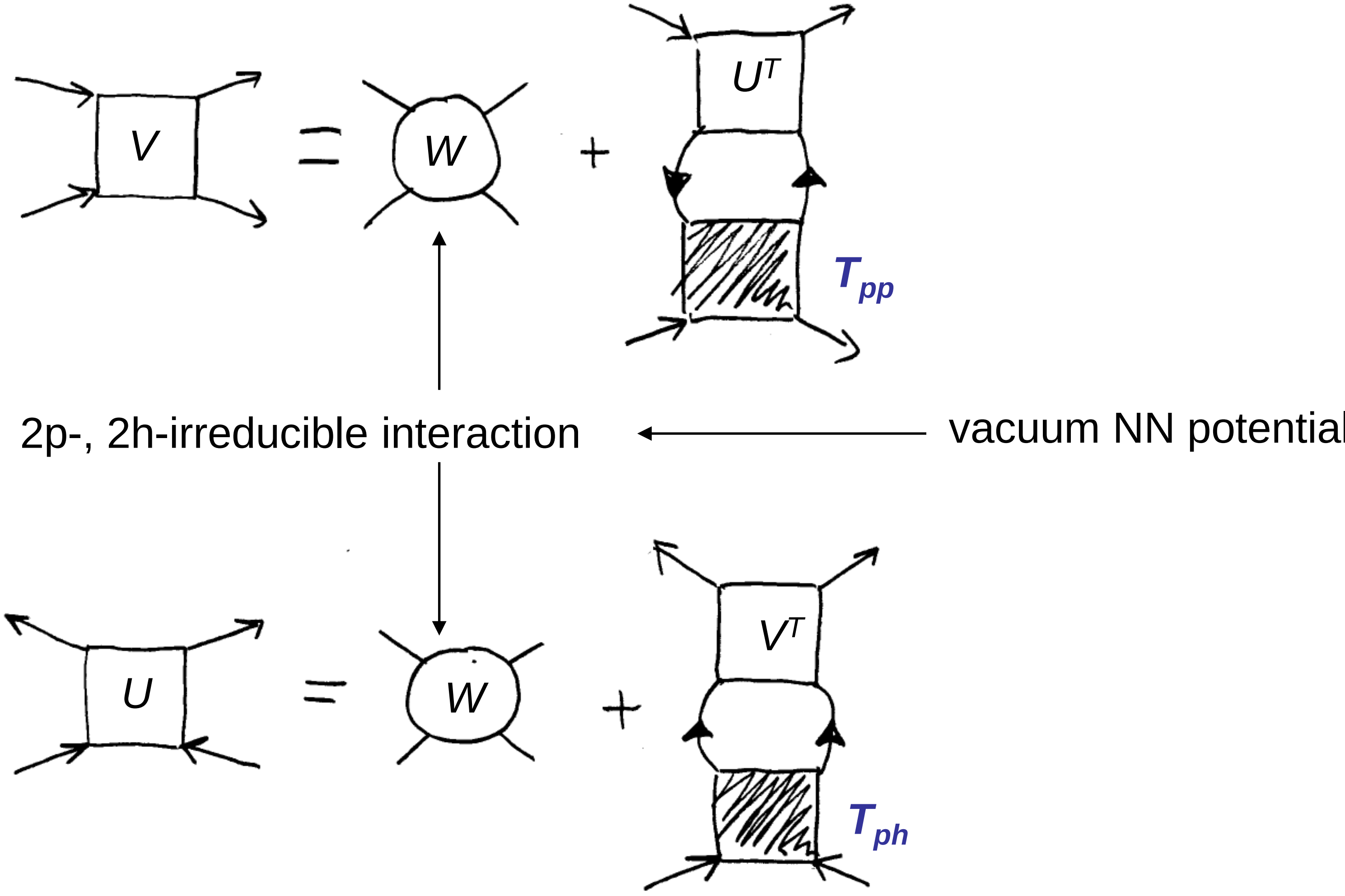
particle-hole irreducible interaction

$$\left[\widehat{U}(p,p',q)\right]_{dc,ab} = U_0(p,p',q)\,\delta_{dc}\delta_{ab} + U_1(p,p',q)\,\boldsymbol{\sigma}_{dc}\,\boldsymbol{\sigma}_{ab}$$



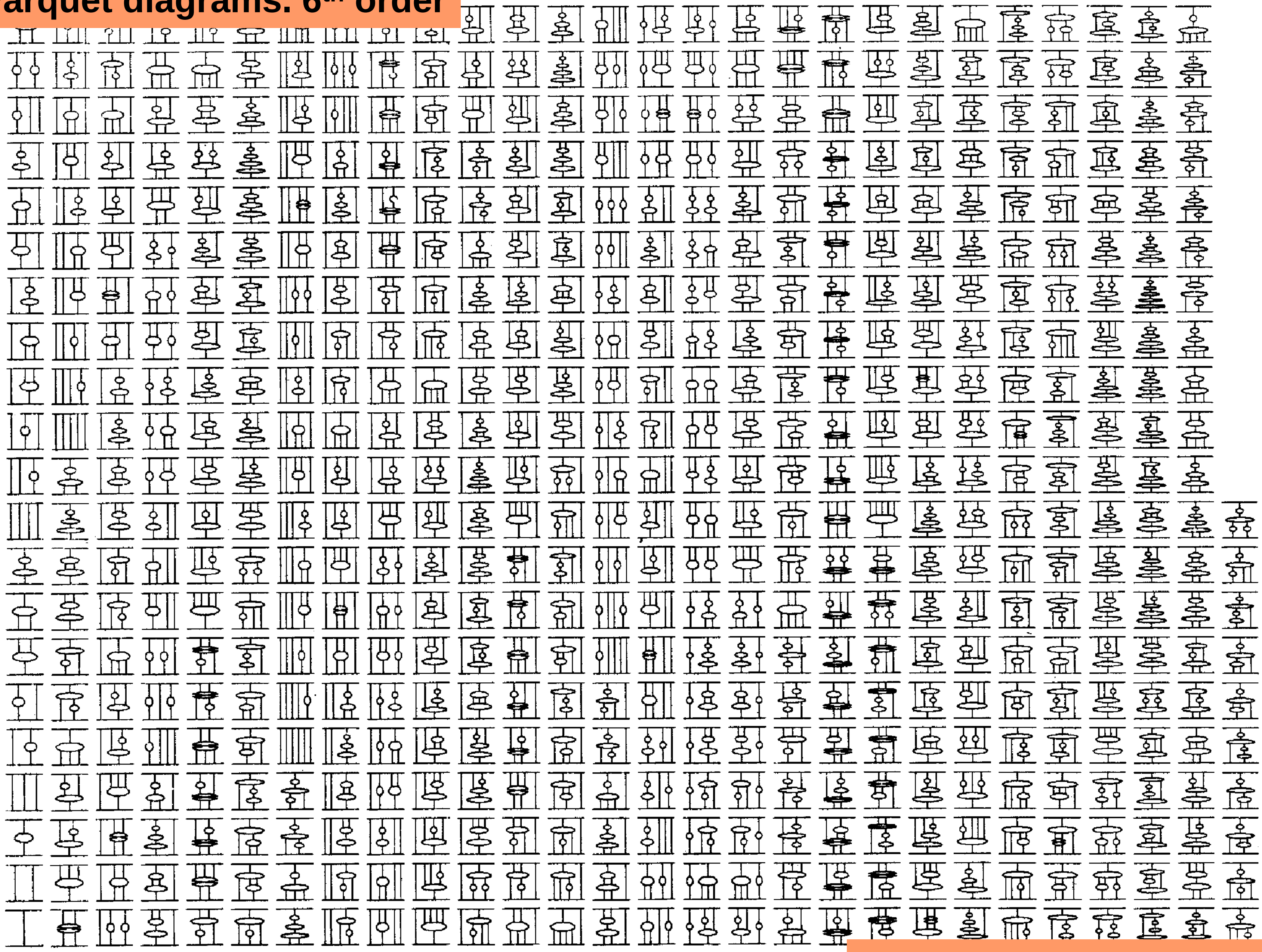


$$\widehat{T}_{\rm ph}(p,p',q) = \widehat{U}(p,p',q) + \int \frac{d^4p''}{(2\pi)^4\,i}\widehat{U}(p,p'',q)\,\widehat{G}(q/2+p'')\,\widehat{G}^h(q/2-p'')\,\widehat{T}_{\rm ph}(p'',p',q)$$



[Wambach, Ainsworth,Pines NPA555]

Parquet diagrams. 6th order



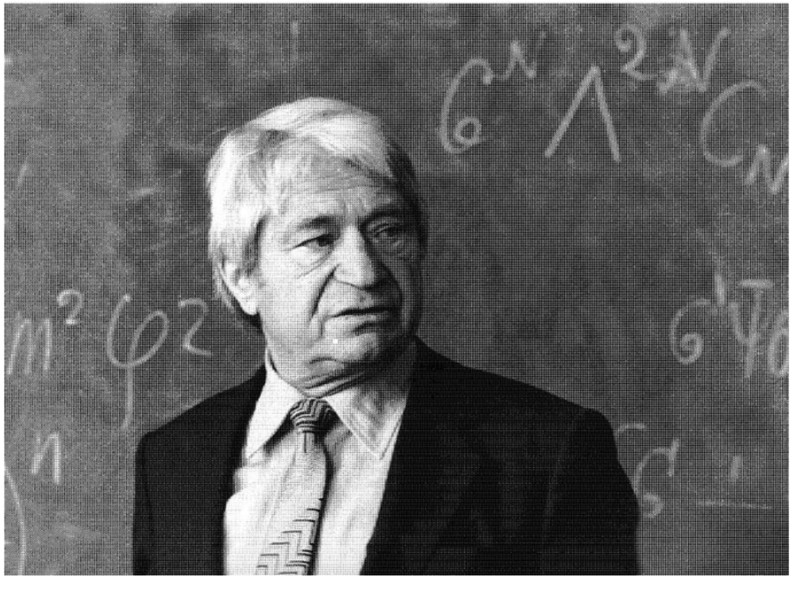
Jackson, Lande, Smith PR86

$$\frac{\delta E}{\delta n(p)} = \varepsilon(p)$$

$$\varepsilon(p) = \varepsilon^{(0)}(p) + \sum_{\boldsymbol{p}'} f(\boldsymbol{p}, \boldsymbol{p}')\,\delta n(\boldsymbol{p}')$$

NUCLEAR FERMI LIQUID

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$



Landau Fermi liquid approach

interacting fermions

system of quasi-particles

quantized excitations in the system

quasi-particles =/= original “bare” fermions [constituents of the system]

Landau wrote the Boltzmann eq.
for q.p distribution function: $n(\boldsymbol{x}, \boldsymbol{p}, t)$

$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \dot{\boldsymbol{x}} \frac{\partial n}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}} \frac{\partial n}{\partial \boldsymbol{p}} = I(n)$$

equations of motion for q.p.

$$\dot{\boldsymbol{x}} = \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -\frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{x}}$$

“generalized” velocity   Newton’s law

$$\frac{\partial n}{\partial t} + \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{p}} \frac{\partial n}{\partial \boldsymbol{x}} - \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{x}} \frac{\partial n}{\partial \boldsymbol{p}} = I(n)$$

$$\boldsymbol{\mathcal{F}} = \int \boldsymbol{p}\, n \frac{d^3p}{(2\pi)^3}$$

momentum flux density

$$\begin{aligned} \frac{\partial \boldsymbol{\mathcal{F}}}{\partial t} &= \int \boldsymbol{p} \frac{\partial n}{\partial t} \frac{d^3p}{(2\pi)^3} = \int \boldsymbol{p}\, I(n) \frac{d^3p}{(2\pi)^3} - \int \boldsymbol{p} \left[\frac{\partial \bar{e}}{\partial \boldsymbol{p}} \frac{\partial n}{\partial \boldsymbol{x}} - \frac{\partial \bar{e}}{\partial \boldsymbol{x}} \frac{\partial n}{\partial \boldsymbol{p}} \right] \frac{d^3p}{(2\pi)^3} \\ &= -\frac{\partial}{\partial x_j} \int \boldsymbol{p}\, n \frac{\partial \bar{e}}{\partial \boldsymbol{p}_j} \frac{d^3p}{(2\pi)^3} - \int n \frac{\partial \bar{e}}{\partial \boldsymbol{x}} \frac{d^3p}{(2\pi)^3} \\ &= \frac{\partial}{\partial x_j} \boldsymbol{\Pi}^j + \int \bar{e} \frac{\partial n}{\partial \boldsymbol{x}} \frac{d^3p}{(2\pi)^3} \end{aligned}$$

momentum conservation $0 = \frac{\partial}{\partial t}\int \boldsymbol{\mathcal{F}} d^3x = \int \frac{\partial}{\partial x_j}\boldsymbol{\Pi}^j \, d^3x + \int d^3x \int \bar{e}\,\frac{\partial n}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3}$

$$\Rightarrow \int d^3x \int \bar{e}\,\frac{\partial n}{\partial \boldsymbol{x}}\,\frac{d^3p}{(2\pi)^3} = 0$$

$$\int \bar{e}\,\frac{\partial n}{\partial \boldsymbol{x}}\,\frac{d^3p}{(2\pi)^3} = \frac{\partial}{\partial \boldsymbol{x}}\,E$$

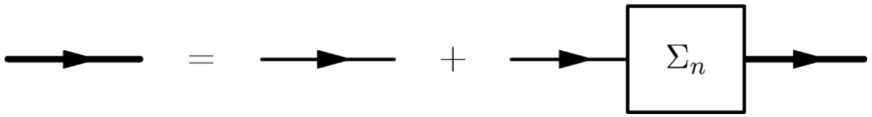
$$\delta E = \int \bar{e}\,\delta n\,\frac{d^3p}{(2\pi)^3}$$

E = energy of the system

$$\frac{\delta E}{\delta n(p)} = \bar{e}(p)$$

single particle mechanism of excitation

[G.E. Brown, RMP 43, 1]



$$\widehat{G} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}\,\widehat{G} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}\right]^{-1}$$

$$T \ll \varepsilon_{\mathrm{F},n}, \varepsilon_{\mathrm{F,p}} \text{ and } \epsilon \sim \epsilon_{\mathrm{F}}\,,\; p \sim p_{\mathrm{F}}$$

pole residue

$$a^{-1} = 1 - \frac{\partial}{\partial \epsilon}\Sigma(\epsilon, 0, T)\Big|_{\epsilon \simeq \epsilon_{\mathrm{F}}}$$

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$

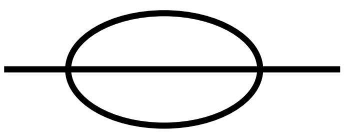
q.p. energy

q.p. width

$$\epsilon_p = \frac{p^2 - 2m_N\mu_N}{2\,m_N^*} \approx \frac{p^2 - p_{\mathrm{F}}^2}{2\,m_N^*} \approx v_{\mathrm{F}}\,(p - p_{\mathrm{F}})$$

$$\gamma = -\lim_{\epsilon \to 0} \mathrm{Im}\,\Sigma^R(\epsilon, p^2 = 2m_N\mu_N, T)/\epsilon^2$$

small for T<<□$_F$



q.p. effective mass

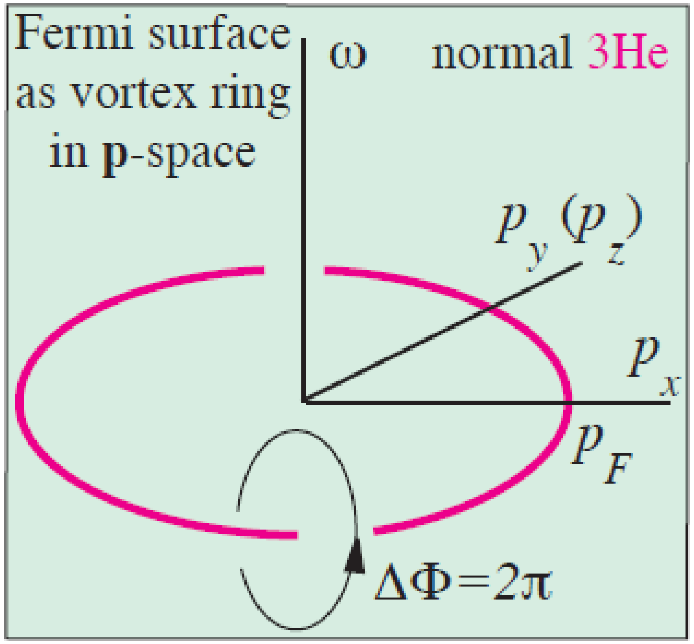
$$\frac{1}{m_N^*} = \frac{a}{m_N} + 2\,a\,\frac{\partial}{\partial p^2}\Sigma(\epsilon, \boldsymbol{p}, T)\Big|_{p=0, \epsilon \simeq \epsilon_{\mathrm{F}}}$$

$G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$ complicated background part

Fermi surface is a topological object.

ideal gas $$G_0(z = i\omega, p) = \frac{1}{i\omega - v_{\rm F}(p - p_{\rm F})} \qquad v_{\rm F} = p_{\rm F}/m_N$$

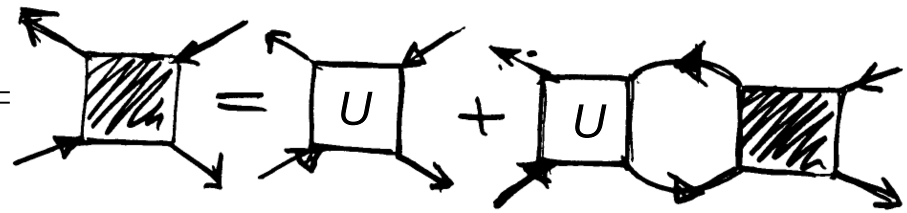
In 4D space (ω,p) there is a singularity at (ω=0,p=p_F) [singular hyperline] where this function is not defined!



