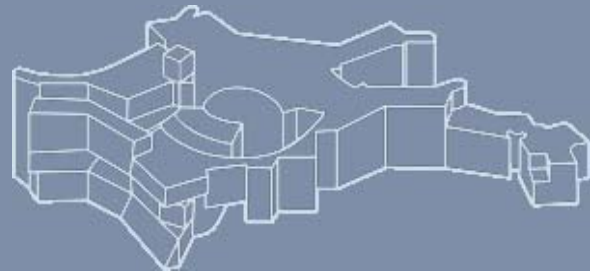




# Type Ia supernovae

Friedrich Röpke

Max-Planck-Institut für Astrophysik, Garching



W. Hillebrandt, S. Woosley, M. Reinecke, M. Gieseler, C. Travaglio, M. Stehle,  
P. Mazzali, J. Niemeyer, W. Schmidt, S. Blinnikov, E. Sorokina, S. Sim, M. Fink,  
R. Pakmor

# Outline

## 1<sup>st</sup> lecture

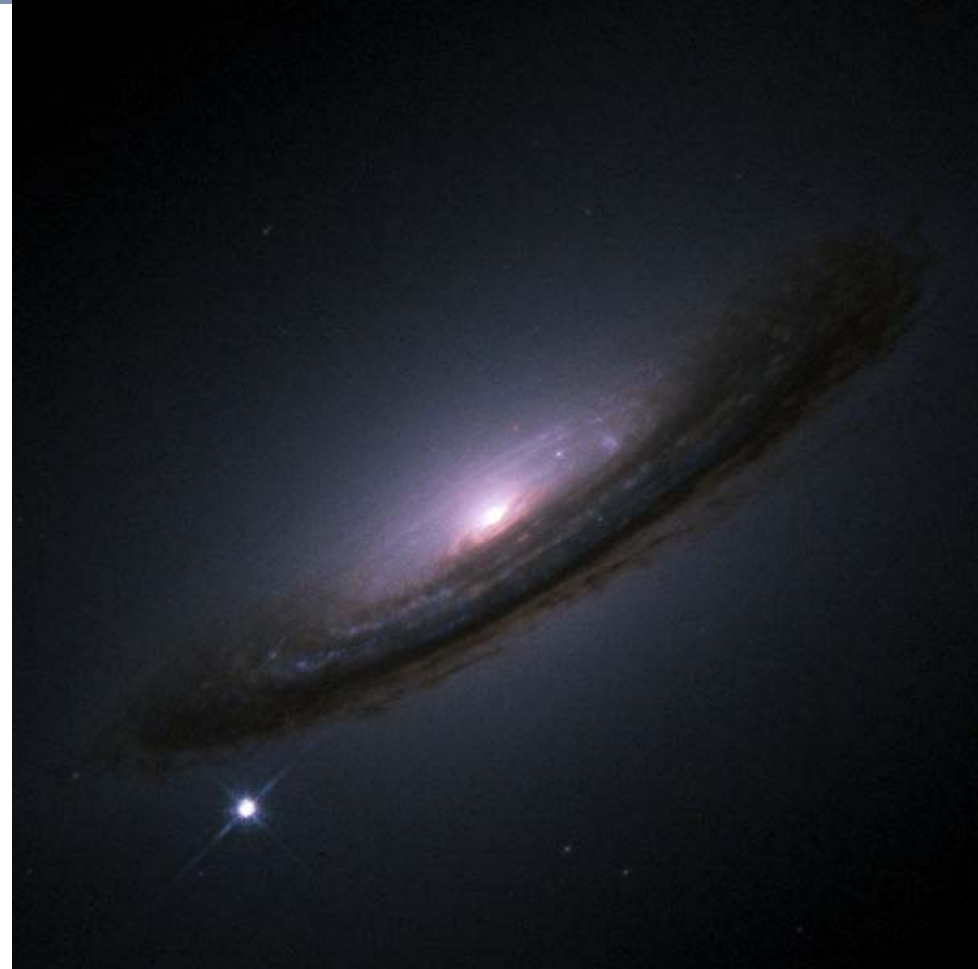
- ▶ supernovae in astronomy
- ▶ Type Ia supernovae: general scenario
- ▶ Modeling Type Ia supernova explosions

## 2<sup>nd</sup> lecture

- ▶ numerical implementation
- ▶ type Ia supernova simulations
- ▶ initial parameters
- ▶ nucleosynthesis
- ▶ reproducing the SN Ia sample
- ▶ implications for cosmology

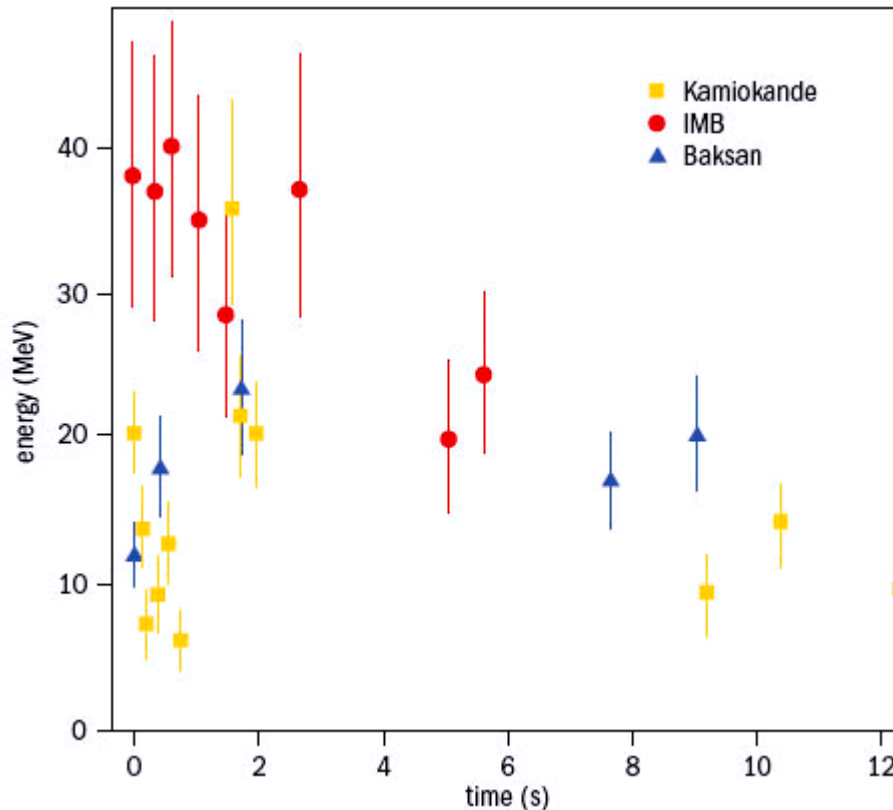
# Supernovae in astronomy

- ▶ stellar explosions
- ▶ brightest explosions in the Universe after Big Bang
- ▶ reach brightness of entire galaxy for some days
- ▶ energy release  $\sim$  solar energy release over 10 billion years



# Supernovae in astronomy

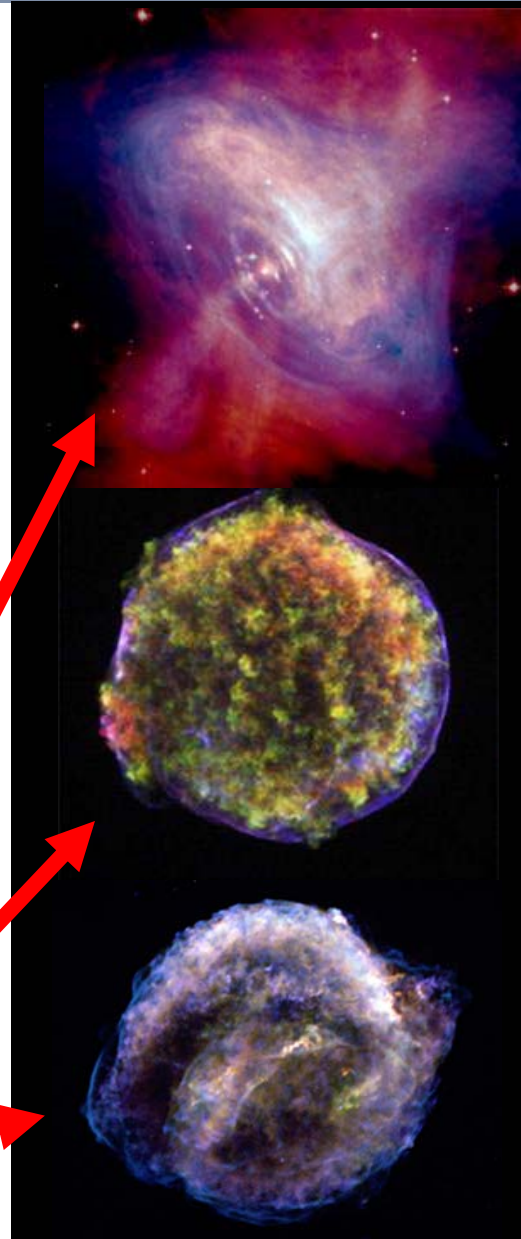
- ▶ most energy goes into kinetic energy of ejected gas (velocity  $\sim 0.1 c$ ,  $E_{\text{kin}} \sim 10^{51}$  erg) and, for some supernovae (explosion of massive star forming neutron star or black hole) into neutrinos ( $E_{\nu} \sim 10^{53}$  erg)



