Nuclear Superfluidity and Thermal Properties of Neutron Stars

N. Sandulescu, Institute of Physics and Nuclear Engineering, Bucharest

Outline

Lecture 1

Nuclear superfluidity in neutron stars

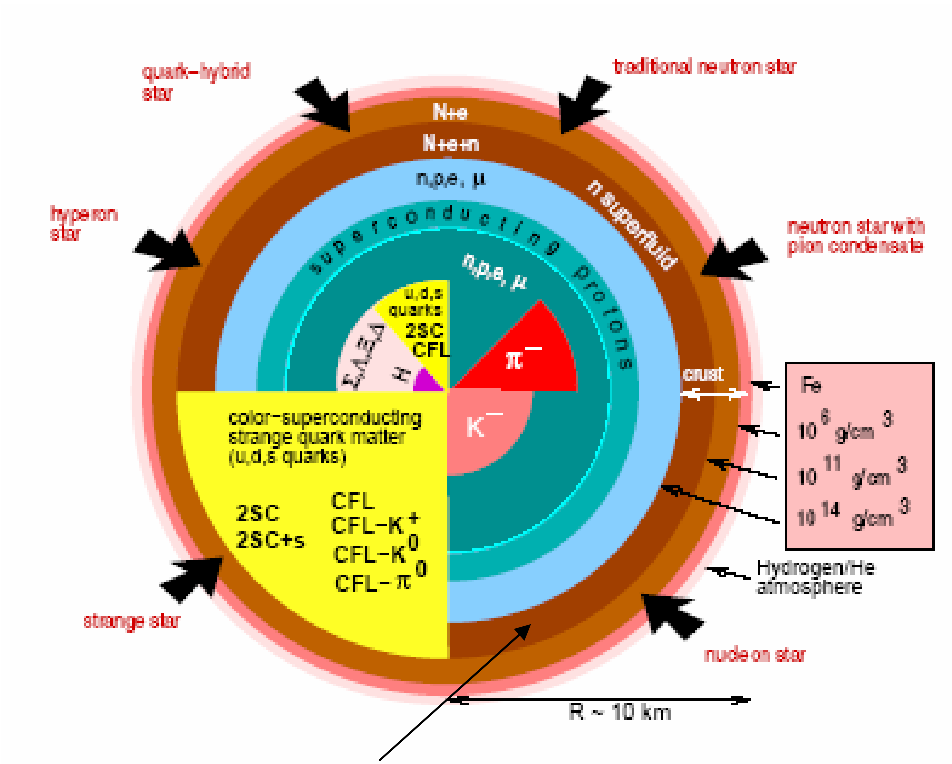
- *generic properties and treatment*
- *vortex motion and giant glitches*

Lecture 2

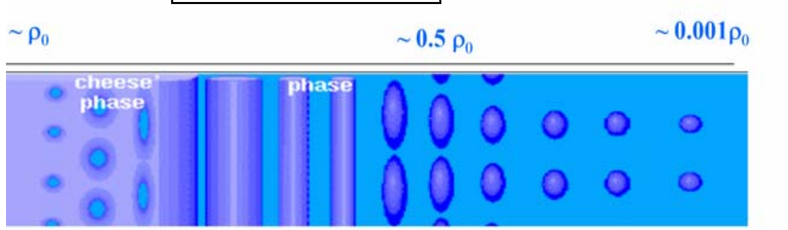
Effects of nuclear superfluidity on thermal properties

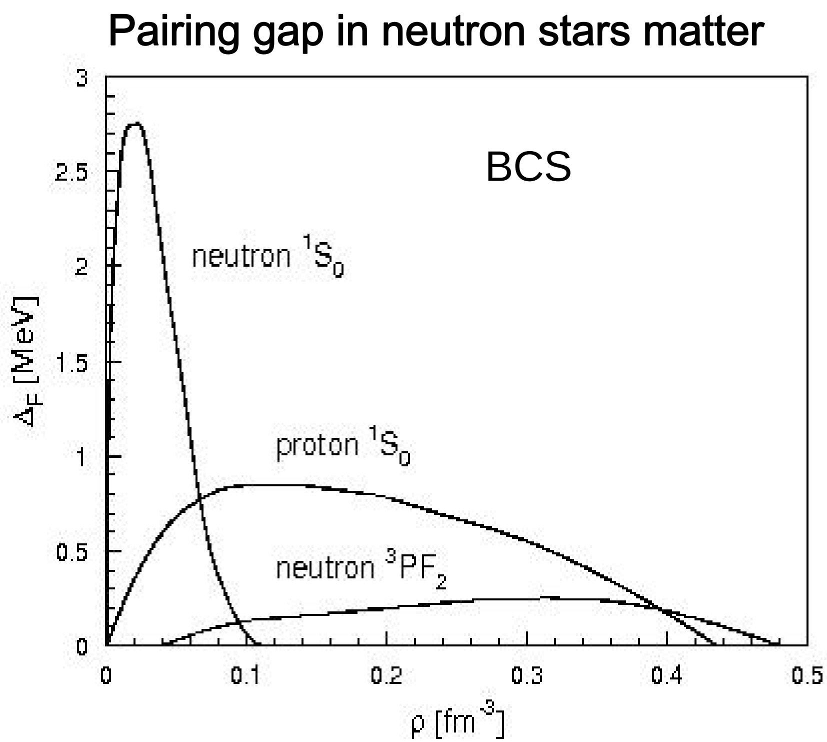
- *specific heat of inner crust matter*
- *crust thermalisation time*

Nuclear Superfluidity in Neutron Stars



Inner Crust

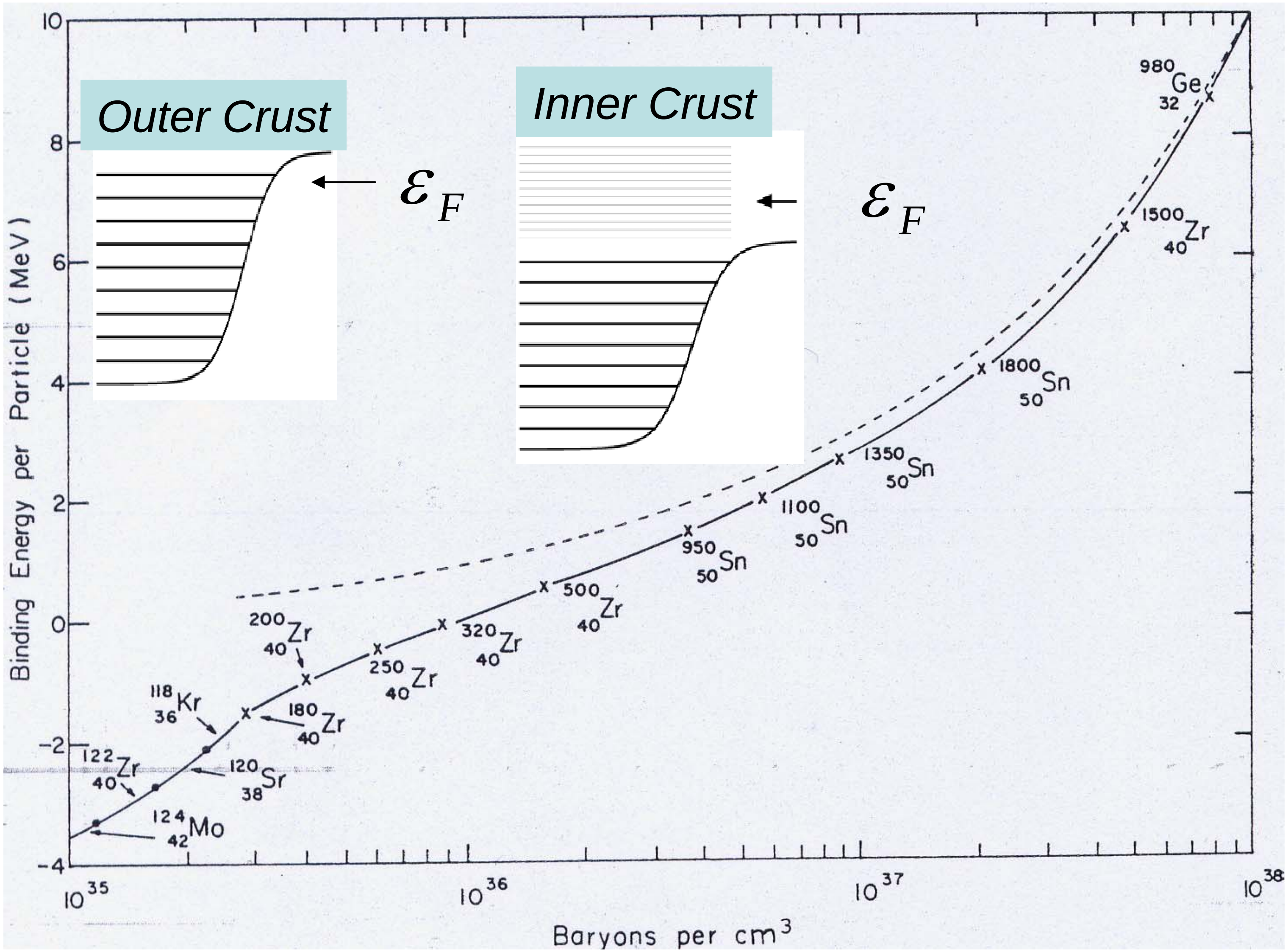




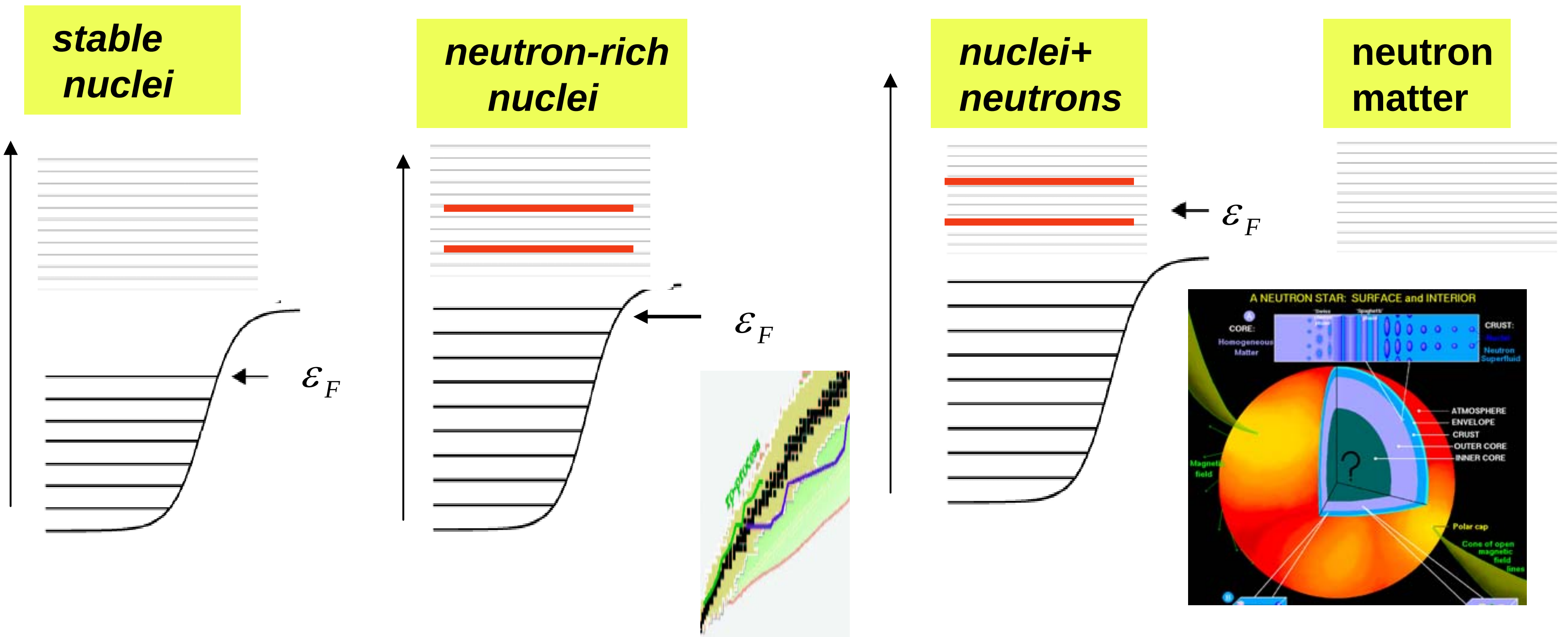
(U.Lombardo, H.-J. Schulze, LNP578, 2001)

Nuclei in the Crust of Neutron Stars

J.W. Negele, D. Vautherin, NPA207 (1973) 298



Nuclear Superfluidity



neutrons: superfluidity of $^1S_0$ type

- ***Consequences***

- excitations (energy gap)

- moment of inertia

- **Crust**: - neutrons: superfluidity of $^1S_0$ type
- ***Core*** : - neutrons: superfluidity of ^3PF type
  - protons: superfluidity of $^1S_0$ type
- ***Conequences*** : - geant glitches
  - cooling

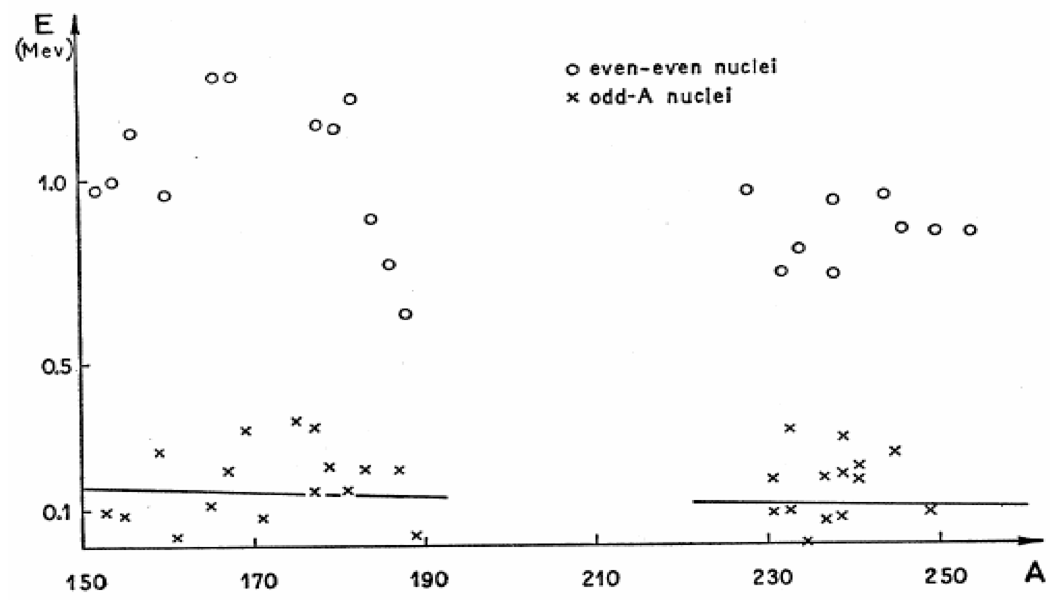
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The correlations giving rise to the energy gap may also affect many other nuclear properties; thus, they appear to be responsible for the observed fact that the rotational moments of inertia are appreciably smaller than the values corresponding to rigid rotation.[3] More-



Moment of inertia : Migdal

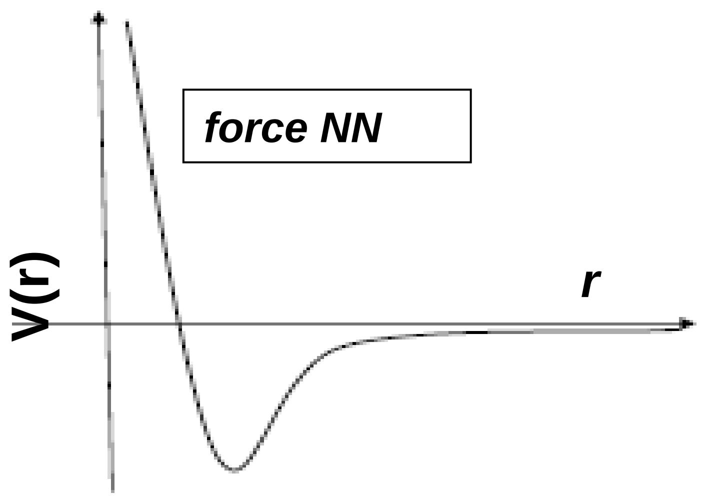
One of the first claims on the nuclear superfluidity in neutron stars

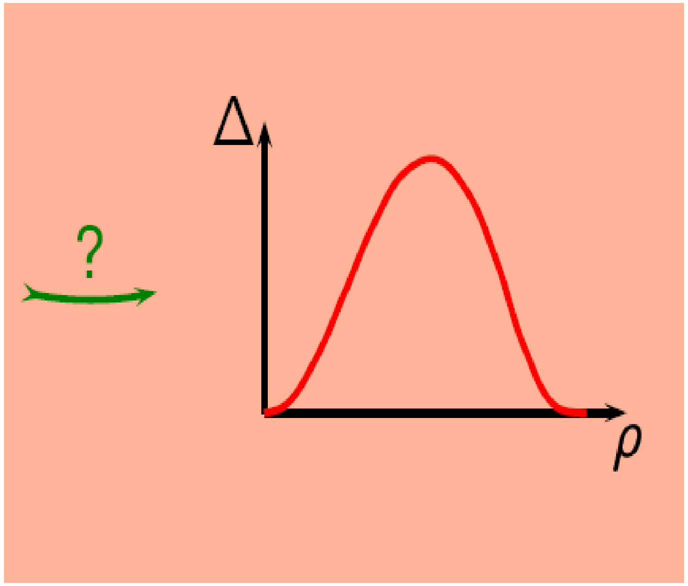
Possible Superfluidity of a System of Strongly Interacting Fermions*†

L. N. Cooper,‡ R. L. Mills, and A. M. Sessler
The Ohio State University, Columbus, Ohio

(Received January 30, 1959)

the superfluid state. We find that a repulsive hard core does not in principle forbid the existence of a superfluid state, but whereas in the absence of a hard core an attractive two-body potential always leads to a superfluid state at sufficiently low temperatures, in the presence of a repulsive core there appears to be a critical strength of attraction needed to form a superfluid state. When the variational principle is applied to liquid He^3 or to nuclear matter, it is found for a wide class of trial functions that the system does not become a superfluid.





There are constraints from neutron stars properties ?

Nuclear Superfluidity: generic properties

- *One- Cooper- pair problem*
- *Condensate of pairs: treatment (BCS and HFB)*
- *Superfluid flow and vortex motion*

Letters to the Editor

Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois

(Received September 21, 1956)

IT has been proposed that a metal would display superconducting properties at low temperatures if the one-electron energy spectrum had a volume-independent energy gap of order $\Delta \simeq kT_c$, between the ground state and the first excited state.[1,2] We should like to point out how, primarily as a result of the exclusion principle, such a situation could arise.