The phase of the Green's function changes by 2p when one goes along a contour encircling this singular line. One can define a topological invariant [see book y G.E. Volovik, The Universe in a helium droplet] The singular-line is topologically protected and thus robust against perturbations

(normal) Fermi liquid $$G(z = i\omega, p) = \frac{a}{i\omega - v_{\rm F}(p - p_{\rm F})} \qquad v_{\rm F} = p_{\rm F}/m_N^*$$

$$-i\,T_{\rm ph}(p,p';q) =$$



$$\widehat{T}_{\rm ph}(p,p',q) = \widehat{U}(p,p',q) + \int \frac{\mathrm{d}^4 p''}{(2\pi)^4\, i} \widehat{U}(p,p'',q)\, \widehat{G}(q/2+p'')\, \widehat{G}^h(q/2-p'')\, \widehat{T}_{\rm ph}(p'',p',q)$$

$$G(q/2+p)\, G^h(q/2-p) = G(q/2+p)\, G(p-q/2)$$

$$= \frac{a}{[\epsilon+\omega/2-\epsilon_{\boldsymbol{p}+\boldsymbol{q}/2}+i\,0\,\mathrm{sign}\,(\epsilon+\omega/2)]}\,\frac{a}{[\epsilon-\omega/2-\epsilon_{\boldsymbol{p}-\boldsymbol{q}/2}+i\,0\,\mathrm{sign}\,(\epsilon-\omega/2)]} + \tilde{B}(p,q)$$

$$\simeq a^2\,\delta(\epsilon)\int d\epsilon\, \frac{1}{[\epsilon+\omega/2-\epsilon_{\boldsymbol{p}+\boldsymbol{q}/2}+i\,0\,\mathrm{sign}\,(\epsilon+\omega/2)]}\,\frac{1}{[\epsilon-\omega/2-\epsilon_{\boldsymbol{p}-\boldsymbol{q}/2}+i\,0\,\mathrm{sign}\,(\epsilon-\omega/2)]} + B(p,q)$$

$$= -2\pi i\, a^2\, \delta(\epsilon) \frac{f(\boldsymbol{p}+\boldsymbol{q}/2) - f(\boldsymbol{p}-\boldsymbol{q}/2)}{\omega - \epsilon_{\boldsymbol{p}+\boldsymbol{q}/2} + \epsilon_{\boldsymbol{p}-\boldsymbol{q}/2} + i\,0} + B(p,q)$$

$$p \sim p_{\rm F}$$

Fermi liquid approximation

particle-hole propagator …. for $q \to 0$

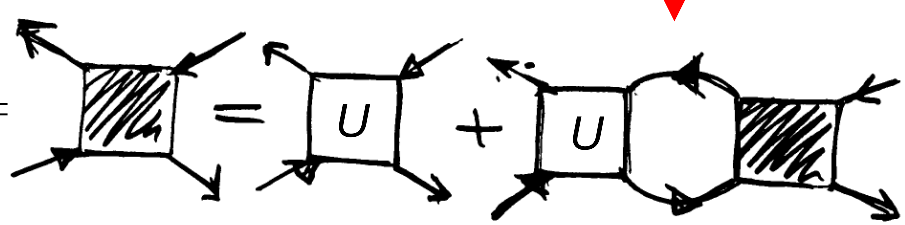
$$\boldsymbol{n} = \boldsymbol{p}/p$$

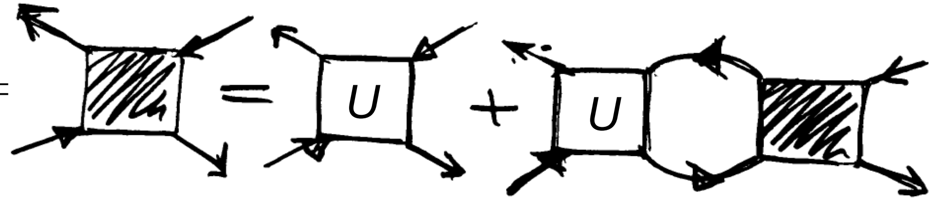
$$G(q/2+p)\,G^h(q/2-p) \simeq 2\pi i\, a^2\, \delta(\epsilon) \frac{v_\mathrm{F}\, \boldsymbol{q}\boldsymbol{n}}{\omega - v_\mathrm{F}\boldsymbol{q}\boldsymbol{n} + i\,0}\, \delta(p - p_\mathrm{F}) + B(p,q)$$

singular pole term

complicated background

$$-i\, T_\mathrm{ph}(p,p';q) =$$



$$-i\,T_{\rm ph}(p,p';q) =$$ 

for $|\boldsymbol{p}| \simeq p_{\rm F} \simeq |\boldsymbol{p}'|$ and $|\boldsymbol{q}\boldsymbol{p}| << \omega << \epsilon_{\rm F}$

$$\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}',q) = \widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}') - \int \frac{d\Omega_{p''}}{4\,\pi}\widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}')\,A(\boldsymbol{n},q)\,\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}',q)$$

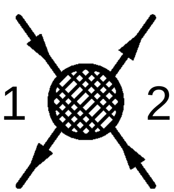
$$A(\boldsymbol{n},q) = a^2\,\frac{m^*\,p_{\rm F}}{\pi^2}\,\frac{v_{\rm F}\,\boldsymbol{q}\boldsymbol{n}}{\omega - v_{\rm F}\boldsymbol{q}\boldsymbol{n} + i\,0}$$

complicated dynamics is here:

$$\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}') = \widehat{U}(\boldsymbol{n},\boldsymbol{n}') - \int \frac{d^4p''}{(2\pi)^4\,i}\widehat{U}(\boldsymbol{n},\boldsymbol{n}')\,B(p,\omega\to 0,\frac{\boldsymbol{q}}{\omega}\to 0)\,\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}')$$

parameterize

Landau-Migdal parameters

1  2 $= f_{12}(\boldsymbol{n},\boldsymbol{n}') + g_{12}(\boldsymbol{n},\boldsymbol{n}')\,\boldsymbol{\sigma_1}\boldsymbol{\sigma_2}$

extracted from experiment

$$a^2 N\,\Gamma_0^\omega(\theta) = f(\theta) = \sum_l f_l P_l(\cos\theta)$$

$$a^2 N\,\Gamma_1^\omega(\theta) = g(\theta) = \sum_l g_l P_l(\cos\theta)$$

$$\theta = \angle(\boldsymbol{n}, \boldsymbol{n}')$$

$$N = \nu m^* p_\mathrm{F}/\pi^2$$

density of states at the Fermi surface

$\nu = 1, 2$ *number of fermion types* $n = \nu p_\mathrm{F}^3/3\pi^2$

neutron matter: $f = f_{nn}$ $g = g_{nn}$ (1 parameter in each channel)

nuclear matter: $f_{nn},\, f_{np},\, f_{pp}$ $g_{nn},\, g_{np},\, g_{pp}$ (3 parameters in each channel)

In matter of arbitrary isospin composition these parameters are independent.
Fermi-liquid renormalization is different for these parameters.

small isospin disballance $f_{nn} = f_{pp} = f + f'$ $f_{np} = f - f'$

$$a^2 N\Gamma^\omega = f + f'\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

(2 parameters in each channel)

In nuclear physics one uses also the normalization on the nuclear Fermi surface

$$\tilde{f}(\boldsymbol{n}', \boldsymbol{n}) = a^2 N_0 \Gamma_0^\omega(\boldsymbol{n}', \boldsymbol{n}) \qquad \tilde{g}(\boldsymbol{n}', \boldsymbol{n}) = a^2 N_0 \Gamma_1^\omega(\boldsymbol{n}', \boldsymbol{n})$$

$N_0 = N(n = n_0)$ constant, independent of density $(N_0^{-1} = 300\,\mathrm{MeV\,fm}^3)$

Density dependence? Residual momentum dependence $\Gamma(\boldsymbol{n}', \boldsymbol{n}; q)$?

There are relations between some Landau parameters and bulk properties of the system

effective mass $m^* = m\left(1 + \frac{2}{3} f_1\right)$

compressibility $K = 6\,\frac{p_{\mathrm{F}}^2}{m^*}\,(1 + 2\,f_0)$

symmetry energy $E_{\mathrm{sym}} = \frac{1}{3}\,\frac{p_{\mathrm{F}}^2}{2\,m^*}\left(1 + 2\,f_0'\right)$

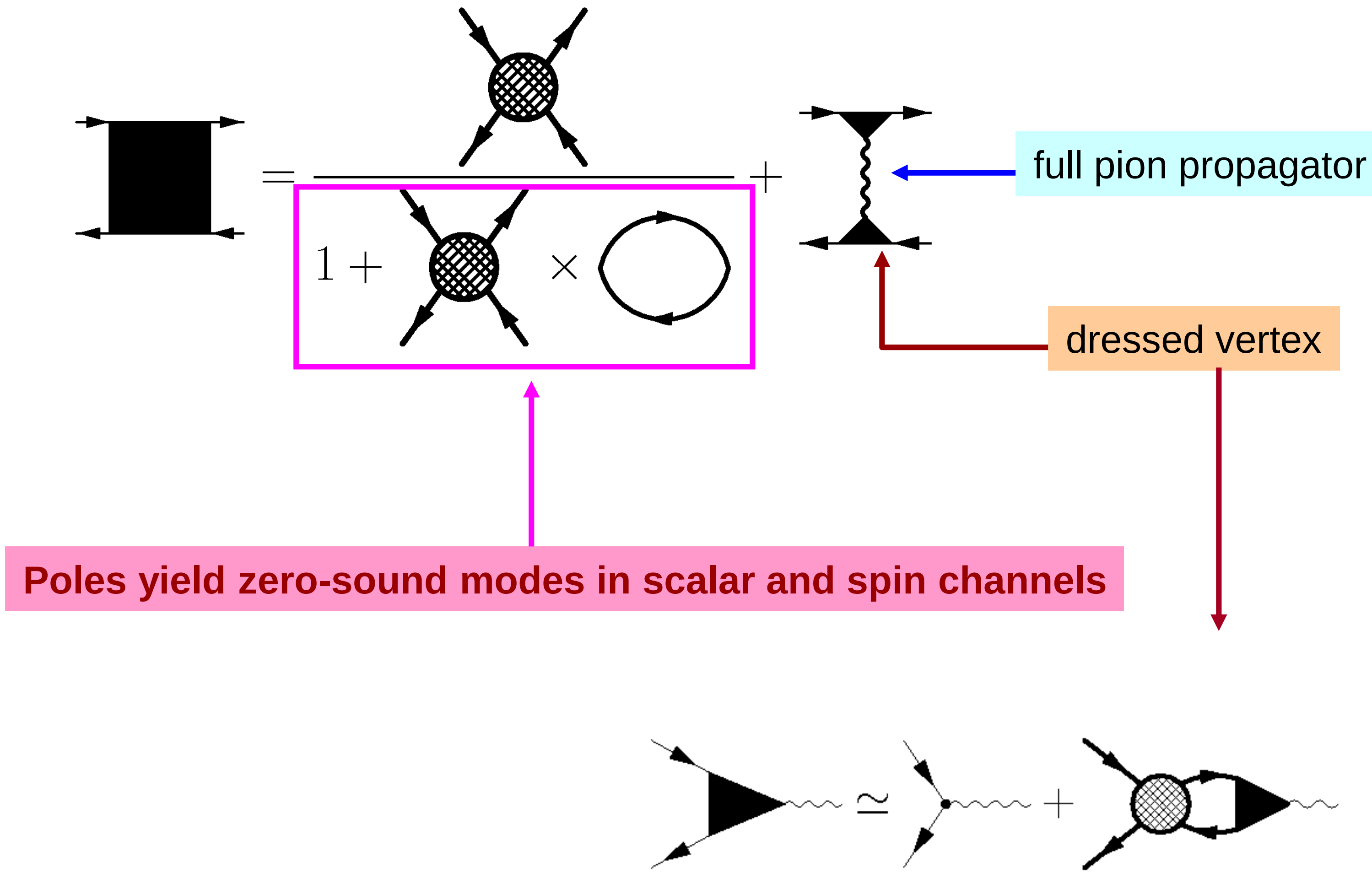
In general Landau parameter are to be fitted to empirical information (nucleus properties)

[Saperstein,Fayans, et al. 1995, 1998]

$$f \simeq 0,\ f' \simeq 0.5 - 0.6,\ g \simeq 0.05 \pm 0.1,\ g' \simeq 1.1 \pm 0.1$$

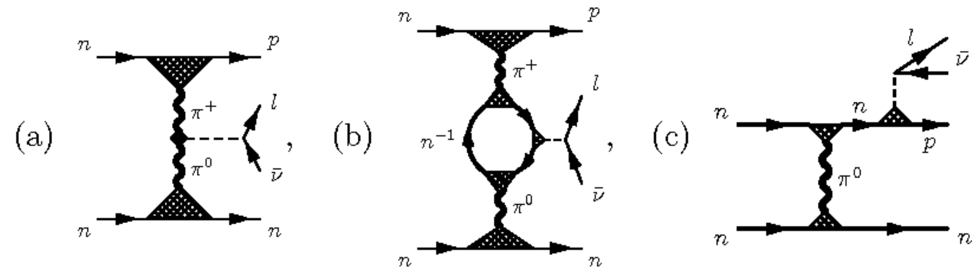
Resummed NN interaction

Graphically, the resummation is straightforward and yields:



• pion softenning in neutrino production

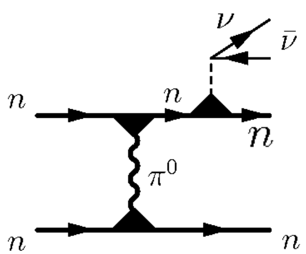
enhancement factors w.r.t. MU emissivity



medium MU reactions

$$F_{\mathrm{MMU}}(n) = 3\left(\frac{n}{n_0}\right)^{10/3} \frac{[\Gamma(n)/\Gamma(n_0)]^6}{(\widetilde{\omega}(n)/m_\pi)^8}$$

medium bremsstrahlung reactions

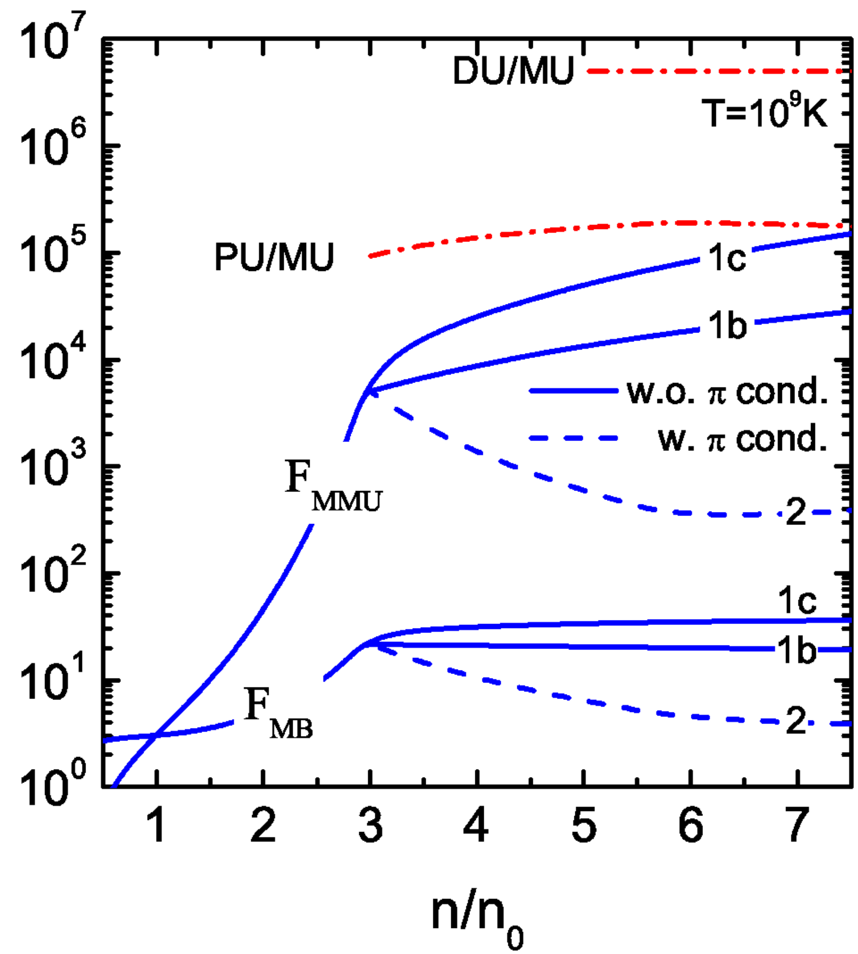


$+ \ldots$

$$F_{\mathrm{MB}}(n) = 3\left(\frac{n}{n_0}\right)^{4/3} \frac{[\Gamma(n)/\Gamma(n_0)]^6}{[\tilde{\omega}(n)/m_\pi]^3}.$$

