Gravitational waves from neutron stars in the era of Advanced LIGO and Advanced Virgo detectors

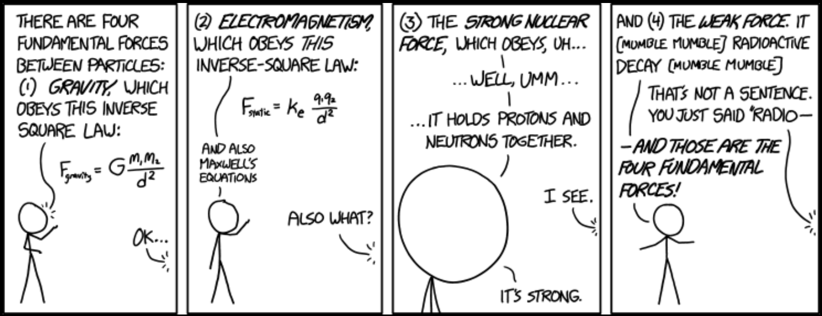
Michał Bejger (CAMK PAN)

Helmholtz International Summer School
”Nuclear Theory and Astrophysical Applications”
Dubna | 17.7.17

Outline

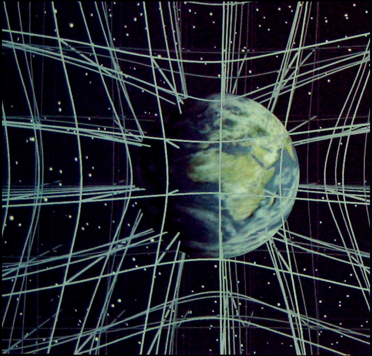
- ⋆ Gravitational waves from Einstein's equations,
- ⋆ Detection principles (what is actually measured by interferometers?)
- ⋆ Newtonian intuitions from inspiralling binary system,
- ⋆ Binary neutron stars and rotating neutron stars.

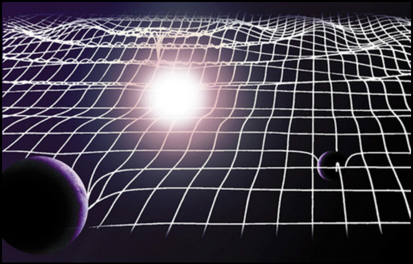
Four fundamental interactions

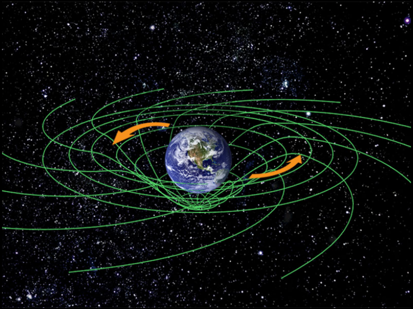


xkcd/1489

Einstein (1915): gravitation *is* the geometry of spacetime







"Mass tells spacetime how to curve, and spacetime tells mass how to move."
(John A. Wheeler)

Gravitation: Newton vs Einstein

- ⋆ Absolute time and space,
- ⋆ deterministic solutions,
- ⋆ Eternal two body systems.

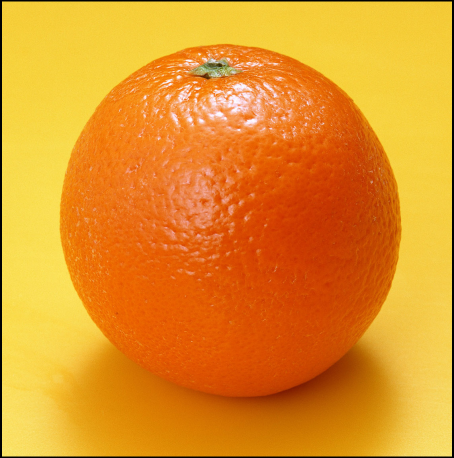
- ⋆ Stable two body system does not exist,
- ⋆ Constant evolution due to the existence of a third ”body”: the spacetime.

Gravitational waves

Einstein (1916) - in linear regime there are wave solutions to GR equations *(time-varying distortions of the curvature propagating with the speed of light)*:

- ⋆ In realistic astrophysical situations, length-scale of the wave λ is much smaller than other important curvatures $\mathcal{L}$,
- ⋆ Split of the Riemann curvature tensor

$$R_{\alpha\beta\gamma\delta} = R^{GW}_{\alpha\beta\gamma\delta} + R^{B}_{\alpha\beta\gamma\delta}$$



"Kip Thorne's orange": B - large-scale background ($\mathcal{L} \simeq$ 10 cm), GW - fine-scale distortions/waves ($\lambda \simeq$ few mm).

Gravitational waves: indirect evidence

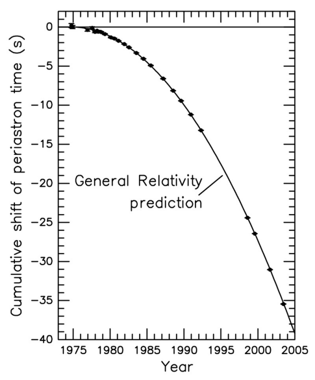
The 50s - breakthrough in theoretical understanding of the nature of the waves:

- ⋆ Herman Bondi, Felix Pirani, Andrzej Trautman (gravitational waves carry energy!)

The 60s - early insight of Bohdan Paczyński:

- ⋆ *"Gravitational Waves and the Evolution of Close Binaries", AcA 1967* - orbital period evolution of WZ Sge and HZ29 driven by the GW emission.

70s - observations of pulsars in relativistic binary systems (e.g. Hulse-Taylor pulsar):



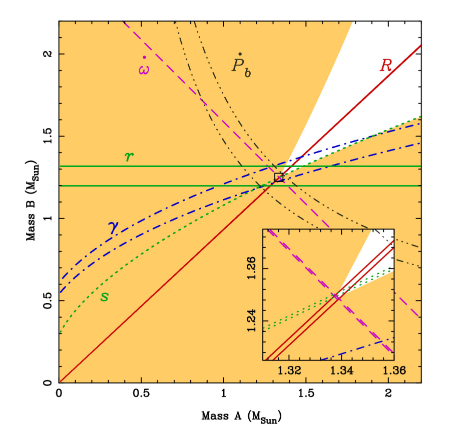
System is losing energy as if by emittion of gravitational waves in concordance with GR.

Neutron stars in relativistic binaries: PSR J0737-3039

- ⋆ Periastron advance:
  $\dot{\omega} = 3 \left(\frac{P_b}{2\pi}\right)^{-5/3} (T_\odot M)^{2/3}(1-e^2)^{-1}$
- ⋆ Orbit decay:
  $\dot{P}_b = -\frac{192\pi m_p m_c}{5M^{1/3}} \left(\frac{P_b}{2\pi}\right)^{-5/3} \times$
  $\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)(1-e^2)^{-7/2} T_\odot^{5/3}$
- ⋆ Shapiro effect:
  $r = T_\odot m_c,$
  $s = \frac{a_p \sin i}{c m_c} \left(\frac{P_b}{2\pi}\right)^{-2/3} T_\odot^{-1/3} M^{2/3}$
- ⋆ Gravitational redshift:
  $\gamma =$
  $e \left(\frac{P_b}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3} m_c (M + m_c)$

where $T_\odot = GM_\odot/c^3$, $M = m_p + m_c$.

Relativistic binaries show a number of effects compatible with GR!

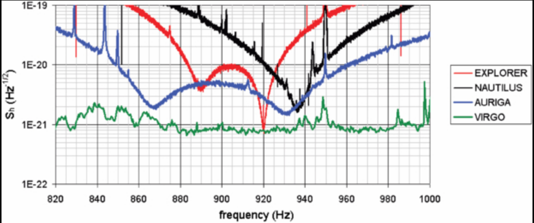


- ⋆ Pulsar A: $P = 22.7$ ms, pulsar B: $P = 2.77$ s,
- ⋆ Orbital period $\simeq$ 2.4 h,
- ⋆ eccentricity $\simeq$ 0.08,
- ⋆ Orbit decay $\simeq$ 7 mm/day.

