# **Fermionic Molecular Dynamics for Nuclear Structure and Reactions**

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# Overview Introduction Unitary Correlation Operator Method Fermionic Molecular Dynamics Cluster degrees of freedom, Reactions

## Introduction

**Nuclear Degrees of Freedom** 

**Many-body Methods** 

**Two-Nucleon System** 

**Nucleon-Nucleon Interaction** 

• Introduction

# **Nuclear Degrees of Freedom**



• Introduction

# Nucleons as effective Degrees of Freedom





cm-coordinates and spins

- at low energies nuclei can be described as a system of nucleons
- nucleons are not point-like particles, proton radius  $\sqrt{r_p^2} \approx 0.89$  fm
- nucleon-nucleon force is something like the van-der-Waals force between atoms

• Introduction

# Quest for a unified Description of Nuclei



#### Introduction

**Exotica:** Special Challenges



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- Introduction Two-Nucleon System (Relative Motion)
  - Couple Spin and Isospin

$$|S, M_S\rangle = \sum_{m_{s1}, m_{s2}} C \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & |S| \\ m_{s1} & m_{s2} & |M_S\rangle \\ |\frac{1}{2}, m_{s1}\rangle \otimes |\frac{1}{2}, m_{s2}\rangle, \qquad S = 0, 1$$

$$|T, M_T\rangle = \sum_{m_{t1}, m_{t2}} C \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & |T| \\ m_{t1} & m_{t2} & |M_T\rangle \\ |\frac{1}{2}, m_{t1}\rangle \otimes |\frac{1}{2}, m_{t2}\rangle, \qquad T = 0, 1$$

Spin/Isospin Singlet, Triplet

• Couple Orbital Angular Momentum with Spin

$$\left\langle \mathbf{r} \middle| \alpha, (LS)JM; TM_T \right\rangle = \sum_{M_L, M_S} C \begin{pmatrix} L & S & J \\ M_L & M_S & M \end{pmatrix} \phi_{\alpha}(r) Y_{LM_L}(\hat{\mathbf{r}}) \left| S, M_S \right\rangle \otimes \left| T, M_T \right\rangle$$

• Antisymmetry

$$(S,T) = (0,1) \text{ or } (1,0)$$
 $\longrightarrow$  $L = 0, 2, 4, \dots$ Even channels $(S,T) = (0,0) \text{ or } (1,1)$  $\longrightarrow$  $L = 1, 3, 5, \dots$ Odd channels

#### Introduction

## Nucleon-Nucleon Force



#### **Realistic Interactions**

- describe NN phaseshifts  $(\chi^2/datum \approx 1)$
- describe deuteron properties
- short-range (high-momentum) and off-shell behavior not constrained by data
- nucleon-nucleon force not completely constrained

#### **Some Realistic Interactions**

- Bonn-Potentials (based on meson-exchange)
- Argonne V18 (local, phenomenological ansatz)
- Potentials based on
   Chiral Perturbation Theory

#### • (almost) local interaction in coordinate space

$$V(\mathbf{r}_{1}, \mathbf{r}_{2}, \mathbf{p}_{1}, \mathbf{p}_{2}, \sigma_{1}, \sigma_{2}, \tau_{1}, \tau_{2}) = V(r) + V^{\sigma}(r)\sigma_{1} \cdot \sigma_{2} + V^{\tau}(r)\tau_{1} \cdot \tau_{2} + V^{\sigma\tau}(r)\sigma_{1} \cdot \sigma_{2}\tau_{1} \cdot \tau_{2} + C^{\sigma\tau}(r)\sigma_{1} \cdot \sigma_{2}\tau_{1} \cdot \tau_{2}\mathbf{L}^{2} + V^{\sigma\tau}_{l^{2}}(r)\mathbf{L}^{2} + V^{\sigma\tau}_{l^{2}}(r)\mathbf{L}^{2} + V^{\sigma\tau}_{l^{2}}(r)\mathbf{L} \cdot \mathbf{S} + V^{\tau}_{l^{2}}(r)\mathbf{L} \cdot \mathbf{S} + V^{\tau}_{l^{2}}(r)\mathbf{L} \cdot \mathbf{S} + V^{\tau}_{l^{2}}(r)\tau_{1} \cdot \tau_{2}(\mathbf{L} \cdot \mathbf{S})^{2} + V^{\tau}_{l^{2}}(r)(\mathbf{L} \cdot \mathbf{S})^{2} + V^{\tau}_{l^{2}}(r)\tau_{1} \cdot \tau_{2}S_{12} + V^{\tau}_{t}(r)\tau_{1} \cdot \tau_{2}S_{12} + C^{\sigma\tau}_{t^{2}}(r)\tau_{1} \cdot \tau_{2}S_{12} + C^{\sigma\tau}_{t^{2}}(r)\tau_{1}^{2}(r)\tau_{$$