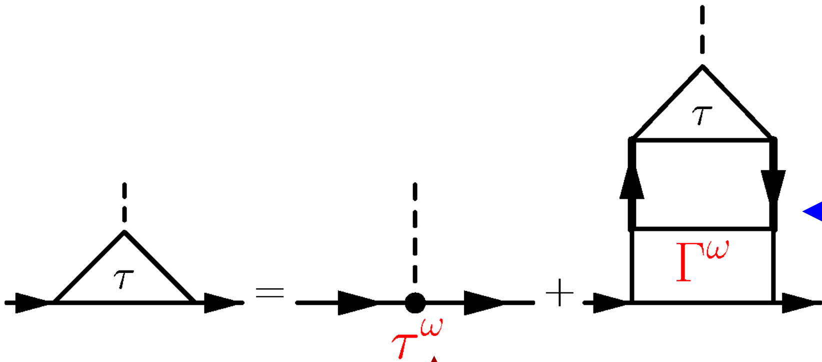
vertex correlation function

$$\Gamma(n) \simeq \frac{1}{1 + 1.6(n/n_0)^{1/3}}$$



Vertex renormalization in FL

Coupling of an external field to a particle

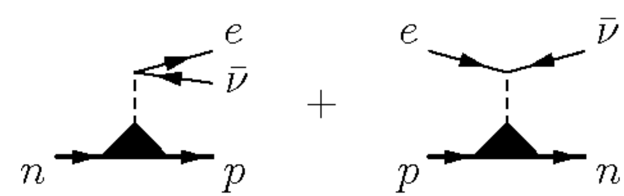


pole parts of Green's functions only

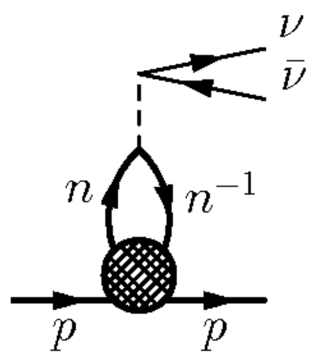
"bare" (FL renormalized) vertex

Effects: Reduce couplings. "A shield" against pion condensation

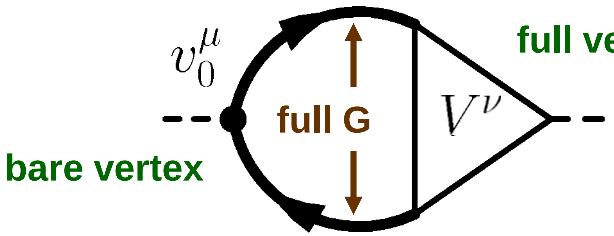
Produce sound modes contributing to response functions



Enhance reactions in some channels



Vector current conservation



Current is conserved if $\Pi^{\mu\nu}\, q_\nu = 0$

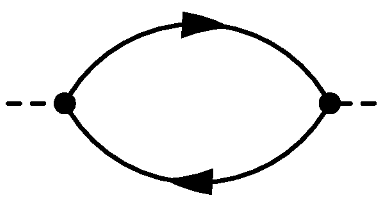
$$\Pi^{\mu\nu} \propto \int d^4p\, \mathrm{Tr}\left\{\gamma^\mu\, G(p+q/2)\, V^\nu(p,-q)\, G(p-q/2)\right\}$$

If the relation $q_\mu\, V^\mu(p,q) = G^{-1}(p+q/2) - G^{-1}(p-q/2)$ Is fulfilled

$$\Pi^{\mu\nu} q^\nu \propto \int d^4p\, \mathrm{Tr}\left\{\gamma^\mu\left[G(p-q/2) - G(p+q/2)\right]\right\} = 0$$

The Ward identities impose non-trivial relations between vertex functions and Green's functions, which synchronize any modification of the Green's function with a corresponding change in the vertex function.

in non-relativistic limit for free G and vertices: $\tau_0^\mu = (1, \boldsymbol{v})$ $\quad G(p) = (\epsilon - p^2/2m)^{-1}$



$$q\cdot\tau_0 = \omega - \boldsymbol{v}\boldsymbol{q} \equiv G_0^{-1}(p+q/2) - G_0^{-1}(p-q/2)$$

The Ward identity is fulfilled and the current is conserved

• "Bare" vertices

"bare" vertex after the Fermi-liquid renormalization $\tau_a^\omega = [1 + \Gamma_0^\omega\,(G_+G_-)^\omega]\,\tau_a^0$

$$V_\mu^{nn} \approx g_V \chi_p^\dagger(p')\big(1, \boldsymbol{v}\big)\chi_n(p)$$

$$A_\mu^{nn} \approx g_A\, \chi_p^\dagger(p')\big(\boldsymbol{\sigma}\cdot\boldsymbol{v}, \boldsymbol{\sigma}\big)\chi_n(p)$$

$$B = (G_+G_-)^\omega = \lim_{\boldsymbol{q}\to 0}\int \frac{2\,\mathrm{d}^4p}{(2\,\pi)^4\,i}\,G_+G_-$$

$$\tau_V^0 - \tau_A^0 = (V_\mu - A_\mu)\,l^\mu$$

weak interactions

$$\hat{\tau}_V^\omega = g_V\,\big(\tau_{V,0}^\omega\, l_0 - \boldsymbol{\tau}_{V,1}^\omega\,\boldsymbol{l}\,\big) \qquad \tau_{V,0}^\omega = \frac{e_V}{a}\,, \qquad \boldsymbol{\tau}_{V,1}^\omega = \frac{e_V}{a}\,\boldsymbol{v}$$

$$\hat{\tau}_A^\omega = -g_A\,\big(\boldsymbol{\tau}_{A,1}^\omega\boldsymbol{\sigma}\, l_0 - \tau_{A,0}^\omega\,\boldsymbol{\sigma}\boldsymbol{l}\,\big) \qquad \tau_{A,0}^\omega = \frac{e_A}{a}\,, \qquad \boldsymbol{\tau}_{A,1}^\omega = \frac{e_A}{a}\,\boldsymbol{v}$$

e_A e_V effective charges

$e_V = 1$ $$\omega\,\tau_{V,0}^\omega - \boldsymbol{q}\,\boldsymbol{\tau}_{V,1}^\omega = G^{(\mathrm{pole}),-1}(p+q/2) - G^{(\mathrm{pole}),-1}(p-q/2)$$

$e_A = 0.8$—0.95 experiment: Gamov-Teller transitions in nuclei $g_A^* \simeq 1$

SUPERFLUID MATTER

Pairing in nuclei

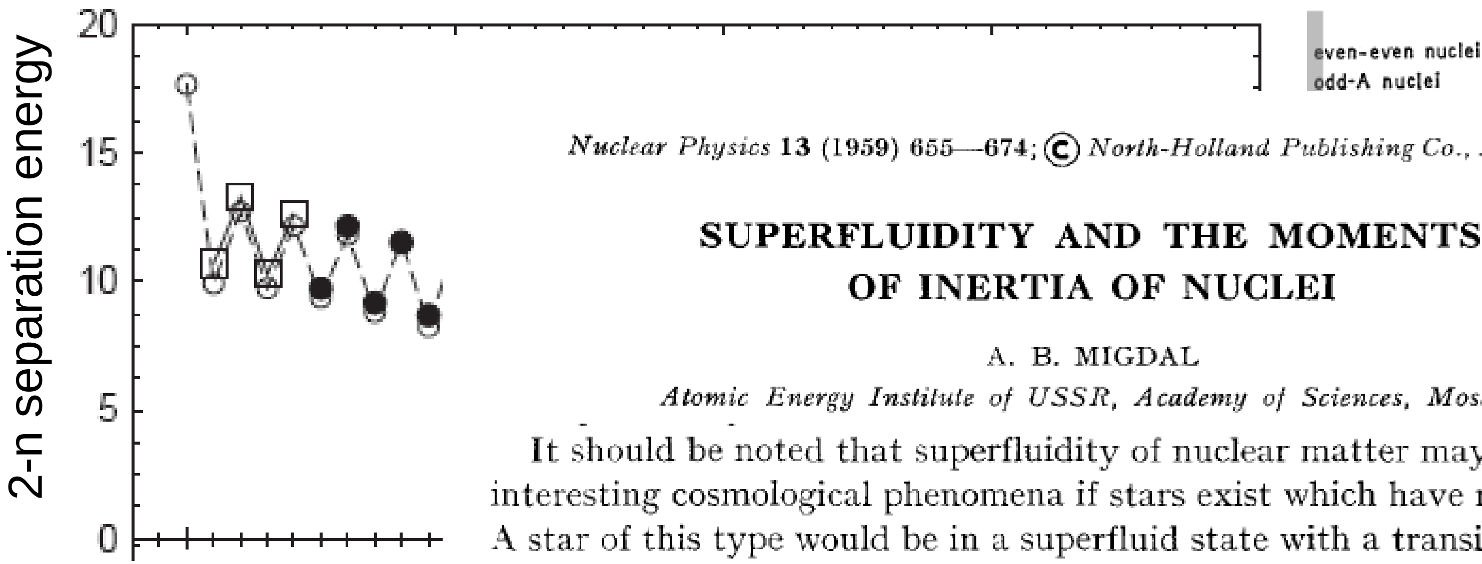
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.



Nuclear Physics **13** (1959) 655—674; © *North-Holland Publishing Co., Amsterdam*

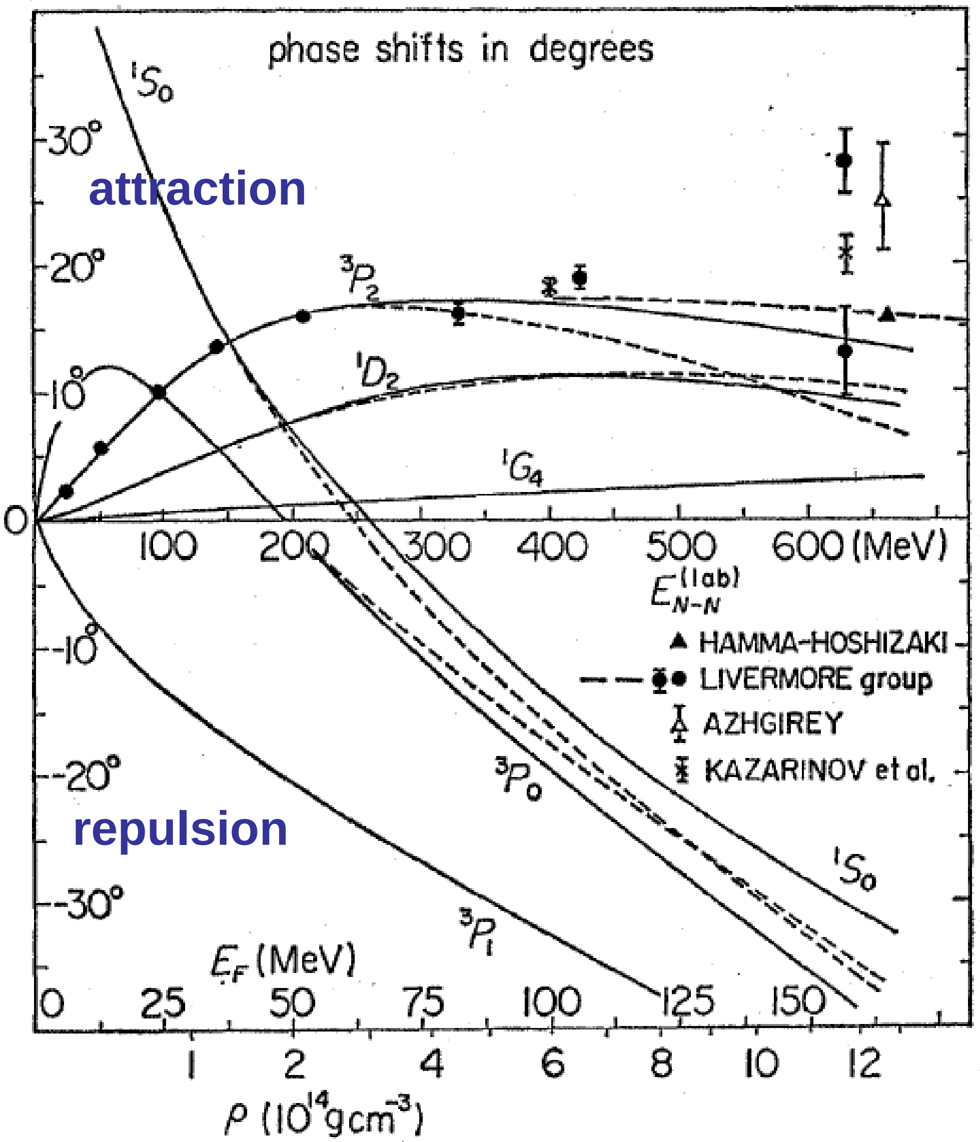
SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL
Atomic Energy Institute of USSR, Academy of Sciences, Moscow

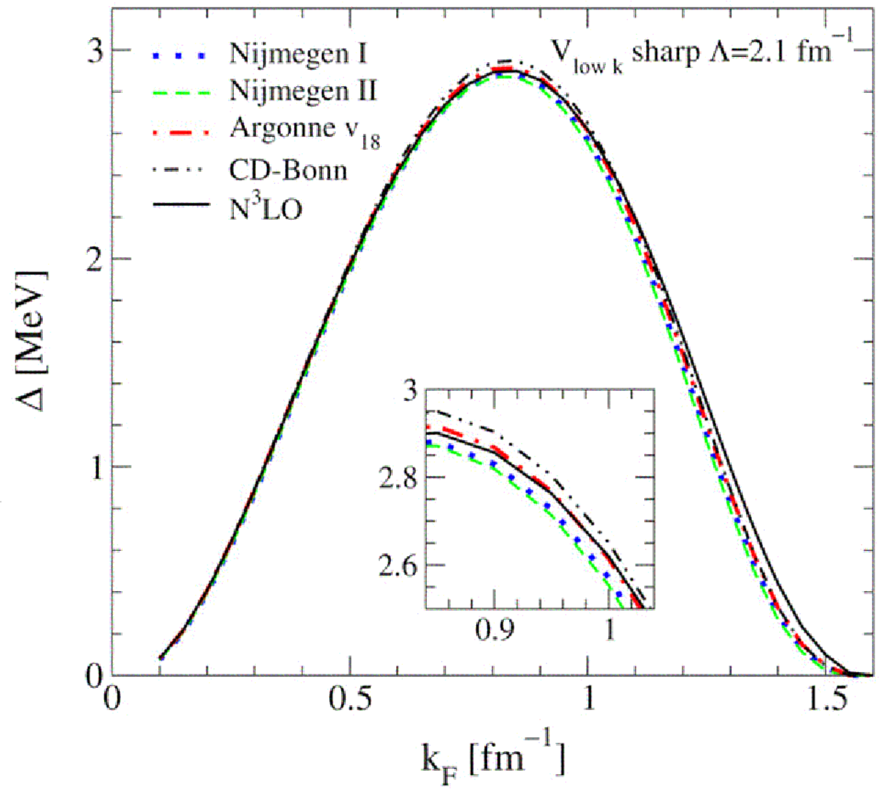
It should be noted that superfluidity of nuclear matter may lead to some interesting cosmological phenomena if stars exist which have neutron cores. A star of this type would be in a superfluid state with a transition temperature corresponding to 1 MeV.