Detection principle: resonant bars

Pioneered by Joseph Weber in the 1960s:

- ⋆ Passing gravitational wave carries energy → induces mechanical vibrations
- ⋆ A narrow-band detector (sensitive near characteristic frequencies of the bar)



Gravitational waves: weak field wave zone

"Ripples" in the "nearly flat" spacetime metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where e.g., $\eta_{\mu\nu} = \mathrm{diag}\,(-1, 1, 1, 1)$, and $|h_{\mu\nu}| \ll 1$ for all μ, ν.

In the weak-field limit h is small, 1st order (linear) sufficient: $h_{\mu\nu} = \eta_{\mu\alpha}\eta_{\beta\nu}h^{\alpha\beta}$

Coordinate transformations that preserve "nearly flat" (nearly Lorentz) spacetime:

- ⋆ background Lorentz transformations (boosts with $v \ll 1$),

$$g'_{\mu\nu} = \eta'_{\mu\nu} + \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}h_{\alpha\beta} = \eta'_{\mu\nu} + h'_{\mu\nu}$$

- ⋆ Gauge transformations (ξ^μ, $|\xi^\mu_{,\nu}|$, $|\xi_{,\mu\nu}| \ll 1$):

$$x'^\mu = x^\mu + \xi^\mu\,(x^\nu)\,, \quad \text{so that}$$

$$g'_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu} \ll 1.$$

Gravitational waves: wave equation

In linear regime, weak field the Riemann tensor is

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\alpha\gamma,\beta\delta} - h_{\beta\delta,\alpha\gamma}\right).$$

Ricci tensor: $$R_{\mu\nu} = \frac{1}{2}\left(h^{\alpha}_{\mu,\nu\alpha} + h^{\alpha}_{\nu,\mu\alpha} - h_{\mu\nu,\alpha}^{,\alpha} - h_{,\mu\nu}\right),$$

where $h \equiv h^{\mu}_{\mu} = \eta^{\mu\nu} h_{\mu\nu}$, $\quad h_{\mu\nu,\alpha}^{,\alpha} = \eta^{\alpha\gamma} h_{\mu\nu,\alpha\gamma}$.

And so... Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}\left(h^{\alpha}_{\mu,\nu\alpha} + h^{\alpha}_{\nu,\mu\alpha} - h_{\mu\nu,\alpha}^{,\alpha} - h_{,\mu\nu} - \eta_{\mu\nu}\left(h_{\alpha\beta}^{,\alpha\beta} - h_{,\beta}^{,\beta}\right)\right).$$

Using trace-reversed form, $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{1}{2}\left(\bar{h}_{\mu\nu,\alpha}^{,\alpha} + \eta_{\mu\nu} \bar{h}_{\alpha\beta}^{,\alpha\beta} - \bar{h}_{\mu\alpha,\nu}^{,\alpha} - \bar{h}_{\nu\alpha,\mu}^{,\alpha}\right) \stackrel{vacuum}{=} 0.$$

'Good choice' of gauge (Lorentz gauge $\bar{h}^{\mu\alpha}_{,\alpha} = 0$) reduces it to

$$\bar{h}_{\mu\nu,\alpha}^{,\alpha} \equiv \eta^{\alpha\alpha} \bar{h}_{\mu\nu,\alpha\alpha} = \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}_{\mu\nu} = 0.$$

Plane gravitational waves

$$\bar{h}_{\mu\nu} = Re\left(A_{\mu\nu} \exp\left(i k_\alpha x^\alpha\right)\right),$$

$$\text{with} \quad k_\alpha k^\alpha = 0 \quad \rightarrow \quad \omega = k^t = \sqrt{k_x^2 + k_y^2 + k_z^2}.$$

From the choice of Lorentz gauge: $A_{\mu\alpha} k^\alpha = 0$.

Using remaining freedom, apply the transverse-traceless gauge for a wave traveling in the z direction:

- $\star$ $k^t = k^z = \omega, k^x = k^y = 0, \quad A_{\alpha z} = 0,$
- $\star$ $A^\mu_\mu = \eta^{\mu\nu} A_{\mu\nu} = 0, \quad A_{\alpha t} = 0.$

In the TT gauge, $\bar{h}^{(TT)}_{\mu\nu} = A^{(TT)}_{\mu\nu} \cos\left(\omega\left(t - z\right)\right)$, with

$$A^{(TT)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A^{(TT)}_{xx} & A^{(TT)}_{xy} & 0 \\ 0 & A^{(TT)}_{xy} & -A^{(TT)}_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{Also,} \quad \bar{h}^{(TT)}_{\mu\nu} = h^{(TT)}_{\mu\nu}.$$

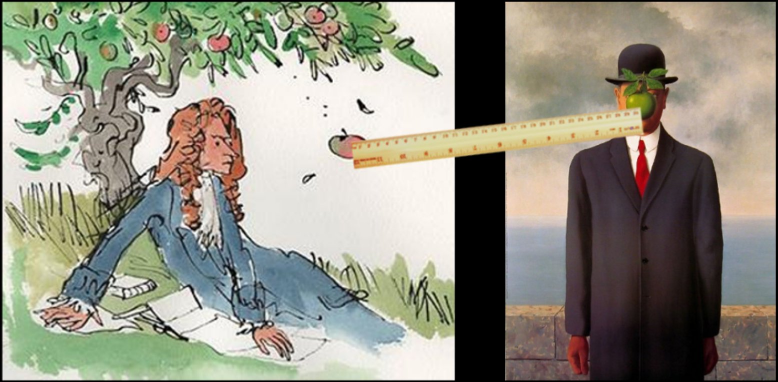
Gravitational waves: TT gauge

For a free test particle initially at rest, in the coordinate system corresponding to the TT gauge, it stays at rest: coordinates do not change, particles remain attached to initial positions.

TT gauge represents a coordinate system that is comoving with freely-falling particles.

What about the **proper distance** between neighbouring particles?

Detection principle: spacetime distance measurement



(Quentin Blake "Izaak Newton")

(Rene Magritte "The Son of Man")

"How to measure distance when the ruler also changes length?"

Proper distance between test particles

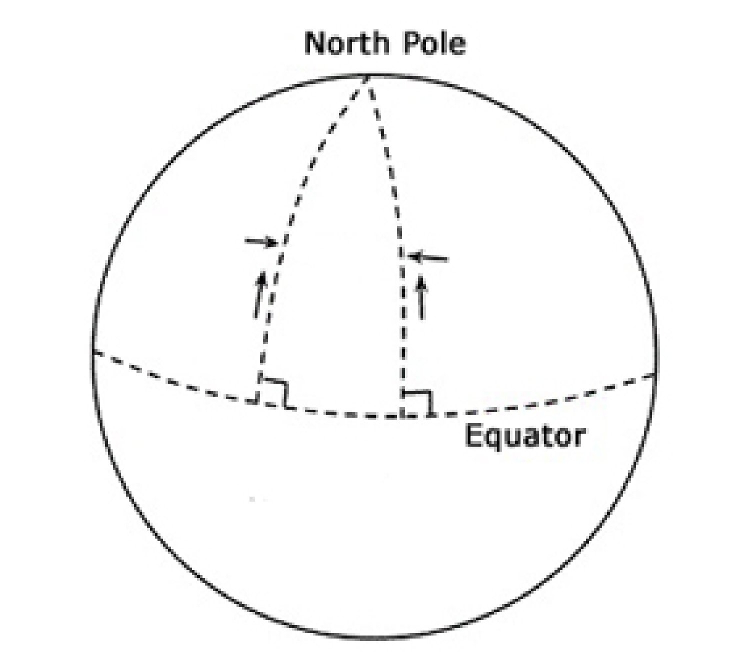
Consider two test particles, both initially at rest, one at $x = 0$ and the other at $x = \epsilon$. The proper distance is

$$\Delta s = \int |g_{\mu\nu} dx^\mu dx^\nu|^{1/2} \to \int_0^\epsilon |g_{xx}|^{1/2} \approx \epsilon \sqrt{g_{xx}\left(x = 0\right)}.$$