Consider a pair of electrons which interact above a quiescent Fermi sphere with an interaction of the kind that might be expected due to the phonon and the screened Coulomb fields. If there is a net attraction between the electrons, it turns out that they can form a bound state, though their total energy is larger than zero. The properties of a noninteracting system of such

$=(1/V)\exp[i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)]$ which satisfy periodic boundary conditions in a box of volume V, and where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of electron one and electron two. (One can use antisymmetric functions and obtain essentially the same results, but alternatively we can choose the electrons of opposite spin.) Defining relative and center-of-mass coordinates, $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$, $\mathbf{r}=(\mathbf{r}_2-\mathbf{r}_1)$, $\mathbf{K}=(\mathbf{k}_1+\mathbf{k}_2)$ and $\mathbf{k}=\frac{1}{2}(\mathbf{k}_2-\mathbf{k}_1)$, and letting $\mathcal{E}_K+\epsilon_k=(\hbar^2/m)(\frac{1}{4}K^2+k^2)$, the Schrödinger equation can be written

$$(\mathcal{E}_K+\epsilon_k-E)a_{\mathbf{k}}+\sum_{\mathbf{k}'} a_{\mathbf{k}'}(\mathbf{k}|H_1|\mathbf{k}') \times\delta(\mathbf{K}-\mathbf{K}')/\delta(0)=0 \quad (1)$$

where

$$\Psi(\mathbf{R},\mathbf{r})=(1/\sqrt{V})e^{i\mathbf{K}\cdot\mathbf{R}}\chi(\mathbf{r},K),\quad \chi(\mathbf{r},K)=\sum_{\mathbf{k}}(a_{\mathbf{k}}/\sqrt{V})e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (2)$$

and

$$(\mathbf{k}|H_1|\mathbf{k}')=\left(\frac{1}{V}\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}H_1e^{i\mathbf{k}'\cdot\mathbf{r}}\right)_{0\text{ phonons}}.$$

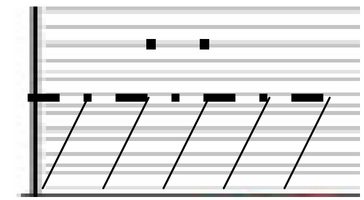
We have assumed translational invariance in the metal. The summation over $\mathbf{k}'$ is limited by the exclusion principle to values of k_1 and k_2 larger than q_0, and by the delta function, which guarantees the conservation of the total momentum of the pair in a single scattering. The K dependence enters through the latter restriction.

Bardeen and Pines[3] and Fröhlich[4] have derived approximate formulas for the matrix element $(\mathbf{k}|H_1|\mathbf{k}')$; it is thought that the matrix elements for which the two electrons are confined to a thin energy shell near the Fermi surface, $\epsilon_1\simeq\epsilon_2\simeq\epsilon_F$, are the principal ones

One- Cooper-pair problem

***Physical system**: two fermions subjected to an atractive interaction and situated on top ($k > k_F$) of a free gas of fermions*

- Free states (box of length L) $\varphi(r,k_i) = \frac{1}{L^{3/2}}\, e^{ik_i r}$



- Two-electrons with CM at rest, i.e., $k_1 = -k_2 \equiv k$

$$\phi(r_1,r_2) = \sum_{k>k_F} g(k) e^{ikr_1} e^{-ikr_2} \uparrow\downarrow$$

$$[-\frac{\hbar^2}{2m}\Delta_1 - \frac{\hbar^2}{2m}\Delta_2 + V(r_1,r_2)]\phi = (E + 2\varepsilon_F)\phi$$

$$V_{k-k} \approx -\frac{G}{L^3} \quad \text{if} \quad \varepsilon_F < \varepsilon_k < \varepsilon_F + \varepsilon_{cut}$$

$$E \equiv -\Delta \approx -2\varepsilon_{cut} e^{\frac{-2}{GN(\varepsilon_F)}}$$

$$\text{if} \;\; GN(\varepsilon_F) << 1$$

A solution with $E < 0$ exists for an arbitrarily small interaction strength, in variance with the two - body problem. This fact is due to the condition $k > k_F$ and due to the degeneracy of $N(\varepsilon_F)$.

The "condensate" of pairs

Physical system: **N** *fermions in the presence of an attractive force*

Cooper pair instability >>> system of ***identical*** pairs

$$\Psi_N(r_1,\ldots,r_N) = A\,\phi(r_1,r_2)\ldots\phi(r_{N-1},r_N)(1\uparrow,2\downarrow)\ldots(N-1\uparrow,N\updownarrow)$$

• each pair is described by *the same wave function* ϕ

$$\phi(r_1,r_2) = \sum_k g(k)e^{ikr_1}e^{-ikr_2}$$

*k is **not** restricted by $k>k_F$*

$$\Psi_N(r_1,\ldots r_N) = \sum_{k_1,..k_{N/2}} g(k_1)\ldots g(k_{N/2})A\,e^{ik_1r_1}(1\uparrow)e^{-ik_1r_2}(2\downarrow)\ldots\ldots\ldots$$

Slater determinant

$$|\Psi_N\rangle = \sum_{k_1\ldots k_{N/2}} g(k_1)\ldots g(k_{N/2})c^+_{k_1\uparrow}c^+_{-k_1\downarrow}\ldots c^+_{k_{N/2}\uparrow}c^+_{-k_{N/2}\downarrow}|-\rangle$$

$$|\Psi_N\rangle \propto (S^+)^{N/2}|-\rangle;\ \text{with } S^+ \equiv \sum_k g(k)c^+_{k\uparrow}c^+_{-k\downarrow}$$

<u>Note</u>: *S^+ is not a boson operator, so the wave function is not a Bose-Einstein condensate*

Pair condensate: BCS ansatz

- *Condensate with a given number of pairs*

$$|\Psi_N> \propto (S^+)^{N/2}|->; \text{ with } S^+ \equiv \sum_k g(k) c^+_{k\uparrow} c^+_{-k\downarrow}$$

- *BCS ansatz : ("coherent") distribution of pairs*

$$|BCS> \propto \sum_{\nu=0}^{\nu=\infty} \frac{1}{\nu!}(S^+)^\nu |-> = e^{S^+} \qquad g_k \equiv \frac{\mathrm{v}_k}{\mathrm{u}_k}$$

$$\delta_{u_k,\mathrm{v}_k} < BCS|\hat{H} - \mu\hat{N}|BCS> = 0 \qquad \text{condition: } N = < BCS|\hat{N}|BCS>$$

- *BCS equations*

$$\mathrm{v}_k^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \mu}{\sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}}\right)$$

$$\Delta_k = -\sum_l V_{kl} \frac{\Delta_l}{2\sqrt{(\varepsilon_k - \mu)^2 + \Delta_l^2}}$$

Pairing in non-uniform systems: Bogoliubov approach

How to form Cooper pairs in inhomogeneous systems ?

use intrinsic properties of the condensate (in "field" picture)

$$\hat{H} = \sum_{\sigma} \int dr \psi^{+}(r\sigma) \hat{T} \psi(r\sigma) - \frac{1}{2} V_0 \sum_{\sigma,\sigma'} \int dr \psi^{+}(r\sigma) \psi^{+}(r\sigma') \psi(r\sigma') \psi(r\sigma)$$

$$H_{eff} = \int dr \{ \sum_{\sigma} [\psi^{+} \hat{T} \psi + \Gamma(r) \psi^{+} \psi] + \Delta(r) \psi^{+}(r \frac{1}{2}) \psi^{+}(r - \frac{1}{2}) + \Delta^{*}(r) \psi(r - \frac{1}{2}) \psi(r, \frac{1}{2}) \}$$

• ***Spectrum***: *canonical transformation*

$$\psi(r\sigma) = \sum_i U_i(r\sigma)\beta_i + V_i^*(r\sigma)\beta_i^\dagger$$

$$\psi^\dagger(r\sigma) = \sum_i U_i^*(r\sigma)\beta_i^\dagger + V_i(r\sigma)\beta_i$$

$$\hat{H}_{eff} = E_g + \sum_i E_i \beta_i^{+} \beta_i$$

Bogoliubov Equations

$$\begin{pmatrix} T+\Gamma-\lambda & \Delta \\ -\Delta^* & -T-\Gamma+\lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

consistency:

$$\Gamma(r) = -V_0 \sum |V(r)|^2$$

$$\Delta(r) = -V_0 \sum U_i(r) V_i^*(r)$$

• Pairing density:

$$\kappa(r) \equiv < \psi(r, \frac{1}{2}) \psi(r, -\frac{1}{2}) > = -\sum \mathrm{U_i} V_i^*$$

κ(r) describes the center of mass motion of the condensed pairs !

• a stationary state of a homogeneous system: **κ(r) =constant**

• a stationary state of an inhomogeneous system: **κ(r) =κ* (r)**

• *translation of pairs with a total momentum **2q**:* $\kappa_q = |\kappa_q| e^{2iqr}$ $\qquad E_k = E_k^{q=0} - \frac{\hbar^2 k}{m} q$

• *vortex type motion :* $\kappa(r) = |\kappa| e^{in\varphi}$

Superfluid Flow

- ***Center of mass at rest:***

$$\phi(r_1,r_2)=\sum g(k)e^{ikr_1}e^{-ikr_2}$$

$$\Psi_N^0(r_1,\ldots,r_N)=A\,\phi(r_1,r_2)\ldots.\phi(r_{N-1},r_N)(1\uparrow,2\downarrow)\ldots(N-1\uparrow,N\updownarrow)$$

- ***Translation motion***: *if the system is Galilean invariant, one can simply shift the CM of each pair with a given amount in momentum space*

$$\Psi_N^q(r_1,\ldots,r_N)=A\phi(r_1,r_2)e^{\frac{i}{2}q(r_1+r_2)}\ldots.\phi(r_{N-1},r_N)e^{\frac{i}{2}q(r_{N-1}+r_N)}\ldots\equiv e^{\frac{i}{2}\sum_{j=1}^{N}qr_j}\Psi_N^0$$

*valid in general if **q** does not change much over a coherence length (London)*

- ***Superfluid flow***: *arbitrary motion (London & Feynman)*

$$\Psi_N^s(r_1,\ldots,r_N)\equiv e^{\frac{i}{2}\sum_{j=1}^{N}s(j)}\Psi_N^0$$

$$\vec{v}_s=\frac{\hbar}{m}grad(s);$$

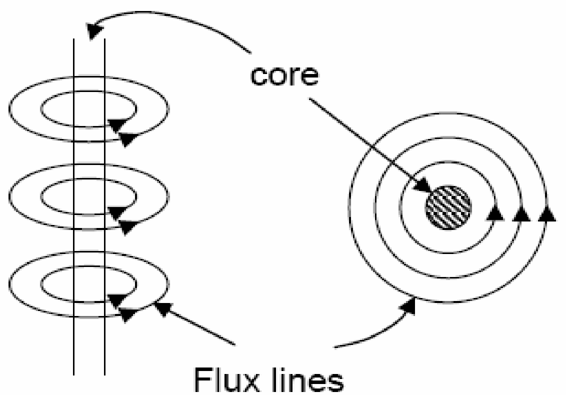
$$\mathrm{curl}(\vec{v}_s)=0$$

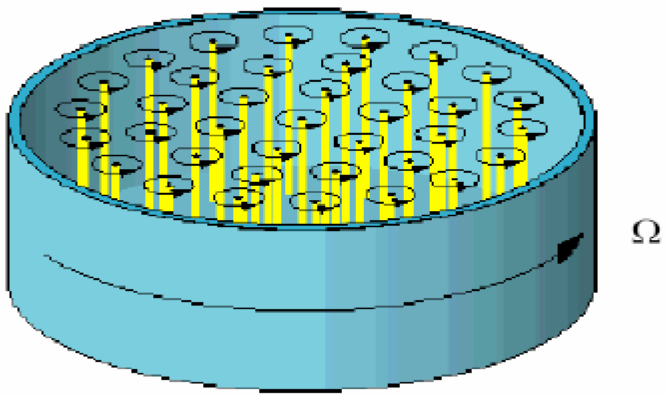
Note*: a condensate corresponds to a* ***metastable*** *equilibrium since a change of it would involve a* ***simultaneous*** *transition of many pairs.*

Rotation of uniform superfluids

superfluid $\rightarrow$ irrotational flow $\rightarrow$ $\nabla \times \vec{v}_s = 0$

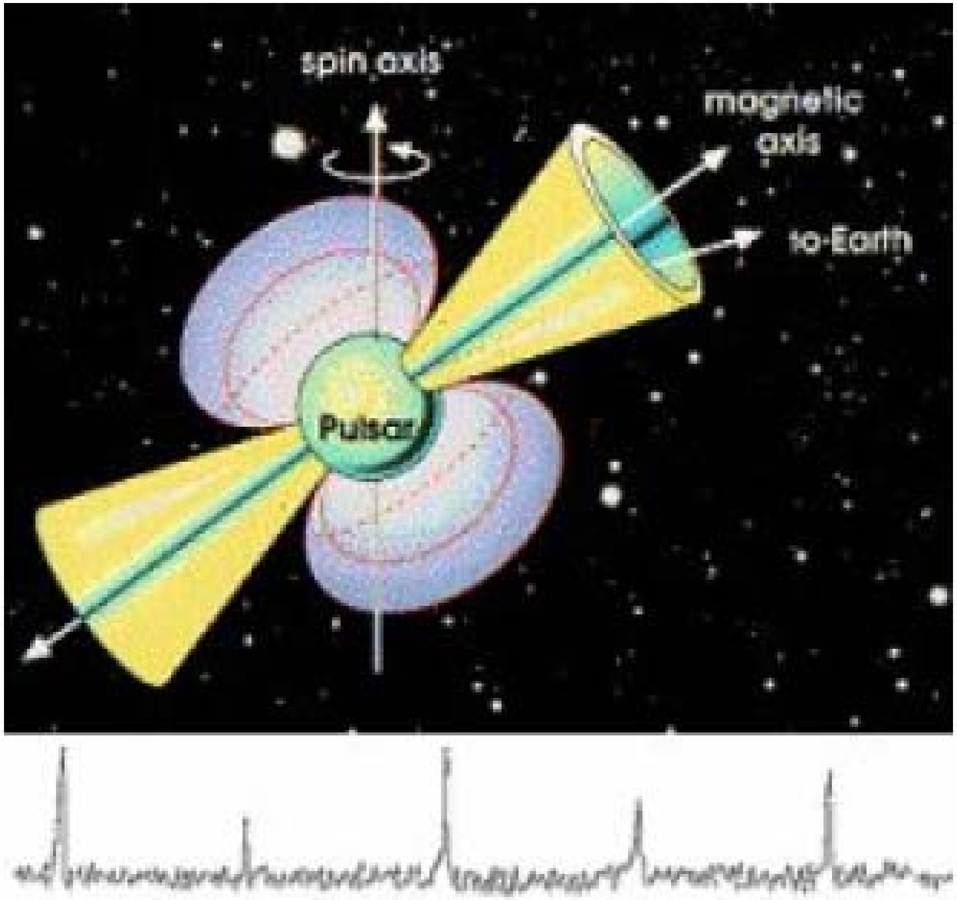
classical vortex $\rightarrow$ $\vec{v}_s = \dfrac{C}{r} \hat{e}_\theta$





quantized vorticity $\rightarrow$ $\oint \vec{v}_s \cdot d\vec{l} = k \dfrac{h}{2m_N} \quad (k = 1, 2, \cdots)$