Pairing gaps in nuclear matter

✔ nucleon-nucleon interaction



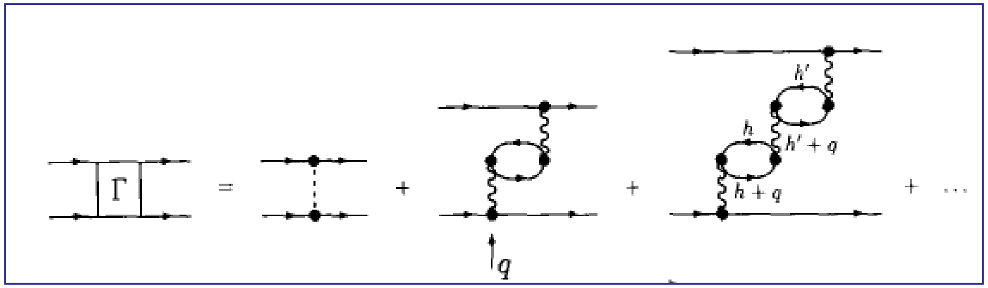
✔ $1S_0$ pairing gaps

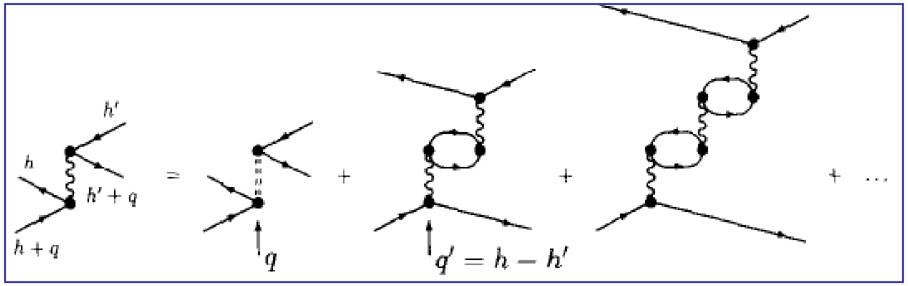


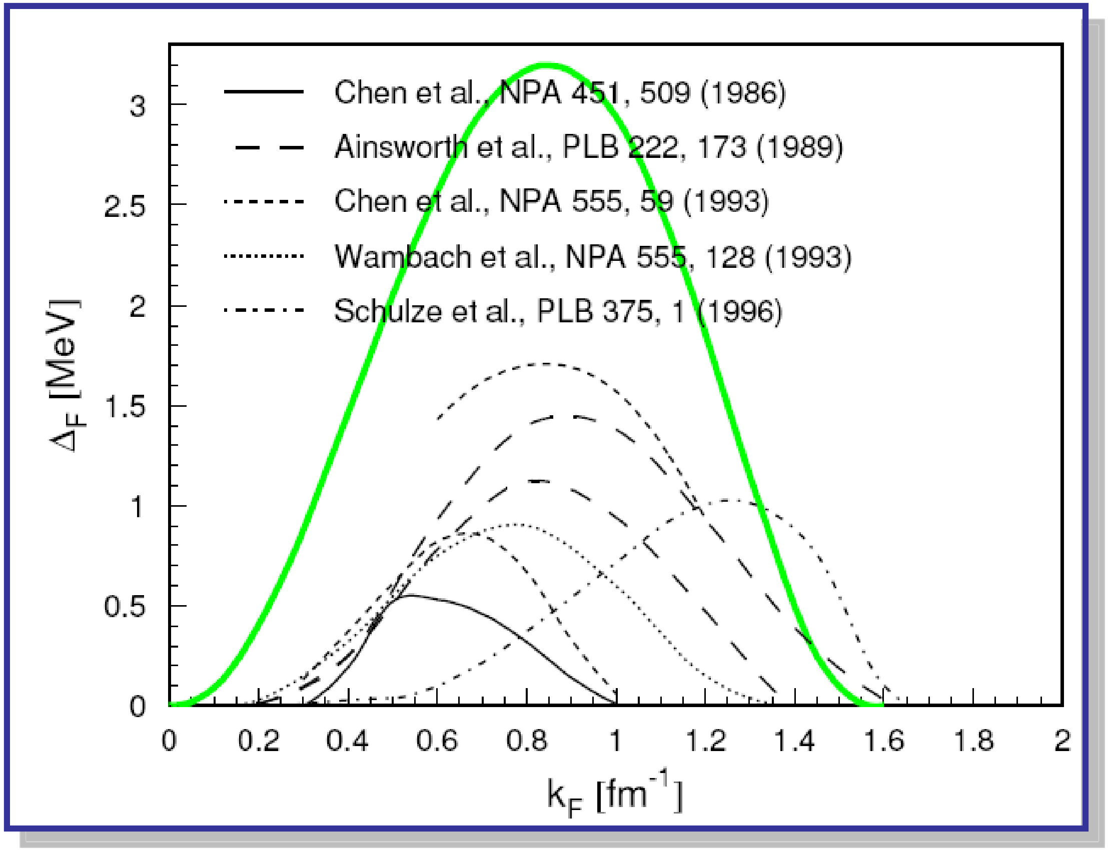
Hebeler, Schwenk, and Friman,
PLB 648 (2007) 176

✔ medium polarization effects on superfluidity

U. Lombardo and H.-J. Schulze, astro-ph/0012209

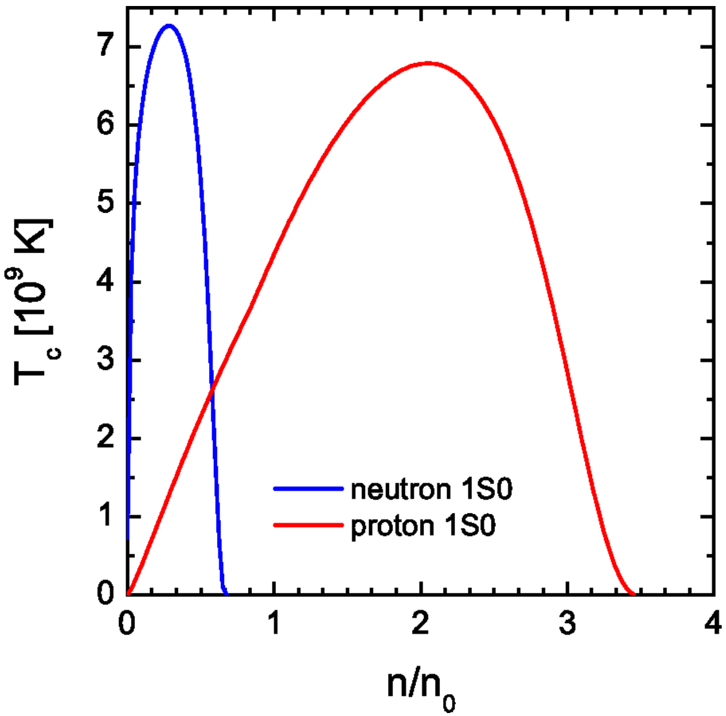




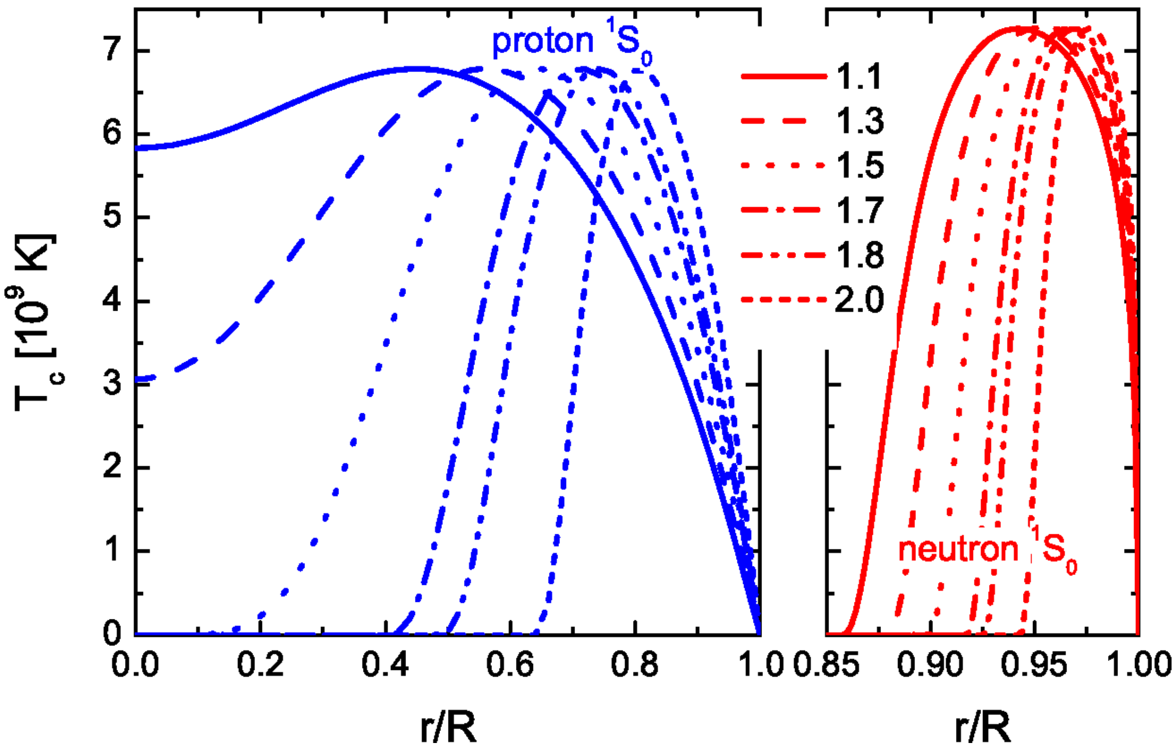


Medium effects strongly suppress triplet pairing!

✔ T_c in the neutron star matter



for HDD EoS from
[Blaschke, Grigorian, Voskresnesky PRC 88, 065805(2013)]



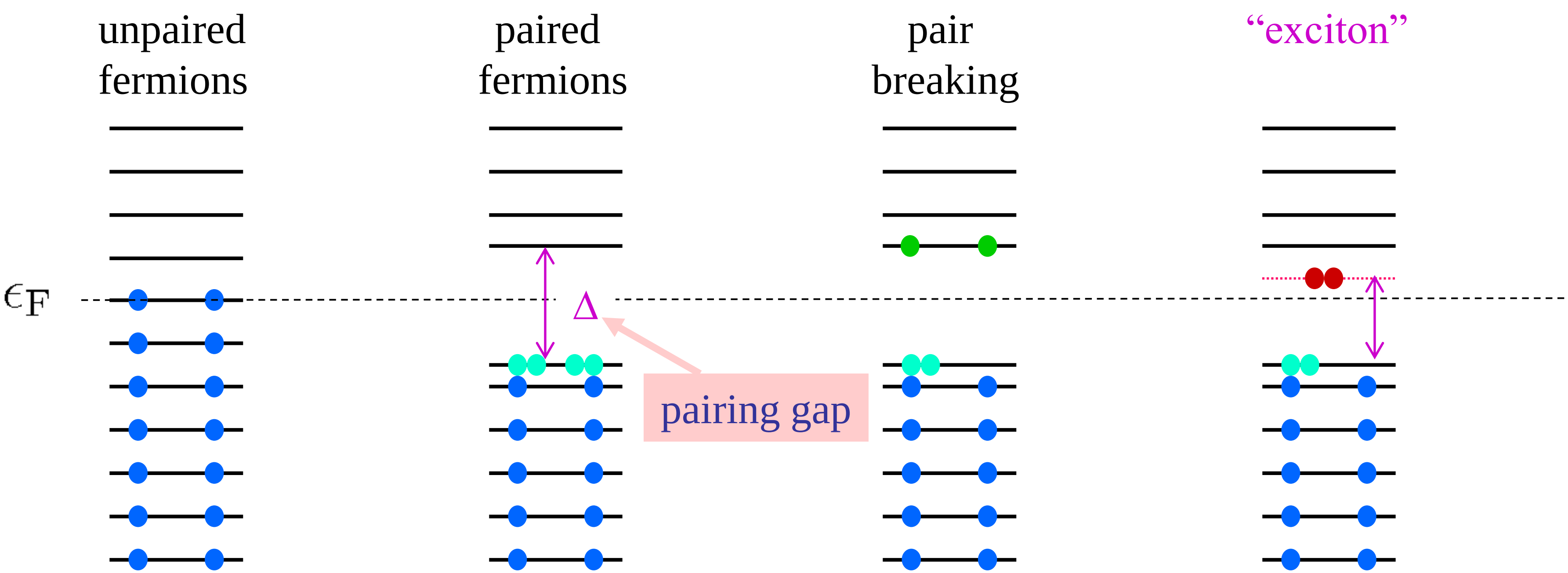
For the s-wave paring

$$T_c = 0.5669\,\Delta(T=0)$$

$$\Delta(T=0) = 1.764\,T_c$$

Fermi system with pairing

Ground state | Excited state



- **quasiparticle spectrum**

$$\xi_p = \frac{p^2}{2\,m^*} - \epsilon_F \simeq v_F\,(p - p_F)$$

$$E_p = \pm\sqrt{\xi_p^2 + \Delta^2}$$

“AB mode”

- **excitation spectrum**

$$\epsilon > 2\,\Delta$$

$$\epsilon \sim \omega_c < 2\,\Delta$$

excitation spectrum → emission spectrum

Superfluid Fermi liquid

- **Green's functions**

<u>normal Green's function</u>

particle $i\,\hat{G} =$ 

hole $i\,\hat{G}^h =$ 

<u>anomalous Green's function</u>

particle → hole $i\,\hat{F}^{(2)} =$ 

hole →particle $i\,\hat{F}^{(1)} =$ 

Number of excitations is not conserved !

$-i\,\hat{\Delta}^{(2)} =$ 

amplitude of 2 particle annihilation

$-i\,\hat{\Delta}^{(1)} =$ 

amplitude of 2 particle creation

*F*s, *G*s and Δs are connected by the Gorkov's and gap equations

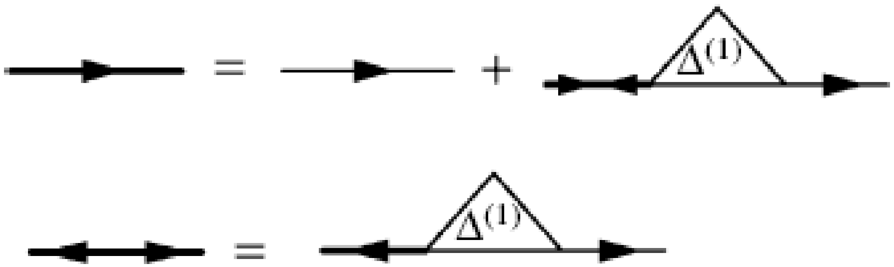
---

<u>S wave pairing</u>

$$\hat{\Delta}^{(1)} = \hat{\Delta}^{(2)} = \Delta\, i\,\sigma_2 \qquad \hat{F}^{(1)} = \hat{F}^{(2)} = F\, i\sigma_2$$

$$G(p) = \frac{a\,(\epsilon + \epsilon_p)}{\epsilon^2 - E_p^2 + i0\,\mathrm{sgn}\epsilon} \qquad F(p) = \frac{-a\,\Delta}{\epsilon^2 - E_p^2 + i0\,\mathrm{sgn}\epsilon}$$

- **Gor'kov equations**



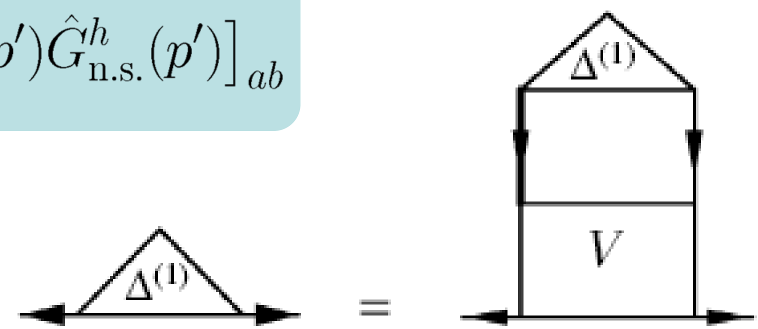
$$\hat{G}(p) = \hat{G}_{\text{n.s.}}(p) + \hat{G}_{\text{n.s.}}(p)\,\hat{\Delta}^{(1)}(p)\,\hat{F}^{(2)}(p)$$

$$\hat{F}^{(2)}(p) = \hat{G}^{h}_{\text{n.s.}}(p)\,\hat{\Delta}^{(2)}(p)\,\hat{G}(p)$$

- **Gap equation**

$$\left[\hat{\Delta}^{(1)}\right]_{cd} = \int \frac{d^4p'}{(2\pi)^4 i}\left[\widehat{V}(p,p')\right]_{cd,ab}\left[\hat{G}(p')\hat{\Delta}^{(1)}(p')\hat{G}^{h}_{\text{n.s.}}(p')\right]_{ab}$$

$$\left[\widehat{V}\right]_{cd,ab} = V_0\,(i\sigma_2)_{dc}\,(i\sigma_2)_{ab} + V_1\,(i\sigma_2\boldsymbol{\sigma})_{dc}\,(\boldsymbol{\sigma}\,i\sigma_2)$$



attractive interaction in paired
particle-particle channel

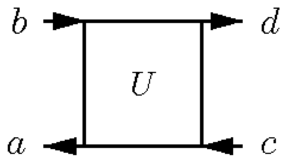
Superfluid Fermi liquid

- **quasiparticle interaction**

T<<ε_F Only particles on the Fermi surface take part in reactions.

particle-hole interaction:

$$\hat{\Gamma}^{\omega}_{dc,ab} = i$$

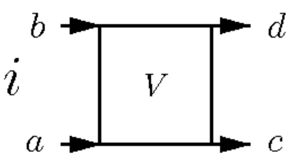


$$= \Gamma^{\omega}_{0}(\boldsymbol{n}', \boldsymbol{n})\, \delta_{dc}\delta_{ab} + \Gamma^{\omega}_{1}(\boldsymbol{n}', \boldsymbol{n})\, (\boldsymbol{\sigma})_{dc}\, (\boldsymbol{\sigma})_{ab}$$

particle-particle interaction:

Interaction in this two channels can be essentially different !