If $g_{xx}\left(x = 0\right) = \eta_{xx} + h_{xx}^{(TT)}\left(x = 0\right)$, then

$$\Delta s \approx \epsilon \left(1 + \frac{1}{2} h_{xx}^{(TT)}(x = 0)\right),$$

which, in general, is time-varying $\ddot\smile$


North Pole
Equator

Geodesic deviation - effect of tidal forces

Consider two test particles, both initially at rest ($u^\alpha = (1,0,0,0)$) one at $x = 0$ and the other at $x = \epsilon$ (distance between particles $\xi^\alpha = (0,\epsilon,0,0)$). Geodesic deviation equation in the weak field (proper time $\tau \approx$ coordinate time t),

$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = R^\alpha_{\beta\gamma\delta} u^\beta u^\gamma \xi^\delta$$

simplifies further to

$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = \epsilon R^\alpha_{ttx} = -\epsilon R^\alpha_{txt},$$

with $R^x_{txt} = \eta^{xx} R_{xtxt} = -\frac{1}{2} h^{(TT)}_{xx,tt}$, $R^y_{txt} = \eta^{yy} R_{ytxt} = -\frac{1}{2} h^{(TT)}_{xy,tt}$,

$$\frac{\partial^2 \xi^x}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2 h^{(TT)}_{xx}}{\partial t^2}, \quad \frac{\partial^2 \xi^y}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2 h^{(TT)}_{xy}}{\partial t^2}.$$

Geodesic deviation - effect of tidal forces

More general case; $x = \epsilon\cos\theta,\ y = \epsilon\sin\theta,\ z = 0$:

$$\frac{\partial^2\xi^x}{\partial t^2} = \frac{1}{2}\epsilon\cos\theta\frac{\partial^2 h_{xx}^{(TT)}}{\partial t^2} + \frac{1}{2}\epsilon\sin\theta\frac{\partial^2 h_{xy}^{(TT)}}{\partial t^2},$$

$$\frac{\partial^2\xi^y}{\partial t^2} = \frac{1}{2}\epsilon\cos\theta\frac{\partial^2 h_{xy}^{(TT)}}{\partial t^2} - \frac{1}{2}\epsilon\sin\theta\frac{\partial^2 h_{xx}^{(TT)}}{\partial t^2}.$$

with solutions, for the plane wave in the z direction,

$$\xi^x = \epsilon\cos\theta + \frac{1}{2}\epsilon\cos\theta A_{xx}^{(TT)}\cos\omega t + \frac{1}{2}\epsilon\sin\theta A_{xy}^{(TT)}\cos\omega t,$$

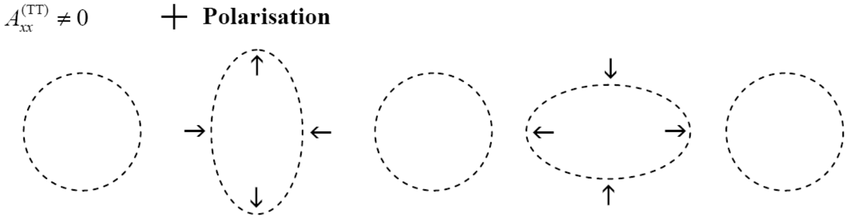
$$\xi^y = \epsilon\sin\theta + \frac{1}{2}\epsilon\cos\theta A_{xy}^{(TT)}\cos\omega t - \frac{1}{2}\epsilon\sin\theta A_{xx}^{(TT)}\cos\omega t.$$

The + polarisation

$A_{xx}^{(TT)} \neq 0$, $A_{xy}^{(TT)} = 0$

$$\xi^x = \epsilon \cos\theta \left(1 + \frac{1}{2} A_{xx}^{(TT)} \cos\omega t\right),$$

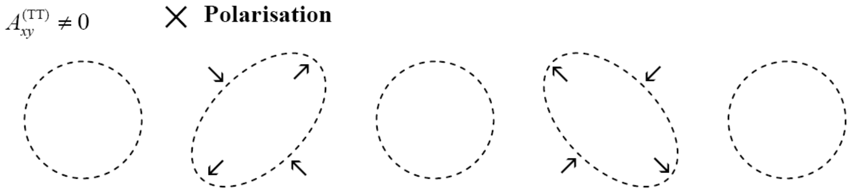
$$\xi^y = \epsilon \sin\theta \left(1 - \frac{1}{2} A_{xx}^{(TT)} \cos\omega t\right).$$



The × polarisation

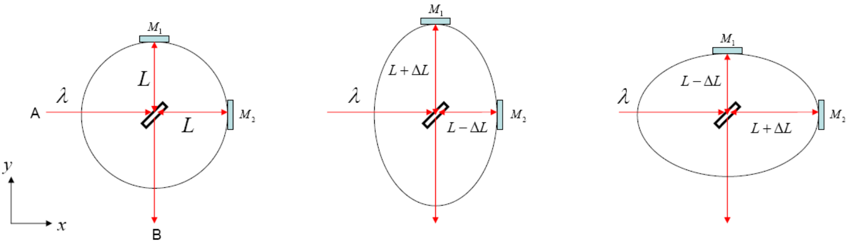
$A^{(TT)}_{xy} \neq 0$, $A^{(TT)}_{xx} = 0$

$$\begin{aligned}
\xi^x &= \epsilon \cos\theta + \frac{1}{2}\epsilon \sin\theta A^{(TT)}_{xy} \cos\omega t, \\
\xi^y &= \epsilon \sin\theta - \frac{1}{2}\epsilon \cos\theta A^{(TT)}_{xy} \cos\omega t.
\end{aligned}$$



For purely $+$ mode wave ($\mathbf{h} = h\mathbf{e}_+$), fractional change in proper distance is

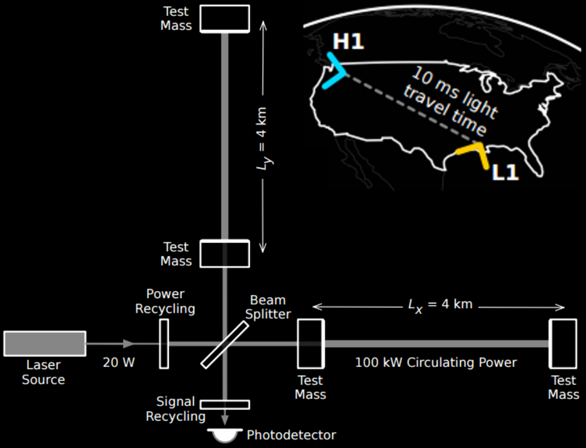
$$\frac{\Delta L}{L} = \frac{h}{2}$$



Gertsenshtein & Pustovit (1962) were first to suggest an interferometer to detect GWs. In the 70s Rainer Weiss (MIT) had the same idea $\rightarrow$ LIGO

Detection principle: laser interferometry

"How to measure distance when the ruler also changes length?"



Changes in arms length are **very** small: $\delta L_x - \delta L_y = \Delta L < 10^{-18}$ m (smaller than the size of the proton). Wave amplitude $h = \Delta L / L \leq 10^{-21}$.