Pulsars



emission of e.m. and gravitational waves

$\Downarrow$

decrease of rotational frequency $\rightarrow$ $\dot{\Omega} < 0$

Superfluidity and Giant Glitches

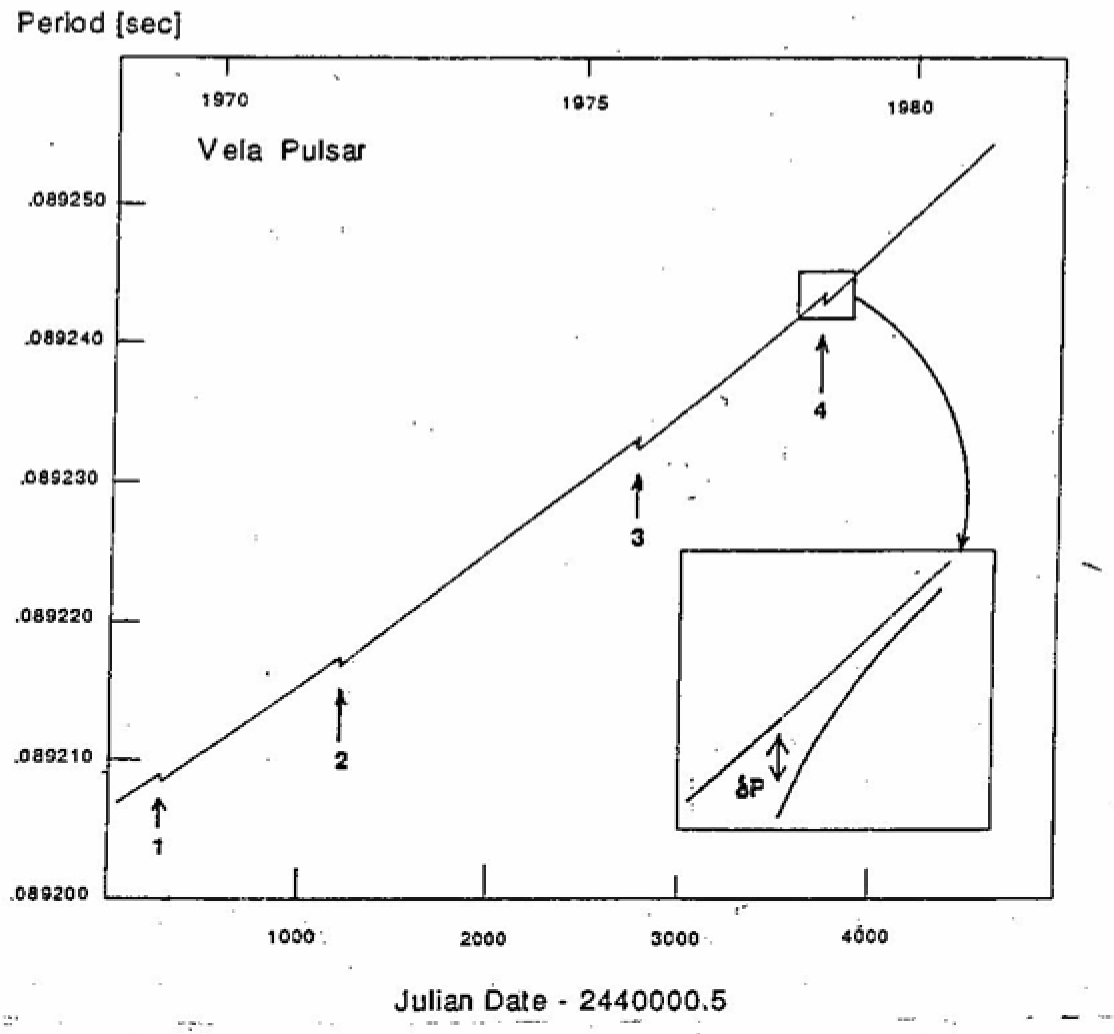
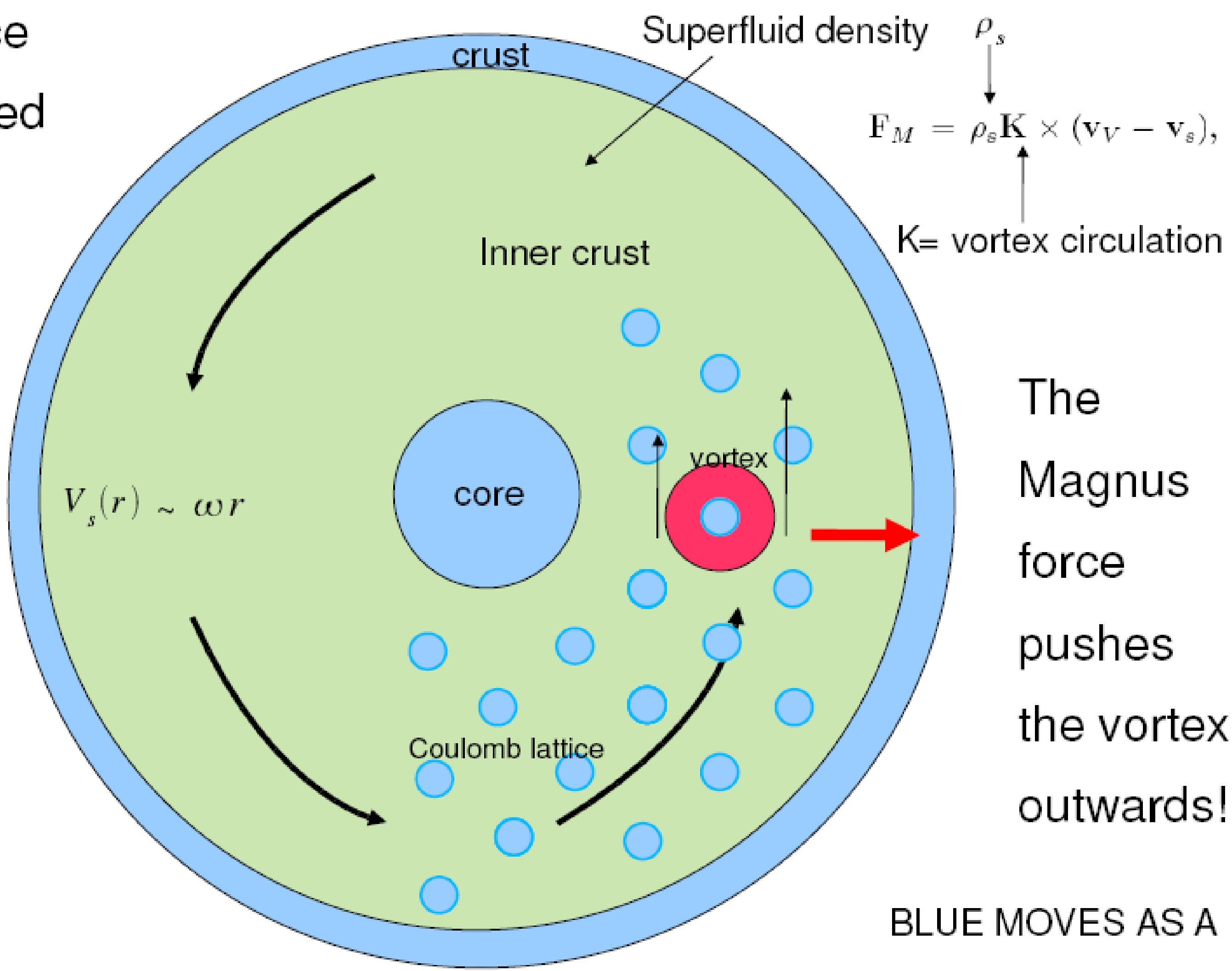


Figure 4. The first four giant glitches of Vela pulsar [from G. Downs (1981)]. The change in period due for each glitch is of order $\Delta P/P \sim 10^{-6}$, and the timescale for the recovery of the glitch is estimated to be of order $\tau \sim 1-3$ months; however, the post-glitch behavior of the period is more complicated than simple exponential recovery of the period and spin-down rate.

- Spin-up: $\Delta P/P \sim 10^{-6}$
- Recovery: ~ 1-3 months
- Energy : 10^{43} erg
- Scenario: vortex depinning

Anderson & Itoh, 1975

Reference frame fixed with the Coulomb lattice



The Magnus force pushes the vortex outwards!

BLUE MOVES AS A RIGID BODY!!

The crust has been exaggerated!!!

Figure by F. Barranco (INT-Seattle, june 2007)

Vortex configuration: pinning energy ?

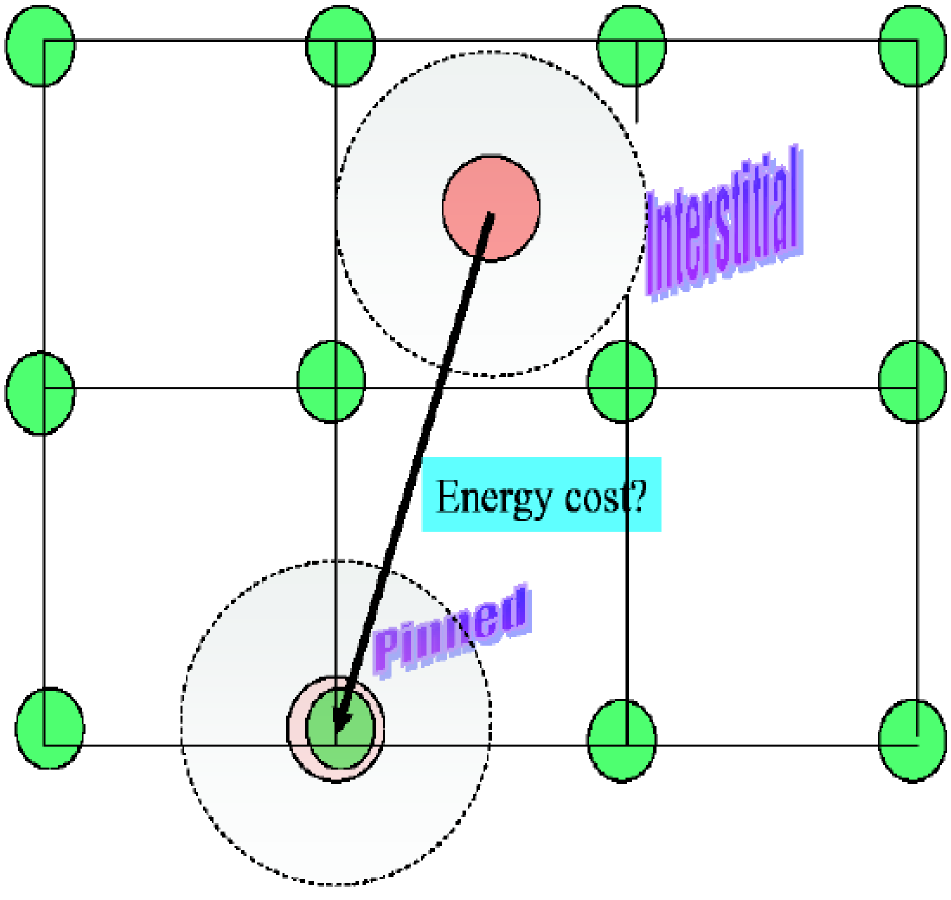
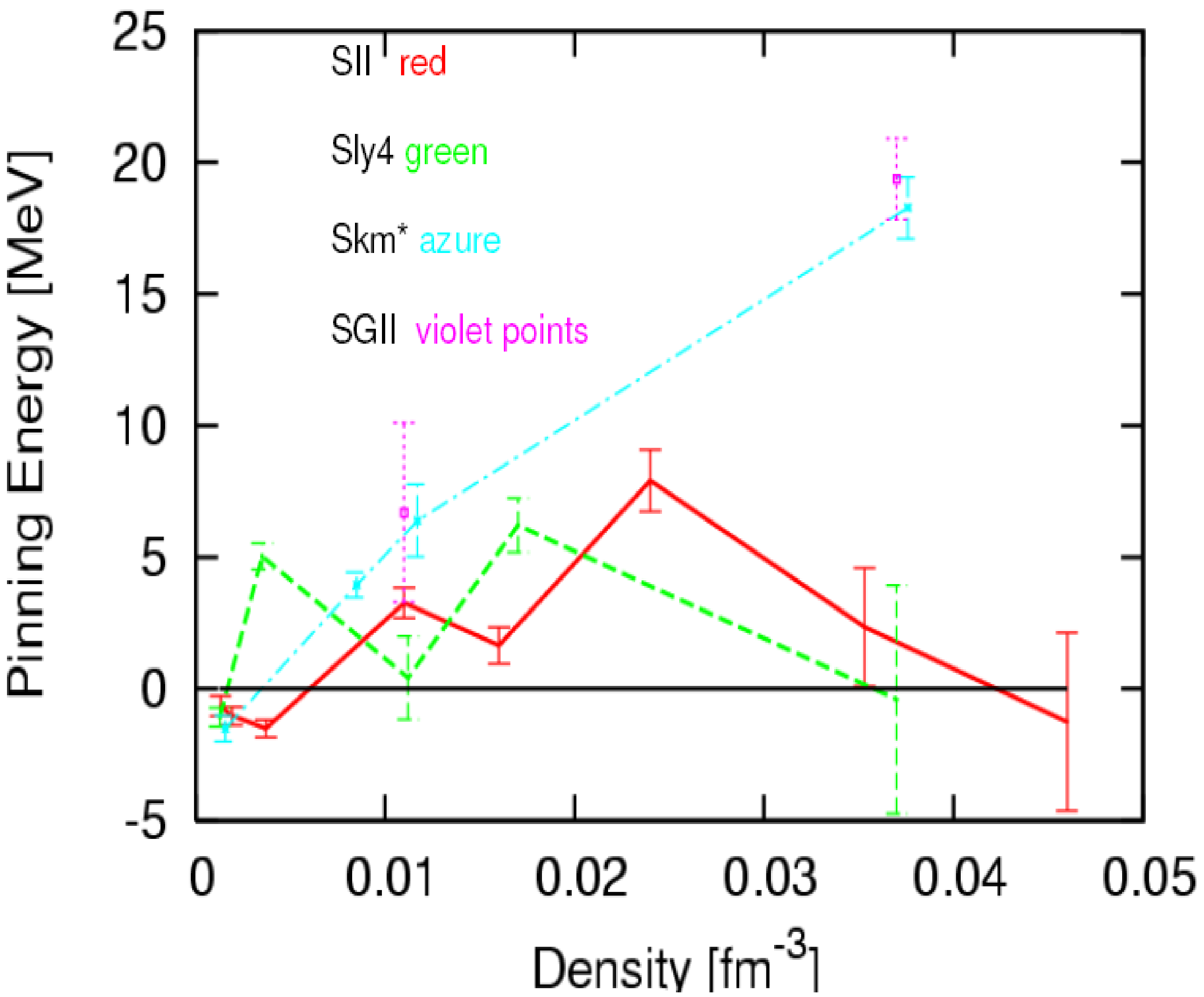


Figure by F. Barranco (INT-Seattle, june 2007)

Pinning Energies



P. Avogadro, F.Barranco, R.A.Broglia, E.Vigezzi **Phys. Rev. C 75 012085 (2007)**

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N. Sandulescu, Institute of Physics and Nuclear Engineering, Bucharest

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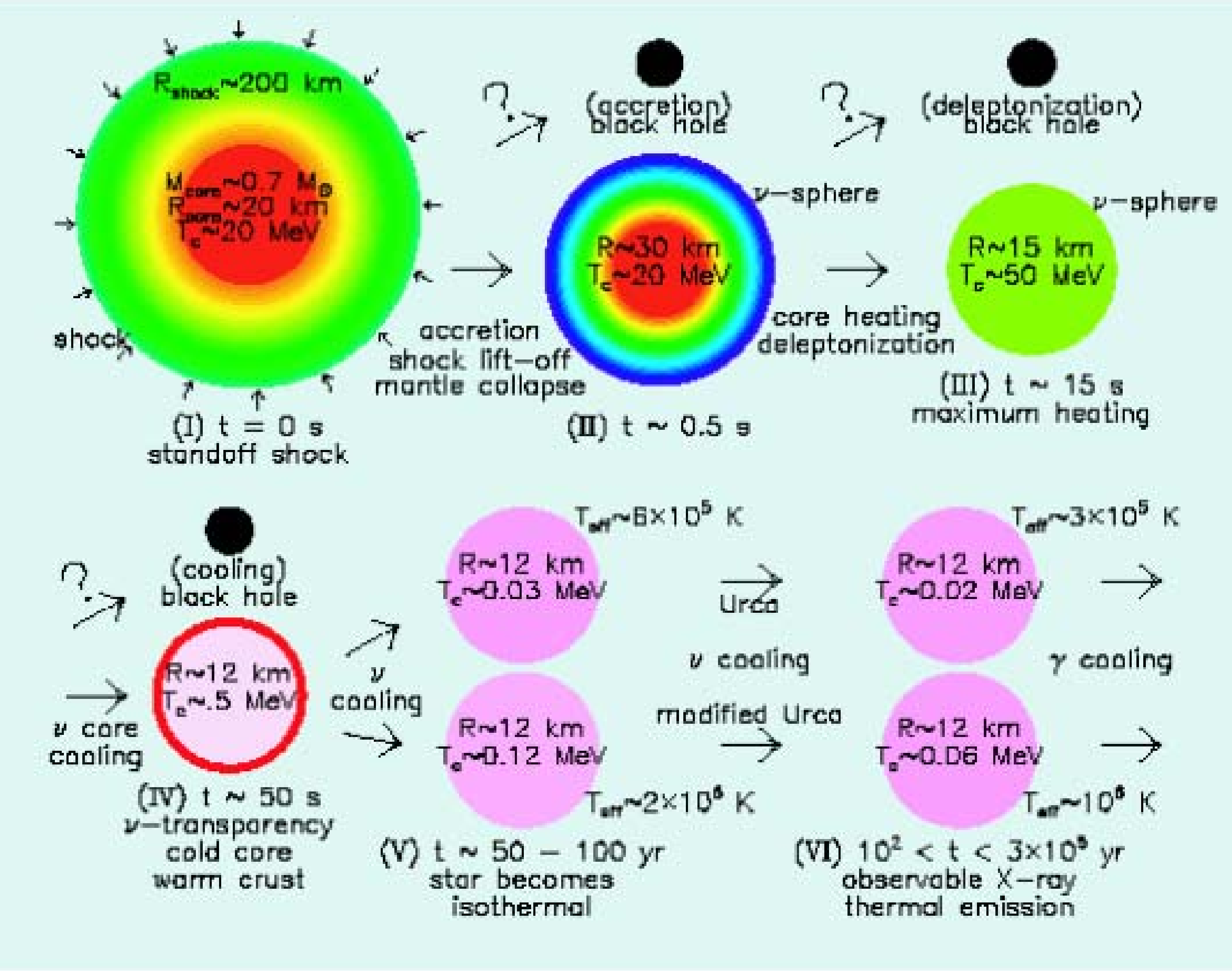
→ ***Lecture 2***

Effects of nuclear superfluidity on thermal properties

- *specific heat of inner crust matter*
- *crust thermalisation time*

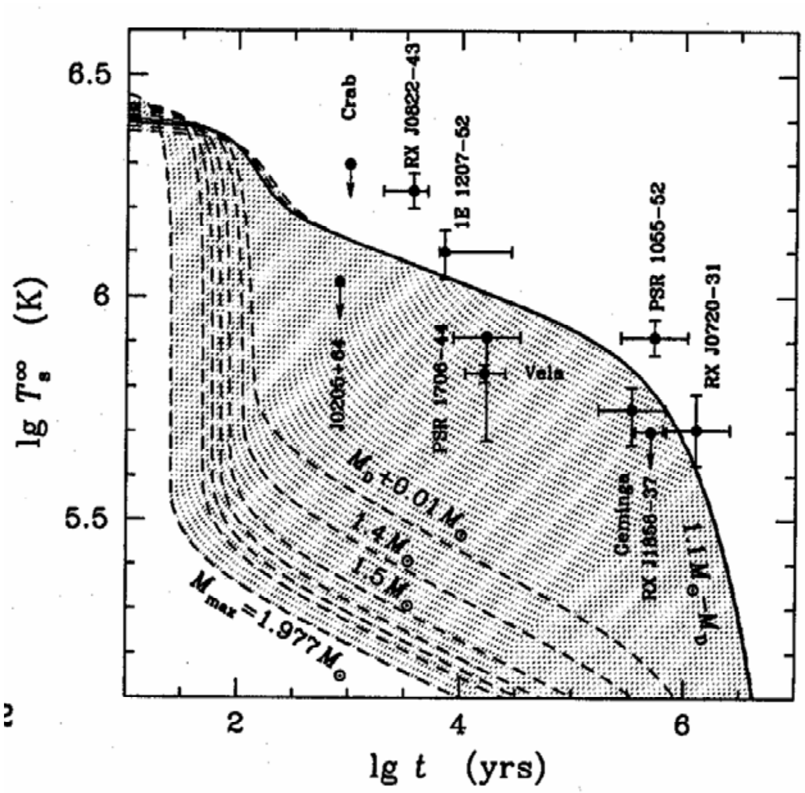
Thermal Evolution of Neutron Stars



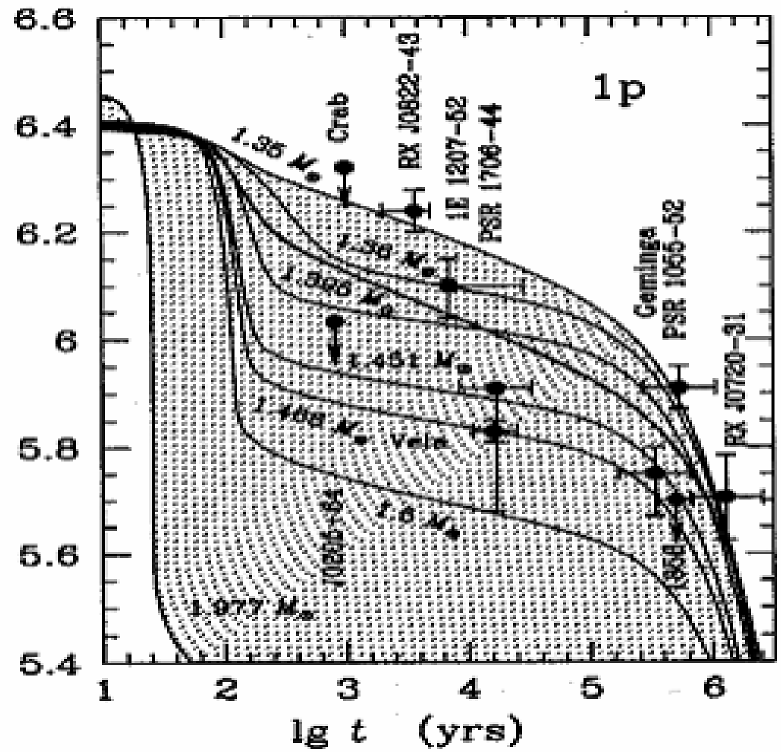
J.M. Lattimer , M. Prakash, Science 304(2004)

