What do you know about weakly bound nuclei?

How do they behave in low energy collisions?

Reaction dynamics of weakly bound nuclei

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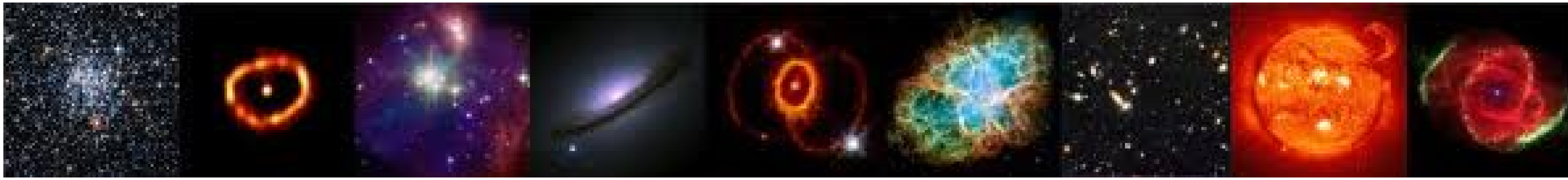
Lecture 1

Fusion dynamics of weakly bound nuclei

What I will tell you next

- Motivation, Important Concepts & Issues
- **Classical & Quantum Dynamical Models**
- Summary & Outlook

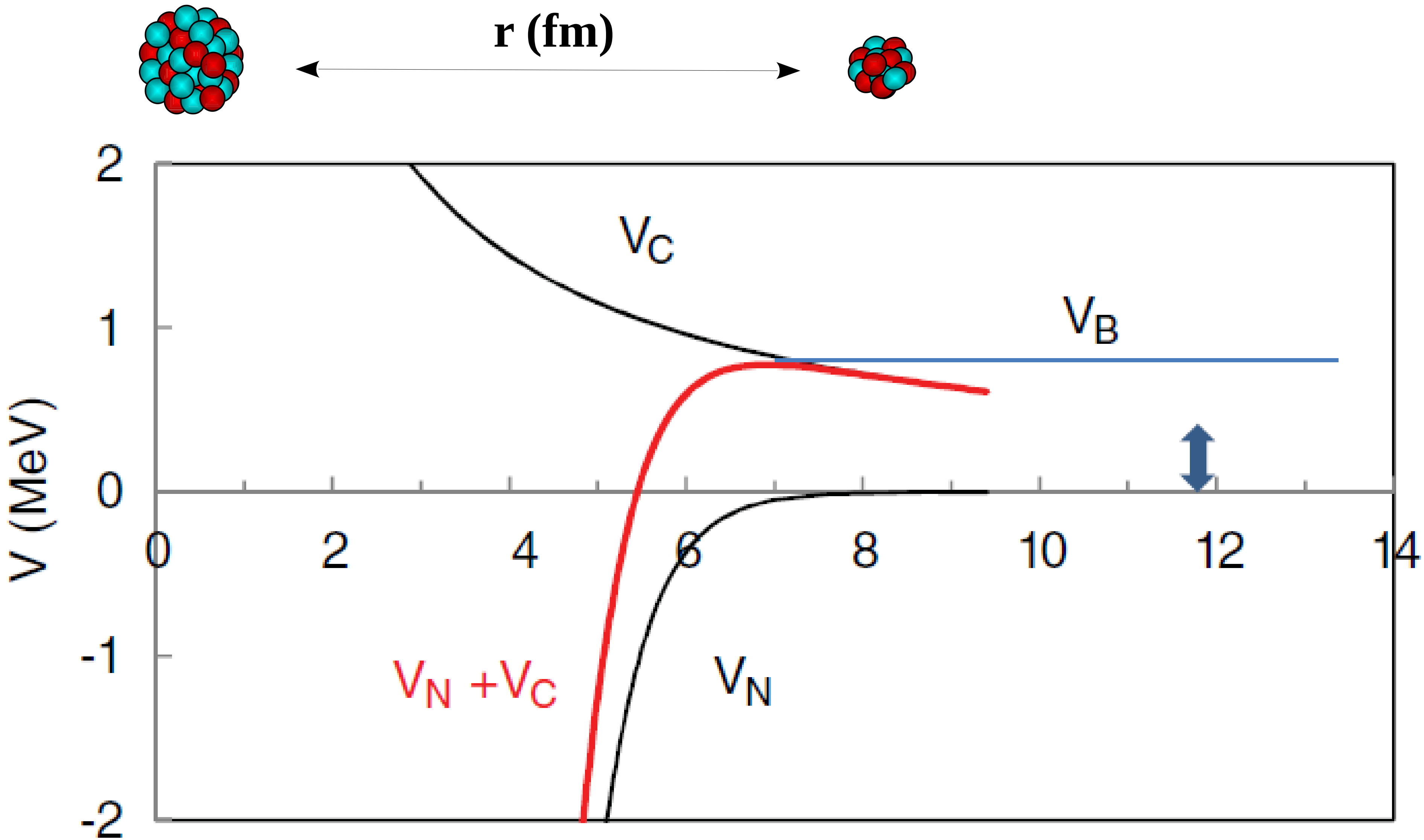
Why I find reaction physics important and exciting



- **The physics of low-energy nuclear reactions** is crucial for understanding **energy production** and **nucleosynthesis** in the Universe

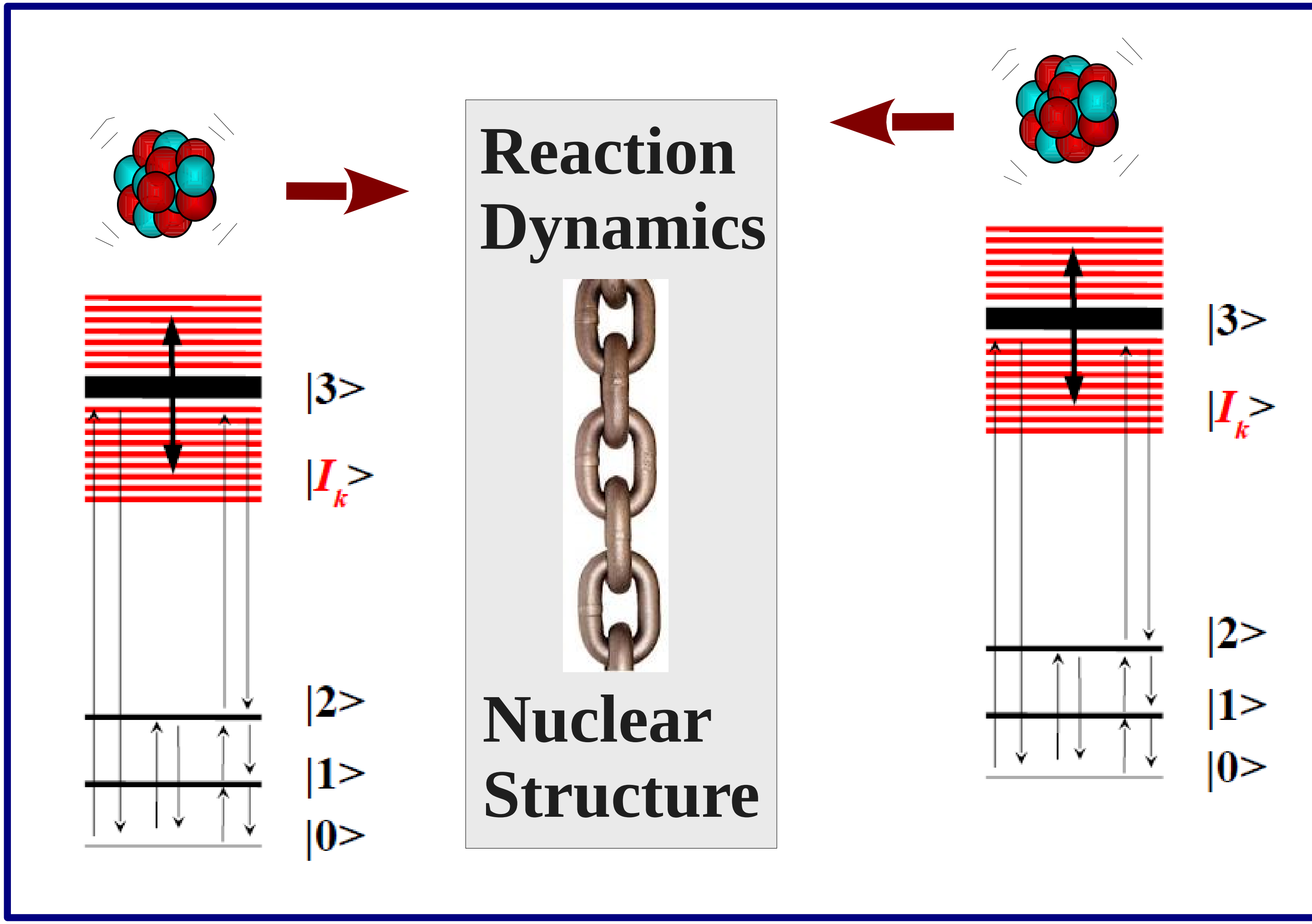
- **Nuclear reactions** are the **primary probe** of the **New Physics**

Interaction Potential between Nuclei & Scales



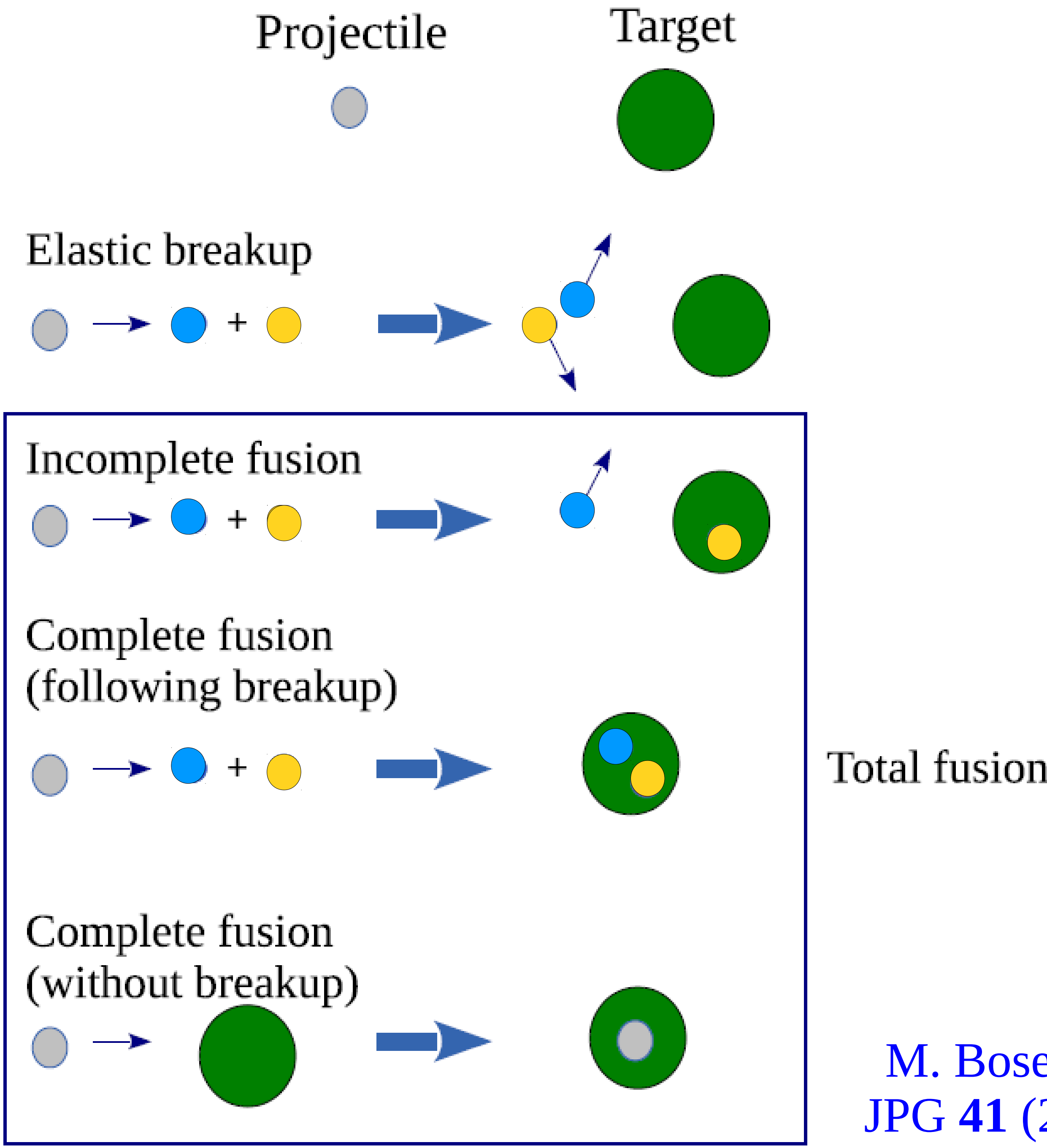
Energy: MeV = 10^6 eV; **Length:** fm = 10^{-15} m; **Time:** 10^{-21} s

Reactions between Complex Nuclei at Low Energy



The interplay between **nuclear structure** & **reaction dynamics** determines the reaction observables (**cross sections**)

Unified description of low-energy reaction processes?



M. Boselli & AD-T,
JPG **41** (2014) 094001

Some examples of low-energy models in the last 17 years

Classical

- AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL **98** (2007) 152701

Hagino, Dasgupta & Hinde, NPA **738** (2004) 475c

Mixed Quantum-Classical

Sargsyan, Adamian, Antonenko, AD-T, Gomes & Lenske, PRC **92** (2015) 054620

Marta, Canto & Donangelo, PRC **89** (2014) 034625; PRC **73** (2005) 034608

Baye, Capel & Melezhik, NPA **722** (2003) 328c

Esbensen & Bertsch, NPA **706** (2002) 383

Quantum Mechanical

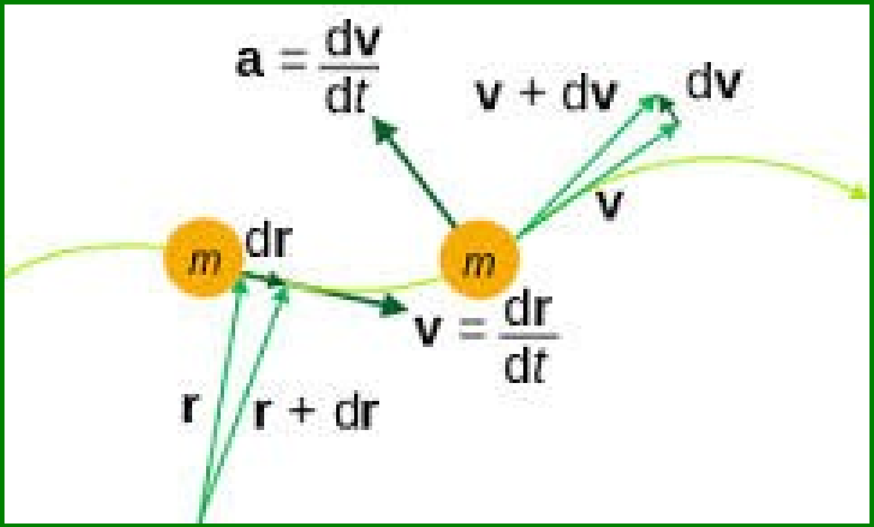
- Boselli & AD-T, PRC **92** (2015) 044610

Descouvemont, Druet, Canto & Hussein, PRC **91** (2015) 024606

Ito, Yabana, Nakatsukasa & Ueda, PLB **637** (2006) 53

AD-T, Thompson & Beck, PRC **68** (2003) 044607; PRC **65** (2002) 024606

Tostevin, Nunes & Thompson, PRC **63** (2001) 024617



Classical Trajectory Monte Carlo Method

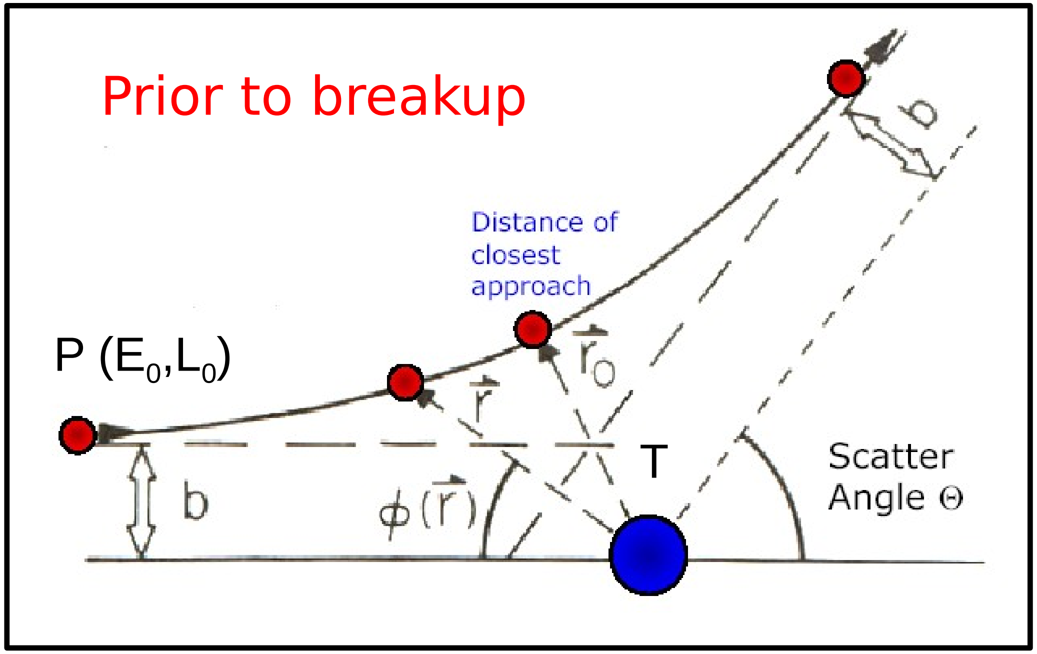
- After breakup, interaction among fragments is crucial
- Useful for interpreting particle-gamma coincidence data
- Transfer triggered breakup enriches the fusion scenario

See e.g., R. E. Olson, CTMC techniques, in Springer Handbook of Atomic, Molecular & Optical Physics (2006) pp. 869-874

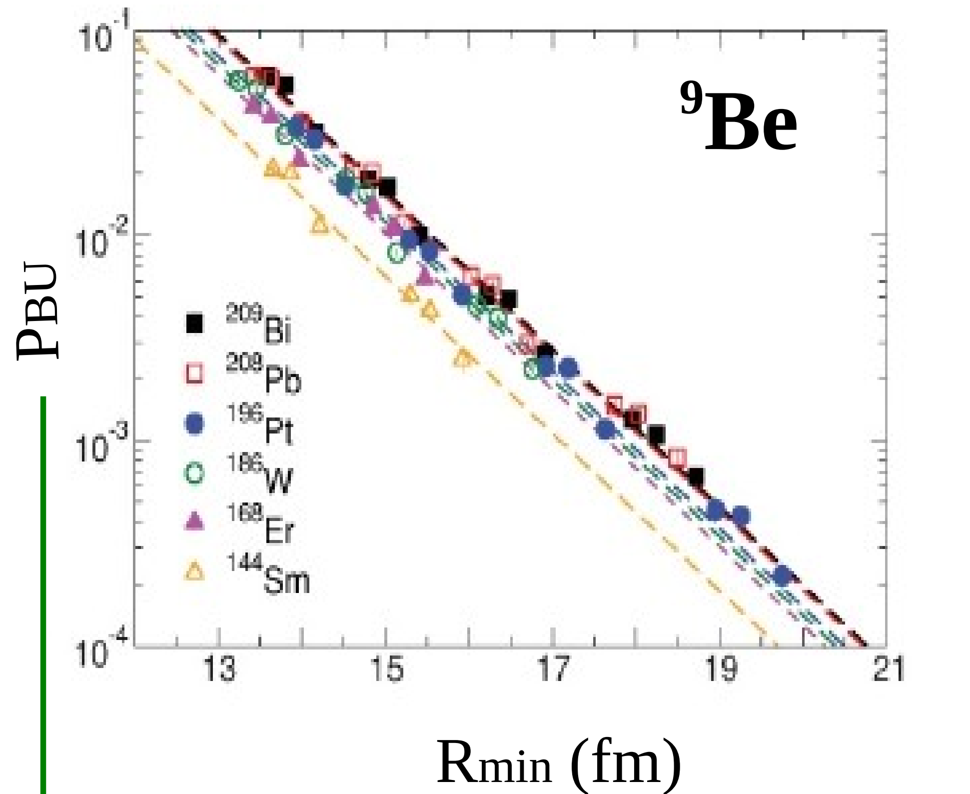
Classical Dynamical Model

AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL **98** (2007) 152701

AD-T, CPC **182** (2011) 1100 (**PLATYPUS** code)