# Historical Supernovae

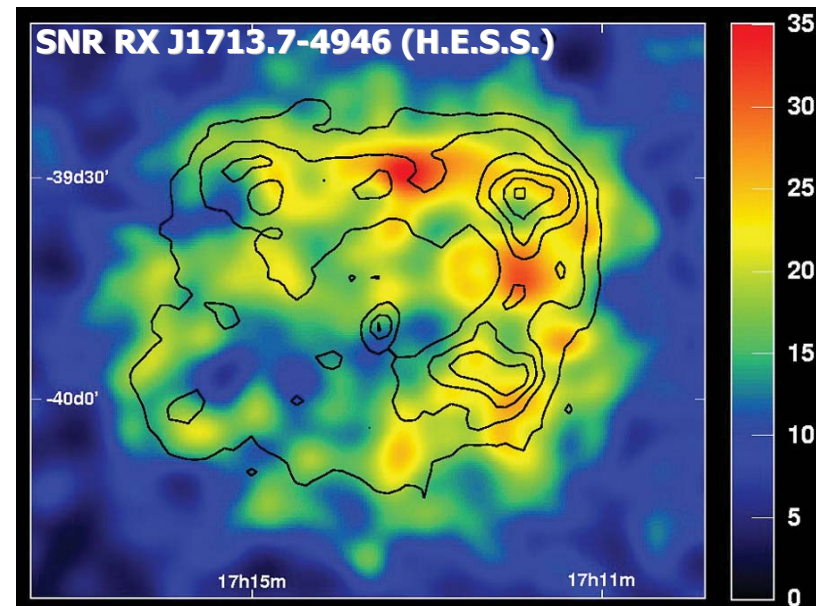
- ▶ rare events: about once per century in Milky Way
- ▶ historically: observations of SNe recorded in ancient texts from China and Korea, Japan, and Europe

year	report	status
185 A.D.	China	identification in doubt
386	China	unknown
393	China	unknown
1006	China, Korea, Japan, Arabia, Europe	identified with radio SNR
1054	China, Japan	Crab Nebula
1181	China, Japan	possible identification with radio SNR
1572	Europe (Brahe), China, Japan	Tycho's remnant
1604	Europe (Kepler), China, Japan, Korea	Kepler's remnant



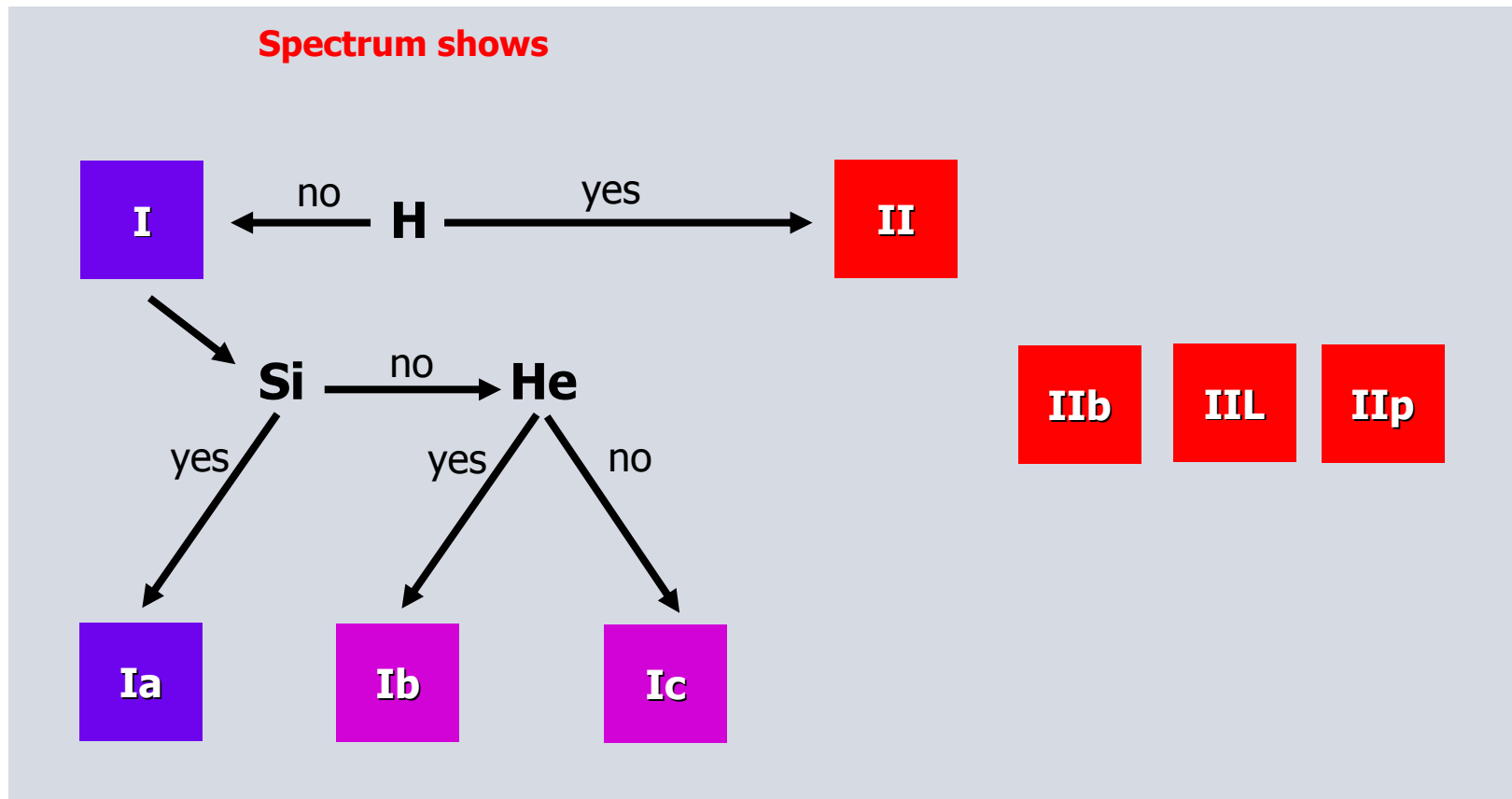
# Supernovae: impact on astrophysics

- ▶ production and distribution of heavy elements (chemical enrichment): since formation of Milky Way (about 12 billion years ago) many supernovae enriched it with Fe, Si, O, C, Ca
- ▶ enabled formation of planets, life on earth
- ▶ explosion waves propagate through interstellar space and compress gas such that new stars form
- ▶ supernovae play an important role in the cycle of matter and in formation and destruction of stars
- ▶ SNe are one of the most important sources of high-energy cosmic rays
- ▶ according to recent research results some kinds of supernovae are associated to cosmic gamma ray bursts



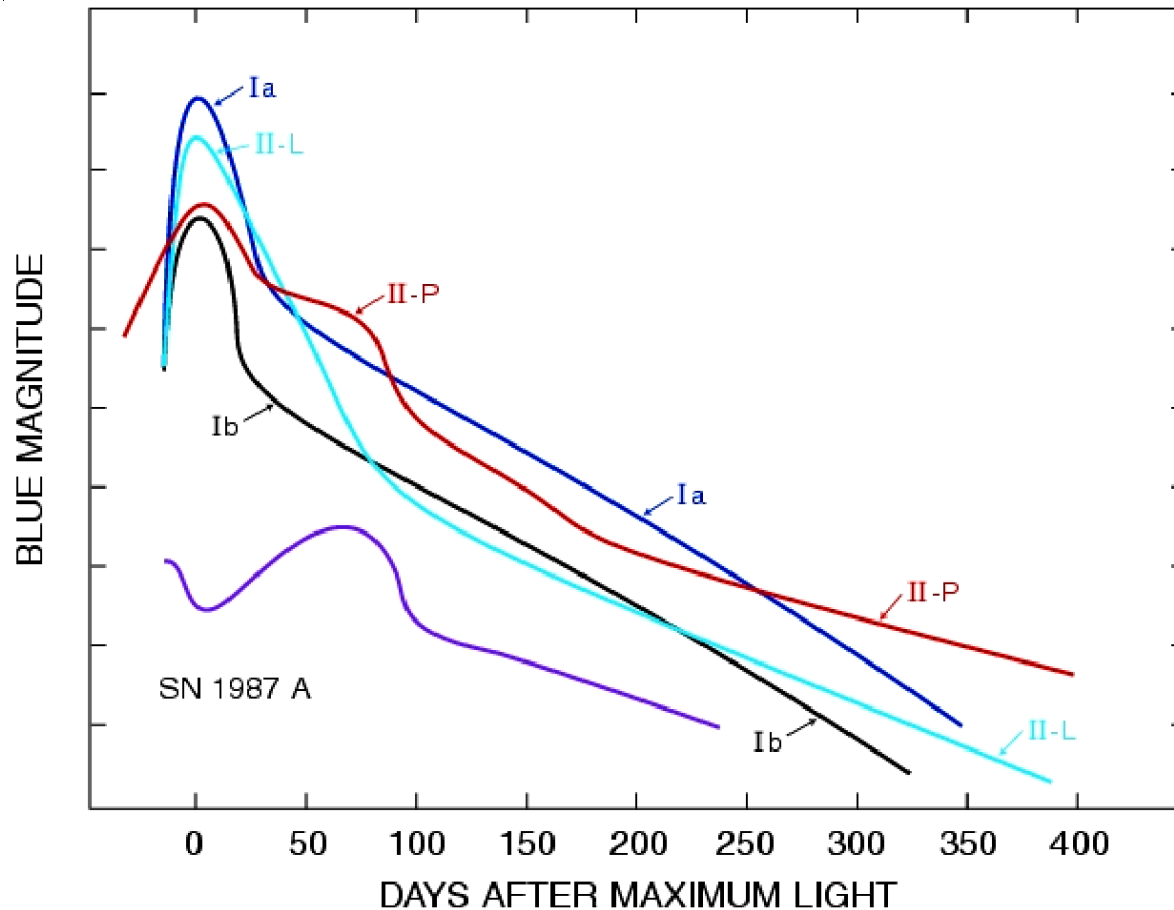
# Supernovae: astronomical classification