Nuclear Superfluidity and Neutron Stars Cooling

No superfluidity



Superfluidity

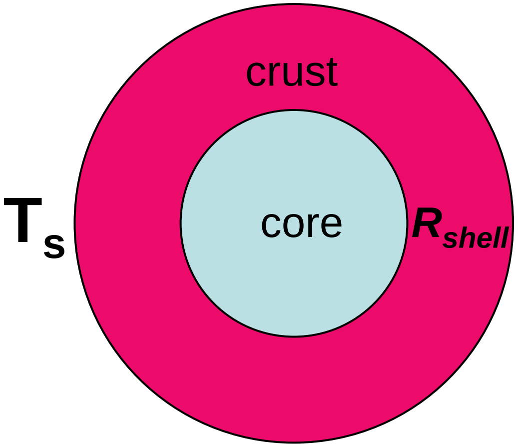


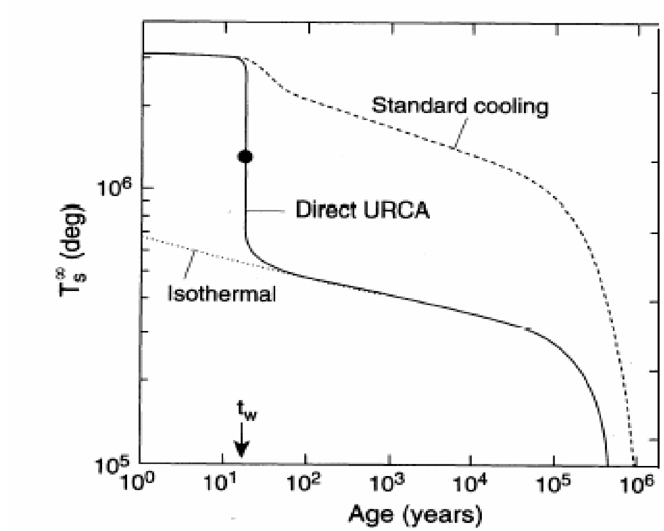
(D.G. Yakovlev et al, ApJ 2004)

- **URCA processes:** ***direct*** **:** **n > p +e +ν; p+e > n+ ν**

  modified **:** **n+n >n+ p +e + ν;n+p+e >n+ n+ ν**

Cooling Time of Neutron-Stars Crust





J.M. Lattimer et al, ApJ 1994

- *G. E. Brown et al, PRD37, 1998*

$$t_w \approx R_{shell}^2 \frac{C_V}{k}$$

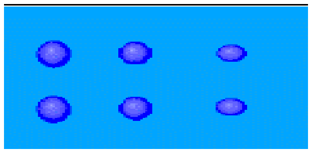
- *J.M. Lattimer et al, ApJ425, 1994*

$$t_w \propto R_{shell}^n \qquad 1.7 \leq n \leq 1.8$$

t_w is strongly affected by the inner crust superfluidity !

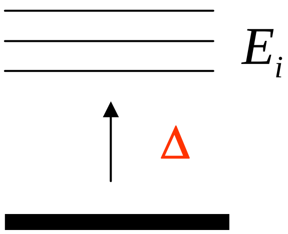
Superfluidity and Specific Heat of Crust Matter

$C^{(t)}_V = \mathbf{C_V\,(n)} + C_V\,(e) + C_V(\text{lattice})$



- *normal phase :* $C_V\,(n) > C_V\,(e)$

- *suprafluid phase :* $C_V(n) \Leftrightarrow C_V(n;\Delta=0)\, e^{-\Delta/kT}$



- issues:

  - cooling time versus pairing intensity ?

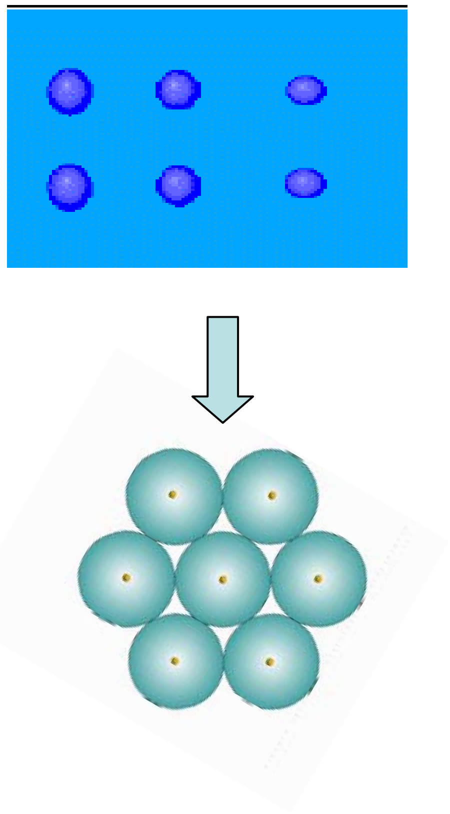
  - effect of nuclear clusters on cooling time ?

  - P.M.Pizzochero et al, ApJ569, 2002

  - effects of the collective excitations ?

Inner crust: microscopic treatment

$$\mathcal{E} = \mathcal{E}_{ph} + \mathcal{E}_{pairing} + \mathcal{E}_{electrons}$$



$\delta\mathcal{F}=0$, β–equilibrium

Self-consistent mean field calculations (HFB)

I) Inner crust structure: N/Z, R_{ws}

*II) Pairing properties : **$\Delta(r,T,\omega)$, E_i***

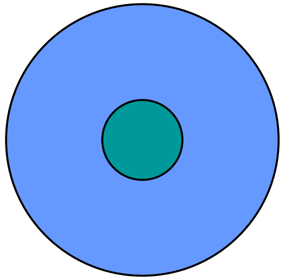
*III) Collective excitations: **QRPA***

→ C_V

Finite-Temperature HFB

$$\mathcal{E}_{nuc} = \mathcal{E}_{ph} + \mathcal{E}_{pair}[\rho, \kappa]$$

$$\begin{pmatrix} h_T(r) - \lambda & \Delta_T(r) \\ \Delta_T(r) & -h_T(r) + \lambda \end{pmatrix} \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix}$$



$$\kappa_T = \frac{1}{4\pi} \sum (2j_i + 1) U_i^*(r) V_i(r) (1 - 2f_i)$$

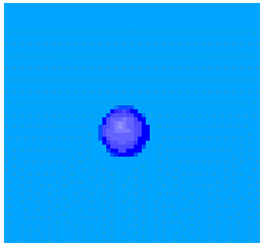
$$\Delta_T(r) = V_{pair}\, \kappa_T(r)$$

$$\rho_T(r) = \frac{1}{4\pi} \sum (2j_i + 1) [V_i^*(r) V_i(r)(1 - f_i) + U_i^*(r) U_i(r) f_i]$$

where : $f_k = \dfrac{1}{1 + \exp(E_k / k_B T)}$

Energy Functional

$$\mathcal{E}_{nuc} = \mathcal{E}_{Skyrme} + \mathcal{E}_{pair}[\rho, \kappa]$$



$$\mathcal{E}_{pair}[\rho, \kappa]$$

nuclei

Pairing in neutron matter ?

neutron matter

Pairing in Neutron Matter : BCS with bare forces

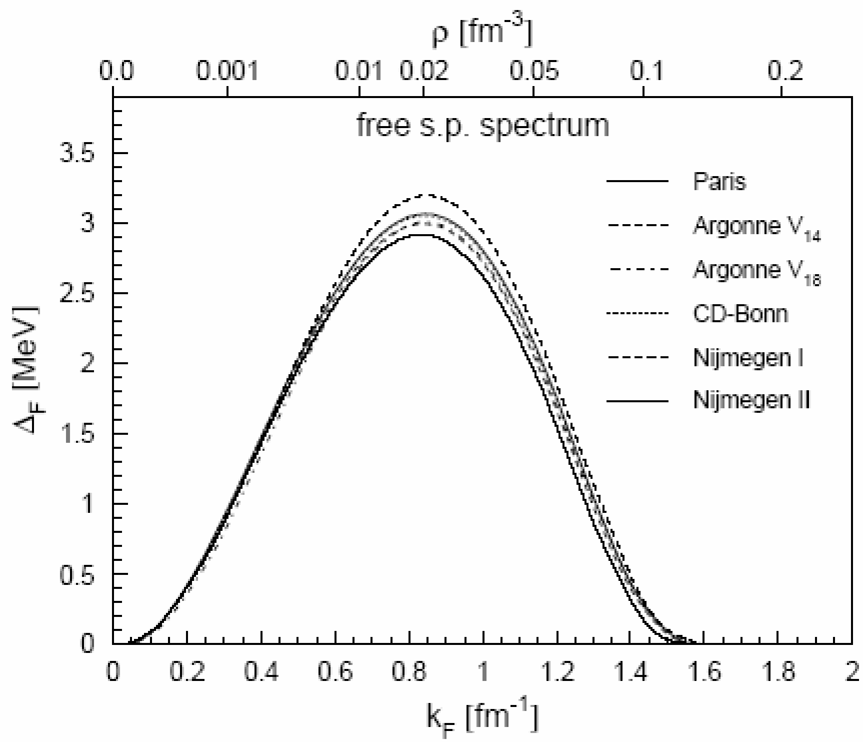
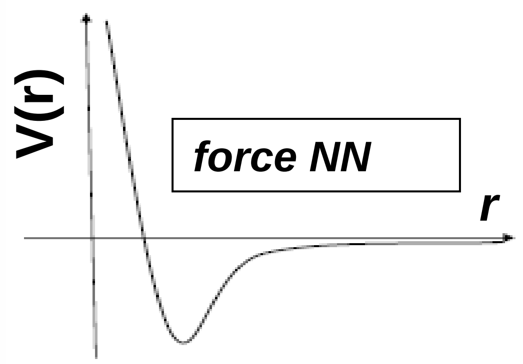


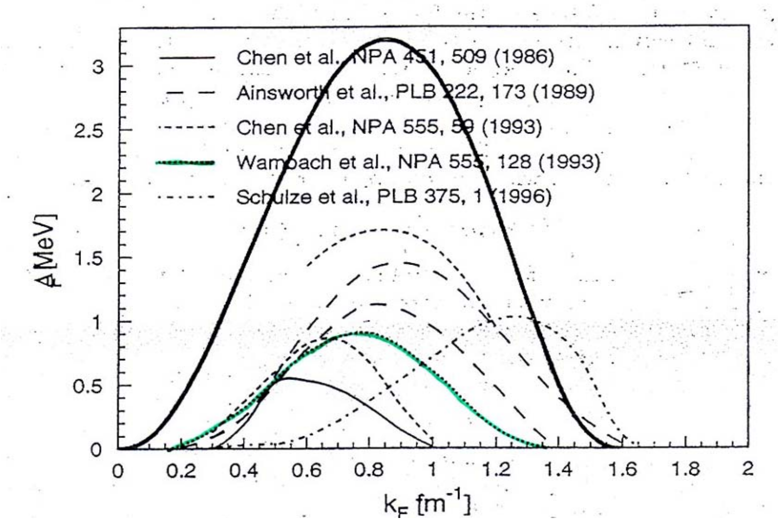
Fig. 2. $^1S_0$ gap evaluated in BCS approximation with free single-particle spectrum and different potentials

$$\Delta_k = -\sum_l V_{kl} \frac{\Delta_l}{2\sqrt{(\varepsilon_k - \mu)^2 + \Delta_l^2}}$$

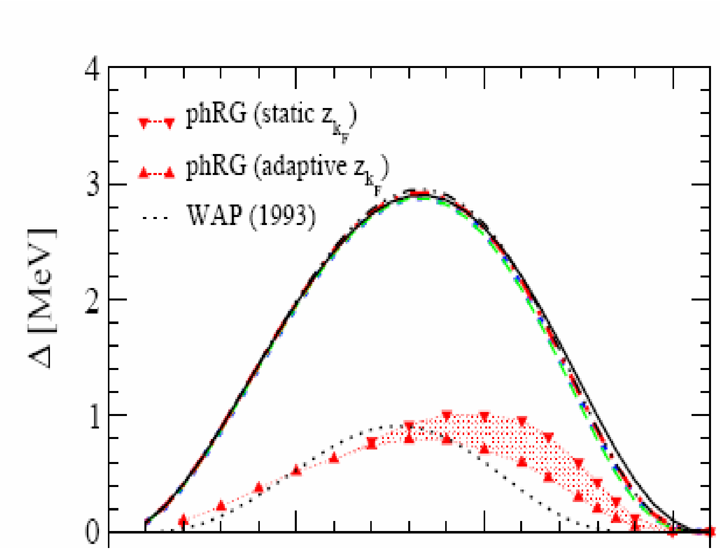


(U.Lombardo, H-J. Schulze, Lect.Notes Phys. 578 ,2001, 30)

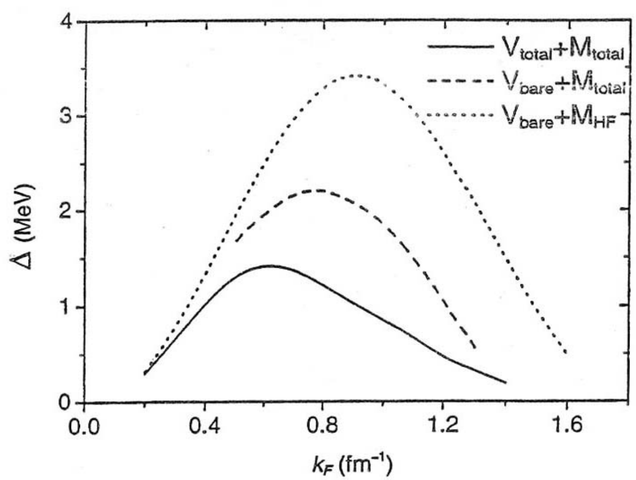
$^1S_0$ Pairing Gap in Neutron Matter: beyond BCS



U.Lombardo, H.-J. Schulze, LNP578, 2001



A. Schwenk et al, NPA713 (2003) 191

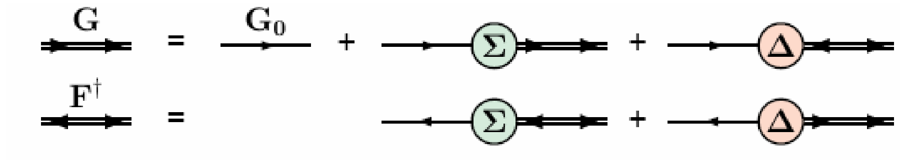


C.Shen et al , Phys.Rev.C67(2003)

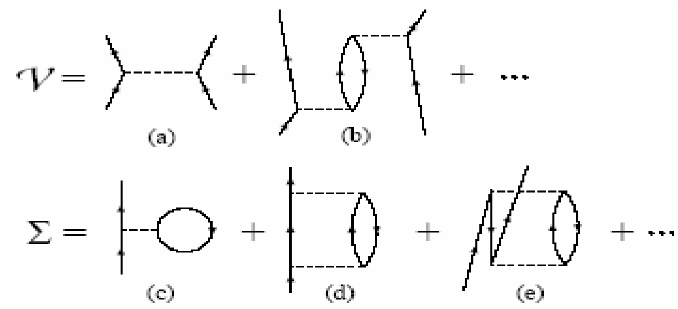
Pairing in Nuclear Matter: beyond BCS

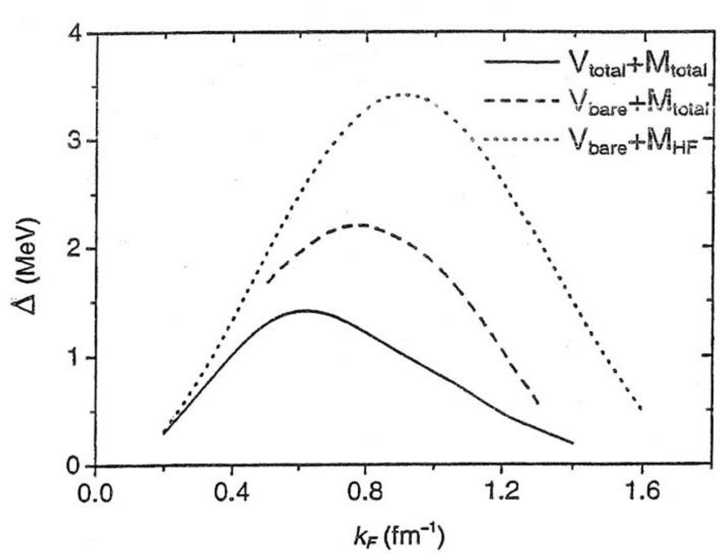
Gorkov equations

$$G = -i < 0 | T(aa^{+}) | 0 >; \qquad F = -i < 0 | T(a^{+}a^{+}) | 0 >$$



$$\Delta_k(\omega) = \sum_{k'} \int d\omega V_{kk'}(\omega, \omega') \frac{\Delta_{k'}(\omega')}{[\omega' - \varepsilon_{k'}(\omega')][\omega' + \varepsilon_{k'}(-\omega')]}$$





C.Shen, U.Lombardo,P.Schuck,W.Zuo, N.S, Phys.Rev.C67(2003)

Self-energy Effects

Pole approximation : $\omega_p = \varepsilon_p(\omega_p)$

$$\tilde{\Delta}_p = -\frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\frac{\mathcal{Z}_p V_{pp'}\mathcal{Z}_{p'}}{\sqrt{\omega_{p'}^2+\tilde{\Delta}_{p'}^2}}\tilde{\Delta}_{p'},$$

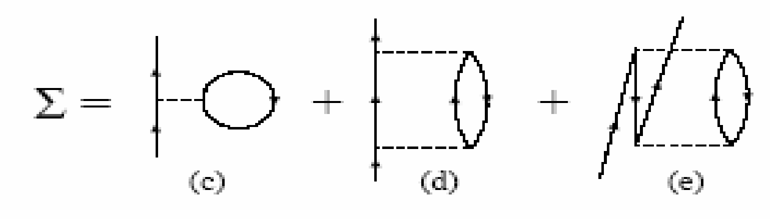
$$\mathcal{Z}_p^{-2} = \left[1-\frac{\partial\Sigma_p(\omega)}{\partial\omega}\right]^2\bigg|_{\omega=\omega_p}$$

$$\tilde{\Delta}_p = \mathcal{Z}_p\Delta_p$$

Expansion around p_F

$$\tilde{\Delta}_p = -\mathcal{Z}_F^2\int \frac{d^3p'}{(2\pi)^3}\frac{V_{p,p'}\tilde{\Delta}_{p'}}{2\sqrt{p_F^2(p'-p_F)^2/m^{*2}+\tilde{\Delta}_{p'}^2}},$$