four charge dependent and charge asymmetric terms

 $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$  $\mathbf{L} = \mathbf{r} \times \mathbf{p}, \qquad \mathbf{S} = \frac{1}{2}(\sigma_1 + \sigma_2)$  $S_{12} = 3(\sigma_1 \cdot \hat{\mathbf{r}})(\sigma_2 \cdot \hat{\mathbf{r}}) - \sigma_1 \cdot \sigma_2$ 

#### • Introduction

# **Green's Function Monte Carlo**



Wiringa, Pieper, PRL 89 (2002) 182501

# **Unitary Correlation Operator Method**

Central and Tensor Correlations Unitary Correlation Operator Interaction in Momentum Space

ab initio Calculations

#### исом Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



• strong repulsive core: nucleons can not get closer than  $\approx 0.5~fm$ 

- central correlations

 strong dependence on the orientation of the spins due to the tensor force

tensor correlations

the nuclear force will induce strong short-range correlations in the nuclear wave function

# **One- and Two-Body Densities**

(Diagonal) One-body density

$$\rho^{(1)}(\mathbf{r}) = \sum_{m_t, m_s} \left\langle \Psi \left| \underset{m_s, m_t}{a}^{\dagger}(\mathbf{r}) \underset{m_s, m_t}{a}(\mathbf{r}) \right| \Psi \right\rangle$$

Probability to find a nucleon at position r

(Diagonal) Two-body density

$$\rho_{S,M_{S};T,M_{T}}^{(2)}(\mathbf{r},\mathbf{r}') = \sum_{m_{s},m_{s}'} C \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ m_{s} & m_{s}' & M_{S} \end{pmatrix} \sum_{m_{t},m_{t}'} C \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & T \\ m_{t} & m_{t}' & M_{T} \end{pmatrix} \langle \Psi | a_{m_{s},m_{t}}^{\dagger}(\mathbf{r}) a_{m_{s}',m_{t}'}^{\dagger}(\mathbf{r}') a_{m_{s},m_{t}}(\mathbf{r}') a_{m_{s},m_{t}}(\mathbf{r}') a_{m_{s},m_{t}}(\mathbf{r}') \langle \Psi | \Psi \rangle$$

$$\rho_{S,M_{S};T,M_{T}}^{(2)}(\mathbf{r}) = \int d^{3}R \ \rho_{S,M_{S};T,M_{T}}^{(2)}(\frac{1}{2}\mathbf{R} + \mathbf{r}, \frac{1}{2}\mathbf{R} - \mathbf{r})$$

Probability to find **two nucleons** at a relative distance r исом Deuteron

Spin-projected two-body density  $\rho_{S=1,M_S,T=0}^{(2)}({\bf r})$  (isodensity plot)

 $egin{aligned} M_{m{S}} &= 0 \ rac{1}{\sqrt{2}}(\ket{\uparrow \downarrow} + \ket{\downarrow \uparrow}) \end{aligned}$ 



"Donut"



"Dumbbell"

density at small distances suppressed

- central correlations

density depends strongly on spin orientation

tensor correlations

these **short-range Correlations** can not be described with product states (Slater determinants)

# Realistic and Effective Nucleon-Nucleon Interactions

#### **Realistic Interactions**

- reproduce scattering data and deutron properties
- meson-exchange (Bonn), phenomenological (AV18),  $\chi$ -PT (Idaho)
- repulsive core and tensor force induce strong short-range correlations