- ▶ empirical classification



# Supernovae: astronomical classification

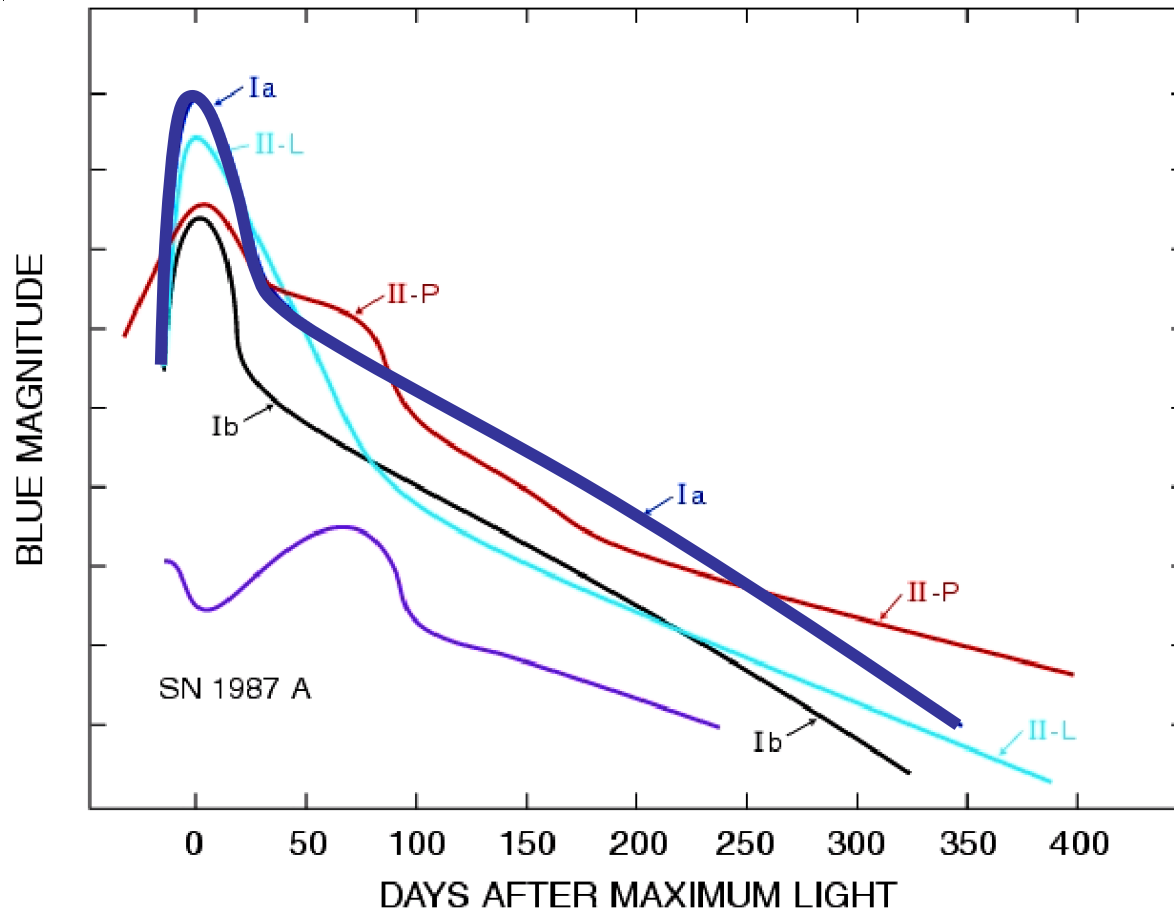
- ▶ blue band light curves (brightness in observational B-band over time) for different types of supernovae





# Homework problem I

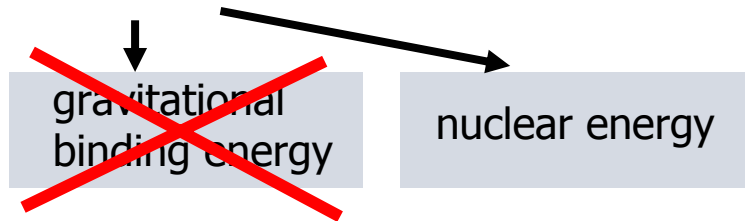
- Explain the shape of the Type Ia supernova light curve qualitatively.



# Supernovae: astrophysical classification

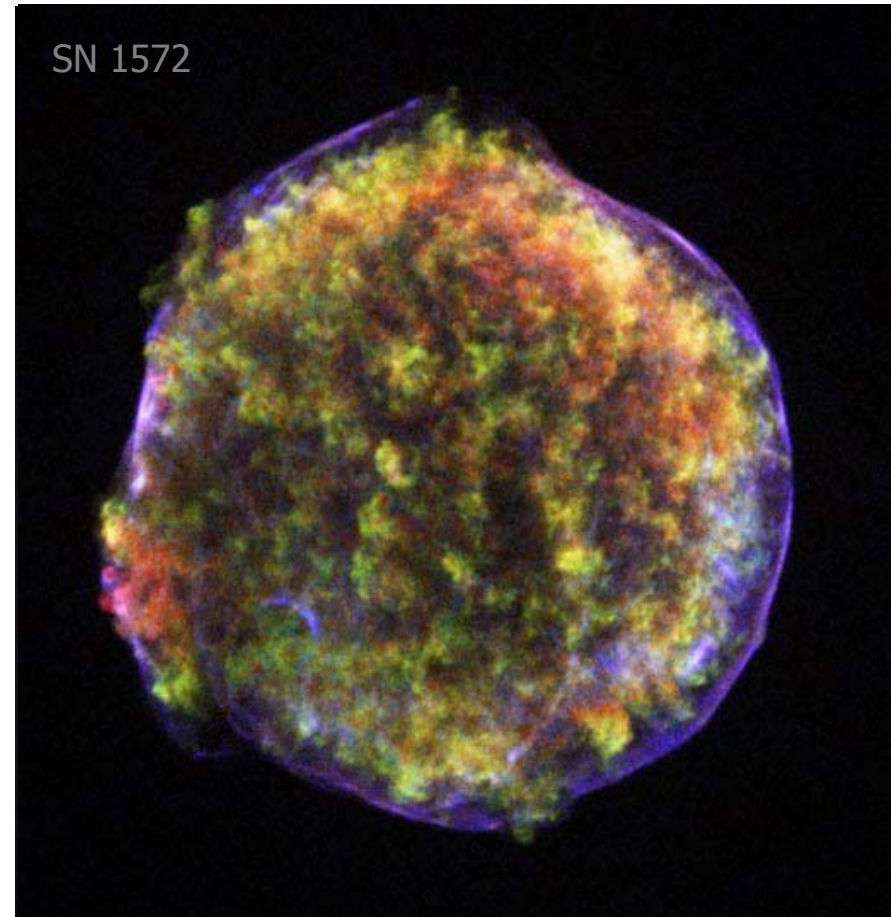
- ▶ astrophysical events of enormous energy release and brightness

- ▶ **energy source:**



- ▶ **Type Ia supernovae:**  
no compact object found in remnant  
→ thermonuclear explosion

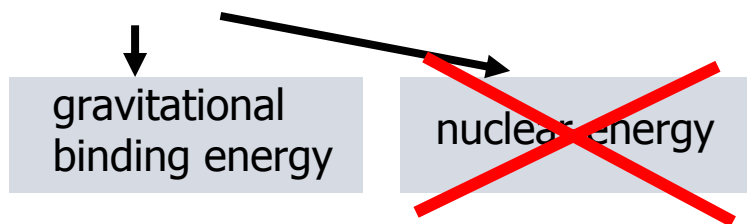
**thermonuclear supernovae**



# Supernovae: astrophysical classification

- ▶ astrophysical events of enormous energy release and brightness

- ▶ **energy source:**



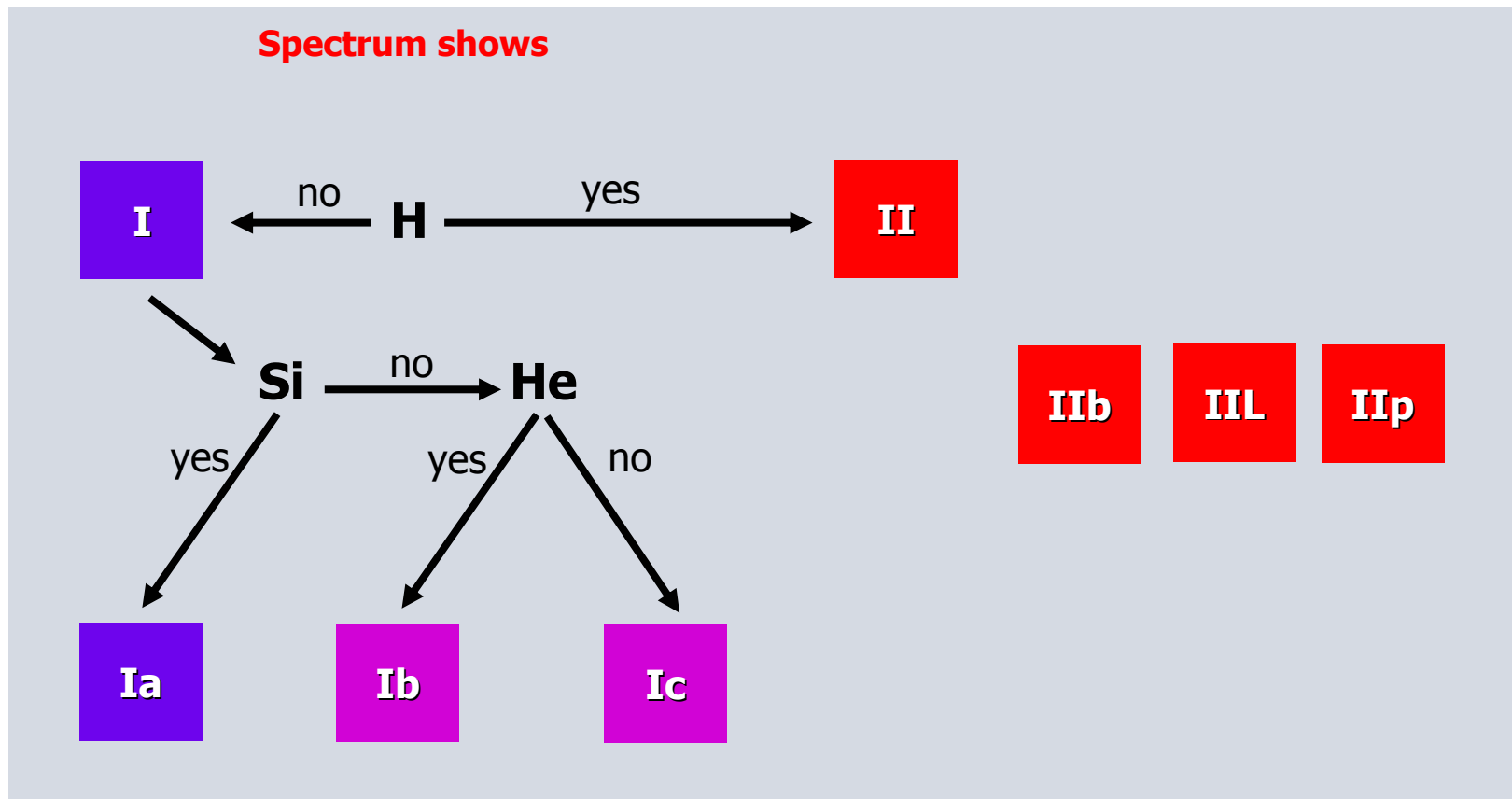
- ▶ **Type Ib, Ic, II supernovae:**  
compact object found in remnant  
(neutron star, black hole)  
→ gravitational collapse of evolved massive star