$$\hat{\Gamma}^{\xi}_{cd,ab} = i$$



$$= \underbrace{\Gamma^{\xi}_{0}(\boldsymbol{n}', \boldsymbol{n})(i\sigma_2)_{dc}(i\sigma_2)_{ab}}_{\text{spin zero}} + \underbrace{\Gamma^{\xi}_{1}(\boldsymbol{n}', \boldsymbol{n})(\boldsymbol{\sigma} i\sigma_2)_{dc}(i\sigma_2\boldsymbol{\sigma})_{ab}}_{\text{spin one}}$$

expansion in Legendre polynomials

$$\Gamma^{\omega(\xi)}_{a}(\boldsymbol{n},'\boldsymbol{n}) = \sum_{l} \Gamma^{\omega(\xi)}_{a;l}\, P_l(\boldsymbol{n}'\boldsymbol{n}).$$

Landau-Migdal constants:

empirical info., calculated from NN potential →

$$a^2\, N\Gamma^{\omega}_{0;l} = \bar{f}^{\omega}_{l} \qquad a^2\, N\Gamma^{\omega}_{1;l} = \bar{g}^{\omega}_{l}$$
$$a^2\, N\Gamma^{\xi}_{0;l} = \bar{f}^{\xi}_{l} \qquad a^2\, N\Gamma^{\xi}_{1;l} = \bar{g}^{\xi}_{l}$$

Fermi liquid approximation

integration over the internal lines is reduced to the Ferm surface

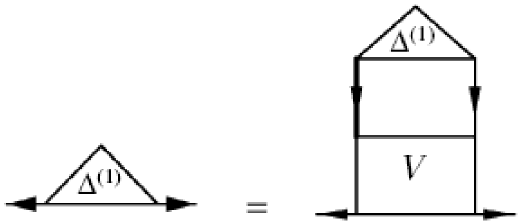
$$\int \frac{2\,\mathrm{d}^4 p}{(2\pi)^4\, i} \simeq \int \frac{\mathrm{d}\Omega_{\boldsymbol{p}}}{4\,\pi} \times \int \mathrm{d}\Phi_p$$

$$\int \mathrm{d}\Phi_p = \rho \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon}{2\,\pi\, i} \int_{-\infty}^{+\infty} \mathrm{d}\epsilon_p$$

can be taken explicitly for *T=0*

$$\rho = \frac{m^*\, p_{\mathrm{F}}}{\pi^2}$$

density states at Fermi surface



- **Gap equation** for $T \ll 2\,\Delta$

$$\Delta(\boldsymbol{n}) \;=\; -A_0\,\langle \Gamma_0^{\xi}(\boldsymbol{n},\boldsymbol{n}')\,\Delta(\boldsymbol{n}')\rangle_{\boldsymbol{n}'}$$

$$\langle \ldots \rangle_{\boldsymbol{n}} = \int \frac{\mathrm{d}\Omega_{\boldsymbol{n}}}{4\pi}\,(\ldots)$$

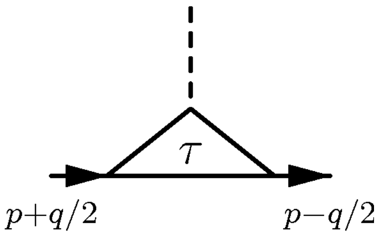
$$A_0 \;= \int \mathrm{d}\Phi_p\, G_{\mathrm{n.s.}}(p)\, G^h(p)\, \theta(\xi - \epsilon_p) \approx a^2\, \rho\, \ln(2\xi/\Delta)$$

Γ^{ξ} in an effective parameterization of a pairing gap

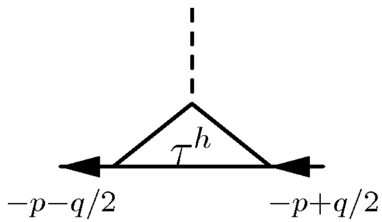
Superfluid system

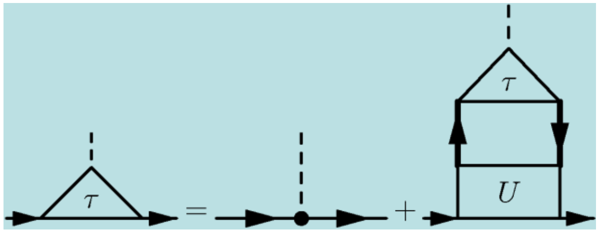
- ## Coupling to an external field

to particle

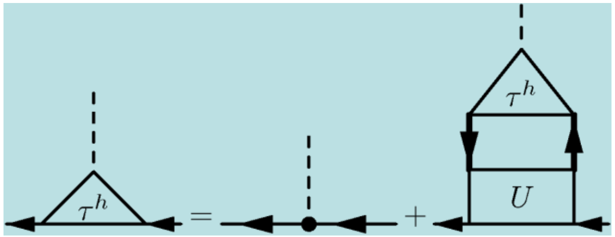


to hole

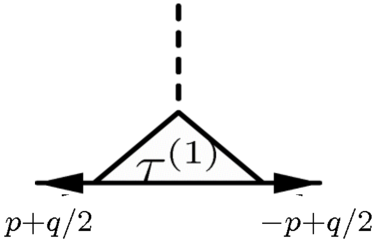




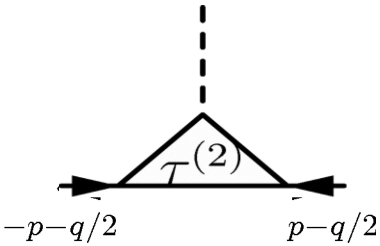
equations for vertices in normal system



In superfluid systems new type of couplings:

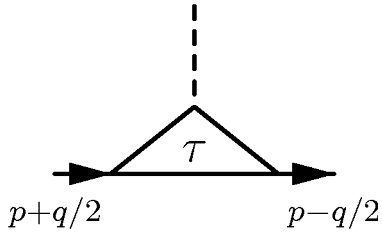


ext. field create 2 holes



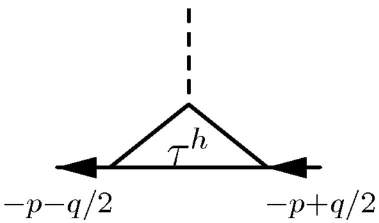
ext. field create 2 particles

- **"Dressed" vertices**

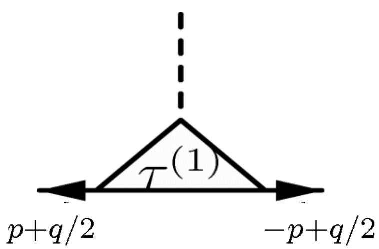


$$\tau_V = g_V\left(\tau_{V,0}\, l_0 - \boldsymbol{\tau}_{V,1}\, \boldsymbol{l}\right)$$
$$\tau_A = -g_A\left(\boldsymbol{\tau}_{A,1}\boldsymbol{\sigma}\, l_0 - \tau_{A,0}\, \boldsymbol{\sigma}\boldsymbol{l}\right)$$

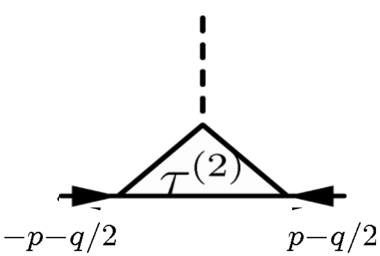
$$\hat{\tau}^h(\boldsymbol{n}, q) = [\hat{\tau}(-\boldsymbol{n}, q)]^{\mathrm{T}}$$
$$\sigma_2 \boldsymbol{\sigma}^{\mathrm{T}} \sigma_2 = -\boldsymbol{\sigma}$$



$$\tau_V^h = g_V\left(\tau_{V,0}\, l_0 + \boldsymbol{\tau}_{V,1}\, \boldsymbol{l}\right)$$
$$\tau_A^h = -g_A\left(-\boldsymbol{\tau}_{A,1}\boldsymbol{\sigma}^{\mathrm{T}}\, l_0 - \tau_{A,0}\, \boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{l}\right)$$



$$\tau_V^{(1)} = -\tau_V^{(2)} = -g_V\left(\widetilde{\tau}_{V,0}\, l_0 - \widetilde{\boldsymbol{\tau}}_{V,1}\, \boldsymbol{l}\right) i\,\sigma_2\,,$$



$$\tau_A^{(1)} = +g_A\left(\widetilde{\boldsymbol{\tau}}_{A,1}\, \boldsymbol{\sigma}\, l_0 - \widetilde{\tau}_{A,0}\, \boldsymbol{\sigma}\, \boldsymbol{l}\right) i\,\sigma_2\,,$$
$$\tau_A^{(2)} = -g_A\, i\,\sigma_2\left(\widetilde{\boldsymbol{\tau}}_{A,1}\, \boldsymbol{\sigma}\, l_0 - \widetilde{\tau}_{A,0}\, \boldsymbol{\sigma}\, \boldsymbol{l}\right)$$

Neutrino emissivity in superfluid Fermi liquid

Consider pure neutron matter at $T<<2\Delta$

$$q = q_1 + q_2$$

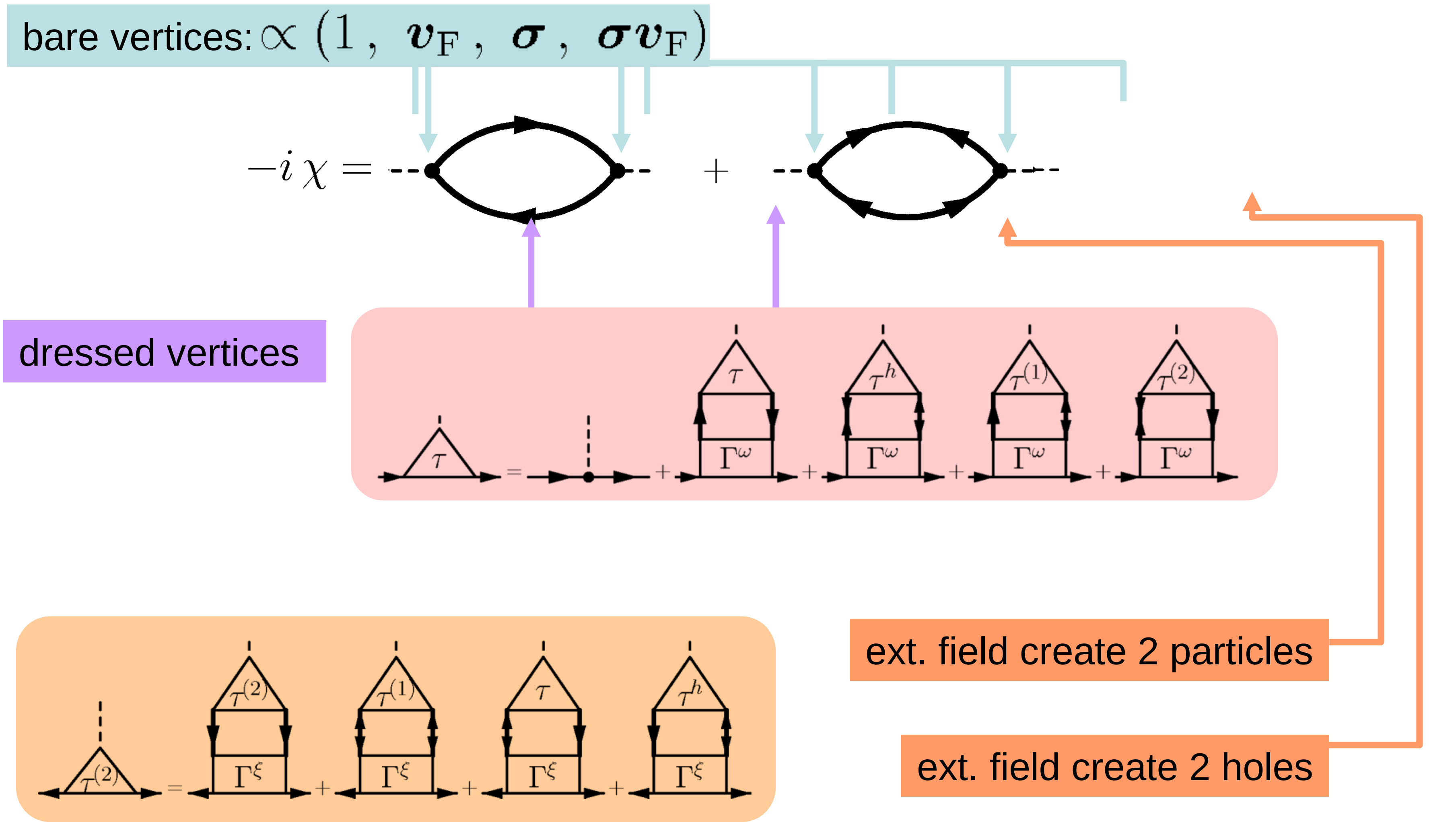
$$\varepsilon_{\nu\bar{\nu}} = \frac{G^2}{2} \int \frac{d^3q_1}{(2\pi)^3\, 2\,\omega_1} \frac{d^3q_2}{(2\pi)^3\, 2\,\omega_2}\, \omega\, n_{\rm bos}(\omega)\, 2\, \Im \sum_{\rm lept.\ spin} \chi(q)$$

produces leading exponential term $\propto e^{-2\Delta/T}$

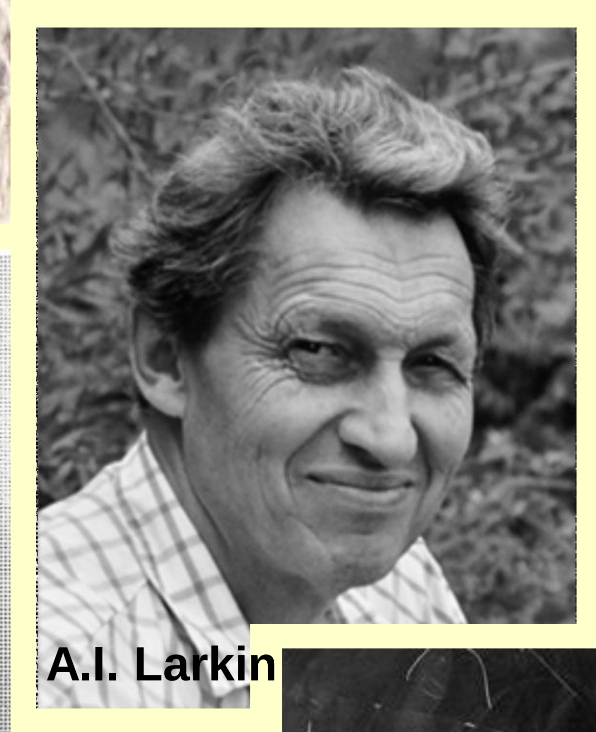
closed diagrams calculated with Green's functions for $T=0$

Superfluid Fermi liquid

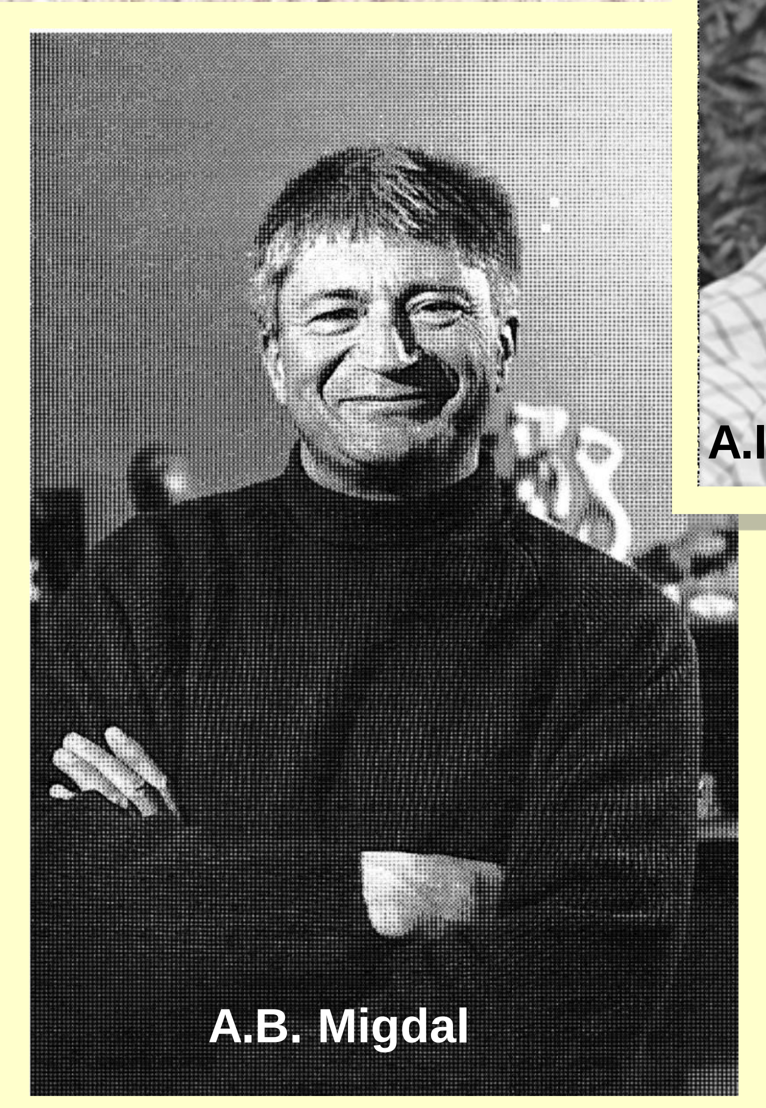
- susceptibility



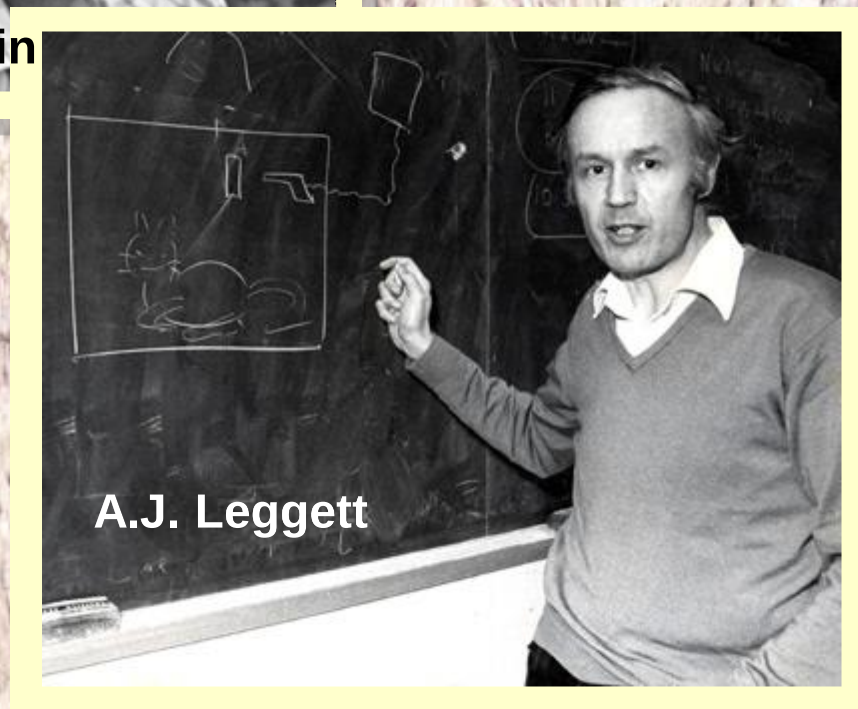
Without vertex corrections the current conservation is violated !