#### **Effective Interactions**

- phenomenological effective interactions describe many properties of nuclear systems like energies, radii, spectra successfully using simple many-body wave functions (HF, shell model, microscopic cluster models)
- No-Core Shell Model uses Lee-Suzuki transformation in oscillator basis
- G-matrix and  $V_{lowk}$  derive effective interaction in momentum space

#### **Our approach**

- derive effective interaction from realistic interaction by explicitly including correlations with unitary correlation operator  $C = C_{\Omega}C_r$  formulated in coordinate space
- correlated (effective) interaction

$$\hat{H} = \hat{C}^{\dagger} H \hat{C}$$

UCOM

# **Unitary Transformation**

transform eigenvalue problem

 $H_{\sim} | \hat{\Psi}_n \rangle = E_n | \hat{\Psi}_n \rangle$ 

with the unitary operator  $\underset{\sim}{C}$ 

$$|\hat{\Psi}_n\rangle = \mathcal{C}|\Psi_n\rangle, \quad \mathcal{C}^{-1} = \mathcal{C}^{\dagger}$$

into the equivalent eigenvalue problem

$$\hat{H} | \Psi_n \rangle = (\underline{C}^{\dagger} \underline{H} \underline{C}) | \Psi_n \rangle = E_n | \Psi_n \rangle$$

finally solve eigenvalue problem in a (small) model space  $\{|\Psi_n\rangle, n = 1, ..., N\}$ 

#### "pre-diagonalization"

correlator  $\underline{C}$  describes short-range correlations that are very similar (for the states in the model space)

correlator  $\underline{C}$  admixes component from outside the model space

it does not project on the model space

# The Unitary Correlation Operator

#### **Two-Body Correlations**

two-body generator

$$C = e^{-iG}, \qquad G = \sum_{i < j} g_{ij}$$

#### **Cluster Expansion**

correlated operators  $\hat{A} = C^{\dagger}AC$  are no longer operators with definite particle number

 decompose correlated operator into irreducible k-body operators

$$\hat{A} = \hat{A}^{[1]} + \hat{A}^{[2]} + \cdots$$

#### **Two-Body Approximation**

$$\hat{T}^{C2} = \hat{T}^{[1]} + \hat{T}^{[2]}, \qquad \hat{V}^{C2} = \hat{V}^{[2]}$$

Correlation range should be smaller than mean distance of nucleons

#### **Correlator** $C_{\sim}$

should conserve translational, rotational and Galilei invariance

cluster decomposition principle should be fulfilled

#### **Spin-Isospin Dependence**

nuclear interaction strongly depends on spin and isospin

$$\underbrace{v}_{\sim} = \sum_{S,T} \underbrace{v}_{ST} \prod_{\sim} I_{ST}$$

 different correlations in the respective channels

$$\underset{\sim}{g} = \sum_{S,T} \underset{\sim}{g}_{ST} \underset{\sim}{\Pi}_{ST}$$

correlated interaction in two-body space

$$\hat{v} = \sum_{S,T} \left( e^{ig_{ST}} v_{ST} e^{-ig_{ST}} \right) \prod_{\sim} ST$$

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# UCOMCentral Correlations

repulsion at short distances

 probability density of nucleons in the repulsive core strongly suppressed



#### **Radial Shift**

- correlator shifts nucleons out of core radial shift generated by radial momentum  $p_r$ 

$$g_r \stackrel{\mathbf{r}}{\Rightarrow} \frac{1}{2} \{ p_r s(r) + s(r) p_r \}, \quad p_r = \frac{1}{i} \left( \frac{1}{r} + \frac{\partial}{\partial r} \right)$$

#### **Correlation Function**

use correlation function  $R_{\pm}(r)$  instead of shift function s(r)

$$\pm 1 = \int_r^{R_{\pm}(r)} \frac{d\xi}{s(\xi)}, \quad R_{\pm}(r) \approx r \pm s(r)$$