Rafiei *et al.*, PRC **81** (2010) 024601



Main Ingredient :

$P^L_{BU}(R)dR$ *probability of breakup on the interval* $R + dR$

$$P_{BU}(R_{min}) = 2\int_{R_{min}}^{\infty} P^L_{BU}(R)dR = A exp(-\alpha R_{min})$$

Constructing Probabilities and Cross Sections

For each projectile angular momentum L_0 we have:

$$Condition: \quad \tilde{P}_i = N_i/N \quad ; \quad \tilde{P}_0 + \tilde{P}_1 + \tilde{P}_2 = 1$$

$$(N_i \text{ is the number of events in which fragments are captured})$$

$$(NCBU): \quad P_0(E_0, L_0) = P_{BU}(R_{min})\tilde{P}_0$$

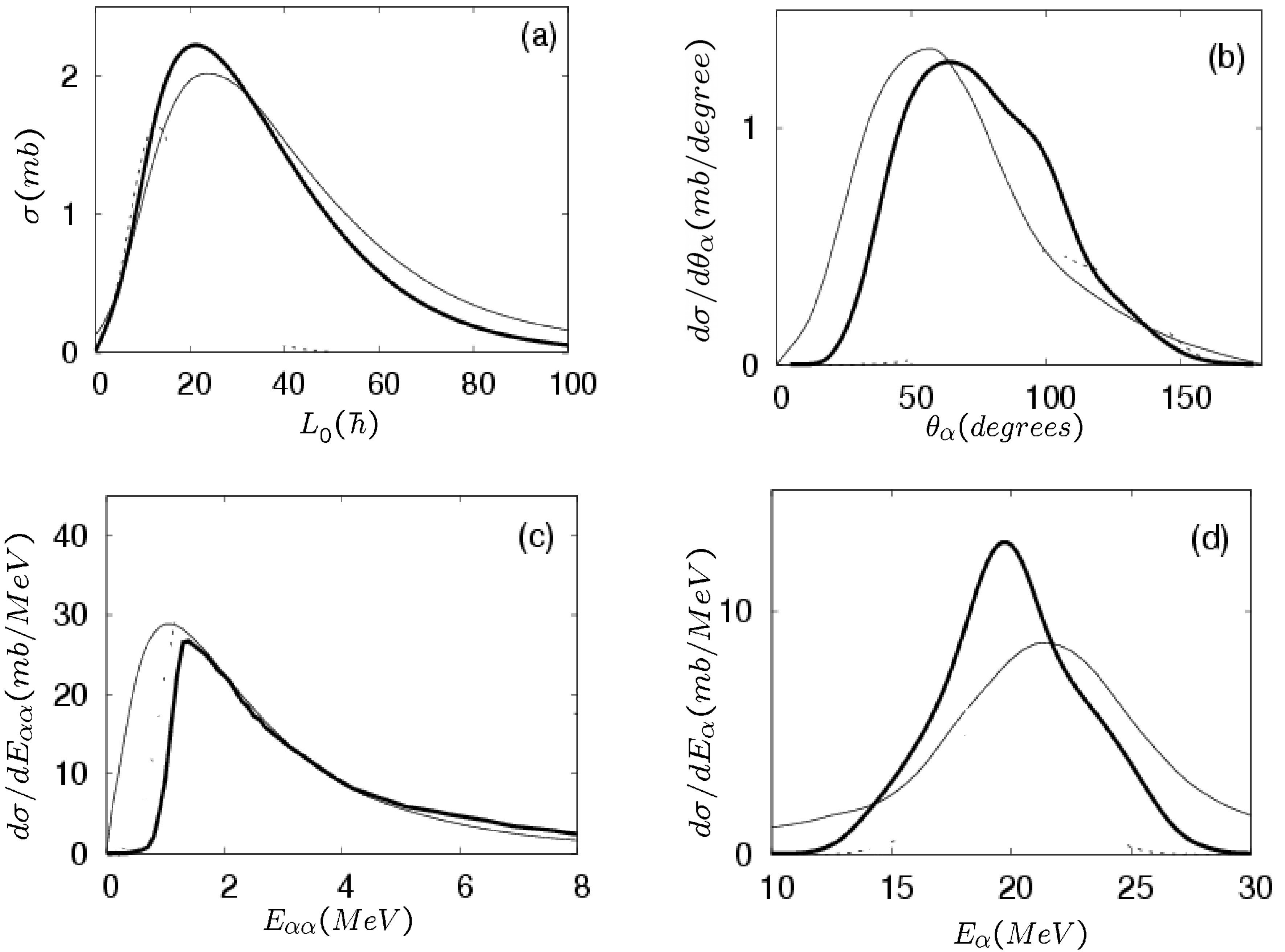
$$(ICF): \quad P_1(E_0, L_0) = P_{BU}(R_{min})\tilde{P}_1$$

$$(CF): \quad P_2(E_0, L_0) = [1 - P_{BU}(R_{min})]H(L_{cr} - L_0) + P_{BU}(R_{min})\tilde{P}_2$$

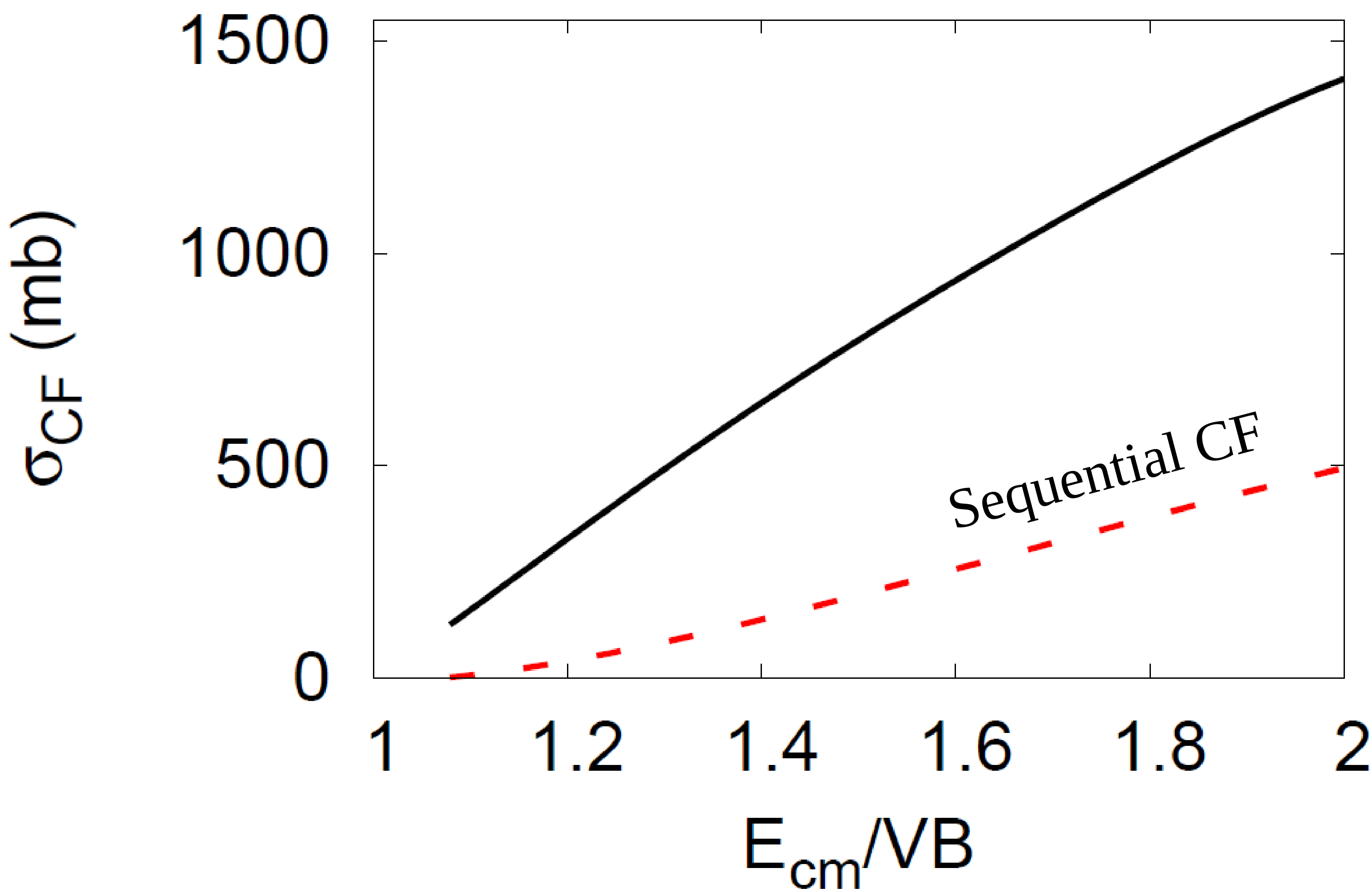
$$\sigma_i(E_0) = \pi\lambda^2 \sum_{L_0} (2L_0 + 1) P_i(E_0, L_0)$$

Classical results vs CDCC outcomes: “ ^{8}Be ” + ^{208}Pb

AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL 98 (2007) 152701



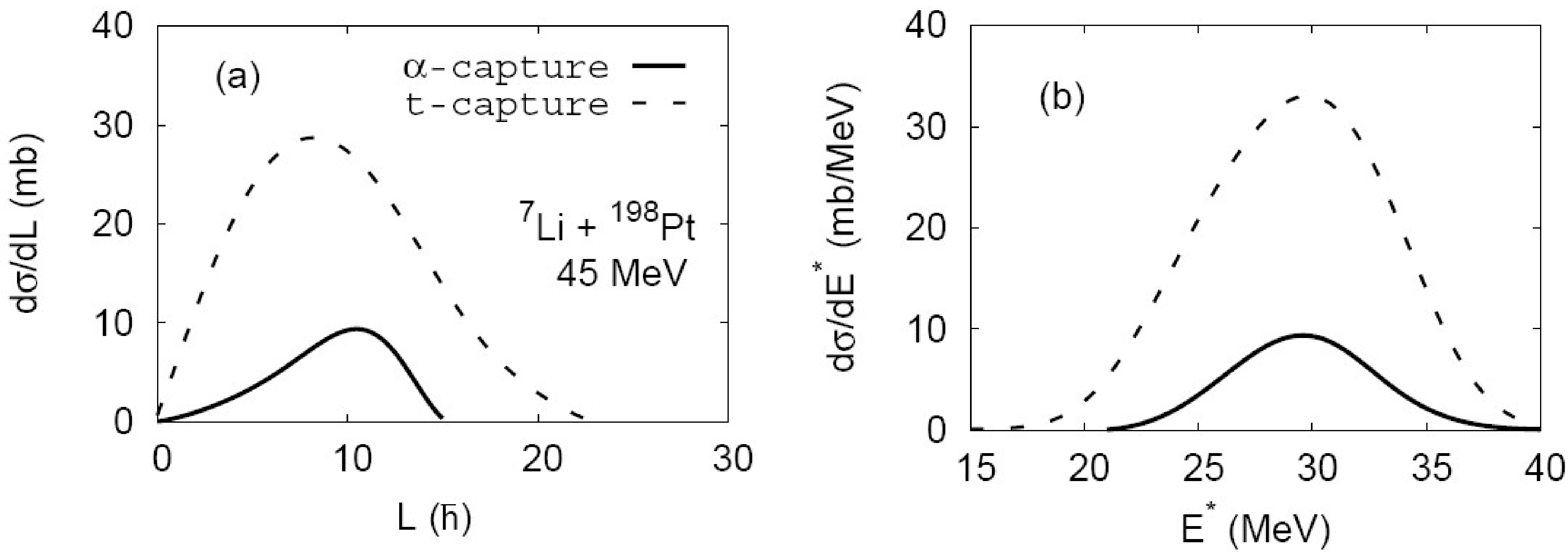
Complete fusion of $^{7}Li + ^{198}Pt$ at above-barrier energy

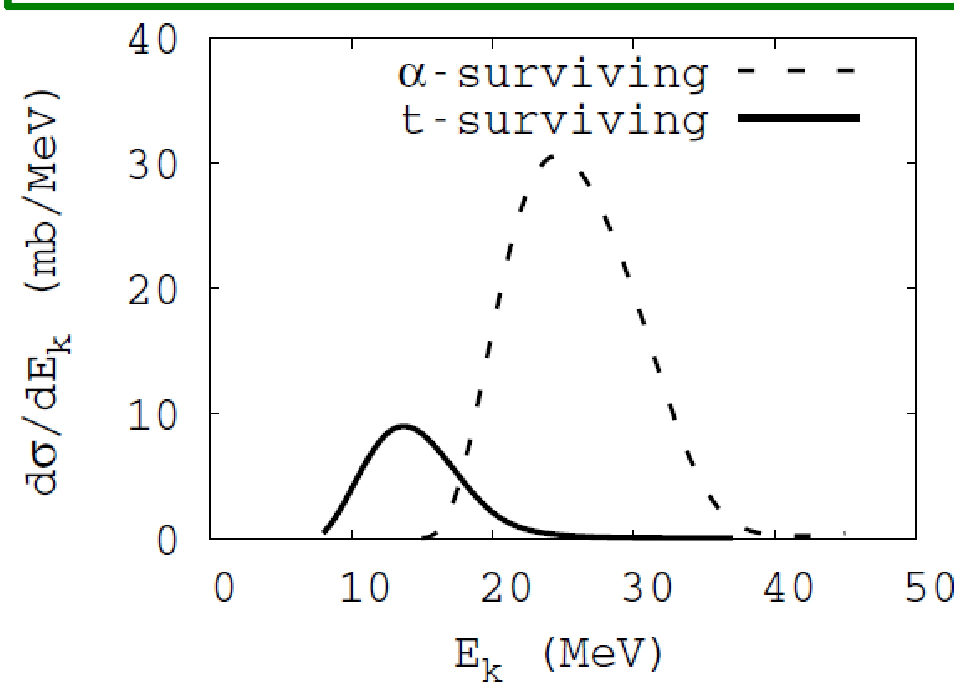


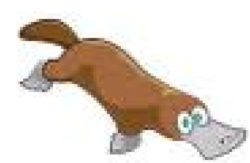
Sequential CF becomes substantial as energy increases

See e.g., Dasgupta *et al*., PRC **66** (2002) 041602 (R), for $^{6,7}Li + ^{209}Bi$

Incomplete fusion of ^{7}Li + ^{198}Pt at above-barrier energy







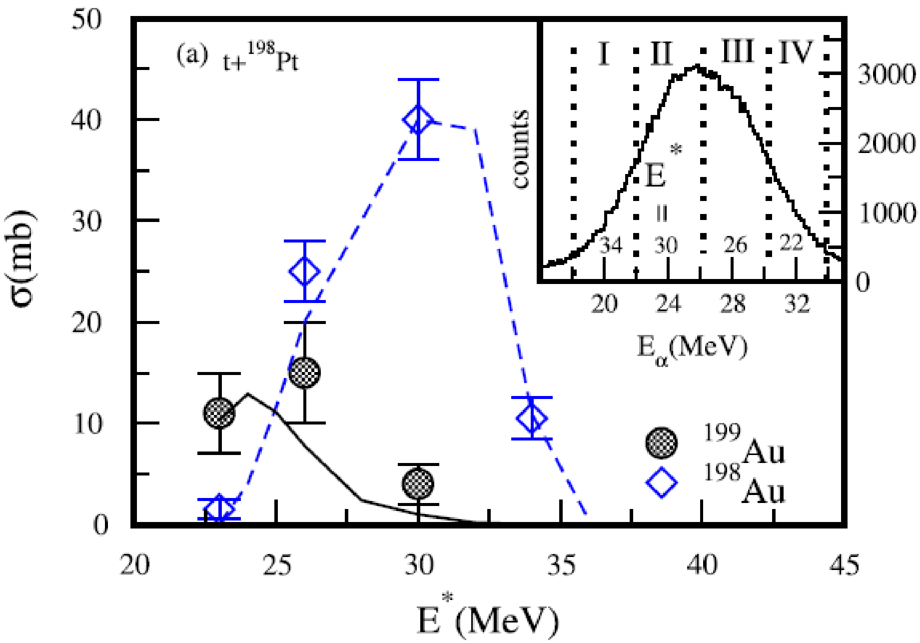
PLATYPUS code

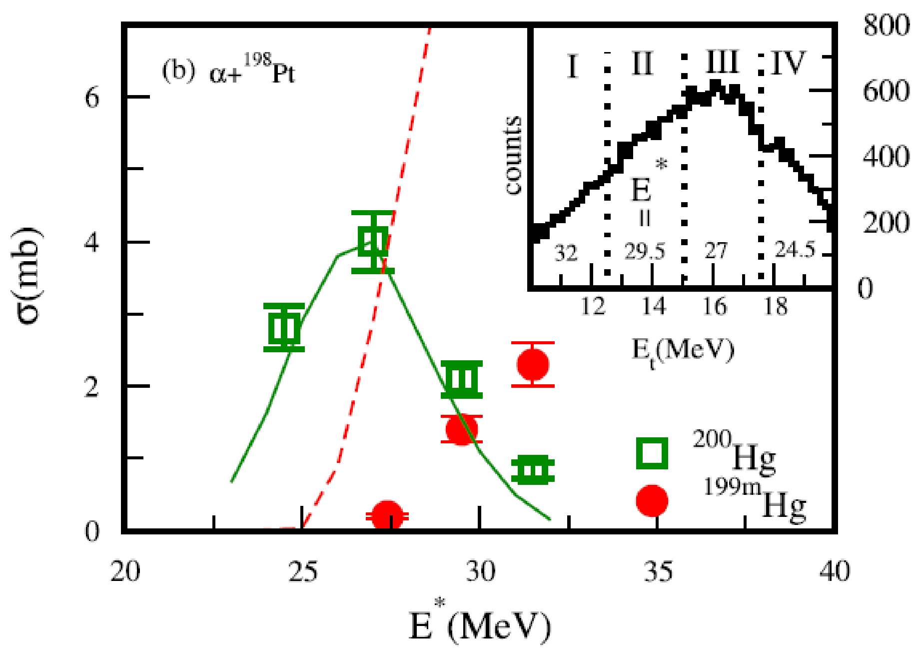
AD-T, CPC **182** (2011) 1100

Useful for planning & interpreting particle-gamma-coincidence measurements

Incomplete fusion measurements vs. **Platypus+PACE** calculations:

