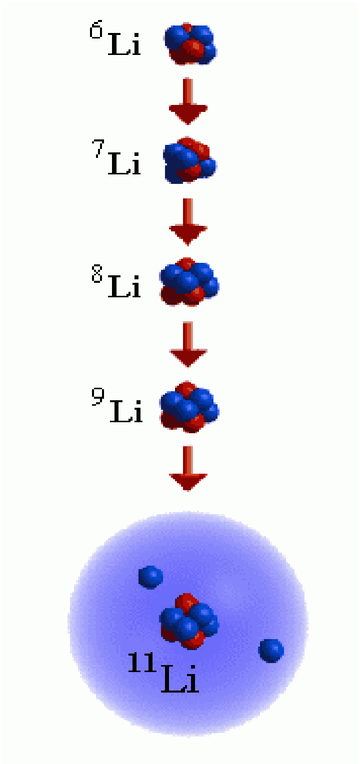
Helmholtz International Summer School

"Nuclear Theory and Astrophysical Applications"

Dubna, Russia, July 10 – 22, 2017



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HALO NUCLEI

Halo is a widespread name for specific phenomena in different fields of physics

in optics:

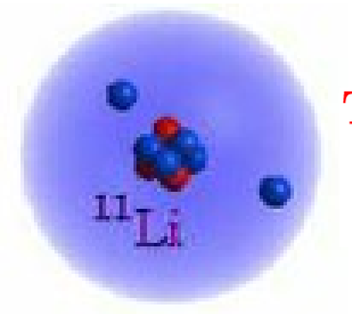
Halo is the name for a family of optical phenomena produced by light interacting with ice crystals suspended in the atmosphere. Halos can have many forms, ranging from colored or white rings to arcs and spots in the sky.

in astrophysics:

Galactic halo is a region of scattered stars that surrounds spiral galaxies.

A dark matter halo is a hypothetical component of a galaxy that envelops the galactic disc and extends well beyond the edge of the visible galaxy.

in nuclear structure:



The new structural dripline phenomenon with clusterization into an ordinary core nucleus and a veil of halo nucleons forming very dilute neutron matter

STRUCTURE

ATOMIC	NUCLEAR
electromagnetic forces between negatively charged electrons and positively charged nucleus	Interactions between neutrons and protons via: • strong forces • electromagnetic forces • weak forces

SPACE SCALES (1 fm = 10^{-13} cm) :

ATOMIC	NUCLEAR
$\sim$ (0.3 - 2.3)$\cdot 10^5$ fm	$\sim$ (2 - 6) fm

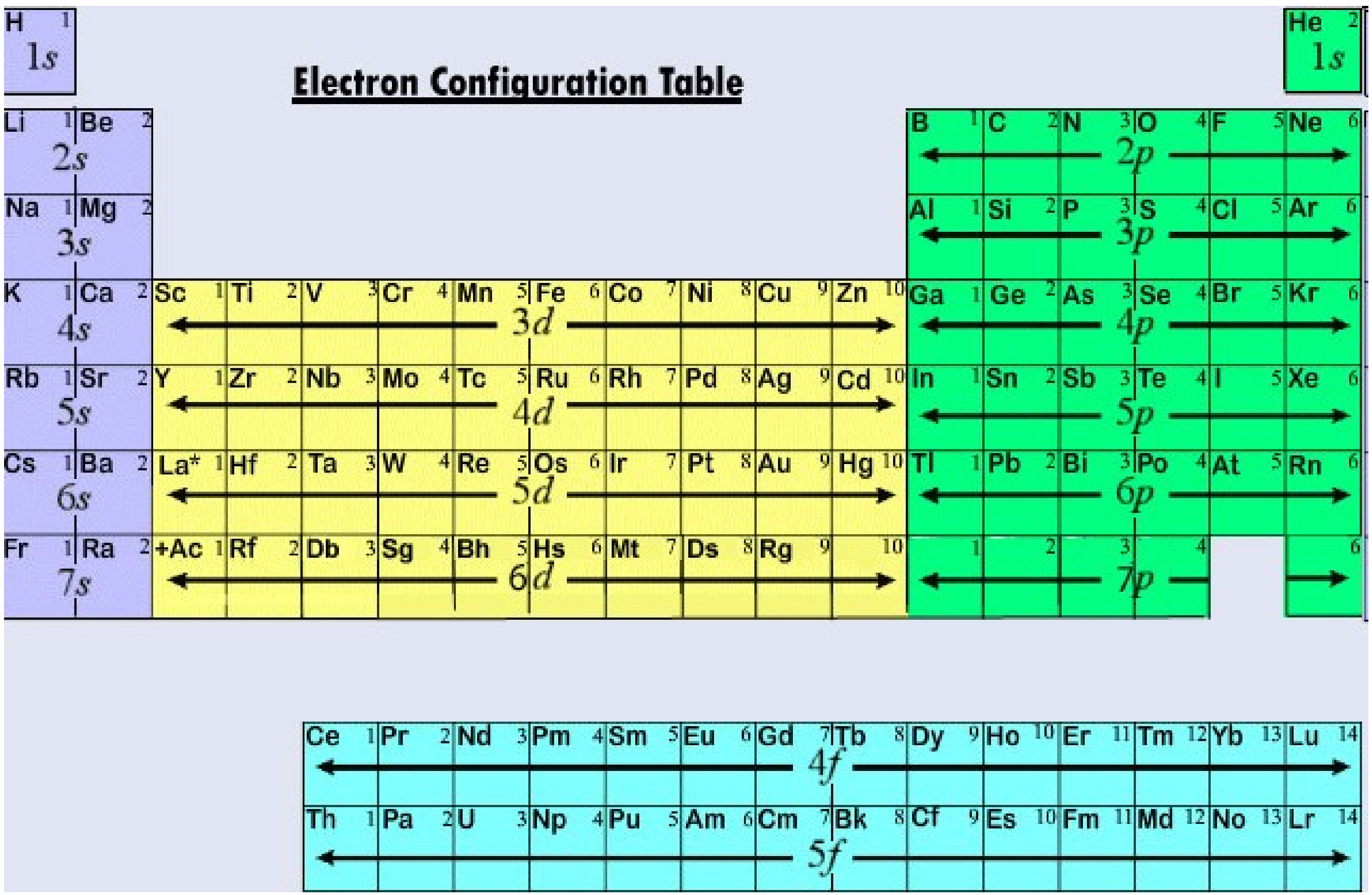
ENERGY SCALES :

ATOMIC	NUCLEAR
$\sim$ electronvolts	$\sim$ MeV = 10^6 electronvolts
(Ionization energy for ^{1}H = 13.6 eV)	(Binding energy for deuteron = 2.2 MeV)

Atom has the very pronounced shell structure.

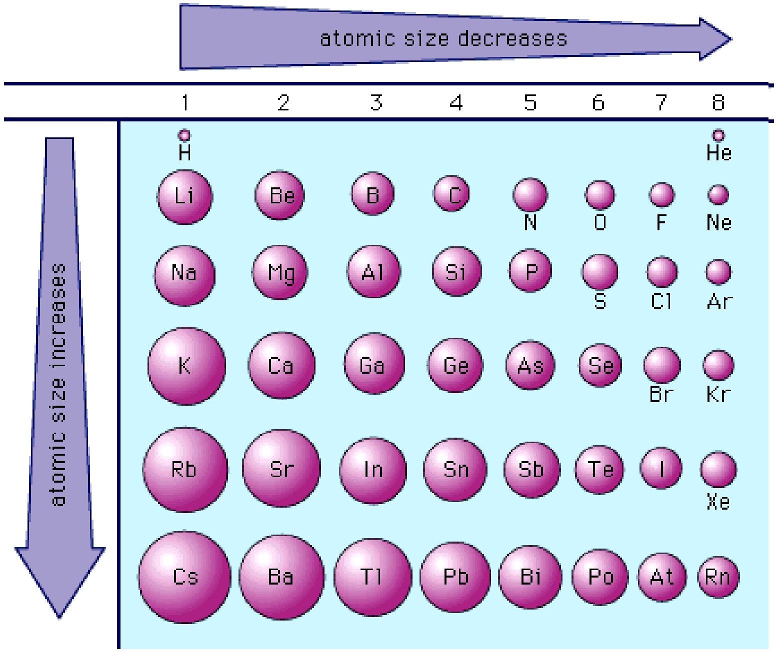
Noble gases correspond to the magic numbers : 2, 10, 18, 36, 54, 86