core collapse supernovae



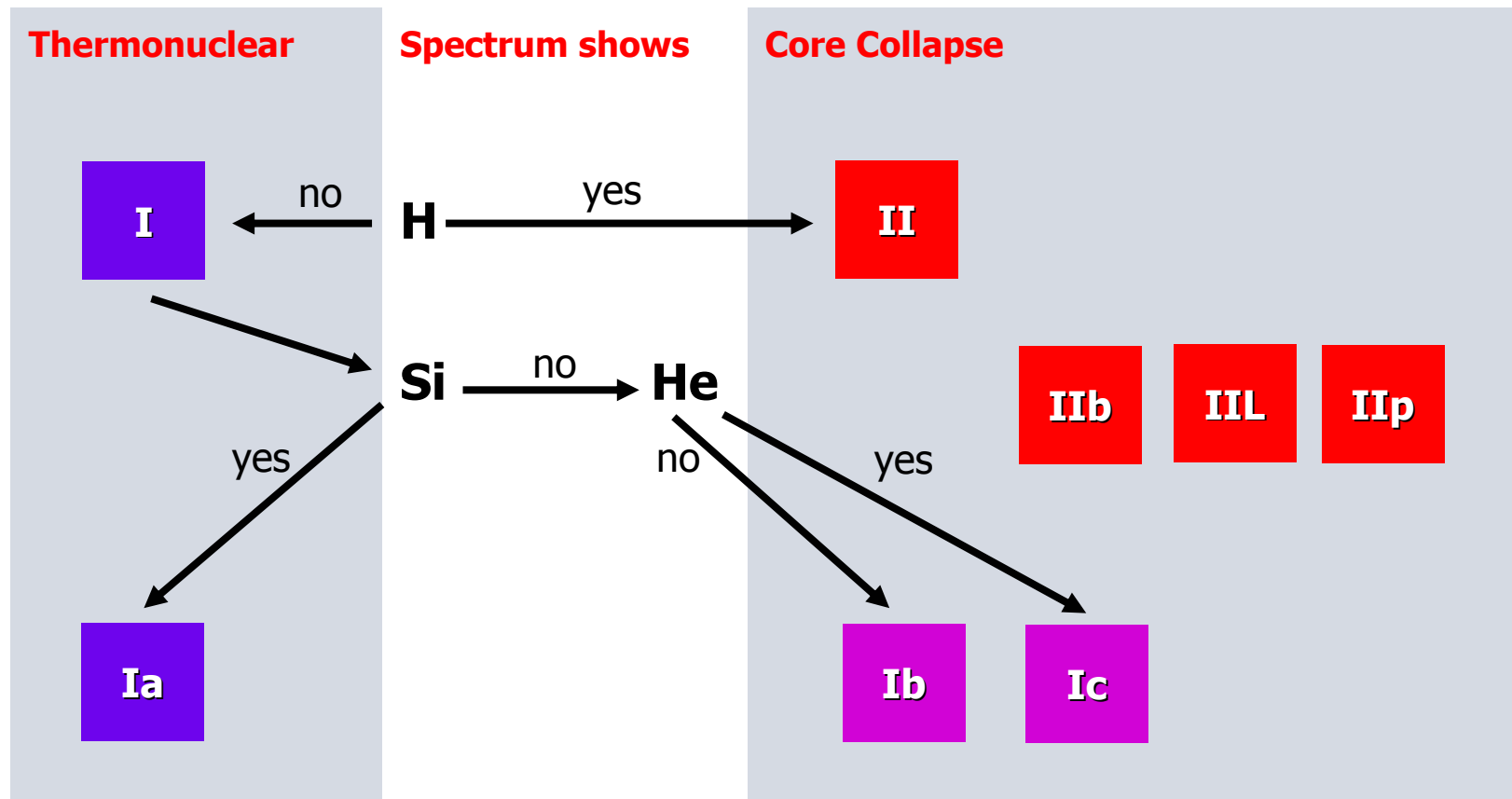
# Supernovae: astronomical classification

- ▶ empirical classification



# Supernovae: astrophysical classification

- ▶ **physical** classification



# SNe Ia from a modeler's perspective

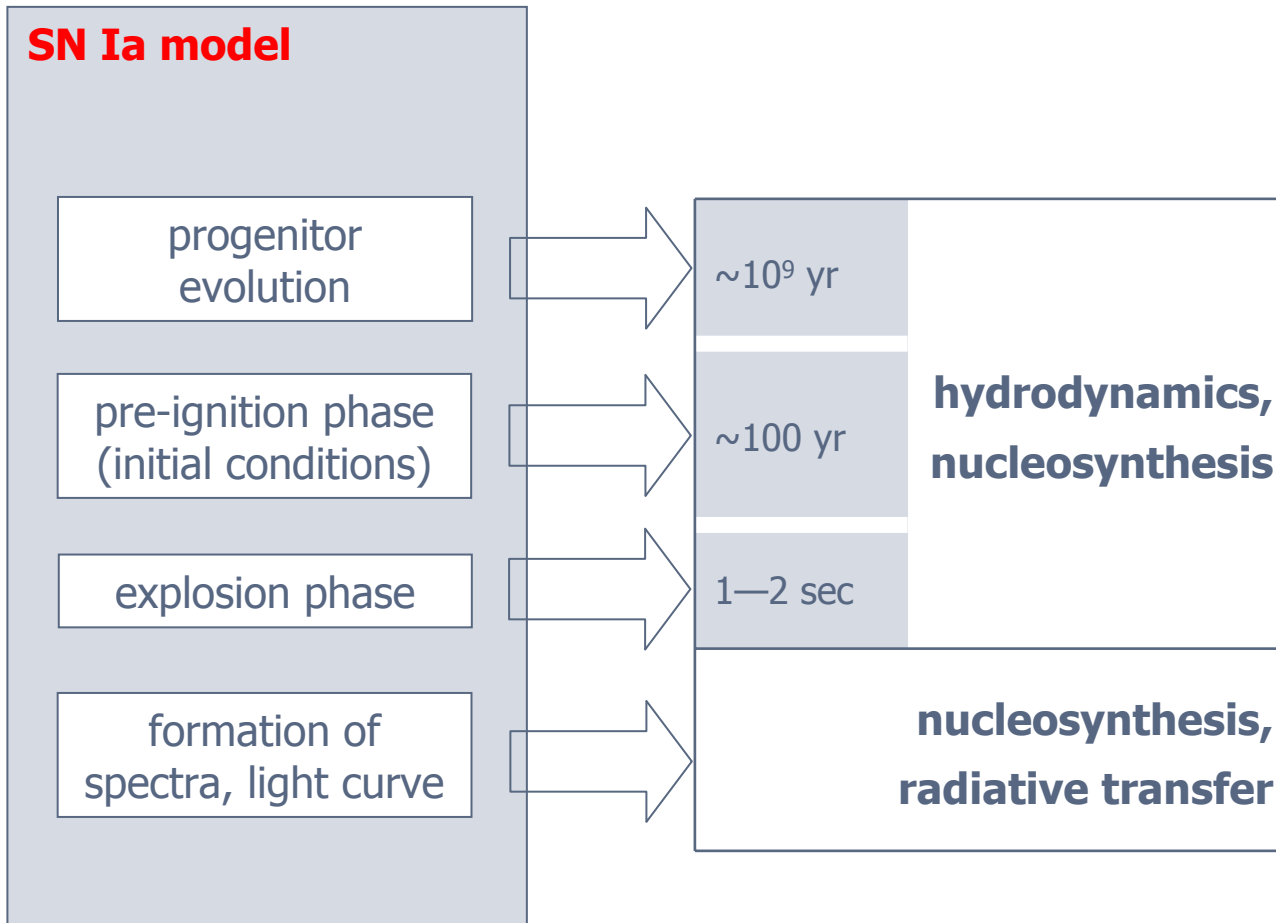
- ▶ energy source: thermonuclear explosion
- ▶ kinetic energy of ejecta:  $\sim 1$  B (Bethe =  $10^{51}$  erg)
- ▶ brightness uniform (at least by astronomical standards)  $M_{\text{bol}} \sim -19$
- ▶ no H, He in spectra  $\rightarrow$  exploding object: **C+O white dwarf star**

Homework problem II:

How much material of the WD must be burned to Ni to power a SN Ia?



# Astrophysical model



# Astrophysical model

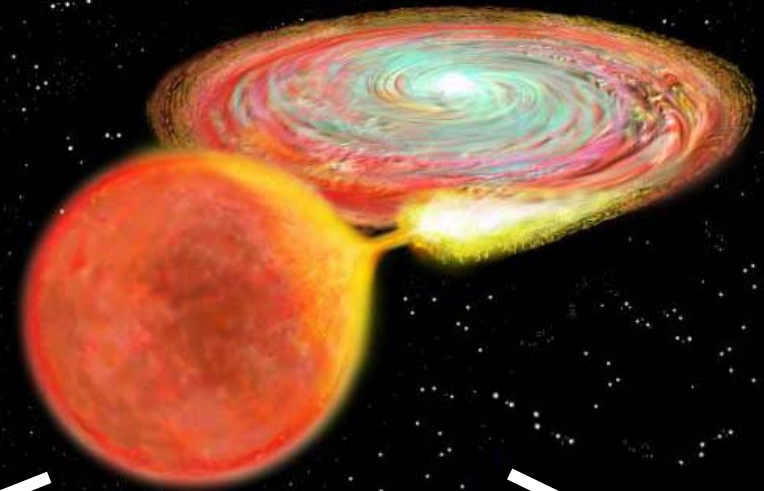
## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve



**sub- $M_{\text{Ch}}$  scenario**

**$M_{\text{Ch}}$  scenario**



# Astrophysical model

## SN Ia model

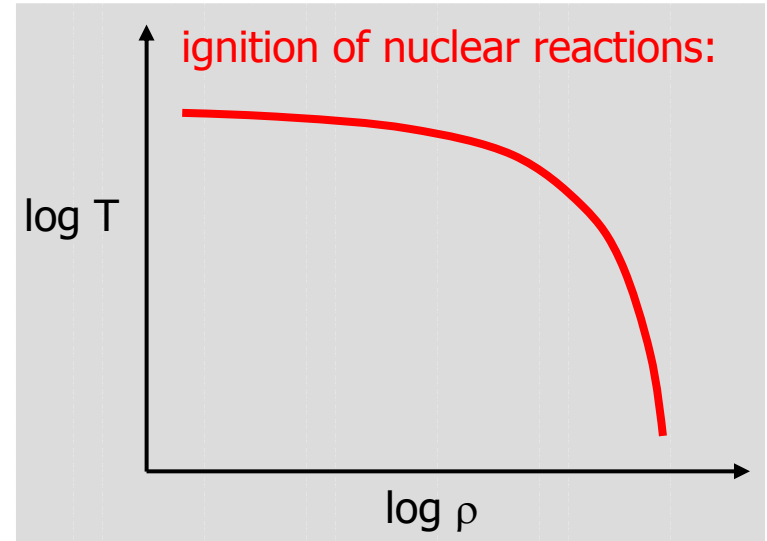
progenitor evolution

pre-ignition phase  
(initial conditions)