E* and spin distribution from **Platypus** fed to evaporation code **PACE**





^{7}Li + ^{198}Pt @ 45 MeV

triton - fusion

α gated γ spectra

t + ^{198}Pt : ^{201}Au*

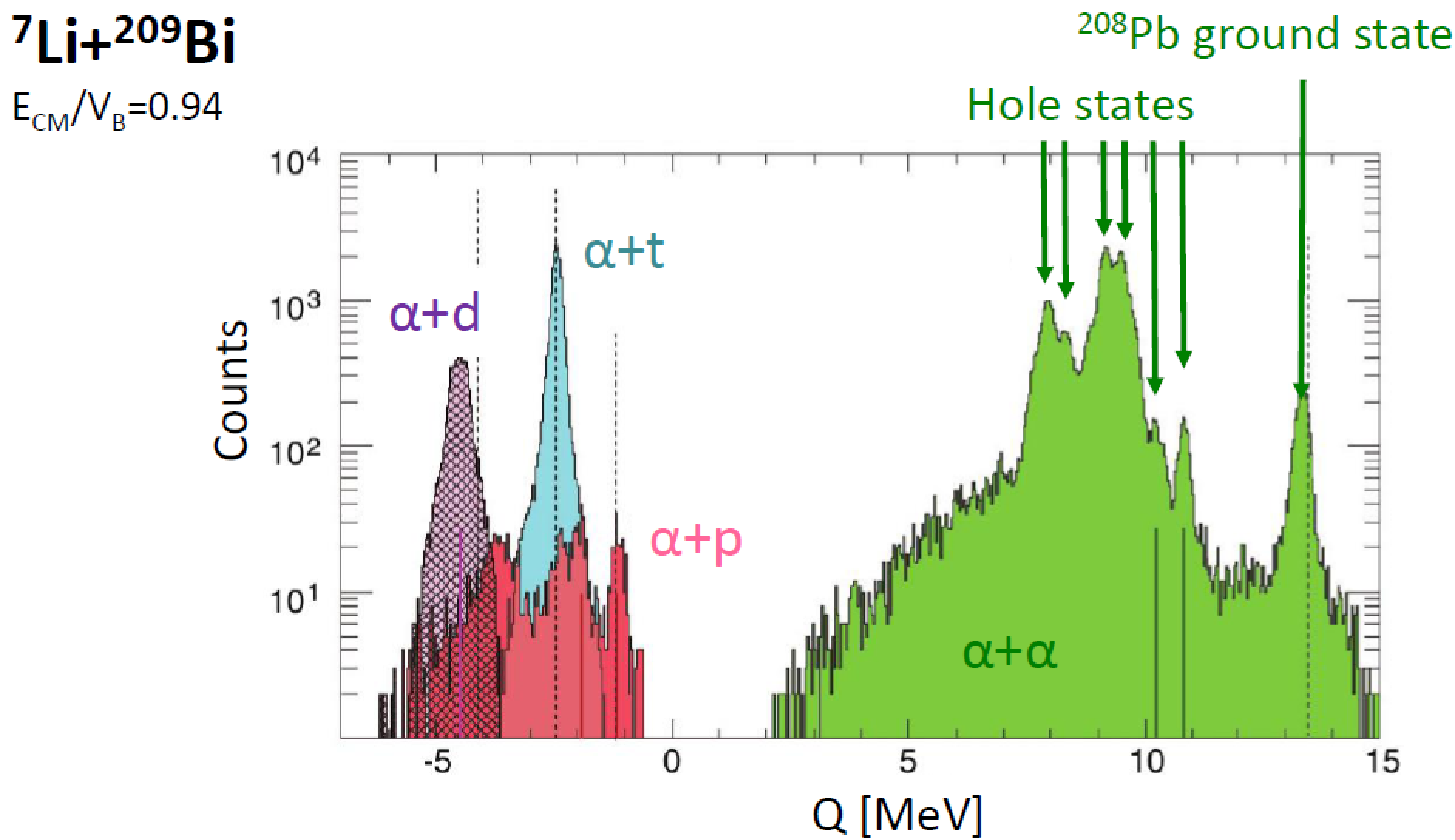
α - fusion

t gated γ spectra

α + ^{198}Pt : ^{202}Hg*

Shrivastava, Navin, AD-T *et al.*, PLB **718** (2013) 931

Q-value spectrum in sub-barrier breakup of ^{7}Li on ^{209}Bi

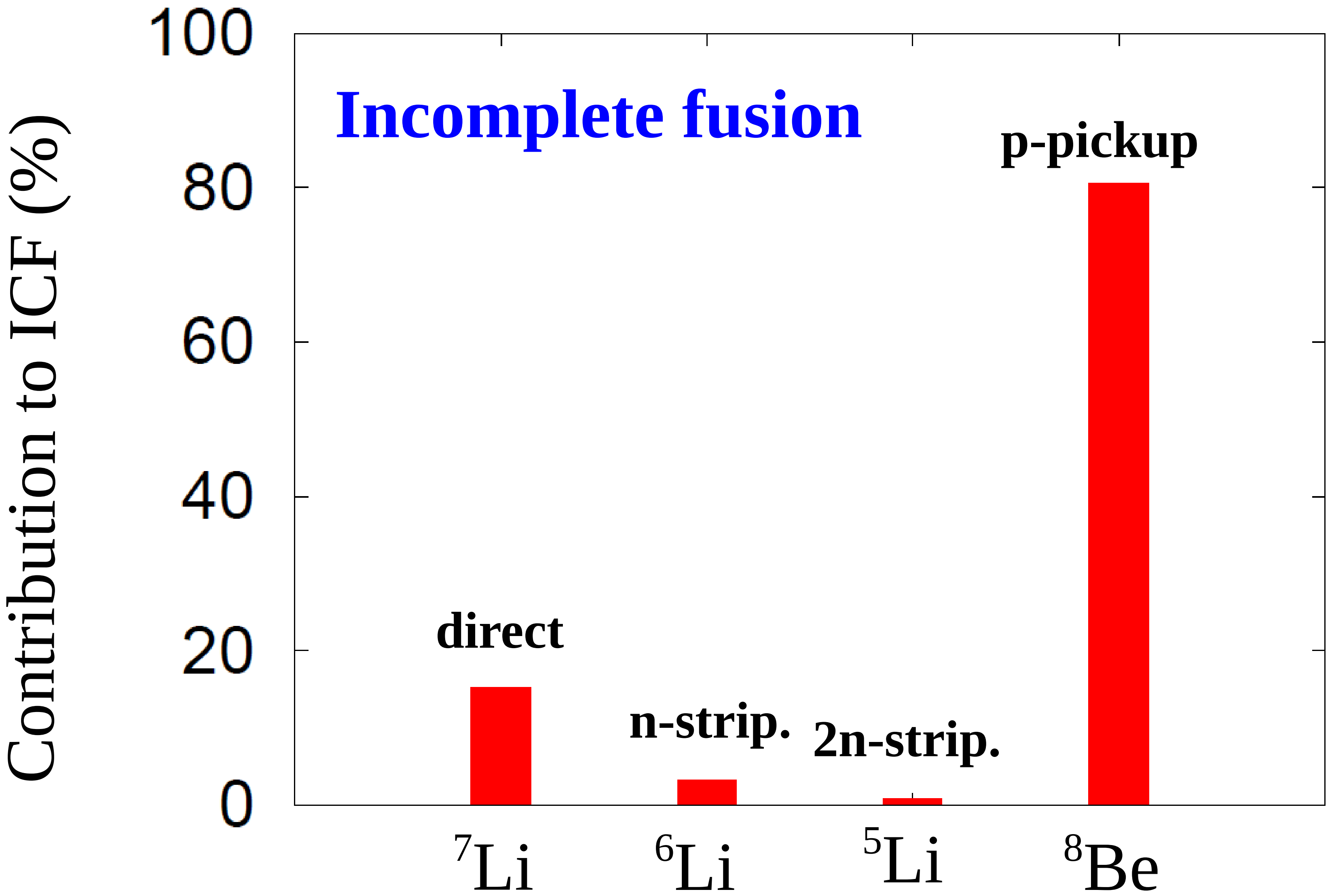


Luong *et al.*, Phys. Rev. C 88, 034609 (2013)

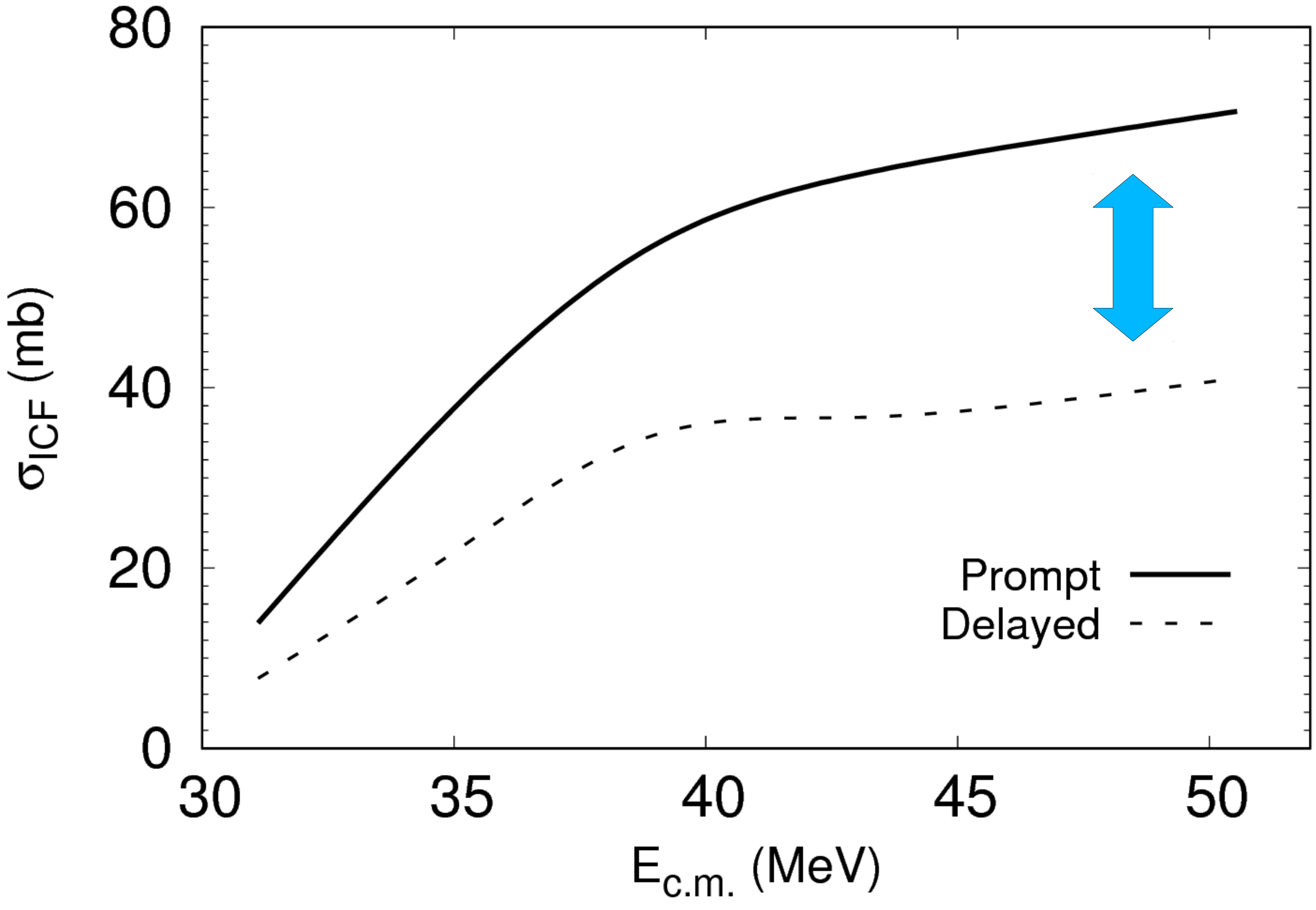
Courtesy of Ed Simpson (ANU)

Breakup triggered by transfer affects above-barrier fusion

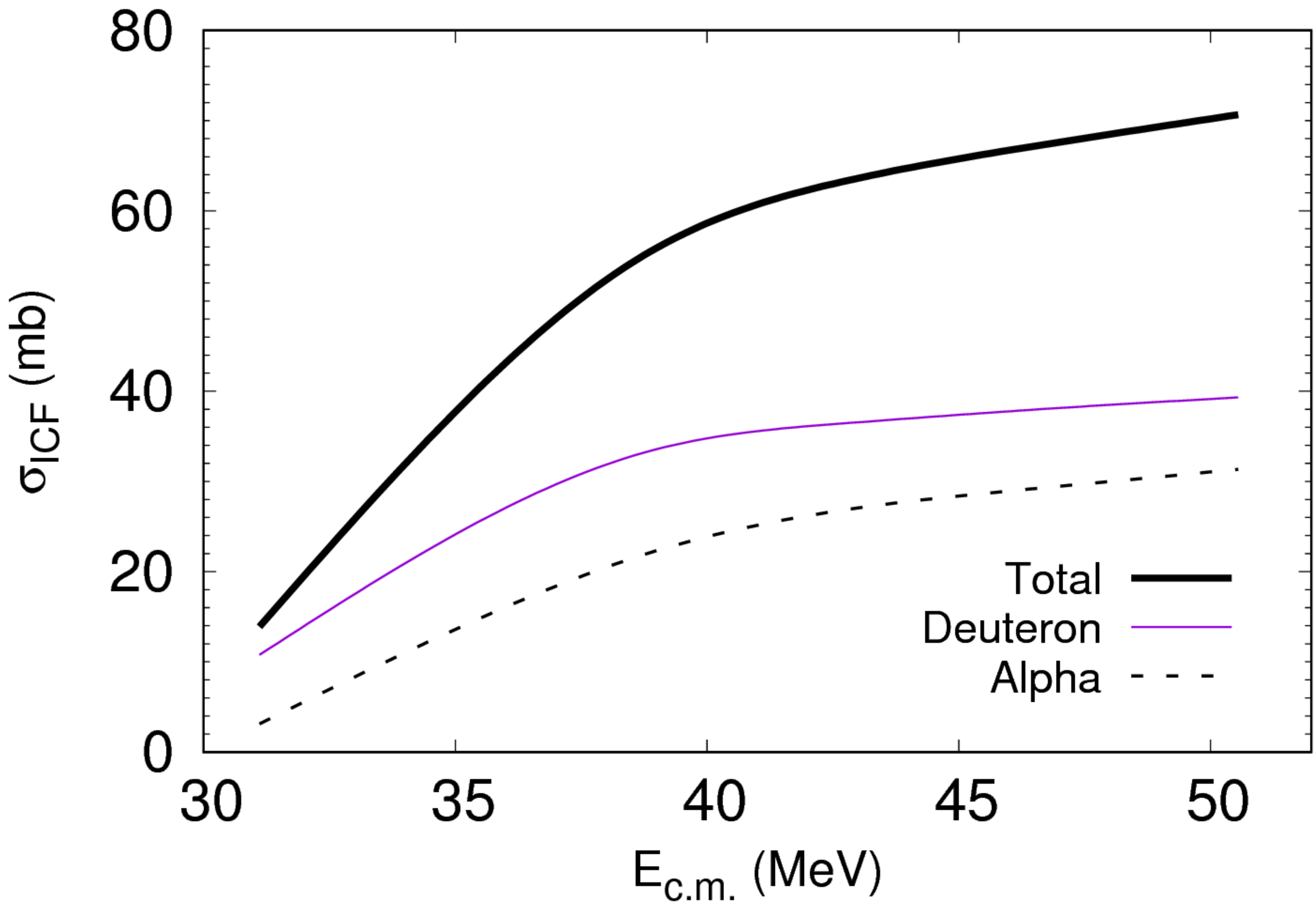
Example: ^{7}Li + ^{209}Bi @ Ecm/VB = 1.2



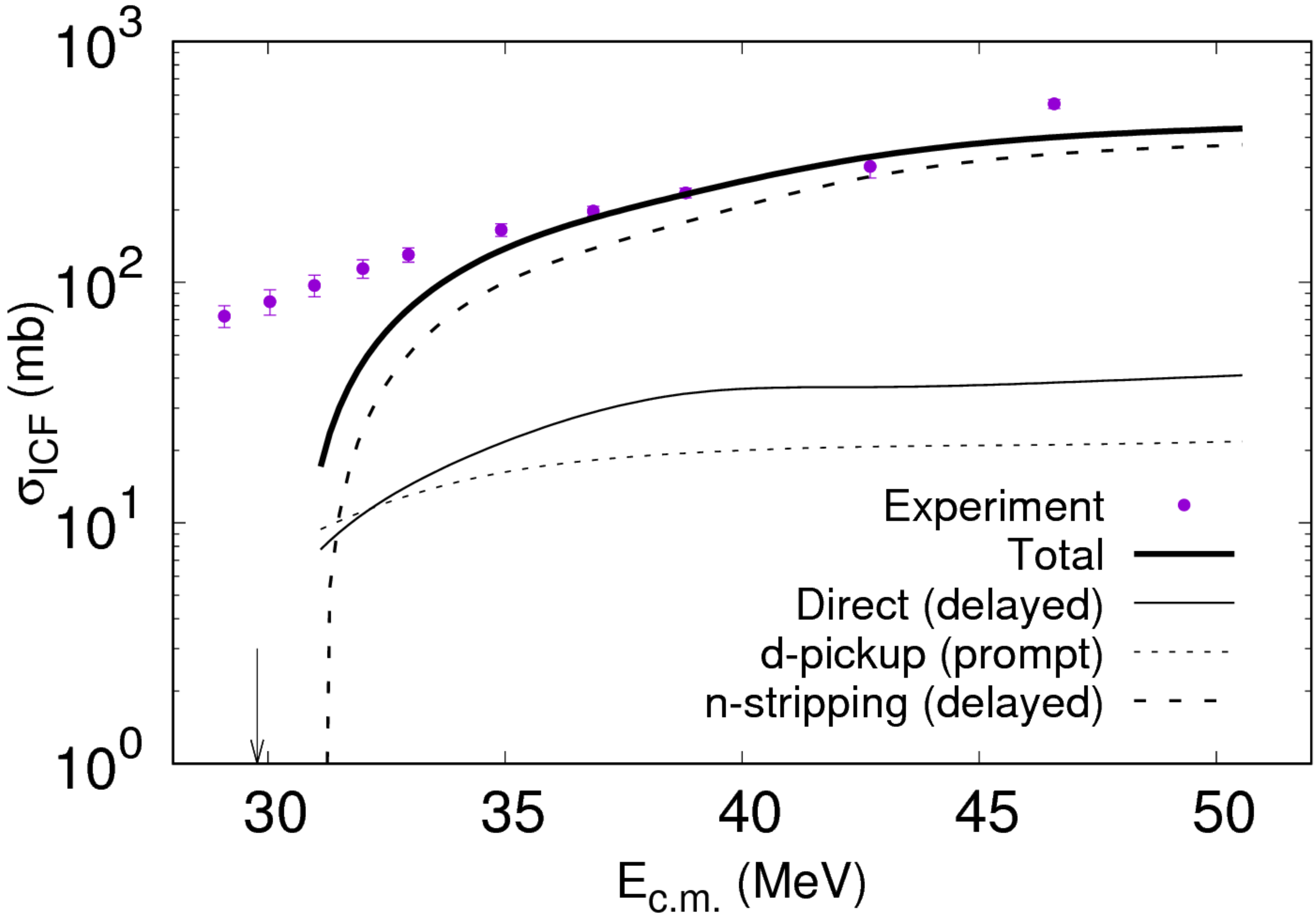
Effects of delayed breakup on incomplete fusion for $^{6}Li + ^{209}Bi$