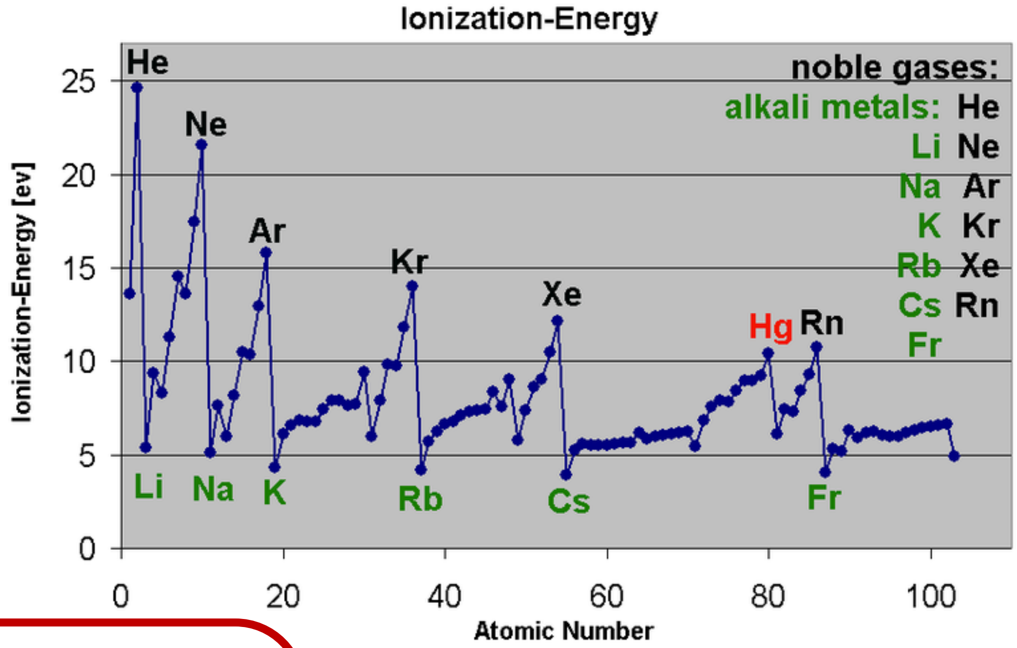
2	8		8		18			18			32				32		
$1s^2$	$2s^2$	$2p^6$	$3s^2$	$3p^6$	$4s^2$	$3d^{10}$	$4p^6$	$5s^2$	$4d^{10}$	$5p^6$	$6s^2$	$4f^{14}$	$5d^{10}$	$6p^6$	$7s^2$	$5f^{14}$	$6d^{10}$ $7p^6$



ATOMIC SIZES

IONIZATION ENERGIES



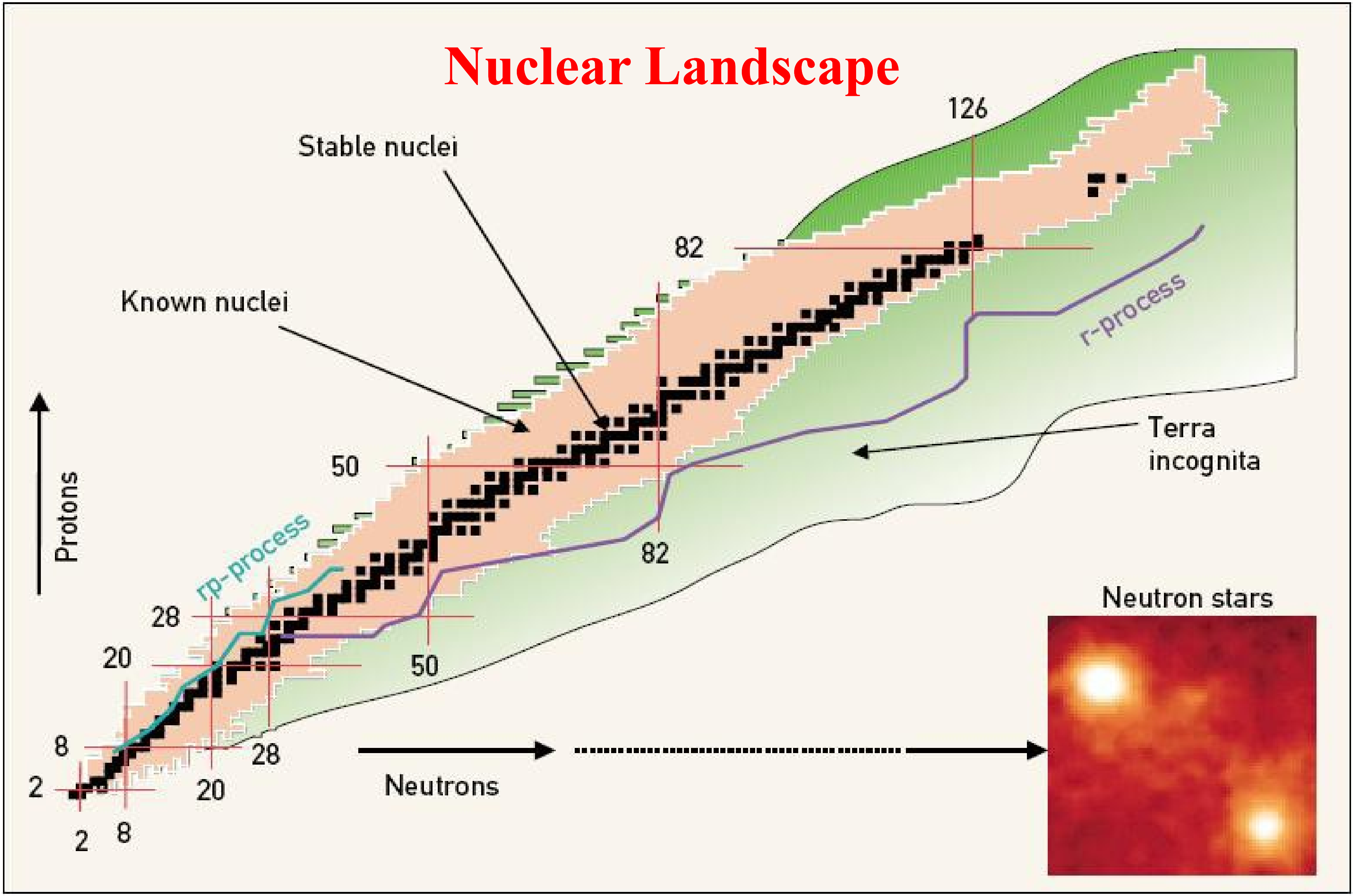


Atoms lack a well-defined outer boundary.
The radius varies with:

- the location of an atom on the atomic chart
- the type of chemical bond
- the number of neighboring atoms

Despite conceptual difficulties atomic radii vary in a predictable and explicable manner across the periodic table

$$H_0 = \sum_{i=1}^{Z} \left(T_i - \frac{Ze^2}{r_i} \right) + \sum_{i<j}^{Z} \frac{e^2}{r_{ij}}$$



Presently $\sim$ 3600 nuclei have been observed,
less than 300 nuclei are stable (with a lifetime greater than 10^9 years)

NUCLEAR STRUCTURE NEAR THE VALLEY OF STABILITY

exhibit similar binding for neutrons and protons
density and diffuseness of the surface are nearly constant

the resulting shell structure is well established

M. Goeppert-Mayer, J.H.D. Jensen, nobel prize in 1963
"for their discoveries concerning nuclear shell structure"

Nobel Prize Medal

magic numbers
(2, 8, 20, 28, 50, 82, 126) are the same for neutrons and protons

stable double-magic nuclei: ${}^{4}_{2}He_{2}$, ${}^{16}_{8}O_{8}$, ${}^{40}_{20}Ca_{20}$, ${}^{48}_{20}Ca_{28}$, ${}^{208}_{82}Pb_{126}$

nuclear potential is well parametrized

pronounced shell closures define the effective degrees of freedom needed for a quantitative understanding of atomic nuclei

A.N. Bohr, B.R. Mottelson, L.J. Raynwater , nobel prize in 1975
"for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection"