A.I. Larkin



A.B. Migdal



A.J. Leggett

Superfluid Fermi liquid

- **Larkin-Migdal equations**

$$\hat{\tau}^{(1)} = \left(\hat{\tau}^{(2)}\right)^{\dagger} = -\widetilde{\tau}$$

<u>bare vertices</u>

$$\tau^{\omega}_{V,0} = \frac{e_V}{a}\,\hat{1}$$

$$\boldsymbol{\tau}^{\omega}_{V,1} = \frac{e_V}{a}\,\boldsymbol{v}$$

$$\tau^{\omega}_{A,0}\,\boldsymbol{\sigma} = \frac{e_A}{a}\,\boldsymbol{\sigma}$$

$$(\boldsymbol{\tau}^{\omega}_{A,1}\boldsymbol{\sigma}) = \frac{e_A}{a}\,(\boldsymbol{\sigma}\boldsymbol{v})$$

→ Larkin-Migdal equations →

<u>dressed vertices</u>

$$\tau_{V,0}\hat{1}, \quad \widetilde{\tau}_{V,0}(i\,\sigma_2)$$

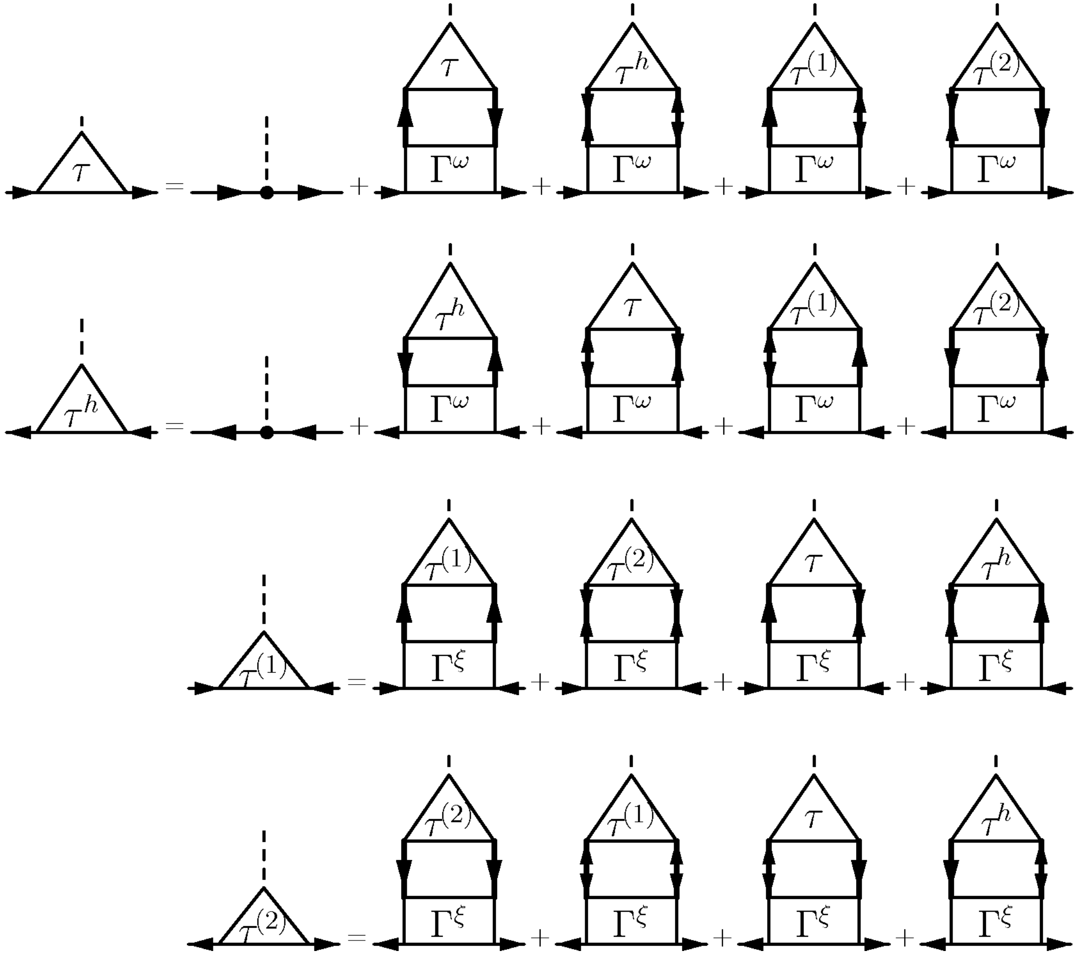
$$\boldsymbol{\tau}_{V,1}, \quad \widetilde{\boldsymbol{\tau}}_{V,1}(i\,\sigma_2)$$

$$\tau_{A,0}\,\boldsymbol{\sigma}, \quad \widetilde{\tau}_{A,0}\,\boldsymbol{\sigma}\,(i\,\sigma_2)$$

$$(\boldsymbol{\tau}_{A,1}\boldsymbol{\sigma}), \quad \widetilde{\boldsymbol{\tau}}_{A,1}\,\boldsymbol{\sigma}(i\,\sigma_2)$$

For T=0 and S-pairing written by Larkin, Migdal 1963 [Sov.Phys.JETP 17, 1146].
For finite T and S-pairing re-derived by Leggett 1965-6 (no Γ^{ω} terms!).
[Phys.Rev. 140, A1869 (1965), Phys.Rev. 147, 119 (1966)].
Applied to weak interactions in [Sedrakian, Muther, Schuck, PRC 76, 055805 (2007); EEK, Voskresensky, Phys.Rev.C 77, 065808 (2008)].
Equivalence of Leggett's and Larkin-Migdal's approaches for finite T
[EEK, Voskresensky, Phys.Rev.C 81, 065801 (2010)].
General structure for arbitrary pairing and non-equlibrium systems
[EEK, Voskresensky, Phys. Atom. Nucl. 74 ,1316 (2011)].

Coupling to an external field



Cannot be written in matrix form in Nambu Gor'kov space since $\Gamma^\omega \neq \Gamma^\xi$

- **equations for dressed vertices** for s-wave pairing

$$\tau_{a,0}(\boldsymbol{n},q)=\tau^{\omega}_{a,0}(\boldsymbol{n},q)+\big\langle\Gamma^{\omega}_{a}(\boldsymbol{n},\boldsymbol{n}')\,\big[L(\boldsymbol{n}',q;\hat{\mathcal{P}}_{a,0})\,\tau_{a,0}(\boldsymbol{n}',q)+M(\boldsymbol{n}',q)\,\tilde{\tau}_{a,0}(\boldsymbol{n}',q)\big]\big\rangle_{\boldsymbol{n}'}$$

$$\tilde{\tau}_{a,0}(\boldsymbol{n},q)=-\big\langle\Gamma^{\xi}_{a}(\boldsymbol{n},\boldsymbol{n}')\,\big[(N(\boldsymbol{n}',q)+A_0)\,\tilde{\tau}_{a,0}(\boldsymbol{n}',q)+O(\boldsymbol{n}',q;\hat{\mathcal{P}}_{a,0})\,\tau_{a,0}(\boldsymbol{n}',q)\big]\big\rangle_{\boldsymbol{n}'}$$

$$\boldsymbol{\tau}_{a,1}(\boldsymbol{n},q)=\boldsymbol{\tau}^{\omega}_{a,1}(\boldsymbol{n},q)+\big\langle\Gamma^{\omega}_{a}(\boldsymbol{n},\boldsymbol{n}')\,\big[L(\boldsymbol{n}',q;\hat{\mathcal{P}}_{a,1})\,\boldsymbol{\tau}_{a,1}(\boldsymbol{n}',q)+M(\boldsymbol{n}',q)\,\tilde{\boldsymbol{\tau}}_{a,1}(\boldsymbol{n}',q)\big]\big\rangle_{\boldsymbol{n}'}$$

$$\tilde{\boldsymbol{\tau}}_{a,1}(\boldsymbol{n},q)=-\big\langle\Gamma^{\xi}_{a}(\boldsymbol{n},\boldsymbol{n}')\,\big[(N(\boldsymbol{n}',q)+A_0)\,\tilde{\boldsymbol{\tau}}_{a,1}(\boldsymbol{n}',q)+O(\boldsymbol{n}',q;\hat{\mathcal{P}}_{a,1})\,\boldsymbol{\tau}_{a,1}(\boldsymbol{n}',q)\big]\big\rangle_{\boldsymbol{n}'}$$

a=V,A $\qquad \langle\ldots\rangle_{\boldsymbol{n}}=\int\frac{\mathrm{d}\Omega_{\boldsymbol{n}}}{4\pi}\,(\ldots)\,,$

- **loop functions** $\qquad G_{\pm}=G(p\pm q/2)$

$$L(\boldsymbol{n},q;P)=\int\mathrm{d}\Phi_p\Big[G_+G_- -(G_+G_-)^{\omega}-F_+F_-P\Big]$$ *p-h channel*

$$N(\boldsymbol{n},q)=\int\mathrm{d}\Phi_p\Big[G_+G^h_- -(G_pG^h_p)\theta(\xi-\epsilon_p)+F_+F_-\Big]$$ *h-h channel*

$$O(\boldsymbol{n},q;P)=-\int\mathrm{d}\Phi_p\Big[G_+F_- +F_+G^h_-P\Big] \qquad M(\boldsymbol{n},q)=\int\mathrm{d}\Phi_p\Big[G_+F_- -F_+G_-\Big]$$

properties of vertices under time inversion

$$P_{V,0}=-P_{V,1}=-P_{A,0}=P_{A,1}=1$$

- **loop functions**

The loop functions L,N,M,O can be expressed algebraically through one function g_T

$$L(\boldsymbol{n},q;P) = a^2\,\rho\left[\frac{\boldsymbol{q}\,\boldsymbol{v}}{\omega-\boldsymbol{q}\,\boldsymbol{v}}\,(1-g_T(\boldsymbol{n},q)) - g_T(\boldsymbol{n},q)\,(1+P)/2\right]$$

$$M(\boldsymbol{n},q) = -a^2\,\rho\,\frac{\omega+\boldsymbol{q}\,\boldsymbol{v}}{2\,\Delta}\,g_T(\boldsymbol{n},q)$$

$$N(\boldsymbol{n},q) = a^2\,\rho\,\frac{\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2}{4\,\Delta^2}\,g_T(\boldsymbol{n},q)$$

$$O(\boldsymbol{n},q;P) = a^2\,\rho\left[\frac{\omega+\boldsymbol{q}\,\boldsymbol{v}}{4\,\Delta}+\frac{\omega-\boldsymbol{q}\,\boldsymbol{v}}{4\,\Delta}\,P\right]g_T(\boldsymbol{n},q) \qquad \boldsymbol{v}=v_{\rm F}\,\boldsymbol{n}$$

$$g_T(\boldsymbol{n},q) = \Delta^2\int\limits_{-\infty}^{+\infty}{\rm d}\epsilon_p\left[\frac{(E_+-E_-)}{E_+\,E_-}\,\frac{(n(E_-)-n(E_+))}{\omega^2-(E_+-E_-)^2} - \frac{(E_++E_-)}{E_+\,E_-}\,\frac{(1-n(E_-)-n(E_+))}{\omega^2-(E_++E_-)^2}\right]$$

$$E_\pm = E_{p\pm q/2} \qquad n(x) = 1/(\exp(x/T)+1)$$

for T=0 $g_T(\boldsymbol{n},q) \longrightarrow g(z^2) = \int_{-1/2}^{+1/2}{\rm d}x[4\,z^2\,x^2 - z^2 + 1 + i\,0]^{-1} \qquad z^2 = \frac{\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2}{4\,\Delta^2} > 1$

For finite number of harmonics in Γs the LM equations can be solved algebraically

[EEK, Voskresensky, Phys.Rev.C 81, 065801 (2010)]. [EEK, Voskresensky, Phys.Rev.C 77, 065808 (2008)]

- **weak currents correlators**

$$\mathcal{L} = -\frac{G}{2\sqrt{2}}\,(\hat{V}^\mu(x) - \hat{A}^\mu(x))\,\hat{l}_\mu(x)$$

<u>lepton current</u>

$$l_\mu = \bar{u}(q_1)\,\gamma_\mu(1-\gamma_5)\,u(q_2)$$

<u>nucleon currents</u>

$$V_\mu = g_V\,(\bar{N}\,\gamma_\mu N) \approx g_V\,\chi_p^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_n(p)$$

$$A_\mu = g_A(\bar{N}\,\gamma_\mu\,\gamma_5 N) \approx g_A \chi^\dagger(p')\big((\boldsymbol{\sigma v}),\boldsymbol{\sigma}\big)\chi$$

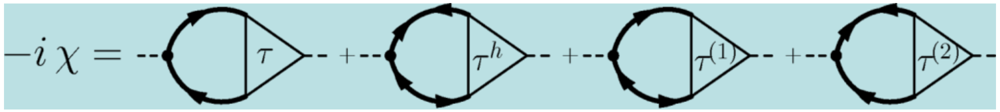
$$g_V = 1 \quad g_A \simeq 1.26$$

$$\chi_V(q) = -i\int \mathrm{d}^4x e^{-i\,q\cdot x}\langle N|(l\cdot\hat{V})(x)(l\cdot\hat{V})(0)|N\rangle$$

$$= g_V^2\,\big\langle\big(\tau^\omega_{V,0}\,l_0 - \boldsymbol{\tau}^{\,\omega}_{V,1}\,\boldsymbol{l}\,\big)\,\big(l_0\,\chi_{V,0}(\boldsymbol{n},q) - \boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\,\boldsymbol{l}\,\big)\big\rangle_{\boldsymbol{n}}$$

$$\chi_A(q) = -i\int \mathrm{d}^4x e^{-i\,q\cdot x}\langle N|(l\cdot\hat{A})(x)(l\cdot\hat{A})(0)|N\rangle$$

$$= g_A^2\,\big\langle\big(\boldsymbol{\tau}^{\,\omega}_{A,1}\,l_0 - \tau^\omega_{A,0}\,\boldsymbol{l}\,\big)\,\big(l_0\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q) - \chi_{A,0}(\boldsymbol{n},q)\boldsymbol{l}\,\big)\big\rangle_{\boldsymbol{n}}$$



a=V,A

$$\chi_{a,0}(\boldsymbol{n},q) = L(\boldsymbol{n},q;\hat{\mathcal{P}}_{a,0})\,\tau_{a,0}(\boldsymbol{n},q) + M(\boldsymbol{n},q)\,\widetilde{\tau}_{a,0}(\boldsymbol{n},q)$$

$$\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q) = L(\boldsymbol{n},q;\hat{\mathcal{P}}_{a,1})\,\boldsymbol{\tau}_{a,1}(\boldsymbol{n},q) + M(\boldsymbol{n},q)\,\widetilde{\boldsymbol{\tau}}_{a,1}(\boldsymbol{n},q)$$