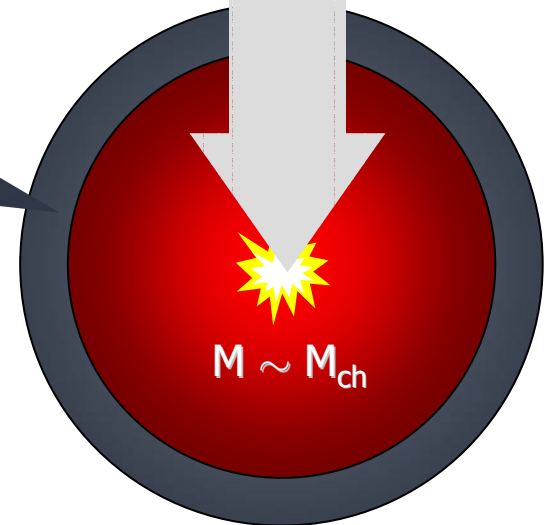
explosion phase

formation of spectra, light curve

Chandrasekhar-mass,  
single degenerate  
progenitor scenario



**He (+H)  
from binary  
companion**



# Astrophysical model

## SN Ia model

progenitor  
evolution

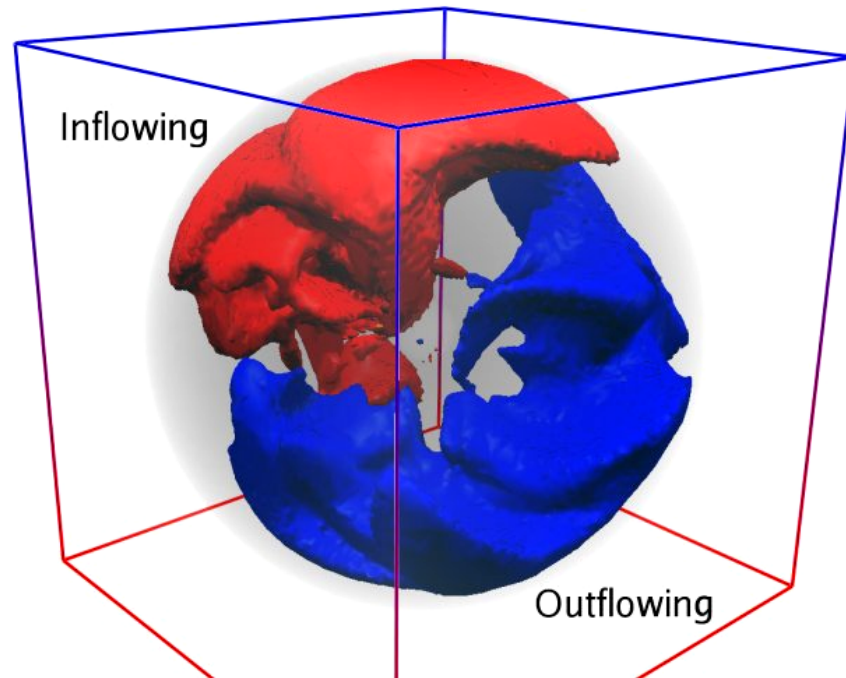
pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve

## convective carbon burning

- ▶ lasts for about a century
- ▶ released energy transported away by convection,
- ▶ cooling due to plasmon neutrinos
- ▶ flow pattern uncertain but determines ignition configuration



Kuhlen et al. 2006

# Astrophysical model

## SN Ia model

progenitor evolution

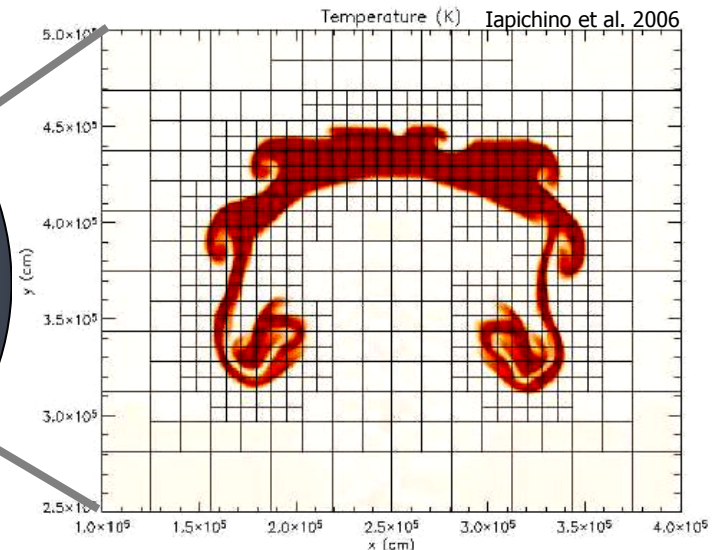
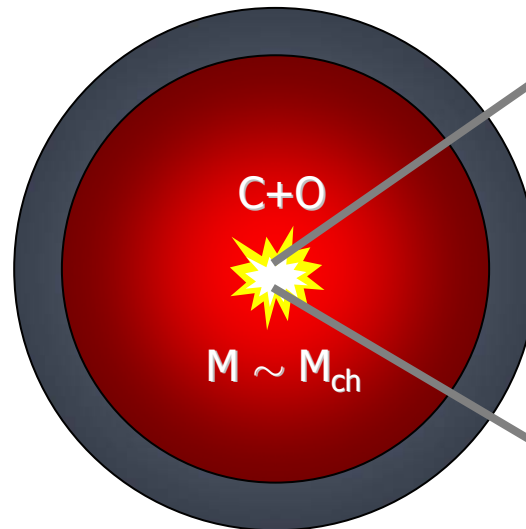
pre-ignition phase  
(initial conditions)

explosion phase

formation of spectra, light curve

## Triggering of explosion:

- ▶ thermonuclear runaway near center of WD
- ▶ fully degenerate material:  $P = P(\rho, T)$   
temperature increase (stochastic fluctuation) → not cooled by expansion
- ▶ C+C reaction rate scales with  $\sim T^{20}$  → thermonuclear runaway in small patch of material → reaction wave starts to propagate outwards → **thermonuclear flame**



# Astrophysical model

## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

**explosion phase**

formation of  
spectra, light curve

## hydrodynamics of flame propagation

**deflagration**  
subsonic

flame mediated by  
thermal conduction  
of degenerate  $e^-$

**detonation**  
supersonic

flame driven by  
shock waves

# Astrophysical model

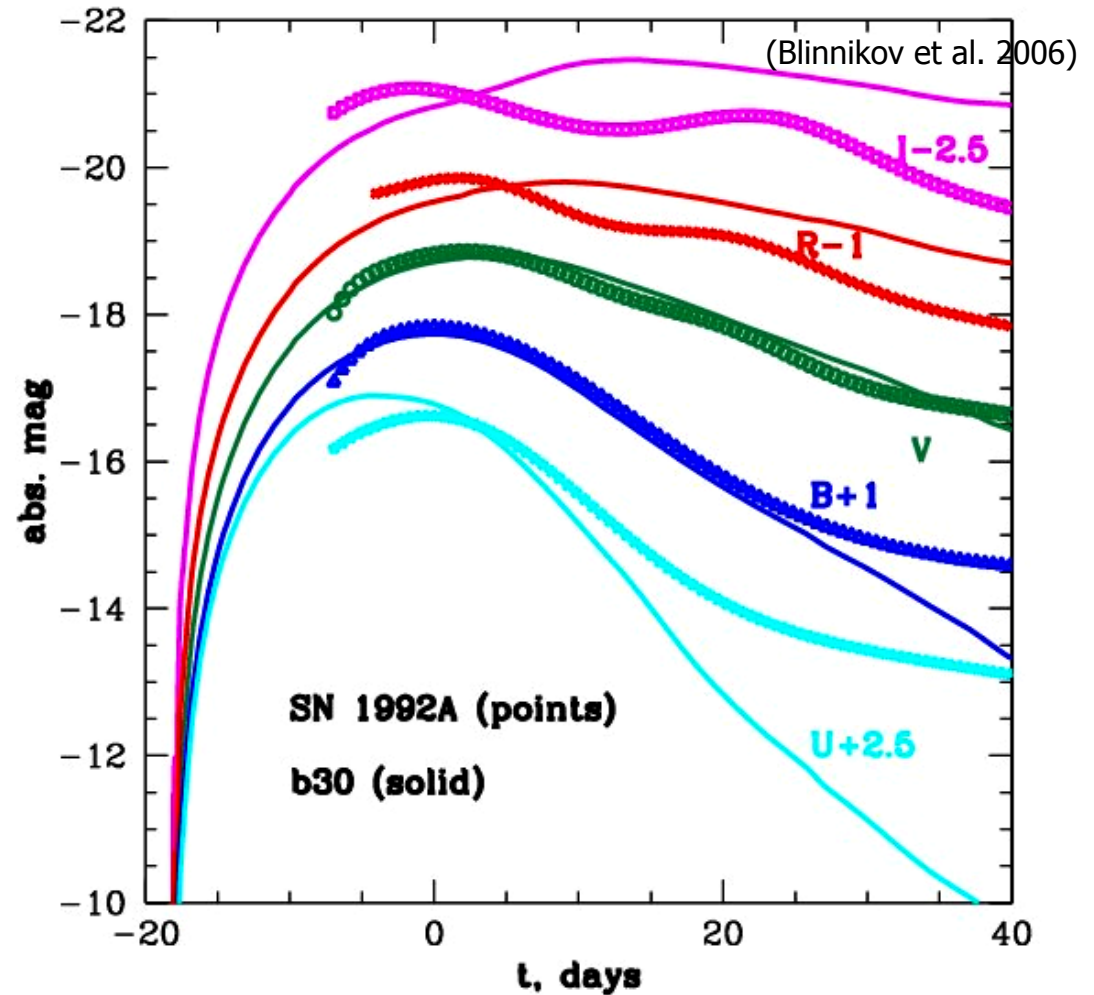
## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve



Homework problem III: Why are Type Ia supernovae bright?