Nuclei with magic numbers of neutrons and protons have

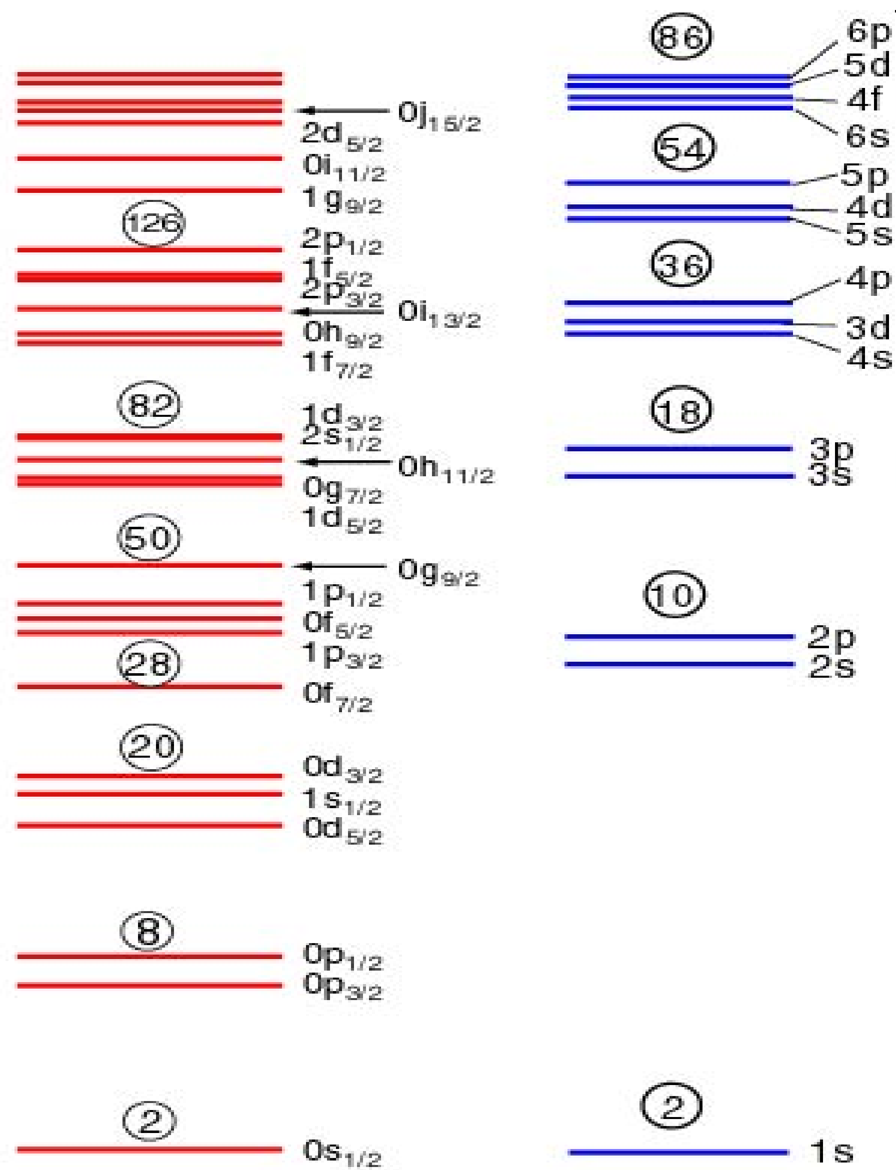
- large binding energies
- high natural abundances
- high energy of first excited state
- ...

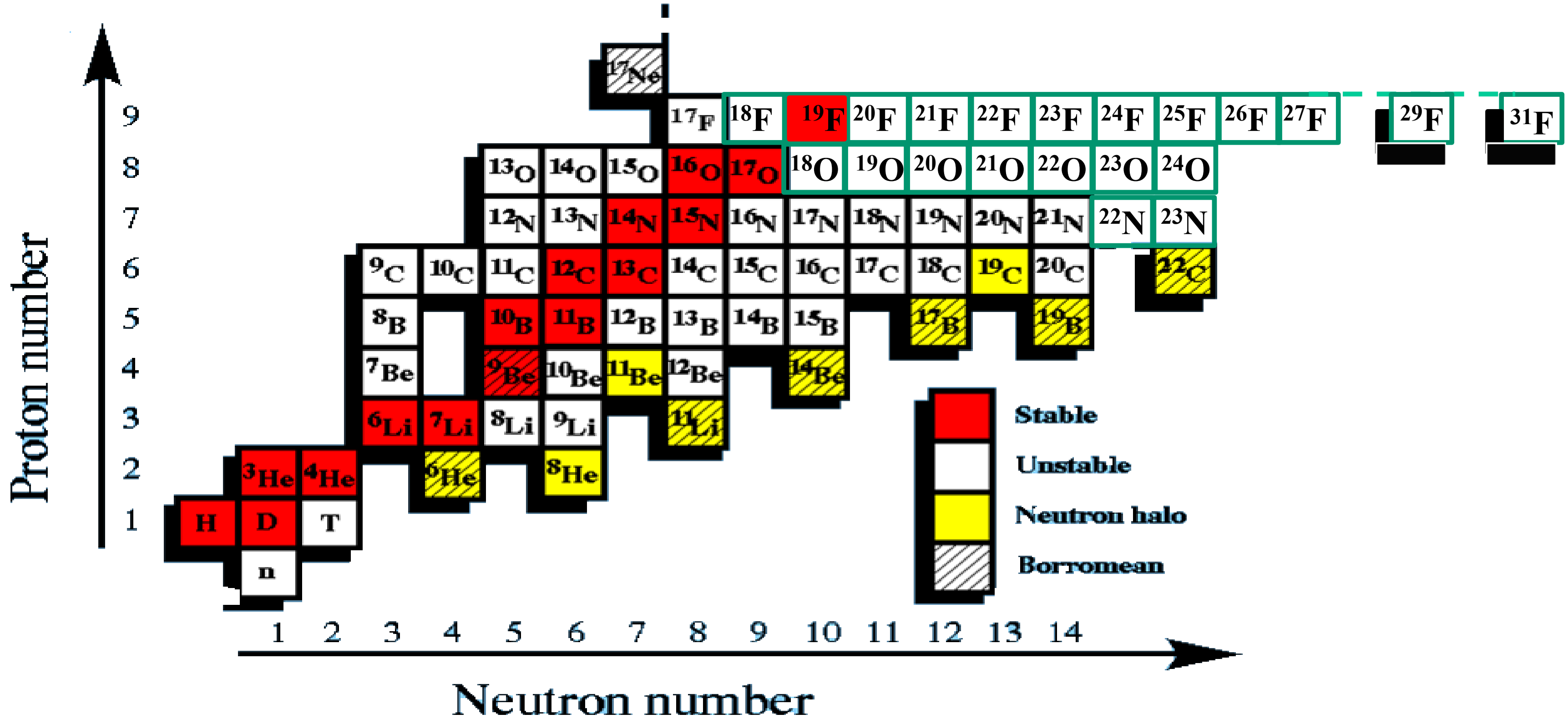
Shell-model potential

$$V(r) = V_c(r) + V_{ls}(L \cdot S)$$

NUCLEUS:
proton/neutron shells

ATOM:
electron shells





Chains of the lightest isotopes (He, Li, Be, B, . . .)
end up with two neutron halo nuclei

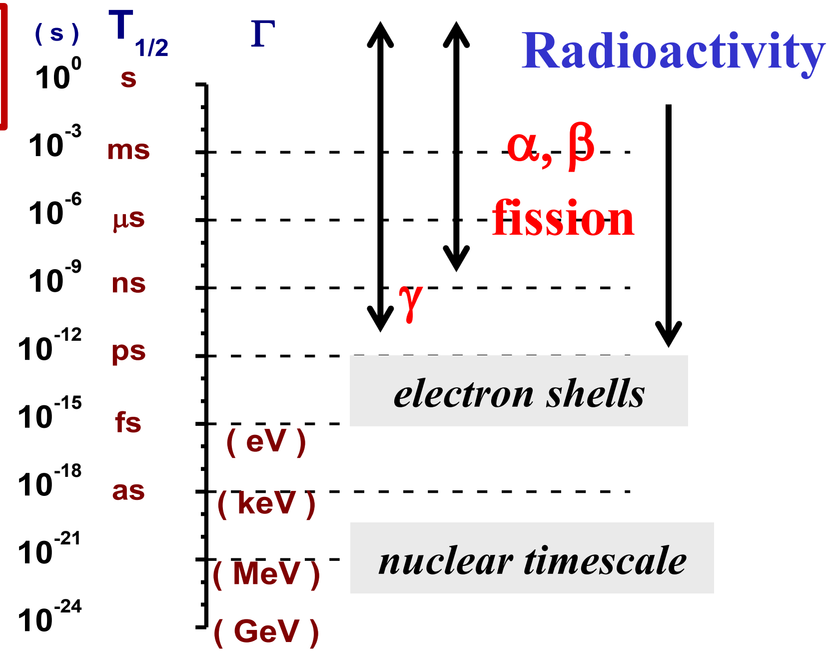
Two neutron halo nuclei (^{6}He , ^{11}Li , ^{14}Be , . . .) are Borromean
systems and break into three fragments

One neutron halo nuclei (^{11}Be , ^{19}C , . . .) break into two fragments

Borromean system: three fragments are bound together
but two-body subsystems are all unbound

DEFINITION OF A NUCLEUS

many combinations of neutrons and protons can make up a nucleus of a given mass



What system of nucleons can we call a nucleus ?