Self- energy



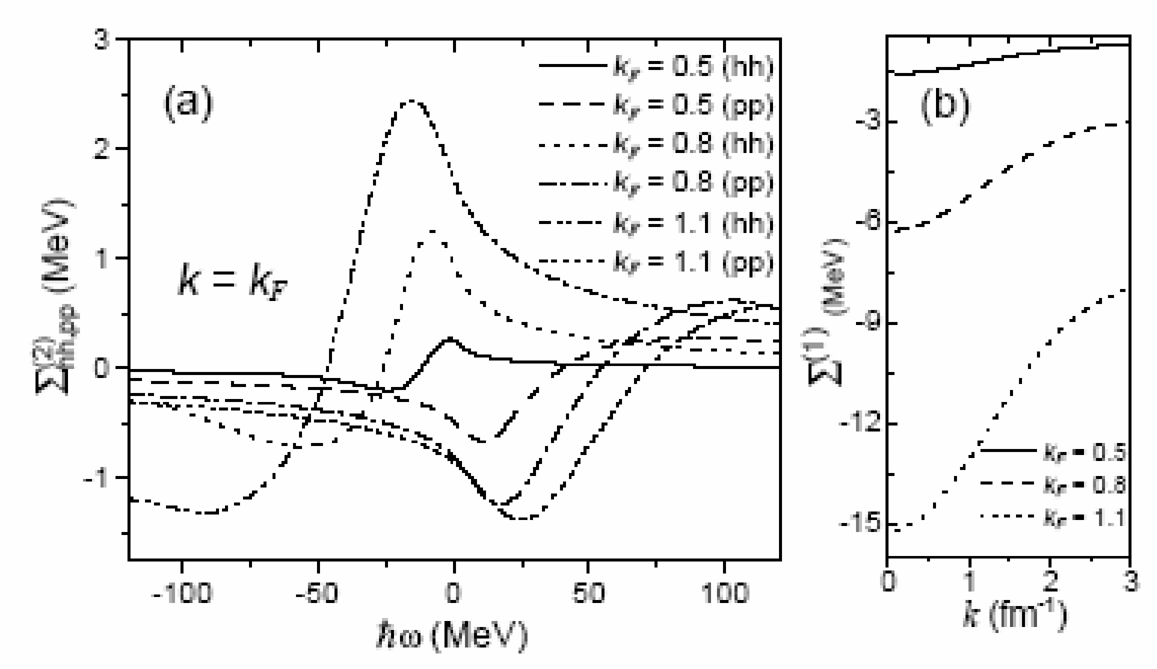
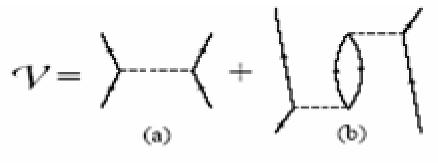


FIG. 2: (a) Rearrangement contributions to the self-energy, where k is fixed to k_F. (b) The HF mean field is plotted vs momentum k at $k_F = 0.5, 0.8, 1.1\ \mathrm{fm}^{-1}$.

C.Shen, U.Lombardo,P.Schuck,W.Zuo, N.S, Phys.Rev.C67(2003)

Screening of Pairing Force



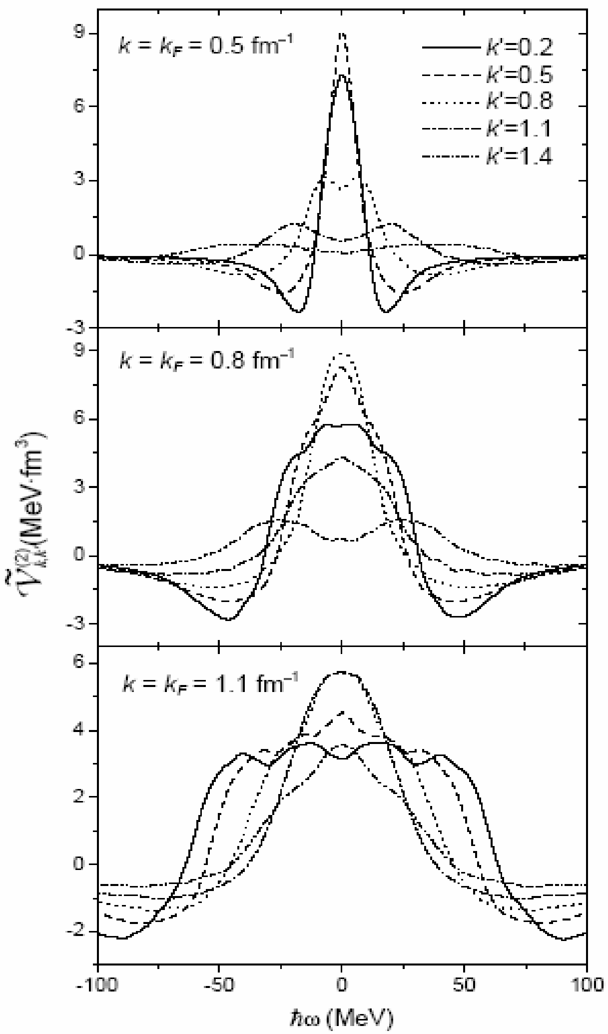
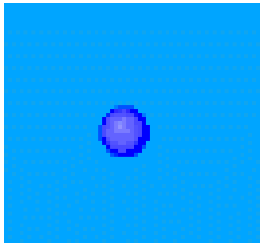


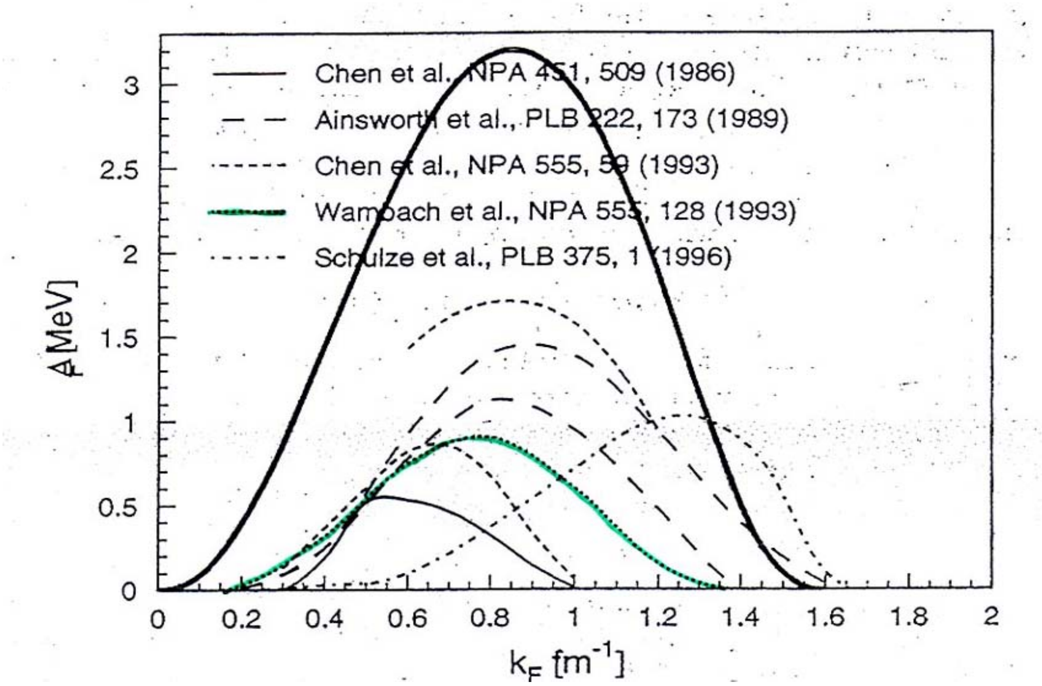
FIG. 3: Screening potential vs energy at $k_F = 0.5, 0.8, 1.1$ fm^{-1}, separately.

Pairing correlations

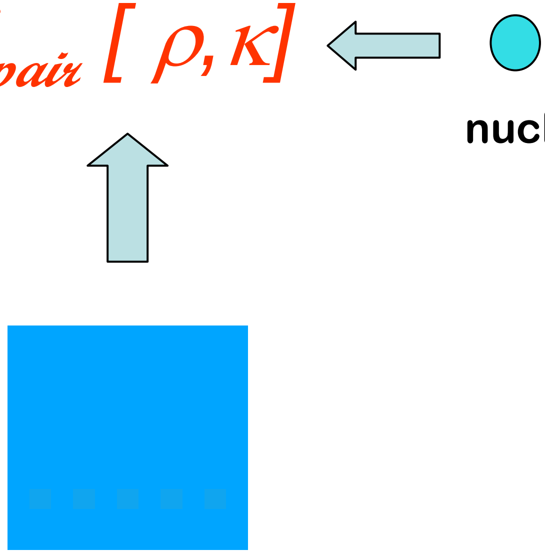
$$\mathcal{E}_{nuc} = \mathcal{E}_{Skyrme} + \mathcal{E}_{pair}[\rho, \kappa]$$



Pairing in uniform neutron matter ?



$$\mathcal{E}_{pair}[\rho, \kappa]$$



Efective Pairing Interactions

V_{bare} ⟺ ($k_F < 0.9$) Gogny force

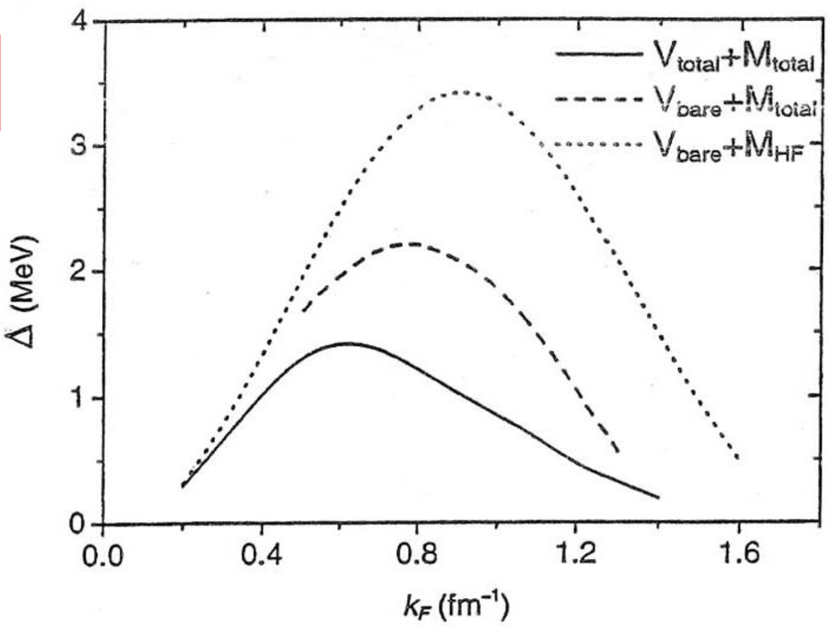
⇕

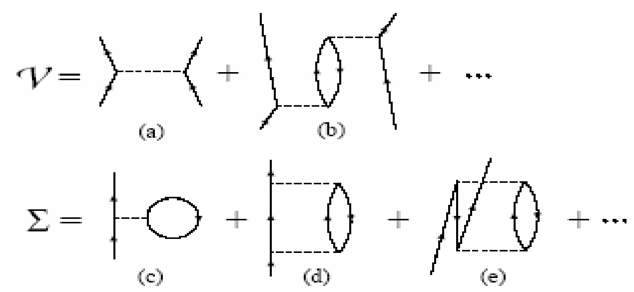
$$V_{pair} = V_0[1-\eta(\rho/\rho_0)^{\alpha}]\delta(r-r')$$

$\alpha=0.45$; $\eta=0.7$

I) $V_0=-430$ ⟺ $\Delta_{max} = 3$ MeV

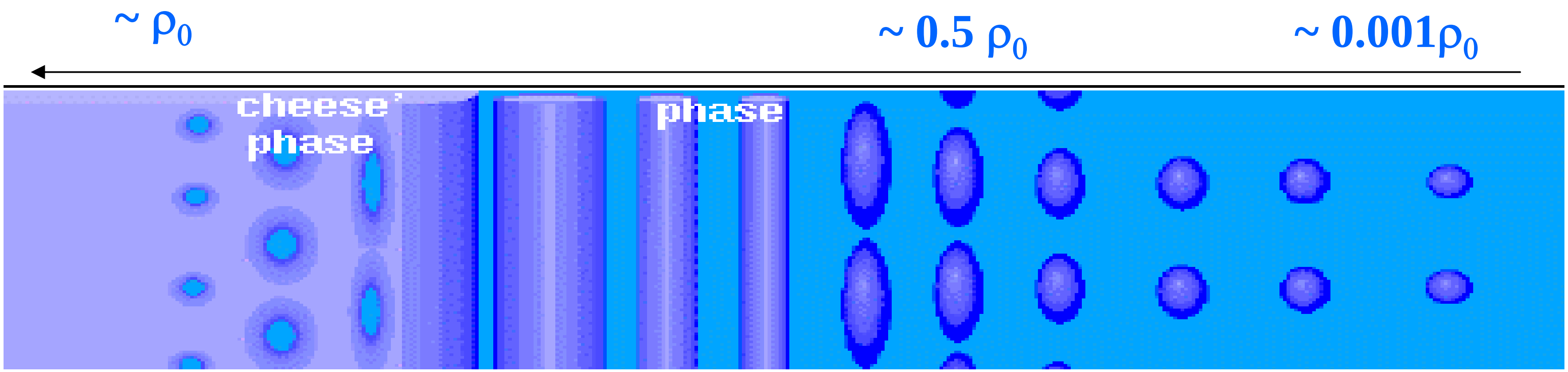
II) $V_0=-330$ ⟺ $\Delta_{max} = 1$ MeV

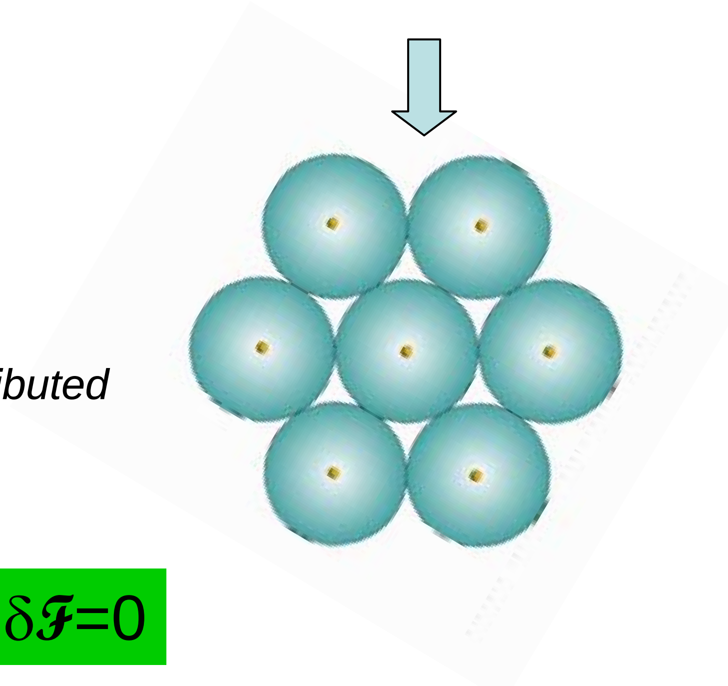




C.Shen et al Phys.Rev.C67(2003)

Inner Crust Structure





Wigner-Seitz approximations

- *Independent spherical cells of radius R_c*
- *Each cell contains: Z protons and N neutrons*
  Z electrons uniformly distributed

What is, for a given density, Z/N and R_c ?

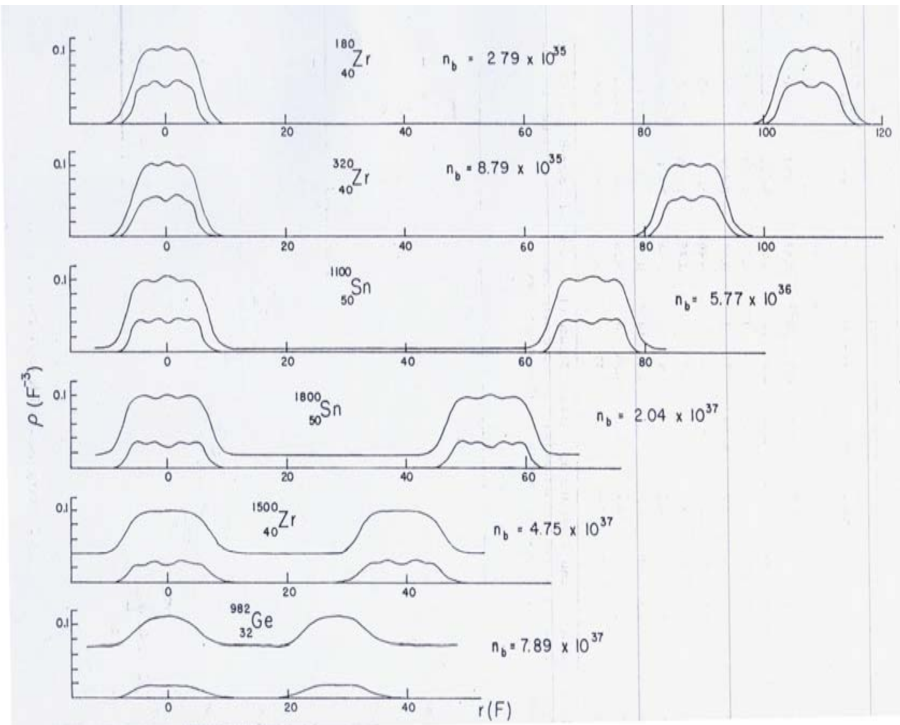
$$\mathcal{F} = \mathcal{E}_{nuc} + \mathcal{E}_{elec} - TS$$