Direct channels of incomplete fusion for $^{6}Li + ^{209}Bi$



Different channels of incomplete fusion for $^{6}Li + ^{209}Bi$



Time-Dependent Wave-Packet Dynamics

Useful for understanding sub-Coulomb fusion data

In collaboration with Maddalena Boselli,

who was my PhD student at the ECT*

Wave-Packet Dynamics

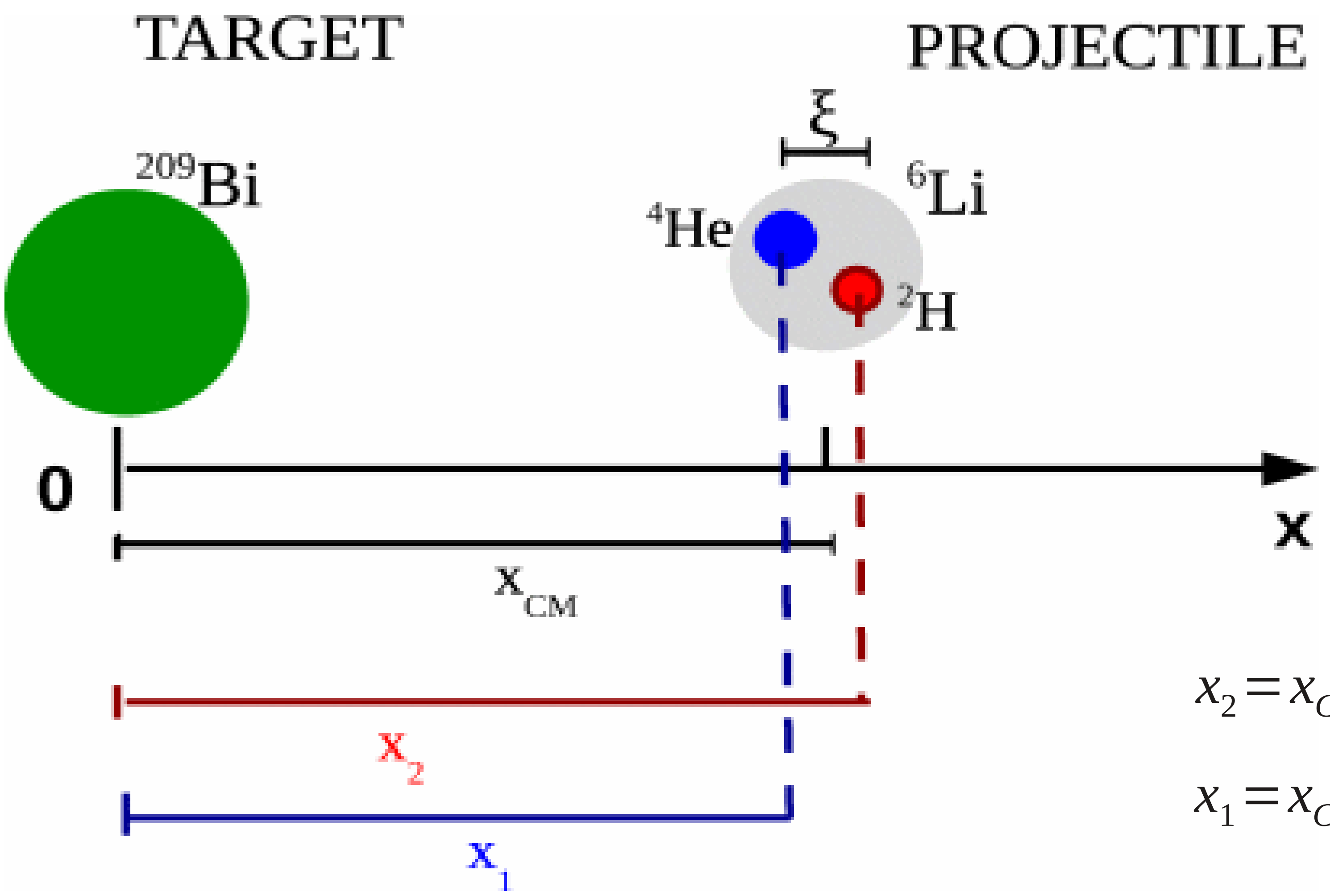
D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

- **Preparation:** the initial state $\Psi(t = 0)$

- **Time propagation:** $\Psi(0) \longrightarrow \Psi(t)$,
  guided by the operator, $\exp(-i\,\hat{H}\,t/\hbar)$
  $\hat{H}$ is the model Hamiltonian

- **Analysis:** extraction of probabilities from the time-dependent wave function

One-Dimensional Toy Model



$$x_2 = x_{CM} + b\,\xi$$

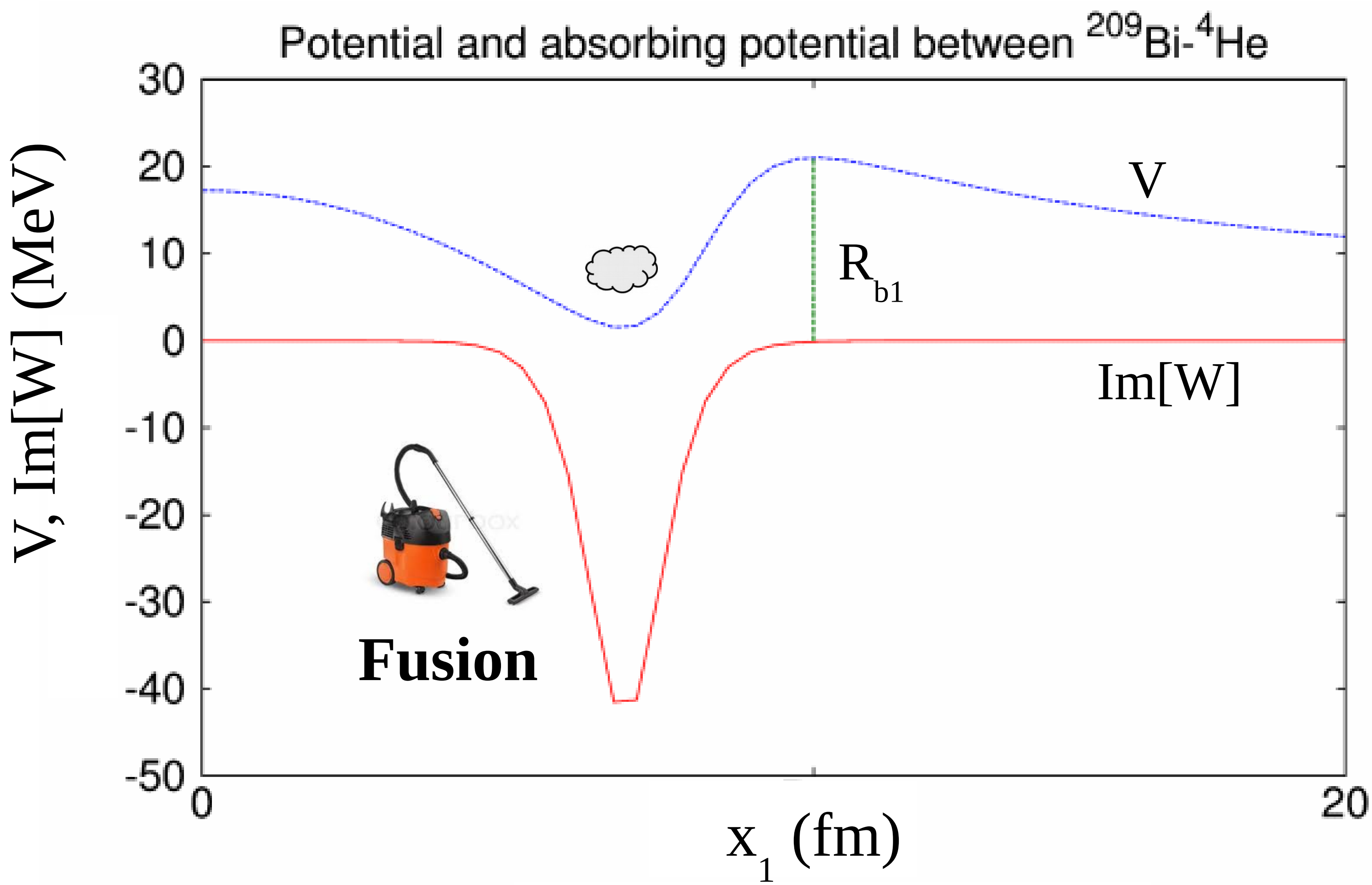
$$x_1 = x_{CM} - a\,\xi$$

$$H = \frac{P_{x_{CM}}^2}{2M_{T12}} + \frac{P_{\xi}^{\ 2}}{2m_{12}} + U_{12}(\xi) + V_{T1}(x_{CM} - a\,\xi) + V_{T2}(x_{CM} + b\,\xi)$$

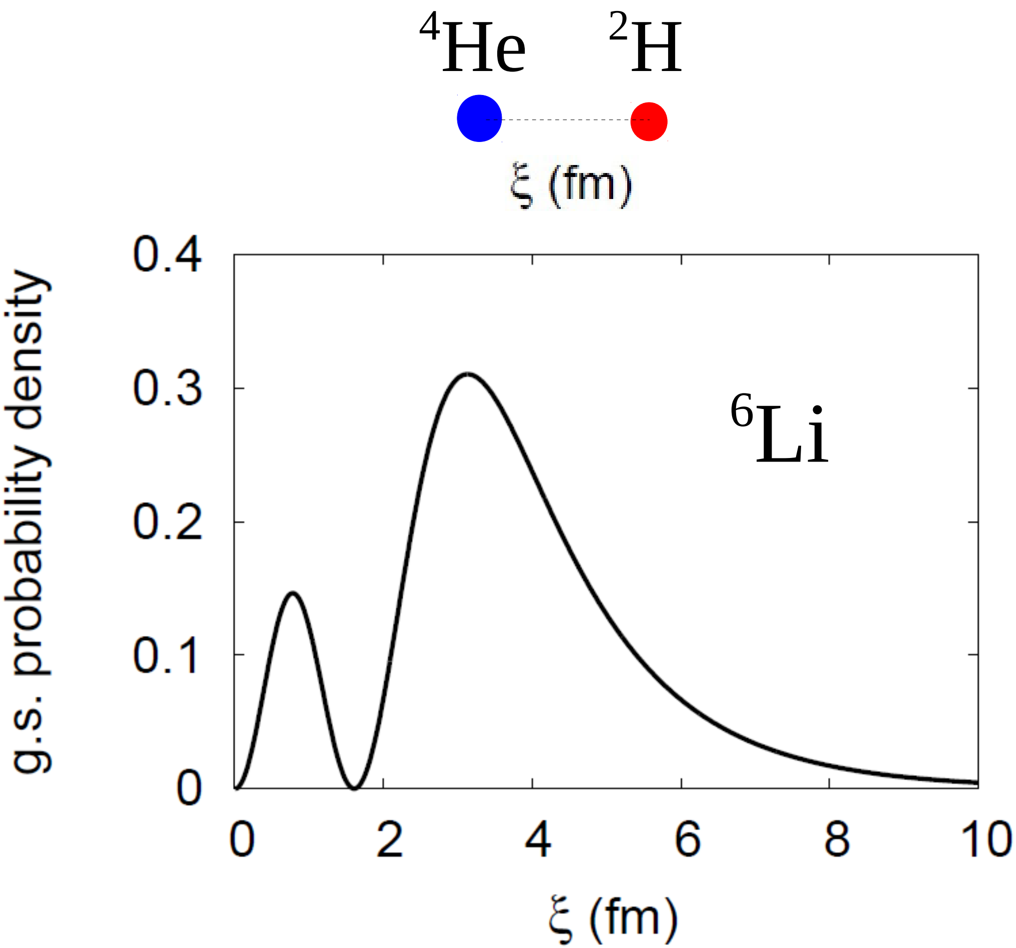
Describing Fusion

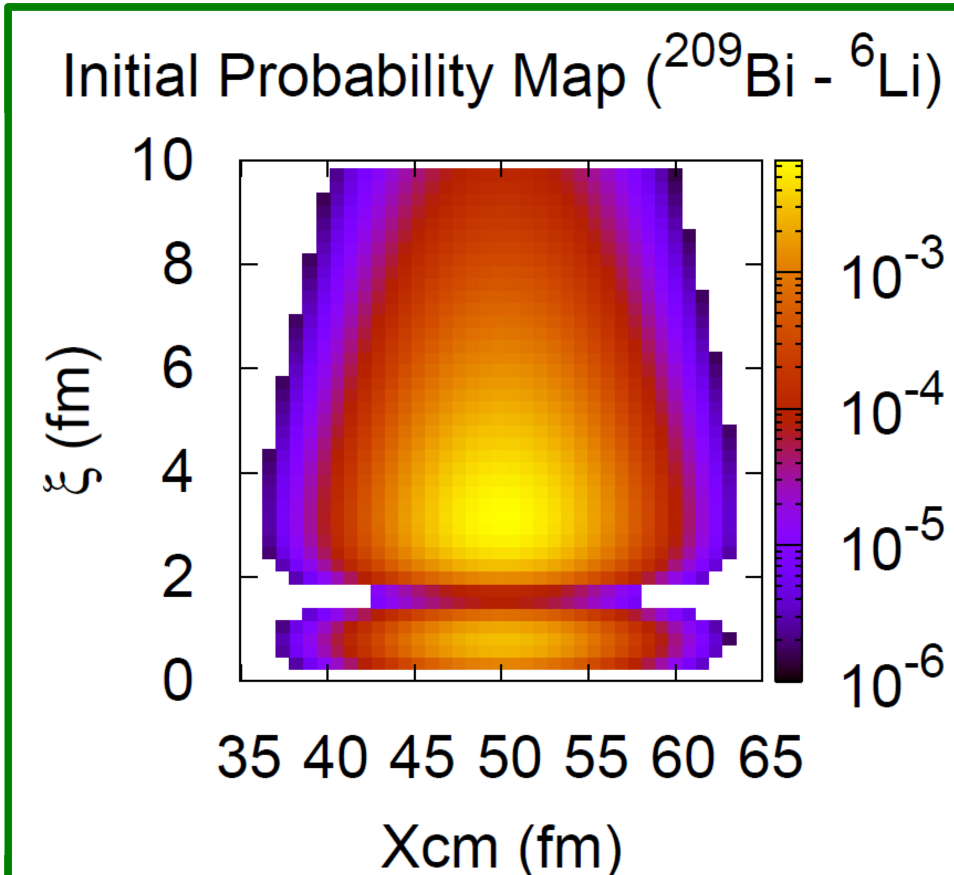
- To simulate fusion (**irreversibility**): acting inside the Coulomb barrier

$-iW_{T1}(x_1)$ & $-iW_{T2}(x_2)$



Preparing the Initial State





Time Propagation

R. Kosloff, Ann. Rev. Phys. Chem. **45** (1994) 145

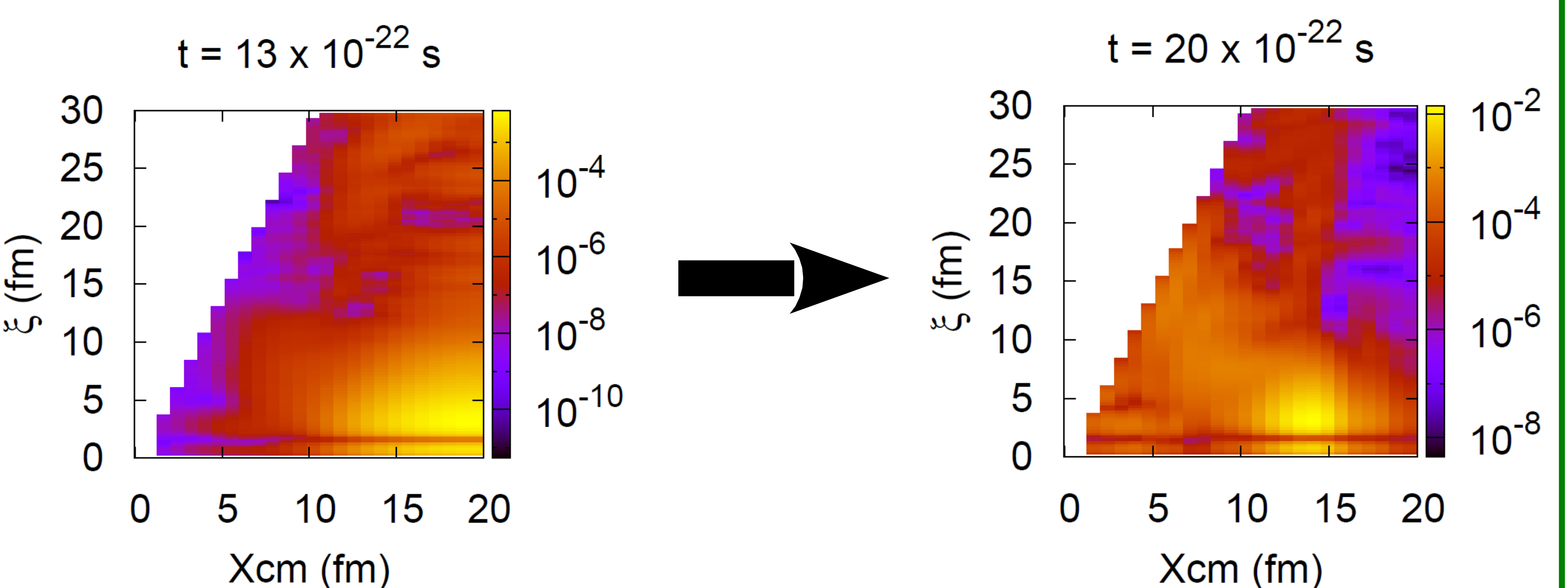
$$\Psi(t+\Delta t) = \exp\Big(-i\frac{\hat{H}\,\Delta t}{\hbar}\Big)\Psi(t)$$

$$\exp\Big(-i\frac{\hat{H}\,\Delta t}{\hbar}\Big) \approx \sum_n a_n\, Q_n(\hat{H}_{norm})$$

$$\hat{H}_{norm} = \frac{(\bar{H}\,\hat{1} - \hat{H})}{\Delta H}$$

The Chebyshev Propagator

$$a_n = i^n(2-\delta_{n0})\exp(-i\frac{\bar{H}\,\Delta t}{\hbar})\,J_n(\frac{\Delta H\,\Delta t}{\hbar})$$



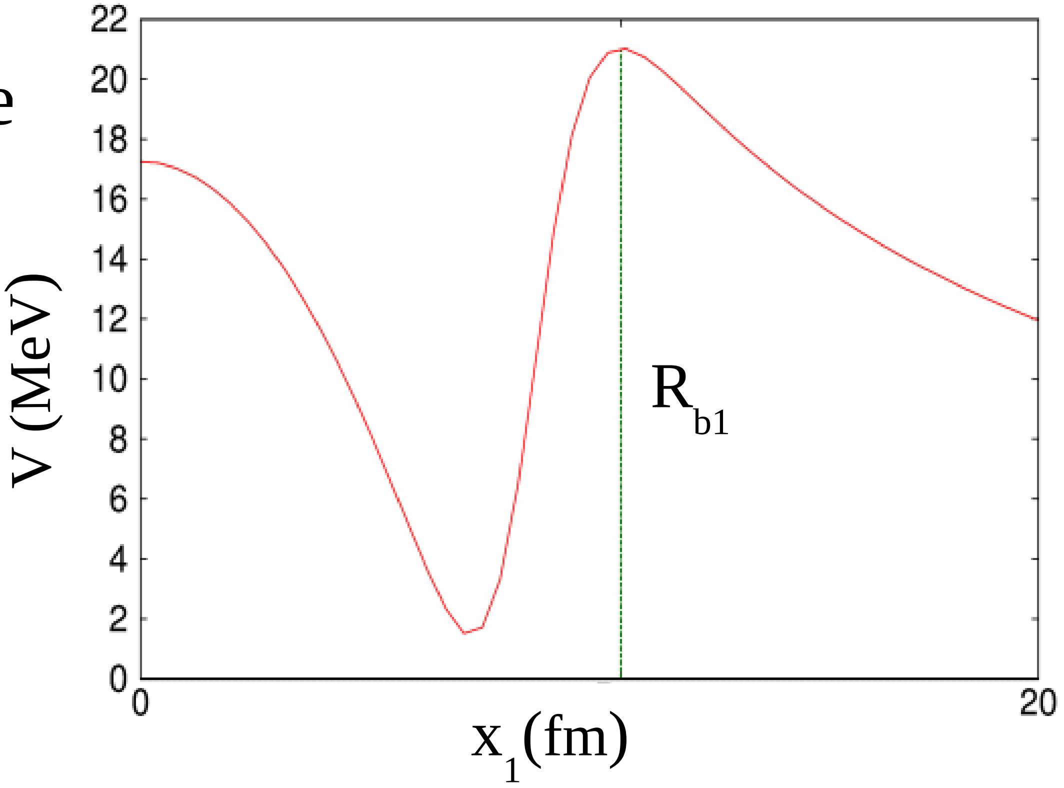
Slicing the Wave Function: A Novel Idea

- Projection operator acting on the position x_i of the fragment relative to the target