Change of arms' length $\leftrightarrow$ variation in light travel time

Change of the x-arm: $ds^2 = -c^2 dt^2 + (1 + h_{xx})\, dx^2 = 0.$

Assume $h(t)$ is constant during light's travel through interferometer, replace $\sqrt{1 + h_{xx}}$ with $1 + h_{xx}/2$, integrate from $x = 0$ to $x = L$:

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{xx}\right) dx \quad \rightarrow \quad t_x = h_{xx} L/2c.$$

Round-trip time in the x-arm: $t_x = h_{xx} L/c.$

Round-trip time in the y-arm: $t_y = -hL/c \qquad (h_{yy} = -h_{xx} = -h)$

Round-trip times difference: $\boxed{\Delta\tau = 2hL/c}$

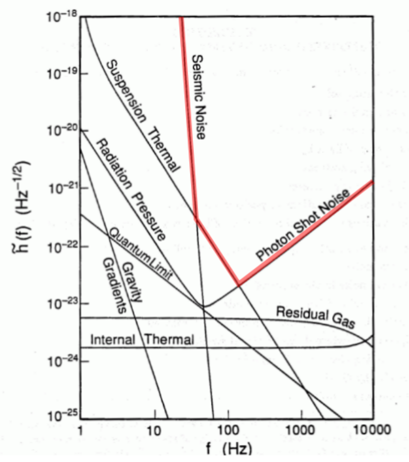
Phase difference (dividing $\Delta\tau$ by the radian period of light $2\pi/\lambda$):

$$\boxed{\Delta\phi = \frac{4\pi}{\lambda} hL = \frac{2\pi c}{\lambda} h\tau}.$$

- Do test masses move in response to a gravitational wave?
  - No, in the TT gauge (free-falling masses define the coordinates),
  - Yes, in the laboratory coordinates (masses move affected by tidal forces).
- Do light wavelength change in response to a gravitational wave?
  - No (see above),
  - Yes, stretch by h as the masses move (as in the cosmological redshift).
- If light waves are stretched by gravitational waves, how can light be used as a ruler?
  - Indeed, the instantaneous response of an interferometer to a gravitational wave is *null*.
  - But the light travels through the arms for some finite time allowing for the phase shift to build up.

See also Saulson, P.R. (1997), *Am. J. Phys.* 65, 501

How the sensitivity curve looks like?

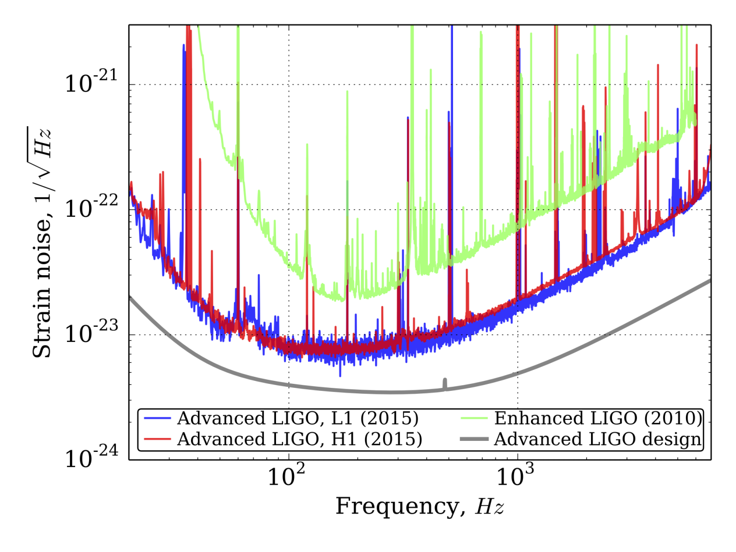


Initial LIGO proposal (1989)

- ⋆ Range of frequencies similar to human ears:

From 20 Hz (H0) to a few thousands Hz (3960 Hz, H7) - 8 octaves.

- ⋆ Poor, like for an ear, angular resolution.



Strain noise, $1/\sqrt{Hz}$
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{2}
10^{3}
Frequency, Hz
Advanced LIGO, L1 (2015)
Advanced LIGO, H1 (2015)
Enhanced LIGO (2010)
Advanced LIGO design

Antenna patterns

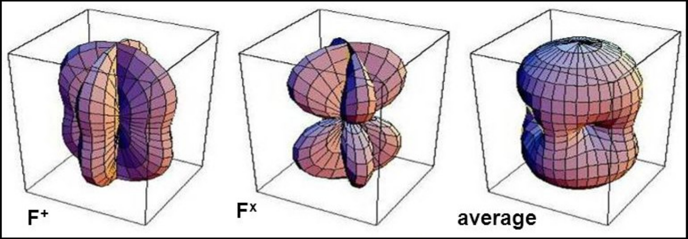
Response of x and y arms to a GW from arbitrary direction

$$h_{xx} = -\cos(\theta)\sin(2\phi)h_\times + (\cos^2(\theta)\cos\phi^2 - \sin\phi^2)h_+$$

$$h_{yy} = \cos\theta\sin 2\phi h_\times + (\cos\theta^2\sin\phi^2 - \cos\phi^2)h_+$$

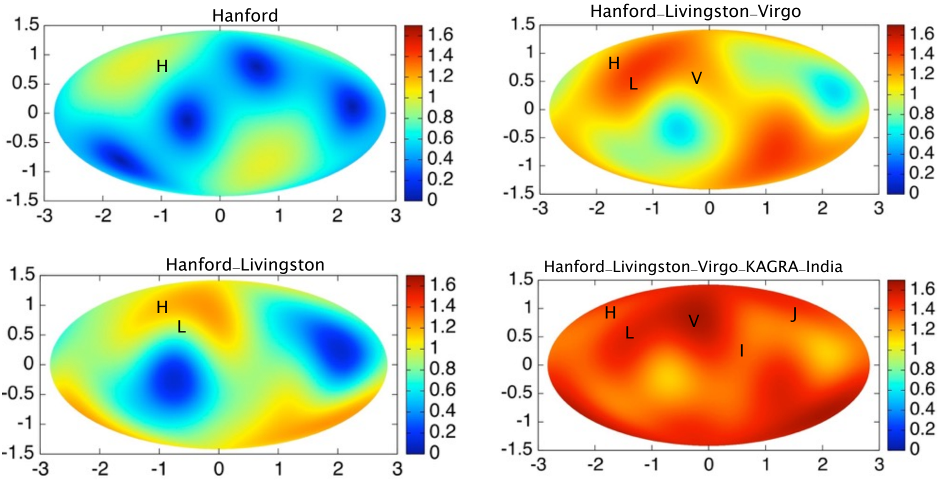
$$|h_{yy} - h_{xx}|$$

$$\frac{\delta L(t)}{L} = h(t) = F^+ h_+(t) + F^\times h_\times(t)$$



- Interferometers have a broad antenna pattern
  - Cannot locate direction of the source with a single detector
  - Can scan large portions of the sky simultaneously

Beam patterns of networks



Orders of magnitude comparison

- ⋆ GW150914: $h = \Delta L/L \simeq 10^{-21}$
- ⋆ Two neutron stars merging near Sgr A*: $\sim 10^{-19}$
- ⋆ Io orbiting Jupiter: $\sim 3 \times 10^{-25}$
- ⋆ Hulse-Taylor pulsar: $\sim 10^{-26}$
- ⋆ Dumbbell 1 tonnes masses, 1 m arm from 300 m: $\sim 10^{-35}$
- ⋆ Collision of two aircraft carriers: 5×10^{-46}
- ⋆ Angry protester shaking her fist: $\sim 7 \times 10^{-52}$
- ⋆ Tennis ball rotating on 1 m string, from 10 m: $\sim 10^{-54}$.

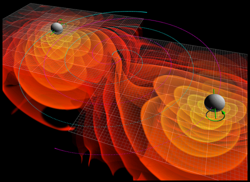
- ⋆ The amplitude $h = \Delta L/L \leq 10^{-21}$ corresponds to the distance measurement between Earth and Sun with the accuracy of the size of the atom (10^{-10} m)
- ⋆ Ground motion amplitude near the detector: $\Delta L \sim 10^{-6}$ m ($10^{12} \times h$)
- ⋆ Laser wavelength: 10^{-6} m ($10^{12} \times h$)

Astrophysical sources: binary systems

(Hokusai "The Great Wave off Kanagawa")

One-time cataclismic events well described by models, e.g. last moments of the binary system of

- ⋆ black holes,
- ⋆ neutron stars,
- ⋆ black hole and a neutron star.