# SN Ia explosion model

## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve

- ▶ flame burns from ignition near center of WD towards surface

## 1. General hydrodynamics

# Hydrodynamics: Euler's equations

- ▶ general form of balance equation for extensive quantity  $A$  in fixed volume  $V$ :

$$\frac{dA}{dt} = \frac{d_f A}{dt} + \frac{d_s A}{dt}$$

- ▶ differential form (density of  $A$ ,  $a$ ):

$$\frac{\partial a}{\partial t} = -\nabla \cdot \vec{j}_a + s(a)$$

- ▶ mass conservation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

- ▶ momentum balance

$$\frac{\partial \rho \vec{v}}{\partial t} = -\nabla \cdot (\rho \vec{v} \vec{v}) - \nabla P + \rho \vec{f}$$

- ▶ energy balance

$$\frac{\partial \rho e_{\text{tot}}}{\partial t} = -\nabla \cdot (\rho e_{\text{tot}} \vec{v}) - \nabla \cdot (P \vec{v}) + \rho \vec{v} \cdot \vec{f}$$

# Euler's equations

Euler's equations describe ideal (inviscid) flow

- ▶ mass conservation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

- ▶ momentum balance

$$\frac{\partial \rho \vec{v}}{\partial t} = -\nabla \cdot (\rho \vec{v} \vec{v}) - \nabla P + \rho \vec{f}$$

- ▶ energy balance

$$\frac{\partial \rho e_{\text{tot}}}{\partial t} = -\nabla \cdot (\rho e_{\text{tot}} \vec{v}) - \nabla \cdot (P \vec{v}) + \rho \vec{v} \cdot \vec{f}$$



# Turbulence

- ▶ real (viscous) flows described by Navier-Stokes equation
- ▶ does not depend individually on density  $\rho$ , velocity  $v$ , viscosity  $\eta$ , and length scale  $l$ , but on dimensionless quantity:

Reynolds number

$$Re(l) = \frac{\rho l v(l)}{\eta}$$

which characterizes flow:

$Re \ll 1$ : viscosity dominates → flow laminar

$Re \gg 1$ : inertia dominates → flow turbulent (common in astrophysics:  $Re \gtrsim 10^{10}$ )

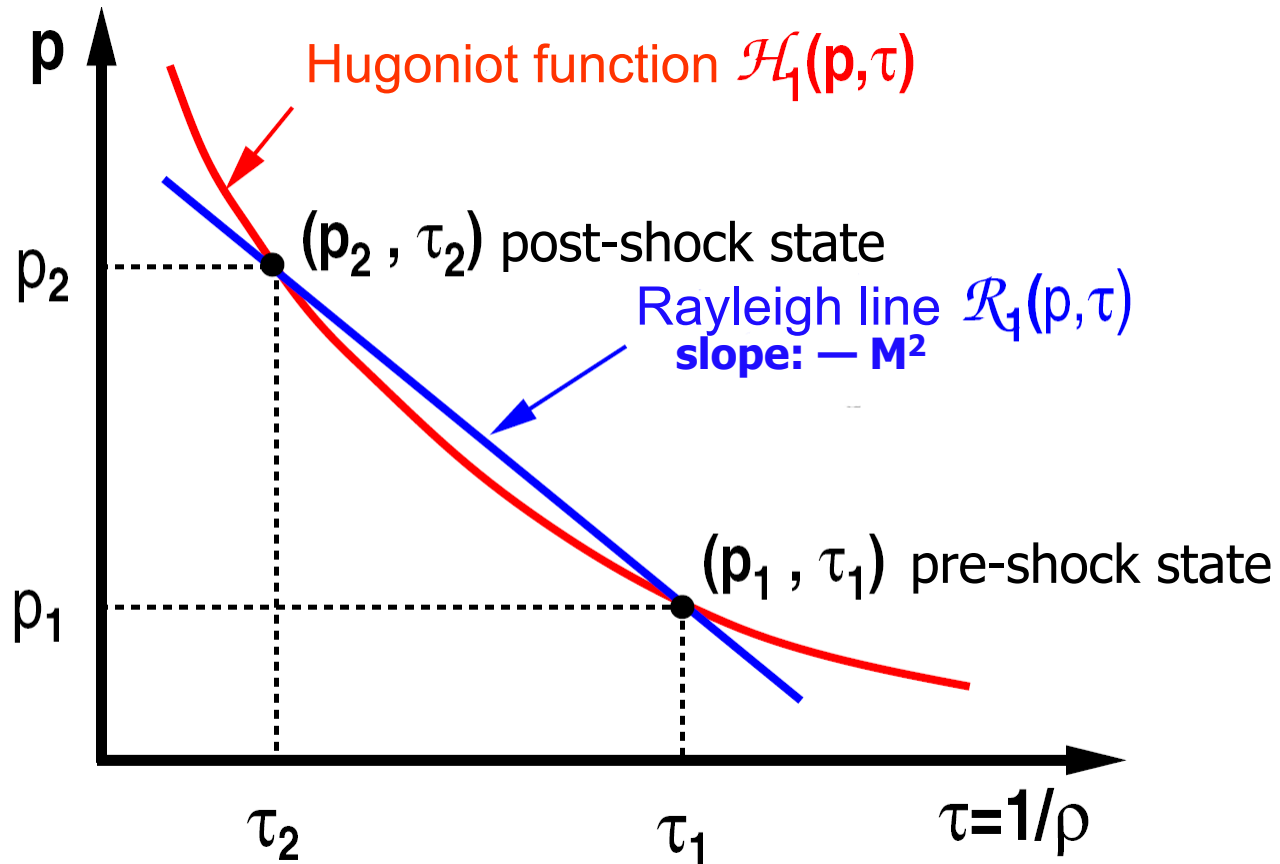
- ▶ **turbulent cascade** (Richardson 1922): turbulent energy transported from large scales to small scales (without loss → "inertial range"), dissipated into heat at Kolmogorov length  $l_K$  ( $Re(l_K) \approx 1$ )

# Flow discontinuities

- ▶ integral form of eqs. of hydro allows for **discontinuous ("weak") solutions**
- ▶ differential form must be augmented by "jump conditions"
- ▶ hydrodynamical fluxes are conserved over discontinuities → Rankine-Hugoniot jump conditions (1-dim form):
  - ▶ mass flux conservation:  
 $\rho_1 u_1 = \rho_2 u_2 = M$       notation:  $[\rho u] = \rho_2 u_2 - \rho_1 u_1 = 0$
  - ▶ momentum flux conservation:  
 $[\rho u^2 + P] = 0$
  - ▶ energy flux conservation:  
 $[(\rho e_{tot}) u] = 0$

# Flow discontinuities

- ▶ mass flux  $M$  over discontinuity:  $M \neq 0$ : **shock waves** → Rankine-Hugoniot jump conditions yield for initial (pre-shock) state  $(p_1, \tau_1 = 1/\rho_1)$ :



# SN Ia explosion model

## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve

- ▶ flame burns from ignition near center of WD towards surface

## 2. Reactive hydrodynamics and flames

# Reactive Euler's equations

- ▶ mass conservation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

- ▶ Euler's equation

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \nabla) \cdot \vec{v} - \frac{\nabla P}{\rho} + \vec{f}$$

- ▶ energy balance

$$\frac{\partial \rho e_{\text{tot}}}{\partial t} = -\nabla \cdot (\rho e_{\text{tot}} \vec{v}) - \nabla \cdot (P \vec{v}) + \rho \vec{v} \cdot \vec{f} + \rho S$$

- ▶ species balance

$$\frac{\partial X}{\partial t} = -\nabla \cdot (\rho X \vec{v}) + r$$

- ▶ energy source term  $S$ , reaction rate  $r$ :

$$S = f(r)$$

$$r = f(\rho, T, X)$$

# Hydrodynamic eqs. for SNe Ia

- ▶ external force: self-gravity (non-relativistic):

$$\Delta\Phi = 4\pi G\rho$$

- ▶ reactive Euler's equations:

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\vec{v})$$

$$\frac{\partial\vec{v}}{\partial t} = -(\vec{v}\nabla)\cdot\vec{v} - \frac{\nabla P}{\rho} + \nabla\Phi$$

$$\frac{\partial\rho e_{\text{tot}}}{\partial t} = -\nabla\cdot(\rho e_{\text{tot}}\vec{v}) - \nabla\cdot(P\vec{v}) + \rho\vec{v}\cdot\nabla\Phi + \rho S$$

$$\frac{\partial\mathbf{X}}{\partial t} = -\nabla\cdot(\rho\mathbf{X}\vec{v}) + \mathbf{r}$$

$$S = f(\mathbf{r})$$

$$\mathbf{r} = \mathbf{f}(\rho, T, \mathbf{X})$$

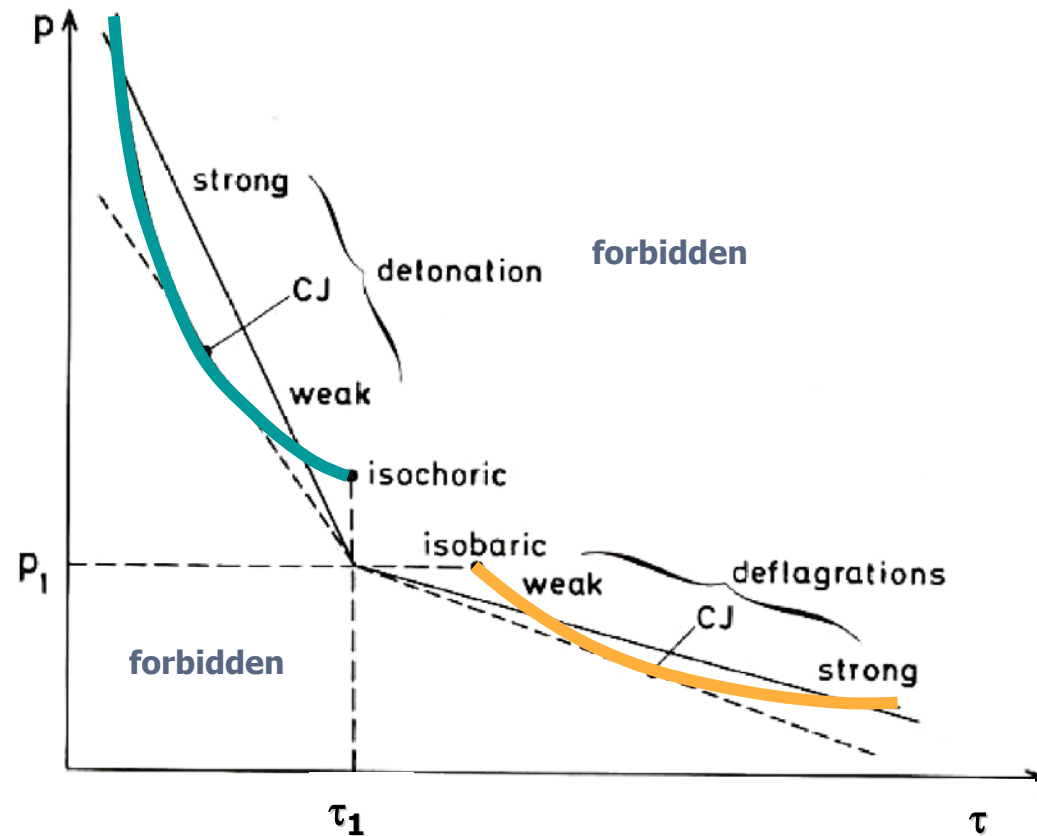
- ▶ closed by equation of state:

$$P = f_{\text{EoS}}(\rho, e_{\text{int}}, \mathbf{X})$$

$$T = f_{\text{EoS}}(\rho, e_{\text{int}}, \mathbf{X})$$

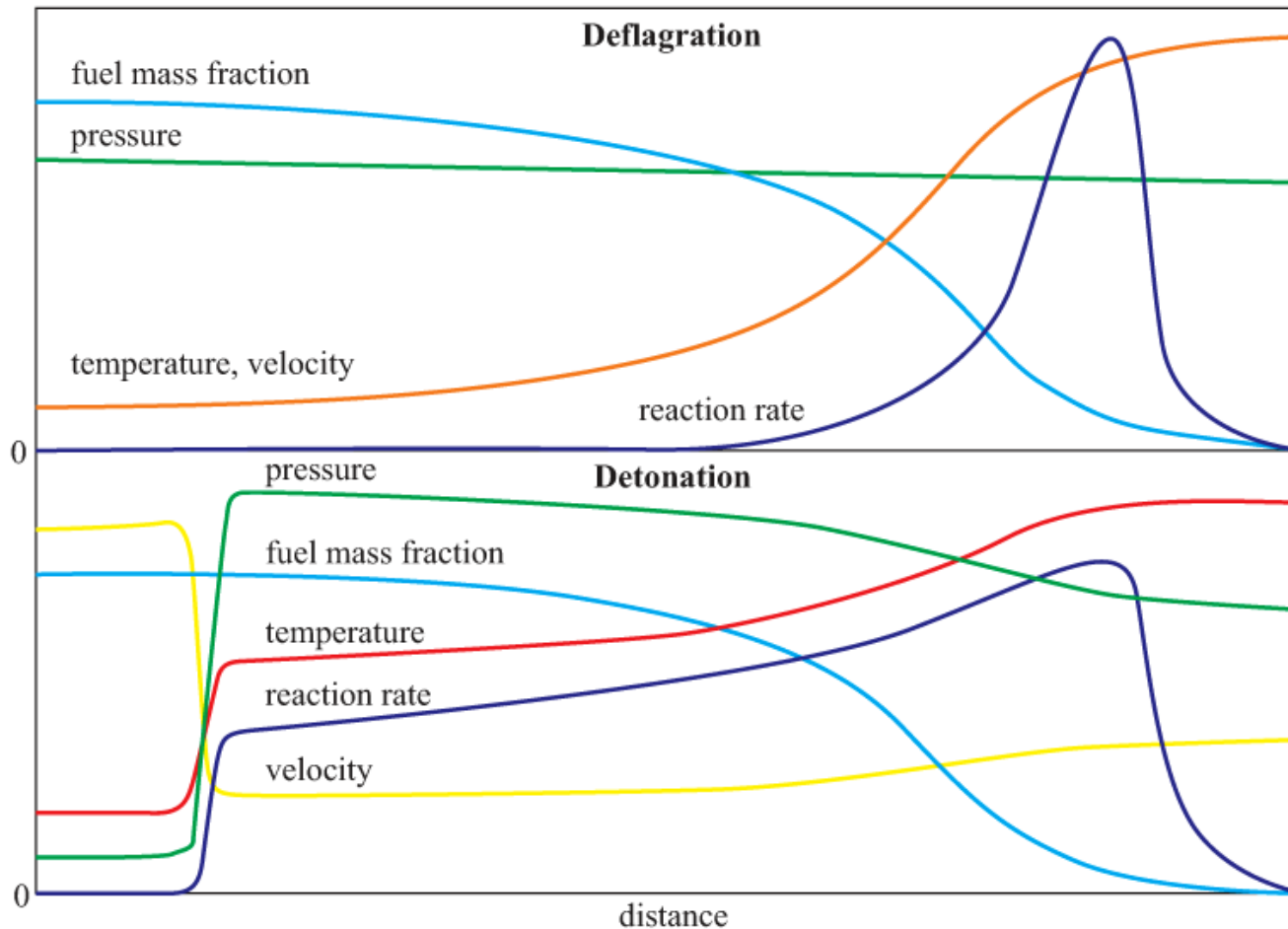
# Thermonuclear supernovae

- ▶ Hugoniot curve separates into two branches:
  - ▶ **detonations** ( $P_2 > P_1, \tau_2 < \tau_1$ )
  - ▶ **deflagrations** ( $P_2 < P_1, \tau_2 > \tau_1$ )
- ▶ **detonations**: supersonic with respect to fuel
- ▶ **deflagrations**: subsonic with respect to fuel



# Deflagrations and detonations

► microscopic structure





# SN Ia explosion model

## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

formation of  
spectra, light curve

- ▶ flame burns from ignition near center of WD towards surface

### 3. Instabilities and production of turbulence

# Instabilities

## Flow instabilities

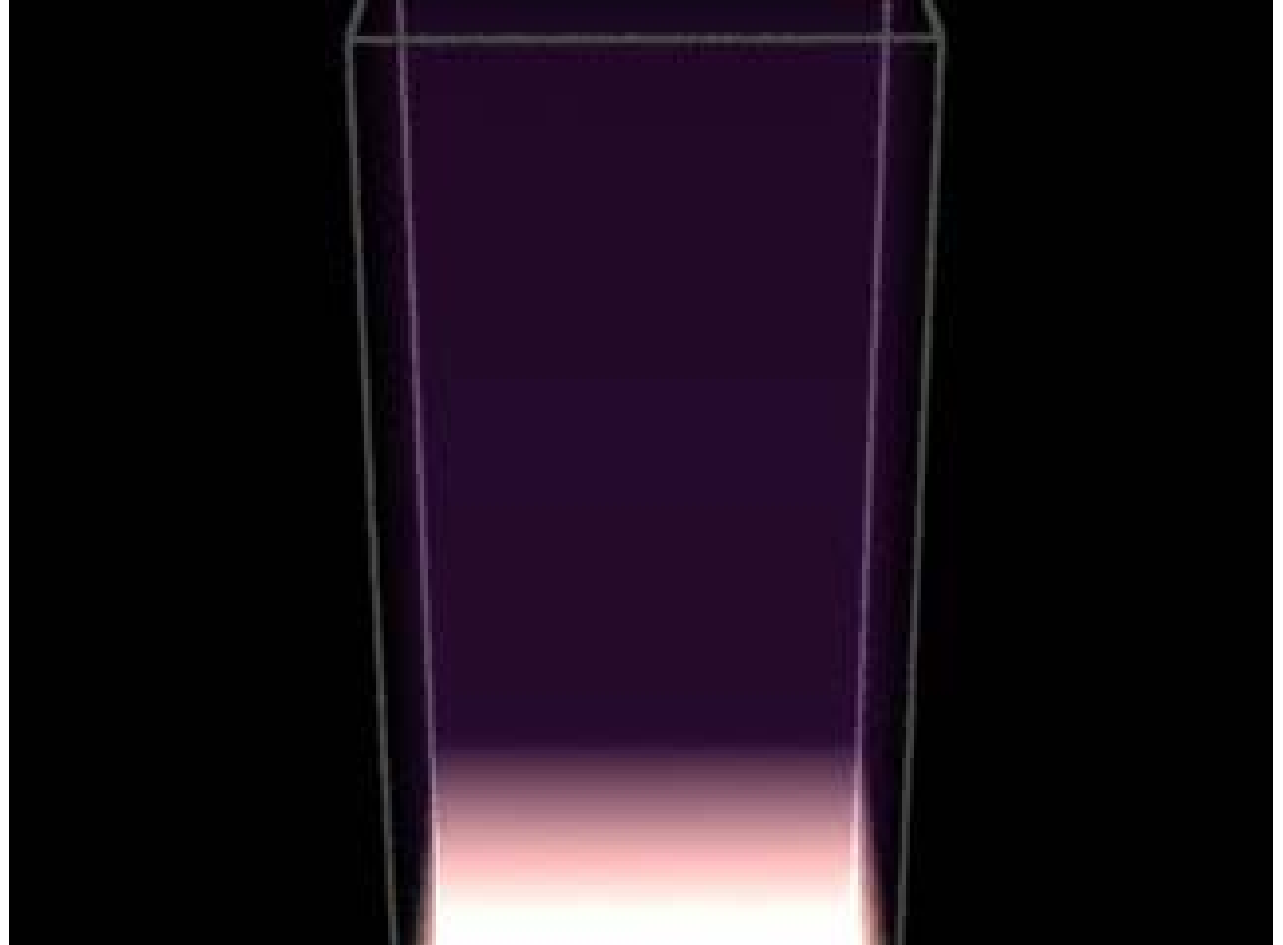
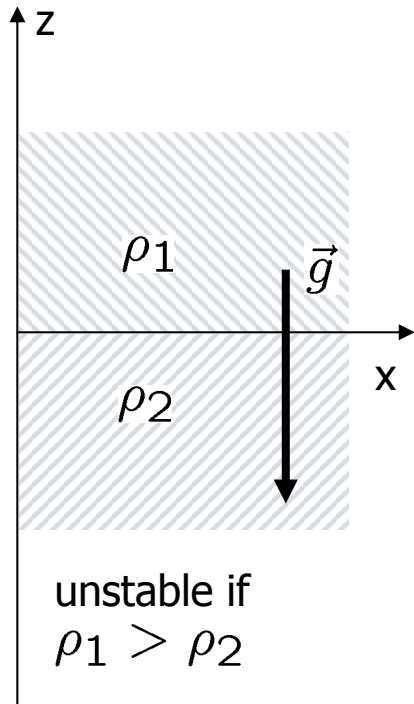
- ▶ Rayleigh-Taylor  
(buoyancy)
- ▶ Kelvin-Helmholtz  
(shear)
- ▶ produce turbulence !!