**Correlated Wave function** 

$$\left\langle \mathbf{X}, \mathbf{r} \middle|_{\sim r}^{c} \middle| \Phi \right\rangle = \frac{R_{-}(r)}{r} \sqrt{R'_{-}(r)} \left\langle \mathbf{X}, R_{-}(r) \hat{\mathbf{r}} \middle| \Phi \right\rangle$$

## **Tensor Operators**

couple two operators

$$\left\{A^{(j_1)} B^{(j_2)}\right\}_q^{(j)} \equiv \sum_{m_1, m_2} C \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & q \end{pmatrix} A^{(j_1)}_{m_1} B^{(j_2)}_{m_2}$$

couple spins to a tensor of rank 1

$$S^{(1)} = \frac{1}{2} \left( \sigma^{(1)} \otimes 1 + 1 \otimes \sigma^{(1)} \right)$$

or to a tensor of rank 2

$$S^{(2)} = \left\{ S^{(1)} \; S^{(1)} \right\}^{(2)}$$

 $S_{12}(\mathbf{a}, \mathbf{b}) = 3(\sigma_1 \cdot \mathbf{a})(\sigma_2 \cdot \mathbf{b}) - (\sigma_1 \cdot \sigma_2)(\mathbf{a} \cdot \mathbf{b})$ =  $3 \{a^{(1)} b^{(1)}\}^{(2)} \cdot S^{(2)}$ =  $3 \sqrt{5} \{(ab)^{(2)} \otimes S^{(2)}\}^{(0)}$ 

$$S_{12}(\hat{\mathbf{r}}, \hat{\mathbf{r}}) = 3 Y^{(2)} \cdot S^{(2)}$$

but they couple different orbital angular momenta  $\langle (L1)JM | S_{12}(\mathbf{a}, \mathbf{b}) | (L'1)JM \rangle \neq 0$  for |L - L'| = 0, 2

tensor operators are scalar operators  $\langle JM | S_{12}(\mathbf{a}, \mathbf{b}) | J'M' \rangle \propto \delta_{J,J'} \delta_{M,M'}$ 

# **Tensor Correlations**

 tensor force admixes higher angular momenta



#### **Perpendicular Shift**

- correlator aligns density with spin perpendicular shift generated by  $s_{12}(\mathbf{r}, \mathbf{p}_{\Omega})$ 

$$\underset{\sim}{\underset{\sim}{g_{\Omega}}} \stackrel{\mathbf{r}}{\Rightarrow} \vartheta(r) s_{12}(\mathbf{r}, \mathbf{p}_{\Omega}), \qquad \mathbf{p} = \mathbf{p}_r + \mathbf{p}_{\Omega}$$

$$s_{12}(\mathbf{r}, \mathbf{p}_{\Omega}) = \frac{3}{2} (\boldsymbol{\sigma}_{1} \cdot \mathbf{p}_{\Omega}) (\boldsymbol{\sigma}_{2} \cdot \mathbf{r}) + \frac{3}{2} (\boldsymbol{\sigma}_{1} \cdot \mathbf{r}) (\boldsymbol{\sigma}_{2} \cdot \mathbf{p}_{\Omega})$$

$$\frac{s_{12}(\mathbf{r}, \mathbf{p}_{\Omega})}{\langle (J-1, 1)J |} \frac{|(J-1, 1)J \rangle}{0} \frac{|(J+1, 1)J \rangle}{\langle (J, 1)J |} \frac{|(J-1, 1)J \rangle}{0} \frac{|(J+1, 1)J \rangle}{\langle (J+1, 1)J |}$$

$$\frac{s_{12}(\mathbf{r}, \mathbf{p}_{\Omega})}{\langle (J-1, 1)J |} \frac{|(J-1, 1)J \rangle}{0} \frac{|(J-1, 1)J \rangle}{0} \frac{|(J-1, 1)J |}{\langle (J, 1)J |} \frac{|(J-1, 1)J \rangle}{0} \frac{|(J-1, 1)J \rangle}{0}$$