Binary black hole merger simulation (C. Henze/NASA

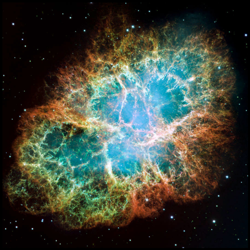
Ames Research Center)

Astrophysical sources: ”bursts”

(Isoda Koryûsai ”The crane, waves and the rising sun”)

One-time events difficult to model, e.g.

- ⋆ supernova explosions,
- ⋆ magnetar & gamma-ray bursts.



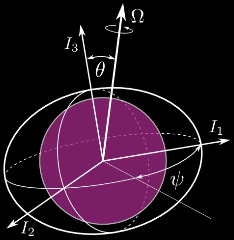
Crab nebula, supernova 1054CE remnant

Astrophysical sources: continuous waves

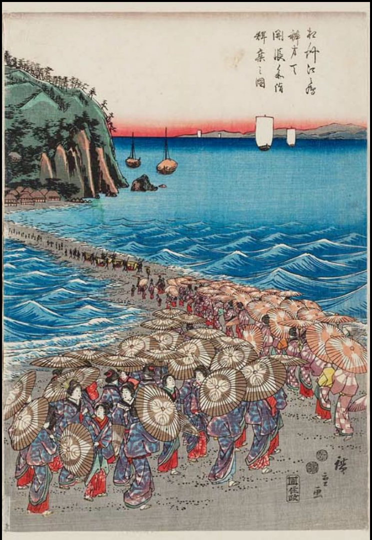
(Shoson "Cranes landing")

Periodic phenomena, e.g.

- ⋆ rotating non-axisymmetric neutron stars ("*gravitational pulsars*").



Astrophysical sources: stochastic background

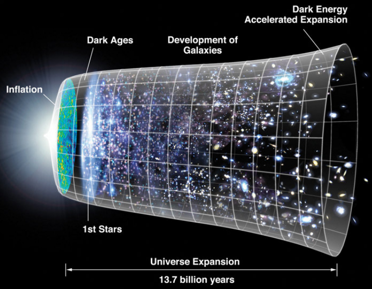


(Utagawa Hiroshige "Crowds Visiting the Shrine of

Benzaiten")

Stochastic background:

- ⋆ waves emitted by the population of objects,
- ⋆ waves from the early Universe.



Gravitational waves: some estimates

For a spherical wave of amplitude $h(r)$, flux of energy is $F(r) \propto h^2(r)$ and the luminosity $L(r) \propto 4\pi r^2 h^2(r)$. Conservation of energy demands

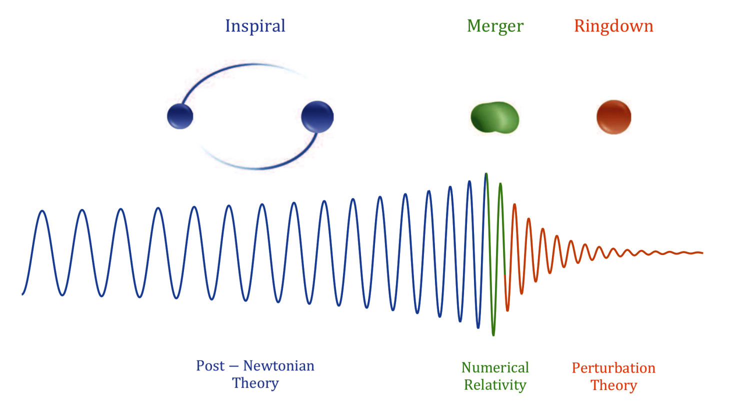
$$\Longrightarrow h(r) \propto 1/r.$$

Radiating modes: quadrupole and higher

For a mass distribution $\rho(r)$, conserved moments:

- ⋆ monopole $\int \rho(r) d^3r$ - total mass-energy (energy conservation),
- ⋆ dipole $\int \rho(r) r d^3r$ - center of mass-energy (momentum conservation).

Evolution of a binary system

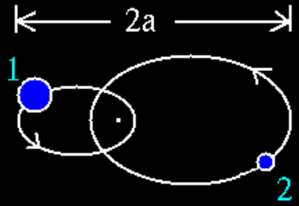


Gravitational waves: some estimates

GWs correspond to accelerated movement of masses.

Consider a binary system of m_1 and m_2, semiaxis a with

- ⋆ total mass $M = m_1 + m_2$,
- ⋆ reduced mass $\mu = m_1 m_2 / M$,
- ⋆ mass quadrupole moment $Q \propto Ma^2$,
- ⋆ Kepler's third law $GM = a^3\omega^2$.



$$h(r) \propto \frac{1}{r}\frac{\partial^2 (Ma^2)}{\partial t^2} = \boxed{\frac{G^2}{c^4}\frac{1}{r}\frac{M\mu}{a} = \frac{G^{5/3}}{c^4}\frac{1}{r}M^{2/3}\mu\,\omega^{2/3}.}$$

Gravitational waves: quadrupole approximation

The quadrupole approximation (slowly-moving sources, Einstein 1918), wave amplitude is

$$h^{\mu\nu} = \frac{2}{r}\frac{G}{c^4}\ddot{Q}^{\mu\nu}, \quad \text{or, in terms of kinetic energy,} \quad h \sim \frac{E_{kin.}^{nsph.}}{r}.$$

Resulting GW luminosity is

$$L_{GW} \equiv \frac{dE_{GW}}{dt} \approx \frac{1}{5}\frac{G}{c^5}\langle \dddot{Q}^{\mu\nu}\dddot{Q}_{\mu\nu}\rangle$$

$$\propto \frac{G}{c^5}Q^2\omega^6 \propto \frac{G^4}{c^5}\left(\frac{M}{a}\right)^5 \propto \frac{c^5}{G}\left(\frac{R_s}{a}\right)^2\left(\frac{v}{c}\right)^6.$$

($R_s = 2GM/c^2$, $c^5/G \simeq 3.6 \times 10^{52}$ Joule/s)

Binary system: evolution of the orbit

Waves are emitted at the expense of the orbital energy:

$$E_{orb} = -\frac{Gm_1 m_2}{2a}, \qquad \frac{dE_{orb}}{dt} \equiv \frac{Gm_1 m_2}{2a^2}\dot{a} = -\frac{dE_{GW}}{dt}.$$

Evolution of the semi-major axis:

$$\frac{da}{dt} = -\frac{dE_{GW}}{dt}\frac{2a^2}{G\underbrace{m_1 m_2}_{\mu M}} \rightarrow \frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^4}{a^3}.$$

The system will coalesce after a time τ,

$$\tau = \frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^4},$$

where a_0 is the initial separation.

Binary system: chirp mass

Waves are emitted at the expense of the orbital energy:

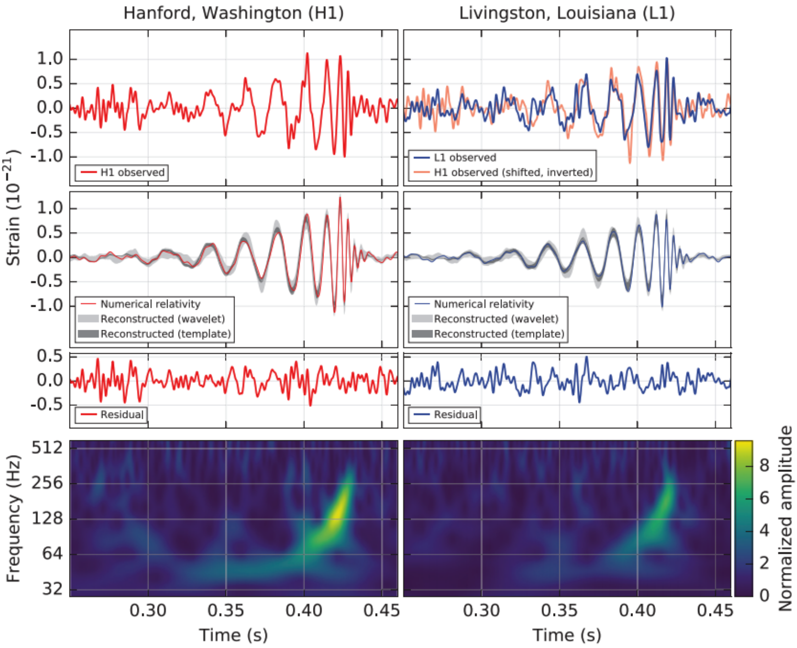
$$E_{orb} = -\frac{Gm_1 m_2}{2a}, \qquad \frac{dE_{orb}}{dt} \equiv \frac{Gm_1 m_2}{2a^2}\dot{a} = -\frac{dE_{GW}}{dt}.$$