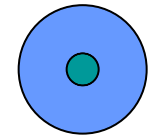
$$\delta\mathcal{F} = 0$$

$$\mu_n - (\mu_p + \mu_e) = 0$$

Inner Crust Structure

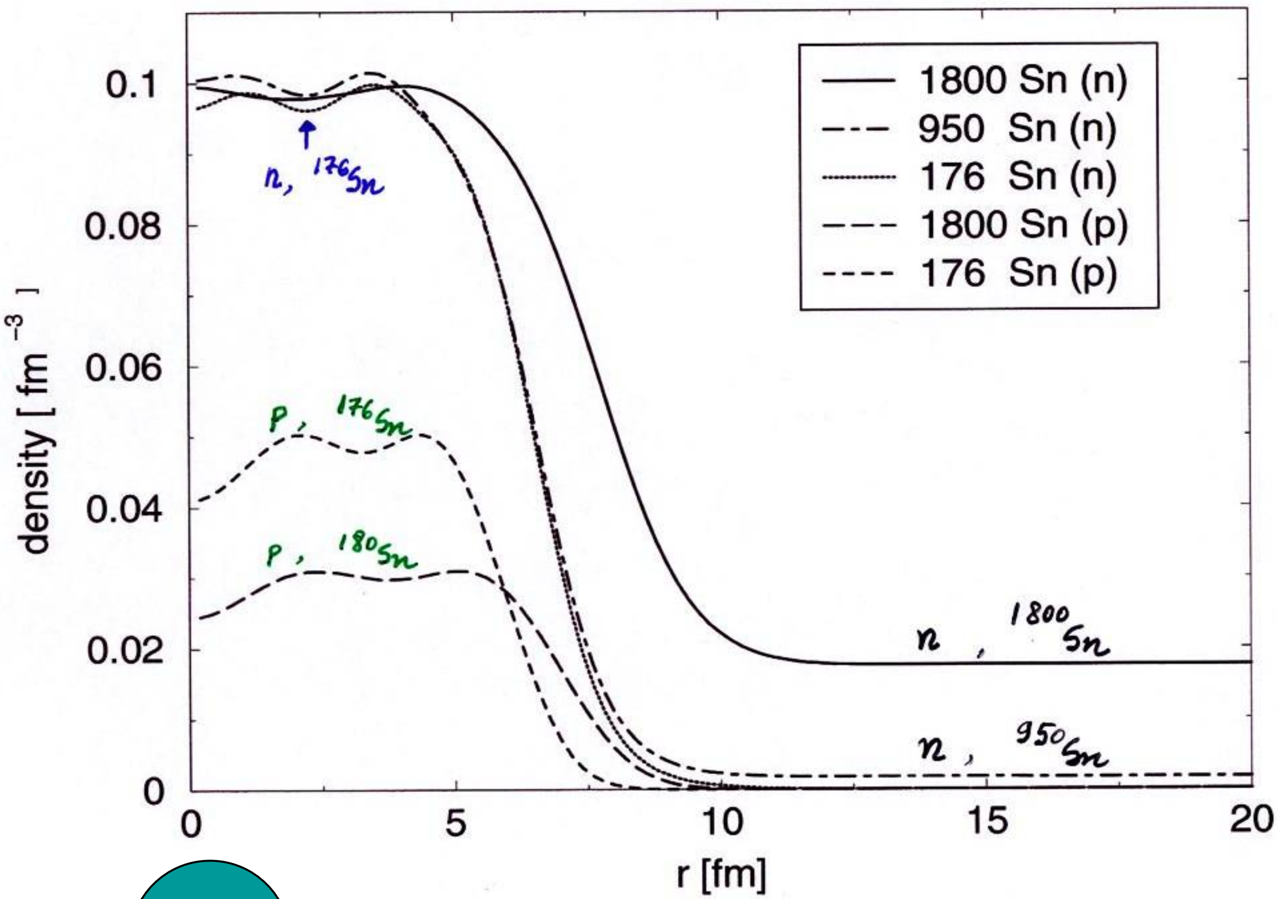
J.W. Negele, D. Vautherin, NPA207 (1973) 298

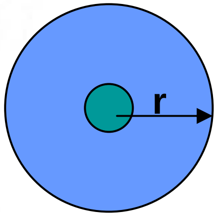


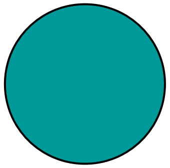


		Z	N	
$\rho_0/100$	⟺	40	460	$^{500}Zr_{40}$
$\rho_0/42.9$	⟺	50	900	$^{950}Sn_{50}$
$\rho_0/7.84$	⟺	50	1750	$^{1800}Sn_{50}$
$\rho_0/3.37$	⟺	40	1460	$^{1500}Zr_{40}$
$\rho_0/2.03$	⟺	32	950	$^{982}Ge_{32}$