#### **Correlated Wave function**

 $\left\langle r \left| c_{\Omega} \right| \varphi; (J, 1)J \right\rangle = \varphi(r) \left| (J, 1)J \right\rangle$  $\left\langle r \left| c_{\Omega} \right| \varphi; (J \neq 1, 1)J \right\rangle = \cos(\theta^{(J)}(r)) \varphi(r) \left| (J \neq 1, 1)J \right\rangle$  $\pm \sin(\theta^{(J)}(r)) \varphi(r) \left| (J \pm 1, 1)J \right\rangle$ 

 $\theta^{(J)}(r) = 3 \sqrt{J(J+1)} \vartheta(r)$ 

# **Tensor Correlated Interaction**

#### **Tensor Correlated Operators**

Baker-Campbell-Hausdorff

$$c_{\Omega \cong \Omega \cong \Omega}^{\dagger} a c_{\Omega} = e^{i g_{\Omega}} a e^{-i g_{\Omega}} = e^{\mathsf{L}_{\Omega}} a, \quad \mathsf{L}_{\Omega} = \left[ g_{\Omega}, \circ \right]_{-}$$

$$s_{12}(\mathbf{l},\mathbf{l}) = 3(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) - (\sigma_1 \cdot \sigma_2)\mathbf{l}^2 .$$
  
$$\bar{s}_{12}(\mathbf{p}_{\Omega},\mathbf{p}_{\Omega}) = 2r^2 s_{12}(\mathbf{p}_{\Omega},\mathbf{p}_{\Omega}) + s_{12}(\mathbf{l},\mathbf{l}) - \frac{1}{2}s_{12}(\hat{\mathbf{r}},\hat{\mathbf{r}})$$

$$\begin{bmatrix} g_{\Omega}, p_r^2 \end{bmatrix}_{-} = i \left( p_r \vartheta'(r) + \vartheta'(r) p_r \right) s_{12}(\mathbf{r}, \mathbf{p}_{\Omega})$$
$$\begin{bmatrix} g_{\Omega}, \left[ g_{\Omega}, p_r^2 \right]_{-} \end{bmatrix}_{-} = -2\vartheta'(r)^2 \left[ (18 + 6\mathbf{l}^2)\Pi_1 + \frac{45}{2}\mathbf{l}\cdot\mathbf{s} + \frac{3}{2}s_{12}(\mathbf{l}, \mathbf{l}) \right]$$
$$\begin{bmatrix} g_{\Omega}, \left[ g_{\Omega}, p_r^2 \right]_{-} \end{bmatrix}_{-} \end{bmatrix}_{-} = 0$$

$$\begin{bmatrix} g_{\Omega}, \Pi_1 \end{bmatrix}_{-} = 0$$
  

$$\begin{bmatrix} g_{\Omega}, s_{12}(\hat{\mathbf{r}}, \hat{\mathbf{r}}) \end{bmatrix}_{-} = i\vartheta(r) \begin{bmatrix} -24 \Pi_1 - 18\mathbf{l}\cdot\mathbf{s} + 3s_{12}(\hat{\mathbf{r}}, \hat{\mathbf{r}}) \end{bmatrix}$$
  

$$\begin{bmatrix} g_{\Omega}, \mathbf{l}\cdot\mathbf{s} \end{bmatrix}_{-} = i\vartheta(r) \begin{bmatrix} -\bar{s}_{12}(\mathbf{p}_{\Omega}, \mathbf{p}_{\Omega}) \end{bmatrix}$$
  

$$\begin{bmatrix} g_{\Omega}, \mathbf{l}^2 \end{bmatrix}_{-} = i\vartheta(r) \begin{bmatrix} 2\bar{s}_{12}(\mathbf{p}_{\Omega}, \mathbf{p}_{\Omega}) \end{bmatrix}$$
  

$$\begin{bmatrix} g_{\Omega}, s_{12}(\mathbf{l}, \mathbf{l}) \end{bmatrix}_{-} = i\vartheta(r) \begin{bmatrix} 7\bar{s}_{12}(\mathbf{p}_{\Omega}, \mathbf{p}_{\Omega}) \end{bmatrix}$$
  