A possible limit could be set by :

- the definition of radioactivity

  lifetimes longer than 10^{-12} s → a lower limit for the process to be called radioactivity

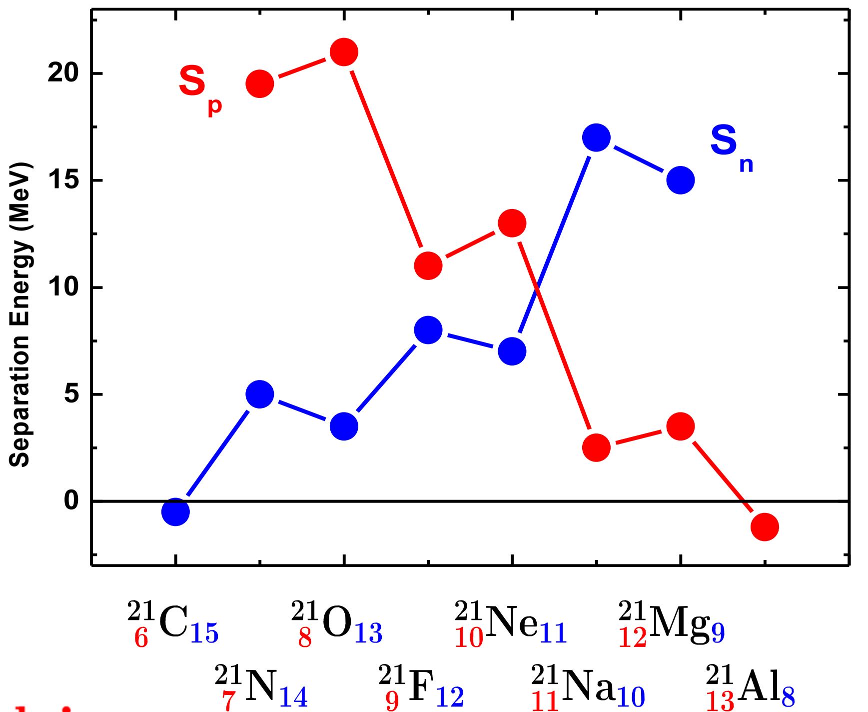
- definition of the International Union of Pure and Applied Chemistry (IUPAC) (Pure Appl. Chem. 63 (1991) 879)

  existing for at least 10^{-14} s → time for a nucleus to acquire its outer electrons

DEFINITION OF THE DRIPLINES

The nuclear binding in nuclei with an extreme excess of neutrons or protons may not be sufficient to bind the last neutron or proton

$$
\begin{aligned}
BE(N,Z) &= ZM_Hc^2 + NM_nc^2 - M(N,Z)c^2 \\
S_n &= BE(N,Z) - BE(N-1,Z) \\
S_p &= BE(N,Z) - BE(N,Z-1)
\end{aligned}
$$



the dripline is the boundary where neutron S_n or proton S_p separation energies cross zero

The unbound nuclei :

- can have fairly long lifetimes along the proton dripline (because of the Coulomb barrier)
- extremely short lived beyond the neutron dripline (absence of the Coulomb barrier)

the proton dripline has been reached for elements up to Z = 83
the neutron dripline has been reached for elements up to Z < 10

Conceptual framework of nuclear structure is the nuclear shell model

Qualitative picture: ⇒ nucleons are moving almost independently in a mean-field (self-consistent) potential obtained by averaging out the interactions between a single nucleon and all remaining protons and neutrons.

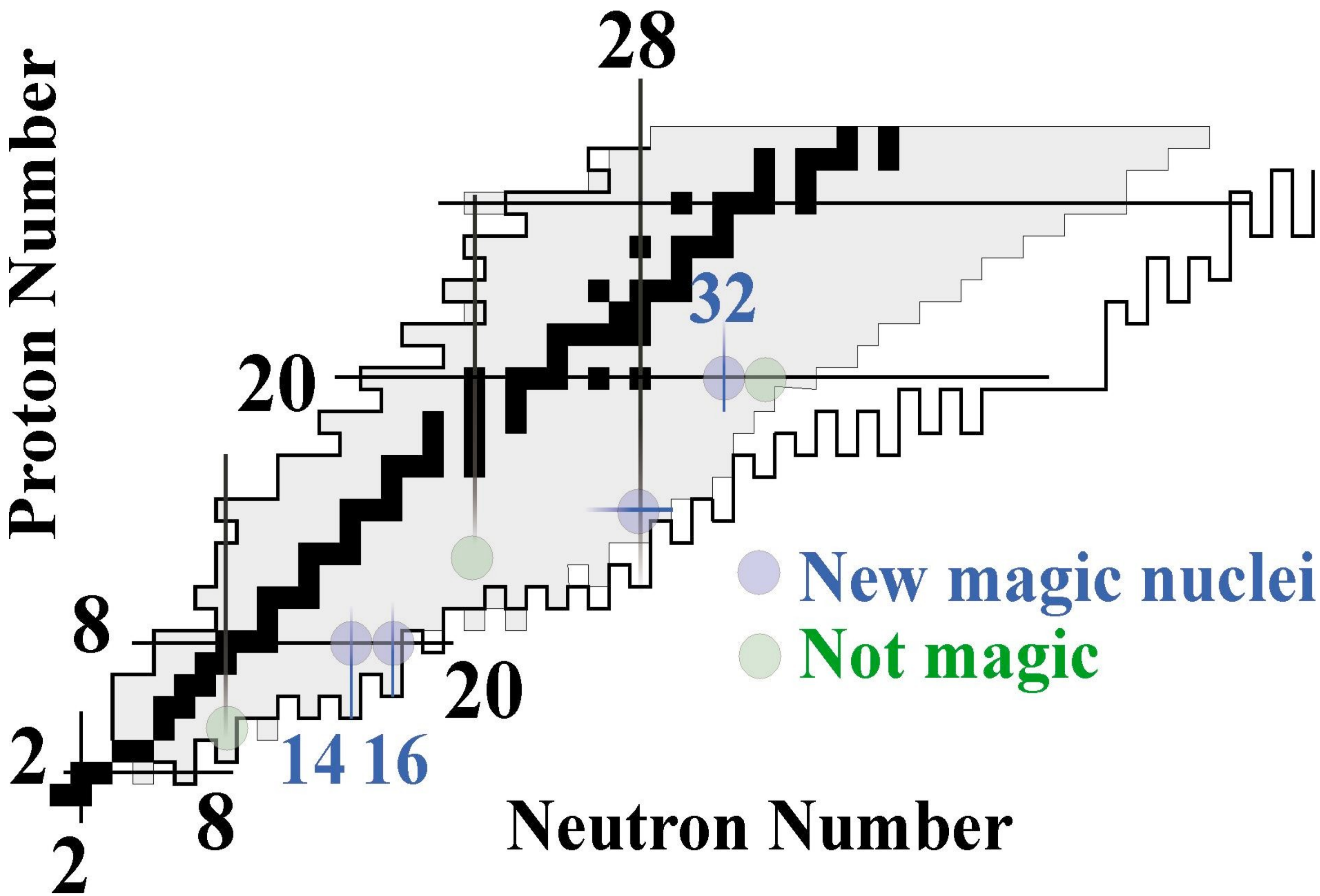
Quantitative results: ⇒ accounting for the residual interaction between the nucleons

Nuclear structure of exotic nuclei is different from that around the stability line and represent a formidable challenge for the nuclear many-body theories and their power to predict nuclear properties

Unique factors for exotic nuclei

- the weak binding, closeness of the particle continuum (a large diffuseness of the nuclear surface, extreme spatial dimensions for the outermost nucleons)
- exotic combinations of proton and neutron numbers (prospects for completely new structural phenomena)

EVOLUTION OF THE SHELL STRUCTURE TOWARDS TO THE DRIPLINE



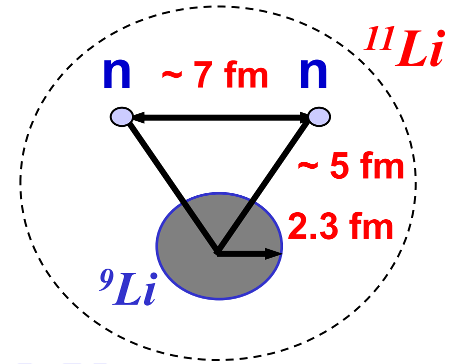
No shell closure for N = 8 and 20 for drip-line nuclei
New shell closure at N = 14, 16, 32, . . .