Resulting evolution of the orbital frequency ω:

$$\dot{\omega}^3 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mu^3 M^2 = \left(\frac{96}{5}\right)^3 \frac{\omega^{11}}{c^{15}} G^5 \mathcal{M}^5,$$

where $\mathcal{M} = \left(\mu^3 M^2\right)^{1/5} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$ is the chirp mass. GWs frequency from a binary system is primarily twice the orbital frequency ($2\pi f_{GW} = 2\omega$). Hence $\mathcal{M}$ is a directly measured quantity:

$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3} f_{GW}^{-11/3} \dot{f}_{GW}\right)^{3/5}.$$


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
8
6
4
2
0

Binary system: emitted energy

End of the chirp f_{GW}^{c} is related to critical distance between masses a_{fin}:

$$a_{fin} = R_{s1} + R_{s2} = \frac{2G}{c^2}\left(m_1 + m_2\right).$$

It can be used to estimate the total mass M:

$$M = m_1 + m_2 \approx \frac{c^3}{2\sqrt{2}G\pi}\frac{1}{f_{GW}^{c}}.$$

Energy emitted during the life of the binary system:

$$E = E_{ms} + E_{orb} = \left(m_1 + m_2\right)c^2 - \frac{Gm_1 m_2}{2a}.$$

(for $m_1 = m_2$, $a_{fin} = 2R_s = 4Gm_1/c^2$, $\Delta E \approx 6\%$).

Parameter estimation basics (GW510914)

GW amplitude dependence for a binary system

$$h \propto \mathcal{M}^{5/3} \times f_{GW}^{2/3} \times r^{-1}$$

where $\mathcal{M}$ is the chirp mass, $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$, known from the observations:

$$\mathcal{M} = \frac{c^3}{G} \left[\frac{5}{96} \pi^{-8/3} f_{GW}^{-11/3} \dot{f}_{GW} \right]^{3/5}$$

From higher-order post-Newtonian corrections: $q = m_2/m_1$, spin components parallel to the orbital angular momentum...

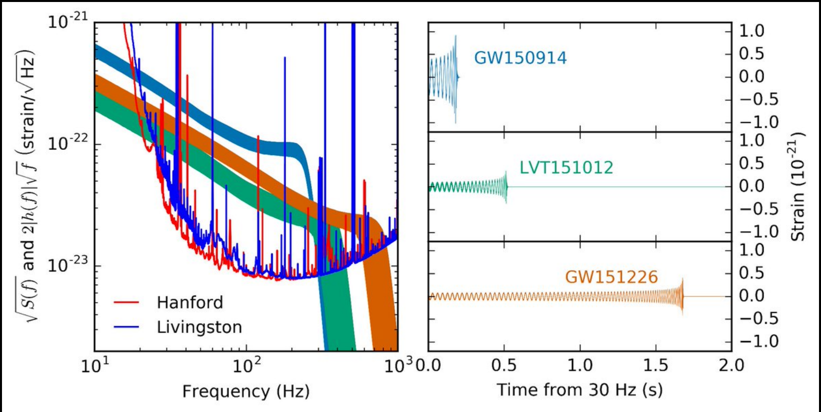
$\mathcal{M} \simeq 30 M_\odot \Longrightarrow M = m_1 + m_2 \simeq 70 M_\odot$ (if $m_1 = m_2,\ M = 2^{6/5}\mathcal{M}$)

8 orbits observed until 150 Hz (orbital frequency 75 Hz):

- ⋆ Double neutron star system compact enough, but too light,
- ⋆ Neutron star-black hole system - black hole too big, would merge at lower frequency.

→ Double black hole binary.

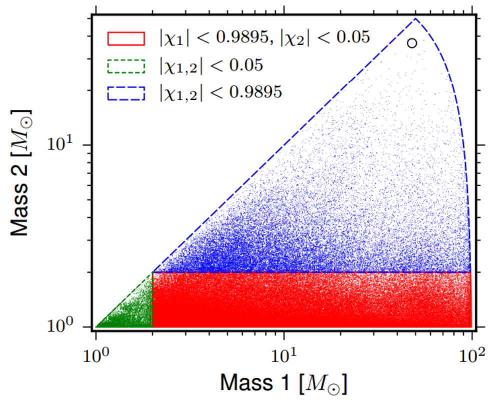
LIGO O1: 2 (”and a half”) events



Optimal signal-to-noise ρ : $$\rho^2 = \int_0^\infty \left(\frac{2|\tilde{h}(f)|\sqrt{f}}{\sqrt{S_n(f)}} \right)^2 d\ln(f)$$

(GW150914: $\rho \simeq 24$, GW151226: $\rho \simeq 13$, LVT151012: $\rho \simeq 10$)

Binary coalescence search



In general, signal model lives in 17D parameter space: masses, spins, eccentricity of the orbit, its orientation, polarization angle, position of the binary, distance, epoch of coalescence and phase of the signal.

Matched filtering

Assuming a signal model h, looking for the "best match" correlation $C(t)$ in data stream x, for a given time offset t

$$C(t) = \int_{-\infty}^{\infty} \underbrace{x(t')}_{\text{Data}} \times \underbrace{h(t'-t)}_{\text{Template with time offset } t} dt'$$

Rewrite correlation using Fourier transforms

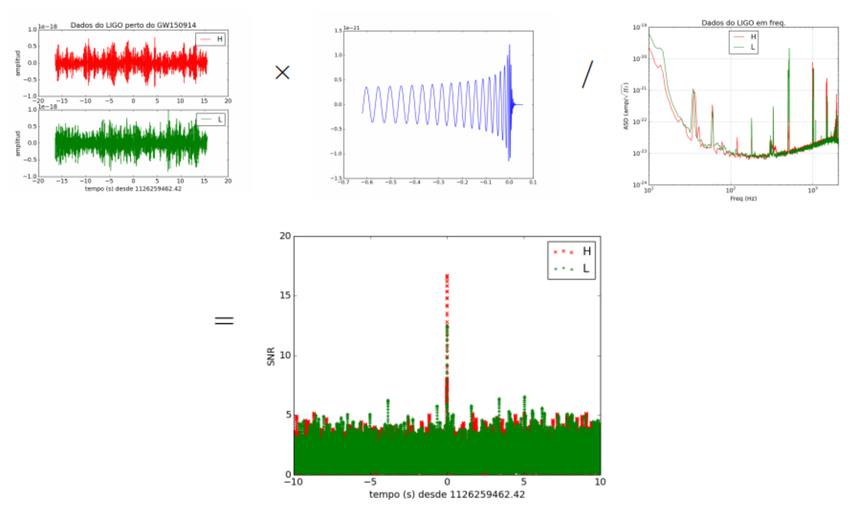
$$C(t) = 4 \int_{0}^{\infty} \tilde{x}(f)\tilde{h}^{*}(f) e^{2\pi i f t} df$$

(an inverse FT of $\tilde{x}(f)\tilde{h}^{*}(f)$). In practice, optimal matched filtering with the frequency weighting

$$C(t) = 4 \int_{0}^{\infty} \frac{\tilde{x}(f)\tilde{h}^{*}(f)}{S_n(f)} e^{2\pi i f t} df$$