$$\begin{bmatrix} g_{\Omega}, \bar{s}_{12}(\mathbf{p}_{\Omega}, \mathbf{p}_{\Omega}) \end{bmatrix}_{-} = i\vartheta(r) \begin{bmatrix} (96 \mathbf{l}^2 + 108)\Pi_1 + (36 \mathbf{l}^2 + 153) \mathbf{l}\cdot\mathbf{s} + 15s_{12}(\mathbf{l}, \mathbf{l}) \end{bmatrix}$$

 evaluate e<sup>L<sub>Ω</sub></sup> in truncated operator space

 use in HF or FMD calculations

# **Determine Correlation Functions**

#### **Central Correlations**



**Tensor Correlations** 



 determine s(r) und θ(r) in each spin-isospin channel by minimizing the energy in the two-body system

 $\min_{s(r),\vartheta(r)} \left\langle \phi_{trial}^{ST} \left| \underset{\sim}{C}_{r}^{\dagger} \underset{\sim}{C}_{\Omega}^{\dagger} \underset{\sim}{HC} \underset{\sim}{C}_{C} \underset{\sim}{C}_{r} \right| \phi_{trial}^{ST} \right\rangle$ 

- correlation functions depend only weakly on the trial wave function
- restrict the range of the tensor correlations in the S = 1, T = 0 channel (parameter  $I_{\vartheta}$ )

#### UCOM Correlated Two-Body Densities and Energies





central correlator  $C_r$ shifts density out of the repulsive core tensor correlator  $C_{\Omega}$ aligns density with spin orientation

both central and tensor correlations are essential for binding





Nucl. Phys. A713 (2003) 311

# **Nucleon Momentum Distributions**



- correlations induce high-momentum components
- large contributions by tensor correlations
- dependence on the correlation range at the Fermi surface

# **Correlated Interaction in Momentum Space**



correlated interaction is **more attractive** at low momenta



 ${}^{3}S_{1}$  correlated



off-diagonal matrix elements connecting low- and highmomentum states are strongly reduced  ${}^{3}S_{1}$  -  ${}^{3}D_{1}$  correlated



Phys. Rev. C72 (2005) 034002

# **Correlated Interaction in Momentum Space**



Bogner, Kuo, Schwenk, Phys. Rept. 386 (2003) 1

#### исом No-Core Shell Model Calculations



- use Jacobi-coordinate NCSM code by Petr Navrátil, LLNL for <sup>3</sup>He and <sup>4</sup>He (don't use Lee-Suzuki transformation)
- dramatically improved convergence compared to bare interaction
- does not converge to exact result for bare interaction due to omitted higher order terms V<sup>[3]</sup><sub>UCOM</sub>, ...
- study the effect of higher order contributions as a function of tensor correlation range *I*<sub>δ</sub>.

### исом Tjon Line with NCSM





- choose tensor correlation range  $I_{\vartheta} = 0.09$  such that **need for three-body forces is minimized**
- different perspective: don't try to reproduce the results with the bare interaction but consider V<sub>UCOM</sub> as a realistic potential to describe experiment

# HF and MBPT calculations



additional attraction mainly by medium to long range tensor forces long-range correlations appear to be perturbative spherical Hartree-Fock in 12  $\hbar\omega$  harmonic oscillator basis

• UCOM

# Hartree-Fock and Many-Body Perturbation Theory



 $E_{HF} + E^{(2)}$   $E_{HF} + E^{(2)} + E^{(3)}$ spherical Hartree-

 $E_{\rm HF}$ 

Fock in 12  $\hbar\omega$ harmonic oscillator basis

additional binding mainly due to medium to long range tensor forces long-range correlations appear to be perturbative

problems with saturation indicate need for three-body forces





- NCSM calculations with "bare"  $V_{\rm UCOM}$  and Lee-Suzuki effective interaction derived from  $V_{\rm UCOM}$  show consistent convergence pattern
- Binding energy close to experiment
- Spectra with  $V_{\text{UCOM}}$  are of similar quality than with other modern NN forces

### исом NCSM <sup>10</sup>В



calculations by Petr Navrátil, LLNL

- correct level ordering without three-body forces
- binding energy not too far from experiment