Neutron halo nuclei

HALO NUCLEI
(^{6}He, ^{11}Li, ^{11}Be, ^{14}Be, ^{17}B, . . .)

weakly bound systems
with large extension
few-body clusterization

"Residence in forbidden regions"
Appreciable probability for dilute nuclear matter extending far out into classically forbidden region



Separation energies of last neutron(s)

halo	*stable*
< 1	6 - 8 MeV

$$\epsilon(^{11}Li) = 0.4 \ \mathrm{MeV}$$
$$\epsilon(^{11}Be) = 0.5 \ \mathrm{MeV}$$
$$\epsilon(^{6}He) = 0.97 \ \mathrm{MeV}$$

Large size of halo nuclei

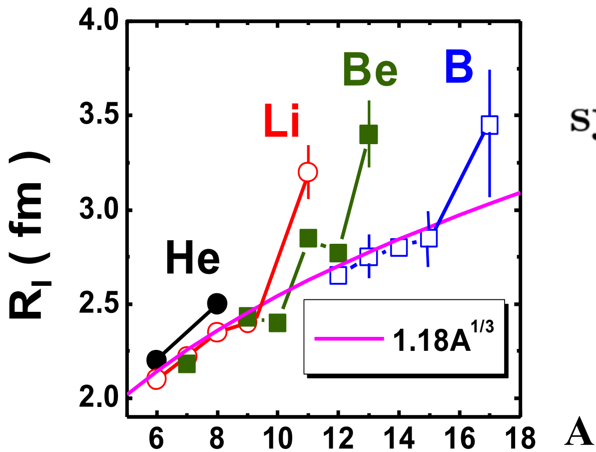
$$\begin{cases} \langle r^2(^{11}Li)\rangle^{1/2} \sim 3.5 fm \\ (r.m.s. \text{ for } A \sim 48) \end{cases}$$

Two-neutron halo nuclei
(^{11}Li, ^{6}He, ^{14}Be, ^{17}B, . . .) → Borromean systems

Peculiarities of halo nuclei: the example of ^{11}Li

(i) weakly bound: the two-neutron separation energy ($\sim$ 400 KeV) $\sim$ 10 times less than the energy of the first excited state in ^{9}Li

(ii) large size: ^{11}Li interaction cross section $\sim$ 30% larger than for ^{9}Li



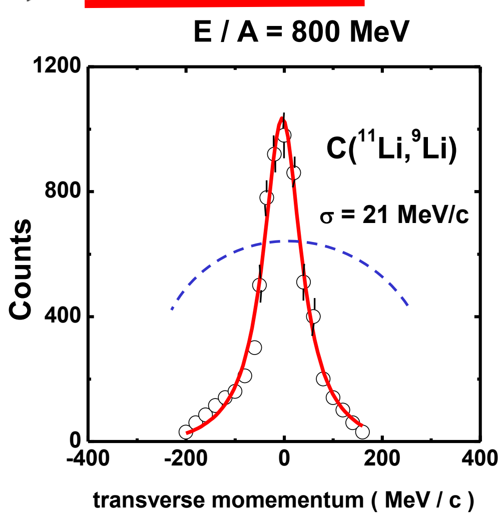
This is very unusual for strongly interacting systems held together by short-range interactions

Interaction radii: $\sigma_I = \pi(R_{proj} + R_{tar})^2$

light targets: E/A = 790 MeV

I. Tanihata et al., PRL 55 (1985) 2676

(iii) very narrow (in comparison to stable nuclei) fragment momentum distributions of both neutrons and ^{9}Li measured in high energy breakup reactions of ^{11}Li



No narrow fragment distributions in breakup on other fragments, say ^{8}Li or ^{8}He

(naive picture)

narrow momentum distributions

$\Downarrow$

large spatial extensions

(IV) Relations between interaction and neutron removal cross sections

A + ^{12}C	σ_I (mb)	σ_{-2n} (mb)	σ_{-4n} (mb)
^{9}Li	796 ± 6		
^{11}Li	1060 ± 10	220 ± 40	
^{4}He	503 ± 5		
^{6}He	722 ± 5	189 ± 4	
^{8}He	817 ± 6	202 ± 17	95 ± 5

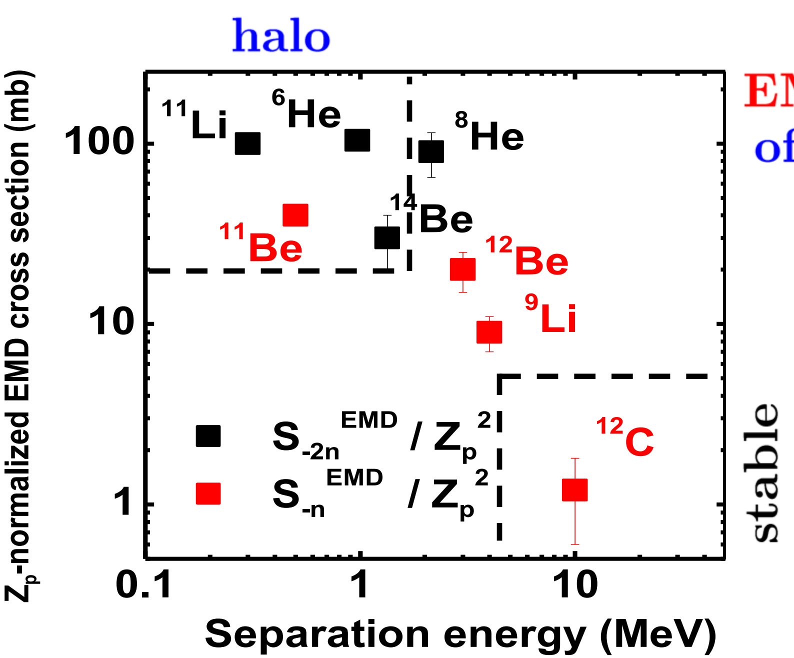
E/A = 790 MeV

$$\sigma_I(A = C + xn) = \sigma_I(C) + \sigma_{-xn}$$

Strong evidence for the well defined clusterization into the core and two neutrons

Tanihata I. et al.
PRL, 55 (1987) 2670; PL, B289 (1992) 263

(V) Electromagnetic dissociation cross sections (EMDC)



EMDC per unit charge for halos are orders of magnitude larger than for stable nuclei

Evidence for a rather large difference between charge and mass centers in a body fixed frame

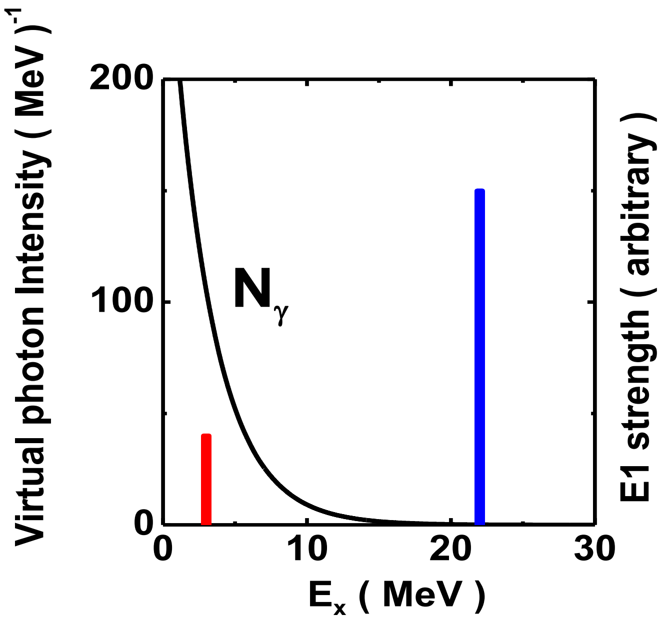
concentration of the dipole strength at low excitation energies