(Heaviside function)

$$P_i = \Theta(R_{bi} - x_i)$$
$$Q_i = 1 - P_i$$

Potential: ^{209}Bi - ^{4}He



	CAPTURED	NON CAPTURED
CF	● ●	
ICF	●	●

- Act with $(P_1+Q_1)(P_2+Q_2)=1$ on the wave function:

$$\tilde{\Psi}(x_1,x_2,t)=(P_1P_2+P_1Q_2+Q_1P_2+Q_1Q_2)\tilde{\Psi}(x_1,x_2,t)=\underline{\Psi_{CF}}+\underline{\Psi_{ICF}}+\underline{\Psi_{SCATT}}$$

Analysis of the Wave Function

Power Spectrum of the Wave Function

$$\mathcal{P}(E) = \langle \Psi(t) | \underbrace{\delta(E - \hat{H})}_{\text{Energy Projector}} | \Psi(t) \rangle$$

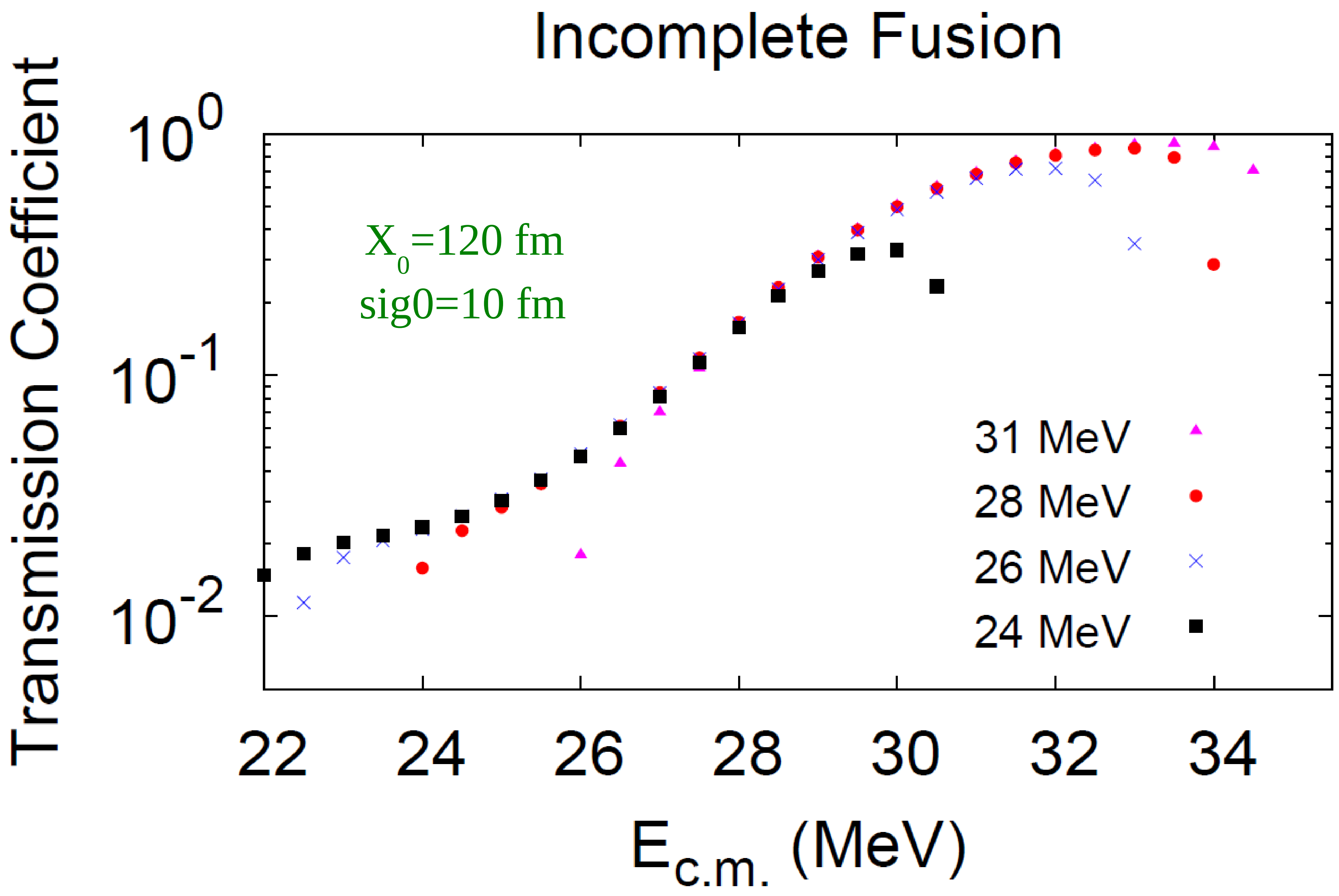
Energy Projector

Reflection & Transmission Coefficients

$$\mathcal{R}(E) = \frac{\mathcal{P}^{final}(E)}{\mathcal{P}^{initial}(E)}$$

$$T(E) = 1 - \mathcal{R}(E)$$

Example



Incomplete Fusion
Transmission Coefficient
10^0
10^{-1}
10^{-2}
X_0=120 fm
sig0=10 fm
31 MeV
28 MeV
26 MeV
24 MeV
22
24
26
28
30
32
34
$E_{c.m.}$ (MeV)

Summary

- **The time-dependent perspective** is useful for understanding and quantifying low-energy reaction dynamics of exotic nuclei.

- **PLATYPUS** is a powerful tool for planning and interpreting (fusion & breakup) measurements that allow its fine tuning.

AD-T, CPC **182** (2011) 1100

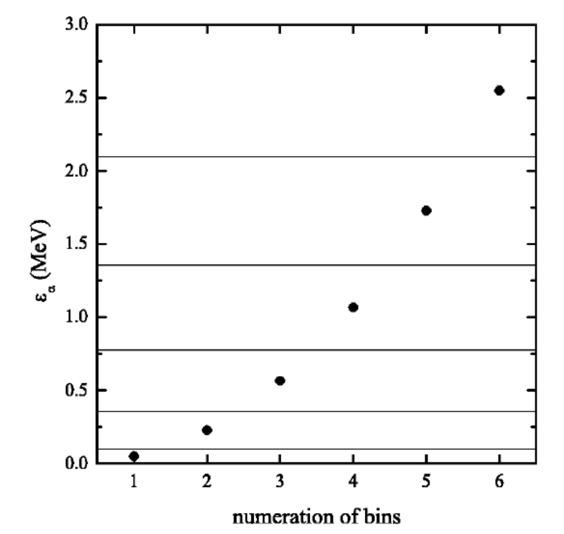
Outlook

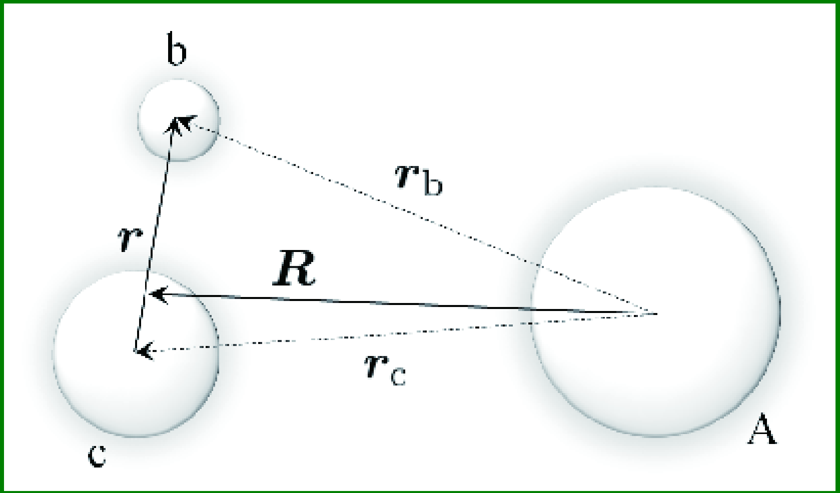
- **A quantum dynamical 3D-model** is being developed.

Maddalena Boselli & AD-T, PRC **92** (2015) 044610

Understanding the breakup mechanisms and their impact on unambiguously separated **CF & ICF** processes could make for further progress in the field.

EXTRA SLIDES





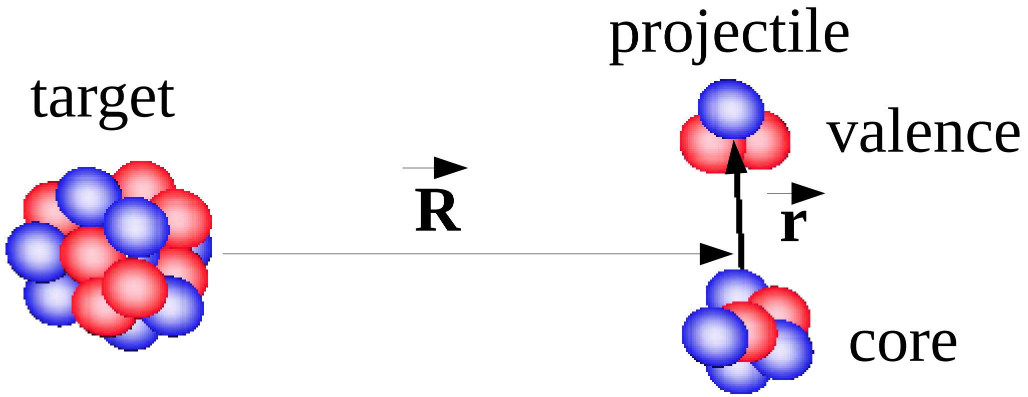
Continuum Discretised Coupled-Channels Method

- Continuum-continuum couplings reduce the fusion cross sections.
- CF and ICF cannot be separated unambiguosly.

See e.g., N. Austern, Physics Report 154 (1987) pp. 125-204.

CDCC Approach: Three-Body Model

Breakup mechanism: Inelastic excitation of the projectile to the continuum.



Potentials:
- Vtc, Vtv real Wood-Saxons
- Vcv Wood-Saxons with SO
- Imaginary short-ranged potentials

$$\sigma_{fus} = \frac{\pi}{2\mu E} \sum_J (2J+1) P_J ,$$

where

$$P_J \sim \sum_\alpha \int_0^\infty |f_{\alpha J}(R)|^2 \underbrace{(-W_F(R))}_{\langle W_1 + W_2 \rangle} dR$$

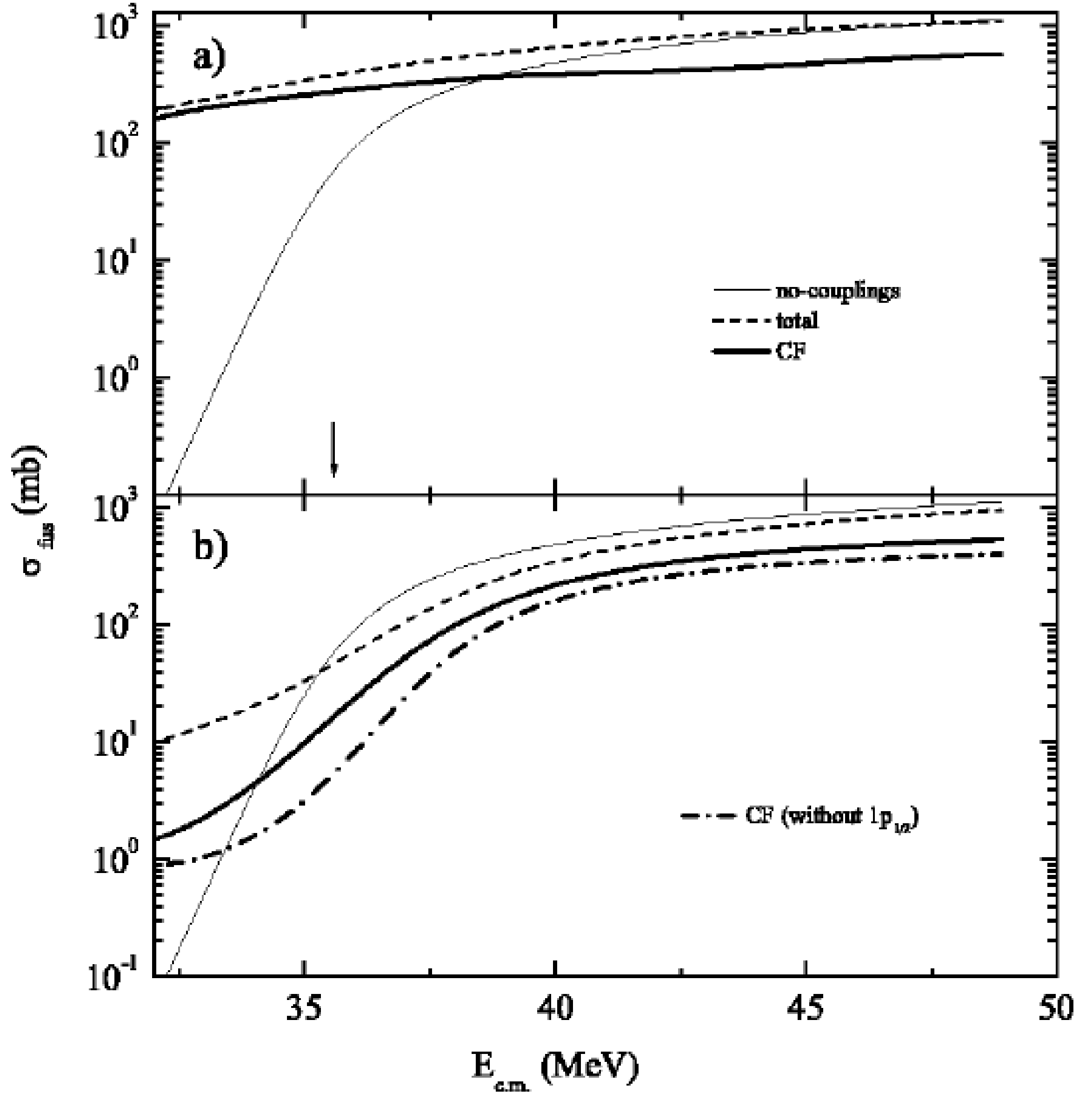
Aplications: $^{11}Be + ^{208}Pb$; $^{6,7}Li + ^{59}Co$, ^{209}Bi; $^{6}He + ^{59}Co$

CF and ICF cannot be separated unambiguously!

AD-T & Thompson, PRC 65 (2002) 024606

AD-T, Thompson & Beck, PRC 68 (2003) 044607

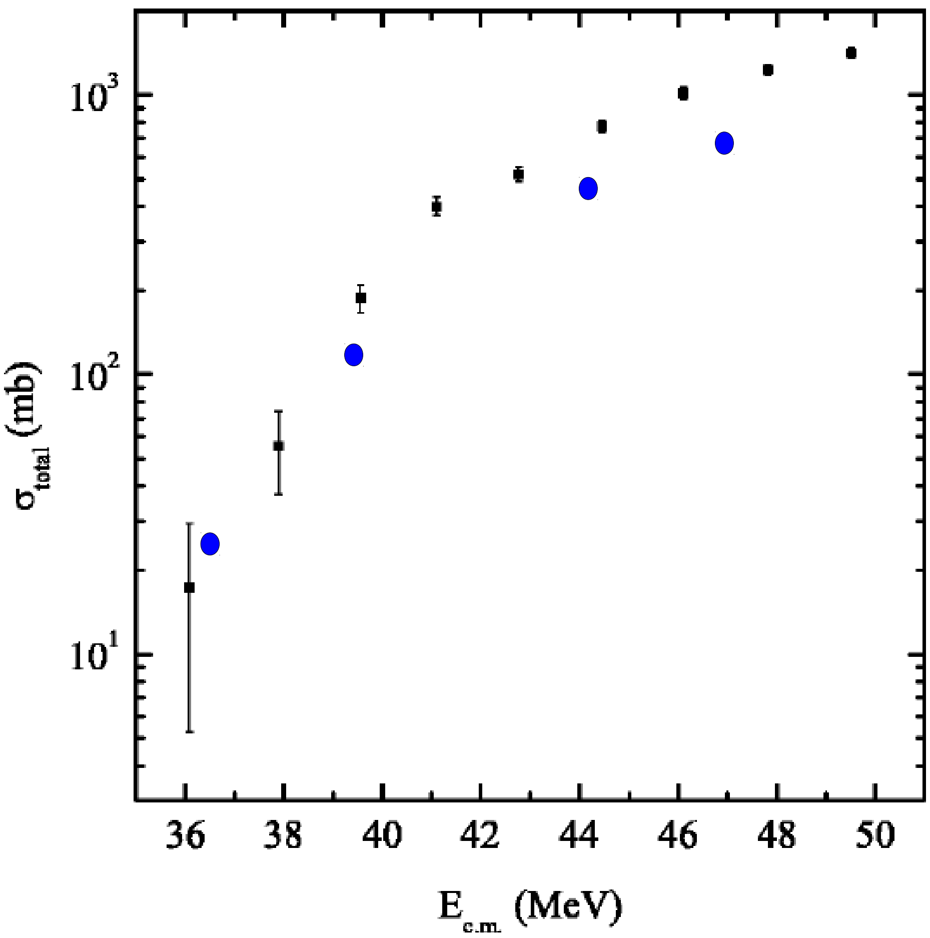
Effects of Continuum Couplings on Fusion