Density in the Wigner-Seitz Cells

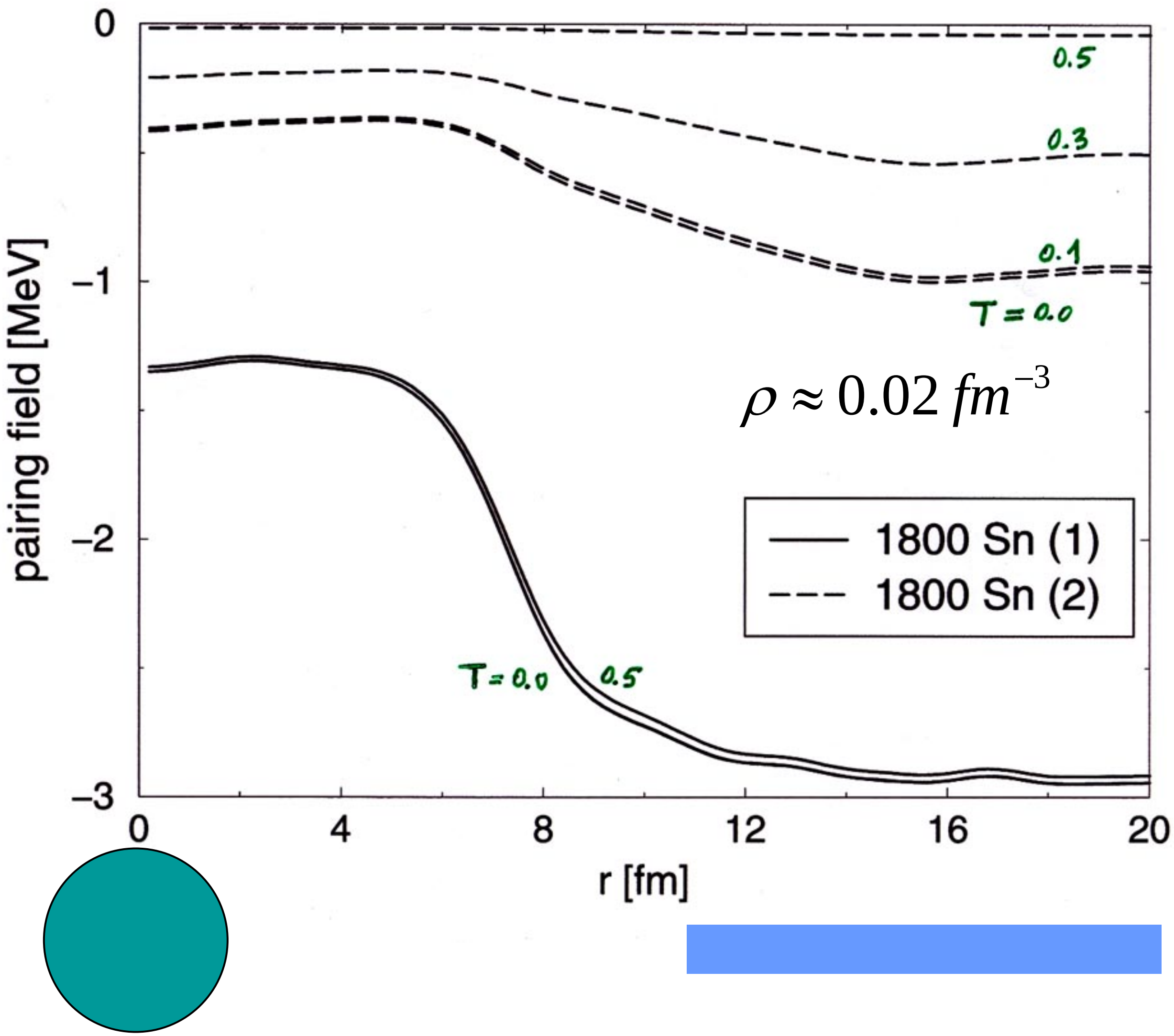


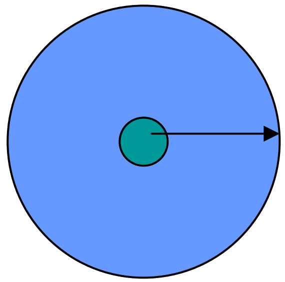




N.S. ,Nguyen Van Giai,R.J.Liotta, Phys.Rev.C69(2004)045802

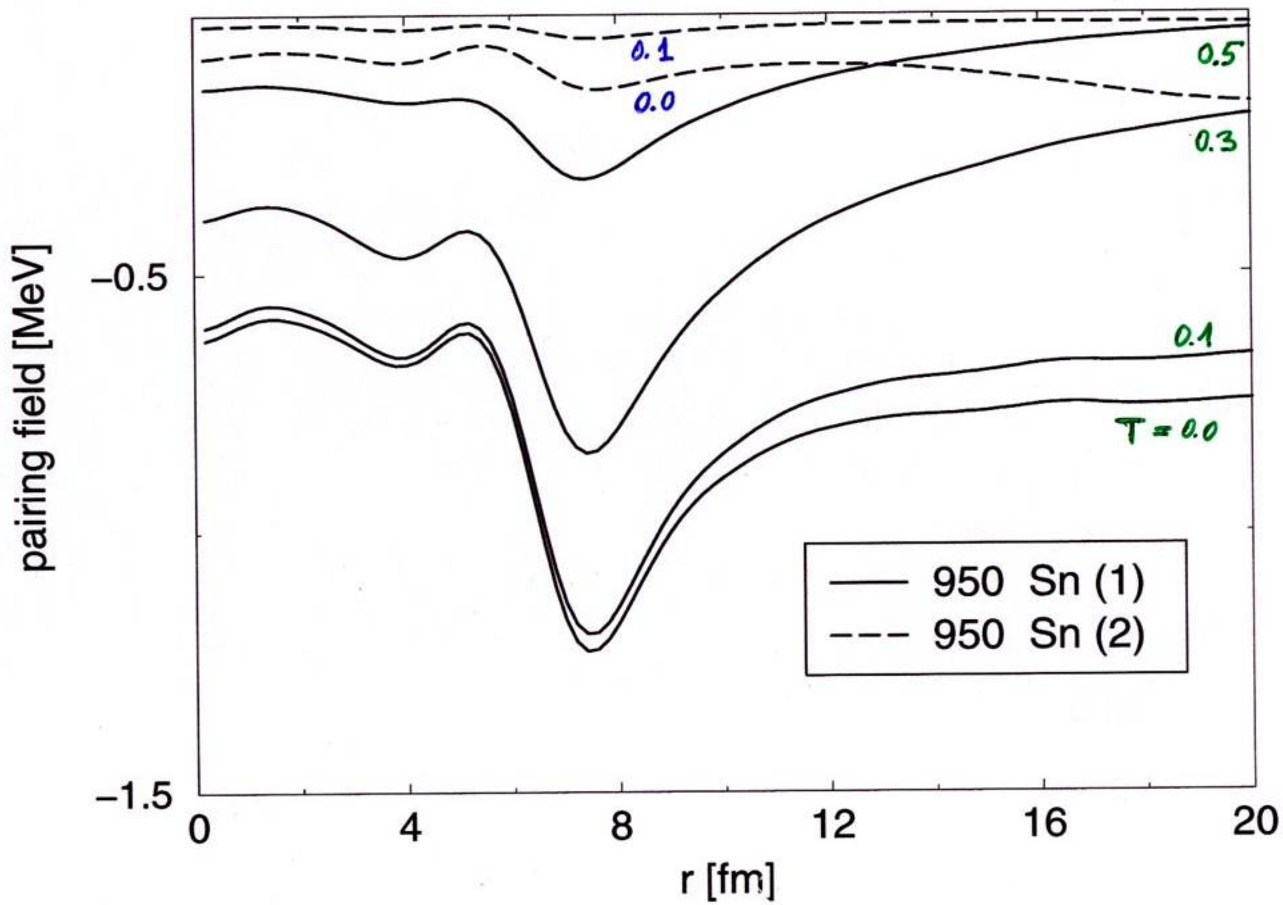
Pairing Field in the Wigner-Seitz Cells





N.S, Phys.Rev.**C70** (2004) 025801

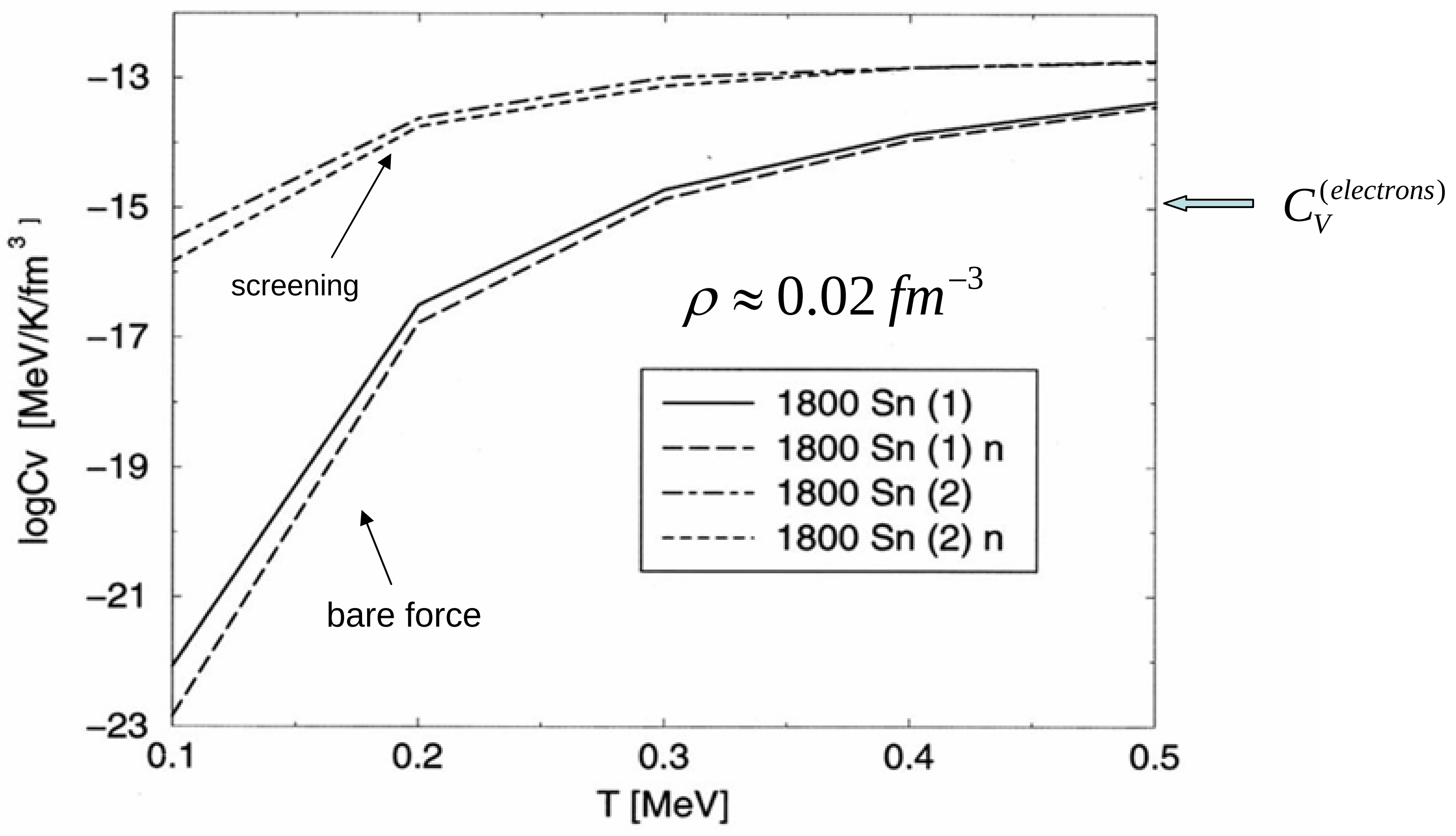
Pairing Field in the Wigner-Seitz Cells



$$\rho \approx 0.0037\ fm^{-3}$$

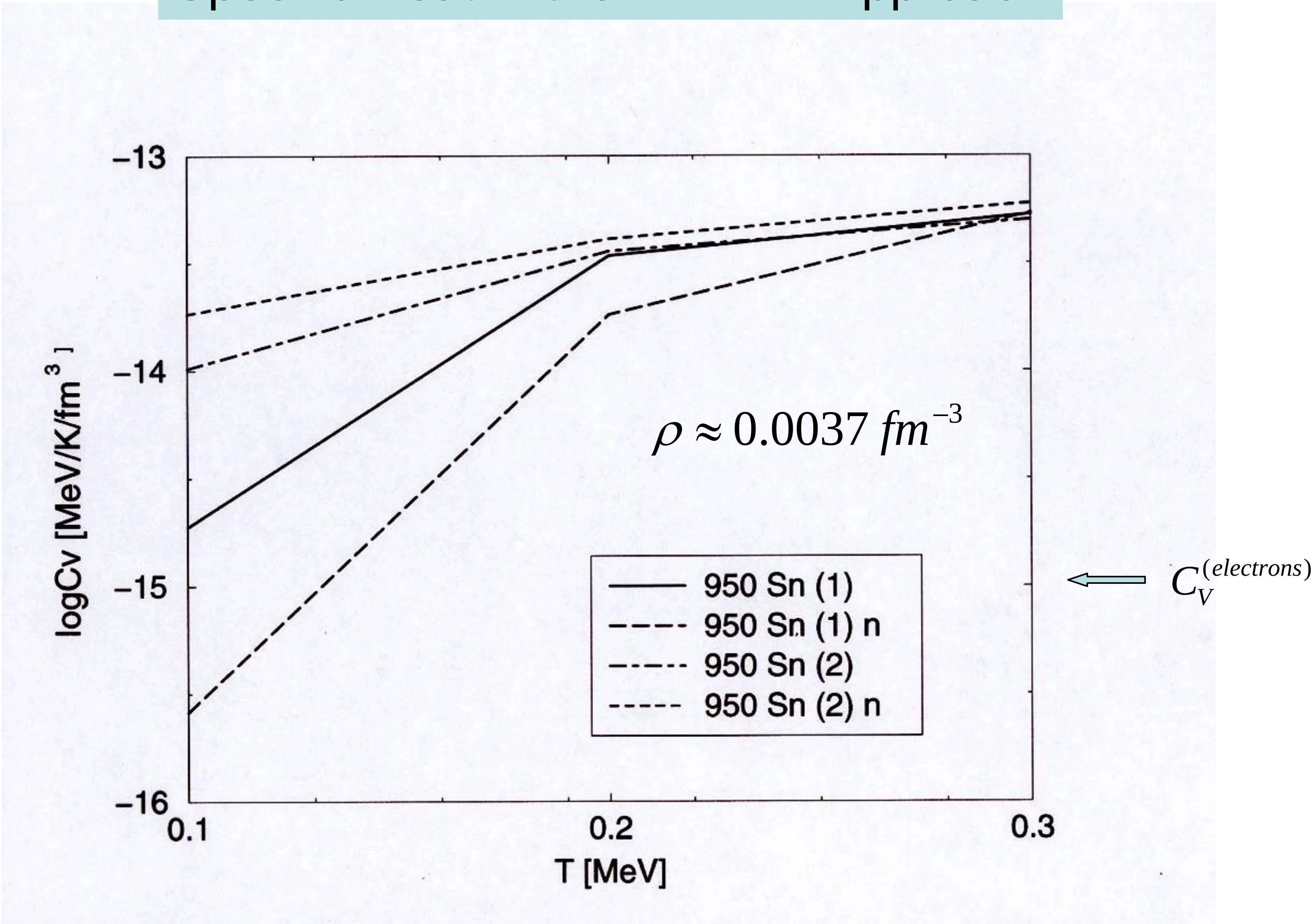
N.S, Phys.Rev.**C70** (2004) 025801

Specific Heat in the FT-HFB Approach



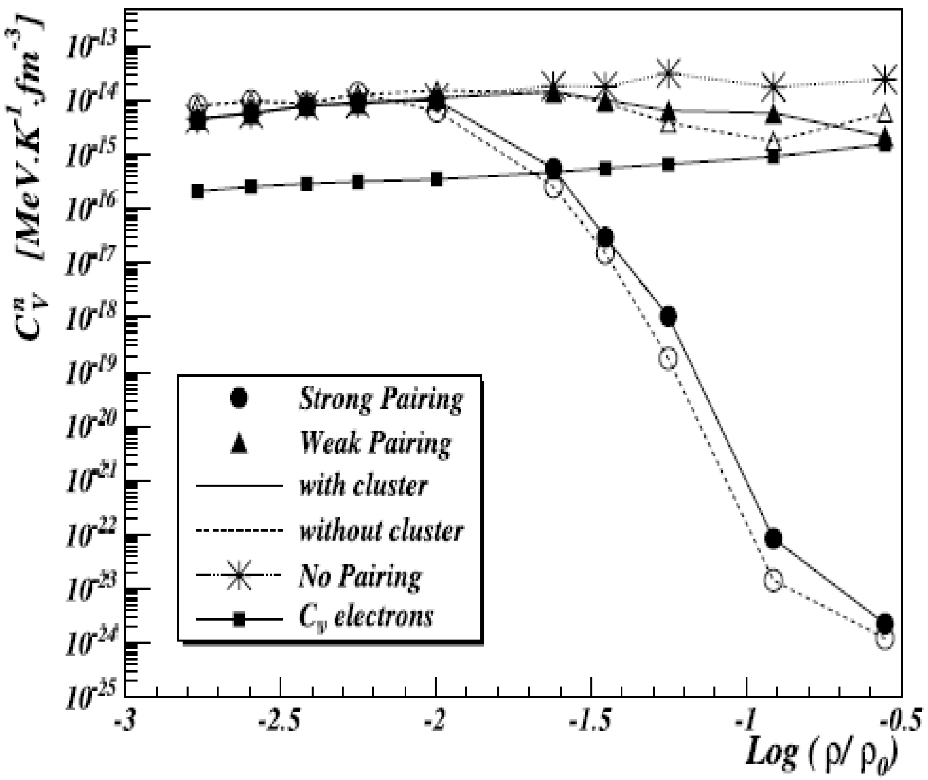
N.S, Phys.Rev.**C70** (2004) 025801

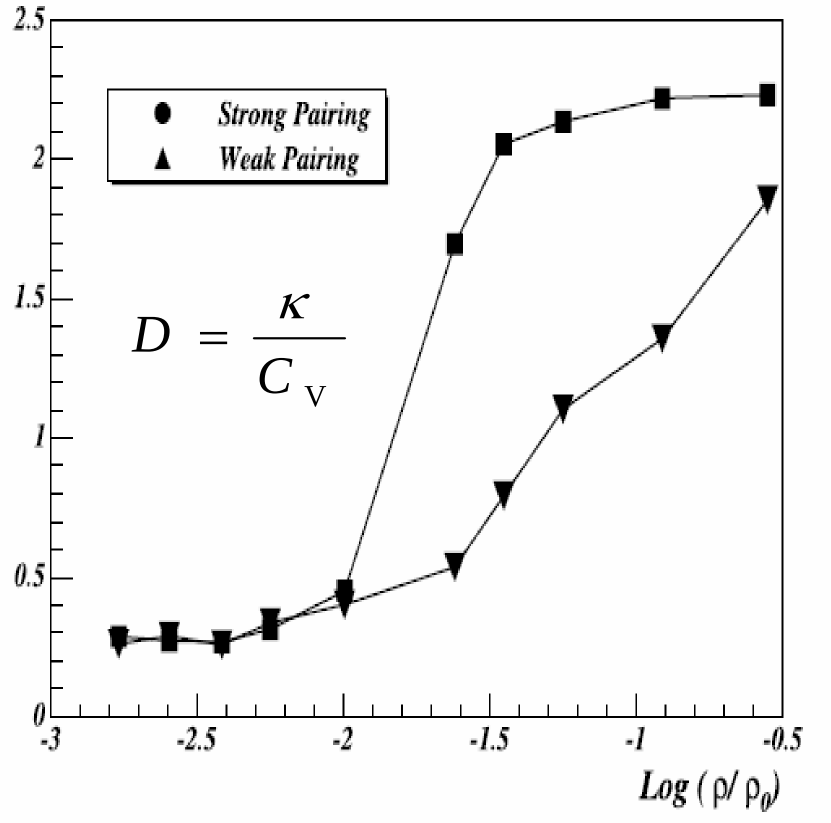
Specific Heat in the FT-HFB Approach



N.S, Phys.Rev.**C70** (2004) 025801

Specific Heat and Diffusivity Across the Inner Crust





C. Monrozeau, J. Margueron, N. S, Phys Rev. C, in press

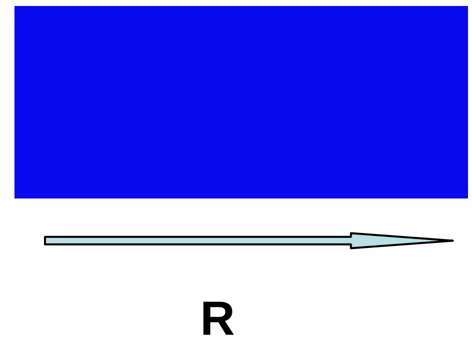
$$k \cong \frac{A_m}{T^m}\left(\frac{\rho}{\rho_0}\right)^s$$

(Lattimer et al, ApJ 425, 1994)

Thermal Diffusivity and Cooling Time

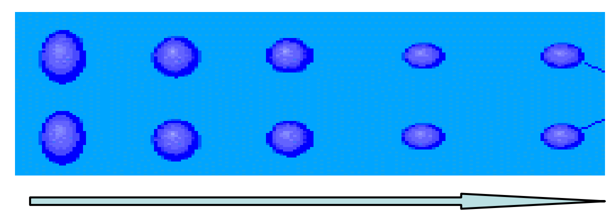
Constant thermal diffusivity

$$t_{diff} = \gamma \frac{R^2}{D} \; ; \; D = \frac{\kappa}{C_V}$$



Non-constant thermal diffusivity

$$t_{diff} = \gamma \int_{\rho_c}^{\rho_{shell}} \frac{1}{D[\rho, T(\rho)]} dR^2[\rho]$$



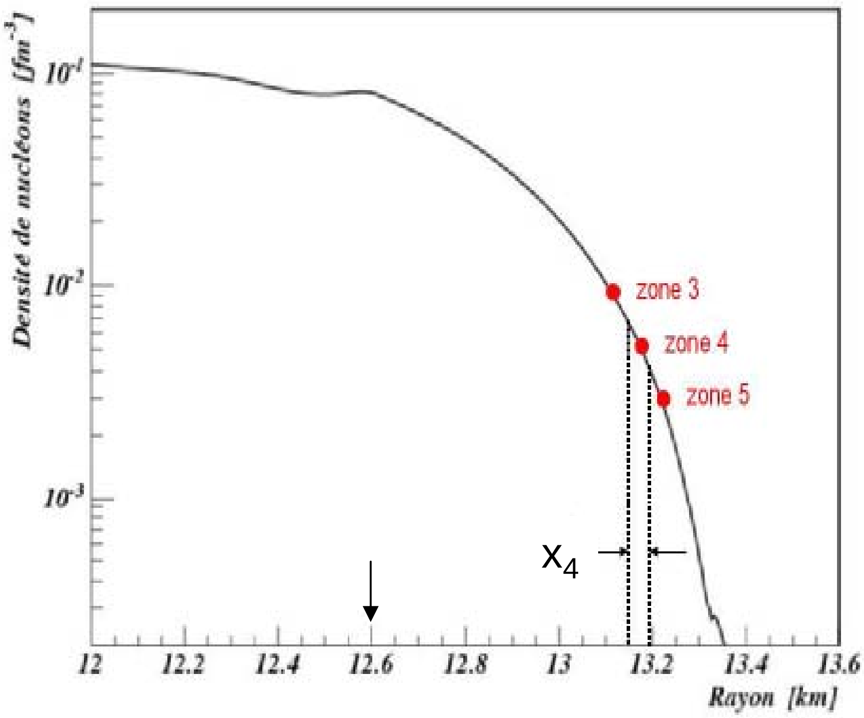
$$R = R[\rho]$$

given by TOV equation

$$t_{diff} = \gamma \sum \frac{x_i^2}{k_i} C_V(i)$$

Tolman – Oppenheimer – Volkov equation

$$\frac{dP}{dr} = -\frac{G[M(r) + 4\pi r^3 P/c^2](P/c^2 + \rho)}{r^2 - 2GM(r)r/c^2}$$

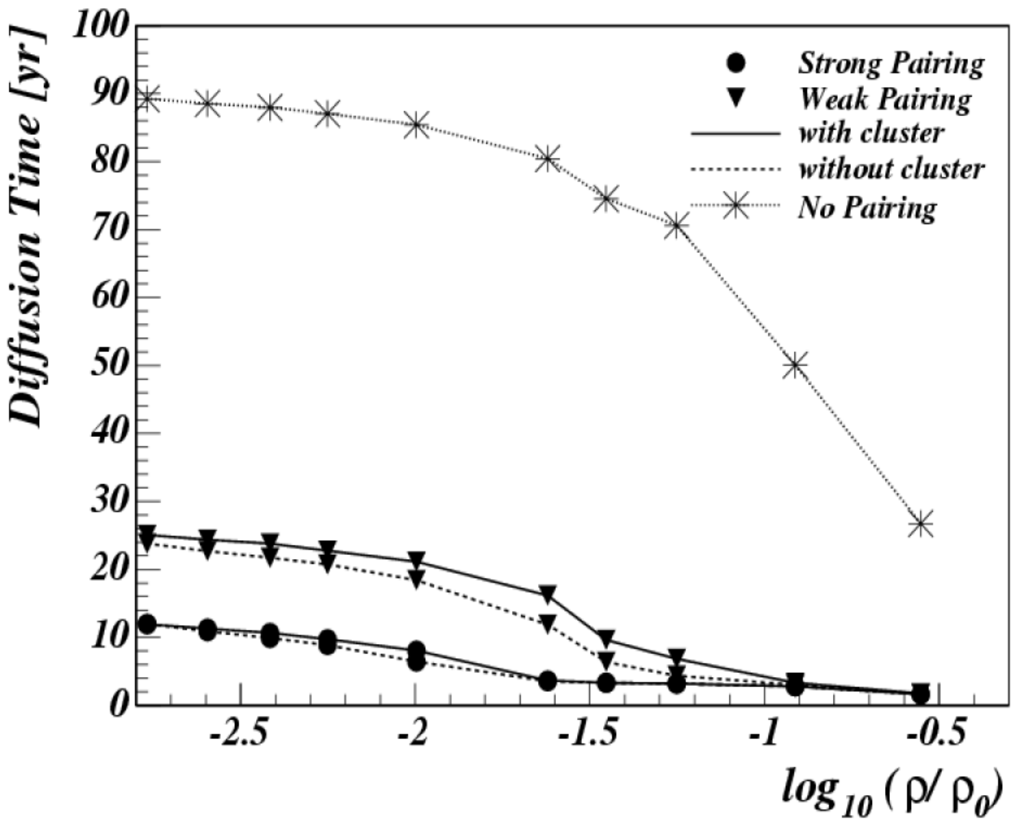


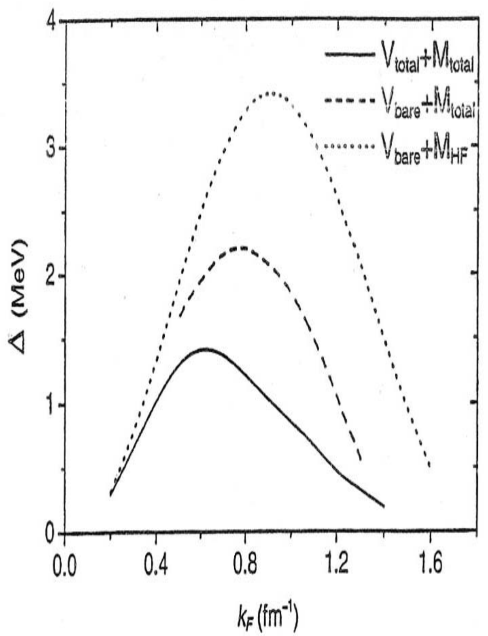
EOS

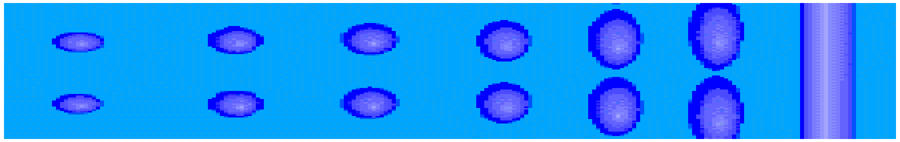
- *outer crust* : Baym-Pethick-Sutherland
- *inner crust* : Negele - Vautherin
- *core* : Glendenning-Moszkowski (GM1)

Results provided by Isaac Vidana

Cooling Time of The Inner Crust

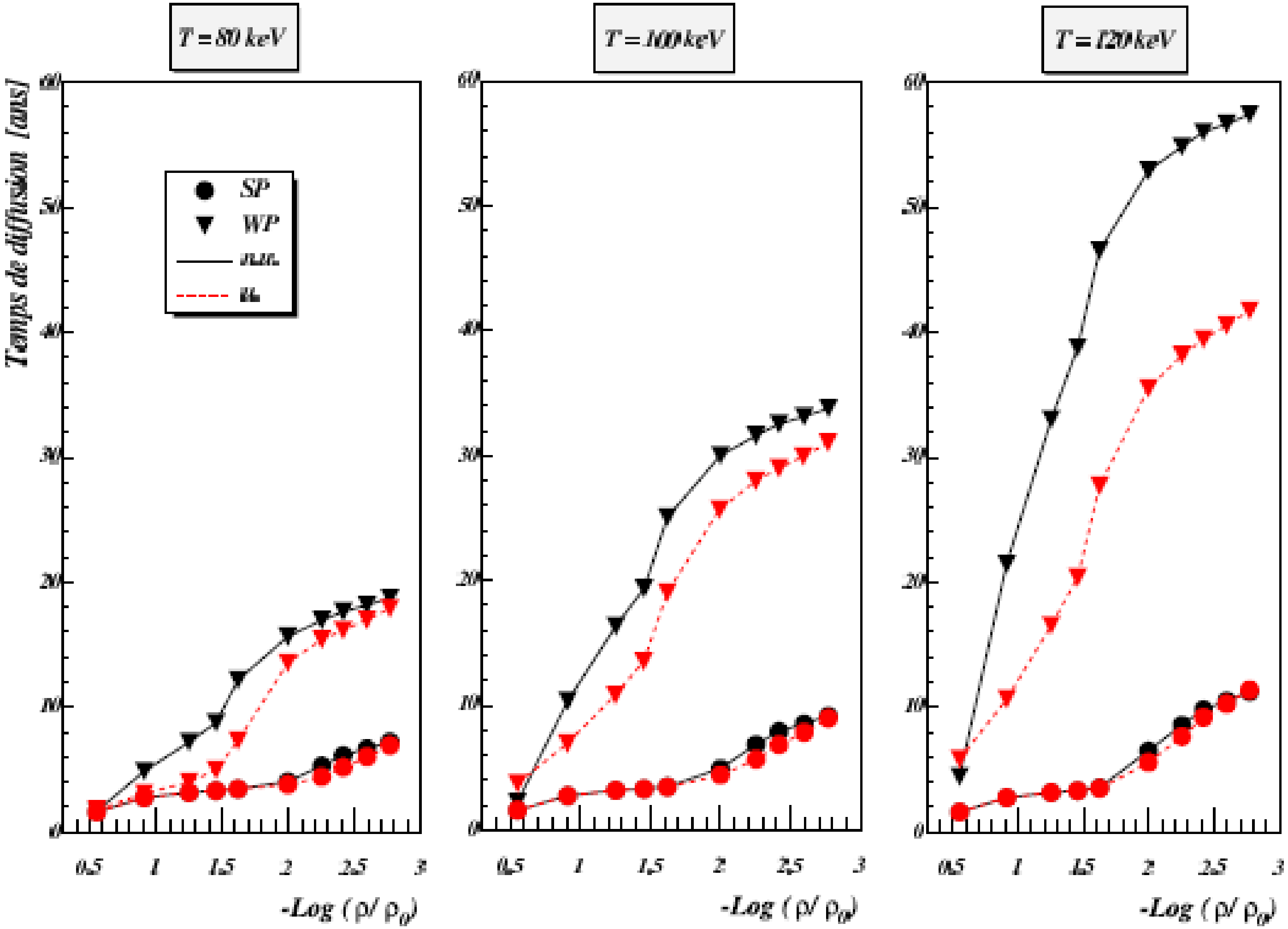






C. Monrozeau, J. Margueron, N. S, Phys Rev. C, in press

Cooling time for various crust temperatures



C. Monrozeau, PhD thesis (Orsay, 2007)

Collective Modes in the Crust of Neutron Stars

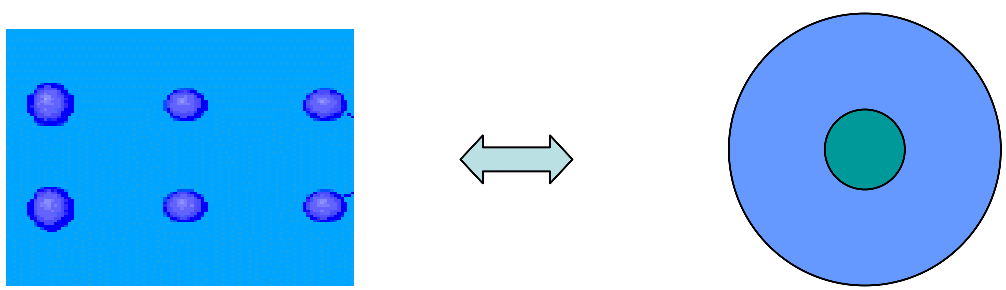
Non-uniform condensate:

coherence length : $\zeta \sim \hbar v_F / \pi \Delta_F$

distance between clusters: **L**

(a) **L >>** ζ : ~ the case of uniform condensate

(b) **L <** ζ : need of microscopic calculations !