T. Kobayashi, Proc. 1st Int. Conf. On Radiactive Nuclear Beams, 1990.

SOFT EXCITATION MODES

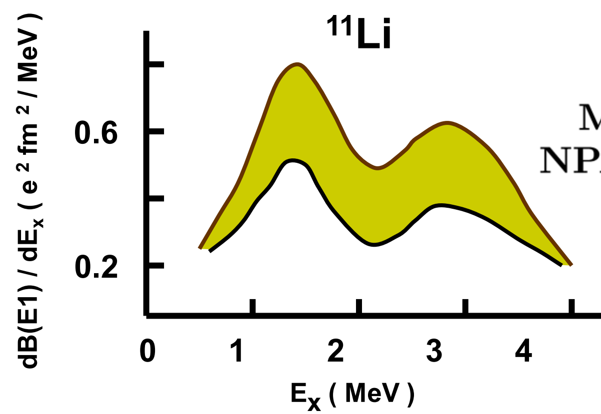
(peculiarities of low energy halo continuum)

Large EMD cross sections ⇒ **specific nuclear property of extremely neutron-rich nuclei**



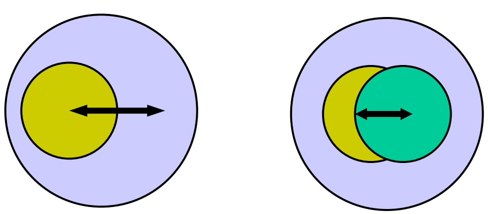
$$\sigma_{EMD} \;=\; \int N(E_x)\, \sigma(E_x)\, dE_x$$

$$\sigma(E_x) \;=\; \frac{16\pi^3}{9\hbar c}\, E_x\, \frac{dB(E1)}{dE_x}$$



M. Zinser et al., NPA619 (1997) 151

soft DR normal GDR



$E_x \sim$ 1 MeV $\sim$ 20 MeV

excitations of soft modes with

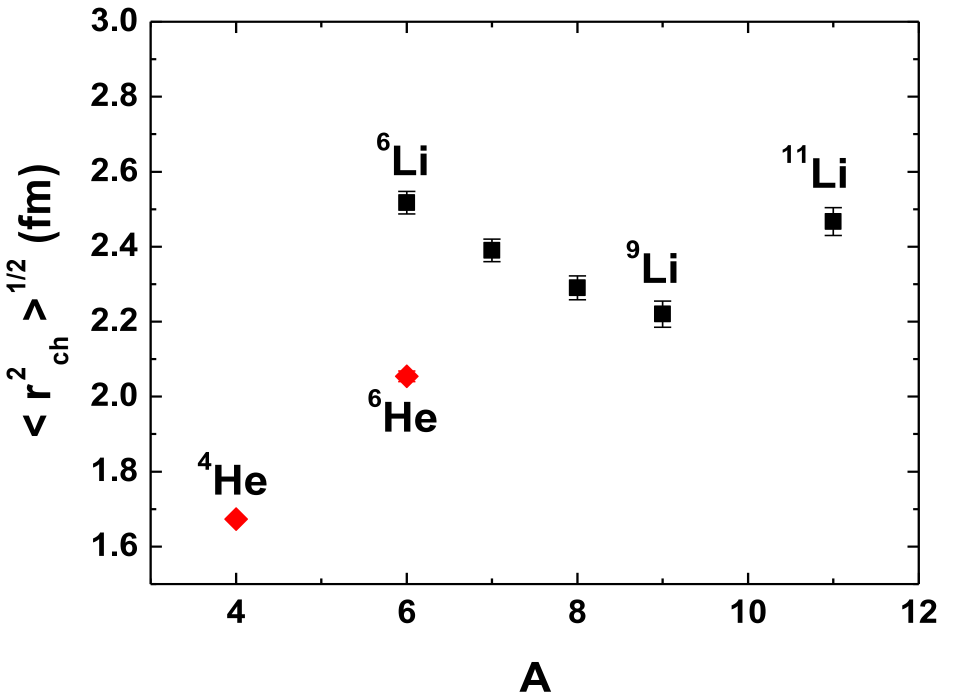
- different multipolarity
- collective excitations versus direct transition from weakly bound to continuum states

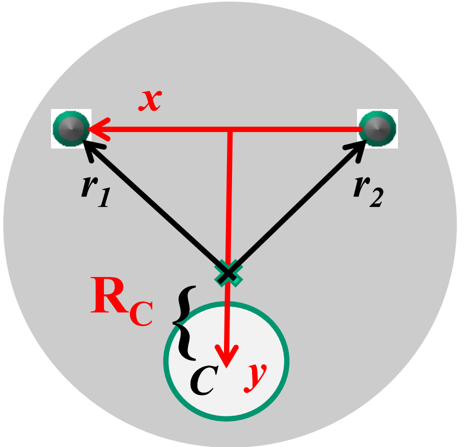
(VI) Ground state properties of ^{11}Li and ^{9}Li

	^{9}Li	^{11}Li
Spin J^π :	$3/2^-$	$3/2^-$
quadrupole moments :	-27.4 $\pm$ 1.0 mb	-31.2 $\pm$ 4.5 mb
magnetic moments :	3.4391 $\pm$ 0.0006 n.m.	3.6678 $\pm$ 0.0025 n.m.

Previous peculiarities cannot arise from large deformations.
Core is not significantly perturbed by the two valence neutrons

Nuclear charge radii by laser spectroscopy

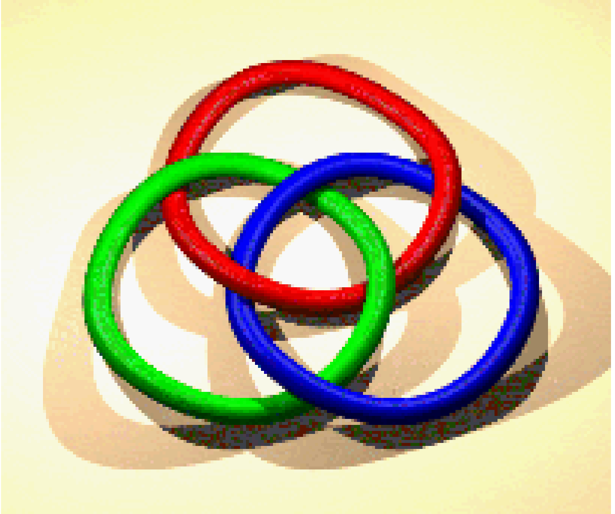




R. Sanches et al., PRL 96 (2006) 033002
L.B. Wang et al., PRL 93 (2004) 142501

(VII) The three-body system ^{11}Li (^{9}Li + n + n) is Borromean
(neither the two neutron nor the core-neutron subsystems are bound)

Three-body correlations are the most important,
due to them the system becomes bound.



"The Borromean rings, the heraldic symbol of the Princes of Borromeo, are carved in the stone of their castle in Lake Maggiore in northern Italy. The three rings are interlocked in such a way that any of them were removed, the other two would also fall apart. In nuclear physics ^{11}Li and ^{6}He have been found to have this property (although for quite different physical reasons) when described in a three-body model. "

M.V. Zhukov et al., Phys. Rep. 231 (1993) 151

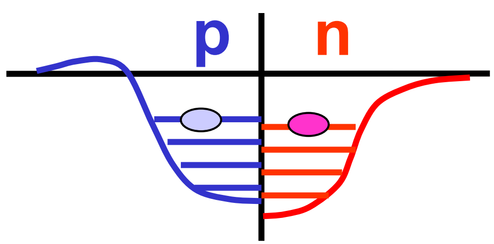
STABLE NUCLEI

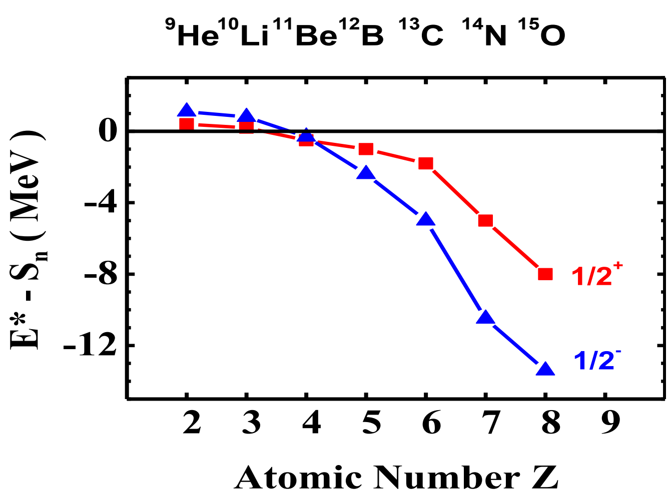
$N / Z \sim 1 - 1.5$

$\epsilon_S \sim 6$ - 8 MeV

$\rho_0 \sim 0.16 \; fm^{-3}$

proton and neutrons homogeneously mixed, no decoupling of proton and neutron distributions