AD-T & Thompson, PRC 65 (2002) 024606

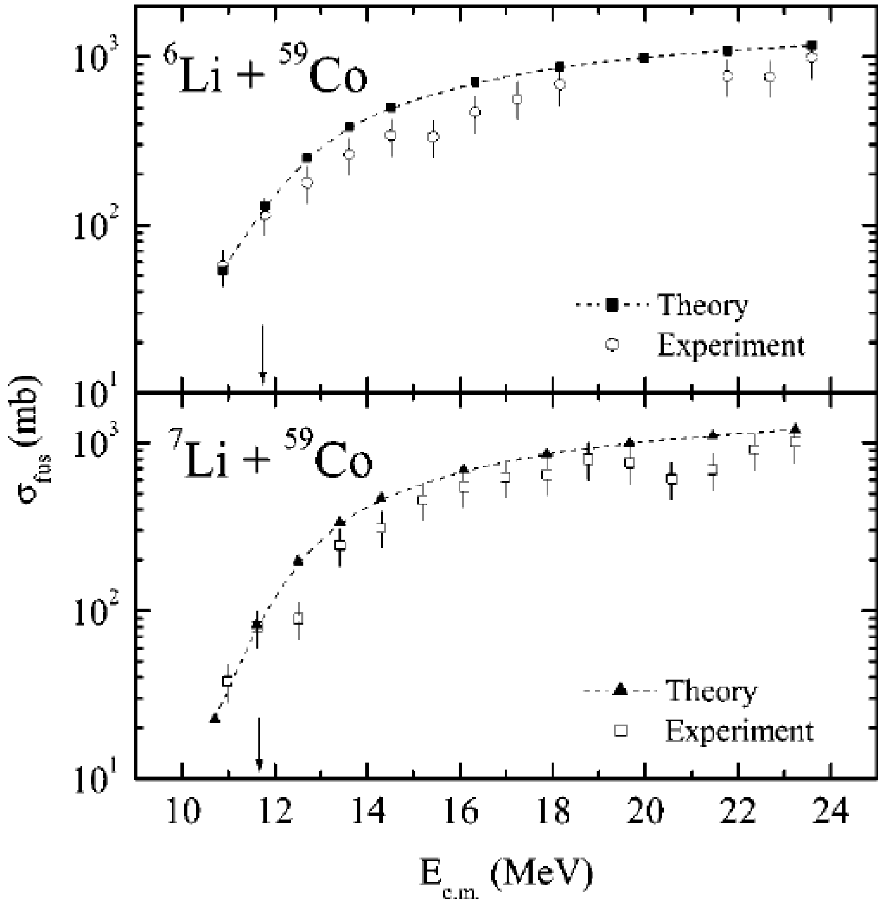
Total Fusion within the CDCC Approach

$^{11}Be + ^{208}Pb$



AD-T & Thompson,
PRC 65 (2002) 024606

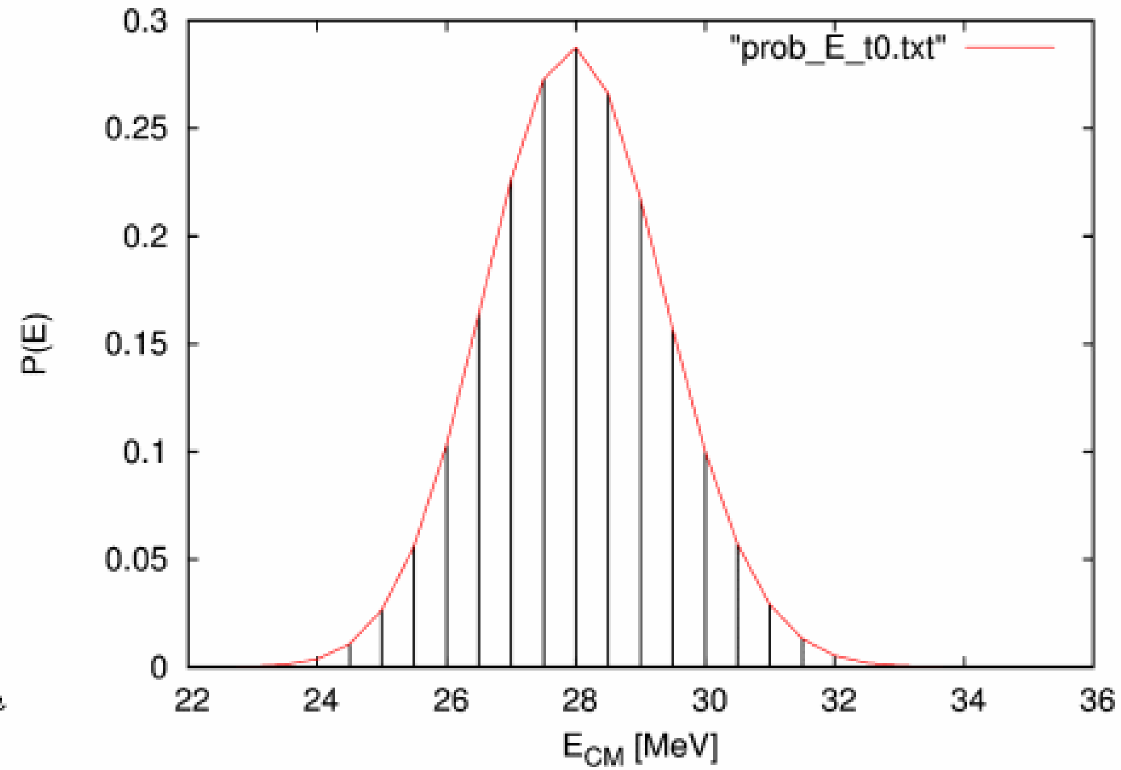
$^{6,7}Li + ^{59}Co$



AD-T, Thompson & Beck,
PRC 68 (2003) 044607

Energy Projection of the Wave Function

Schafer & Kulander,
PRA **42** (1990) 5794



- Energy spectra of $\Psi(t)$ at initial and final time as expectation values of the projection operator

$$\hat{\boldsymbol{\Delta}}(E_k, n, \epsilon) \equiv \frac{\epsilon^{2^n}}{(\hat{\mathcal{H}} - E_k)^{2^n} + \epsilon^{2^n}}$$

$$E_{k+1} = E_k + 2\epsilon$$

- $\mathcal{P}(E_k) = \langle \Psi | \hat{\Delta} | \Psi \rangle$, for instance, **n = 2** :

$$(\hat{\mathcal{H}} - E_k + \sqrt{i}\,\epsilon)(\hat{\mathcal{H}} - E_k - \sqrt{i}\,\epsilon)\,|\chi\rangle = |\Psi\rangle$$

$$\rightarrow \quad \mathcal{P}(E_k) = \epsilon^4 \langle \chi | \chi \rangle$$

Transmission & Reflection Coefficients

$$\mathcal{T}(E_k) = 1 - \mathcal{R}(E_k)$$

$$\mathcal{R}(E_k) = \frac{\mathcal{P}^{final}(E_k)}{\mathcal{P}^{initial}(E_k)}$$

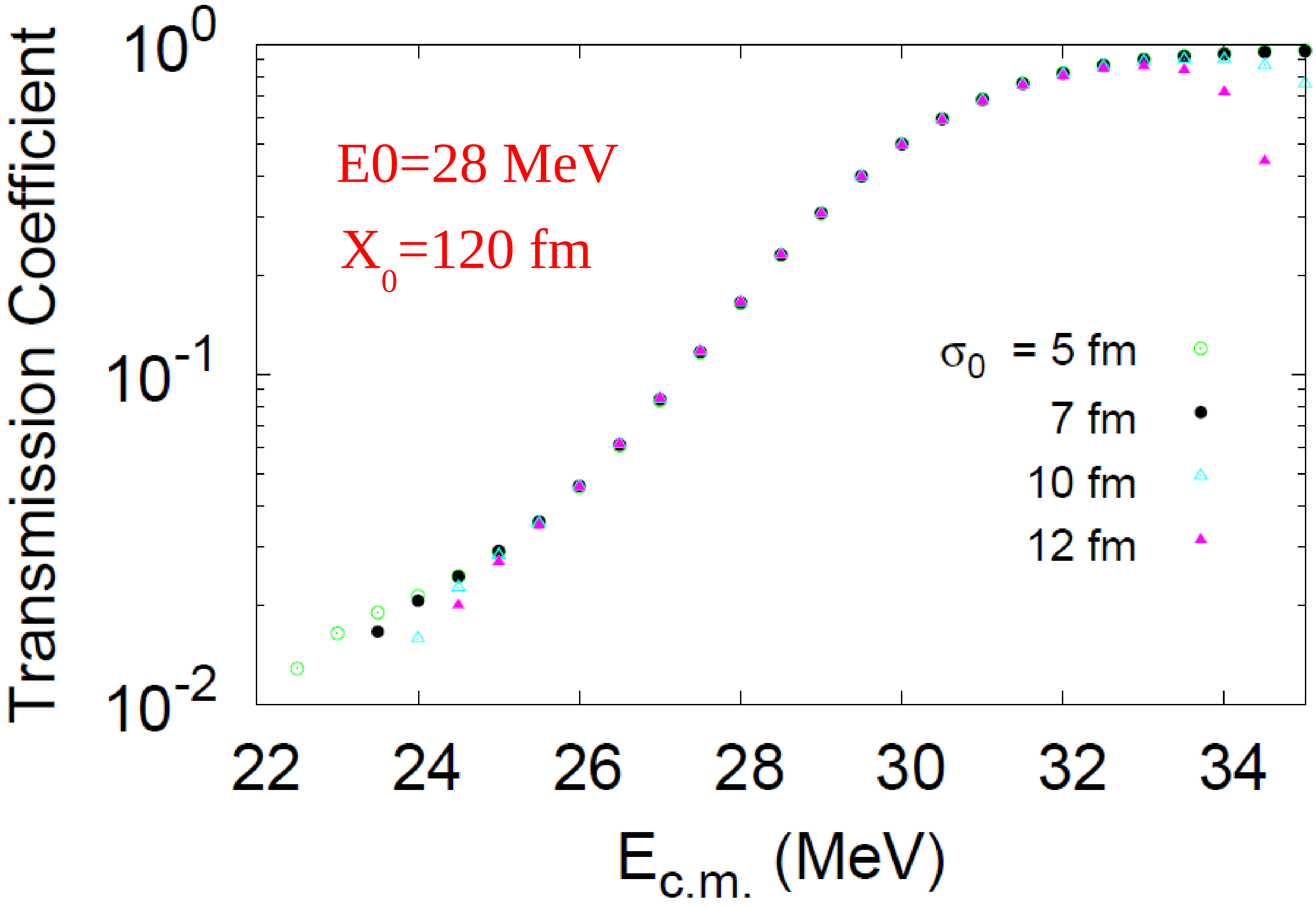
$$\mathcal{P}(E_k) = \langle \Psi | \hat{\Delta} | \Psi \rangle$$

$$\hat{\Delta}(E_k, n, \epsilon) \equiv \frac{\epsilon^{2^n}}{(\hat{\mathcal{H}} - E_k)^{2^n} + \epsilon^{2^n}}$$

$$E_{k+1} = E_k + 2\epsilon$$

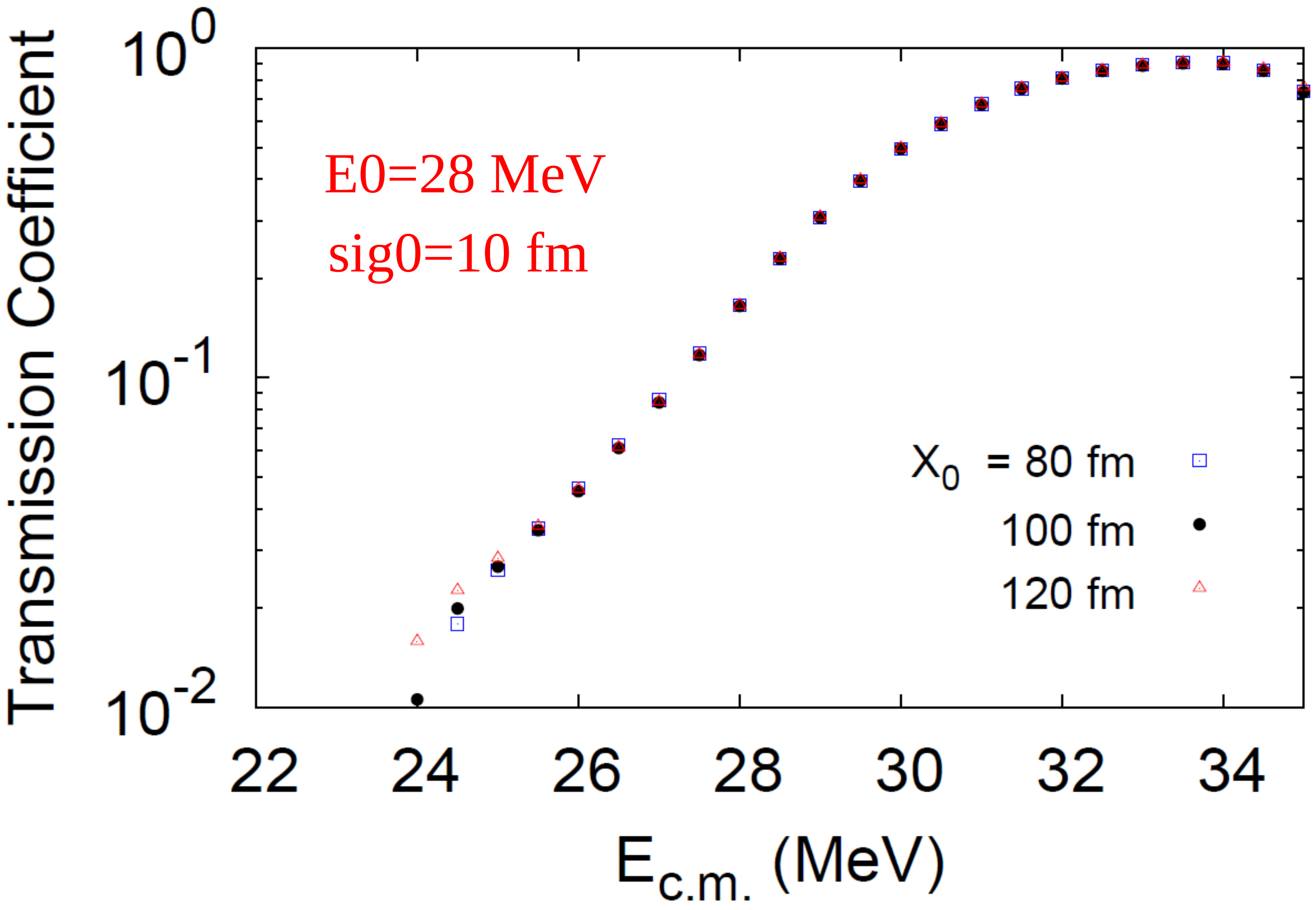
Results

♦ Energy-resolved **total transmission** for different values of the **width** of the initial wave packet



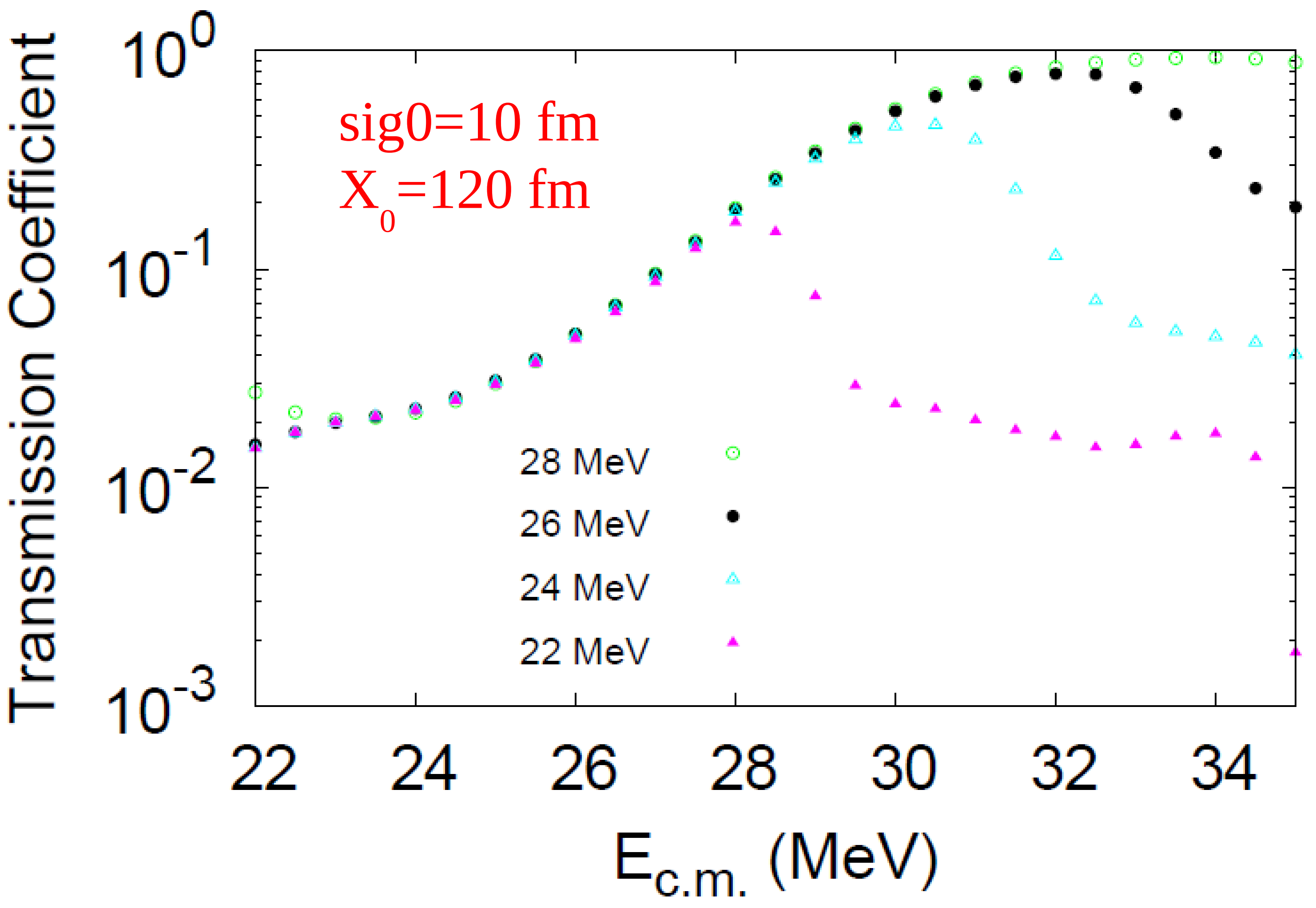
Results

♦ Energy-resolved **total transmission** for different values of the **location** of the initial wave packet



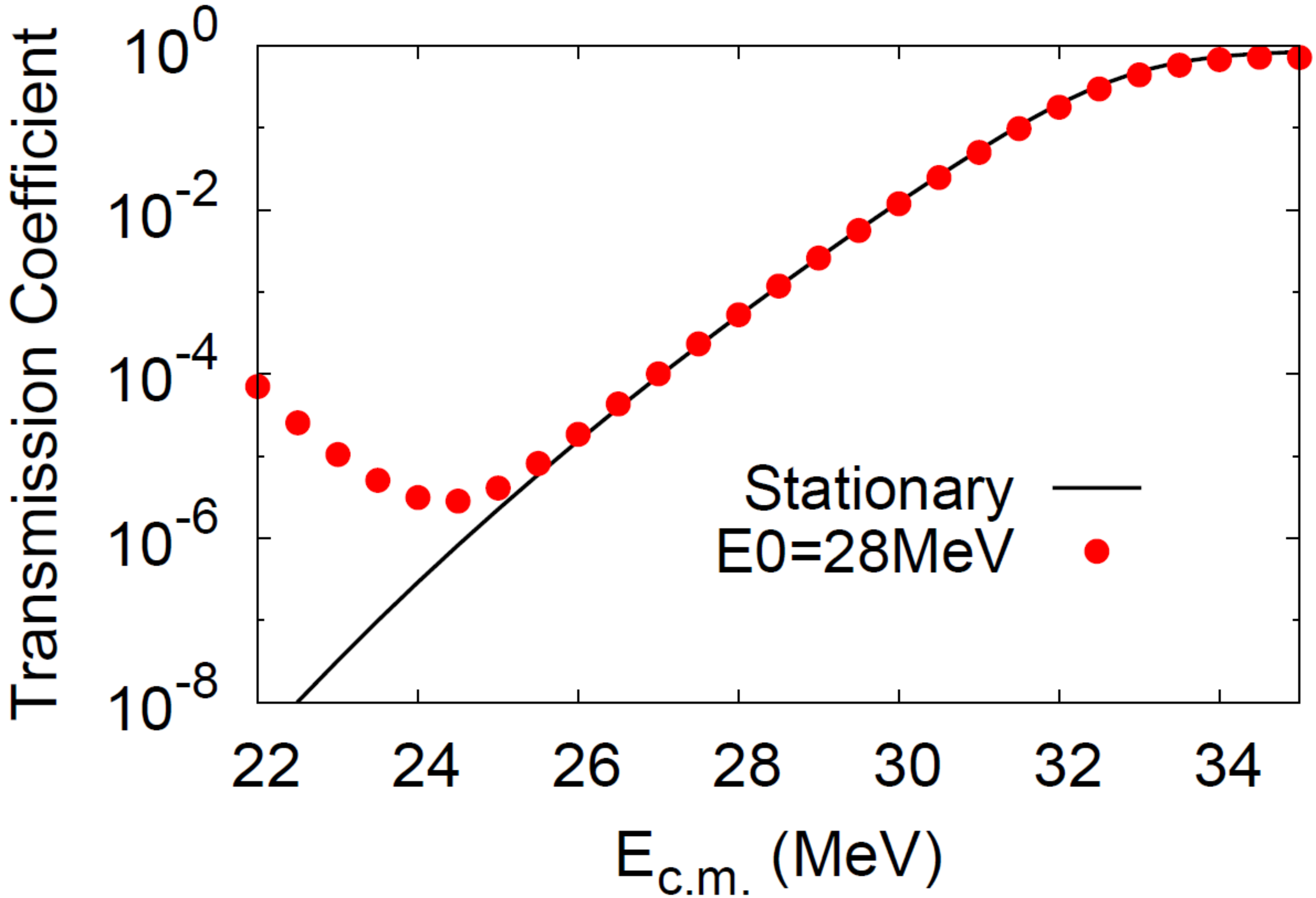
Results

♦ Energy-resolved **total transmission** for different values of the **mean energy** of the initial wave packet