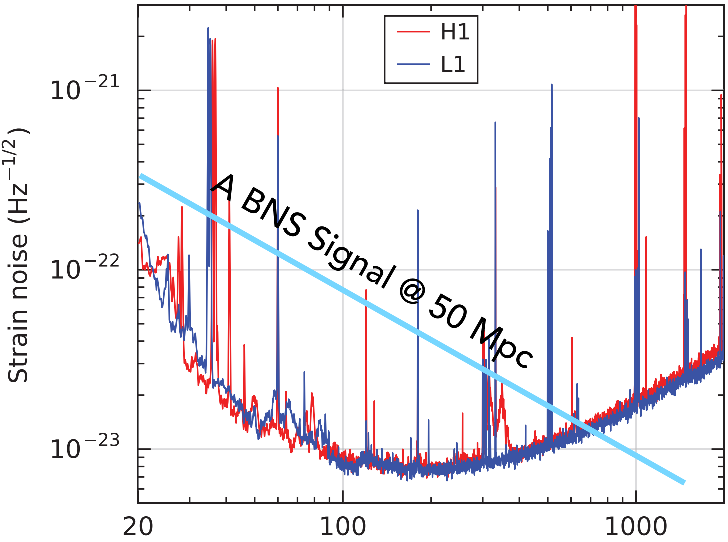
$S_n(f)$ - noise power spectral density

Matched filter in pictures



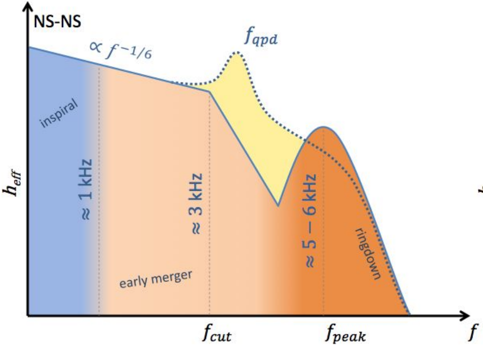
(from Riccardo Sturani's talk)

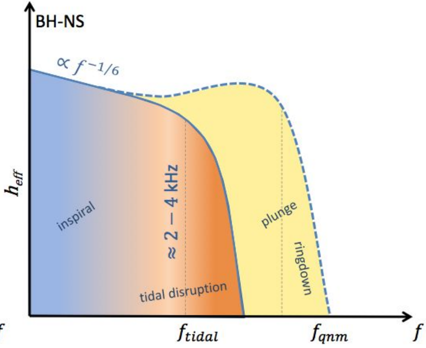
LIGO SENSITIVITY DURING FIRST OBSERVING RUN (O1)



BINARY NEUTRON STARS (BNS)

[Bartos, Brady, Marka, CQG **30**, 123001 (2013)]





Binary inspiral vs the sensitivity curve

The so-called *Newtonian* signal at instantaneous frequency f_{GW} is

$$h = Q(\text{angles}) \times \mathcal{M}^{5/3} \times f_{GW}^{2/3} \times r^{-1} \times e^{-i\Phi}.$$

where the signal's phase is

$$\Phi(t) = \int 2\pi f_{GW}(t')dt'.$$

The relation between f_{GW} and t

$$\pi \mathcal{M} f_{GW}(t) = \left(\frac{5\mathcal{M}}{256(t_c - t)} \right)^{3/8}$$

The orbital velocity

$$v \propto (\pi \mathcal{M} f_{GW})^{1/3}$$

Binary inspiral vs the sensitivity curve

Match filtering means that the signal is integrated as is sweeps through the range of frequencies.

Sensitivity curves most often show the effective (match-filtered) h_{eff}, and not the instantaneous h.

Order-of-magnitude estimation of the frequency slope:

$$h_{eff} \propto \sqrt{N_{cycles}}\, h \propto \sqrt{ft}\, h \propto \sqrt{f \times f^{-8/3}} \times f^{2/3} = f^{-1/6}.$$

Binary inspiral vs the sensitivity curve

Actually used in estimating the SNR is the frequency-domain match-filtering signal model $\tilde{h}(f)$ (Fourier transform of $h(t)$),

$$\tilde{h}(f) = Q(angles)\sqrt{\frac{5}{24}}\pi^{-2/3}\frac{\mathcal{M}^{5/6}}{r}f_{GW}^{-7/6}e^{-i\Psi(f)},$$

where the frequency domain phase Ψ is

$$\Psi(f) \equiv \Psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu v^{5/2}}\sum_{k=0}^{N}\alpha_k v^{k/2}.$$

Note that the above equations are for point particles! Of course, at the end of inspiral, for a few last orbits

$$\Psi(f) = \Psi_{PP}(f) + \Psi_{tidal}(f)$$

Binary system: source distance estimate

- ⋆ At cosmological distances, the observed frequency f_{GW} is redshifted by $(1+z)$

$\rightarrow$ $f \rightarrow f/(1+z)$,

- ⋆ There is no mass scale in vacuum GR, so redshifting of f_{GW} cannot be distinguished from rescaling the masses

$\rightarrow$ expansion in powers of $v \propto (\pi \mathcal{M} f_{GW})^{1/3}$

$\Longrightarrow$ inferred masses are $m = (1+z)m^{source}$

$\rightarrow$ Direct, independent **luminosity distance** measurement (but not z) from GW with f_{GW} and the strain h:

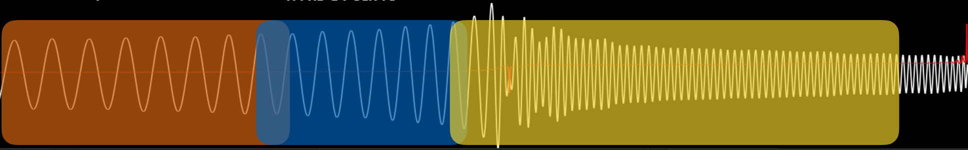
$$r = \frac{5}{96\pi^2} \frac{c}{h} \frac{\dot{f}_{GW}}{f_{GW}^3}.$$

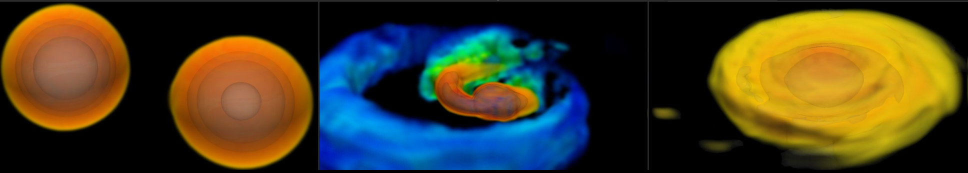
PHYSICAL EFFECTS IN BINARY NEUTRON STAR COALESCENCE WAVEFORMS

dominated by gravitational radiation back reaction - masses and spins

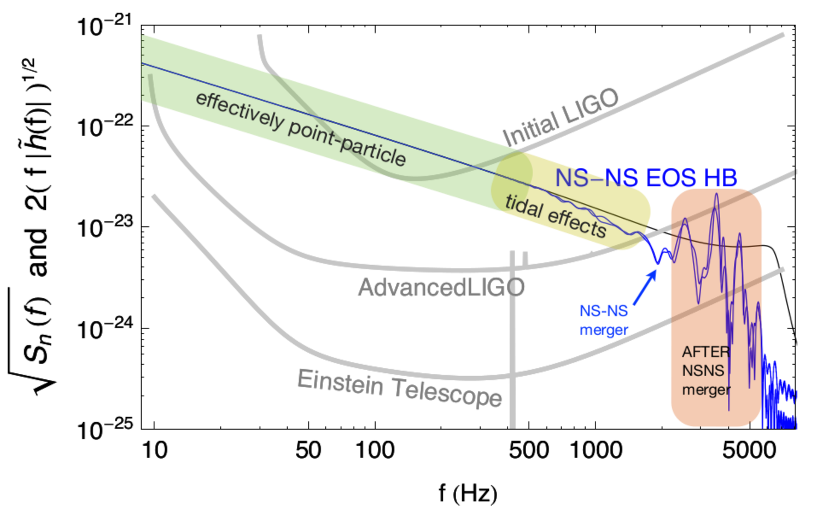
tidal effects appear at high PN order, dynamical tides might be important

complex physics of the merger remnant, multi-messenger source, signature of neutron star EoS