## Burning front instabilities

- ▶ Landau-Darrius  
(stabilized in cellular pattern)
- ▶ Thermal-diffusive  
(irrelevant in SNe Ia)

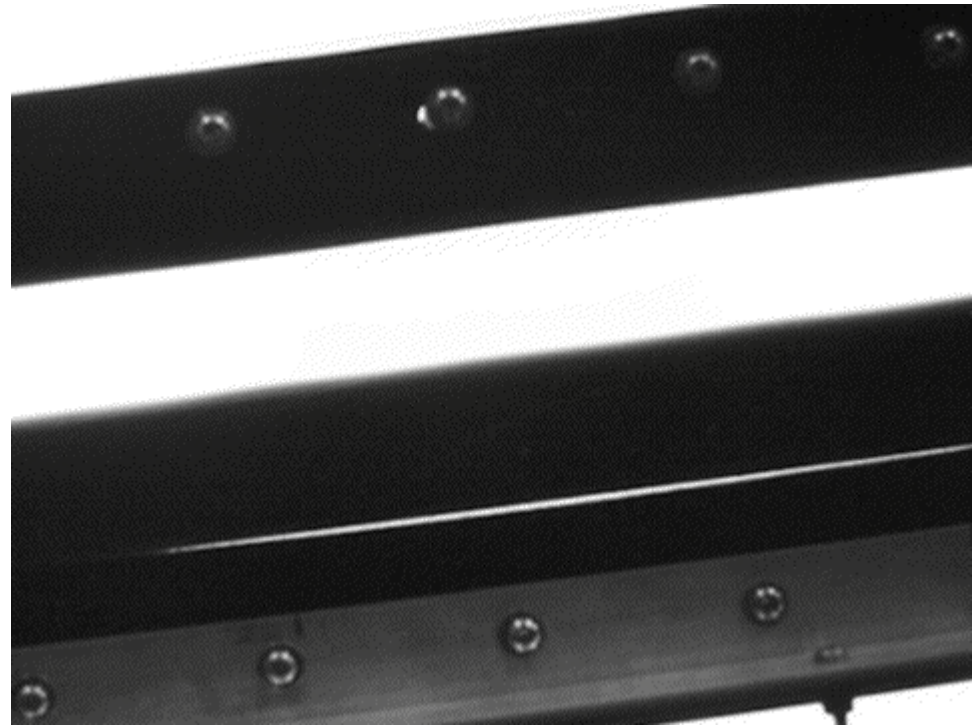
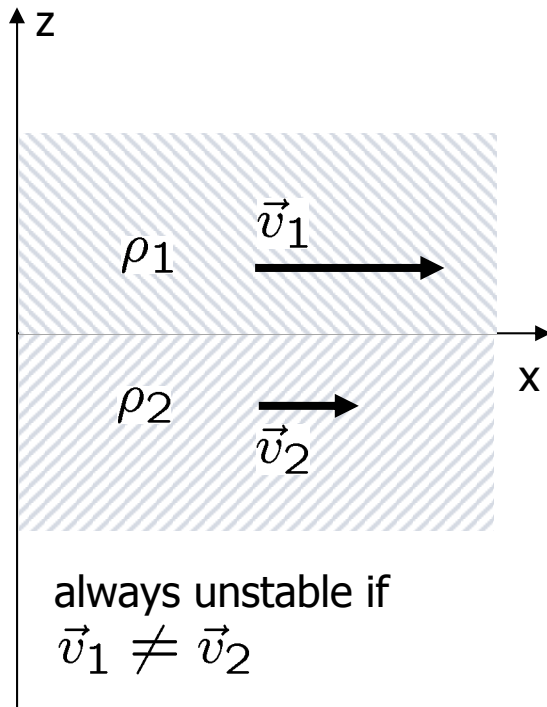
# Rayleigh-Taylor instability

- ▶ buoyancy instability → results from inverse density stratification in gravitational field



# Kelvin-Helmholtz instability

- ▶ shear instability



- ▶ produces turbulence

# Kelvin-Helmholtz instability



**encontro das águas**



**Billow clouds**



# SN Ia explosion model

## SN Ia model

progenitor  
evolution

pre-ignition phase  
(initial conditions)

explosion phase

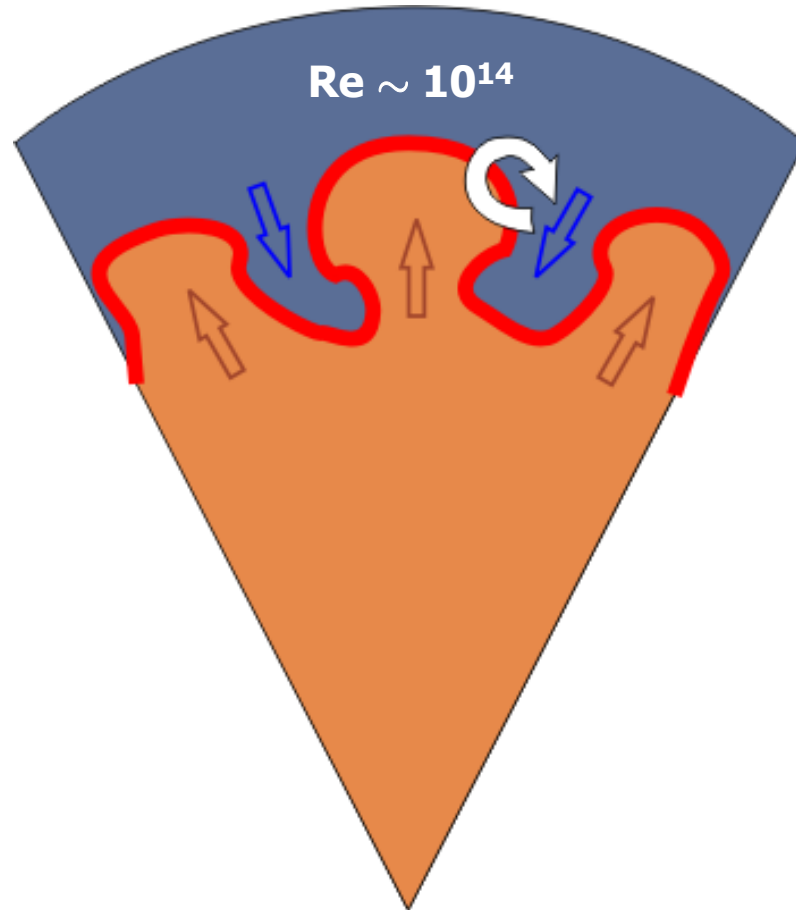
formation of  
spectra, light curve

- ▶ flame burns from ignition near center of WD towards surface

## 4. Turbulent combustion in SNe Ia

# Turbulent combustion in SNe Ia

- ▶ generic turbulence production



# Turbulent deflagration

- ▶ **Turbulent energy cascade:** eddies decay to smaller scales  $\leftrightarrow$  lower velocities





# The Gibson scale

- ▶ in turbulent cascade velocity fluctuations decrease towards smaller scales, e.g. for Kolmogorov scaling:

$$v'(l) \propto l^{1/3}$$

- ▶ at **Gibson scale** turbulent eddies as slow as laminar flame

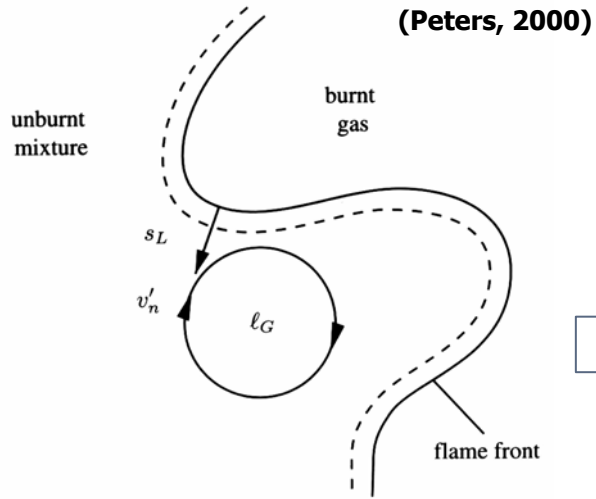
$$v'(l_{\text{Gibs}}) = s_L$$

- ▶ below  $l_{\text{Gibs}}$ : flame burns faster through turbulent eddies than they can deform it
- ▶ flame flow interaction only between integral scale of turbulence and Gibson scale!

# Turbulent burning regimes

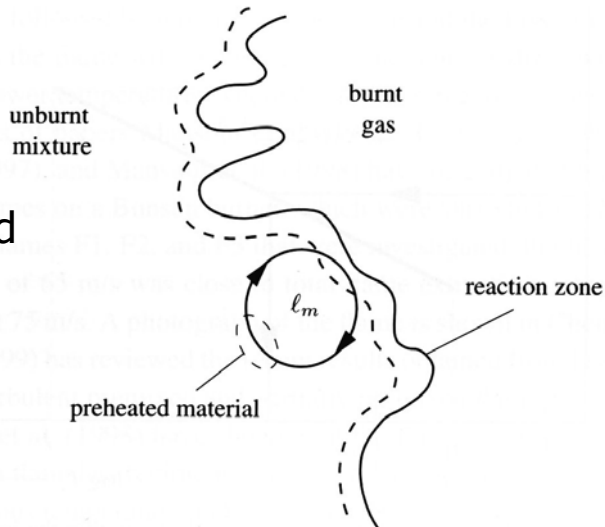
flamelet regime

$$l_{Gibbs} \gg l_f$$

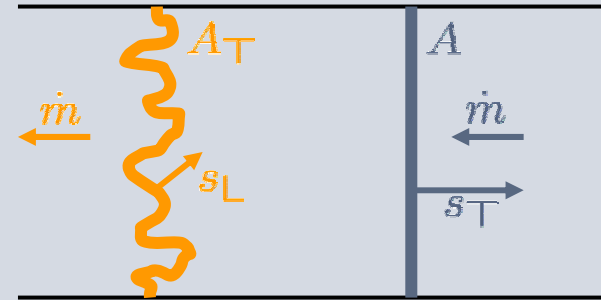


distributed burning regime

$$l_{Gibbs} \lesssim l_f$$



Burning in the flamelet regime:



$$s_T \sim v'$$

(Damköhler 1940)