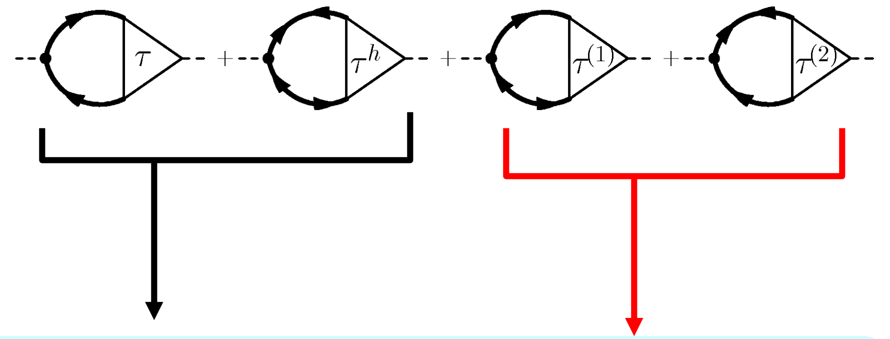
Neutrino emissivity can be expressed through χs

[EEK, Voskresensky, Phys.Rev.C 77, 065808 (2008)]

Solution for correlators

$$\chi_{a,0}(\boldsymbol{n},q) = \gamma_a(q;P_{a,0})\,\mathcal{L}(\boldsymbol{n},q;P_{a,0})$$

$-i\,\chi =$



$$\gamma_a^{-1}(q;P) = 1 - \Gamma_a^{\omega}\,\langle \mathcal{L}(\boldsymbol{n},q;P)\rangle_{\boldsymbol{n}}$$

normal Fermi liquid correlations

$$\Gamma_V^{\omega} = \Gamma_0^{\omega} \qquad\qquad \Gamma_A^{\omega} = \Gamma_1^{\omega}$$

$$\mathcal{L}(\boldsymbol{n},q;P) = L(\boldsymbol{n},q;P) - \frac{\langle O(\boldsymbol{n},q;P)\rangle_{\boldsymbol{n}}}{\langle N(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}\,M(\boldsymbol{n},q)$$

$$\langle L(\boldsymbol{n},q;+1)\rangle_{\boldsymbol{n}} \simeq -g(\omega^2/\Delta^2)\left(1 + O\big(v_{\rm F}^2\,\boldsymbol{q}^{\,2}/\omega^2\big)\right)$$

$$\langle \mathcal{L}(\boldsymbol{n},q;+1)\rangle_{\boldsymbol{n}} \simeq 0 + O\big(g\,v_{\rm F}^4\,\boldsymbol{q}^{\,4}/\omega^4\big)$$

$$\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q) = \gamma_a(q;P_{a,1})\,\boldsymbol{v}\,\mathcal{L}(\boldsymbol{n},q;P_{a,1}) + \delta\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q)$$

$$\delta\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q) = \frac{M(\boldsymbol{n},q)}{\langle N(\boldsymbol{n}',q)\rangle_{\boldsymbol{n}'}}\,\langle O(\boldsymbol{n}',q;P_{a,1})\,(\boldsymbol{v}-\boldsymbol{v}\,')\rangle_{\boldsymbol{n}'} + \mathcal{L}(\boldsymbol{n},q;P_{a,1})\,\gamma_a(q;P_{a,1})\,\Gamma_a^{\omega}\,\langle \widetilde{\mathcal{L}}(\boldsymbol{n}',q;P_{a,1})(\boldsymbol{v}\,'-\boldsymbol{v}\,)\rangle_{\boldsymbol{n}'}$$

$$P_{V,0} = -P_{V,1} = -P_{A,0} = P_{A,1} = 1$$

$$\widetilde{\mathcal{L}}(\boldsymbol{n},q;P) = L(\boldsymbol{n},q;P) - \frac{\langle M(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}{\langle N(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}\,O(\boldsymbol{n},q;P)$$

- **vector current concervation** $j_V^{\mu} \approx g_V\,(\boldsymbol{\tau}^{\omega}_{V,0}, \boldsymbol{\tau}^{\omega}_{V,1})$

$$\tau^{\omega}_{v,0} = \frac{e_V}{a}$$
$$\boldsymbol{\tau}^{\omega}_{V,1} = \frac{e_V}{a}\boldsymbol{v}_{\rm F}$$

effective charge

$e_V = 1$

$$\omega\,\tau^{\omega}_{V,0} - \boldsymbol{q}\,\boldsymbol{\tau}^{\omega}_{V,1} = G^{({\rm pole}),-1}_{\rm n.s.}(p+q/2) - G^{({\rm pole}),-1}_{\rm n.s.}(p-q/2)$$

Ward identity for non superfluid GF

In-medium current $\left(\chi_{V,0}(q,\boldsymbol{n}), \boldsymbol{\chi}_{V,1}(q,\boldsymbol{n})\right)$

$$\mathrm{Im}\langle \tau^{\omega}_{V}\,(\chi^{\nu}_{V}\,q_{\nu})\rangle_{\boldsymbol{n}} = O(f^{\omega}\,g\,\boldsymbol{q}^{\,6}\,v_{\rm F}^{6}/\omega^{6})$$

$$\mathrm{Re}\langle \tau^{\omega}_{V}\,(\chi^{\nu}_{V}\,q_{\nu})\rangle_{\boldsymbol{n}} + \text{[diagram]} = O(f^{\omega}\,g\,\boldsymbol{q}^{\,6}\,v_{\rm F}^{6}/\omega^{6})$$

gauging of nucleon kinetic energy

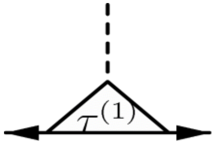
$$\frac{1}{2\,m^{*}}\psi^{\dagger}(\boldsymbol{\nabla} - \boldsymbol{V}\,)^{2}\psi$$

$$q \sim T, \qquad \omega \sim 2\,\Delta$$

$$\Gamma^{\omega,\xi}_{0} = f^{\omega,\xi}/(a^{2}\rho(n_0))$$

Anderson Bogoliubov mode

$$\Gamma_0^\omega = 0$$



anomalous vertices

$$\tilde{\tau}_{V,0} = -\frac{2\,\Delta\,\omega\langle g_T(\boldsymbol{n}\,,\omega,\boldsymbol{q}\,)\rangle_{\boldsymbol{n}}}{\langle[\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2]\,g_T(\boldsymbol{n}\,,\omega,\boldsymbol{q}\,)\rangle_{\boldsymbol{n}}}\,\tau^\omega_{V,0}$$

$$\tilde{\boldsymbol{\tau}}_{V,1} = -\frac{2\,\Delta\langle(\boldsymbol{q}\,\boldsymbol{v})\,g_T(\boldsymbol{n}\,,\omega,\boldsymbol{q}\,)(\boldsymbol{n}\,\boldsymbol{n}_q)\rangle_{\boldsymbol{n}}}{\langle[\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2]\,g_T(\boldsymbol{n}\,,\omega,\boldsymbol{q}\,)\rangle_{\boldsymbol{n}}}\,(\boldsymbol{n}\boldsymbol{\tau}^\omega_{V,1})\,\boldsymbol{n}_q$$

for $\omega, vq \ll 2\Delta$ $\quad \langle[\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2]\,g_T(\boldsymbol{n}\,,\omega,\boldsymbol{q}\,)\rangle_{\boldsymbol{n}} = \omega^2 - \frac{1}{3}v^2q^2 - i\omega\,\gamma(\omega,vq)$

$$\tilde{\tau}_{V,0} = -\frac{2\,\Delta\,\omega}{\omega^2-\frac{1}{3}v^2q^2-i\omega\,\gamma(\omega,vq)}\,\tau^\omega_{V,0}$$

$$(\boldsymbol{q}\tilde{\boldsymbol{\tau}}_{V,1}) = 2\Delta\tau^\omega_{V,0} - \frac{2\,\Delta\omega}{\omega^2-\frac{1}{3}v^2q^2-i\omega\,\gamma(\omega,vq)}\,\omega\tau^\omega_{V,0}$$

$$\gamma(\omega,vq) = \frac{\pi}{2}\frac{\Delta}{T}\frac{\omega^2}{v\,q}\,\theta(vq-\omega)\int_{\frac{v\,q}{\sqrt{v^2q^2-\omega^2}}}^{\infty}\frac{\mathrm{d}y\,e^{-\frac{\Delta}{T}y}}{(y^2-1)^2}$$

width of the AB mode

[Aronov Gurevich (1976)]

S-wave pairing with a residual spin interaction

- **Simple model**

Consider axial weak currents (dominate the emissivity)

$$j_A^\mu \approx g_A \left(\boldsymbol{\sigma} \boldsymbol{\tau}^\omega_{A,1}, \boldsymbol{\sigma}\, \tau^\omega_{A,0} \right) \quad \tau^\omega_{A,0} = \frac{e_A}{a} \quad \boldsymbol{\tau}^\omega_{A,1} = \frac{e_A}{a} \boldsymbol{v}_{\mathrm{F}}$$

particle-particle interaction:

s-wave paring: $a^2 \rho\, \Gamma^\xi_0 = f^\xi_0 < 0 \qquad -1/f^\xi_0 = \ln(2\,\epsilon_{\mathrm{F}}/\Delta)$ **spin zero channel**

next possible harmonics f^ξ_2 , which is expected to be much smaller!

$$a^2 \rho \Gamma^\xi_1(\boldsymbol{n}, \boldsymbol{n}') = g^\xi_1(\boldsymbol{n}\, \boldsymbol{n}')$$ **spin one channel**

next possible harmonics g^ξ_3 , which is expected to be much smaller!

particle-hole interaction: Γ^ω_0 does not contribute to the axial channel

$$a^2 \rho \Gamma^\omega_1 = g^\omega_0 + \boxed{g^\omega_1 (\boldsymbol{n}' \boldsymbol{n})}$$ ← drop here for simplicity ☺

g^ω_0 → corrections $\sim v_{\mathrm{F}}^2$ → neglect

$$\rho = \frac{m^* p_{\mathrm{F}}}{\pi^2}$$ density states at Fermi surface

- solution of Larkin-Migdal equations

Normal vertices remains non-renormalized

$$\boldsymbol{\tau}_{A,1} = \boldsymbol{\tau}^{\omega}_{A,1} \qquad \tau_{A,0} = \tau^{\omega}_{A,0}$$

solutions for anomalous vertices: $\widetilde{\tau}_{A,0}(\boldsymbol{n}, q) = -\frac{(\boldsymbol{v}\,\boldsymbol{q})}{2\,\Delta}\,\tau^{\omega}_{A,0}\,\gamma^{\xi}_{\parallel}(q)\langle g_T(\boldsymbol{n}')\,(\boldsymbol{n}_q\,\boldsymbol{n}')^2\rangle_{\boldsymbol{n}'}$

$$\begin{aligned}\widetilde{\boldsymbol{\tau}}_{A,1}(\boldsymbol{n}, q) &= -\frac{\omega}{2\,\Delta}\,\tau^{\omega}_{A,1}\Big[\gamma^{\xi}_{\perp}(q)\langle g_T(\boldsymbol{n}')\frac{1}{2}[1-(\boldsymbol{n}_q\,\boldsymbol{n}')^2]\rangle_{\boldsymbol{n}'}\,\underbrace{(\boldsymbol{n}-\boldsymbol{n}_q\,(\boldsymbol{n}\,\boldsymbol{n}_q))}_{\perp\,\boldsymbol{q}}\\ &+\frac{\gamma^{\xi}_{\parallel}(q)\,\gamma^{\xi}_{\perp}(q)}{g^{\xi}_1}\langle g_T(\boldsymbol{n}')\frac{1}{2}[1-(\boldsymbol{n}_q\,\boldsymbol{n}')^2]\rangle_{\boldsymbol{n}'}\,\underbrace{\boldsymbol{n}_q\,(\boldsymbol{n}\boldsymbol{n}_q)}_{\parallel\,\boldsymbol{q}}\\ &-\gamma^{\xi}_{\parallel}(q)\langle g_T(\boldsymbol{n}')\,(\frac{3}{2}(\boldsymbol{n}'\boldsymbol{n}_q)^2-\frac{1}{2})\rangle_{\boldsymbol{n}'}\underbrace{\boldsymbol{n}_q\,(\boldsymbol{n}\boldsymbol{n}_q)}_{\parallel\,\boldsymbol{q}}\Big]\end{aligned}$$

$$[\gamma^{\xi}_{\perp}(q)]^{-1} = \frac{1}{3}\,C + \Big\langle\frac{\omega^2-(\boldsymbol{v}\,\boldsymbol{q})^2}{4\,\Delta^2}g_T(\boldsymbol{n})\frac{1}{2}[1-(\boldsymbol{n}'\boldsymbol{n}_q)^2]\Big\rangle_{\boldsymbol{n}'}$$

$$[\gamma^{\xi}_{\parallel}(q)]^{-1} = \frac{1}{3}\,C + \Big\langle\frac{\omega^2-(\boldsymbol{v}\,\boldsymbol{q})^2}{4\,\Delta^2}g_T(\boldsymbol{n})(\boldsymbol{n}'\boldsymbol{n}_q)^2\Big\rangle_{\boldsymbol{n}'}$$

$$C = \frac{3}{g^{\xi}_1} + \frac{1}{|f^{\xi}_0|}$$

Exciton-like collective modes

$$[\gamma_\perp^\xi(q)]^{-1} = \frac{1}{3}C + \langle \frac{\omega^2 - (\boldsymbol{v}\,\boldsymbol{q})^2}{4\,\Delta^2} g_T(\boldsymbol{n}) \frac{1}{2}[1 - (\boldsymbol{n}'\boldsymbol{n}_q)^2] \rangle_{\boldsymbol{n}'}$$

$$[\gamma_\parallel^\xi(q)]^{-1} = \frac{1}{3}C + \langle \frac{\omega^2 - (\boldsymbol{v}\,\boldsymbol{q})^2}{4\,\Delta^2} g_T(\boldsymbol{n}) (\boldsymbol{n}'\boldsymbol{n}_q)^2 \rangle_{\boldsymbol{n}'}$$

$$C = \frac{3}{g_1^\xi} + \frac{1}{|f_0^\xi|}$$

$[\gamma_\perp^\xi(q)]^{-1} = 0$ → transversal

$[\gamma_\parallel^\xi(q)]^{-1} = 0$ → longitudinal

exciton-like modes

For $\boldsymbol{q} = 0$: two modes are degenerated, since $\gamma_\perp^\xi(\omega, 0) = \gamma_\parallel^\xi(0)$

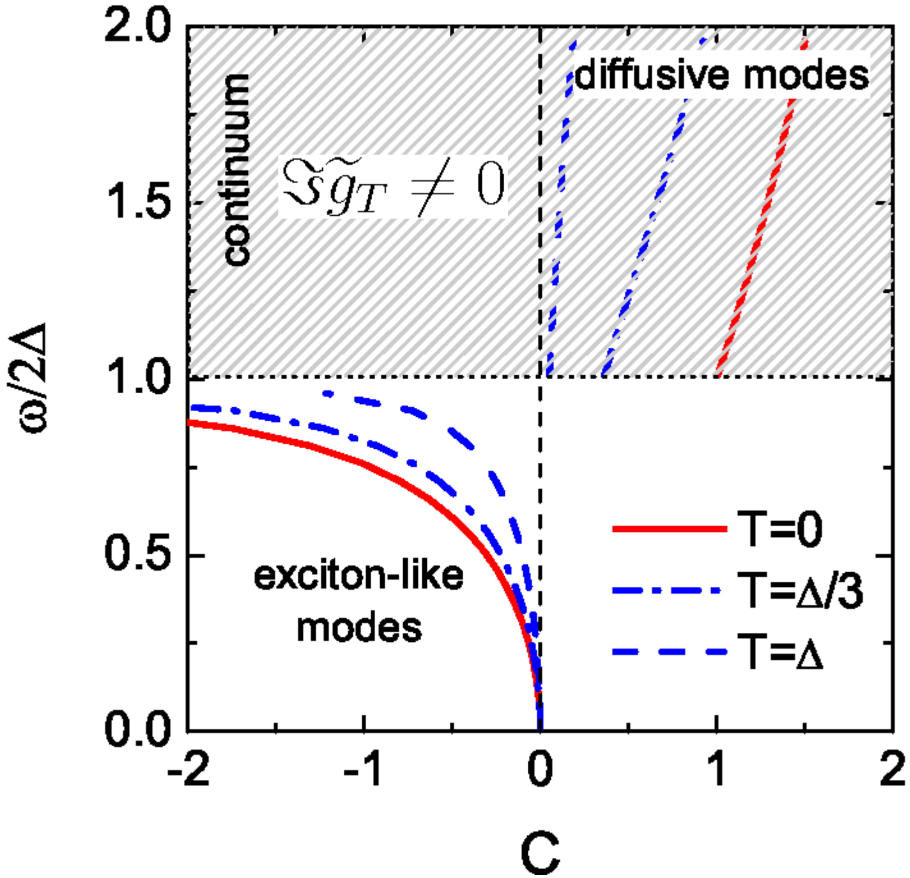
Collective mode spectrum

$$\underbrace{\frac{3}{g_1^\xi} + \frac{1}{|f_0^\xi|}}_{C} + z^2\,\Re g_T(z) = 0$$

$$z = \omega/2\Delta$$

$$\widetilde{g}_T(z) = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dy}{\sqrt{y^2+1}}\,\frac{\mathrm{th}(\sqrt{y^2+1}\Delta/2T)}{y^2+1-z^2+i\,0}$$

$$\widetilde{g}_0(z) = \frac{\arctan\left(\frac{z}{\sqrt{1-z^2}}\right)}{z\sqrt{1-z^2}} \qquad \widetilde{g}_0(0) = 1$$