Results

♦ Transmission coefficients compared with those obtained from a time-independent calculation



Breakup triggered by transfer affects **above-barrier** fusion

Example: ^{7}**Li** + ^{209}**Bi** @ Ec.m. = 36 MeV (about 1.2 times the SP barrier)

PRELIMINARY RESULTS FROM **PLATYPUS**

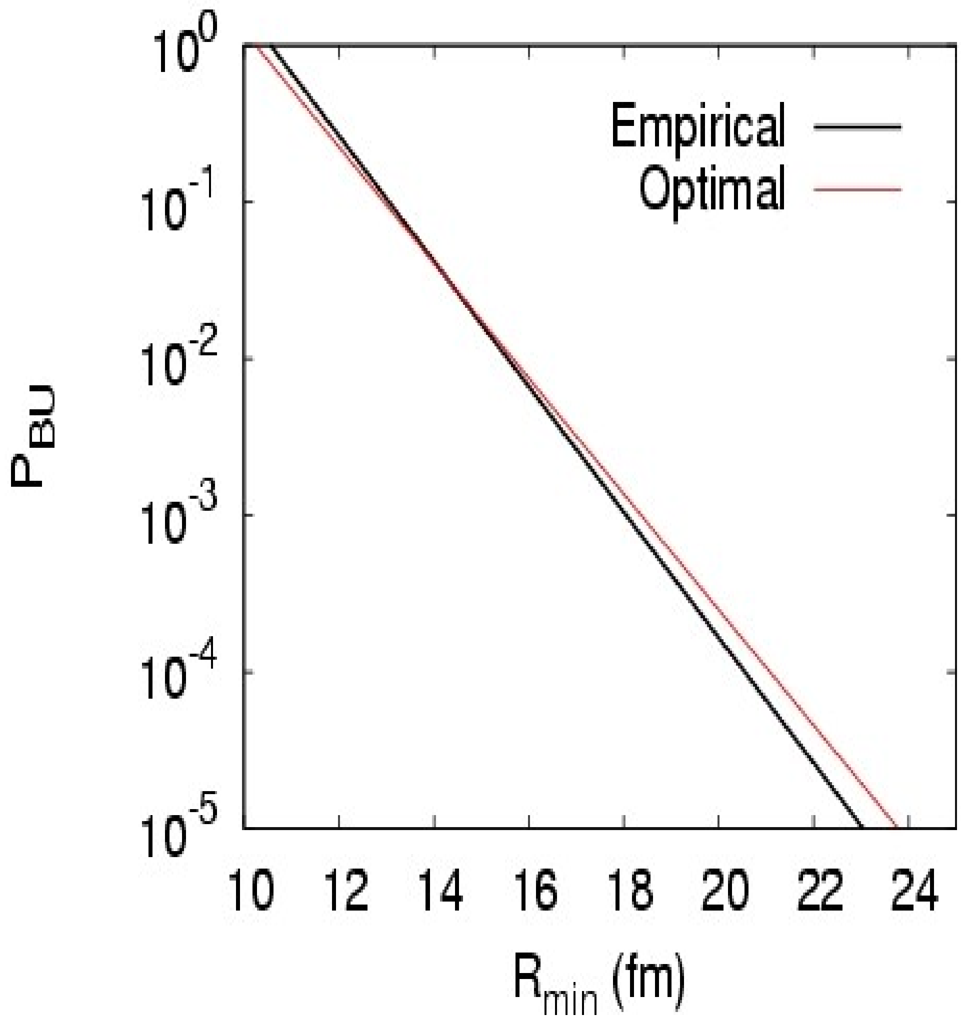
Processes	Reactions	ICF(mb)	CF$^{\cdot}$(mb)	CF$^{seq.}$(mb)	NCBU(mb)
direct	^{7}Li + ^{209}Bi	10.46	687.2	2.48	10.53
n-stripping	^{6}Li + ^{210}Bi	2.24	283.2	0.51	2.4
2n-stripping	^{5}Li + ^{211}Bi	0.67 **0.49**		5.9 x 10^{-2} **1.9 x 10^{-2}**	3.5 3.9
p-pickup	^{8}Be + ^{208}Pb	**55.5**	**767.1**	**12.01**	**42.81**

Prompt vs. delayed breakup of the ^{5}Li projectile-like nucleus

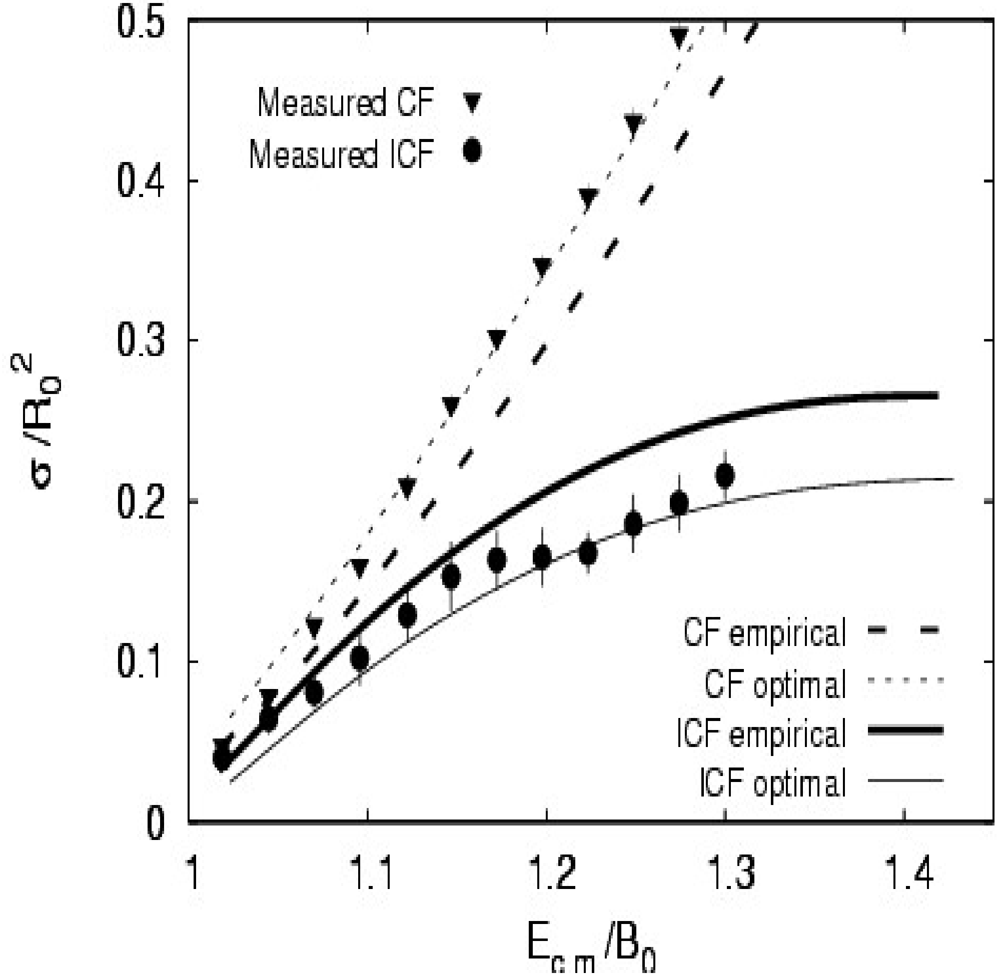
In collaboration with Huy Luong *et al.* (ANU)

Linking breakup and fusion: " ^{8}Be " + ^{208}Pb

Breakup function



CF & ICF excitation functions



AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL 98 (2007) 152701

Classical Dynamical Model

PLATYPUS code

AD-T, CPC 182 (2011) 1100

Main aspects of the approach:

- Projectile-target interaction
- Encoding of breakup
- Initial conditions of breakup events
- Time propagation of breakup fragments & target
- Probabilities and cross sections

AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL 98 (2007) 152701

AD-T, JPG 37 (2010) 075109

Matching Reaction Stages

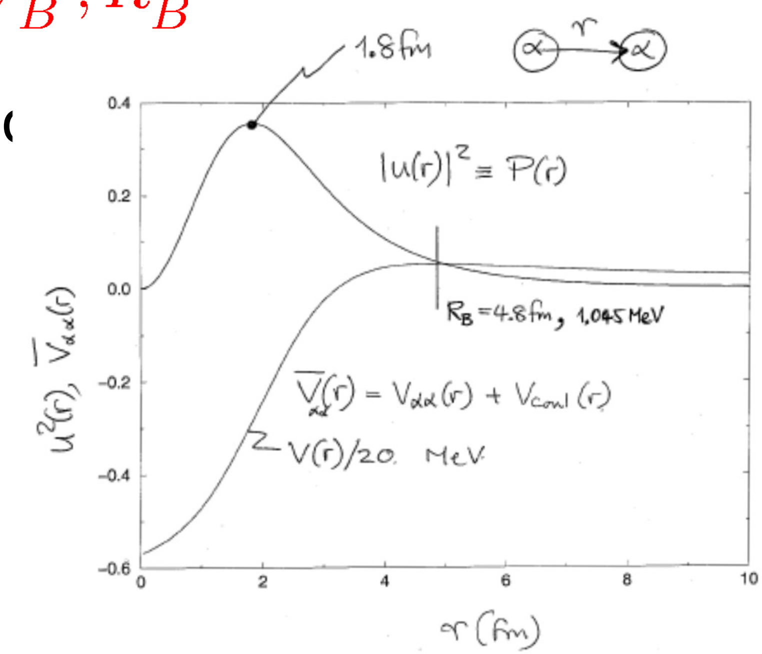
Prior breakup: For each L_0 a sample of N incident projectiles is taken.
R_{BU} is sampled on the interval $[R_{min}(E_0,L_0),\infty]$ with the weighting P^L_{BU}(F

After breakup: fragments F_1 and F_2 interact with T and with each oth

$$i,j = 1,2,T \;;\; (i \neq j) \;\rightarrow\; V_B^{ij}, R_B^{ij}$$

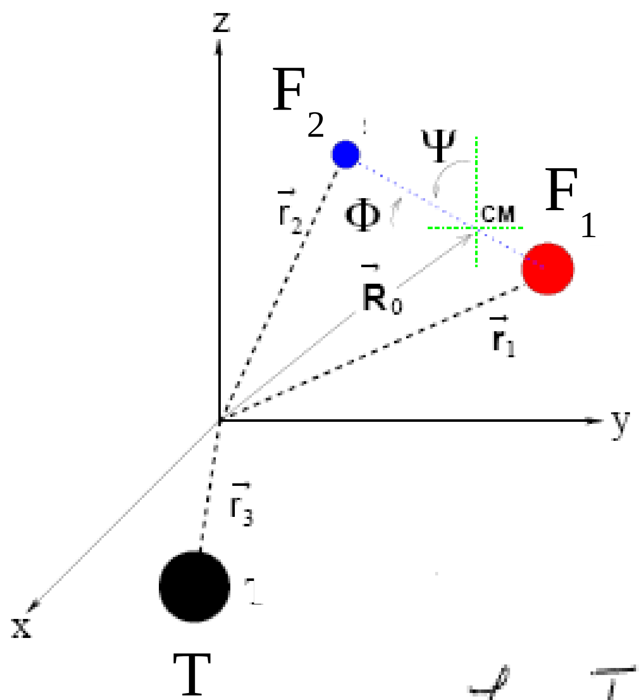
The dynamical variables o
the excited projectile at
breakup are Monte Carlo
sampled as well

$$\epsilon_{12} \;;\; \vec{\ell}_{12} \;;\; \vec{d}_{12}$$



Conservation laws of linear momentum, angular momentum and en
(in the overall CM reference frame) determine the initial conditions
for the three-body propagation in time

Time Propagation



Fragment F_j is assumed to be captured if the classical trajectories take it within the fragment-target barrier radius

(CF, ICF and NCBU events)

$$\mathcal{L} = T - U, \quad \text{where} \quad T = \sum_{i=1}^{3} \frac{\mu_i}{2}\left(\dot{r}_i^2 + r_i^2\dot{\phi}_i^2 + r_i^2 \sin^2\phi_i\, \dot{\varphi}_i^2\right)$$

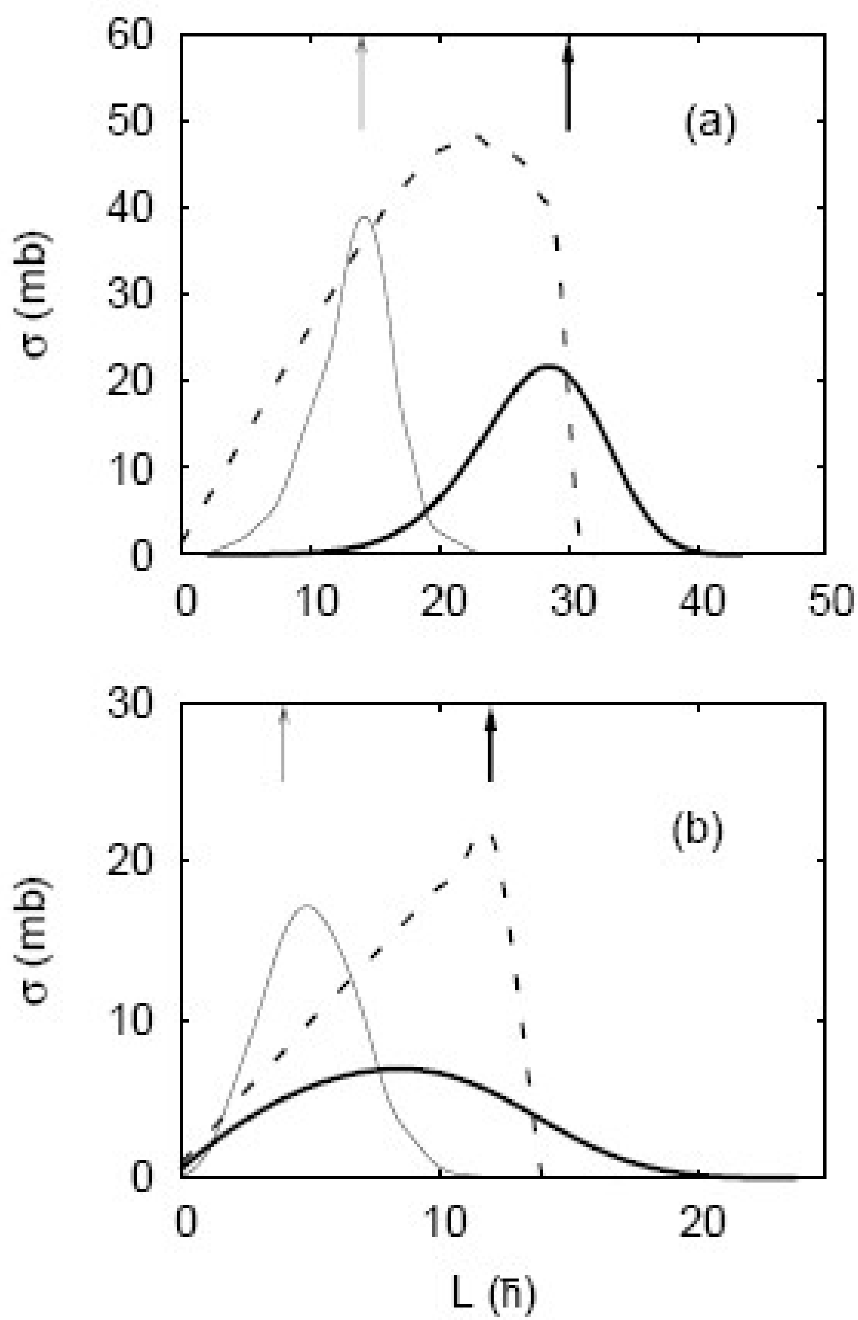
$$U = \sum_{\substack{i,j \\ (i<j)}}^{3} V_{ij}(|\vec{r}_{ij}|)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0, \quad q_i = \{r_i, \phi_i, \varphi_i, \ i = 1,3\}$$

Spin distribution of fusion products: ^{216}Rn & ^{212}Po

Dashed line: **CF**

Solid line: **ICF**



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

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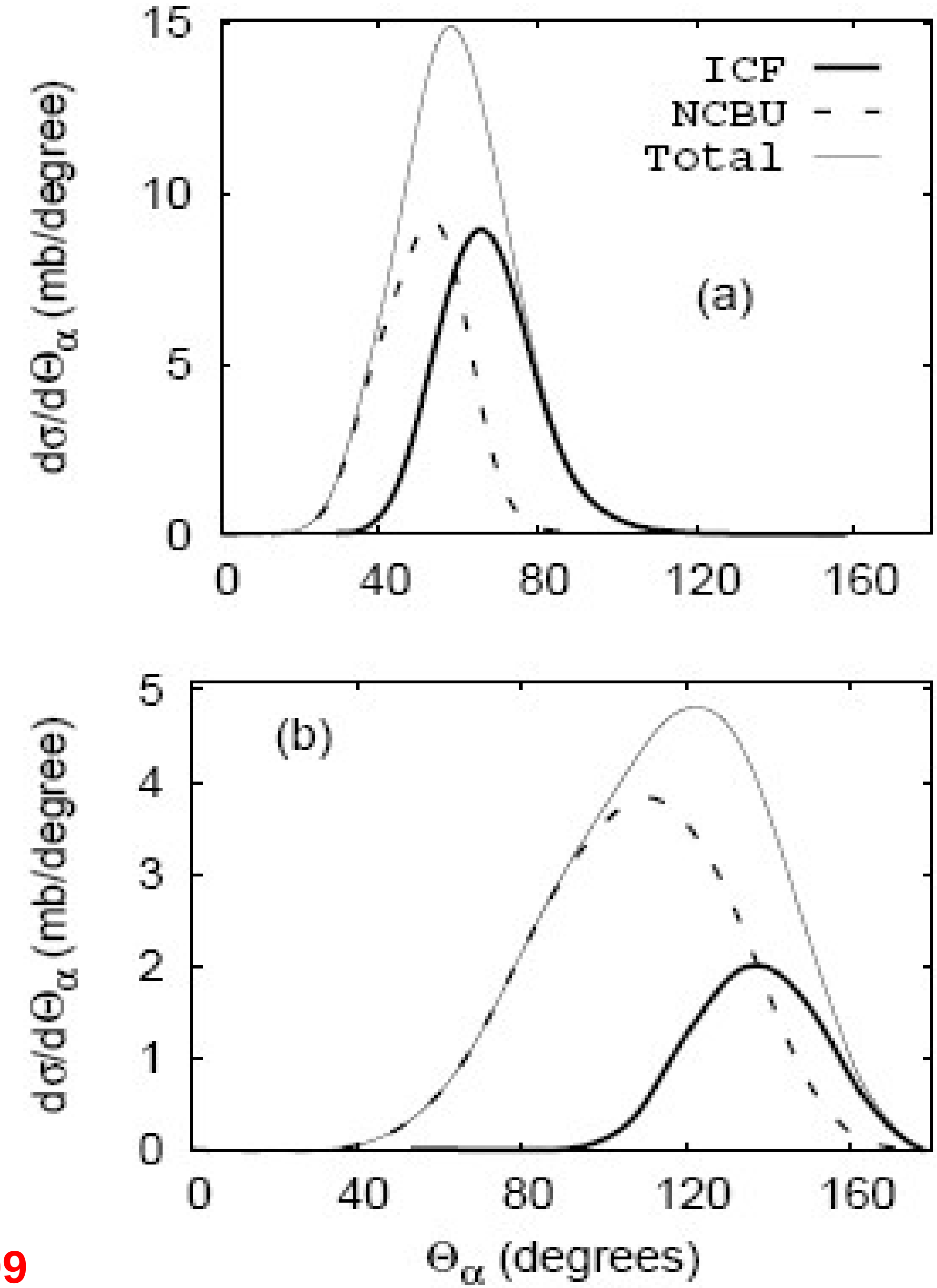
Gasques et al,
PRC **74** (2006) 064615