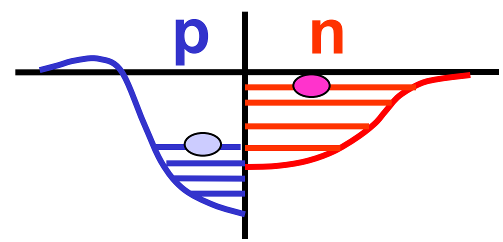
UNSTABLE NUCLEI

$N / Z \sim 0.6 - 4$

$\epsilon_S \sim 0$ - 40 MeV

decoupling of proton and neutron distributions

neutron halos and neutron skins



Prerequisite of the halo formation:

low angular momentum motion for halo particles and few-body dynamics

$1s$ – intruder level

^{11}Be parity inversion of g.s.

^{10}Li g.s. : $\left[\pi 0 p_{3/2} \otimes \nu 1 s_{1/2}\right]_{2^-}$

PECULIARITIES OF HALO

in ground state

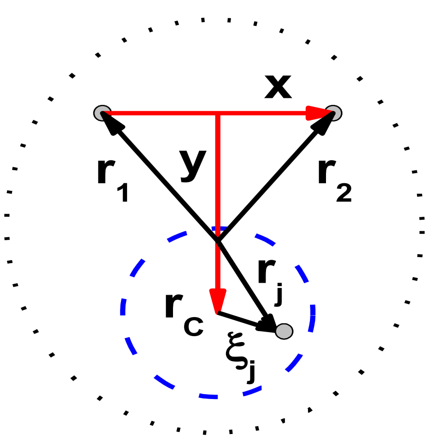
weakly bound,
with large extension
and space granularity

elastic scattering
some inclusive observables
(reaction cross sections, ...)

in low-energy continuum

concentration of the transition
strength near break up threshold
– soft modes

nuclear reactions
(transition properties)

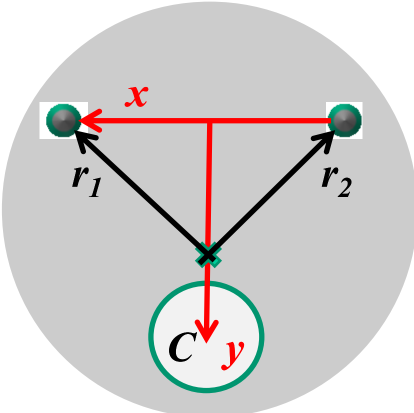


BASIC dynamics of halo nuclei ⇨ Decoupling of halo and nuclear core degrees of freedom

Dominance of few-body dynamics

$$\Phi(\bar{r}_1, \ldots, \bar{r}_A) = \varphi(\bar{\xi}_1, \ldots, \bar{\xi}_C)\, \psi(\bar{x}, \bar{y})$$

FEW-BODY CLUSTER MODELS



The Schrodinger equation

$$H_A \, \Psi(\mathrm{r}_1, \cdots, \mathrm{r}_A) = E \, \Psi(\mathrm{r}_1, \cdots, \mathrm{r}_A)$$

Total hamiltonian of the three-body cluster models $(A = A_C + 2)$

$$H_A = H_{A_C} + T_{x,y} + V(r_1, r_2) + \sum_{i=1}^{A_C} V(r_1, r_i) + \sum_{i=1}^{A_C} V(r_2, r_i)$$

wave function is factorized into a sum of products from two parts

$$\Psi(\mathrm{r}_1, \cdots, \mathrm{r}_A) = \sum_i \phi_i(\mathrm{r}_1, \cdots, \mathrm{r}_{A_C}) \, \psi_i(\mathrm{x}, \mathrm{y})$$

The sum may include core excitations

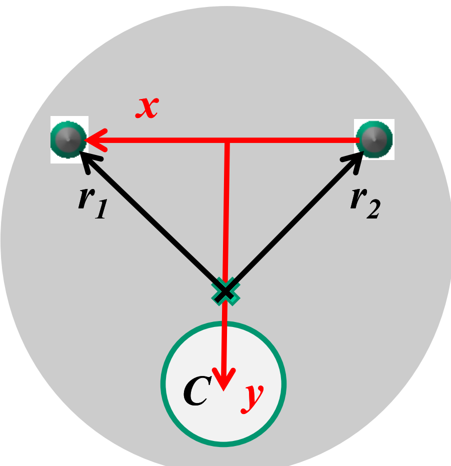
$$H_{A_C} \, \phi_i(\mathrm{r}_1, \cdots, \mathrm{r}_{A_C}) = \epsilon_i \, \phi_i(\mathrm{r}_1, \cdots, \mathrm{r}_{A_C})$$

Calculations of the bound states and continuum wave functions

Borromean nature of halo nuclei

(no bound states between pairs of clusters)

one type of the wave function asymptotic behaviour



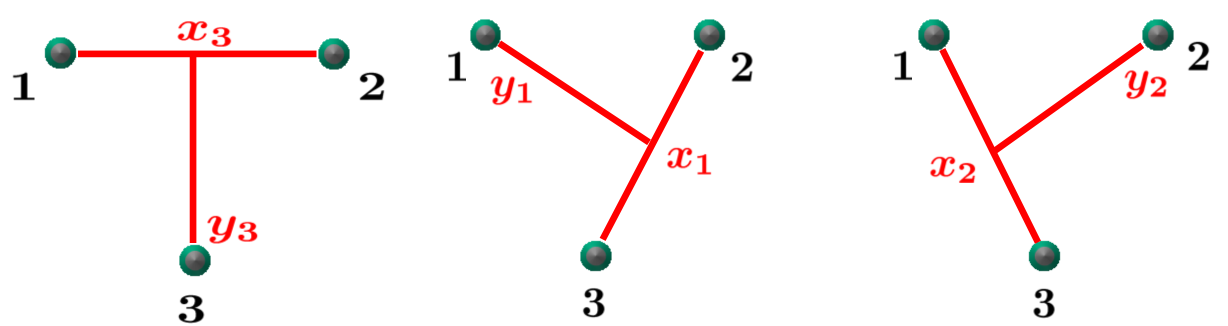
$$\rho^2 = \mu_x \bar{\mathrm{x}}^2 + \mu_y \bar{\mathrm{y}}^2$$
$$\alpha_\rho = \arctan\left(\frac{\sqrt{\mu_x}\, x}{\sqrt{\mu_y}\, y}\right)$$
$$\Omega_5^\rho = \{\alpha_\rho, \hat{x}, \hat{y}\}$$

$$\frac{\kappa^2}{2m} = \frac{\bar{\mathrm{k}}_{\mathrm{x}}^2}{2\mu_x} + \frac{\bar{\mathrm{k}}_{\mathrm{y}}^2}{2\mu_y}$$
$$\alpha_\kappa = \arctan\left(\frac{\sqrt{\mu_y}\, \mathrm{k_x}}{\sqrt{\mu_x}\, \mathrm{k_y}}\right)$$
$$\Omega_5^\kappa = \{\alpha_\kappa, \hat{k}_x, \hat{k}_y\}$$

$$\{\bar{\mathrm{x}}, \bar{\mathrm{y}}\} \Rightarrow \{\rho\,,\, \Omega_5^\rho\}$$

$$\{\bar{\mathrm{k}}_x, \bar{\mathrm{k}}_y\} \Rightarrow \{\kappa\,,\, \Omega_5^\kappa\}$$

$\{\rho, \kappa\}$ are independent of the Jacobi system



The bound state wave function ($\gamma = \{K, l_x, l_y, L, S, I, j\}$)

$$\Psi_{JM}(\mathrm{r}_1, \cdots, \mathrm{r}_A) = \frac{1}{\rho^{5/2}} \sum_{\gamma} \chi^J_\gamma(\rho) \left[\Upsilon^{l_x l_y}_{KL}(\Omega^\rho_5) \otimes [\chi_s \otimes \phi_{nI}]_j \right]_{JM}$$