C<0 exciton modes for ω<2Δ

$$\frac{\omega_0}{2\Delta} = \xi\sqrt{\frac{\frac{|C|}{g_T(0)} + \frac{4\,C^2}{\pi^2\,\mathrm{th}(\Delta/4T)}}{1 + \frac{|C|}{g_T(0)} + \frac{4\,C^2}{\pi^2\,\mathrm{th}(\Delta/4T)}}}$$

$$\xi \approx 1$$

$C>g_T(0)$ diffusive modes for ω>2Δ

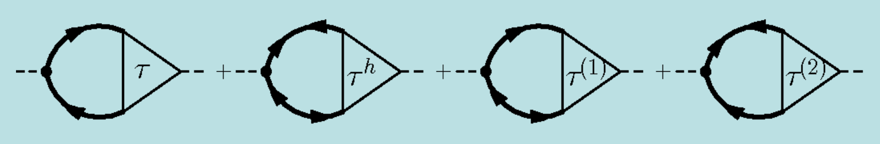
Pair breaking and formation (PBF) reactions

on the axial current

$$\varepsilon_{\nu\bar{\nu},A} = \frac{G^2}{48\,\pi^4}\int_0^\infty \mathrm{d}\omega \frac{\omega\,\Im\overline{K}_A(\omega)}{e^{\omega/T}-1} \quad \text{with} \quad \overline{K}_A(\omega) = \int_0^\omega \mathrm{d}|\boldsymbol{q}\,|\boldsymbol{q}^{\,2}\,K_A(\omega,\boldsymbol{q}\,)$$

- **neutrino source function**

$$\frac{K_A(q)}{g_A^2} = (3\,\omega^2 - 2\,\boldsymbol{q}^{\,2})\,\langle\tau^\omega_{A,0}\,\chi_{A,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + \boldsymbol{q}^{\,2}\langle\boldsymbol{\tau}^\omega_{A,1}\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}$$
$$- \quad \omega\,\langle\tau^\omega_{A,0}\,\boldsymbol{q}\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} - \omega\,\langle(\boldsymbol{\tau}^\omega_{A,1}\,\boldsymbol{q}\,)\,\chi_{A,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}$$



[EEK, Voskresensky, Phys.Rev.C 77, 065808 (2008)]

- **neutrino source function**

$$\frac{K_A(q)}{e_A^2\, g_A^2\, \rho} = \left\langle \frac{g_T(\boldsymbol{n}, (\boldsymbol{v}\boldsymbol{q}), \boldsymbol{q})}{\omega - \boldsymbol{v}\,\boldsymbol{q} - i\,0} \left[(\boldsymbol{v}\,\boldsymbol{q})\,(\boldsymbol{q}^2 v_{\rm F}^2 + (3\,\omega^2 - 2\,\boldsymbol{q}^2)) - 2\omega(\boldsymbol{v}\,\boldsymbol{q})^2 \right] \right\rangle_{\boldsymbol{n}}$$

sound! Imaginary part only for $\omega < q\, v_{\rm F}$; do not contribute to PFB

$$- \left\langle \frac{g_T(\boldsymbol{n}, \omega, \boldsymbol{q})}{\omega - \boldsymbol{v}\,\boldsymbol{q}} \left[\boldsymbol{q}^2 v_{\rm F}^2\, \omega + (\boldsymbol{v}\,\boldsymbol{q})\,((3\,\omega^2 - 2\,\boldsymbol{q}^2) - \omega^2 - \omega\,(\boldsymbol{v}\,\boldsymbol{q})) \right] \right\rangle_{\boldsymbol{n}}$$

the standard term [Yakovlev,Kaminker Levenfish AA343,650;
present form from EEK,Voskresenksy PRC77, 065808]

$$+ \boldsymbol{q}^2 \frac{v_{\rm F}^2}{2\,\Delta^2} \Big[\omega^2\, \gamma_\perp^\xi(q)\, \langle g_T(\boldsymbol{n}', \omega, \boldsymbol{q}) \tfrac{1}{2} [1 - (\boldsymbol{n}_q\, \boldsymbol{n}')^2] \rangle_{\boldsymbol{n}'}^2$$

$$+ (\omega^2 - \boldsymbol{q}^2)\, \gamma_\parallel^\xi(q)\, \langle g_T(\boldsymbol{n}', \omega, \boldsymbol{q}) (\boldsymbol{n}_q\, \boldsymbol{n}')^2 \rangle_{\boldsymbol{n}'}^2 \Big]$$

new terms induced by the spin interaction g_1^ξ in the particle-particle channel

All three terms contribute to neutrino scattering processes

- **neutrino source function for $v_{\rm F} << 1$**

We keep only the leading term.

$$\overline{K}_A(\omega) \simeq -\frac{6}{35}\, g_A^2\, e_A^2\, \rho\, v_{\rm F}^2\, \frac{C\,\omega^5\,\tilde{g}_T(\omega)}{C + \frac{\omega^2}{4\,\Delta^2}\,\tilde{g}_T(\omega)}$$

To calculate imaginary part we use

$$\Im\tilde{g}_T(\omega) = -\frac{2\pi\Delta^2\theta(\omega - 2\,\Delta)}{\omega\sqrt{\omega^2 - 4\,\Delta^2}}\,\tanh\left(\frac{\omega}{4T}\right) - 0$$

and find two contributions:

$$\Im\overline{K}_A(\omega) \simeq \frac{96}{35}\,\pi\, g_A^{*2}\,\rho\, v_{\rm F}^2\,\Delta^5\left[F\Big(\frac{\omega}{2\Delta}, C\Big)\theta(\omega - 2\,\Delta) + C^2\frac{\omega_0^3}{4\Delta^3}\delta\Big(C + \frac{\omega^2}{4\,\Delta^2}\,\Re\tilde{g}_T(\omega)\Big)\right]$$

continuum

exciton mode for C<0

with the modified "continuum functions":

$$F(y, C) = \frac{C^2\,y^4\sqrt{y^2 - 1}\,\tanh(\frac{y\Delta}{2T})}{(y^2 - 1)\big(C + y^2{\rm Re}\,\tilde{g}_T(y)\big)^2 + \frac{\pi^2}{4}y^2\tanh^2(\frac{y\Delta}{2T})}$$

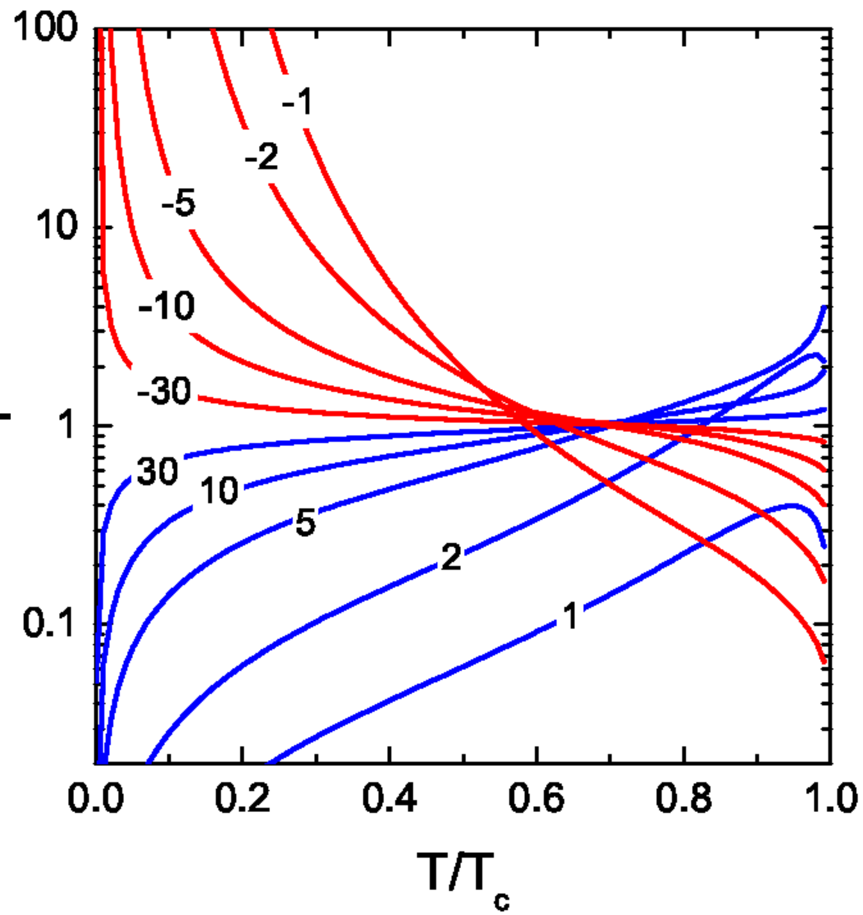
$$g_1^\xi \to 0,\ C \to \infty\,, \qquad F(y, C) \to F_0(y) = \frac{y^4\,\tanh\left(\frac{y\,\Delta}{2\,T}\right)}{\sqrt{y^2 - 1}}$$

the standard result

- neutrino emissivity in pair-formation breaking processes

$$\Delta(T) \simeq 3.1\, T_c\, (1 - T/T_c)^{1/2}$$

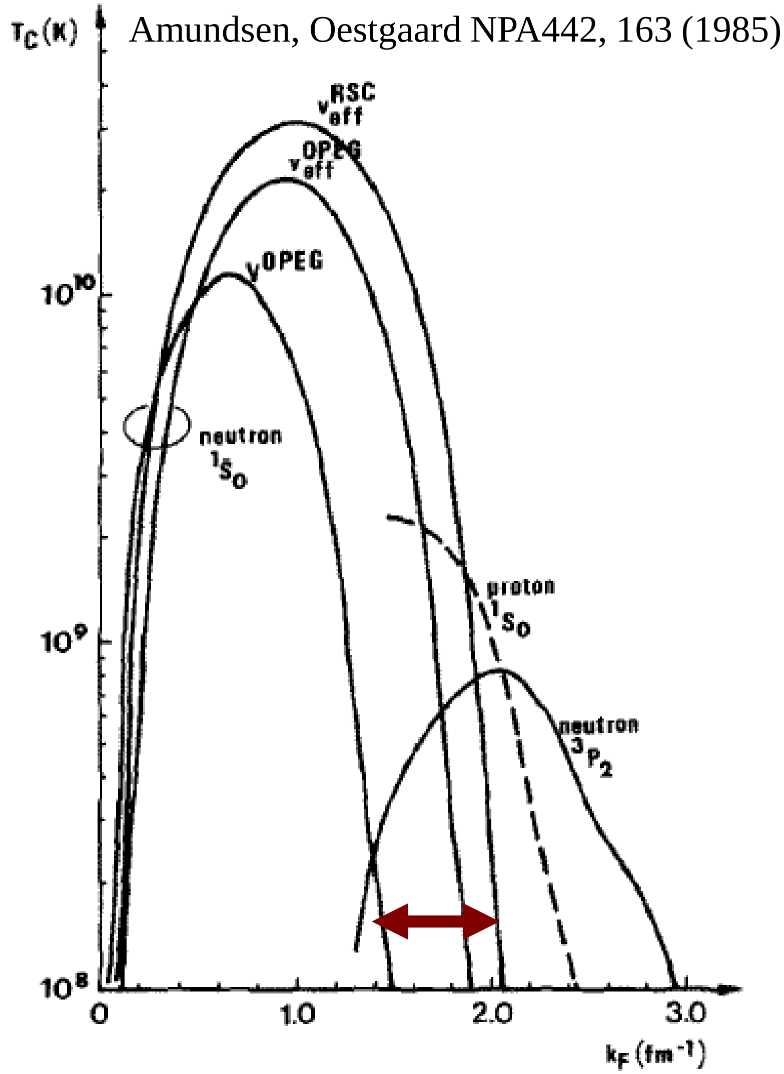
$$\frac{\text{new emissivity}}{\text{standard emissivity}}$$

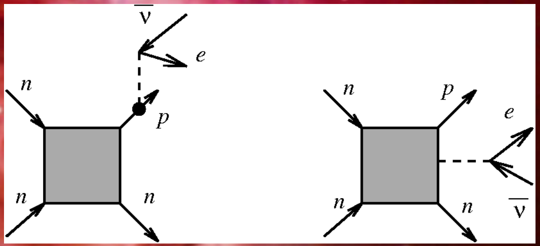


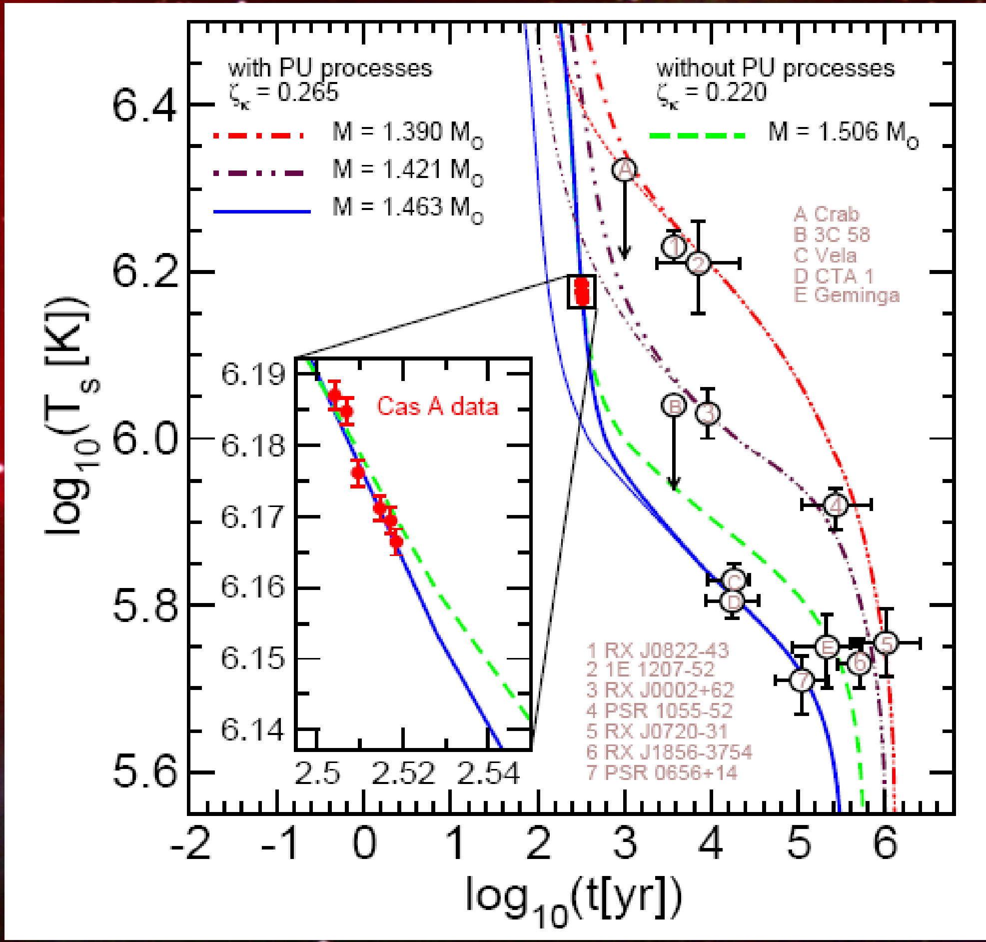
We could be not aware about a factor of ~100.

The parameter g_1^ξ can be negative and $C < 0$

✔ pairing gaps

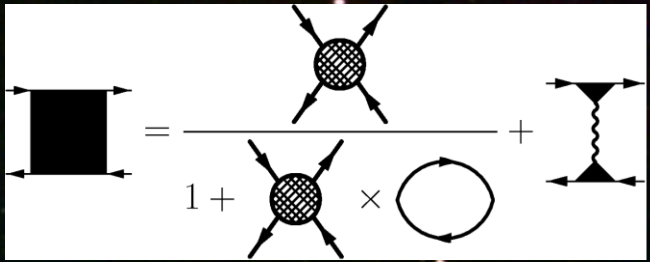






Blaschke et al PRC 88, 065805(2013)

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$



SOVIET PHYSICS JETP VOLUME 17, NUMBER 5 NOVEMBER, 1963

Conclusions ….

THEORY OF SUPERFLUID FERMI LIQUID. APPLICATION TO THE NUCLEUS

A. I. LARKIN and A. B. MIGDAL

Fermi liquid approach is an effective low-energy theory for strongly interacting fermion systems

Interactions in ph and pp channels can very different

Fermi liquid approach can be applied to superfluid systems

It respects the current conservation

I. INTRODUCTION

IN all real many-particle systems the interaction is not small, and therefore in the derivation of quantitative relations one cannot proceed, as is often done, by combining some part of the diagrams of perturbation theory.

superconductors, the Debye temperature). Here one must introduce, in addition to Γ^{ω}, one other function of the angles between the momenta of the quasi-particles, Γ^{ξ}; the spherical harmonic of this function is connected with the width of the energy gap. It is natural to expect that the functions Γ^{ω} and Γ^{ξ} depending on the angles between