Linear Response for Superfluid Systems

• *Probe the system with a weak time-dependent external field*

$$\mathcal{F} = Fe^{-i\omega t} + h.c.. \qquad F = \sum_{ii} F^{11}_{ij} c^\dagger_i c_j + \sum_{ii} (F^{12}_{ij} c^\dagger_i c^\dagger_j + F^{21}_{ij} c_i c_j),$$

pair transfer

• *The external field induces strong oscillations of the nuclear densities whenever the frequency is close to an eigenmode of the system*

$$\rho(t) = \rho^{(0)} + \rho^{(\nu)} e^{-i\omega_\nu t} + c.c\,;$$

$$\kappa(t) = \kappa^{(0)} + \kappa^{(\nu)} e^{-i\omega_\nu t} + c.c\,;$$

$$\overline{\kappa}(t) = \overline{\kappa}^{(0)} + \overline{\kappa}^{(\nu)} e^{-i\omega_\nu t} + c.c\,;$$

$\rho^{(\nu)} \Leftrightarrow$ 'transition density'

• *The external field produces small changes which can be treated in the linear order*

$$\hat{\rho}^{(\nu)} = \hat{G}\hat{F} \qquad \begin{pmatrix} \rho^{(\nu)} \\ \kappa^{(\nu)} \\ \overline{k}^{(\nu)} \end{pmatrix} = \begin{pmatrix} G^{11} & G^{12} & G^{13} \\ G^{21} & G^{22} & G^{23} \\ G^{31} & G^{32} & G^{33} \end{pmatrix} \begin{pmatrix} F^{11} \\ F^{12} \\ F^{21} \end{pmatrix}$$

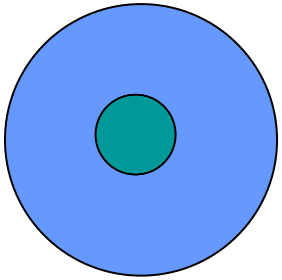
response function

• *Advantage: a method to derive the (Q)RPA equations for density-dependent forces*

QRPA response

$$\rho' = \mathbf{GF} \quad \Longleftrightarrow \quad G = \left(1 - G_0 V\right)^{-1} G_0 = G_0 + G_0 V G.$$

$$G_0^{\alpha\beta}\left(\mathbf{r}\sigma, \mathbf{r}'\sigma'; \omega\right) = \sum_{ij} \frac{\mathcal{U}_{ij}^{\alpha 1}(\mathbf{r}\sigma)\bar{\mathcal{U}}_{ij}^{*\beta 1}(\mathbf{r}'\sigma')}{\hbar\omega - \left(E_i + E_j\right) + i\eta} - \frac{\mathcal{U}_{ij}^{\alpha 2}(\mathbf{r}\sigma)\bar{\mathcal{U}}_{ij}^{*\beta 2}(\mathbf{r}'\sigma')}{\hbar\omega + \left(E_i + E_j\right) + i\eta},$$

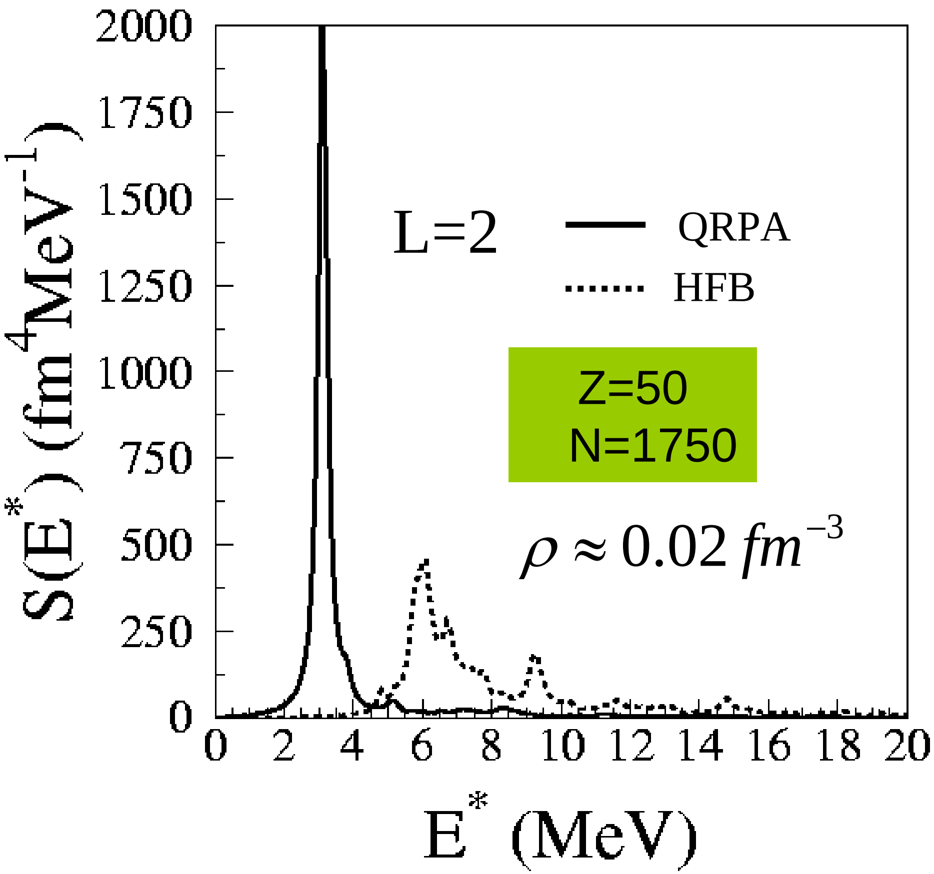


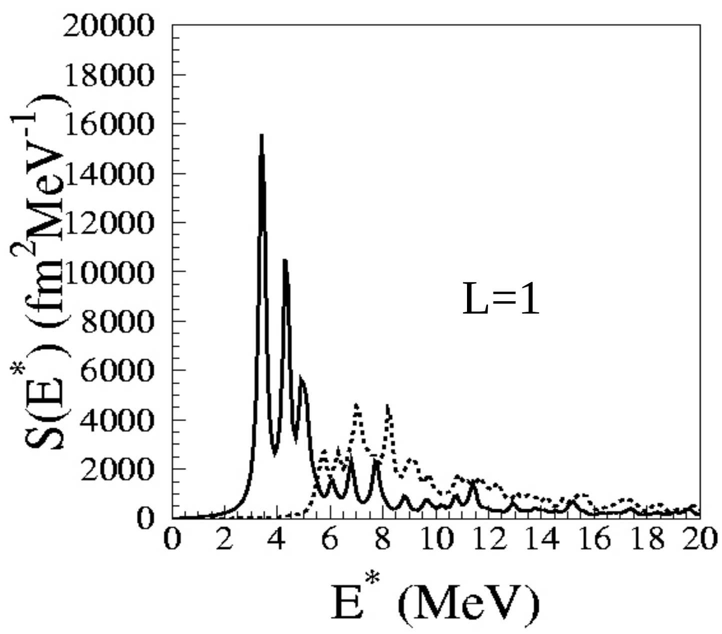
Rezidual interaction:

$$V^{\alpha\beta}\left(\mathbf{r}\sigma, \mathbf{r}'\sigma'\right) = \frac{\partial^2 \mathcal{E}}{\partial\rho_\beta(\mathbf{r}'\sigma')\,\partial\rho_{\bar{\alpha}}(\mathbf{r}\sigma)} \quad \Longleftrightarrow \quad V = \begin{pmatrix} V^{ph,ph} & V^{ph,pp} & V^{ph,hh} \\ V^{pp,ph} & V^{pp,pp} & V^{pp,hh} \\ V^{hh,ph} & V^{hh,pp} & V^{hh,hh} \end{pmatrix}$$

E.Khan, N. Sandulescu .,M.Grasso, Nguyen Van Giai, Phys. Rev. C66 (2002)024309

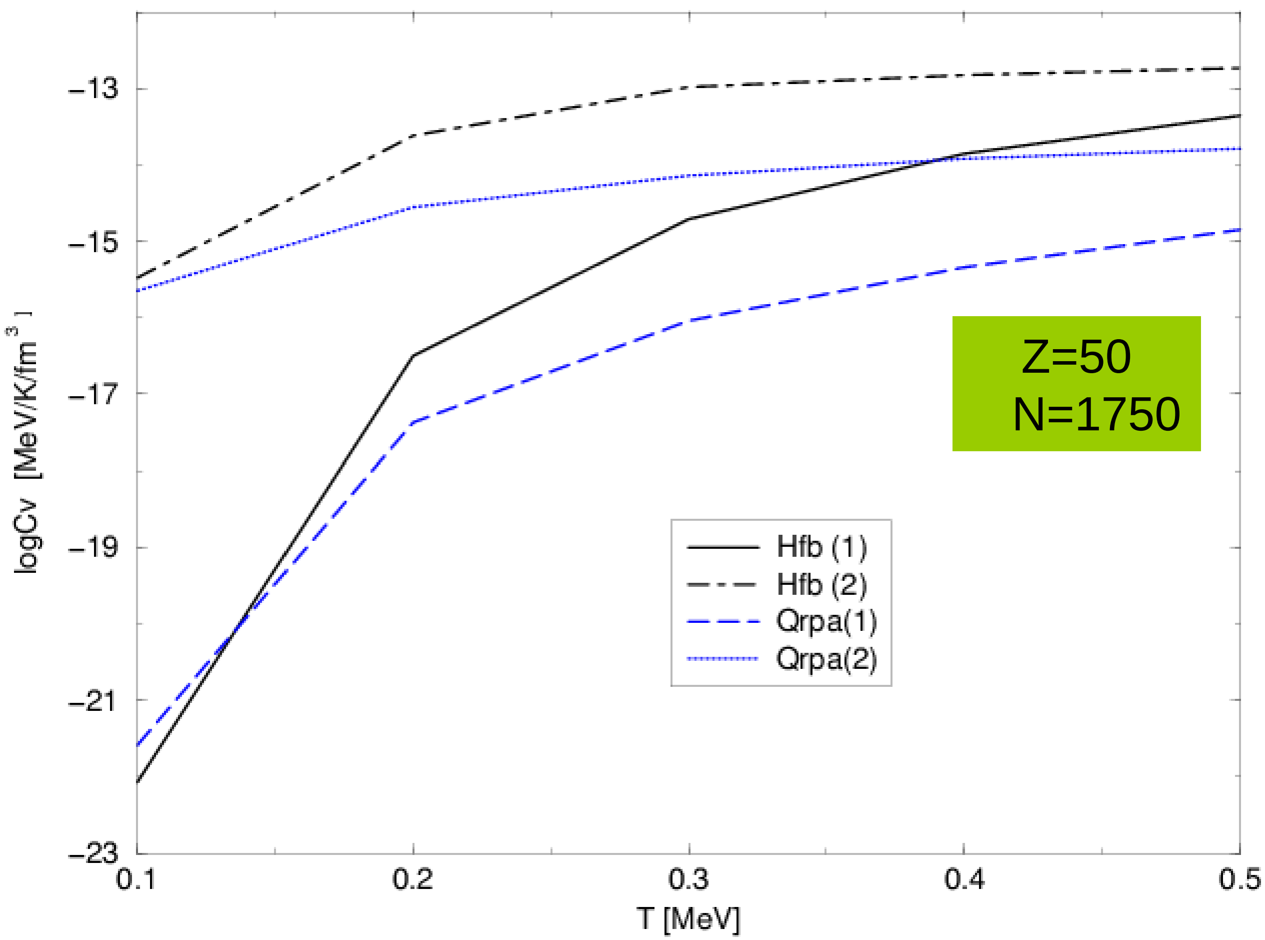
Supergiant resonances in the crust of neutron stars





E.Khan, N.Sandulescu, Nguyen Van Giai, Phys.Rev.C71 (2005) 042801R

Specific heat of collective modes



(N. Sandulescu. , nul-th/061201)

Mass-Radius Constraints from Cooling Time

Lattimer et al, ApJ425(1994)802

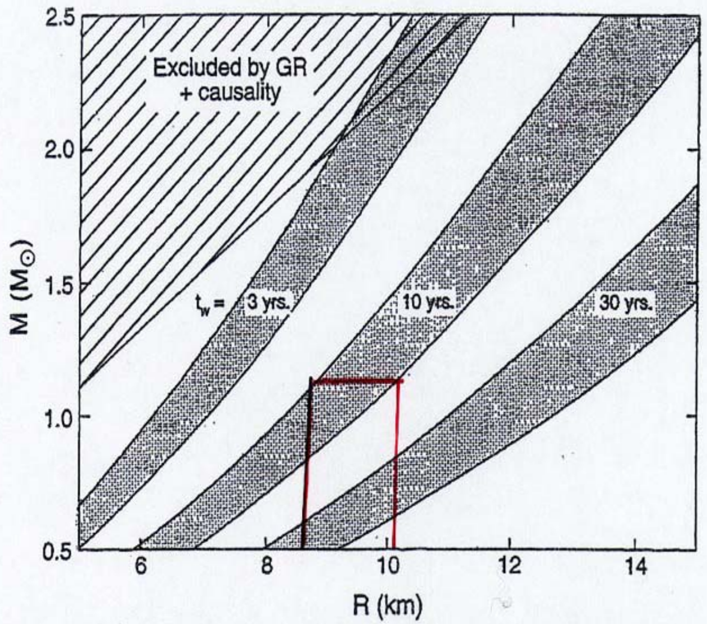


FIG. 12.—Shaded areas are the allowed regions of mass and radius for a neutron star observed to have the indicated values of the rapid cooling time t_w. It is assumed that $\rho_{core} = 0.5\rho_0$ and that the neutron star crust is superfluid. The mass-radius region excluded by general relativity and causality is indicated by the hatched region.

$1.15\ M_0 < M < 1.5\ M_0$

t_W= 10 years

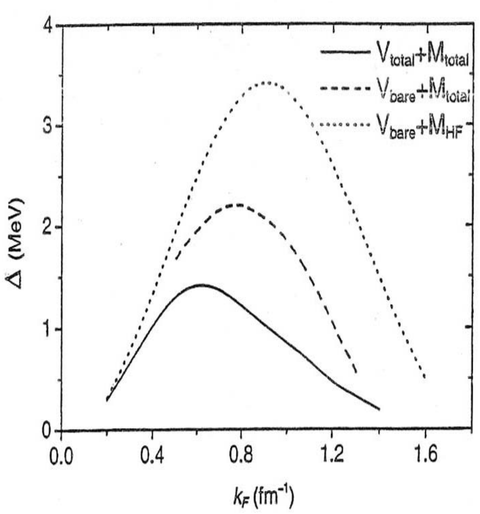
No Superfluidity:

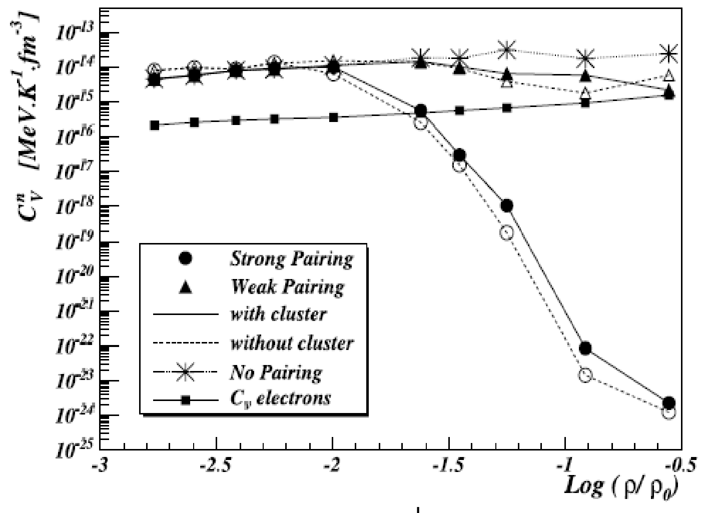
6.8 km < R < 8.5 km

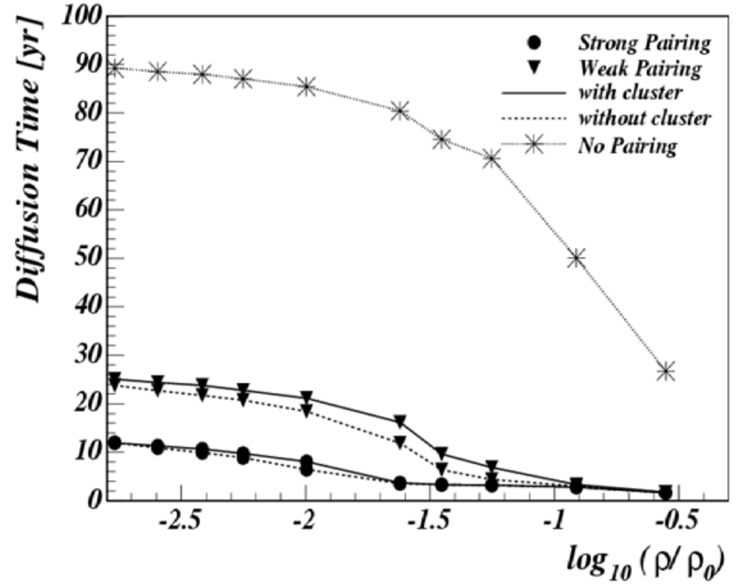
Superfluidity:

9 km < R < 11.5 km

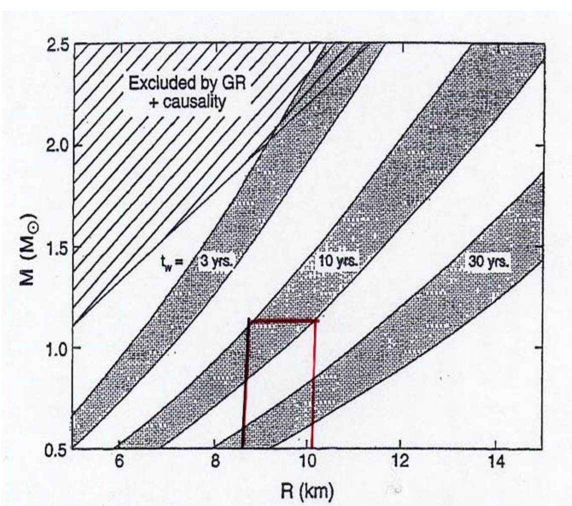
Summary and (No) Conclusions







consequences ?



Lattimer et al, ApJ425(1994)802