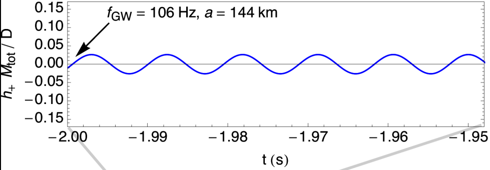


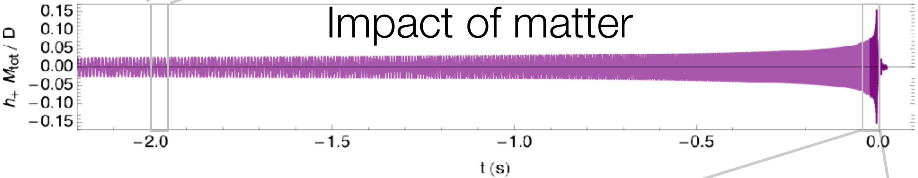
Gravitational-wave spectrum of binary NSs



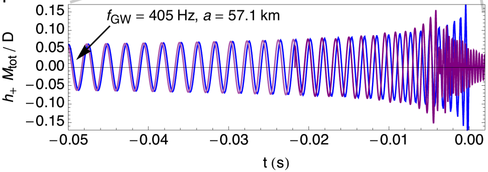
Impact of matter

Hard to modify inspiral: transfer of ~10^{46} erg at ~100 Hz modifies phase by 10^{-3} radians (Crust shattering, Tsang et al 1110.0467)





Tidal interactions lead to accumulated phase shift at higher frequencies.

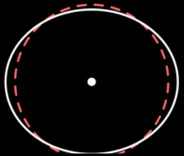


Signature of EOS in binary NSs waveforms

- Tidal tensors $\mathcal{E}_{ij}$ of one of the component of the binary induces quadrupole moment Q_{ij} in the other
- variation in the quadrupole moment causes GW emission
- in the adiabatic approximation

$$Q_{ij} = -\lambda(m)\,\mathcal{E}_{ij}, \qquad \lambda(m) = (2/3)\,k_2(m)\,R^5(m)$$

- where λ(m) is EoS dependent tidal deformability, k_2(m) is the Love number and R is the NS radius
- Just from the scaling this is a 5-PN effect $(v/c)^{10}$



$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

Tidal deformability

Love number k_2

Radius R

$$\lambda = \frac{2}{3} k_2 R^5 \quad (G = c = 1)$$

$$\Lambda \equiv G\lambda (G m_{\rm NS}/c^2)^{-5}$$

$$\Lambda \in [300, 600]$$

(from B.S. Sathyaprakash slides)

Cosmology from tidal interactions & microphysics

- Post-Newtonian phasing formula has binary M and freq. f together

$$\Psi(f) = 2\pi f t_C - \phi_C + \sum_{k=0}^{7} \alpha_k \left(\pi M f\right)^{(k-5)/3}$$

- So it is possible to scale away cosmological frequency redshift: $f \to f\,/\,(1+z)$ and $M \to M\,(1+z)$

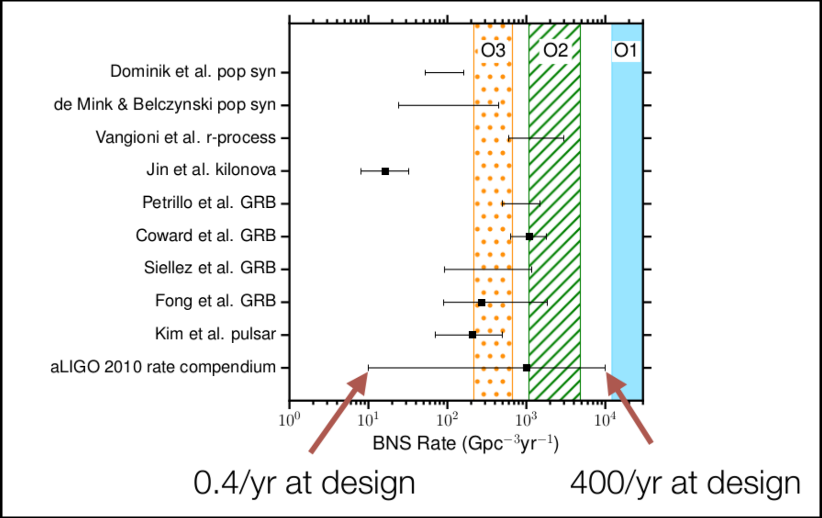
- The tidal term, on the other hand, cannot be scaled away

$$\Psi_{\rm Tide}(f) = -\frac{1250 k_2 \alpha_0}{3} \left(\pi M f\right)^{(k-5)/3} \left(\frac{R}{M}\right)^5$$

- This helps measure neutron star radius and cosmological redshift directly from GW observations

(from B.S. Sathyaprakash slides)

Binary NSs: rates predictions



An unexpected lack of neutron-star mergers?

- ⋆ Salpeter initial mass function, $\xi(M) \propto M^{-2.35}$, for BHs and NSs progenitor stars:

$$\frac{N(M > 80M_\odot)}{N(M > 10M_\odot)} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06$$

- ⋆ If one assumes the same merger rates

$$\frac{\mathcal{R}_{BH}}{\mathcal{R}_{NS}} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \simeq 0.06$$

- ⋆ Signal-to-noise $\propto \mathcal{M}^{5/6}$, detection volume $\propto SNR^3 \propto r^3$

$$\frac{\mathcal{D}_{BH}}{\mathcal{D}_{NS}} = \frac{\mathcal{R}_{BH}}{\mathcal{R}_{NS}} \left(\frac{\mathcal{M}_{BH}}{\mathcal{M}_{NS}}\right)^{5/2} = \left(\frac{80M_\odot}{10M_\odot}\right)^{-1.35} \left(\frac{8.7M_\odot}{1.4M_\odot}\right)^{5/2} \simeq 5.8$$

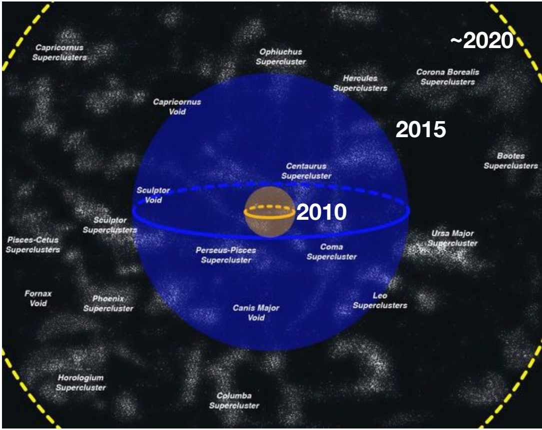
(Phys. Usp. 44 1 2001 [astro-ph/0008481])

Detection prospects of Advanced LIGO design

- binary neutron star mergers to ~200 Mpc
- neutron star–(10 M_{sun}) black hole mergers to ~0.5 Gpc
- (10-10 M_{sun}) binary black hole mergers to ~1 Gpc

(LIGO White Paper: https://dcc.ligo.org/LIGO-T1400054/public, rates above sky-averaged)

initial LIGO BNS range: up to 20 Mpc
image: Shane Larson, Northwestern University



BNS expected 0.4 - 400 yr^{-1}
NSBH expected 0.2 - 300 yr^{-1}
LSC/Virgo 1003.2480

BBH expected 9 – 240 Gpc^{-3} yr^{-1}
LSC/Virgo 1606.04856