The continuum wave function at the positive energy

$$\Psi^{(\pm)}_{s\nu I M_I}(\mathrm{k_x}, \mathrm{k_y}; \mathrm{r}_i) = \sum_{\gamma} \imath^K (s\nu I M_I \mid j m_j)(L M_L\, j m_j \mid J M_J)\, \Upsilon^{l_x l_y *}_{KL}(\Omega^\kappa_5) \times$$

$$\times \frac{1}{\rho^{5/2}} \sum_{\gamma'} \chi^J_{\gamma', \gamma}(\kappa,\ \rho) \left[\Upsilon^{l'_x l'_y}_{K' L'}(\Omega^\rho_5) \otimes [\chi_{s'} \otimes \phi_{n' I'}]_{j'} \right]_{JM}$$

Set of coupled Schrödinger equations for radial wave functions

$$\left(-\frac{\hbar^2}{2\mu} \left[\frac{d^2}{d\rho^2} - \frac{(K+3/2)(K+5/2))}{\rho^2} \right] + \epsilon_\gamma - E \right) \chi^J_{\gamma,\gamma'}(\rho) = - \sum_{\gamma''} V^J_{\gamma,\gamma''}(\rho)\, \chi^J_{\gamma'',\gamma'}(\rho)$$

Hyperspherical harmonics $\Upsilon^{l_x, l_y}_{KLM}(\Omega_5)$ ($K = 2n + l_x + l_y$)

$$\Upsilon^{l_x, l_y}_{KLM}(\Omega^\rho_5) = \psi^{l_x, l_y}_K(\alpha_\rho) \left[Y_{l_x}(\hat{x}) \otimes Y_{l_y}(\hat{y}) \right]_{LM}$$

$$\psi^{l_x, l_y}_K(\alpha) = N^{l_x, l_y}_K\ (\sin\alpha)^{l_x}\ (\cos\alpha)^{l_y}\ P^{(l_x+1/2, l_y+1/2)}_n(\cos 2\alpha)$$

$$\Upsilon^{l'_x, l'_y}_{KLM}(\Omega'_5) = \sum_{l_x, l_y} \left\langle l_x, l_y \mid l'_x, l'_y \right\rangle_{KL}\ \Upsilon^{l_x, l_y}_{KLM}(\Omega_5)$$

boundary condition of the radial wave function at the origin

$$\chi_\gamma^J(\rho \to 0) \to 0$$

asymptotic behaviour of the bound state radial wave function

$$\chi_\gamma^J(\rho \to 0) \to \exp(-\kappa_n \rho), \quad \kappa_n = \sqrt{2m \mid E_n - \epsilon_\gamma \mid /\hbar^2}$$

asymptotic behaviour of the continuum radial wave function

$$\chi_{\gamma',\gamma}^J(\kappa, \rho \to \infty) \to \frac{\imath}{\sqrt{2\pi}} \frac{1}{\sqrt{k_\gamma \, k_{\gamma'}}} \left(H_{K+2}^{(-)}(k_\gamma \, \rho) \, \delta_{\gamma,\gamma'} - H_{K'+2}^{(+)}(k_{\gamma'} \, \rho) \, S_{\gamma',\gamma} \right)$$

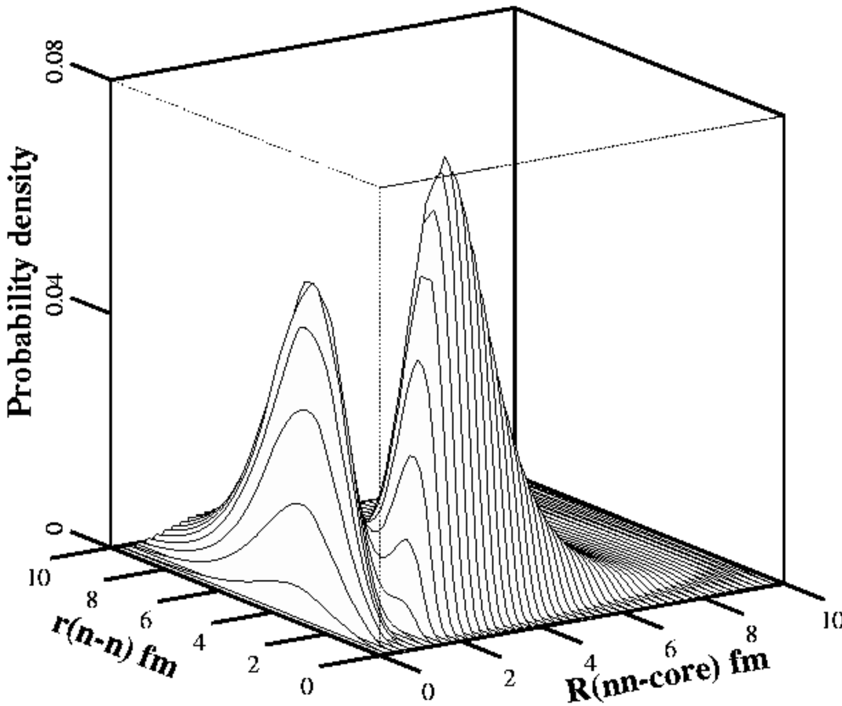
$$k_\gamma = \sqrt{2m \mid E - \epsilon_\gamma \mid /\hbar^2}$$

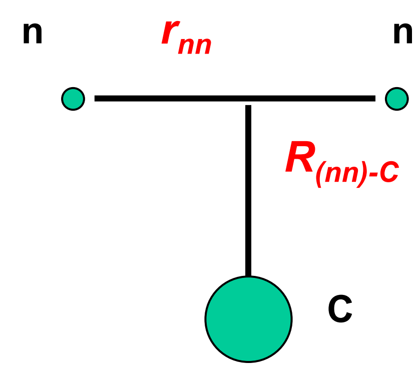
In collisions we explore the transition properties of nuclei from ground state to continuum states

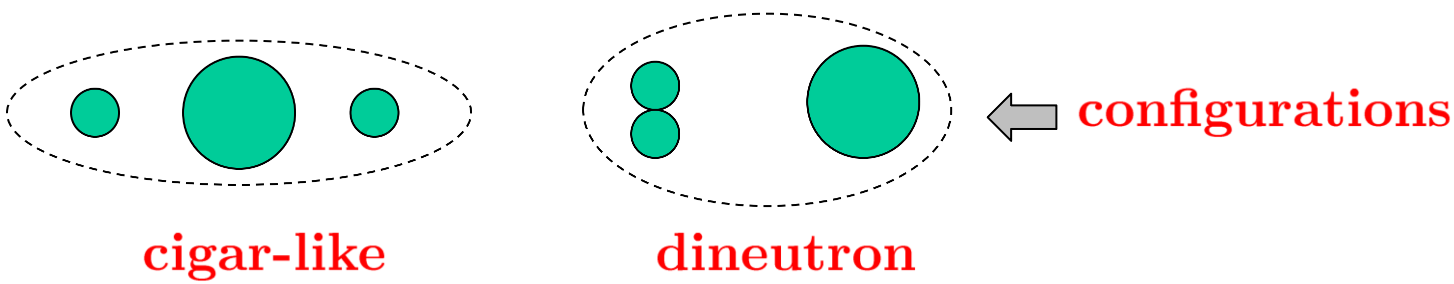
$$\langle \Psi^{(-)}(\bar{k}_x, \bar{k}_y) \mid\mid \sum_p \frac{\delta(r - r_p)}{r r_p} \left[Y_L(\hat{r}_p) \times \sigma_p \right]_J \mid\mid \Psi_{gr.st.} \rangle$$

Correlation density for the ground state of ^{6}He

$$P(r_{nn}, R_{nn-C}) = r_{nn}^2 R_{nn-C}^2 \frac{1}{2J+1} \sum_M \int d\Omega_{nn}\, d\Omega_{nn-C} \mid \Phi_{JM}(\bar{r}_{nn}^2, \bar{R}_{nn-C}^2) \mid^2$$







CONCLUSIONS

- **Near the dripline nuclear structure may be dramatically different. Reaching the limits of nuclear stability offers unique opportunities to understand basic nuclear properties**

- **Halo – new structural dripline phenomenon with clusterization into an ordinary core nucleus and a veil of halo nucleons forming very dilute neutron matter, have already been discovered in reaction with radioactive ion beams**

- **Development of new experimental techniques for production and/or detection of radioactive beams is the way to unexplored**

  "TERRA INCOGNITA"