Direct alpha-production yields: “ ^{8}Be ” + ^{208}Pb



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

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Breakup probability function

Let us define two probabilities: (i) the probability of breakup between R and $R + dR$, $\rho(R)dR$ [being $\rho(R)$ a density of probability], and (ii) the probability of the weakly-bound projectile's survival from ∞ to R, $S(R)$. The survival probability at $R + dR$, $S(R + dR)$, can be written as follows

$$S(R + dR) = S(R)\,[1 - \rho(R)dR]. \tag{A.1}$$

Expression (A.1) suggests the following differential equation for the survival probability $S(R)$,

$$\frac{dS(R)}{dR} = -S(R)\,\rho(R), \tag{A.2}$$

whose solution is $[S(\infty) = 1]$:

$$S(R) = \exp(-\int_{\infty}^{R} \rho(R)dR). \tag{A.3}$$

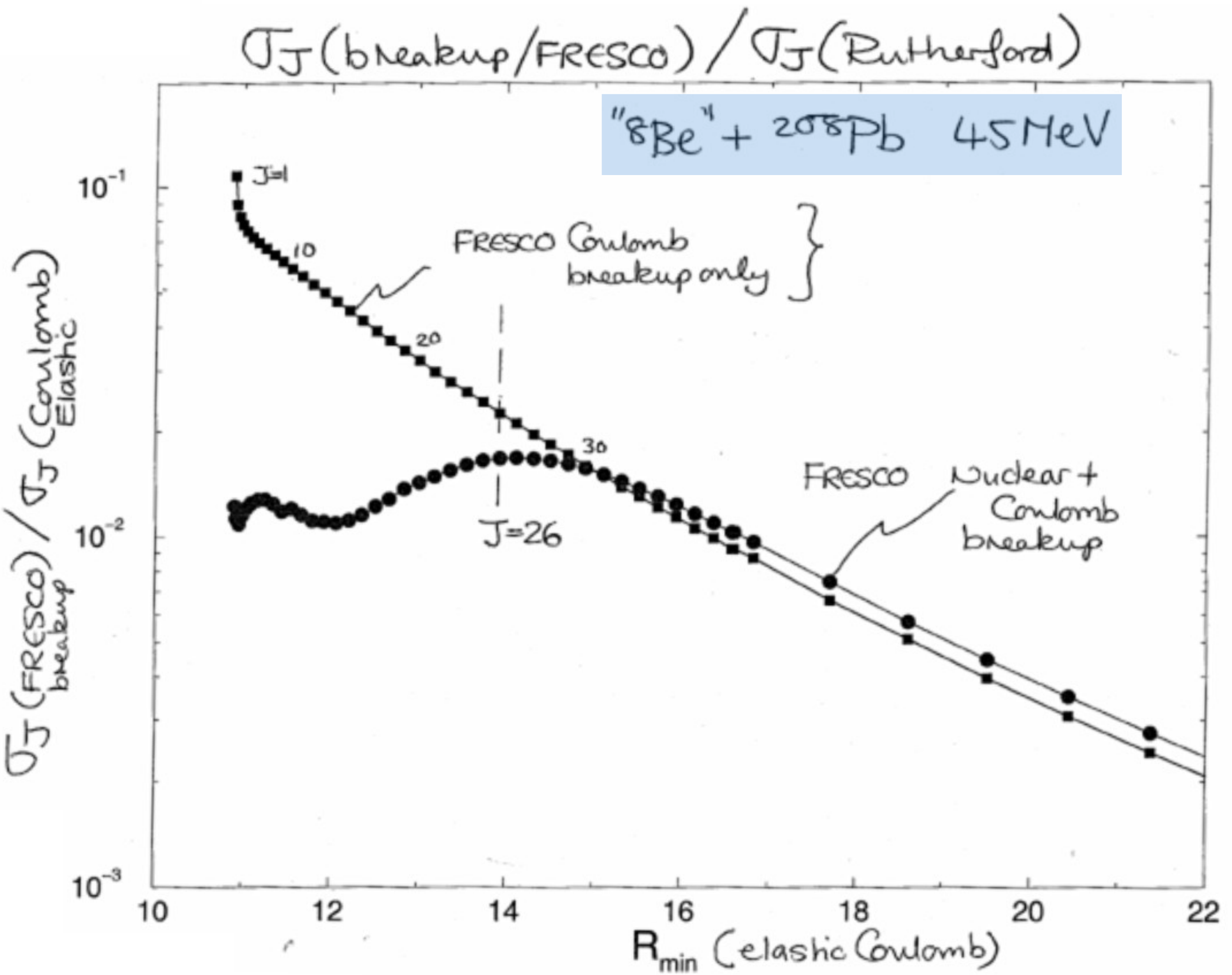
From (A.3), the breakup probability at R, $B(R) = 1 - S(R)$. If $\int_{\infty}^{R} \rho(R)dR \ll 1$, $B(R)$ can be written as

$$B(R) \approx \int_{\infty}^{R} \rho(R)dR. \tag{A.4}$$

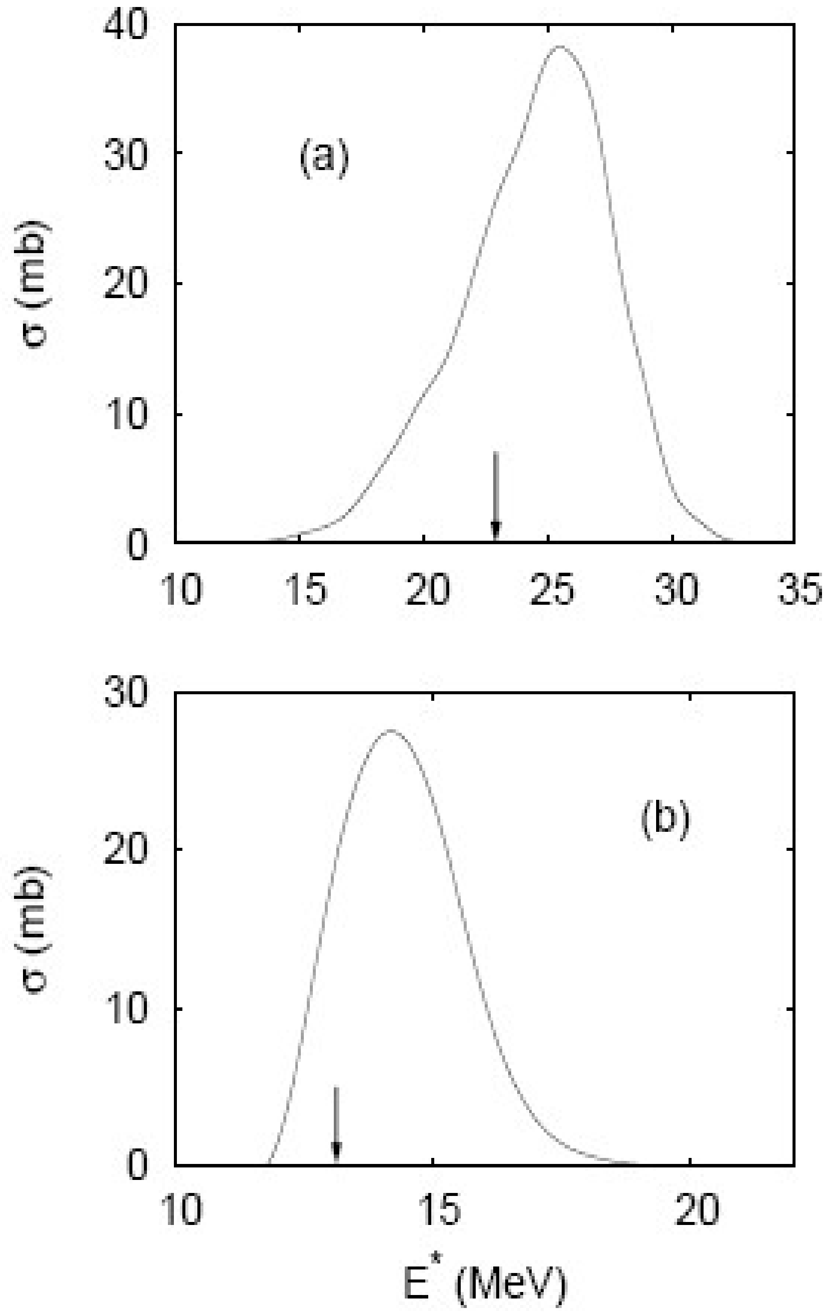
From (A.4), identifying $\rho(R)$ with $\mathcal{P}^{L}_{BU}(R)$, we obtain expression (1) for the breakup probability integrated along a given classical orbit.

Breakup Function from CDCC Calculations

CDCC calculations give an exponential function for the breakup prob



Excitation energy distribution of ICF product ^{212}Po



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

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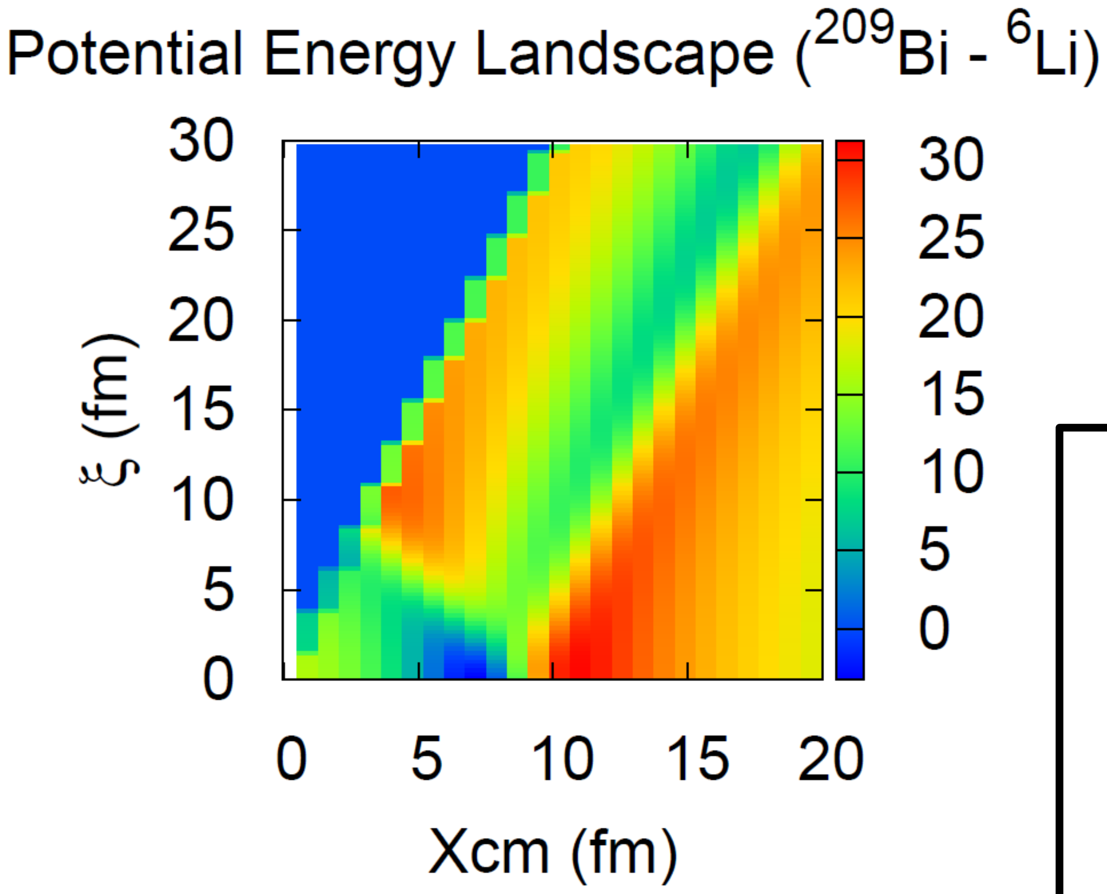
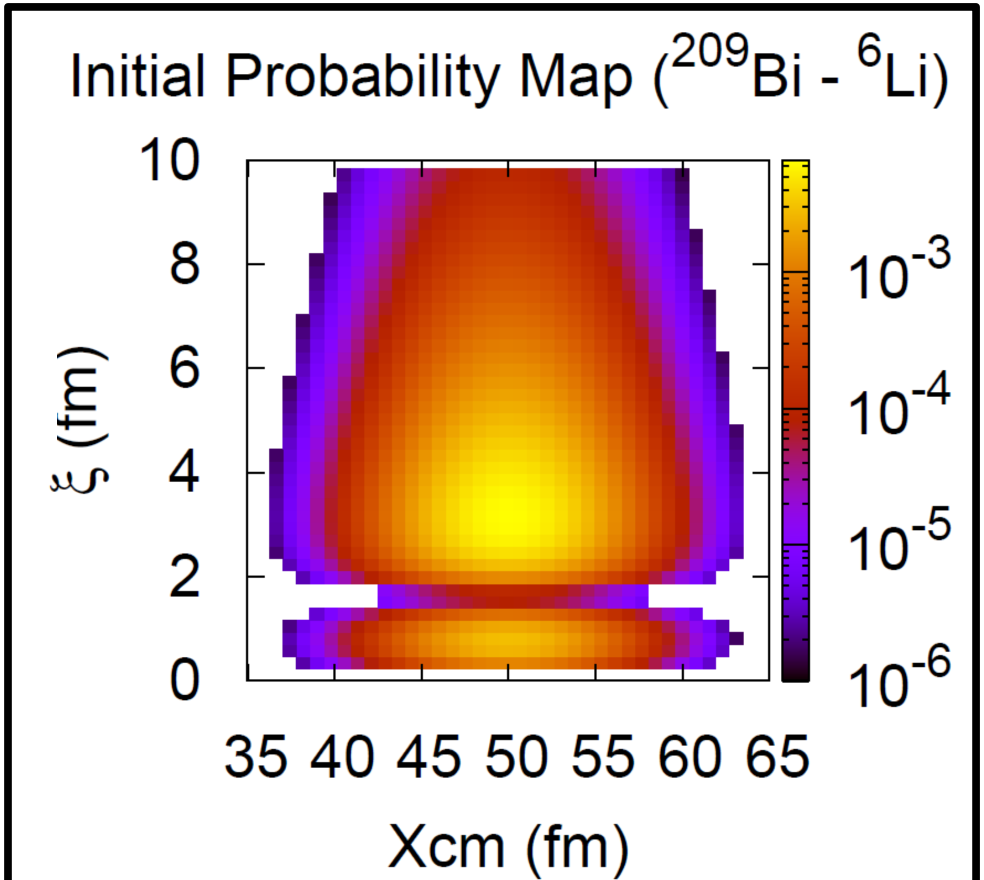
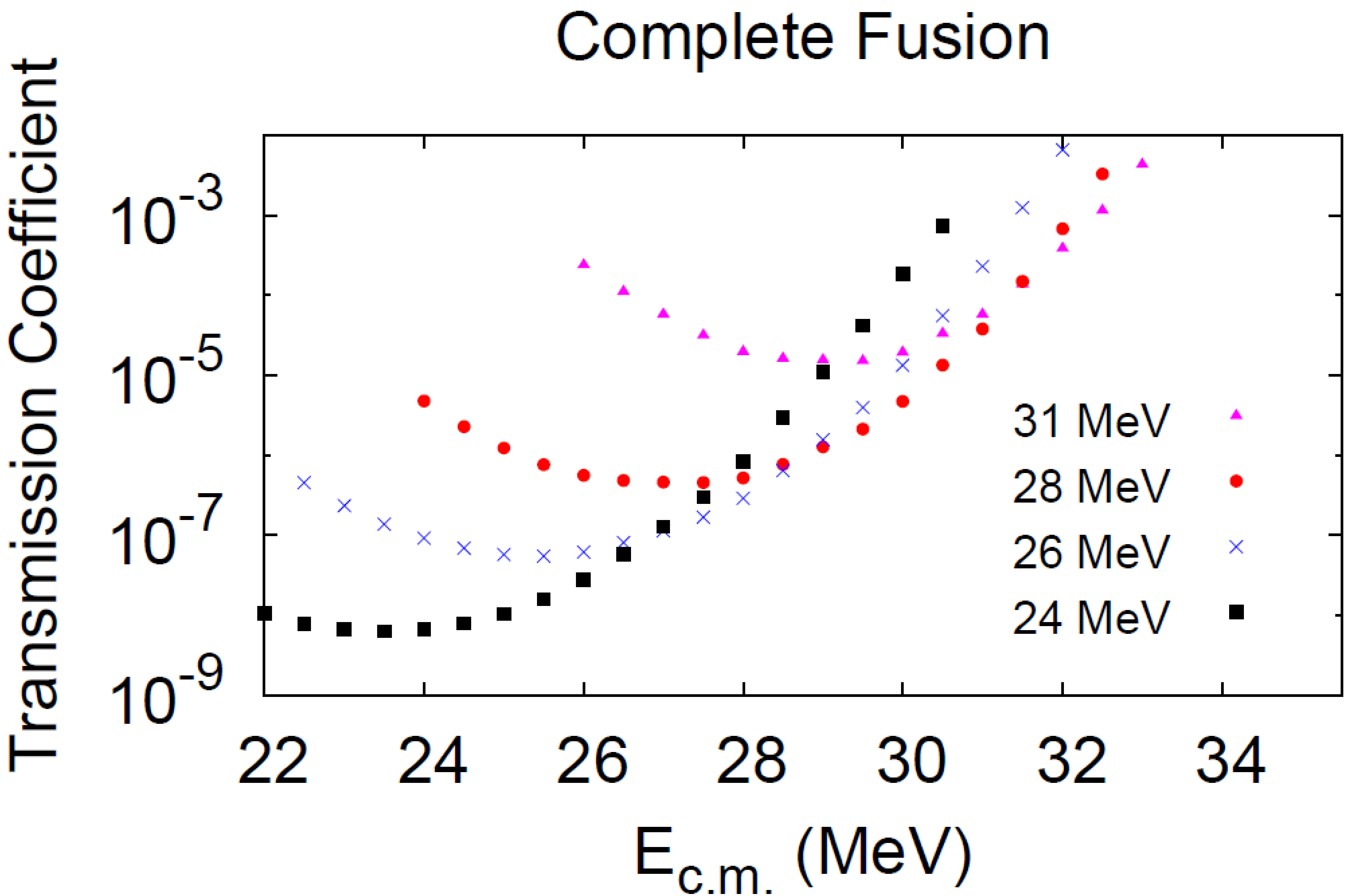
Initial conditions







Complete Fusion
Transmission Coefficient
10-3
10-5
10-7
10-9
22
24
26
28
30
32
34
Ec.m. (MeV)
31 MeV
28 MeV
26 MeV
24 MeV